1.4 VELOCITY DISTRIBUTION

The presence of corners and boundaries in an open channel causes the velocity vectors of the flow to have components not only in the longitudinal and lateral direction but also in normal direction to the flow. In a macro-analysis, one is concerned only with the major component, viz., the longitudinal component, v_x. The other two components being small are ignored and v_x is designated as v. The distribution of v in a channel is dependent on the geometry of the channel. Figure 1.2(a) and (b) show isovels (contours of equal velocity) of v for a natural and rectangular channel respectively. The influence of the channel geometry is apparent. The velocity v is zero at the solid boundaries and gradually increases with distance from the boundary. The maximum velocity of the cross-section occurs at a certain distance below the free surface. This dip of the maximum velocity point, giving surface velocities which are less than the maximum velocity, is due to secondary currents and is a function of the aspect ratio (ratio of depth to width) of the channel. Thus for a deep narrow channel, the location of the maximum velocity point will be much lower from the water surface than for a wider channel of the same depth. This characteristic location

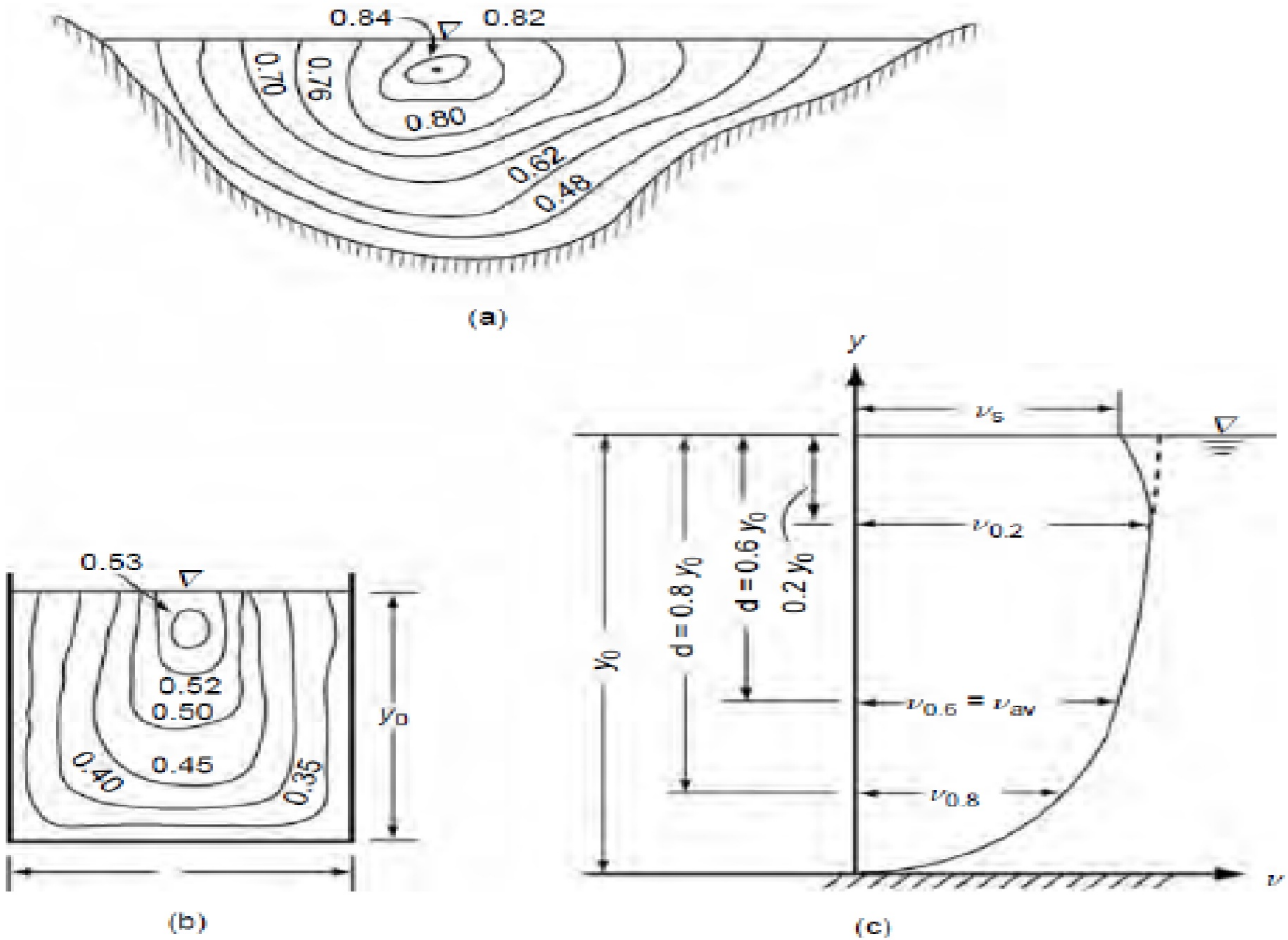

Fig. 1.2 *Velocity distribution in open channels: (a) Natural channel (b) Rectangular channel and (c) Typical velocity profile*

1.11 MOMENTUM EQUATION

Steady Flow Momentum is a vector quantity. The momentum equation commonly used in most of the open channel flow problems is the *linear-momentum equation*. This equation states that the algebraic sum of all external forces, acting in a given direction on a fluid mass equals the time rate of change of linear-momentum of the fluid mass in the direction. In a steady flow the rate of change of momentum in a given direction will be equal to the net flux of momentum in that direction.

Figure 1.17 shows a *control volume* (a volume fixed in space) bounded by Sections 1 and 2, the boundary and a surface lying above the free surface. The various forces acting on the control volume in the longitudinal direction are as follows:

(i) Pressure forces acting on the control surfaces, F_1 and F_2.
(ii) Tangential force on the bed, F_3,
(iii) Body force, i.e., the component of the weight of the fluid in the longitudinal direction, F_4.

