SCIENCE EAGLE
விஞ்ஞானக் கழுகு

இலங்கையின் உயர்தர கணித விஞ்ஞான
பிரிவிற்கான இணையதளம்

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01. $f(n) = 7^n$

$f(1) = 7 = 6\times1+1$ 5

$f(p) = 7^p = 6k+1,\ k\in\mathbb{Z}^+$ 5

$f(p+1) = 7(6k+1)$ 5

$= 6(7k+1)+1,\ 7k+1\in\mathbb{Z}^+$ 5

Conclusion 5

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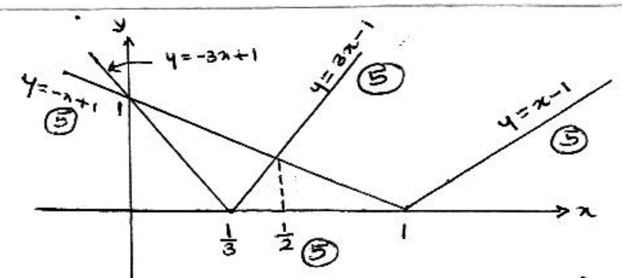
02.



Required solution $\left\{x \mid 0<x<\frac{1}{2}\right\}$ 5

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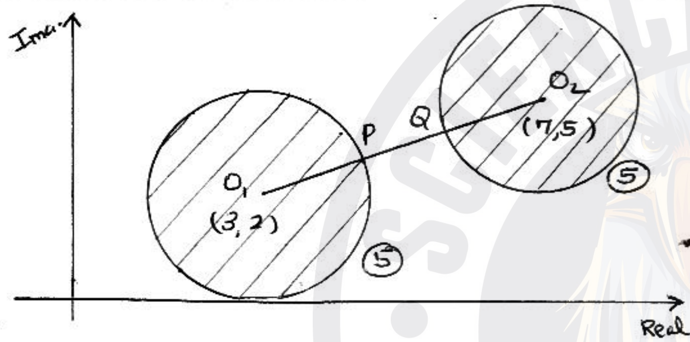
03.



$|z-\omega|_{min} = PQ$ 5

$= 5-3$ 5

$= 2$ 5

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04. $f(x) = k(x^2-4kx+4k^2)+k^2-2$ 5

$= k(x-2k)^2+k^2-2$ 5

$= a(x-b)^2+c$; where $a=k,\ b=2k$

$c=k^2-2$ 5

$k>0$ and $f(x)|_{min} = k^2-2>2$ 5

$\therefore k>2$ 5

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05. $\lim\limits_{x\to0} \dfrac{1+\sin4x-(\sin^2x+\cos^2x)}{x\{\sqrt{1+\sin4x-\sin^2x}+\cos x\}}$ 10

$= 4\lim\limits_{4x\to0}\dfrac{\sin4x}{4x}\cdot\lim\limits_{x\to0}\dfrac{1}{\{\sqrt{1+\sin4x-\sin^2x+\cos x}\}}$ 5 5

$= 4\cdot1\cdot\dfrac{1}{\{\sqrt{1}+1\}}$ 5

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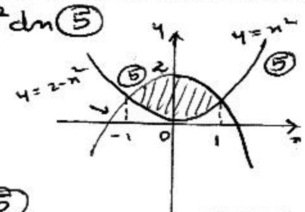
06. $A = \int_{-1}^{1}(2-x^2)\,dx - \int_{-1}^{1}x^2\,dx$ 5



$= \left\{2x-\dfrac{2x^3}{3}\right\}_{-1}^{1}$ 5

$= \left(2-\dfrac{2}{3}\right)-\left(-2+\dfrac{2}{3}\right)$ 5

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07. $\dfrac{dx}{d\theta} = 3a\sec^3\theta\tan\theta$ 5

$\dfrac{dy}{d\theta} = 3a\tan^2\theta\sec^2\theta$ 5

$\dfrac{dy}{dx} = \dfrac{\tan\theta}{\sec\theta}$ 5

Equation of the tangent

$y-a\tan^3\alpha = \dfrac{\tan\alpha}{\sec\alpha}(x-a\sec^3\alpha)$ 5

$x\tan\alpha - y\sec\alpha = a\sec\alpha\tan\alpha$ 5

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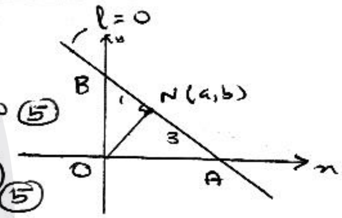
08. $M_{AB} = -a/b$ 5



$l \equiv ax+by-(a^2+b^2)=0$ 5

$A \equiv \left(\dfrac{a^2+b^2}{a},0\right),\ B\equiv\left(0,\dfrac{a^2+b^2}{b}\right)$ 5

$AN:NB = 3:1$ 5

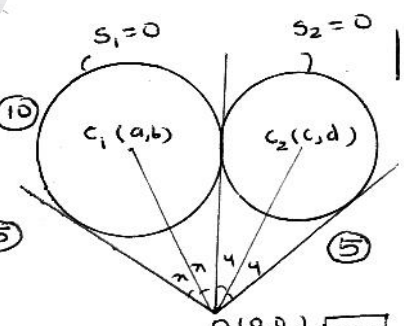
$a = \dfrac{3(0)+1\left(\frac{a^2+b^2}{a}\right)}{4}$ 5

