

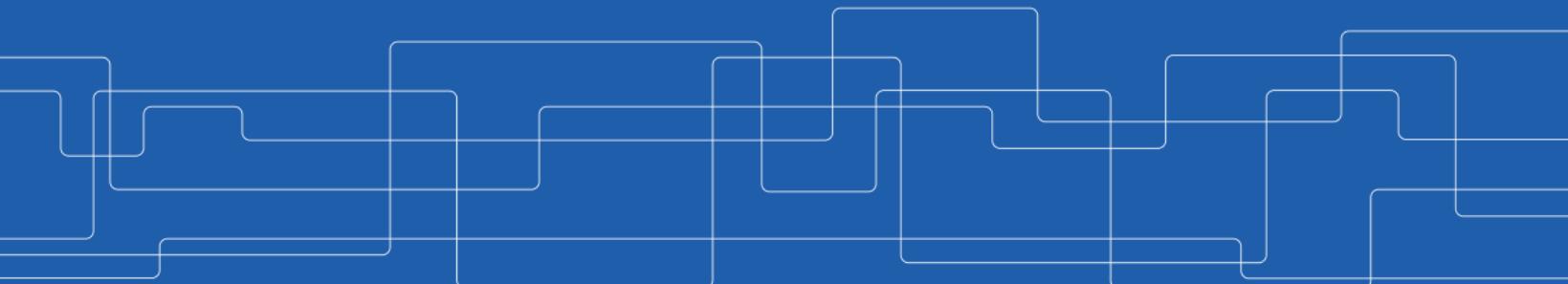


# Collective decision-making on networked systems in presence of antagonistic interactions

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# Outline

- ▶ **Background**

Motivation and problem statement

- ▶ **Signed networks**

Notions of structural balance and frustration

- ▶ **Model for collective decision-making over signed networks**

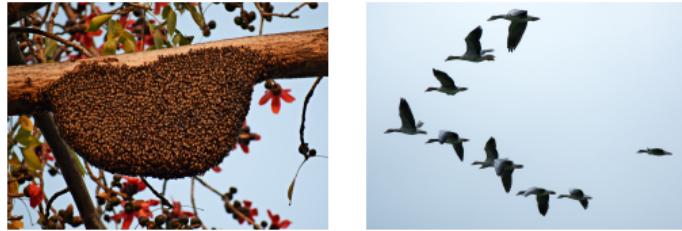
Bifurcation analysis on structurally balanced and structurally unbalanced networks

- ▶ **Application**

Process of government formation over signed “parliamentary networks”

# Background

## Motivation



Animal groups

⇒ decision reached through **collaboration**

# Background

## Motivation

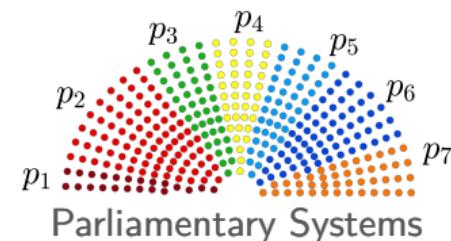


Animal groups

⇒ decision reached through **collaboration**



Social Networks

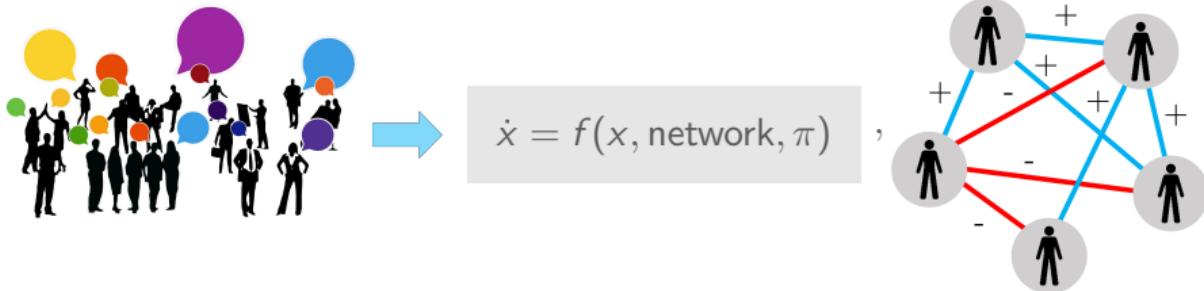


Parliamentary Systems

⇒ both **cooperative** and **antagonistic** interactions may coexist

# Background

Problem: collective decision-making in presence of antagonism



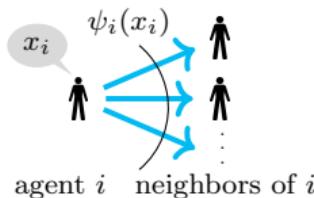
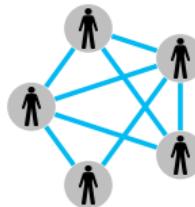
1. Signed networks
  - positive weight: **cooperative** interaction
  - negative weight: **antagonistic** interaction
2. Model for collective decision-making
  - $x$ : vector of opinions
  - equilibrium points: possible decisions

# Model for collective decision-making

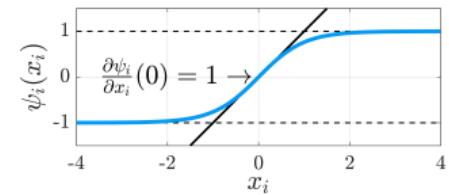
$$\dot{x} = -\Delta x + \pi A \psi(x)$$

- ▶  $n$  agents,  $x \in \mathbb{R}^n$  vector of opinions
- ▶ “inertia” of the agents:  $\Delta = \text{diag}\{\delta_1, \dots, \delta_n\}$ ,  $\delta_i > 0$
- ▶ interactions between the agents:

unsigned (connected) network  $\mathcal{G}(A)$



$$\psi(x) = [\psi_1(x_1) \dots \psi_n(x_n)]^T$$



and  $\pi > 0$  scalar parameter

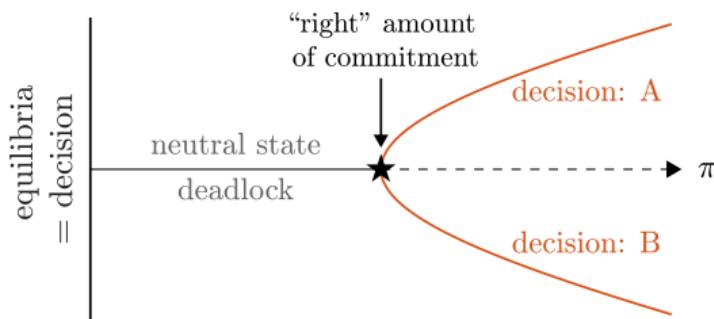
# Model for collective decision-making

$$\dot{x} = -\Delta x + \pi A \psi(x) \quad (*)$$

- ▶  $\pi$  = “social effort” or “strength of commitment” among the agents
- ▶ equilibria = decisions

**Assumption:**  $\delta_i = \sum_j a_{ij} \Rightarrow L = \Delta - A$ : Laplacian of  $\mathcal{G}(A)$

**Task:** Study qualitative behavior of (\*) as social effort parameter  $\pi$  is varied

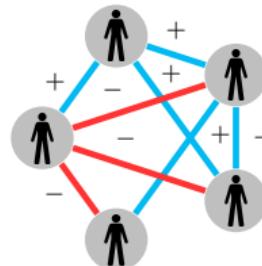


# Model for collective decision-making over signed networks

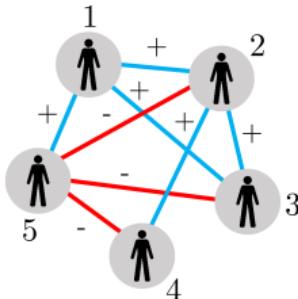
**Task:** Study the decision-making process in a community of agents where **both cooperative and antagonistic interactions coexist**

