1. $1+2+3+4+\cdots+2n = n(2n+1)$

For $n=1$, L.H.S $=1+2=3$, R.H.S $=1(3)=3$

$\therefore$ The result is true for $n=1$. (5)

Take any $p \in \mathbb{Z}^+$ and assume that the result is true for $n=p$

$1+2+3+4+\cdots+2p = p(2p+1)$ (5)

For $n=p+1$

$1+2+3+4+\cdots+2p+(2p+1)+(2p+2)$

$= p(2p+1)+(2p+1)+(2p+2)$ (5)

$= 2p^2+5p+3$

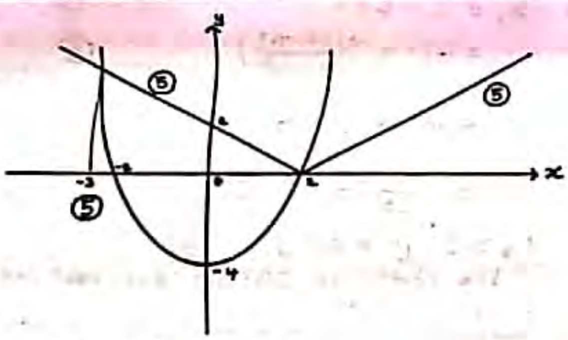
$= (p+1)(2p+3)$

$\therefore$ The result is true for $n=p+1$ (5)

Hence, by the principle of mathematical Induction, the result is true for all $n \in \mathbb{Z}^+$. (5)

$\boxed{25}$

2.



$2x^2-2 \geq |x-1|$

$4x^2-4 \geq |2x-2|$

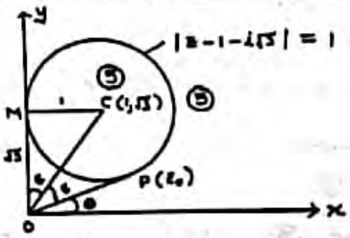
$(2x)^2-4 \geq |(2x)-2|$ (5)

$2x \leq -3$ or $2x \geq 2$

$x \leq -\frac{3}{2}$ or $x \geq 1$ (5)

$\boxed{25}$

3.



$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\theta = \frac{\pi}{2} - 2\alpha = \frac{\pi}{6}$

$z_0 = \sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ (5)

$\text{Arg}(z)\big|_{min} = \theta = \frac{\pi}{6}$ (5)

$OP = OM = \sqrt{3}$ (5)

$\boxed{25}$

4. $(1-kx^2)^2\left(1+\frac{1}{x}\right)^6$

$= (1-2kx^2+k^2x^4)\sum_{r=0}^{6} {}^6C_r \frac{1}{x^r}$ (5)

The term independent of x $= 1$

${}^6C_0 - 2k\,{}^6C_2 + k^2\,{}^6C_4 = 1$ (10)

$1 - 30k + 15k^2 = 1$ (5)

$k(k-2) = 0$

$k=0$ or $k=2$ (5)

$\boxed{25}$

5. $\lim_{x\to 0} \frac{1-\cos\frac{x}{2}}{\sqrt{x^3+x^2+4}-2}$

$= \lim_{x\to 0} \frac{\left(1-\cos\frac{x}{2}\right)\left(\sqrt{x^3+x^2+4}+2\right)}{x^3+x^2}$ (5)

$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{4}\left(\sqrt{x^3+x^2+4}+2\right)}{x^2(x+1)}$ (5)

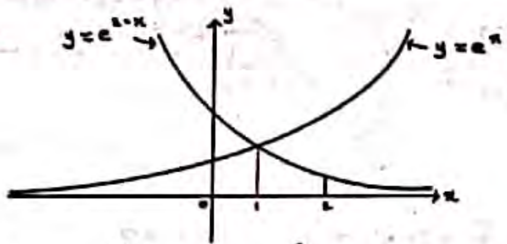
$= \left(\lim_{\frac{x}{4}\to 0} \frac{\sin\frac{x}{4}}{\frac{x}{4}}\right)^2 \lim_{x\to 0} \frac{\sqrt{x^3+x^2+4}+2}{8(x+1)}$ (5)

$= 1^2 \times \frac{4}{8}$ (5) (5)

$= \frac{1}{2}$

$\boxed{25}$

6.



Volume $= \int_0^1 \pi(e^x)^2dx + \int_1^2 \pi(e^{2-x})^2dx$ (5)

$= \pi\int_0^1 e^{2x}dx + \pi\int_1^2 e^{4-2x}dx$ (5)

$= \pi\left[\frac{e^{2x}}{2}\right]_0^1 + \pi\left[\frac{e^{4-2x}}{-2}\right]_1^2$ (5) (5)

$= \frac{\pi}{2}(e^2-1) + \frac{\pi}{2}(e^2-1)$ (5)

$= \pi(e^2-1)$

$\boxed{25}$

7. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

L.H.S $= (a\cos^3\theta)^{\frac{2}{3}} + (a\sin^3\theta)^{\frac{2}{3}}$

$= a^{\frac{2}{3}}(\cos^2\theta + \sin^2\theta)$

$= a^{\frac{2}{3}} = $ R.H.S (5)

$x = a\cos^3\theta$ $\quad\quad$ $y = a\sin^3\theta$

$\frac{dx}{d\theta} = a\,3\cos^2\theta(-\sin\theta)$ $\quad\quad$ $\frac{dy}{d\theta} = a\,3\sin^2\theta\cos\theta$ (5)

$= -3a\cos^2\theta\sin\theta$ (5)

$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$

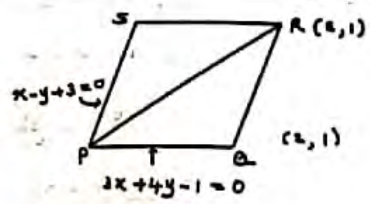
$= \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$

$= -\tan\theta$ (5)

$-\tan\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$ $[\because 0<\theta<\frac{\pi}{2}]$ (5)

$\boxed{25}$



Equation of PR is

$(3x+4y-1)+\lambda(x-y+3)=0$ (5)

$(2,1)$ // $6+4-1+\lambda(2-1+3)=0$

$\lambda = -\frac{9}{4}$ (5)

PR: $3x+4y-1-\frac{9}{4}(x-y+3)=0$

$3x+25y-31=0$ (5)

Equation of SR is $3x+4y+c=0$ (5)

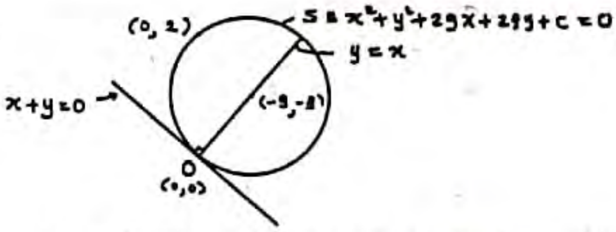
$(2,1)$ // $6+4+c=0$

$\Rightarrow c=-10$

SR: $3x+4y-10=0$ (5)

$\boxed{25}$

9.



$S \equiv x^2+y^2+2gx+2fy+c=0$

$(0,0)$ // $0+0+0+0+c=0$

$c=0$ (5)

$(0,2)$ // $0+4+0+4f+c=0$

$f=-1$ (5)

centre $(-g,-f)$ lies on $y=x$ (5)

$-f=-g$

$\Rightarrow g=-1$ (5)

$x^2+y^2-2x-2y=0$ (5)

$\boxed{25}$

10. $\sqrt{2}(\sin 2x - \sin x) + (2\cos x - 1) = 0$

$\sqrt{2}(2\sin x\cos x - \sin x) + (2\cos x - 1) = 0$ (5)

$\sqrt{2}\sin x(2\cos x - 1) + (2\cos x - 1) = 0$

$(2\cos x - 1)(\sqrt{2}\sin x + 1) = 0$ (5)

$\cos x = \frac{1}{2}$ (5) $[\because 0<x<\frac{\pi}{2} \Rightarrow \sqrt{2}\sin x + 1 > 0]$ (5)

$x = \frac{\pi}{3}$ (5)

$\boxed{25}$

11. (a) $f(x)=ax^2+2x+c$, $g(x)=bx^2+x+c$

Since α is a common root of $f(x)=0$ and $g(x)=0$,

we have $a\alpha^2+2\alpha+c=0$ — (1) (5)

$b\alpha^2+\alpha+c=0$ — (2) (5)

$(2)-(1) \Rightarrow (b-a)\alpha^2-\alpha=0$

$\alpha = \frac{1}{b-a}$ $[\because c\neq 0 \Rightarrow \alpha\neq 0]$ (5)

$\boxed{15}$

$c = -b\alpha^2-\alpha$ (5)

$= -\alpha(b\alpha+1)$

$= -\frac{1}{b-a}\left(\frac{b}{b-a}+1\right)$ (5)

$= -\frac{(2b-a)}{(b-a)^2}$

$\boxed{10}$

(i) $\Delta_1 = 4-4ac$ (5)

$= 4\left(1+\frac{a(2b-a)}{(b-a)^2}\right)$ (5)

$= 4\left(\frac{b^2-2ba+a^2+2ab-a^2}{(b-a)^2}\right)$

$= \frac{4b^2}{(b-a)^2}$ (5)

$\Delta_1>0$ $[\because b\neq 0]$ (5)

$\therefore$ the roots of $f(x)=0$ are real and distinct.