By the linear-momentum equation in the longitudinal direction for a steady-flow discharge of Q,

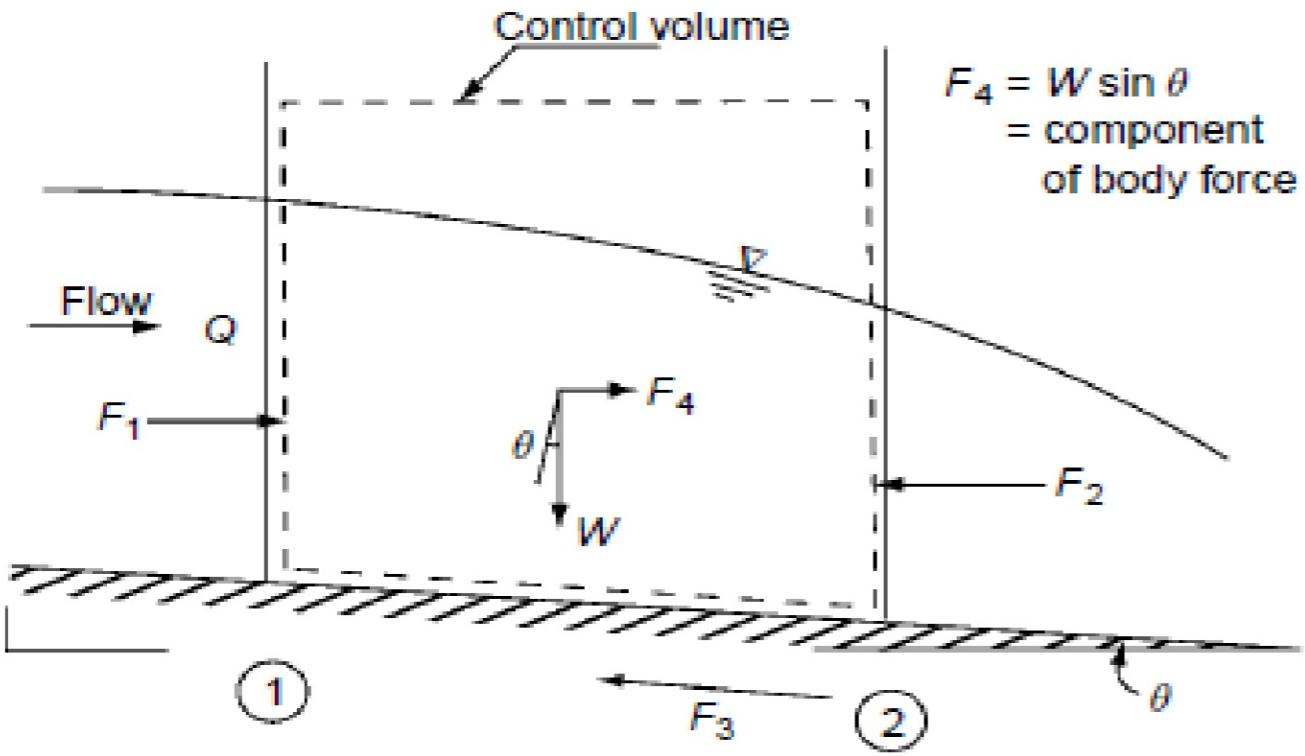

Fig.1.17 *Definition sketch for the momentum equation*

$$\Sigma F_1 = F_1 - F_2 - F_3 + F_4 = M_2 - M_1 \tag{1.45}$$

in which $M_1 = \beta_1 \rho Q V_1$ = momentum flux entering the control volume, $M_2 = \beta_2 \rho Q V_2$ = momentum flux leaving the control volume.

In practical applications of the momentum equation, the proper identification of the geometry of the control volume and the various forces acting on it are very important. The momentum equation is a particularly useful tool in analysing rapidly varied flow (RVF) situations where energy losses are complex and cannot be easily estimated. It is also very helpful in estimating forces on a fluid mass. Detailed information on the basis of the momentum equation and selection of the control volume are available in books dealing with the mechanics of fluids.[5,6]

Specific Force The steady-state momentum equation (Eq. 1.45) takes a simple form if the tangential force F_3 and body force F_4 are both zero. In that case

$$F_1 - F_2 = M_2 - M_1$$

or

$$F_1 + M_1 = F_2 + M_2$$

Denoting $\frac{1}{\gamma}(F + M) = P_s$

$$(P_s)_1 = (P_s)_2 \tag{1.50}$$

The term P_s is known as the *specific force* and represents the sum of the pressure force and momentum flux per unit weight of the fluid at a section. Equation (1.50) states that the specific force is constant in a horizontal, frictionless channel. This fact can be advantageously used to solve some flow situations. An application of the specific force relationship to obtain an expression for the depth at the end of a hydraulic jump is given in Section 6.4. In a majority of applications the force F is taken as due to hydrostatic pressure distribution and hence is given by, $F = \gamma A\bar{y}$ where $\bar{y}$ is the depth of the centre of gravity of the flow area.

IN Unsteady Flow

The linear-momentum equation will have an additional term over and above that of the steady flow equation to include the rate of change of momentum in the control volume.

The momentum equation would then state that in an unsteady flow

The algebraic sum of all external forces in a given direction on a fluid mass equals the net change of the linear-momentum flux of the fluid mass in that direction plus the time rate of increase of momentum in that direction within the control volume.

An application of the momentum equation in unsteady flows is Discussed Later..

2.1 SPECIFIC ENERGY

The *total energy* of a channel flow referred to a datum is given by Eq.1.39 as

$$H = Z + y\cos\theta + \alpha\frac{V^2}{2g}$$

If the datum coincides with the channel bed at the section, the resulting expression is know as *specific energy* and is denoted by E. Thus

$$E = y\cos\theta + \alpha\frac{V^2}{2g} \tag{2.1}$$

When $\cos\theta = 1.0$ and $\alpha = 1.0$,

$$E = y + \frac{V^2}{2g} \tag{2.2}$$

The concept of specific energy, introduced by Bakhmeteff, is very useful in

The concept of specific energy, introduced by Bakhmeteff, is very useful in defining critical depth and in the analysis of flow problems. It may be noted that while the total energy in a real fluid flow always decreases in the downstream direction, the specific energy is constant for a uniform flow and can either decrease or increase in a varied flow, since the elevation of the bed of the channel relative to the elevation of the total energy line, determines the specific energy. If the frictional resistance of the flow can be neglected, the total energy in non-uniform flow will be constant at all sections while the specific energy for such flows, however, will be constant only for a horizontal bed channel and in all other cases the specific energy will vary.

To simplify the expressions it will be assumed, for use in all further analysis, that the specific energy is given by Eq. 2.2, i.e., $\cos\theta = 1.0$ and $\alpha = 1.0$. This is with the knowledge that $\cos\theta$ and α can be appended to y and $(V^2/2g)$ terms respectively, without difficulty if warranted.

1.10 ENERGY EQUATION

In the one-dimensional analysis of steady open-channel flow, the energy equation in the form of the Bernoulli equation is used. According to this equation, the total energy at a downstream section differs from the total energy at the upstream section by an amount equal to the loss of energy between the sections.