**Model:**  $\dot{x} = -\Delta x + \pi A\psi(x)$

**Assumptions:**  $\mathcal{G}(A)$  is **signed**,  $\pi$ : “social effort” between the agents



# Signed networks and signed Laplacian matrix



$$A = \begin{bmatrix} 0 & + & + & 0 & + \\ + & 0 & + & + & - \\ + & + & 0 & 0 & - \\ 0 & + & 0 & 0 & - \\ + & - & - & - & 0 \end{bmatrix} \Rightarrow \delta_1 \quad \dots \quad \mathcal{L} = \begin{bmatrix} 1 & - & - & 0 & - \\ - & 1 & - & - & + \\ - & - & 1 & 0 & + \\ 0 & - & 0 & 1 & + \\ - & + & + & + & 1 \end{bmatrix}$$

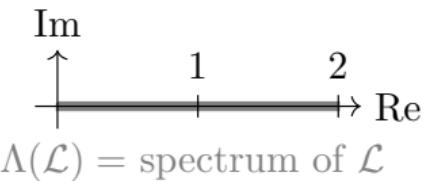
Signed Laplacian:

$$L = \Delta - A$$

$$\Delta = \text{diag}\{\delta_1, \dots, \delta_n\} : \delta_i = \sum_{j=1}^n |a_{ij}| > 0 \quad \forall i$$

Focus on:

normalized signed Laplacian:  $\mathcal{L} = I - \Delta^{-1}A$



$\Lambda(\mathcal{L})$  = spectrum of  $\mathcal{L}$

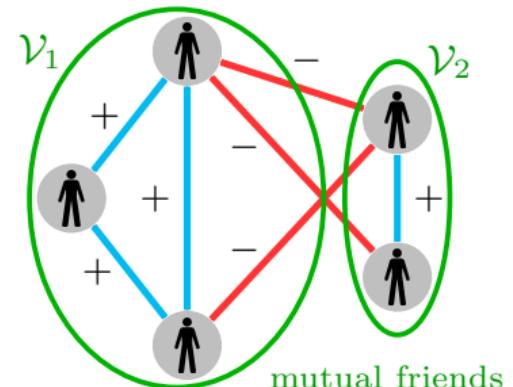
# Structural balance

A connected signed graph is **structurally balanced**

if  $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$  s.t. every edge:

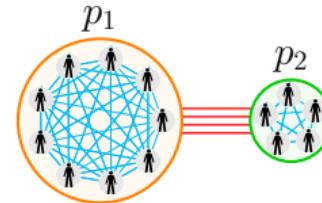
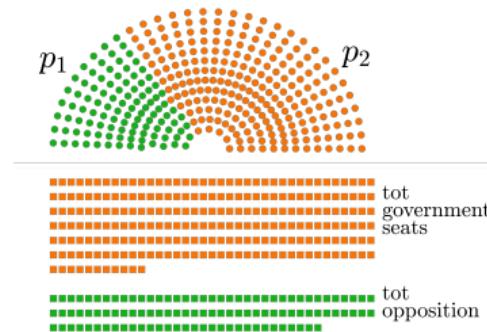
- between  $\mathcal{V}_1$  and  $\mathcal{V}_2$  is negative
- within  $\mathcal{V}_1$  or  $\mathcal{V}_2$  is positive

It is **structurally unbalanced**  
otherwise

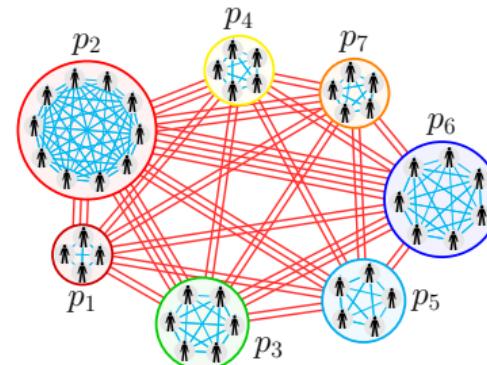
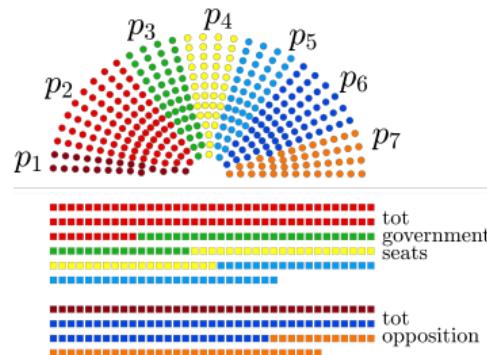


## Example: Parliamentary systems

Structurally balanced network



Structurally unbalanced network



## Structural balance: equivalent conditions

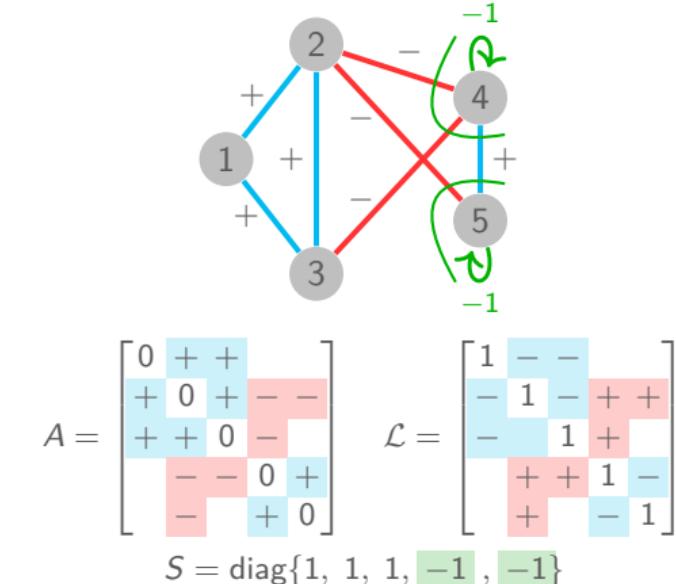
$\mathcal{G}(A)$  connected signed graph is  
**structurally balanced** iff

1.  $\exists$  signature matrix

$$S = \text{diag}\{s_1, \dots, s_n\}, s_i = \pm 1, \text{ s.t.}$$

$S\mathcal{L}S$  has all nonpositive

off-diagonal entries ( $SAS \geq 0$ )



# Structural balance: equivalent conditions

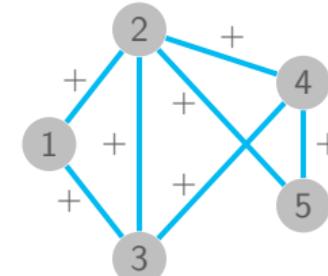
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$$A = \begin{bmatrix} 0 & + & + & & \\ + & 0 & + & - & - \\ + & + & 0 & - & \\ - & - & 0 & + & \\ - & & + & 0 & \end{bmatrix} \quad \mathcal{L} = \begin{bmatrix} 1 & - & - & & \\ - & 1 & - & + & + \\ - & 1 & + & - & \\ + & + & 1 & - & \\ + & & - & 1 & \end{bmatrix}$$

$$S = \text{diag}\{1, 1, 1, \boxed{-1}, \boxed{-1}\}$$

$$SAS = \begin{bmatrix} 0 & + & + & & \\ + & 0 & + & + & + \\ + & + & 0 & + & \\ + & + & 0 & + & \\ + & & + & 0 & \end{bmatrix} \quad S\mathcal{L}S = \begin{bmatrix} 1 & - & - & & \\ - & 1 & - & - & - \\ - & - & 1 & - & \\ - & - & 1 & - & \\ - & & - & 1 & \end{bmatrix}$$

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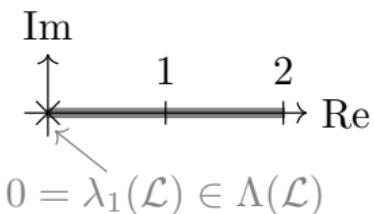
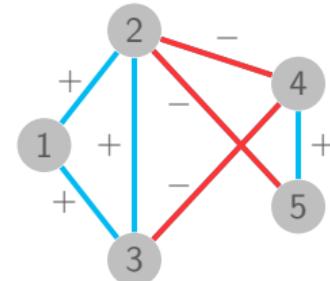
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2.  $\lambda_1(\mathcal{L}) = 0$