$\boxed{20}$

(ii) $\Delta_2 = 1-4bc$ (5)

$= 1+\frac{4b(2b-a)}{(b-a)^2}$ (5)

$= \frac{b^2-2ba+a^2+8b^2-4ab}{(b-a)^2}$

$= \frac{a^2-6ab+9b^2}{(b-a)^2}$

$= \left(\frac{a-3b}{b-a}\right)^2$ (5)

$\Delta_2=0 \Rightarrow a=3b$ (5)

$\boxed{20}$

$\alpha+\beta = -\frac{2}{a}$ (5) $\quad\quad$ $\alpha+\gamma = -\frac{1}{b}$ (5)

$\Rightarrow \beta = -\frac{2}{a}-\frac{1}{b-a}$ (5) $\quad\quad$ $\Rightarrow \gamma = -\frac{1}{b}-\frac{1}{b-a}$ (5)

$= \frac{-2b+2a-a}{a(b-a)}$ $\quad\quad$ $= \frac{-b+a-b}{b(b-a)}$

$= \frac{a-2b}{a(b-a)}$ (5) $\quad\quad$ $= \frac{a-2b}{b(b-a)}$ (5)

$\boxed{30}$

(b) $h(x) = ax^3 + bx^2 + cx + 1$

Since x^2-4 is a factor of $h(x)$,

$x-2$ and $x+2$ are both factors of $h(x)$.

$\Rightarrow h(2) = 0$ (5) and $h(-2) = 0$ (5)

$\Rightarrow 8a + 4b + 2c + 1 = 0$ —① (5) and $-8a + 4b - 2c + 1 = 0$ —② (5)

①+② $\Rightarrow 8b + 2 = 0$

$\Rightarrow b = -\frac{1}{4}$ (5)

$h(x) = (x^2-1)\phi(x) + x + k$ (5)

$h(1) = k \Rightarrow a + b + c + 1 = 1 + k$ —③ (5)

$h(-1) = k \Rightarrow -a + b - c + 1 = -1 + k$ —④ (5)

③+④ $\Rightarrow 2b + 2 = 2k$

$b + 1 = k$

$-\frac{1}{4} + 1 = k$

$k = \frac{3}{4}$ (5)

③ $\Rightarrow a + c = 1$
① $\Rightarrow c = -4a$ $\Big\}$ $a = -\frac{1}{3}$ (5), $c = \frac{4}{3}$ (5)

$\boxed{55}$

12. (a)

(i) $^{10}C_6 = 210$ (5) (5) $\boxed{10}$

(ii) $^5C_3 \cdot {^5C_3} = 100$ (5) (5) $\boxed{10}$

(iii) Students (M-2, F-2) Teachers (M-2, F-2) Non-Academic Staff (M-1, F-1)

Number of selection that do not include Teachers $= {^6C_6}$ (5)

Number of selection that do not include Students $= {^6C_6}$ (5)

Number of selection that do not include Non-Academic staff $= {^8C_6}$ (5)

$\therefore$ The required number of ways $= {^{10}C_6} - {^6C_6} - {^6C_6} - {^8C_6}$ (5)

$= 210 - 1 - 1 - 28$ (5)

$= 180$ (5) $\boxed{30}$

(iv) No. of selection of 3 males and 3 females that do not include teachers $= {^3C_3}\cdot{^3C_3}$ (5)

No. of selection of 3 males and 3 females that do not include students $= {^3C_3}\cdot{^3C_3}$ (5)

No. of selection of 3 males and 3 females that do not include Non-Academic staff $= {^4C_3}\cdot{^4C_3}$ (5)

$\therefore$ The required number of ways $= {^5C_3}{^5C_3} - {^3C_3}{^3C_3} - {^3C_3}{^3C_3} - {^4C_3}{^4C_3}$ (5)

$= 100 - 1 - 1 - 16$ (5)

$= 82$ (5) $\boxed{30}$

(b) $V_r - V_{r+1}$

$= \frac{1}{2(2r-1)} - \frac{4}{3}(r-1)r(r+1) - \frac{1}{2(2r+1)} + \frac{4}{3}r(r+1)(r+2)$ (5)

$= \frac{1}{2}\left\{\frac{2r+1-(2r-1)}{(2r-1)(2r+1)}\right\} + \frac{4}{3}r(r+1)\{r+2-(r-1)\}$ (5)

$= \frac{1}{(2r-1)(2r+1)} + 4r(r+1)$ (5) $\boxed{15}$

$= U_r$

$U_r = V_r - V_{r+1}$

$r=1;\ U_1 = V_1 - \cancel{V_2}$ (5)

$r=2;\ U_2 = \cancel{V_2} - \cancel{V_3}$

$\vdots$

$r=n-1;\ U_{n-1} = \cancel{V_{n-1}} - \cancel{V_n}$ (5)

$r=n;\ U_n = \cancel{V_n} - V_{n+1}$

$\sum_{r=1}^{n} U_r = V_1 - V_{n+1}$ (5)

$= \frac{1}{2}$ (5) $- \frac{1}{2(2n+1)} + \frac{4}{3}n(n+1)(n+2)$ (5) $\boxed{25}$

$\lim_{n\to\infty} \sum_{r=1}^{n} U_r = \lim_{n\to\infty}\left\{\frac{1}{2} - \frac{1}{2(2n+1)} + \frac{4}{3}n(n+1)(n+2)\right\}$

$= \lim_{n\to\infty}\left[\frac{1}{2} - \frac{\frac{1}{n}}{2(2+\frac{1}{n})} + \frac{4}{3}n^3(1+\frac{1}{n})(1+\frac{2}{n})\right] = \infty$ (5)

Infinite series $\sum_{r=1}^{\infty} U_r$ does not converge (5) $\boxed{10}$

$W_r = \{r(r+2)\}^{(-1)^r}$

$\sum_{r=1}^{2n} W_r = \sum_{r=1}^{n}(W_{2r-1} + W_{2r})$ (5)

$= \sum_{r=1}^{n}\left\{\frac{1}{(2r-1)(2r+1)} + 2r(2r+2)\right\}$ (5)

$= \sum_{r=1}^{n} U_r$ (5)

$= \frac{1}{2} - \frac{1}{2(2n+1)} + \frac{4}{3}n(n+1)(n+2)$ (5) $\boxed{20}$

13. (a) $A = \begin{pmatrix} a & 0 \\ 2 & -2 \\ b & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 5 \\ 4 & -1 \\ 2 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$

$A^T B = C$

$\begin{pmatrix} a & 2 & b \\ 0 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 4 & -1 \\ 2 & 1 \end{pmatrix}$ (5) $= \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$

$\begin{pmatrix} a+2b+8 & 5a+b-2 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$ (10)

(5) $a + 2b + 8 = 15$
(5) $-2 = c$
(5) $5a + b - 2 = 6$ $\Bigg\}$ $a = 1$, $b = 3$, $c = -2$

$C = \begin{pmatrix} 15 & 6 \\ -2 & 5 \end{pmatrix}$

$C^{-1} = \frac{1}{87}\begin{pmatrix} 5 & -6 \\ 2 & 15 \end{pmatrix}$ (5)

$C(P + 2I) = 3C + I$

$C^{-1}C(P+2I) = 3C^{-1}C + C^{-1}I$ (5)