Figure 1.14 shows a steady varied flow in a channel. If the effect of the curvature on the pressure distribution is neglected, the total energy head (in N.m/newton of fluid) at any point A at a depth d below the water surface is

$$H = Z_A + d\cos\theta + \alpha\frac{V^2}{2g} \tag{1.41}$$

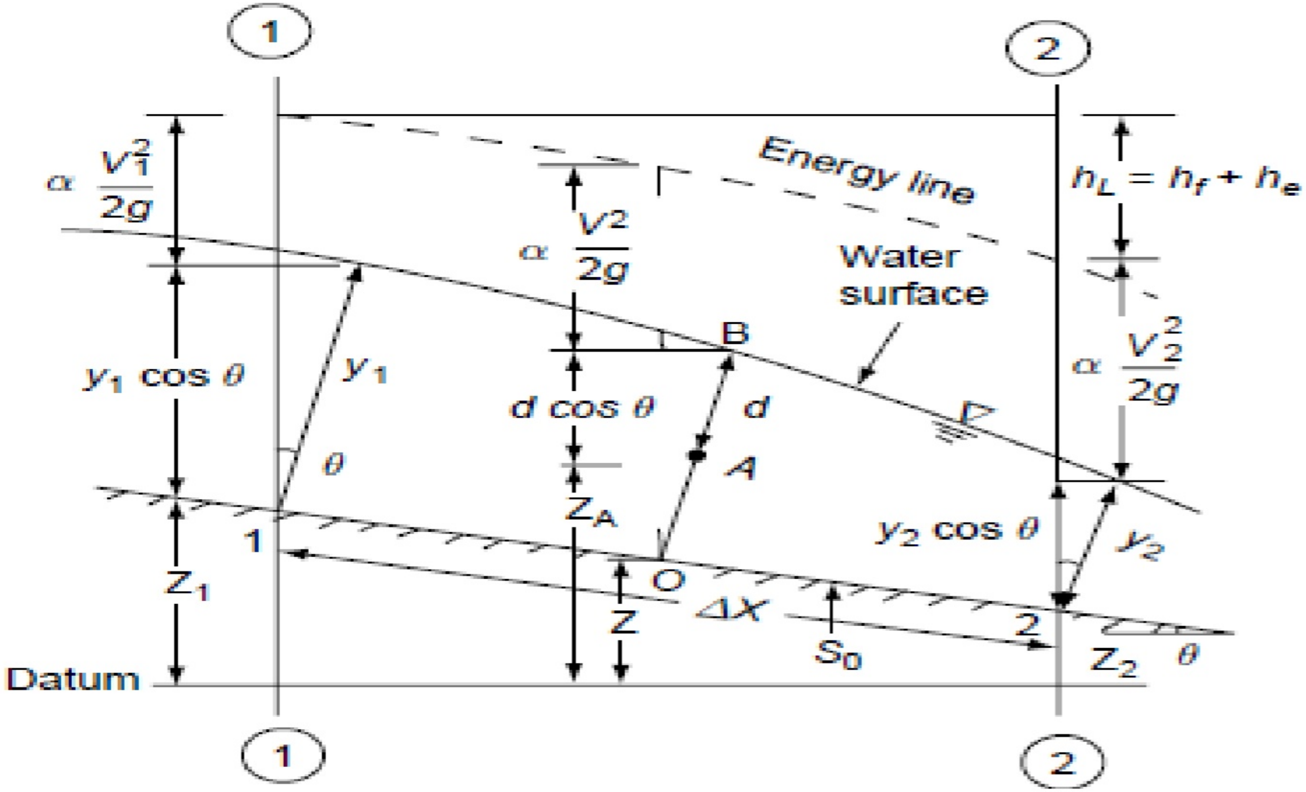

Fig.1.14 *Definition sketch for the energy equation*

Fig.1.14 *Definition sketch for the energy equation*

This total energy will be constant for all values of d from zero to y at a normal section through point A (i.e. Section OAB), where y = depth of flow measured normal to the bed. Thus the total energy at any section whose bed is at an elevation Z above the datum is

$$H = Z + y\cos\theta + \alpha V^2/2g \tag{1.42}$$

2.2 CRITICAL DEPTH

Constant Discharge Situation Since the specific energy

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} \tag{2.2a}$$

for a channel of know geometry, $E = f(y, Q)$ keeping Q = constant = Q_1 the variation of E with y is represented by a cubic parabola Fig. 2.1. It is seen that there are two positive roots for the equation of E indicating that any seen that there are two positive roots for the equation of E indicating that any particular discharge Q_1 can be passed in a given channel at two depths and still maintain the same specific energy E. In Fig. 2.1 the ordinate PP' represents the condition for a specific energy of E_1. The depths of flow can be either $PR = y_1$ or $PR' = y'_1$. These two possible depths having the same specific energy are know as *alternate depths*. In Fig. 2.1, a line (OS) drawn such that $E = y$ (i.e. at 45° to the abscissa) is the asymptote of the upper limb of the specific energy curve. It may be noticed that the intercept $P'R'$ or $P'R$ represents the velocity head. Of the two alternate depths, one ($PR = y_1$) is smaller and has a large velocity head while the other ($PR' = y'_1$) has a larger depth and consequently a smaller velocity head. For a given Q_1 as the specific energy is increased the difference between the two alternate depths increases. On the other hand, if E is decreased, the difference ($y'_1 - y_1$) will decrease and at a certain value $E = E_c$, the two depths will merge with each other (point C in Fig. 2.1). No value for y can be obtained when $E < E_c$, denoting that the flow under the given conditions is not possible in this region. The condition of minimum specific energy is known as the *critical-flow condition* and the corresponding depth y_c is known as the *critical depth*.