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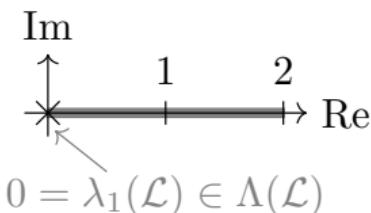
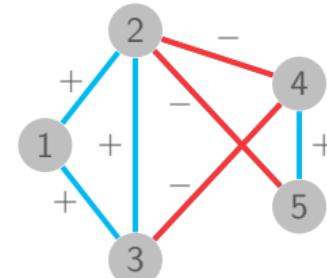
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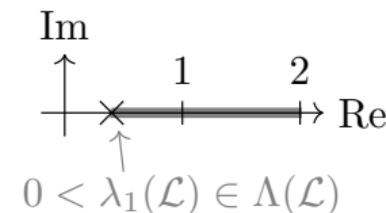
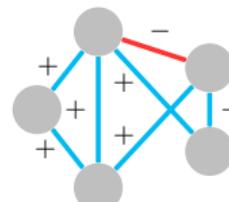
$$S = \text{diag}\{s_1, \dots, s_n\}, s_i = \pm 1, \text{ s.t.}$$

$S\mathcal{L}S$  has all nonpositive off-diagonal entries ( $SAS \geq 0$ )

2.  $\lambda_1(\mathcal{L}) = 0$



$\Rightarrow \mathcal{G}(A)$  connected signed graph is **structurally unbalanced** iff  $\lambda_1(\mathcal{L}) > 0$





# Frustration index and algebraic conflict

**Task:** characterize the graph distance from structurally balanced state

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- ▶ Frustration Index  
(computation: NP-hard problem)

$$\epsilon(\mathcal{G}) = \min_{\substack{S=\text{diag}\{s_1, \dots, s_n\} \\ s_i = \pm 1}} \frac{1}{2} \cdot \underbrace{\sum_{i \neq j} [|\mathcal{L}| + S\mathcal{L}S]_{ij}}_{=e(S): \text{"energy functional"}}$$

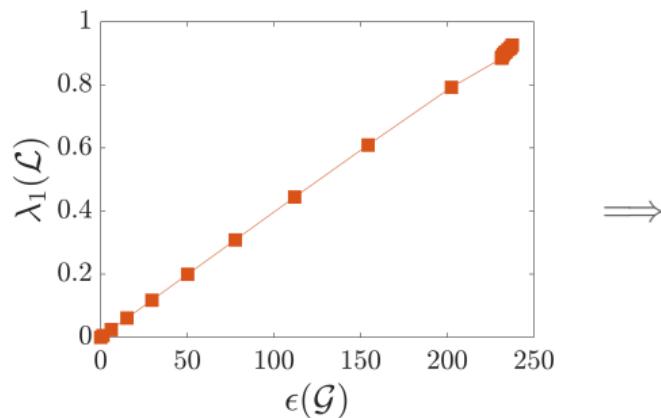
# Frustration index and algebraic conflict

**Task:** characterize the graph distance from structurally balanced state

- ▶ Frustration Index  
(computation: NP-hard problem)
- ▶ Algebraic Conflict

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$$\xi(\mathcal{G}) = \lambda_1(\mathcal{L})$$



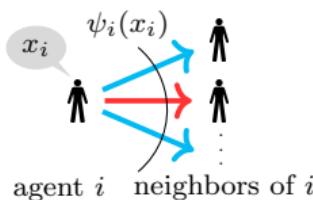
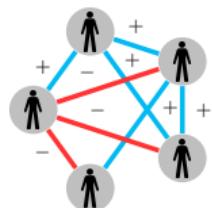
$\lambda_1(\mathcal{L})$  good  
approximation of  $\epsilon(\mathcal{G})$

# Model for collective decision-making over signed networks

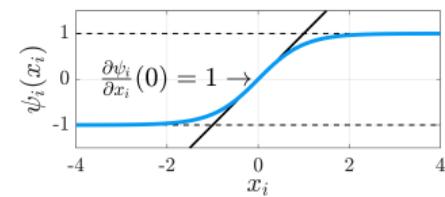
$$\dot{x} = -\Delta x + \pi A\psi(x)$$

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- “inertia” of the agents:  $\Delta = \text{diag}\{\delta_1, \dots, \delta_n\}$ ,  $\delta_i > 0$
- interactions between the agents:

**signed** (connected) network  $\mathcal{G}(A)$



$$\psi(x) = [\psi_1(x_1) \dots \psi_n(x_n)]^T$$



and  $\pi > 0$  “social effort” (or “strength of commitment”)

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A. Fontan and C. Altafini, “The role of frustration in collective decision-making dynamical processes on multiagent signed networks”, IEEE TAC, 2022

## Dynamical interpretation of structural balance

$$\dot{x} = -\Delta x + \pi A \psi(x) = \Delta(-x + \pi \underbrace{\Delta^{-1} A}_{:=H} \psi(x)) \quad (*)$$

“Laplacian” assumption:  $\delta_i = \sum_j |a_{ij}| > 0 \ \forall i \Rightarrow \mathcal{L} = I - H$

Then at the origin for  $\pi = 1$ :

Jacobian:  $J = -L = \Delta(-\mathcal{L})$

and

$(*)$  is monotone  $\Leftrightarrow \mathcal{G}(A)$  is structurally balanced  $\Leftrightarrow \lambda_1(\mathcal{L}) = 0$ .



## Task

$$\dot{x} = -\Delta x + \pi A \psi(x) = \Delta(-x + \pi H \psi(x)) \quad (*)$$

Investigate how:

- ▶ the **social effort parameter  $\pi$**  affects the existence and stability of the equilibrium points of the system (\*)  
Tool: bifurcation theory ( $\mathcal{L} = I - H$  has simple eigenvalues)
- ▶ the presence of **antagonistic** interactions affects the behavior of (\*)  
Tool: signed networks theory (frustration)

# Bifurcation analysis: structurally balanced networks

$$\dot{x} = \Delta(-x + \pi H\psi(x)), \quad x \in \mathbb{R}^n$$

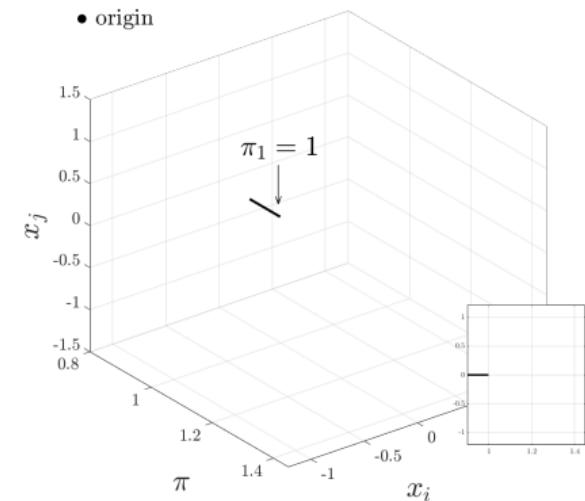
$\pi < 1$ :  $x = 0$  only eq. point (GAS)

$\pi = 1$ : pitchfork bifurcation

- ▶  $x = 0$  saddle point
- ▶ new equilibria:  $x^*, -x^*$  (loc. AS  $\forall \pi > 1$ )

$\pi = \pi_2 = \frac{1}{1-\lambda_2(\mathcal{L})}$ : pitchfork bifurcation

- ▶ new equilibria (stable/unstable for  $\pi > \pi_2$ )



Bifurcation diagram

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A. Fontan and C. Altafini, "Multiequilibria analysis for a class of collective decision-making networked systems", IEEE TCNS, 2018