$P + 2I = 3I + C^{-1}$

$P = I + C^{-1}$ (5)

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{87}\begin{pmatrix} 5 & -6 \\ 2 & 15 \end{pmatrix}$

$= \frac{1}{87}\begin{pmatrix} 92 & -6 \\ 2 & 102 \end{pmatrix}$ (5) $\boxed{50}$

(b) $|z-2i|^2$

$= (z-2i)\overline{(z-2i)}$

$= (z-2i)(\bar{z}+2i)$ (5)

$= z\bar{z}+2iz-2i\bar{z}-4i^2$ (5)

$= |z|^2+2i(z-\bar{z})+4$

$= |z|^2+2i(2i\,Im\,z)+4$ (5)

$= |z|^2-4\,Im(z)+4$ (5) $\boxed{20}$

$|1+2iz|^2$

$= (1+2iz)\overline{(1+2iz)}$

$= (1+2iz)(1-2i\bar{z})$ (5)

$= 1-2i\bar{z}+2iz-4i^2z\bar{z}$ (5)

$= 1+2i(z-\bar{z})+4|z|^2$

$= 1+2i(2i\,Im\,z)+4|z|^2$ (5)

$= 1-4\,Im(z)+4|z|^2$ (5) $\boxed{20}$

$\left|\frac{1+2iz}{z-2i}\right| = 1$

$\Leftrightarrow |1+2iz| = |z-2i|$ (5)

$\Leftrightarrow |1+2iz|^2 = |z-2i|^2$

$\Leftrightarrow 1-4\,Im(z)+4|z|^2 = |z|^2-4\,Im(z)+4$ (5)

$\Leftrightarrow |z|^2 = 1$ (5)

$\Leftrightarrow |z| = 1$ (5) $\boxed{20}$

$\left|\frac{1+2iz}{z-2i}\right| = 1$ and $Arg(2iz) = \pi/6$

$\Rightarrow |z| = 1$ and $\arg z + \arg(2i) = \frac{\pi}{6}$ (5)

$\Rightarrow |z| = 1$ and $\arg z + \pi/2 = \pi/6$ (5)

$\Rightarrow |z| = 1$ and $\arg z = -\pi/3$ (5)

$z = 1(\cos(-\pi/3)+i\sin(-\pi/3))$ (5) $\boxed{20}$

$\sqrt{6}+\sqrt{2}i$

$= 2\sqrt{2}\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)$ (5)

$= 2\sqrt{2}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ (5) $\boxed{10}$

$(\sqrt{6}+\sqrt{2}i)^6 = (2\sqrt{2})^6\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^6$

$= 512\left(\cos 6\frac{\pi}{6}+i\sin 6\frac{\pi}{6}\right)$ (5)

$= 512\,(-1)$ (5)

$= -512.$ $\boxed{10}$

14. (a) $f(x) = \frac{x-2}{(x-1)^2}$

$f'(x) = \frac{(x-1)^2(1)-(x-2)2(x-1)}{(x-1)^4}$ (10)

$= \frac{x-1-2(x-2)}{(x-1)^3}$ (5)

$= \frac{3-x}{(x-1)^3}$ $\boxed{20}$

$f'(x) = 0 \Leftrightarrow x = 3$ (5)

	$-\infty < x < 1$	$1 < x < 3$	$3 < x < \infty$
Sign of $f'(x)$	(−)	(+)	(−)
$f(x)$ is	Decreasing	Increasing	Decreasing
	(5)	(5)	(5)

Turning point $(3, \frac{1}{4})$ is a local maximum (5)

$f(x)$ is increasing on $(1,3]$ and decreasing on $(-\infty,1)$ and $[3,\infty)$ (5) $\boxed{30}$

$f''(x) = 0 \Leftrightarrow x = 4$ (5)

	$-\infty < x < 1$	$1 < x < 4$	$4 < x < \infty$
Sign of $f''(x)$	(−)	(−)	(+)
concavity	Concave down	Concave down	Concave up
		(5)	(5)

$(4, \frac{2}{9})$ is a inflection point $\boxed{15}$

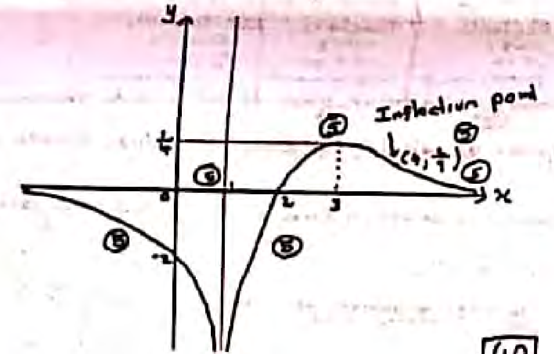
As $x \to \pm\infty$, $y \to 0$

$y = 0$ is a horizontal asymptote (5)

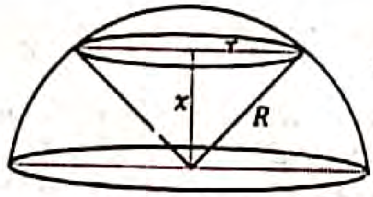
$x = 1$ is a vertical asymptote (5)

When $x = 0$, $y = -2$

when $y = 0$, $x = 2$



$\boxed{40}$



$r = \sqrt{R^2-x^2}$ (5)

$V = \frac{1}{3}\pi(\sqrt{R^2-x^2})^2x$ (5)

$= \frac{1}{3}\pi(R^2x-x^3)$

$\frac{dV}{dx} = \frac{1}{3}\pi(R^2-3x^2)$ (5)

$= -\pi\left(x^2-\frac{R^2}{3}\right)$

$\frac{dV}{dx} = 0 \Leftrightarrow x = \frac{R}{\sqrt{3}}$ (5)

For $0 < x < \frac{R}{\sqrt{3}}$, $\frac{dV}{dx} > 0$ and For $\frac{R}{\sqrt{3}} < x < R$, $\frac{dV}{dx} < 0$ (5) (5)

17. (a) $\frac{\sin x\cos 3x}{\sin 3x\cos x}$

$= \frac{2\sin x\cos 3x}{2\sin 3x\cos x}$

$= \frac{\sin 4x + \sin(-2x)}{\sin 4x + \sin(2x)}$ (10)

$= \frac{2\sin 2x\cos 2x - \sin 2x}{2\sin 2x\cos 2x + \sin 2x}$ (5)

$= \frac{2\cos 2x - 1}{2\cos 2x + 1}$ (5) [20]

Put $x = 15^\circ$ (5)

$\frac{\sin 15^\circ \cos 45^\circ}{\sin 45^\circ \cos 15^\circ} = \frac{2\cos 30^\circ - 1}{2\cos 30^\circ + 1}$ (5)

$\tan 15^\circ = \frac{\sqrt{3}-1}{\sqrt{3}+1}$ (5)

$= \frac{(\sqrt{3}-1)^2}{3-1}$ (5)

$= 2-\sqrt{3}$ (5) [25]

$y = \frac{2\cos 2x - 1}{2\cos 2x + 1}$

$\Rightarrow 2y\cos 2x + y = 2\cos 2x - 1$

$\Rightarrow 2(1-y)\cos 2x = y+1$

$\Rightarrow \cos 2x = \frac{y+1}{2(1-y)}$ (5)

but $|\cos 2x| \le 1$ (5)

$\therefore \left|\frac{y+1}{2(1-y)}\right| \le 1$

$\Rightarrow (y+1)^2 \le 4(1-y)^2$ (5)

$\Rightarrow y^2+2y+1 \le 4(1-2y+y^2)$

$\Rightarrow 3y^2 - 10y + 3 \ge 0$ (5)

$\Rightarrow (3y-1)(y-3) \ge 0$ (5)

$\Rightarrow y \le \frac{1}{3}$ or $y \ge 3$ (5)

$\therefore \frac{\sin x\cos 3x}{\sin 3x\cos x}$ does not lie between $\frac{1}{3}$ and 3 (5) [35]

(b) $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ (5) [5]

Let $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$

$\frac{\sin A + \sin B}{\sin C} = \frac{ka+kb}{kc} = \frac{a+b}{c}$ (5)

$\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\frac{C}{2}\cos\frac{C}{2}}$ (10) $= \frac{2c}{c}$ (5) $[\because a+b=2c]$

$\frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}} = 2$ (5)

$\cos 45^\circ = 2\sin\frac{C}{2}$ (5)