25

09. $x+y=45^\circ$ 5

$\tan C_1\hat{O}C_2 = \left|\dfrac{b/a-d/c}{1+b/a\cdot d/c}\right|$ 10

$|bc-ad| = |ac+bd|$ 5



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10. $\cot A = -3/4$ 5

$\cos A = -3/5$ 5

$\sin A = 4/5$ 5

$2\cot A - 5\cos A + \sin A$

$= 2(-3/4)-5(-3/5)+4/5$ 5

$= \dfrac{23}{10}$ 5

25

11. (a) (i) $P(-2)=0$ (5) $P(2)=-48$ (5)

$2a-b=5$ (5) $2a+b=-1$ (5)

$\Rightarrow a=1,\ b=-3$ (10)

(ii) $P(x) = (x+2)(x-6)(x+1)$ (10)

$\boxed{40}$

(b) $y = \dfrac{x^2+34x-71}{x^2+2x-7}$ (5)

$(y-1)x^2+2(y-17)x-7y+71=0$ (5)

If $y=1$, $x=2$ (5)

Let $y \neq 1$, For all real values of x $\Delta \geq 0$ (5)

$\Rightarrow (y-5)(y-9) \geq 0$ (15)

$y \leq 5$ or $y \geq 9$ (5)

$\boxed{40}$

(c) (i) $\alpha+\beta=b,\ \alpha\beta=c$ (10)

$\dfrac{\alpha^2}{\beta}+\dfrac{\beta^2}{\alpha} = \dfrac{b(b^2-3c)}{c}$ (15)

$\dfrac{\alpha^2}{\beta}\cdot\dfrac{\beta^2}{\alpha} = c$ (10)

$\therefore$ Required equation $cx^2-b(b^2-3c)x+c^2=0$ (5)

(ii) $\alpha+\alpha^2=b,\ \alpha^2-b\alpha=-c$ (10)

$\dfrac{b+c}{b+1}+\left(\dfrac{b+c}{b+1}\right)^2 = b$ (10)

$\Rightarrow b^3 = c(3b+c+1)$ (10)

$\boxed{70}$

12. (a) (i) ${}^9C_3 = 84$ (15)

(ii) ${}^5C_3\cdot{}^4C_3 + {}^5C_3\cdot{}^3C_3 + {}^4C_3\cdot{}^3C_3 +$
${}^5C_3\cdot{}^4C_2\cdot{}^3C_1 + {}^5C_3\cdot{}^4C_1\cdot{}^3C_2 +$
${}^5C_2\cdot{}^4C_3\cdot{}^3C_1 + {}^5C_1\cdot{}^4C_3\cdot{}^3C_2 + {}^5C_1\cdot{}^4C_2\cdot{}^3C_3$
${}^5C_2\cdot{}^4C_1\cdot{}^3C_3 + {}^5C_2\cdot{}^4C_2\cdot{}^3C_2 = 784$ (60)

$\boxed{75}$

(b) If n is even Let $n=2m$

$S = 1^3+3\cdot 2^2+3^3+\cdots+(2m-1)^3+3(2m)^2$ (5)

$= (1^3+3^3+\cdots+(2m-1)^3)+3(2^2+4^2+\cdots(2m)^2)$ (5)

$= \sum_{r=1}^{m}(2r-1)^3 + 3\cdot 4\sum_{r=1}^{m} r^2$ (10)

$= 8\sum_{r=1}^{m} r^3 + 6\sum_{r=1}^{m} r - \sum_{r=1}^{m} 1$ (10)

$= 8\,\dfrac{m^2(m+1)^2}{4} + 6\,\dfrac{m(m+1)}{2} - m$ (15)

$= \dfrac{n}{\ }(n^3+4n^2+10n+8)$ (10)

If n is odd, $(n+1)$ is even

$S_n = S_{n+1} - T_{n+1}$ (10)

$= \dfrac{(n+1)}{8}\left[(n+1)^3+4(n+1)^2+10(n+1)+8\right]$

$-3(n+1)^2$ (10)

$\boxed{75}$

13. (i) (a) $|z_1| = \sqrt{26}$ (5) $|z_2| = \sqrt{13}$ (5)

$\therefore |z_1|^2 = 2|z_2|^2$ (5)

(b) $z_1\cdot z_2 = -13+13i$ (5)

$= 13\sqrt{3}\left(\cos 3\pi/4 + i\sin 3\pi/4\right)$ (10)

$\therefore \arg(z_1\cdot z_2) = 3\pi/4$ (5)

$\boxed{35}$

(ii) Let $\sqrt{16-30i} = x+iy$ (5)

$16-30i = x^2-y^2+2xyi$ (5)

$\Rightarrow x = -15/y$ (5)

$x^2+y^2 = 16$ (5)

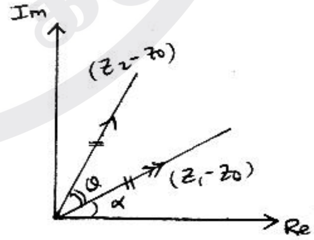
$\Rightarrow y = \pm 3$ (5)

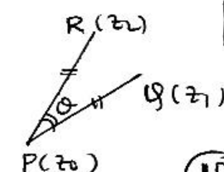
$\Rightarrow x = \mp 5$ (10)

$\therefore \sqrt{16-30i} = -5+3i,\ 5-3i$ (5)

$\boxed{40}$

(iii)