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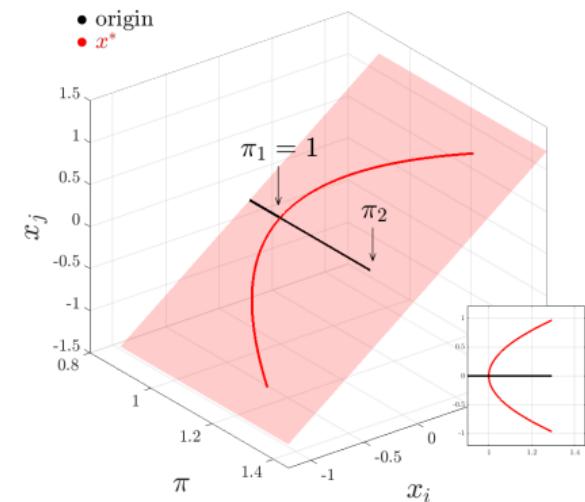
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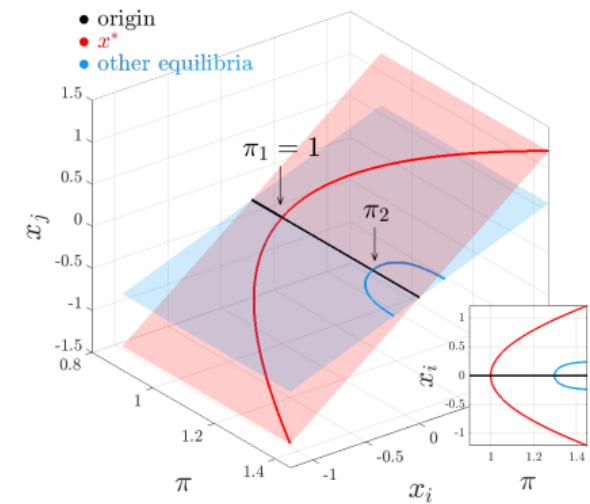
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Bifurcation diagram

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# Bifurcation analysis: structurally unbalanced networks

$$\dot{x} = \Delta(-x + \pi H\psi(x)), \quad x \in \mathbb{R}^n$$

$$\pi_1 = \frac{1}{1 - \lambda_1(\mathcal{L})} \quad \pi_2 = \frac{1}{1 - \lambda_2(\mathcal{L})}$$

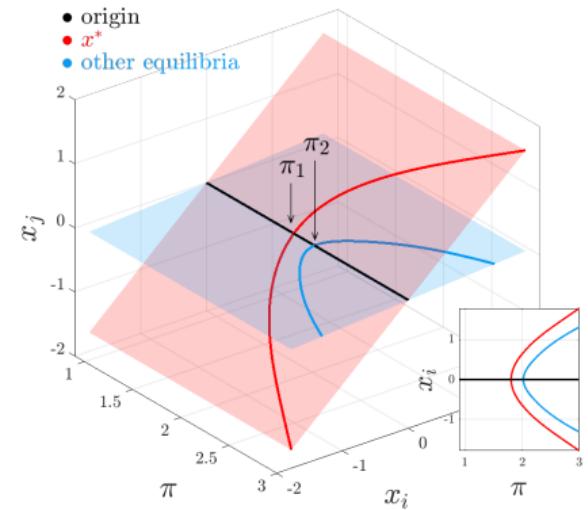
$\pi < \pi_1$ :  $x = 0$  only eq. point (GAS)

$\pi = \pi_1$ : pitchfork bifurcation

- ▶  $x = 0$  saddle point
- ▶ new equilibria:  $x^*, -x^*$  (loc. AS)

$\pi = \pi_2$ : pitchfork bifurcation

- ▶ new equilibria (stable/unstable for  $\pi > \pi_2$ )



Bifurcation diagram

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## Sketch of the proof: first bifurcation

Theorem

Assuming:

- S-shaped  $\psi$ :  $\forall i \psi_i$  is odd, saturated, sigmoidal, monotonically increasing with  $\frac{\partial \psi_i}{\partial x_i}(0) = 1$
- $\lambda_1(\mathcal{L}) > 0$  simple

Then:

$$x^* \neq 0 \text{ is equilibrium point of} \quad \iff \quad \pi > \pi_1 = \frac{1}{1 - \lambda_1(\mathcal{L})}$$
$$\dot{x} = \Delta(-x + \pi H\psi(x))$$

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Assuming:

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Then:

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$$\dot{x} = \Delta(-x + \pi H \psi(x))$$

**Proof: Sufficiency** [ $x = 0$  is GAS when  $\pi \leq \pi_1$ ]

Lyap. function  $V : \mathbb{R}^n \rightarrow \mathbb{R}_+$ ,  $V(x) = \sum_i \int_0^{x_i} \psi_i(s) ds \geq 0$  (radially unbounded)

$$\begin{aligned} \dot{V}(x) &= \psi(x)^T \dot{x} = -\underbrace{\psi(x)^T \Delta x}_{>\psi(x)^T \Delta x} + \underbrace{\psi(x)^T \Delta (\pi H) \psi(x)}_{=\Delta^{\frac{1}{2}} (\pi \Delta^{\frac{1}{2}} H \Delta^{-\frac{1}{2}}) \Delta^{\frac{1}{2}}} \\ &< -\psi(x)^T \Delta^{\frac{1}{2}} \underbrace{(I - \pi \Delta^{\frac{1}{2}} H \Delta^{-\frac{1}{2}})}_{\text{symmetric, psd } (\succeq 0)} \Delta^{\frac{1}{2}} \psi(x) \leq 0 \quad \forall x \neq 0 \end{aligned}$$

## Sketch of the proof: first bifurcation

**Proof: Necessity** [pitchfork bifurcation when  $\pi = \pi_1 = \frac{1}{1-\lambda_1(\mathcal{L})} = \frac{1}{\lambda_n(H)}$ ]

$$\Phi(x, \pi) = -x + \pi H \psi(x) = 0, \quad \textcolor{red}{J} := \frac{\partial \Phi}{\partial x}(0, \pi_1) = -I + \pi_1 H$$

Lyapunov-Schimdt reduction:

- ▶  $v$  (right),  $w$  (left) eigenvectors of  $J$  relative to 0  $\Rightarrow$   $E = I - vw^T : \mathbb{R}^n \rightarrow \text{range}(J)$   
 $I - E : \mathbb{R}^n \rightarrow \ker(J)$
- ▶ split  $x = yv + r$ ,  $y \in \mathbb{R}$  and  $r = Ex \Rightarrow$  near  $(0, \pi_1)$ :  $\begin{cases} 0 = E \Phi(yv + r, \pi) \\ 0 = (I - E) \Phi(yv + r, \pi) \end{cases}$
- ▶ implicit function theorem:  $\exists! r = R(yv, \pi) : E \Phi(yv + R(yv, \pi), \pi) = 0$
- ▶ define center manifold  $g : \mathbb{R} \times \mathbb{R} \mapsto \mathbb{R}$  by:  $\textcolor{red}{g}(y, \pi) := w^T(I - E) \Phi(yv + R(yv, \pi), \pi)$
- ▶ partial derivatives at  $(0, \pi_1)$  satisfy

$\textcolor{red}{g}_y = g_{yy} = g_\pi = 0$ ,  $g_{\pi y} > 0$ ,  $g_{yyy} < 0$   $\Rightarrow$  **pitchfork bifurcation** at  $\pi = \pi_1$ !