$\sin\frac{C}{2} = \frac{1}{2\sqrt{2}}$ (5) [35]

(c) $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}(2\sin^2 x)$

Let $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$, $\beta = \tan^{-1}\left(\frac{1}{3}\right)$

$\alpha + \beta = \tan^{-1}(2\sin^2 x)$

$\Rightarrow \tan(\alpha+\beta) = 2\sin^2 x$ (5)

$\Rightarrow \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = 2\sin^2 x$ (5)

$\Rightarrow \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = 2\sin^2 x$ (5)

$\Rightarrow \sin^2 x = \frac{1}{2}$

$\Rightarrow \sin x = \pm\frac{1}{\sqrt{2}}$ (5)

$\Rightarrow \sin x = \sin\left(\pm\frac{\pi}{4}\right)$ (5)

$\Rightarrow x = n\pi + (-1)^n\left(\pm\frac{\pi}{4}\right)$; $n \in \mathbb{Z}$ (5)

$\Rightarrow x = n\pi \pm \frac{\pi}{4}$; $n \in \mathbb{Z}$ [30]

17 a) I Bayes Law. (10)

A - A [illegible]

B - B " "

C - C " "

$P(A) = \frac{50}{100}$, $P(B) = \frac{30}{100}$, $P(C) = \frac{20}{100}$ (5)

D - [illegible]

$P(D/A) = \frac{3}{100}$, $P(D/B) = \frac{2}{100}$, $P(D/C) = \frac{1}{100}$ (5)

i) $P(D) = P(A).P(D/A) + P(B)P(D/B) + P(C).P(D/C)$ (10)

$= \frac{50}{100}\cdot\frac{3}{100}$ (5) $+ \frac{30}{100}\cdot\frac{2}{100}$ (5) $+ \frac{20}{100}\cdot\frac{1}{100}$ (5)

$= \frac{230}{100\times100}$

$= 0.023$ (5)

ii) $P(A/D) = \frac{P(A).P(D/A)}{P(D)}$ (5)

$= \dfrac{\frac{50}{100}\cdot\frac{3}{100}}{\frac{230}{100\times100}}$ (5) (5)

$= \frac{150}{230}$

$= \frac{15}{23}$ (5)

$\boxed{70}$

17 b)

Range	x (5)	f	d (5)	fd (5)	fd² (5)
10 - 20	15	5	-3	-15	45
20 - 30	25	12	-2	-24	48
30 - 40	35	x	-1	-x	x
40 - 50	$\boxed{45}$	20	0	0	0
50 - 60	55	y	1	y	y
60 - 70	65	10	2	20	40
70 - 80	75	4	3	12	36
		100		y-x-7 (5)	129+x+y=218 (5)

x+y=49 (5)

Median 44

$44 = 40 + \frac{10}{20}\left(\frac{100}{2} - (17+x)\right)$ (10)

$4 = \frac{1}{2}(33-x)$

$8 = 33 - x$

$x = 25$ (5)

$\therefore y = 24$ (5)

$\boxed{55}$

$\bar{x} = 45 + 10\frac{\sum fd}{100}$ (5)

$= 45 + 10\times\frac{(y-x-7)}{100}$

$= 45 + \frac{24-25-7}{10}$

$= 45 - \frac{8}{10}$

$= 44.2$ (5)

$s^2 = 10^2\left[\frac{\sum fd^2}{100} - \left(\frac{\sum fd}{100}\right)^2\right]$ (5)

$= 10^2\left[\frac{218}{100} - \left(\frac{-8}{100}\right)^2\right]$ (5)

$= 10^2[2.18 - 0.0064]$

$s = 14.6$ (5)

$\boxed{25}$

10 (a)(I) (i) A ⇒ anode – Titanium / Ti

75

B ⇒ cathode – Nickel / Ni

$2\times(03+03) = 12$

(ii) At A / anode : $2Cl^-_{(aq)} \rightarrow Cl_{2(g)} + 2e$

At B / cathode: $2H_2O_{(l)} + 2e \rightarrow 2OH^-_{(aq)} + H_{2(g)}$

$2\times 04 = 08$

C ⇒ $Cl_{2(g)}$ / chlorine

D ⇒ $H_{2(g)}$ / hydroge

$2\times 04 = 08$

(iii) P ⇒ concentrated NaCl / (brine) solution

Q ⇒ used (salt) solution

R ⇒ water

S ⇒ NaOH solution

$4\times 03 = 12$

(iv) Overall reaction $2NaCl_{(aq)} + 2H_2O_{(l)} \rightarrow 2NaOH_{(aq)} + Cl_{2(g)} + H_{2(g)}$

– – – – 04

(v) E – selective membrane – – – 04

Permeable only to + ions and ∴ allows migration of Na^+ – – – 04

(vi) sea water – – – 03

(vii) Mg^{2+}, Ca^{2+} and SO_4^{2-} – – – $3\times 01 = 03$

(ix) Uses :- use of NaOH – Production of soap

· as a strong base

· production of paper

– – – any one – –

$3\times 01 = 03$

Use of by-products

H_2 – manufacture of HCl

production of margarine

production of NH_3

– – – any one –

use of Cl_2 – . purifying drinking water

$\therefore$ V is minimum when $x = \frac{R}{\sqrt{3}}$ (5)

$V_{max} = \frac{2\pi R^3}{9\sqrt{3}}$ (5)

$= \frac{1}{3\sqrt{3}}\left(\frac{2}{3}\pi R^3\right)$

$= \frac{1}{3\sqrt{3}}$ (volume of the semi sphere) (5)

$\boxed{45}$

15. (a) $3x^2+7x \equiv A(x^2+4x+5)+(x-1)(Bx+5)$

$x^2;\ 3 = A+B$ (5)
$x^0;\ 0 = 5A-5$ (5)
$A=1,\ B=2$ (10)

$$\frac{3x^2+7x}{(x-1)(x^2+4x+5)} = \frac{x^2+4x+5+(x-1)(2x+5)}{(x-1)(x^2+4x+5)}$$

$$= \frac{1}{x-1} + \frac{2x+5}{x^2+4x+5} \quad (10)$$

$$\int\frac{3x^2+7x}{(x-1)(x^2+4x+5)}dx = \int\frac{1}{x-1}dx + \int\frac{2x+5}{x^2+4x+5}dx \quad (5)$$

$$= \int\frac{1}{x-1}dx + \int\frac{2x+4}{x^2+4x+5}dx + \int\frac{1}{(x+2)^2+1}dx \quad (5)$$

$$= \ln|x-1| + \ln(x^2+4x+5) + \tan^{-1}(x+2) + c$$
(5) (5) (5) (5)

$\boxed{60}$

(b) $\int x^3(\ln x)^2 dx$

$= (\ln x)^2\frac{x^4}{4} - \int\frac{x^4}{4}\,2(\ln x)\frac{1}{x}dx$ (5) (10)

$= \frac{1}{4}x^4(\ln x)^2 - \frac{1}{2}\int x^3\ln x\,dx$

$= \frac{1}{4}x^4(\ln x)^2 - \frac{1}{2}\left\{\ln x\,\frac{x^4}{4} - \int\frac{x^4}{4}\frac{1}{x}dx\right\}$ (5) (10)

$= \frac{1}{4}x^4(\ln x)^2 - \frac{1}{8}x^4\ln x + \frac{1}{32}x^4 + c$ (5) (5)

$\boxed{40}$

(c) $I = \int_0^{\pi/2}\cos^4 x\sin^2 x\,dx$

$= \int_0^{\pi/2}\cos^4\left(\frac{\pi}{2}-x\right)\sin^2\left(\frac{\pi}{2}-x\right)dx$ (5)

$= \int_0^{\pi/2}\sin^4 x\cos^2 x\,dx = J$ (5)

$I + J = \int_0^{\pi/2}(\cos^4 x\sin^2 x + \cos^2 x\sin^4 x)\,dx$ (5)

$2I = \int_0^{\pi/2}\cos^2 x\sin^2 x(\cos^2 x+\sin^2 x)\,dx$ (5)

$I = \frac{1}{2}\int_0^{\pi/2}\cos^2 x\sin^2 x\,dx$ (5)

$= \frac{1}{8}\int_0^{\pi/2}\sin^2 2x\,dx$ (5)

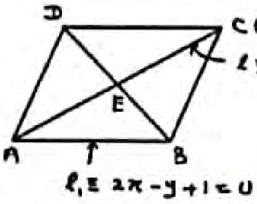
$= \frac{1}{16}\int_0^{\pi/2}(1-\cos 4x)\,dx$ (5)

$= \frac{1}{16}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi/2}$ (5) (5)

$= \frac{\pi}{32}$ (5)

$\boxed{50}$

16.