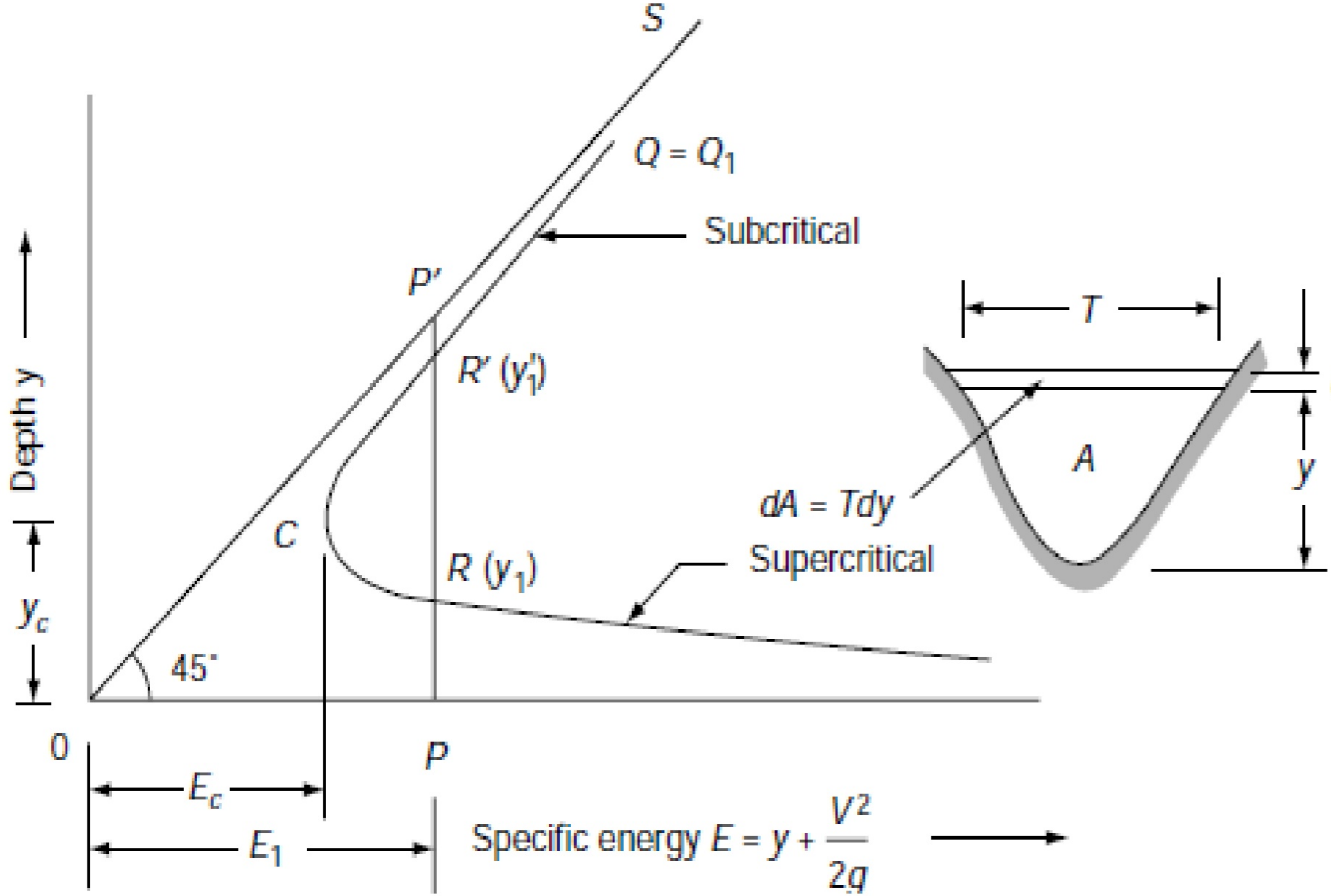

Fig. 2.1 *Definition sketch of specific energy*

At critical depth, the specific energy is minimum. Thus differentiating Eq. 2.2a with respect to y (keeping Q constant) and equating to zero,

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy} = 0 \tag{2.3}$$

But $\dfrac{dA}{dy} = T =$ top width, i.e. width of the channel at the water surface.

Designating the critical-flow conditions by the suffix 'c',

$$\frac{Q^2 T_c}{gA_c^3} = 1 \tag{2.4}$$

or

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} \tag{2.4a}$$

If an α value other than unity is to be used, Eq. 2.4 will become

$$\frac{\alpha Q^2 T_c}{gA_c^3} = 1.0 \tag{2.5}$$

Equation 2.4 or 2.5 is the basic equation governing the critical-flow conditions in a channel. It may be noted that the critical-flow condition is governed solely by the channel geometry and discharge (and α). Other channel properties such as the bed slope and roughness do not influence the critical-flow condition for any given Q. If the Froude number of the flow is define as

$$F = V/\left(\sqrt{gA/T}\right) \tag{2.6}$$

it is easy to see that by using F in Eq. 2.4, at the critical flow $y = y_c$ and $F = F_c = 1.0$. We thus get an important result that the critical flow corresponds to the minimum specific energy and at this condition the Froude number of the flow is unity. For a channel with large longitudinal slope θ and having a flow with an energy correction

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$$F = V/\left(\sqrt{\frac{1}{\alpha} g \frac{A}{T} \cos\theta}\right) \tag{2.6a}$$

Referring to Fig. 2.1, considering any specific energy other than E_c, (say ordinate PP' at $E = E_1$) the Froude number of the flow corresponding to both the alternate depths will be different from unity as y_1 or $y'_1 \neq y_c$. At the lower limb, CR of the specific-energy curve, the depth $y_1 < y_c$. As such, $V'_1 > V_c$ and $F_1 > 1.0$. This region is called the *supercritical flow* region. In the upper limb CR', $y'_1 > y_c$. As such $V'_1 < V_c$ and $F'_1 < 1.0$. This denotes the *subcritical flow* region.

Discharge as a Variable In the above section the critical-flow condition was derived by keeping the discharge constant. The specific-energy diagram can be plotted