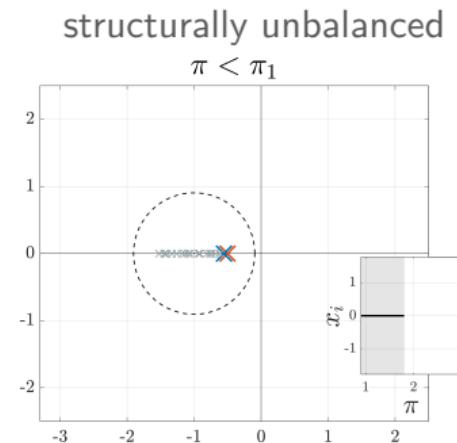
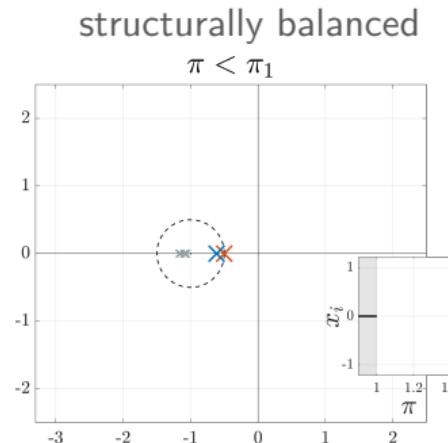
□

## Interpretation of the results.. as we vary $\pi$

Pitchfork bifurcation at:  $\pi_1 = \frac{1}{1-\lambda_1(\mathcal{L})}$ ,  $\pi_2 = \frac{1}{1-\lambda_2(\mathcal{L})}$

(norm.) linearization at 0:  
 $-I + \pi H = -I + \pi(I - \mathcal{L})$

- $\times \lambda_1(-I + \pi(I - \mathcal{L}))$
- $\times \lambda_2(-I + \pi(I - \mathcal{L}))$



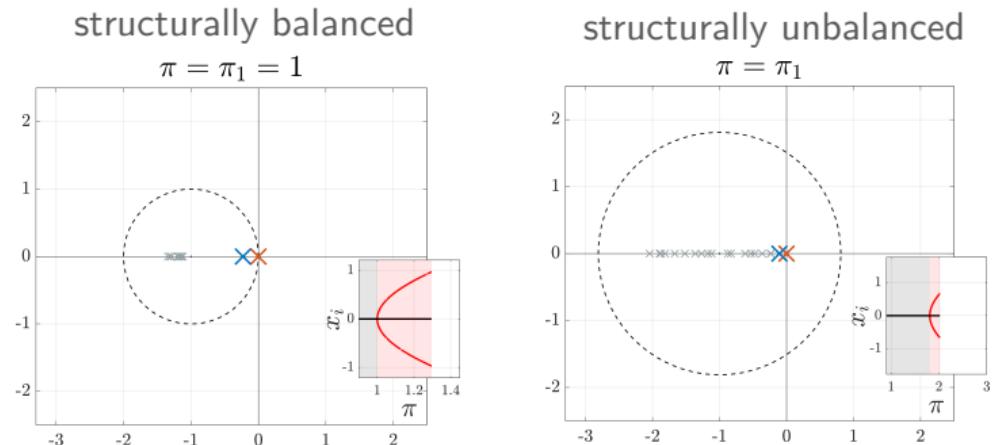
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 (small effort)  
■ no decision (deadlock)

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(small effort)  
 ■ no decision (deadlock)

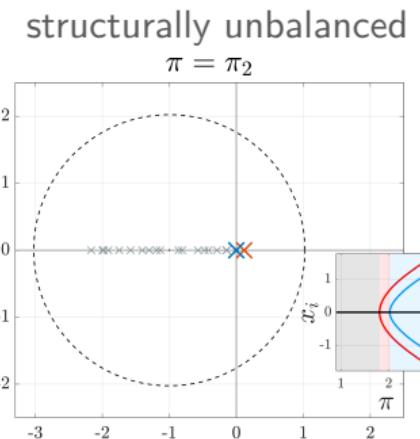
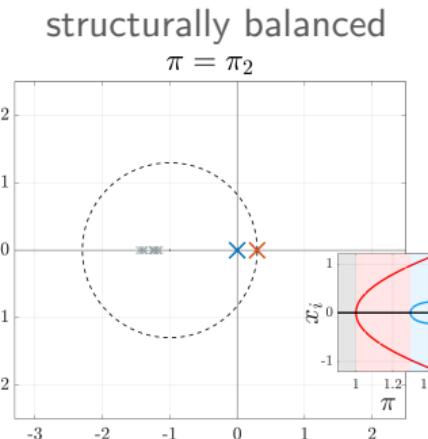
$\pi \in (\pi_1, \pi_2)$   
“right” commitment  
 ■ two (alternative) decisions

## Interpretation of the results.. as we vary $\pi$

Pitchfork bifurcation at:  $\pi_1 = \frac{1}{1-\lambda_1(\mathcal{L})}$ ,  $\pi_2 = \frac{1}{1-\lambda_2(\mathcal{L})}$

(norm.) linearization at 0:  
 $-I + \pi H = -I + \pi(I - \mathcal{L})$

- $\times \lambda_1(-I + \pi(I - \mathcal{L}))$
- $\times \lambda_2(-I + \pi(I - \mathcal{L}))$



$\pi < \pi_1$   
(small effort)  
■ no decision (deadlock)

$\pi \in (\pi_1, \pi_2)$   
“right” commitment  
■ two (alternative) decisions

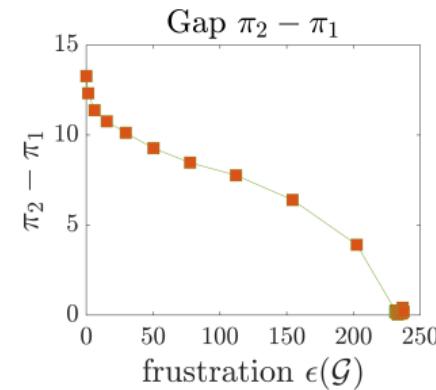
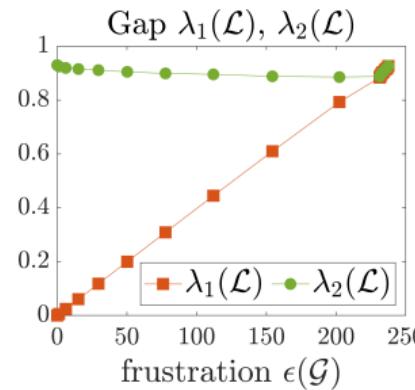
$\pi > \pi_2$   
“overcommitment”  
■ several decisions

## Interpretation of the results.. as we vary the frustration

Signed network  $\mathcal{G}$  with frustration  $\epsilon(\mathcal{G})$

$$\pi_1 = \frac{1}{1 - \lambda_1(\mathcal{L})} \begin{cases} = 1 \text{ fixed,} & \text{structurally balanced } \mathcal{G} \\ \text{depends on } \epsilon(\mathcal{G}), & \text{structurally unbalanced } \mathcal{G} \end{cases}$$

$$\pi_2 = \frac{1}{1 - \lambda_2(\mathcal{L})} \begin{cases} \text{depends on algebraic connectivity,} & \text{structurally balanced } \mathcal{G} \\ \text{independent from } \epsilon(\mathcal{G}), & \text{structurally unbalanced } \mathcal{G} \end{cases}$$



# Summary

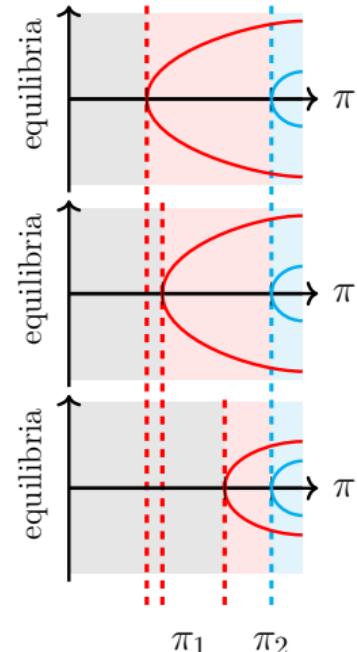
- ▶  $\pi_1 = \frac{1}{1-\lambda_1(\mathcal{L})}$  grows with  $\lambda_1(\mathcal{L})$
- ▶  $\lambda_1(\mathcal{L}) \approx$  frustration
- ▶ the higher the frustration:
  - the higher the social effort needed to achieve a decision
  - the smaller the interval for which only two alternative decisions exist