(i) $A \equiv (0,1)$ (5)

$E \equiv \left(\frac{0+2}{2}, \frac{1-3}{2}\right) \equiv (1,-1)$ (5) $\boxed{10}$

(ii) $m_{l_2} = \frac{1}{2}$ (5)

Equation of l_2 is $y+1 = \frac{1}{2}(x-1)$ (5)

$l_2 \equiv x-2y-3=0$ (5) $\boxed{15}$

(iii) $2x+y-1=0$

$x = \frac{y-1}{-2} = t$ (say) (5)

$x = t,\ y = 1-2t$

$P \equiv (t, 1-2t)$ (5)

$0<x<1 \Rightarrow 0<t<1$ (5) $\boxed{15}$

(iv) $\frac{|2(t)-(1-2t)+1|}{\sqrt{5}} = \frac{|t-2(1-2t)-3|}{\sqrt{5}}$ (10) (10)

$|4t| = |5t-5|$ (5)

$4t = \pm(5t-5)$ (5)

$t = \frac{5}{9}$ or $t = 5$ (5)

$0<t<1 \Rightarrow t = \frac{5}{9}$ (5) $\boxed{40}$

(v) $P \equiv \left(\frac{5}{9}, -\frac{1}{9}\right)$ (5)

radius $= \frac{\left|2\left(\frac{5}{9}\right)+\frac{1}{9}+1\right|}{\sqrt{5}} = \frac{4\sqrt{5}}{9}$ (5) (5)

Equation of S is

$\left(x-\frac{5}{9}\right)^2 + \left(y+\frac{1}{9}\right)^2 = \left(\frac{4\sqrt{5}}{9}\right)^2$ (10)

$x^2+y^2-\frac{10}{9}x+\frac{2}{9}y+\frac{25}{81}+\frac{1}{81} = \frac{80}{81}$ (5)

$9x^2+9y^2-10x+2y-6=0$ (5) $\boxed{35}$

(vi) $B \equiv \left(-\frac{5}{3}, -\frac{7}{3}\right)$ (5) $A \equiv (0,1)$

Equation of S' is

$(x-0)\left(x+\frac{5}{3}\right) + (y-1)\left(y+\frac{7}{3}\right) = 0$ (10)

$x^2+y^2+\frac{5}{3}x+\frac{4}{3}y-\frac{7}{3}=0$ (5) $\boxed{20}$

(vii) $2g_1g_2 + 2f_1f_2$

$= 2\left(-\frac{5}{9}\right)\left(\frac{5}{6}\right) + 2\left(\frac{1}{9}\right)\left(\frac{4}{6}\right)$ (5)

$= -\frac{7}{9}$

$c_1 + c_2 = -\frac{6}{9} - \frac{7}{3} = -3$ (5)

$\therefore 2g_1g_2 + 2f_1f_2 \neq c_1 + c_2$ (5)

$\therefore$ S and S' do not intersect orthogonally.

$\boxed{15}$

13)

$T = 2mg$

$\frac{\lambda}{x} \cdot 2x = 2mg$

$\lambda = mg$ (5)

05

This string and mass satisfy the above condition $\Rightarrow \lambda = mg$ (5)

$\downarrow \; 2mg - T = 2m\ddot{x}$ (10)

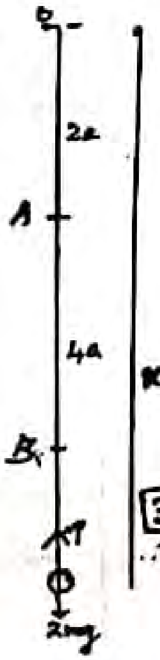
$2mg - \frac{mg}{2a}(x-2a) = 2\ddot{x}$ (5)

$\ddot{x} = -\frac{g}{4a}(x-6a)$ (5)

$X = x - 6a$

$\ddot{X} = \ddot{x}$ (5)

$\ddot{X} = -\frac{g}{4a} X$



This S.H.M

centre $X = 0$, $x = 6a$ (B) (5)

35

$\dot{X}^2 = w^2(b^2 - X^2)$

$2\dot{X}\ddot{X} = w^2(-2X\dot{X})$ (5)

$\ddot{X} = -w^2 X$

$w = \sqrt{\frac{g}{4a}}$ (5)

$X = 0 \Rightarrow \dot{x} = 3\sqrt{ag}$ (5)

$9ag = \frac{g}{4a}(b^2 - 0)$

20

$b = 6a$ (5) (amplitude)

I Using conservation of energy

$\frac{1}{2}2m\dot{y}^2 - 2mgy + \frac{1}{2}\frac{mg}{2a}(y-2a)^2 = \frac{1}{2}m\frac{32ag}{4} - 2mg3a + \frac{1}{2}\frac{mg}{2a}(3a)^2$ (20)

$\dot{y}^2 - 2gy + \frac{g}{4a}(y-2a)^2 = 0$ (5)

25

$\dot{y}^2 = \frac{g}{4a}[32a^2 - (y-6a)^2]$

$= \frac{g}{4a}[(4\sqrt{2}a)^2 - Y^2]$ (10)

$\dot{Y}^2 = \frac{g}{4a}[c^2 - Y^2]$

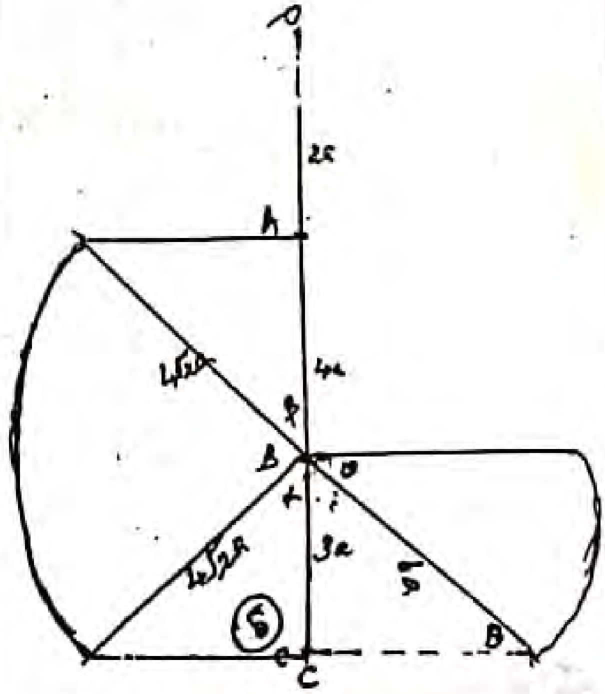
$\dot{Y}^2 = \frac{g}{4a}[c^2 - Y^2]$

$c = 4\sqrt{2}a$, $w_1 = \sqrt{\frac{g}{4a}}$ (5)

$\ddot{Y} = -\frac{g}{4a}Y$ (5)

$Y = 0$ centre $y = 6a$ + amplitude $4\sqrt{2}a$ (5)

25



$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$ (5)

$\cos\alpha = \frac{3}{4\sqrt{2}}$ (5)

$\cos\beta = \frac{1}{\sqrt{2}} \Rightarrow \beta = \frac{\pi}{4}$ (5)

$T_{BC} = \frac{\theta}{w} = \frac{\pi}{6}\sqrt{\frac{4a}{g}}$ (5)

$T_{CA} = \frac{\pi - \beta - \alpha}{w_1}$ (5)

$= (\pi - \frac{\pi}{4} - \alpha)\sqrt{\frac{4a}{g}}$

$= (\frac{3\pi}{4} - \cos^{-1}\frac{3}{4\sqrt{2}})\sqrt{\frac{4a}{g}}$

$\dot{y}_A^2 = \frac{g}{4a}[(4\sqrt{2}a)^2 - (-4a)^2]$

$V_A = 2\sqrt{ag}$ (5)

Motion under gravity

$\uparrow \; v^2 = u^2 + 2as$

$v^2 = 4ag - 2gh$

$h = 2a \Rightarrow v = 0$

$\uparrow \; v = u + at$ (5)