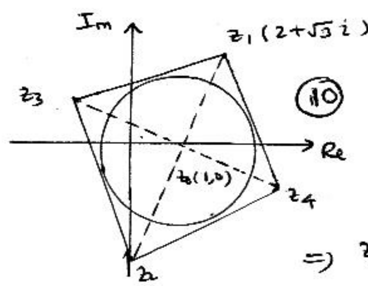
(10)

Let $z_1-z_0 = r(\cos\alpha + i\sin\alpha)$

$(z_2-z_0) = r\left[\cos(\theta+\alpha)+i\sin(\theta+\alpha)\right]$ (5)

$= (z_1-z_0)(\cos\theta+i\sin\theta)$ (10)

$\boxed{25}$



(10)

$z_2 = -\sqrt{3}i$ (10)

$(z_3-z_0) = (z_1-z_0)(\cos\pi/2 + i\sin\pi/2)$ (10)

$\Rightarrow z_3 = 1-\sqrt{3}i$ (5)

$(z_4-z_0) = (z_2-z_0)(\cos\pi/2+i\sin\pi/2)$ (10)

$\Rightarrow z_4 = (1+\sqrt{3})-i$ (5)

$\boxed{50}$

14.(a) $\frac{dy}{dx} = e^{\tan^{-1}x}\cdot\frac{1}{(1+x^2)} + e^{-\tan^{-1}x}\left(\frac{-1}{1+x^2}\right)$ 10

$(1+x^2)\frac{dy}{dx} = e^{\tan^{-1}x} - e^{-\tan^{-1}x}$ 5

$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot 2x = e^{\tan^{-1}x}\cdot\frac{1}{1+x^2} - e^{-\tan^{-1}x}\left(\frac{-1}{1+x^2}\right)$ 10

$\boxed{25}$

(b) $f'(x) = \frac{(x^2-5x+4) - x(2x-5)}{(x^2-5x+4)^2}$ 10

At the turning point $f'(x) = 0$ 5

$\Rightarrow x = 2$ or $x = -2$ 5

$x = -2-\delta \Rightarrow \frac{dy}{dx} < 0$ minimum point

$x = -2+\delta \Rightarrow \frac{dy}{dx} > 0$ 5 $(-2, -1/9)$ 5

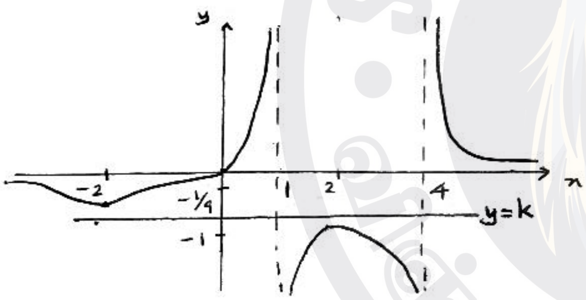
$x = 2-\delta \Rightarrow \frac{dy}{dx} > 0$ maximum point

$x = 2+\delta \Rightarrow \frac{dy}{dx} < 0$ 5 $(2, -1)$ 5

$x = 1-\delta \Rightarrow y \to \infty$ 5 $x = 4-\delta \Rightarrow y \to -\infty$

$x = 1+\delta \Rightarrow y \to -\infty$ $x = 4+\delta \Rightarrow y \to \infty$ 5

$x \to \pm\infty \Rightarrow y \to 0$ 5



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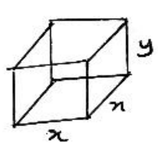
$\frac{x}{(x-1)(x-4)} = k$ 5

If $-1 < k < -1/9$ both graphs will not intersect. 5

$\boxed{75}$

(c)



$x^2 + 4xy = a^2$ 5

$V = x^2 y$ 5

$= \frac{1}{4}\{a^2x - x^3\}$ 5

$\frac{dV}{dx} = \frac{1}{4}\{a^2 - 3x^2\}$ 5

For maximum or minimum $\frac{dV}{dx} = 0$ 5

$\Rightarrow x = a/\sqrt{3}$ 5 ($\because x \neq -a/\sqrt{3}$ not valid) 5

$x = a/\sqrt{3} - \delta \Rightarrow \frac{dV}{dx} > 0$ 5 At $x = a/\sqrt{3}$ V_{max} 5

$x = a/\sqrt{3} + \delta \Rightarrow \frac{dV}{dx} < 0$

$\Rightarrow V_{max} = \frac{a^3}{6\sqrt{3}}$ 5

$\boxed{50}$

15.(a) To show $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ 15

$I = \int_0^{\pi/2} \sin^4(\pi/2 - x)\cos^2(\pi/2 - x)\,dx$ 5

$= \int_0^{\pi/2} \cos^4 x \sin^2 x\,dx$ 5

$2I = I + I$

$= \int_0^{\pi/2} \sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)\,dx$ 5

$= \int_0^{\pi/2} \sin^2 x \cos^2 x\,dx$ 5

$= \frac{1}{4}\int_0^{\pi/2} \sin^2 2x\,dx$ 5

$= \frac{1}{4}\int_0^{\pi/2}\left(\frac{1-\cos 4x}{2}\right)dx$ 5

$= \frac{1}{8}\left\{x - \frac{\sin 4x}{4}\right\}_0^{\pi/2}$ 5

$= 1/8\left\{\pi/2 - \frac{\sin 2\pi}{4}\right\}$ 5

$\boxed{55}$

(b) $t = x^2$ 5

$\Rightarrow \int t^2 \cos t\,\frac{dt}{2}$ 5

$= \frac{1}{2}\left\{t^2\sin t - \int \sin t\cdot 2t\,dt\right\}$ 15

$= \frac{1}{2}\ t^2 \sin t - \left\{t(-\cos t) - \int -\cos t\cdot dt\right\}$ 10