SIGNED GRAPH    DYNAMICAL SYSTEM

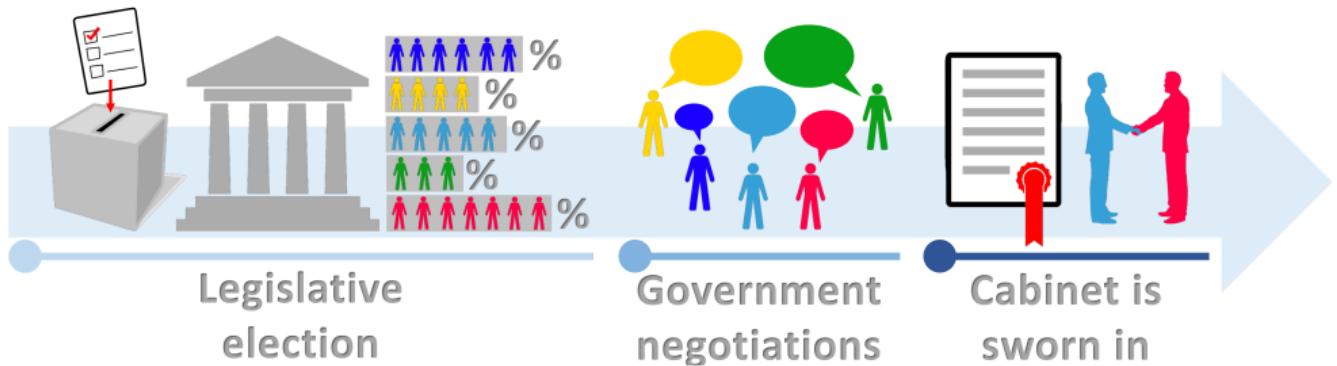
zero  
frustration

low  
frustration

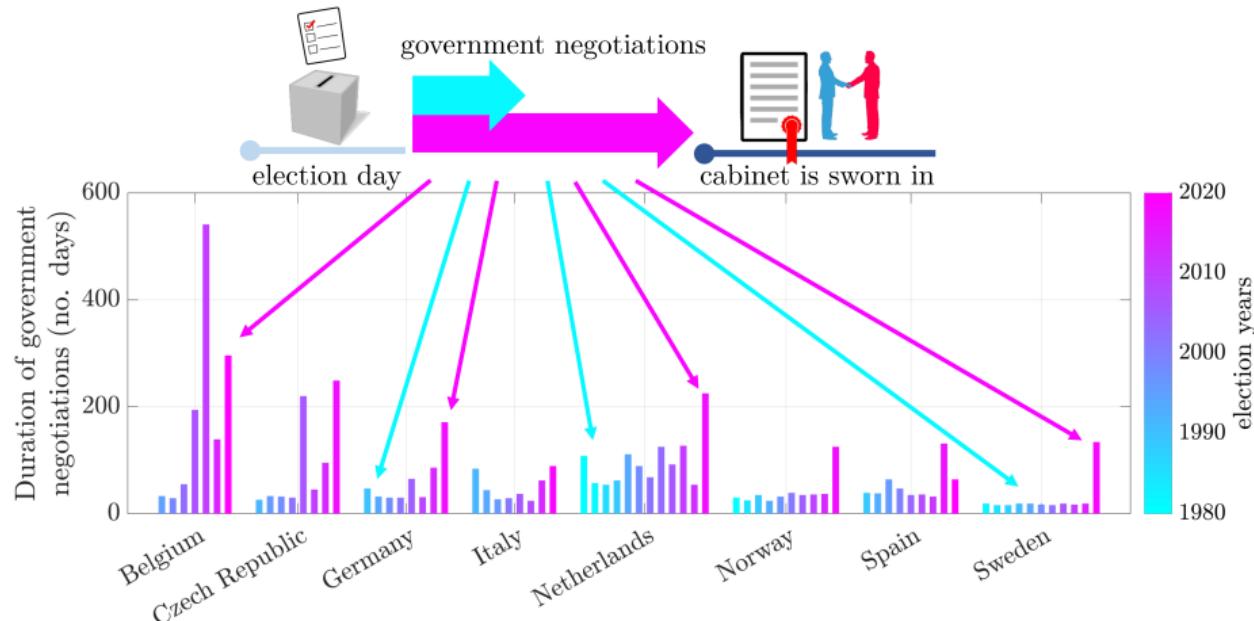
high  
frustration



# Government formation in parliamentary democracies



## Duration of government negotiation phase



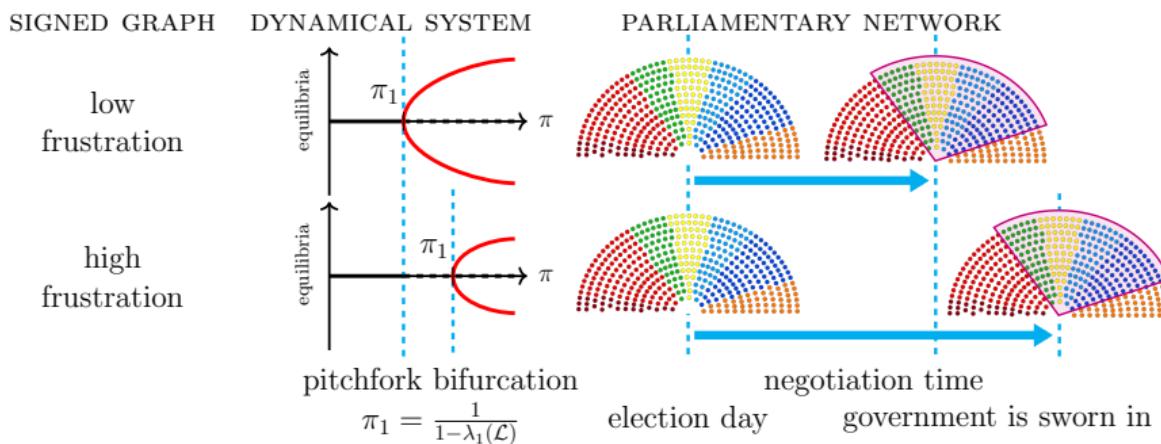
**Question:** can we use our model to explain this behavior?

# Dynamics of the formation of a government

- signed network: **parliament**
- decision: **vote of confidence** of the parliament
- social effort: **duration** of the government negotiation phase

$$\lambda_1(\mathcal{L}) \sim \text{frustration} + \pi_1 \sim \text{duration of negotiations} + \pi_1 = \frac{1}{1-\lambda_1(\mathcal{L})}$$

$\Rightarrow \text{duration of negotiations} \sim \text{frustration}$

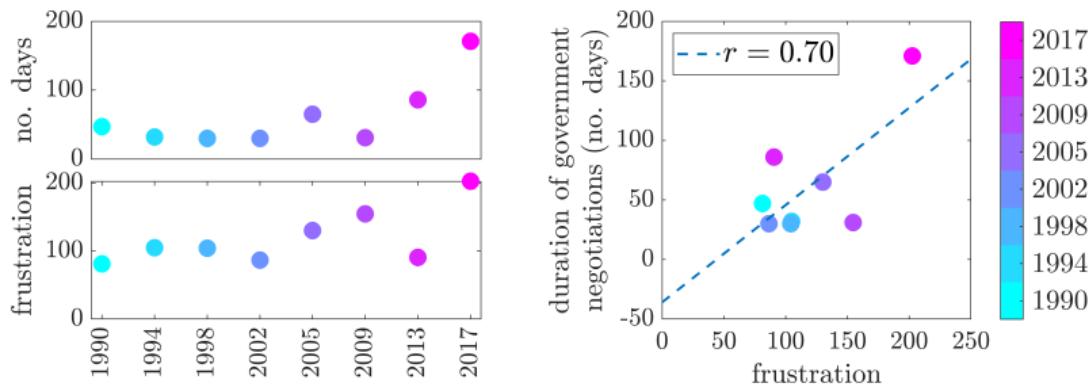


# Frustration vs duration of government negotiations

**Task:** show that the government formation process is influenced by the frustration of the parliamentary network

- ▶ Data: elections in 29 European countries (election years: 1978 - 2020)
- ▶ Method: Pearson's correlation index ( $r$ ), frustration vs duration of negotiations