$t = \frac{2\sqrt{ag}}{g} = 2\sqrt{\frac{a}{g}}$

40

Total time $= \frac{\pi}{6}\sqrt{\frac{4a}{g}} + (\frac{3\pi}{4} - \cos^{-1}\frac{3}{4\sqrt{2}})\sqrt{\frac{4a}{g}} + 2\sqrt{\frac{a}{g}}$

$= 2\sqrt{\frac{a}{g}}[\frac{11\pi}{12} - \cos^{-1}(\frac{3}{4\sqrt{2}}) + 1]$

① $\downarrow$ $v^2 = u^2 + 2as$
$u^2 = 2gh$ ⑤

$\Delta E = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$\frac{1}{4}mgh = \frac{1}{2}m \cdot 2gh - \frac{1}{2}mv^2$ ⑩

$gh = 4gh - 2v^2$

$v^2 = \frac{3gh}{2}$

$v = eu$ ⑤

$\sqrt{\frac{3gh}{2}} = e\sqrt{2gh}$

$e = \frac{\sqrt{3}}{2}$ ⑤ $\boxed{25}$

② $\uparrow$ $-h = u\sin\theta t - \frac{1}{2}gt^2$ ⑤

$\rightarrow$ $x = u\cos\theta . t$ ⑤

$gx^2\tan^2\theta - 2u^2x\tan\theta + gx^2 - 2u^2h = 0$ ⑤

for real value of $\tan\theta$

$\Delta \geq 0$ ⑤

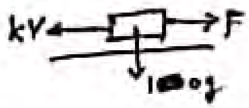
$\Rightarrow$ $4u^4x^2 - 4gx^2(gx^2 - 2u^2h) \geq 0$

$x^2 \leq \frac{u^4 + 2ghu^2}{g^2}$ ⑤

$x \leq \frac{u}{g}\sqrt{u^2 + 2gh}$

$\therefore x_{max} = \frac{u}{g}\sqrt{u^2 + 2gh}$ ⑤ $\boxed{25}$

③



$P = F.V$

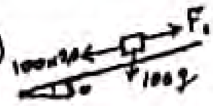
$40 \times 1000 = F.20$

$F = 2000$ ⑤

$\rightarrow$ $F - kV = 0$

$2000 - k.20 = 0$

$k = 100$ ⑤



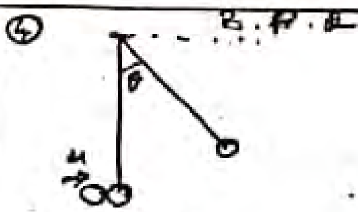
$80 \times 1000 = F_1 . 20$

$F_1 = 4000$ ⑤

$\nearrow$ $4000 - 2000 - 100g\sin 30 = 100f$ ⑤

$f = 15\ ms^{-2}$ ⑤ $\boxed{25}$

④



$mu = 2mv$

$v = \frac{u}{2}$ ⑤

$u = 2\sqrt{2ag}$

$\frac{1}{2}.2mv^2 - 2mga = \frac{1}{2}m.0 - 2mga\cos\theta$ ⑩

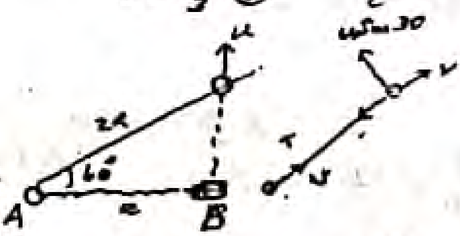
$\frac{u^2}{8} - 2ag + 2ag\cos\theta = 0$

$\frac{8ag}{8} - 2ag + 2ag\cos\theta = 0$

$\cos\theta = \frac{1}{2}$ ⑤

$\theta = \frac{\pi}{3}$ ⑤ $\boxed{25}$

⑤



for $B \nearrow$ $-T = 2mv - 2mu\cos 30$ ⑤

for $A \nearrow$ $T = mv$ ⑤

$\Rightarrow$ $3v = 2u\cos 30$

$v = \frac{u}{\sqrt{3}}$ ⑤

$w^2 = v^2 + \frac{u^2}{4}$

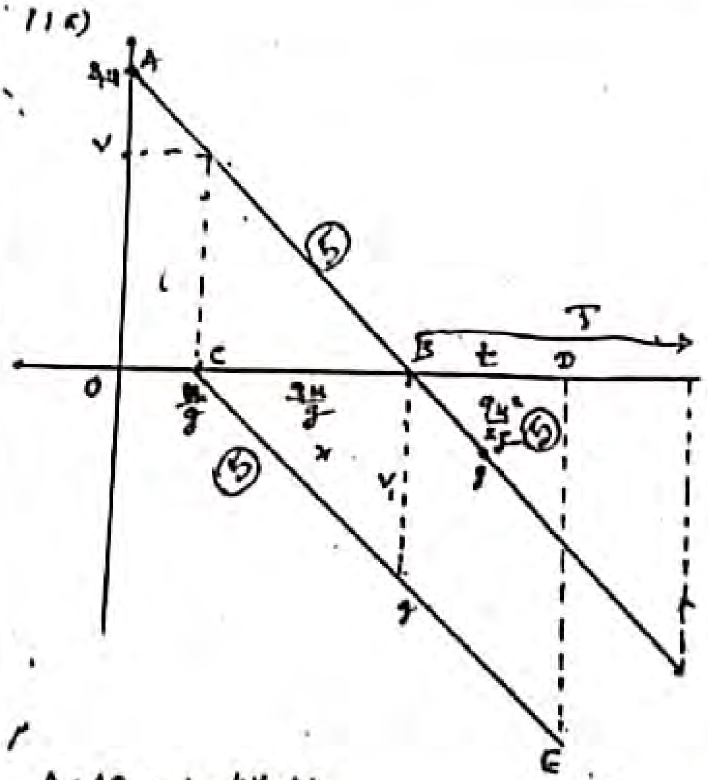
$= \frac{u^2}{3} + \frac{u^2}{4}$

$w = \frac{u}{2}\sqrt{\frac{7}{3}}$ ⑤

$\tan\alpha = \frac{\frac{u}{2}}{v} = \frac{\frac{u}{2}}{\frac{u}{\sqrt{3}}} = \frac{\sqrt{3}}{2}$

$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ ⑤ $\boxed{25}$

11 a)



$\Delta OAB = \frac{1}{2}\cdot\frac{4u}{g}\cdot 4u$

$= \frac{16u^2}{2g} = \frac{8u^2}{g}$

$\frac{9u^2}{2g} = \frac{1}{2}gt^2$

$t = \frac{3u}{g}$

I $\quad \frac{4u-v}{\frac{u}{g}} = \frac{v}{\frac{3u}{g}}$ 5

$v = 3u$ 5

II $\quad v_1 = v = 3u$ 5

$x = \frac{1}{2}\cdot 3u\cdot\frac{3u}{g} = \frac{9u^2}{2g}$ 5

III $\quad h = \Delta CDE$

$= \frac{1}{2}g\cdot\left(\frac{3u}{g}+t\right)^2$ 5

$= \frac{1}{2}g\left(\frac{3u}{g}+\frac{3u}{g}\right)^2 = \frac{36u^2}{2g}$ 5

$= \frac{18u^2}{g}$ 5

IV) $\quad \frac{1}{2}gT^2 = h + \frac{8u^2}{g}$ 5

$= \frac{18u^2}{g} + \frac{8u^2}{g}$ 5

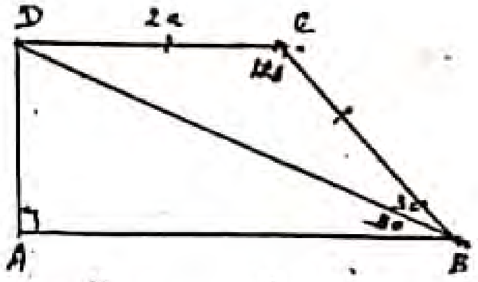
$T = 2\sqrt{13}\frac{u}{g}$ 5

Req Time $= \frac{4u}{g} + 2\sqrt{13}\frac{u}{g}$

$= \frac{2u}{g}(2+\sqrt{13})$ 5

$\boxed{70}$

b)