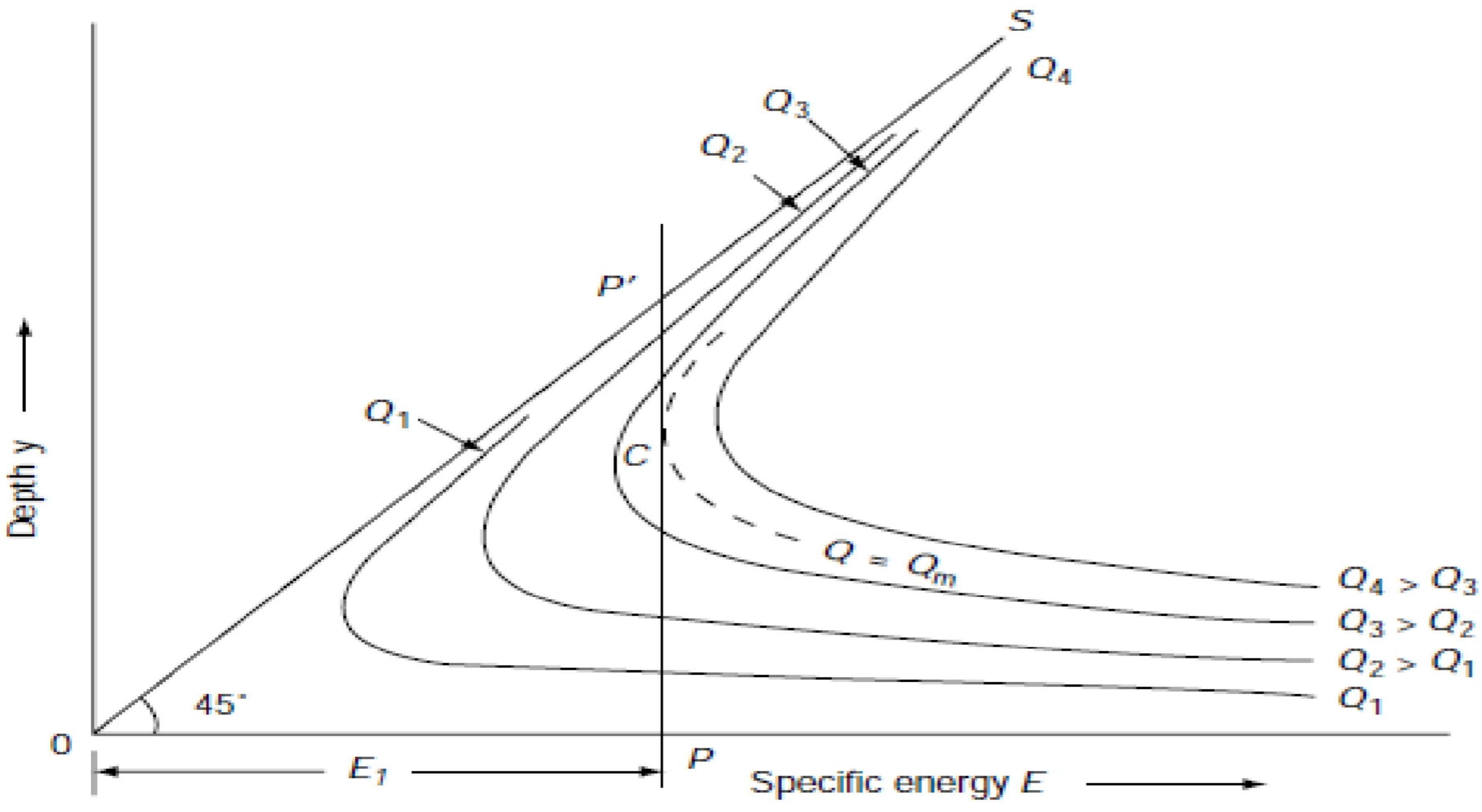

Fig. 2.2 *Specific energy for varying discharges*

for different discharges $Q = Q_1$ = constant ($i = 1, 2, 3 \ldots$), as in Fig. 2.2. In this figure, $Q_1 < Q_2 < Q_3 < \ldots$ and is constant along the respective E vs y plots. Consider a section PP' in this plot. It is seen that for the ordinate PP', $E = E_1$ = constant. Different Q curves give different intercepts. The difference between the alternate depths decreases as the Q value increases. It is possible to imagine a value of $Q = Q_m$ at a point C at which the corresponding specific-energy curve would be just tangential to the ordinate

curves give different intercepts. The difference between the alternate depths decreases as the Q value increases. It is possible to imagine a value of $Q = Q_m$ at a point C at which the corresponding specific-energy curve would be just tangential to the ordinate PP'. The dotted line in Fig. 2.2 indicating $Q = Q_m$ represents the maximum value of discharge that can be passed in the channel while maintaining the specific energy at a constant value (E_1). Any specific energy curve of higher Q value (i.e $Q > Q_m$) will have no intercept with the ordinate PP' and hence there will be no depth at which such a discharge can be passed in the channel with the given specific energy.
Since by Eq. 2.2a

$$E = y + \frac{Q^2}{2gA^2}$$

$$Q = A\sqrt{2g(E - y)} \tag{2.7}$$

The condition for maximum discharge can be obtained by differentiating Eq. 2.7 with respect to y and equating it to zero while keeping E = constant.

Thus $$\frac{dQ}{dy} = \sqrt{2g(E-y)}\,\frac{dA}{dy} - \frac{gA}{\sqrt{2g(E-y)}} = 0$$

By putting $$\frac{dA}{dy} = T \text{ and } \frac{Q}{A} = \sqrt{2g(E-y)} \text{ yields}$$

$$\frac{Q^2 T}{gA^3} = 1.0 \tag{2.8}$$

This is same as Eq. 2.4 and hence represents the critical-flow conditions. Hence, the critical-flow condition also corresponds to the condition for maximum discharge in a channel for a fixed specific energy.

Example 2.1 *A 2.5-m wide rectangular channel has a specific energy of 1.50 m when carrying a discharge of 6.48 m³/s. Calculate the alternate depths and corresponding Froude numbers.*

Solution From Eq. 2.2a

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gB^2 y^2}$$

$$1.5 = y + \frac{(6.48)^2}{2 \times 9.81 \times (2.5)^2 y^2}$$

$$= y + \frac{0.34243}{y^2}$$

Solving this equation by trial and error, the alternate depths y_1 and y_2 are obtained as $y_1 = 1.296$ m and $y_2 = 0.625$ m.

Froude number $$F = \frac{V}{\sqrt{gy}} = \frac{6.48}{(2.5y)\sqrt{9.81y}} = \frac{0.82756}{y^{3/2}},$$

At $y_1 = 1.296$ m, $F_1 = 0.561$; and
at $y_2 = 0.625$ m, $F_2 = 1.675$

The depth $y_1 = 1.296$ m is in the subcritical flow region and the depth $y = 0.625$ m

2.3 CALCULATION OF THE CRITICAL DEPTH

Using Eq. 2.4, expressions for the critical depth in channels of various geometric shapes can be obtained as follows:

Rectangular Section For a rectangular section, $A = By$ and $T = B$ (Fig. 2.3). Hence by Eq. 2.4

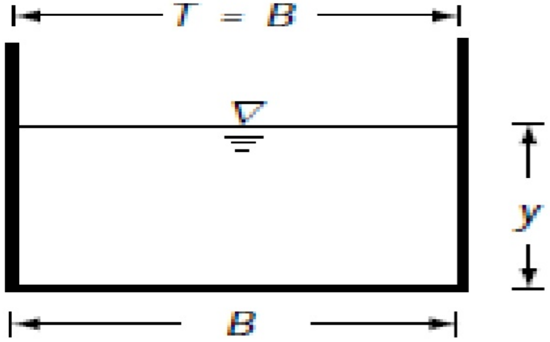

Fig. 2.3 *Rectangular channel*

$$\frac{Q^2 T_c}{g A_c^3} = \frac{V_c^2}{g y_c} = 1$$

or
$$\frac{V_c^2}{2g} = \frac{1}{2} y_c \tag{2.9}$$

Specific energy at critical depth $E_c = y_c + \dfrac{V_c^2}{2g} = \dfrac{3}{2} y_c$ (2.10)

Note that Eq. 2.10 is independent of the width of the channel.
Also, if q = discharge per unit width = Q/B,

$$\frac{q^2}{g} = y_c^3$$

i.e.
$$y_c = \left(\frac{q^2}{g}\right)^{1/3} \tag{2.11}$$

Since $A/T = y$, from Eq. 2.6, the Froude number for a rectangular channel will be defined as

$$F = \frac{V}{\sqrt{gy}} \tag{2.12}$$

Triangular Channel For a triangular channel having a side slope of m horizontal: 1 vertical (Fig. 2.4), $A = my^2$ and $T = 2my$.
By Eq. 2.4a,