Example: German elections

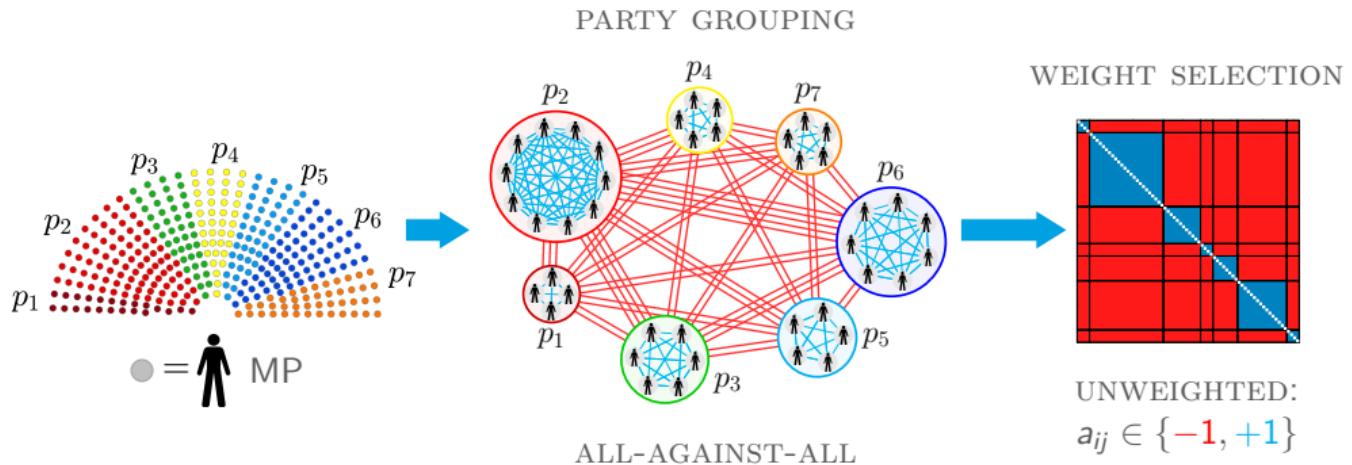


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A. Fontan and C. Altafini, "A signed network perspective on the government formation process in parliamentary democracies", Scientific Reports, 2021

# Construction of the parliamentary networks

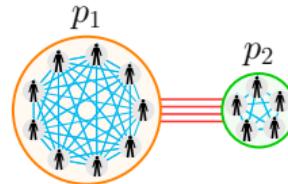
**Definition:** complete, undirected, signed graph in which each MP is a node



UNWEIGHTED:  
 $a_{ij} \in \{-1, +1\}$

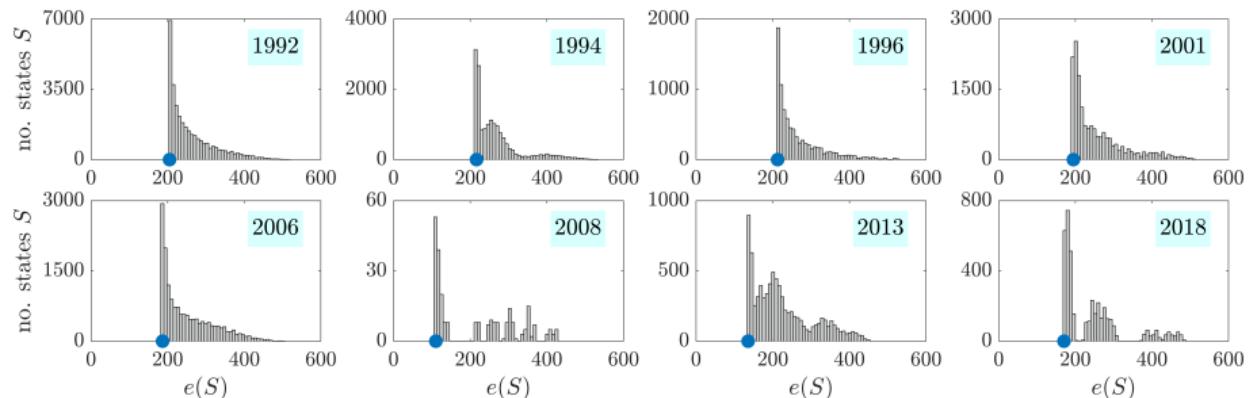
**collaboration:** MPs belong to the **same** party  
**rivalry:** MPs belong to **different** parties

# Are the parliamentary networks structurally balanced?



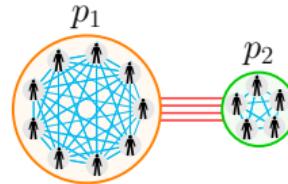
Structurally balanced  
parliamentary network

The parliamentary networks have (in general) nonzero frustration..



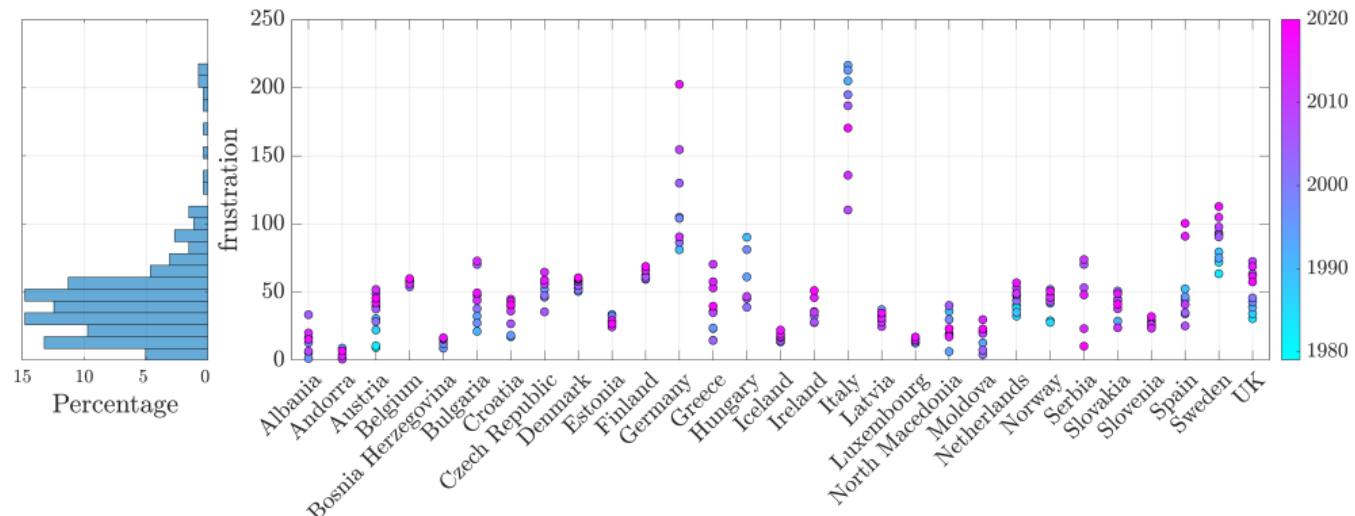
Energy landscape of the Italian parliamentary elections:  
 ● = frustration