$V_{RE} \quad \xrightarrow{u}$

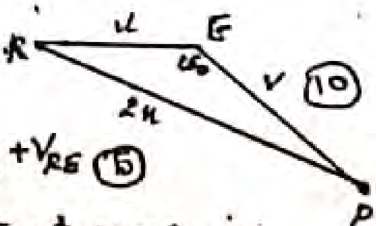
$V_{PR} \quad 2u$

$V_{QR} \quad \sqrt{5}u$

V_{PE} 5

$V_{QE} \quad \xleftarrow[\sqrt{5}u-u]{}$

for P



$V_{PE} = V_{PR} + V_{RE}$ 5

$4u^2 = u^2 + v^2 - 2uv\cos 150$ 5

$v^2 - \sqrt{3}uv - 3u^2 = 0$

$v = \frac{\sqrt{3}}{2}u(\sqrt{5}+1)$ 5

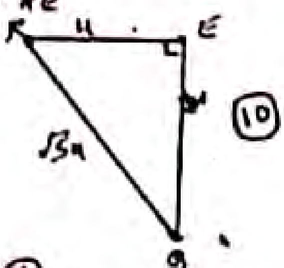
$T_{BD} = \frac{2\sqrt{3}a}{v}$

$= \frac{4a}{u(\sqrt{5}+1)}$ 5

for Q $\quad T_{BA} = \frac{3a}{u(\sqrt{3}-1)}$ 5 + 5

$A \longrightarrow D$

$V_{QE} = V_{QR} + V_{RE}$



$V = 2u$ 5

$T_{AD} = \frac{\sqrt{3}a}{2u}$ 5

$T_Q = T_{BA} + T_{AD}$

$= \frac{3a}{u(\sqrt{3}-1)} + \frac{\sqrt{3}a}{2u}$ 5

$T_Q - T_P$ 5

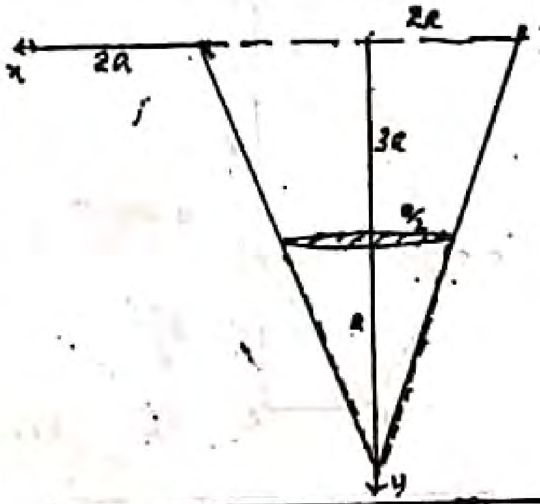
$= \frac{3a}{u(\sqrt{3}-1)} + \frac{\sqrt{3}a}{2u} - \frac{4a}{u(\sqrt{5}+1)}$

$= \frac{a}{2u}[14+4\sqrt{3}-2\sqrt{5}] > 0 \Rightarrow T_Q > T_P$ 5

$\boxed{80}$

(16) Theory (40)

$\boxed{40}$



Body	Mass	$\bar{x}$	$\bar{y}$
Large Cone	(5) $\pi \cdot 2a \cdot 2a \cdot 2\sqrt{3} a\rho = 16M$	0	$4a/3$ (5)
Small Cone	(5) $\pi \frac{a}{2} \frac{\sqrt{3}}{2} a\rho = M$	0	$10a/3$ (5)
Cir. Plat	$\frac{\sqrt{3}}{4} \pi a^2 \rho = M$	0	$3a$ (5)
Rod	(5) $\frac{\sqrt{3}}{2} \pi a^2 \rho = 2M$	$3a$ (5)	0 (5)
Tea Spoon	(5) $18M$ =	$\bar{x}$	$\bar{y}$

$18M\bar{x} = 2M \cdot 3a$ (10)

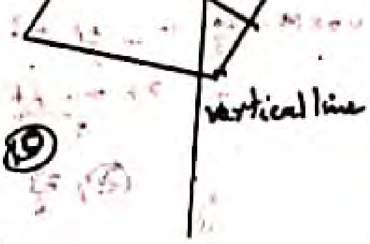
$\bar{x} = \frac{a}{3}$ (5)

$18M\bar{y} = 16M \cdot \frac{4a}{3} - M \cdot \frac{10a}{3} + M \cdot 3a$ (15)

$= \frac{64 - 10 + 9}{3} \cdot Ma$

$\bar{y} = \frac{7a}{6}$ (5)

$\boxed{80}$



(10)

$\tan\theta = \frac{\bar{y}}{4a - \bar{x}}$ (10)

$= \frac{7a/6}{4a - \frac{a}{3}}$

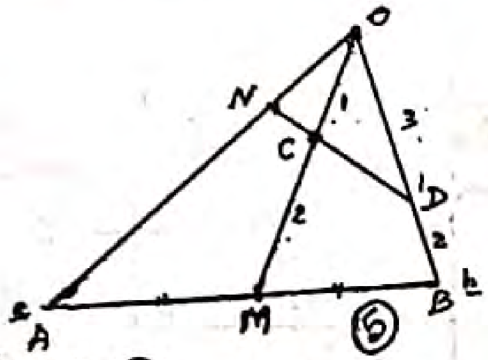
$= \frac{7a}{6} \cdot \frac{3}{11a}$ (5)

$= \frac{7}{22}$

$\theta = \tan^{-1}\left(\frac{7}{22}\right)$ (5)

$\boxed{30}$

14(a)



$\vec{OM} = \vec{OA} + \vec{AM}$ (5)

$= \underline{a} + \frac{1}{2}\vec{AB}$

$= \underline{a} + \frac{1}{2}(\underline{b} - \underline{a})$

$= \frac{\underline{a}+\underline{b}}{2}$ (5)

$\vec{OC} = \frac{1}{3}\vec{OM}$ (5)

$= \frac{1}{6}(\underline{a}+\underline{b})$

$\vec{OD} = \frac{3}{5}\vec{OB} = \frac{3}{5}\underline{b}$ (5)

$\vec{ON} = \lambda\vec{OA} = \lambda\underline{a}$

$\vec{NC} = \mu\vec{CD}$ (5)

$= \mu(\vec{CO} + \vec{OD})$

$= \mu[-\frac{1}{6}(\underline{a}+\underline{b}) + \frac{3}{5}\underline{b}]$

$= \mu[-\frac{1}{6}\underline{a} + \frac{13}{30}\underline{b}]$ (5) —— ①

$\vec{NC} = \vec{NO} + \vec{OC}$ (5)

$= -\lambda\underline{a} + \frac{1}{6}(\underline{a}+\underline{b})$

$= (\frac{1}{6}-\lambda)\underline{a} + \frac{1}{6}\underline{b}$ (5) —— ②

①+② ⇒ $(\frac{1}{6}-\lambda)\underline{a} + \frac{1}{6}\underline{b} = \mu[-\frac{1}{6}\underline{a} + \frac{13}{30}\underline{b}]$ (5)

$\frac{1}{6} - \lambda = -\frac{\mu}{6}$ & $\frac{13}{30}\mu = \frac{1}{6}$ (5)

$\mu = \frac{5}{13}$ (5)

$\lambda = \frac{3}{13}$ (5)

$ON = \frac{3}{13}OA$

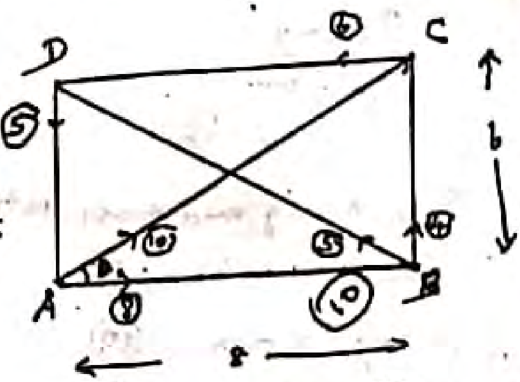
$ON : NA = 3:10$ (5)

$NC = \frac{5}{13}CD$

$NC : CD = 5:13$ (5)

$\boxed{75}$

14(b)



$\cos\theta = \frac{4}{5}$

$\rightarrow X = 8 - 6 + 10\cos\theta - 5\cos\theta$ (5)

$= 2 + 5\times\frac{4}{5}$

$= 6$ (5)