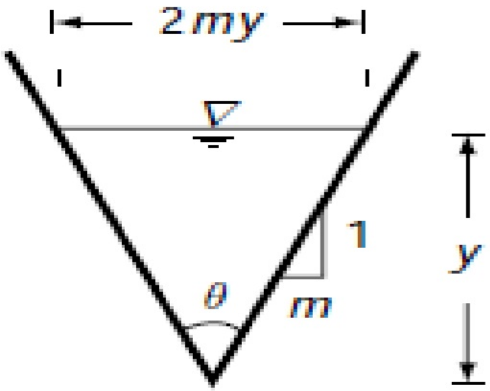

Fig. 2.4 *Triangular channel*

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{m^3 y_c^6}{2my_c} = \frac{m^2 y_c^5}{2} \tag{2.13}$$

$$\text{Hence} \quad y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5} \tag{2.14}$$

The specific energy at critical depth $E_c = y_c + \dfrac{V_c^2}{2g}$

$$= y_c + \frac{Q^2}{2gA_c^2} = y_c + \frac{m^2 y_c^5}{4m^2 y_c^4}$$

i.e.

$$E_c = 1.25 y_c \tag{2.15}$$

It is noted that Eq. 2.15 is independent of the side slope m of the channel. Since $A/T = y/2$, the Froude number for a triangular channel is defined by using Eq. 2.6 as

$$F = \frac{V\sqrt{2}}{\sqrt{gy}} \tag{2.16}$$

Circular Channel Let D be the diameter of a circular channel (Fig. 2.5) and 2θ be the angle in radians subtended by the water surface at the centre.

A = area of the flow section

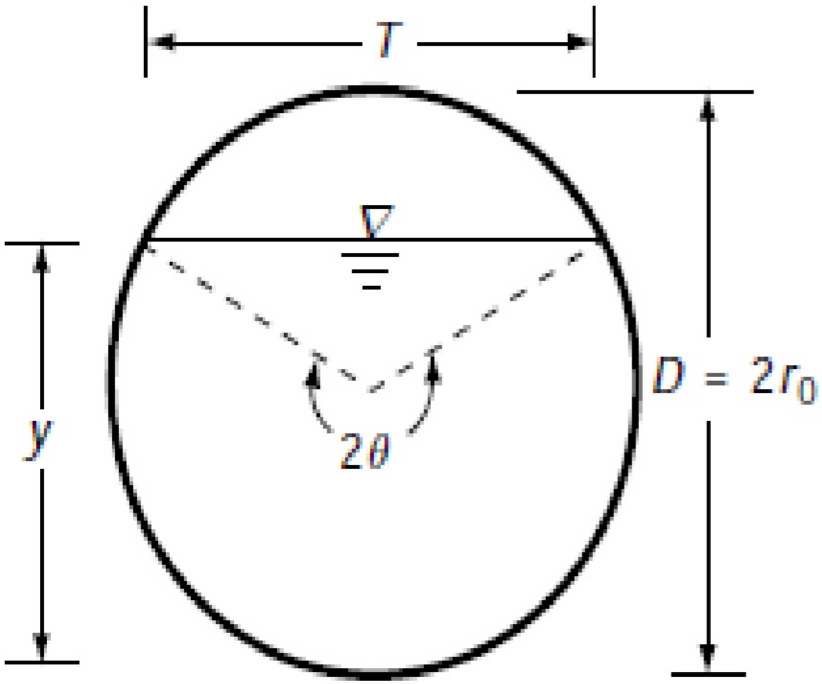

Fig. 2.5 *Circular channel*

= area of the sector + area of the triangular portion

$$= \frac{1}{2} r_0^2\, 2\theta + \frac{1}{2} \cdot 2r_0 \sin(\pi - \theta)\, r_0 \cos(\pi - \theta)$$

$$= \frac{1}{2}(r_0^2\, 2\theta - r_0^2 \sin 2\theta)$$

$$A = \frac{D^2}{8}(2\theta - \sin 2\theta)$$

Top width $\quad T = D \sin\theta$

and $\quad 2\theta = 2\cos^{-1}\left(1 - \frac{2y}{D}\right) = f(y/D)$

Substituting these in Eq. 2.4a yields

$$\frac{Q^2}{g} = \frac{\left[\frac{D^2}{8}(2\theta_c - \sin 2\theta_c)\right]^3}{D \sin\theta_c} \tag{2.17}$$

Since explicit solutions for y_c cannot be obtained from Eq. 2.17, a non-dimensional representation of Eq. 2.17 is obtained as

$$\frac{Q^2}{\sqrt{gD^5}} = \frac{0.044194\ (2\theta_c - \sin 2\theta_c)^{3/2}}{(\sin\theta_c)^{1/2}} = f(y_c/D) \tag{2.18}$$

This function is evaluated and is given in Table 2A.1 of Appendix 2A at the end of this chapter as an aid for the estimation of y_c.

Since $A/T = fn\left(\frac{y}{D}\right)$, the Froude number for a given Q at any depth y will be

$$F = \frac{V}{\sqrt{g(A/T)}} = \frac{Q}{\sqrt{g(A^3/T)}} = fn(y/D)$$

The following are two empirical equations that have been proposed for quick and accuarate estimation of critical depth in circular channels:

Empirical relationships for critical depth in circular channels

Sl.No	*Equation*	*Details*
1	$\frac{y_c}{D} = \left[0.77 F_D^{-6} + 1.0\right]^{-0.085}$ **where** $F_D = \frac{Q}{D^2\sqrt{gD}} = \frac{Z}{D^{2.5}}$	*Swamee P K (1993)(Ref. 4).*
2	$y_c = \frac{1.01}{D^{0.265}}\left(\frac{Q}{\sqrt{g}}\right)^{0.506}$ *for* $0.02 < \frac{y_c}{D} \le 0.85$	*Straub W O (1978) (Ref. 5).*

Trapezoidal Channel For a trapezoidal channel having a bottom width of B and side slopes of m horizontal: 1 vertical (Fig. 2.6)

Area $\qquad A = (B + my)y$

and Top width $\qquad T = (B + 2my)$

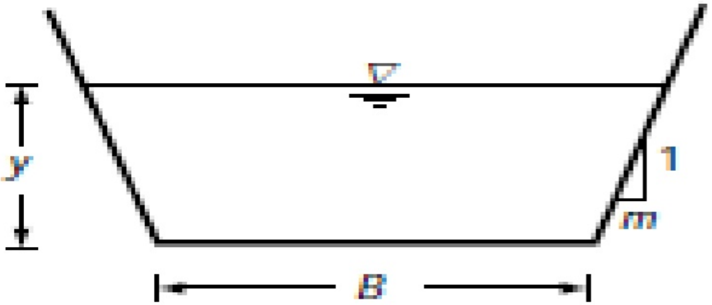

Fig. 2.6 *Trapezoidal channel*

At the critical flow

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{(B + my_c)^3\, y_c^3}{(B + 2my_c)} \tag{2.19}$$