$= \frac{1}{2}\ x^4\sin x^2 + x^2\cos x^2 - \sin x^2 + C$ 10

$\boxed{45}$

(c) $\frac{x^2+2}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4}$ 10

$\Rightarrow A = 0,\ B = \frac{1}{3},\ C = 0,\ D = \frac{2}{3}$ 20

$\int \frac{x^2+2}{(x^2+1)(x^2+4)}\,dx = \int \frac{1/3}{(x^2+1)}\,dx +$ 5

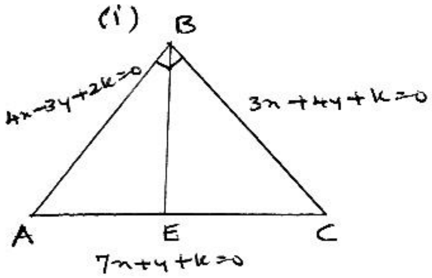
$\int \frac{2/3}{(x^2+4)}\,dx$

$= \frac{1}{3}\tan^{-1}x + \frac{2}{3}\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$ 15

$\boxed{50}$

16. (a) To prove that $\dfrac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \dfrac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}}$ (20)

(i)



$M_{AB} = 4/3$

$M_{BC} = -3/4$ (5)

$M_{AB} \times M_{BC} = -1$ (5)

(ii) Equations of the bisectors

$$\frac{4x-3y+2k}{5} = \pm \frac{3x+4y+k}{5} \quad (10)$$

$x-7y+k=0$, $7x+y+3k=0$ (10)

(iii) Bisector of $A\hat{B}C$ $x-7y+k=0$ (5)

$E \equiv (-4, 3)$ (10) $k = 25$ (5)

(iv) $D \equiv (3,4)$ (5)

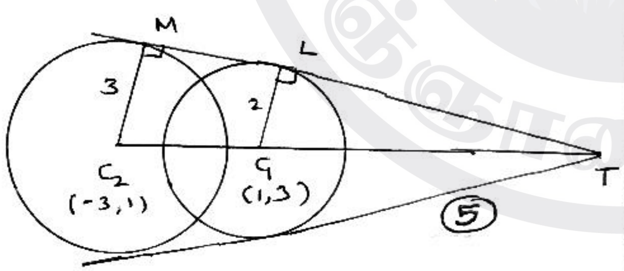
$\boxed{75}$

(b) $C_1 \equiv (1,3)$ $C_2 \equiv (-3,1)$

$r_1 = 2$ $r_2 = 3$ (5)

$C_1C_2 = \sqrt{20}$ (5)

$3-2 < \sqrt{20} < 3+2$ (5)



(5)

$\dfrac{TC_1}{TC_2} = \dfrac{2}{3}$ (10) $T \equiv (9,7)$ (10)

Let m be the gradient of tangent.

Equation of the tangent

$mx-y+7-9m=0$ (10)

Perpendicular distance = radius

$$\frac{|m(1)-3+7-9m|}{\sqrt{m^2+1}} = 2 \quad (10)$$

$15m^2-16m+3=0$ (10)

Equations $(8+\sqrt{19})x-15y+33-9\sqrt{19}=0$ (5)

$(8-\sqrt{19})x-15y+33+9\sqrt{19}=0$

17. (a) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$ (15)

$$\frac{\tan 3A}{\tan A} = \frac{3-\tan^2 A}{1-3\tan^2 A} = k \quad (5)$$

$$\Rightarrow \tan^2 A = \frac{k-3}{3k-1} \quad (5)$$

$$\frac{\sin 3A}{\sin A} = \frac{3\sin A - 4\sin^3 A}{\sin A} \quad (5)$$

$$= 3 - \frac{4}{1+\cot^2 A} \quad (10)$$

$$= \frac{2k}{k-1} \quad (5)$$

$\boxed{45}$

(b) $\sin\alpha = x$, $\tan\beta = x$

$\cos(\alpha+\beta) = \cos \pi/2$ (5)

$$\sqrt{1-x^2}\cdot\frac{1}{\sqrt{1+x^2}} = x \cdot \frac{x}{\sqrt{1+x^2}} \quad (10)$$

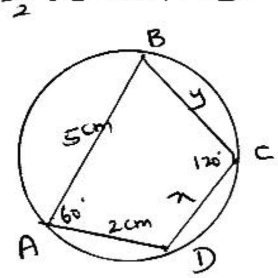
$x^4+x^2-1=0$ (5)

$$x^2 = \frac{-1+\sqrt{5}}{2} \quad (10)$$

$\boxed{30}$

(c) State cosine rule (5)

Prove Area $= \frac{1}{2}bc\sin A$ (10)



$\triangle ABD = \dfrac{5\sqrt{3}}{2}$ (10)

$\triangle BCD = \dfrac{3\sqrt{3}}{2}$ (5)

$\triangle BCD = \dfrac{\sqrt{3}}{4}xy$ (5)

$\therefore xy = 6$ (5)

In $\triangle ABD$ cos rule

$$\frac{1}{2} = \frac{2^2+5^2-BD^2}{2\cdot 2\cdot 5} \quad (10)$$

$BD^2 = 19$ (5)