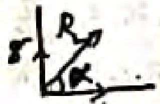
$\uparrow Y = 4 - 5 + 10\sin\theta + 5\sin\theta$ (5)

$= 8$ (5)

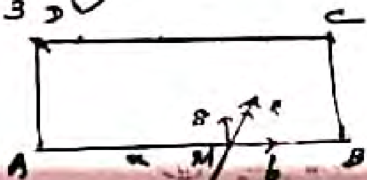
$R^2 = 8^2 + 6^2$

$R = 10$ (5)

$\tan\alpha = \frac{4}{3}$ (5)



$\boxed{40}$



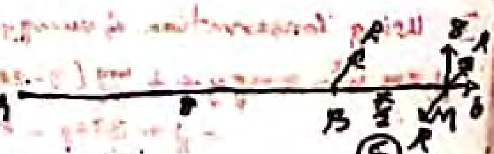
A↺ $8x = 4\cdot 8 + 6\cdot 6 + 5\cdot 8\sin\theta$ (10)

$= 32 + 36 + 24$

$= 92$ (5)

$x = \frac{92}{8} = 11\frac{1}{2}$ (5)

$\boxed{20}$

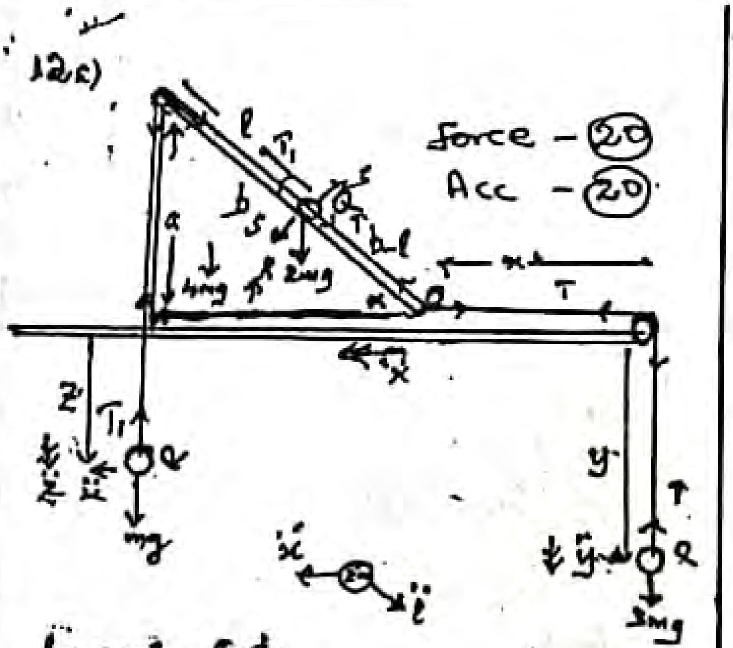


$G↺ = 8\cdot\frac{7}{2}$ (5)

$= 28$ Nm (5)

$\boxed{15}$

12(a)



Force – 20
Acc – 20

$l + a + z =$ Const.

$\ddot{l} + 0 + \ddot{z} = 0$ —— (1)

$b - l + x + y =$ Const

$0 - \ddot{l} + \ddot{x} + \ddot{y} = 0$ —— (2)

for m ↓ $mg - T_1 = m\ddot{z}$ —— (3) (10)

for 3m ↓ $3mg - T = 3m\ddot{y}$ —— (4) (10)

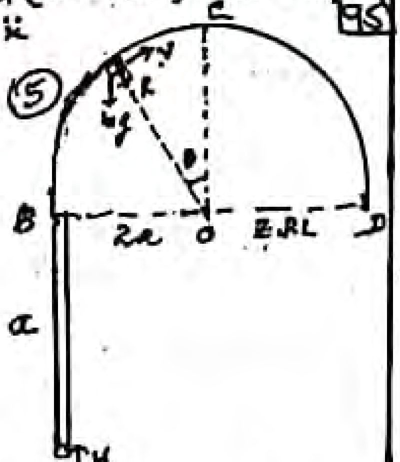
for 2m ↘ $2mg\sin\alpha + T - T_1 = 2m(\ddot{l} - \ddot{x}\cos\alpha)$ – (5) (15)

for wedge + 2m + m

← $-T = 4m\ddot{x} + 2m(\ddot{x} - \ddot{l}\cos\alpha) + m\ddot{x}$ (20) – (6)

$\boxed{95}$

12(b)



Conservation of energy

$\frac{1}{2}mv^2 + mg\cdot 2a\cos\theta = \frac{1}{2}mu^2 - mga$ (15)

$v^2 = u^2 - 2ag - 4ag\cos\theta$ (5)

↘ $R + mg\cos\theta = m\frac{v^2}{2a}$; $u = 2\sqrt{2ag}$ (10)

$R = \frac{m}{2a}[u^2 - 2ag - 6ag\cos\theta]$ (5)

$= 3mg(1 - \cos\theta)$ (5)

$R = 0 \Rightarrow \theta = 0$ (at C) (5)

$\theta = 0 \Rightarrow v^2 = 2ag > 0$

∴ particle continues moving on the surface. (5)

$\boxed{55}$

$OC = 24\underline{b} - 3\underline{c}$

$\overrightarrow{AB} = 15\underline{b} - 5\underline{c}$

$= 5(3\underline{b} - \underline{c})$ (5)

$\overrightarrow{AC} = 24\underline{b} - 3\underline{c} - 5\underline{c}$

$= 24\underline{b} - 8\underline{c}$

$= 8(3\underline{b} - \underline{c})$ (5)

$= 8 \cdot \frac{1}{5}\overrightarrow{AB}$ (5)

$\Rightarrow AC // AB \Rightarrow A, B, C$ collinear (5)

$\frac{AC}{AB} = \frac{8}{5} \Rightarrow AB : BC = 5 : 3$ (5)

$\boxed{25}$

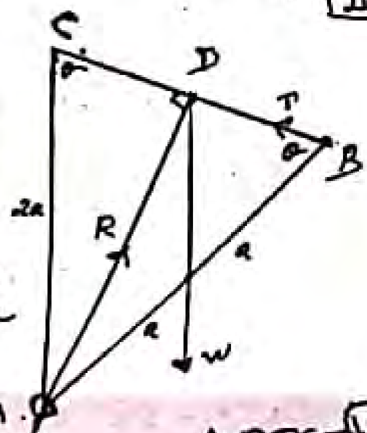
7)

AD//R

DC//T

CA//W

∴ ADC triangle of Force



$\frac{R}{AD} = \frac{T}{DC} = \frac{W}{CA}$ ΔADC – (10)

$\frac{R}{2a\sin\theta} = \frac{T}{2a\cos\theta} = \frac{W}{2a}$ (5)

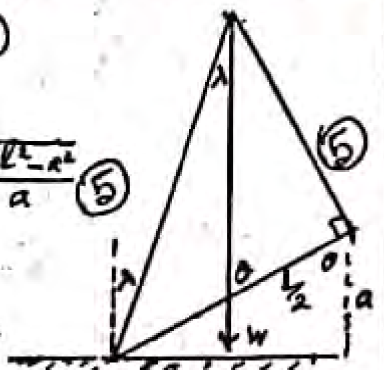
$\boxed{25}$

$R = W\sin\theta$ (5)

$T = W\cos\theta$ (5)

8)

$\tan\theta = \frac{\sqrt{l^2 - a^2}}{a}$ (5)



$2\cot\theta = 1\cot\lambda - 1\cot(90 - \theta)$ (10)

$\frac{2}{\tan\theta} = \frac{1}{\mu} - \tan\theta$

$\frac{1}{\mu} = \frac{2}{\tan\theta} + \tan\theta$ (5)

$\mu = \frac{a\sqrt{l^2 - a^2}}{l^2 + a^2}$

$\boxed{25}$

B = {Houses with Computer}

$P(A) = \frac{20}{30}$ (5)

$P(B) = \frac{15}{30}$ (5)

$P(A \cap B) = \frac{10}{30}$ (5)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (5)

$= \frac{20}{30} + \frac{15}{30} - \frac{10}{30}$

$= \frac{5}{6}$ (5)

10) Let m, n are the numbers of Males and females respectively

Totals marks of females = 62m (5)

" " of males = 52n (5)

Total marks of the class = 62m + 52n

$62m + 52n = (m+n)60$ (10)

$2m = 8n$

$m : n = 4 : 1$ (5)

$\boxed{25}$