Here also an explicit expression for the critical depth y_c is not possible. The non-dimensional representation of Eq. 2.19 facilitates the solution of y_c by the aid of tables or graphs. Rewriting the right-hand side of Eq. 2.19 as

$$\frac{(B + my_c)^3\, y_c^3}{B + 2my_c} = \frac{B^3\left(1 + \dfrac{my_c}{B}\right)^3 y_c^3}{B\left(1 + \dfrac{2my_c}{B}\right)}$$

$$= \frac{B^6}{m^3}\frac{(1 + \zeta_c)^3 \zeta_c^3}{(1 + 2\zeta_c)} \quad \text{where } \zeta_c = \frac{m\, y_c}{B}$$

gives

$$\frac{Q^2 m^3}{gB^5} = \frac{(1 + \zeta_c)^3 \zeta_c^3}{(1 + 2\zeta_c)} \tag{2.20}$$

or
$$\frac{Qm^{3/2}}{\sqrt{g}B^{5/2}}=\psi=\frac{(1+\zeta_c)^{3/2}\zeta_c^{\ 3/2}}{(1+2\zeta_c)^{1/2}} \tag{2.20a}$$

Equation 2.20a can easily be evaluated for various value of ζ_c and plotted as ψ vs ζ_c. It may be noted that if $\alpha > 1$, ψ can be defined as

$$\psi=\left(\frac{\alpha Q^2 m^3}{gB^5}\right)^{1/2} \tag{2.21}$$

Table 2A - 2 which gives values of ψ for different values of ζ_c is provided at the end of this chapter. This table is very useful in quick solution of problems related to critical depth in trapezoidal channels.

Since $A/T=\frac{(B+my)\,y}{(B+2my)}=\frac{\left(1+\frac{my}{B}\right)y}{\left(1+2\frac{my}{B}\right)}$ the Froude number at any depth y is

$$F=\frac{V}{\sqrt{gA/T}}=\frac{Q/A}{\sqrt{gA/T}}=fn\,(my/B)\text{ for a given discharge }Q$$

Further the specific energy at critical depth, E_c is a function of (my_c/B) and it can be shown that (Problem 2.7)

$$\frac{E_c}{y_c}=\frac{1}{2}\frac{(3+5\zeta_c)}{(1+2\zeta_c)}$$

where
$$\zeta_c=\frac{my_c}{B}$$

2.4 SECTION FACTOR Z

The expression $A\sqrt{A/T}$ is a function of the depth y for a given channel geometry and is known as the *section factor Z*.

Thus
$$Z = A\sqrt{A/T} \tag{2.22}$$

At the critical-flow condition, $y = y_c$ and

$$Z_c = A_c\sqrt{A_c/T_c} = Q/\sqrt{g} \tag{2.23}$$

which is a convenient parameter for analysing the role of the critical depth in a flow problem.

As a corollary of Eq. 2.23, if Z is the section factor for any depth of flow y, then

$$Q_c = \sqrt{g}\,Z \tag{2.24}$$

where Q_c represents the discharge that would make the depth y critical and is known as the *critical discharge*.

Note that the left-hand side of Eq. 2.18 is a non-dimensional form of the section factor (as $Z/D^{2.5}$) for circular channels.

2.6 COMPUTATIONS

The problems concerning critical depth involve the following parameters: geometry of the channel, Q or E *or* y_c. For rectangular and triangular channel sections, most of the problems involve explicit relationships for the variable and a few problems involve trial and error solutions. However, for trapezoidal, circular and most other regular geometrical shapes of channel sections, many of the problems have to be solved by trial and error procedure. Tables 2A.1 and 2A.2 are helpful

in problems connected with circular and trapezoidal channels respectively. Examples 2.5 to 2.8 illustrate some of the typical problems and the approach to their solutions. The graphical solutions and monographs which were in use some decades back are obsolete now. With the general availability of computers, a large number of elegant numerical methods are available to solve non-linear algebraic equations and the solutions of critical depth and related critical flow problems in channels of all shapes, including natural channels, is no longer difficult.

Example 2.5 *Calculate the critical depth and the corresponding specific energy for a discharge of 5.0 m³/s in the following channels:*

(a) Rectangular channel, B = 2.0 m
(b) Triangular channel, m = 0.5
(c) Trapezoidal channel, B = 2.0 m, m = 1.5
(d) Circular channel, D = 2.0 m

Solution (a) *Rectangular Channel*

$$q = Q/B = \frac{5.0}{2.0} = 2.5 \text{ m}^3/\text{s/m}$$

$$y_c = \left(q^2/g\right)^{1/3} = \left[\frac{(2.5)^2}{9.81}\right]^{1/3} = 0.860 \text{ m}$$

Since for a rectangular channel $\frac{E_c}{y_c} = 1.5$, $E_c = 1.290$ m

(b) *Triangular Channel*

From Eq. 2.14 $$y_c = \left(\frac{2Q^2}{gm^2}\right)^{1/5}$$

$$= \left[\frac{2\times(5)^2}{9.81\times(0.5)^2}\right]^{1/5} = 1.828 \text{ m}$$

Since for a triangular channel $\frac{E_c}{y_c} = 1.25$, $E_c = 2.284$ m

(c) *Trapezoidal Channel*

$$\psi = \frac{Qm^{3/2}}{\sqrt{g}B^{5/2}} = \frac{5.0\times(1.5)^{3/2}}{\sqrt{9.81}\times(2.0)^{5/2}} = 0.51843$$

Using Table 2A.2 the corresponding value of

$$\zeta_c = \frac{my_c}{B} = 0.536$$

$$y_c = 0.715\,\text{m}$$

$$A_c = (2.0+1.5\times0.715)\times0.715 = 2.197\,\text{m}^2$$

$$V_c = 5.0/2.197 = 2.276\,\text{m/s}$$

$$V_c^2/2g = 0.265\,\text{m}$$

$$E_c = y_c + \frac{V_c^2}{2g} = 0.715+0.264 = 0.979\,\text{m}$$

3.2 CHEZY EQUATION

By definition there is no acceleration in uniform flow. By applying the momentum equation to a control volume encompassing Sections 1 and 2, distance L apart, as shown in Fig. 3.1,

$$P_1 - W \sin\theta - F_f - P_2 = M_2 - M_1 \tag{3.1}$$

where P_1 and P_2 are the pressure forces and M_1 and M_2 are the momentum fluxes at Sections 1 and 2 respectively W = weight to fluid in the control volume and F_f = shear force at the boundary.