In $\triangle BCD$ $-\dfrac{1}{2} = \dfrac{x^2+y^2-19}{2xy}$ (5) $\Rightarrow x^2+y^2=13$ (5)

$\Rightarrow$ $x=3$, $y=2$

OR (10)

$x=2$, $y=3$

$\boxed{75}$

01. For 1st particle $t_1 = \frac{2u\sin\theta}{g}$ (5)

$R = u\cos\theta\, t$ (5)

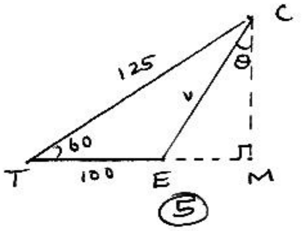
For 2nd particle $t_2 = \frac{2u\sin\alpha}{g}$, $R = u\cos\alpha\, t_2$ (5)

$\frac{g^2 t_1^2}{4u^2} + \frac{R^2}{u^2 t_1^2} = 1$ & $\frac{g^2 t_2^2}{4u^2} + \frac{R^2}{u^2 t_2^2} = 1$ (5)

$\Rightarrow R = \frac{1}{2} g\, t_1 t_2$ (5)

$\boxed{25}$

02. $V_{CE} = V_{CT} + V_{TE} = \underset{30°}{\overset{125}{\nearrow}} + \overset{100}{\longrightarrow}$ (5)



(5)

$v^2 = 125^2 + 100^2 - 2\cdot125\cdot100\cos60$ (5)

$v = 25\sqrt{21}$ (5)

$\tan\theta = -\frac{\sqrt3}{5}$, $\theta = 50E$ (5)

$\boxed{25}$

03.

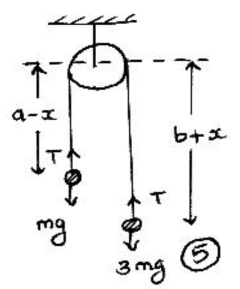
$m\cdot3u + mu = mu + mku$ (10)

$k = 3$ (5)

$3u - u = e(3u - u)$ (10)

$\boxed{25}$

04.



(5)

$-3mg\,b - mg\,a =$

$-mg(a-x) - 3mg(b+x)$

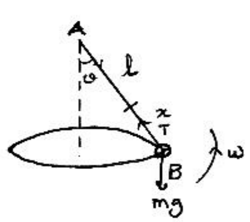
$+ \frac{1}{2}m\dot{x}^2 + \frac{1}{2}3m\dot{x}^2$ (10)

$\Rightarrow \ddot{x} = \frac{g}{2}$ (5)

$T = \frac{3mg}{2}$ (5)

$\boxed{25}$

05.



$3mg = \frac{2mg}{l}x$ (5)

$x = \frac{3l}{2}$ (5)

$\underline{F} = m\underline{a}$

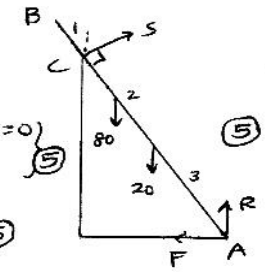
$\leftarrow \quad T\sin\theta = m\frac{5l}{2}\sin\theta\, \omega^2$ (10)

$\Rightarrow \omega = \sqrt{\frac{6g}{5l}}$ (5)

$\boxed{25}$

06. $\rightarrow S\cos30 - F = 0$

$\uparrow R + S\sin30 - 80 - 20 = 0$ (5)



(5)

A) $S\cdot5 - 20\cdot3\cos60 - 80\cdot4\cos60 = 0$ (5)

$S = 95$

$F = 95\sqrt3/2$, $R = \frac{295}{2}$ (5)

$\mu = F/R = \frac{19\sqrt3}{59}$ (5)

$\boxed{25}$

07. $\underline{a}^2 + \underline{a}\cdot\underline{b} + \underline{a}\cdot\underline{c} = 0$ (5)

$\underline{a}\cdot\underline{b} + \underline{b}^2 + \underline{b}\cdot\underline{c} = 0$ (5)

$\underline{a}\cdot\underline{c} + \underline{b}\cdot\underline{c} + \underline{c}^2 = 0$ (5)

$\Rightarrow 2(\underline{a}\cdot\underline{b} + \underline{b}\cdot\underline{c} + \underline{c}\cdot\underline{a}) = -(a^2 + b^2 + c^2)$ (5)

$= -(1^2 + 2^2 + 3^2)$ (5)

$\boxed{25}$

08. (i) $P(A|B) = \frac{P(A\cap B)}{P(B)}$ (5)

$P(A\cup B) = P(A) + P(B) - P(A|B)\cdot P(B) \le 1$ (5)

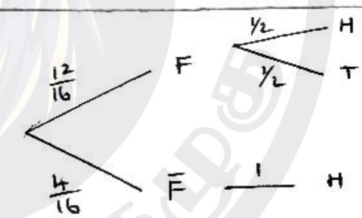
(ii) $P(A\cap B) = P(A)\,P(B)$ (5)

$P(A\cup B) = P(A)\,P(\bar{B}) + 1 - P(\bar{B})$ (5)

$= 1 - P(\bar{B})\,P(\bar{A})$ (5)

$\boxed{25}$

09.