# Are the parliamentary networks structurally balanced?



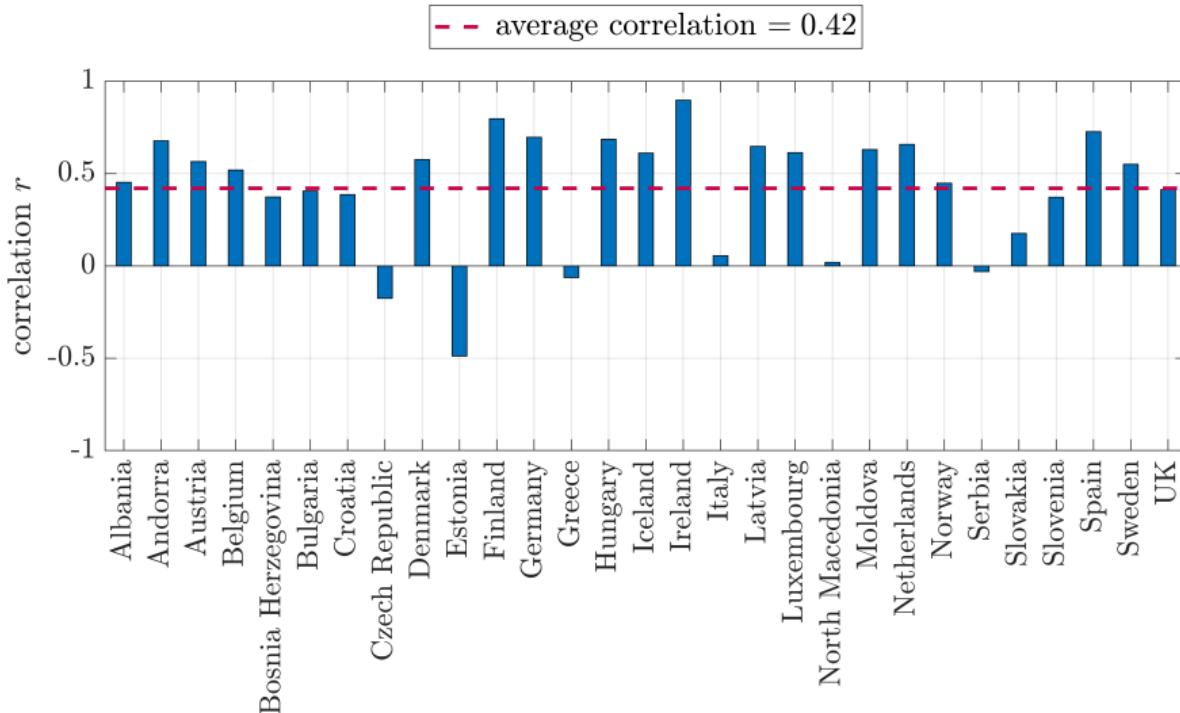
Structurally balanced  
parliamentary network

The parliamentary networks have (in general) nonzero frustration..

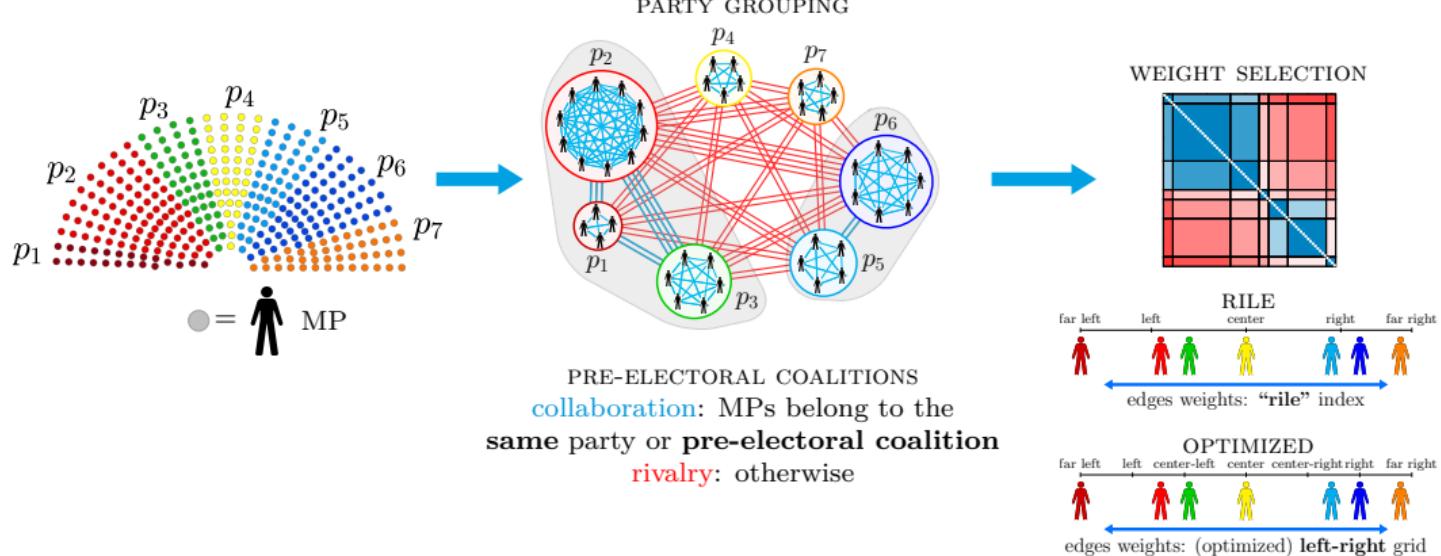


# Correlation for all 29 European countries

Duration of the government negotiations vs frustration of the parliamentary networks

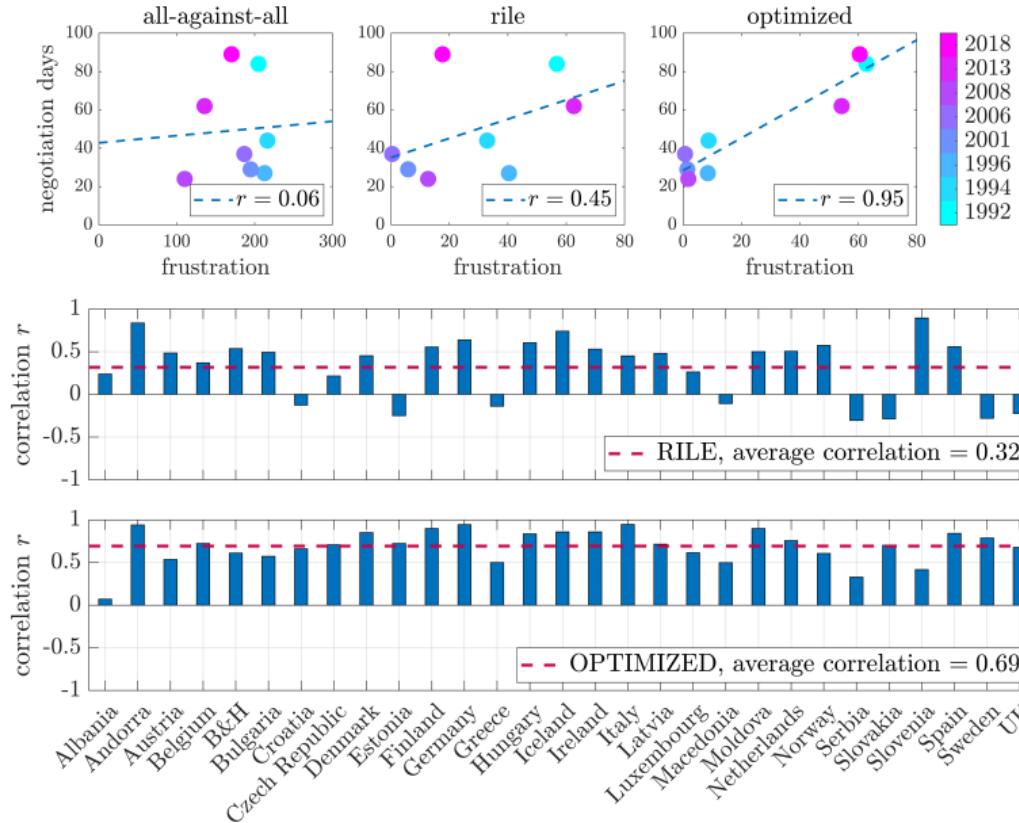


# Coalitions and ideological differences in the networks



"Rile" Data: Manifesto Project Database

# Correlation for all 29 European countries



Example:  
Italian  
elections

Results on  
average correlation:  
0.42, 0.32, 0.69

# Conclusion

**Task:** Study the decision-making process in a community of agents where **both cooperative and antagonistic interactions coexist**

As we vary the **social effort**: pitchfork bifurcation behavior

- ▶ “right” commitment: 2 alternative decisions
- ▶ “overcommitment”: several (more than 2) alternative decisions

As we vary the **frustration** (i.e., amount of disorder) of the signed networks

- ▶ frustration influences the level of commitment required from the agents to reach a decision

**Application:** Government formation process

- ▶ frustration correlates well with duration of government negotiation phase



# Thanks!

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<https://angelafontan.github.io/>