Since the flow is uniform,

$$P_1 = P_2 \quad \text{and} \quad M_1 = M_2$$

Also,

$$W = \gamma AL \quad \text{and} \quad F_f = \tau_0 PL$$

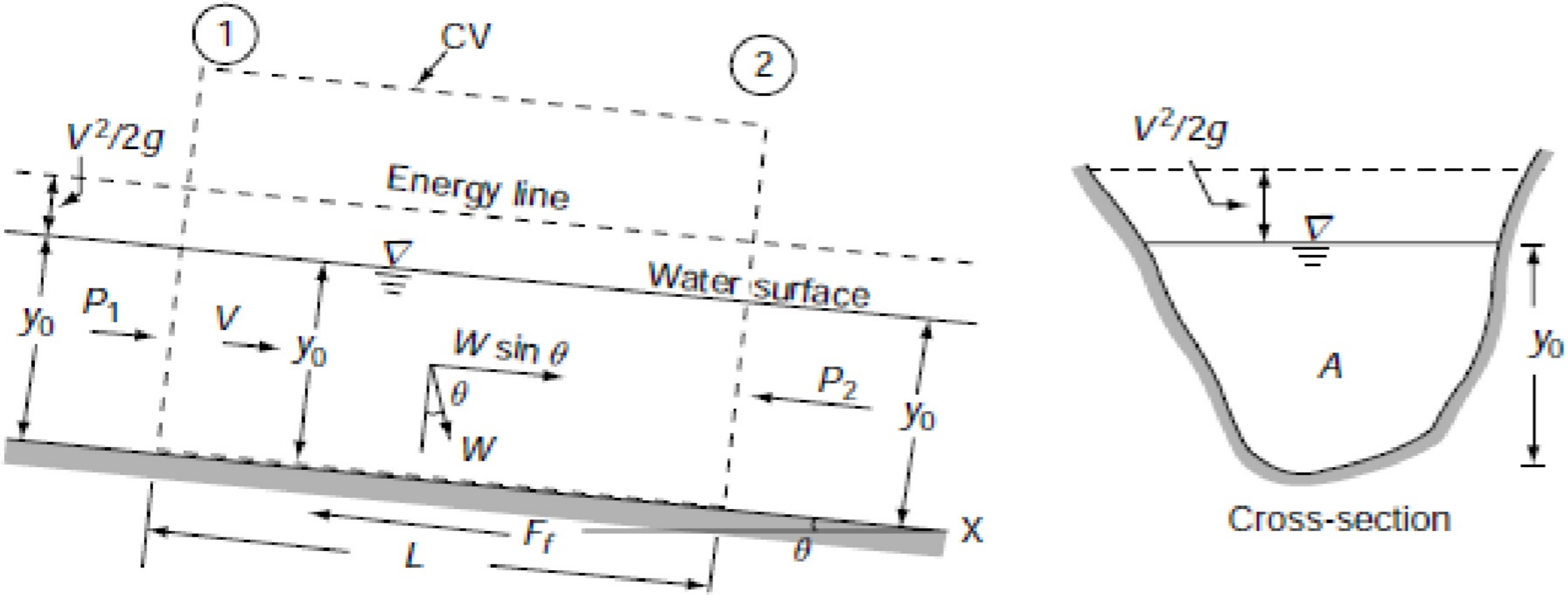

Fig. 3.1 *Uniform flow*

where τ_0 = average shear stress on the wetted perimeter of length P and γ = unit weight of water. Replacing $\sin\theta$ by S_0 (= bottom slope), Eq. 3.1 can be written as

$$\gamma ALS_0 = \tau_0 PL$$

or

$$\tau_0 = \gamma \frac{A}{P} S_0 = \gamma RS_0 \tag{3.2}$$

where $R = A/P$ is defined as the *hydraulic radius*. R is a length parameter accounting [redacted]

$$\tau_0 = \gamma \frac{A}{P} S_0 = \gamma R S_0 \tag{3.2}$$

where $R = A/P$ is defined as the *hydraulic radius*. R is a length parameter accounting for the shape of the channel. It plays a very important role in developing flow equations which are common to all shapes of channels.

Expressing the average shear stress τ_0 as $\tau_0 = k\rho V^2$, where k = a coefficient which depends on the nature of the surface and flow parameters, Eq. 3.2 is written as

$$k\rho V^2 = \gamma R S_0$$

leading to

$$V = C\sqrt{RS_0} \tag{3.3}$$

where $C = \sqrt{\frac{\gamma}{\rho}\frac{1}{k}}$ = a coefficient which depends on the nature of the surface and the flow. Equation 3.3 is known as the *Chezy formula* after the French engineer Antoine Chezy, who is credited with developing this basic simple relationship in 1769. The dimensions of C are $[L^{1/2}\, T^{-1}]$ and it can be made dimensionless by dividing it by $\sqrt{g}$. The coefficient C is known as the Chezy coefficient.

3.5 OTHER RESISTANCE FORMULAE

Several forms of expressions for the Chezy coefficient C have been proposed by different investigators in the past. Many of these are archaic and are of historic interest only. A few selected ones are listed below:

1. ***Pavlovski formula***

$$C = \frac{1}{n} R^x \tag{3.14}$$

in which $x = 2.5 \sqrt{n} - 0.13 - 0.75 \sqrt{R} (\sqrt{n} - 0.10)$ and n = Manning's coefficient. This formula appears to be in use in Russia.

2. ***Ganguillet and Kutter Formula***

$$C = \frac{23 + \dfrac{1}{n} + \dfrac{0.00155}{S_0}}{1 + \left[23 + \dfrac{0.00155}{S_0}\right] \dfrac{n}{\sqrt{R}}} \tag{3.15}$$

in which n = Manning's coefficient.

3. ***Bazin's formula***

$$C = \frac{87.0}{1 + M / R} \tag{3.16}$$

in which M = a coefficient dependent on the surface roughness.

Gradually Varied Flow Theory

INTRODUCTION

A steady non-uniform flow in a prismatic channel with gradual changes in its water surface elevation is termed as *gradually varied flow (GVF).*

Examples of GVF

1)The backwater produced by a dam

2)weir across a river and the drawdown produced at a sudden drop in a channel are few typical.

In a GVF, the velocity varies along the channel and consequently the bed slope, water surface slope, and energy slope will all differ from each other.

Regions of high curvature are excluded in the analysis of this flow.

The two basic assumptions involved in the analysis of GVF are the following:

1. The pressure distribution at any section is assumed to be hydrostatic. This follows from the definition of the flow to have a gradually-varied water surface. As gradual changes in the surface curvature give rise to negligible normal accelerations, the departure from the hydrostatic pressure distribution is negligible. The exclusion of the region of high curvature from the analysis of GVF, as indicated earlier, is only to meet this requirement.

2. The resistance to fl ow at any depth is assumed to be given by the corresponding uniform fl ow equation, such as the Manning's formula, with the condition that the slope term to be used in the equation is the energy slope and not the bed slope. Thus, if in a GVF the depth of fl ow at any section is *y, the energy* slope *Sf is given by*

S