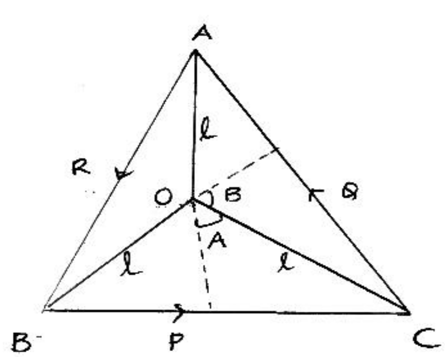
(10)

$P(H) = \frac{12}{16}\cdot\frac{1}{2} + \frac{4}{16}\cdot1$ (10)

$= \frac{5}{8}$ (5)

$\boxed{25}$

10.



(10)

$O \circlearrowleft \quad P\cdot l\cos A + Q\cdot l\cos B + R\cdot l\cos C = 0$ (15)

$\boxed{25}$

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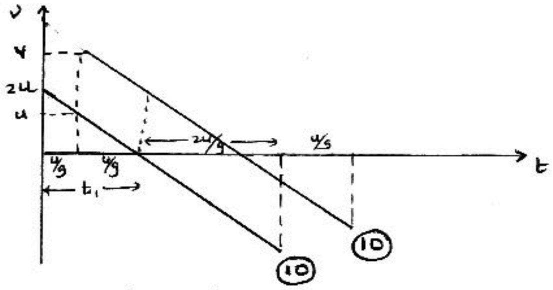
11. (a)



(i) $g = \frac{2u}{t_1}$ 5

(ii) $\frac{1}{2} t_1 \times 2u = \frac{2u^2}{g}$ 10

(iii) (α) $(v-u)\frac{u}{g} = \frac{2u^2}{g} - \frac{1}{2}\frac{u^2}{g}$ 10

$v = \frac{5u}{2}$ 5

(β) Q will hit the ground after time $\left(\frac{5u}{g} - \frac{4u}{g}\right) + \frac{u}{g}$ in which P hits the ground. 10

(γ) Time Q attains the highest point $= \frac{5u}{2g}$ 5

Time P moves at this time $= \frac{u}{g} + \frac{5u}{2g}$ 10

$< \frac{4u}{g}$ 5

80

(b) (i) $\underline{v}_A = -4\underline{i} + 2\underline{j}$ 10

$\underline{v}_B = 6\underline{i} - 4\underline{j}$ 10

(ii) $\underline{r}_A = (-4\underline{i} + 2\underline{j})t + (\underline{i} + 2\underline{j})$ 10

$\underline{r}_B = (6\underline{i} - 4\underline{j})t + (-\underline{i} + \underline{j})$ 10

(iii) $\overrightarrow{BA} = \underline{r}_A - \underline{r}_B = (2-10t)\underline{i} + (1+6t)\underline{j}$ 5

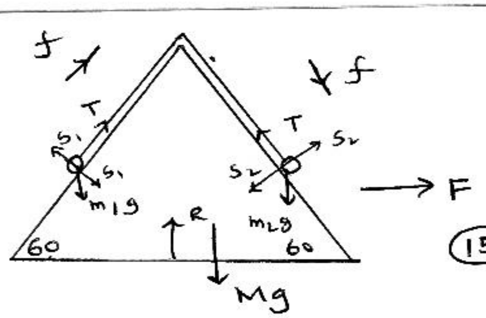
(iv) $AB^2 = (2-10t)^2 + (1+6t)^2$ 10

$= 136\left[\left(t - \frac{7}{68}\right)^2 + \frac{5}{136} - \left(\frac{7}{68}\right)^2\right]$ 10

$AB_{min} \Rightarrow t = \frac{7}{68}$ 5

70

12. (a) (i)



15

$\underline{F} = m\underline{a}$ 15

For the system $0 = MF + m_1(F + f\cos 60) + m_2(F + f\cos 60)$

For m_1 $T - m_1 g\sin 60 = m_1(f + F\cos 60)$ 10

For m_2 $m_2 g\sin 60 - T = m_2(f + F\cos 60)$ 10

$\Rightarrow F = \frac{\sqrt{3}(m_1 - m_2)g}{3m_1 + 3m_2 + 4M}$ 10

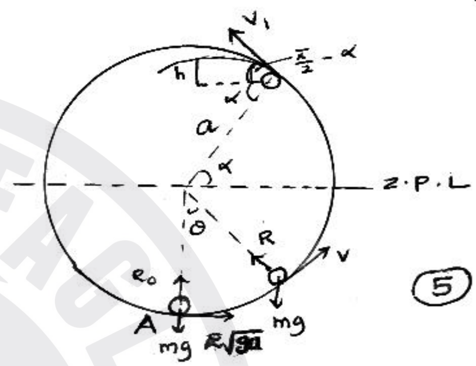
(ii) when $M = 3m$, $m_1 = 2m$ & $m_2 = m$

$F = \frac{\sqrt{3}g}{21}$ 5 $f = -\frac{4\sqrt{3}g}{21}$ 5

$T = \frac{2\sqrt{3}mg}{3}$ 10

80

(b)



5

(i) Principle of conservation of energy 15

$\frac{1}{2}mv^2 - mga\cos\theta = \frac{1}{2}m(4ga) - mga$

$v^2 = 2ga(1+\cos\theta)$ 5

$\underline{F} = m\underline{a}$

$R - mg\cos\theta = m \cdot v^2/a$ 10

$R = m(2 + 3\cos\theta)$ 5

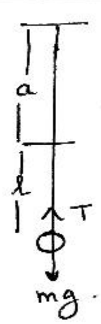
when $\theta = \pi/2 + \alpha$; $R = 0$ 5

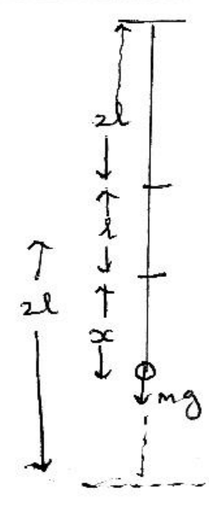
$\Rightarrow \sin\alpha = 2/3$ 5 $a + a\sin\alpha = \frac{5a}{3}$ 5

$\Rightarrow$ Maximum height $= \frac{50a}{27}$ 15

70

13.





$T = mg$

$\frac{2mgl}{a} = mg \Rightarrow a = 2l$ 20

$x = 2l \cos \omega t$

when $x = -l \Rightarrow$ 10 $\cos \omega t = -\frac{1}{2}$ 5

$\omega t = \frac{2\pi}{3}$ 5

$t = \frac{2\pi}{3\omega}$ 10

$\underline{F} = m\underline{a}$

$mg - T = m\ddot{x}$ 15 $\Rightarrow \ddot{x} = -\frac{g}{l}x \longrightarrow *$ 15

$x = A \cos \omega t + B \sin \omega t \longrightarrow (1)$

$\dot{x} = -A\omega \sin \omega t + B\omega \cos \omega t \longrightarrow (2)$ 10

$\ddot{x} = -A\omega^2 \cos \omega t - B\omega^2 \sin \omega t \longrightarrow (3)$ 10

$= -\omega^2 x \longrightarrow (3)$ 10

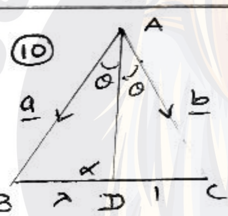
$t = 0, \ x = 2l, \ \dot{x} = 0$ 10

$(1) \Rightarrow A = 2l, \ (2) \Rightarrow B = 0$ 20

$(3) \Rightarrow \omega = \sqrt{g/l}$ 10

$\boxed{150}$

14. (a) (i) $\overrightarrow{BD} = \frac{\lambda}{\lambda+1}(\underline{b} - \underline{a})$ 10



$\overrightarrow{DC} = \frac{1}{\lambda+1}(\underline{b} - \underline{a})$ 10

(ii) $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD} = \frac{\lambda \underline{b} + \underline{a}}{\lambda+1}$ 10

(iii) $\underline{a} \cdot \overrightarrow{AD} = \frac{\lambda \underline{a}\cdot\underline{b} + a^2}{\lambda+1} = a \cdot AD \cos\theta$ 10

$\underline{b} \cdot \overrightarrow{AD} = \frac{\lambda b^2 + \underline{a}\cdot\underline{b}}{\lambda+1} = b \cdot AD \cos\theta$ 10

$\Rightarrow (\lambda b - a)(\underline{a}\cdot\underline{b} - ab) = 0$ 10

$\Rightarrow \frac{a}{b} = \lambda$ 10

$\boxed{70}$

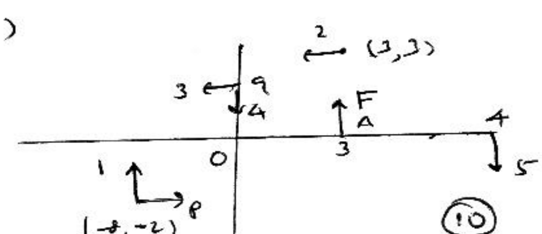
(b) (i) $\underline{R} = (-2\underline{i}) + (-5\underline{j}) + (P\underline{i} + \underline{j}) + (-3\underline{i} - 4\underline{j}) = (P-5)\underline{i} - 8\underline{j}$ 10

(ii) $R = F$ & $F \parallel y$-axis

$\Rightarrow P - 5 = 0$ 10

$P = 5$ 5

(iii)



10

$A \circlearrowleft = 0$ 5

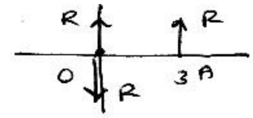
$3 \cdot 4 + 9 \cdot 3 + 3 \cdot 2 - 1 \cdot 5 - 11 \cdot 1 + 2P = 0$ 15

$\Rightarrow q = -4$ 5

(iv) $G \circlearrowleft = R \cdot 3$

$= (-8) \cdot 3$ 10

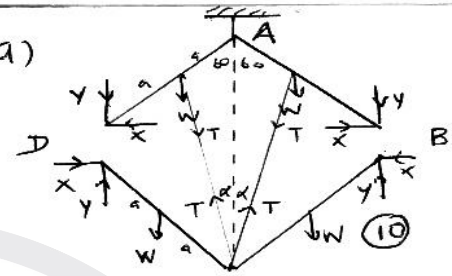
$= -24$



$\therefore G \circlearrowright = 24$ 10

$\boxed{80}$

15. (a)



10

$\alpha = 30°$ 5

$BCD: \uparrow \ 2Y + 2T \cdot \frac{\sqrt{3}}{2} = 2W$ 5

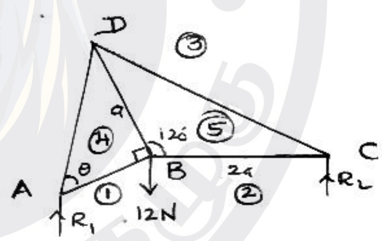
$CB: C \circlearrowleft \ Y \cdot 2a \cdot \frac{\sqrt{3}}{2} + X \cdot 2a \cdot \frac{1}{2} = W \cdot \frac{a\sqrt{3}}{2}$ 10

$AB: A \circlearrowleft \ X \cdot 2a \cdot \frac{1}{2} = Y \cdot 2a \cdot \frac{\sqrt{3}}{2} + W \cdot \frac{a\sqrt{3}}{2} + T \cdot a$ 10

$\Rightarrow T = \sqrt{3} W$ 20

$\boxed{60}$

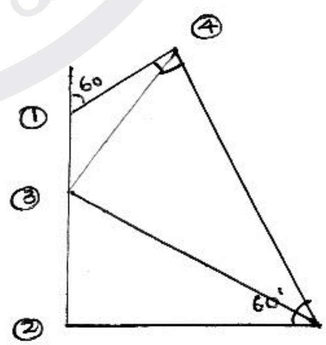
(b)



$R_1 = 6 N$ 5

$R_2 = 6 N$ 5

$\tan\theta = \sqrt{3}/4$

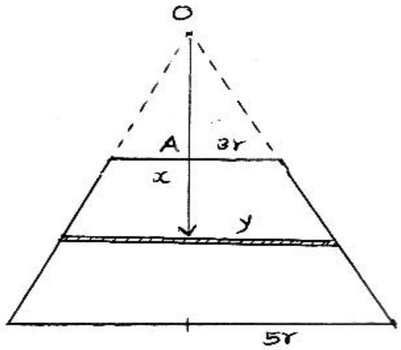


30

Rod	Tension	Thrust	
AB	9	—	10
BC	$10\sqrt{3}$	—	10
CD	—	$4\sqrt{21}$	10
AD	—	$3\sqrt{19}$	10
BD	$11\sqrt{3}$	—	10

$\boxed{90}$

16. (i)



$x = y$ (10)

O) $\bar{x} \int \pi y^2 dx \rho = \int \pi y^2 x \, dx \rho$ (10)

$\bar{x} \int \pi x^2 dx = \int \pi x^3 dx$ (10)

$\bar{x} \, \pi \left[\frac{x^3}{3} \right]_{3r}^{5r} = \pi \left[\frac{x^4}{4} \right]_{3r}^{5r}$ (10)

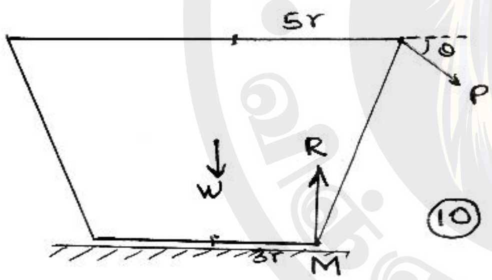
$\bar{x} \frac{\pi}{3} \left(125 r^3 - 27 r^3 \right) = \frac{\pi}{4} \left(625 r^4 - 81 r^4 \right)$ (10)

$\Rightarrow \bar{x} = \frac{204}{49} r$ (10)

The distance of the centre of gravity from A $= \frac{57}{49} r$ (10)

$\boxed{70}$

(ii)



(10)

At the position of toppling

M) = 0 (5)

$P \cos\theta \cdot 2r + P \sin\theta \cdot 2r - W \cdot 3r = 0$ (15)

$P = \frac{3W}{2(\cos\theta + \sin\theta)}$ (10)

$\boxed{40}$

(iii) $P = \frac{3W}{2\sqrt{2}\left(\frac{1}{\sqrt{2}} \cos\theta + \frac{1}{\sqrt{2}} \sin\theta \right)}$ (10)

$= \frac{3W}{2\sqrt{2} \cos(\theta - \pi/4)}$ (10)

$P_{min} = \frac{3W}{2\sqrt{2}}$ (10) when $\theta = \frac{\pi}{4}$ (10)

$\boxed{40}$

17. (a) To show that

$P(A' \cap B) = P(A') \cdot P(B)$

$\boxed{40}$

(b) (i) $\frac{^2C_1}{^9C_1} \cdot \frac{1}{^{10}C_1} = \frac{2}{90}$ (10)

$\frac{^3C_1}{^9C_1} \cdot \frac{^3C_1}{^{10}C_1} = \frac{9}{90}$ (10)

$\frac{^2C_1}{^9C_1} \cdot \frac{^3C_1}{^{10}C_1} = \frac{6}{90}$ (10)

$\frac{1}{^9C_1} \cdot \frac{^2C_1}{^{10}C_1} = \frac{2}{90}$ (10)

Adding $\frac{19}{90}$ (10)

$\boxed{50}$

(ii) $\frac{2}{9} \cdot \frac{2}{10}$

$\frac{1}{9} \cdot \frac{1}{10}$ (15)

Adding $\frac{1}{18}$ (5)

$\boxed{20}$

(iii) $\frac{1}{18}$ (5)

$\frac{2}{9} \cdot \frac{1}{10}$ (5)

$\frac{1}{9} \cdot \frac{2}{10}$ (5)

Adding $\frac{1}{10}$ (5)

$\boxed{20}$

(iv) $\frac{3}{9} \cdot \frac{3}{10}$

$\frac{2}{9} \cdot \frac{3}{10}$ (15)

Adding $\frac{1}{6}$ (5)

$\boxed{20}$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