Grade - 13 (2021) **இணைந்த கணிதம் I** Marking Scheme

1. Let $f(n) = n^3 - n$.

When $n=1$, $f(1) = 1^3 - 1 = 0 = 6 \times 0$

Hence, the result is true for $n=1$. (5)

Take any $p \in \mathbb{Z}^+$ and assume that the result is true for $n=p$. (5)

i.e., $f(p) = p^3 - p = 6k,\ k \in \mathbb{Z}$ ——(*)

when $n = p+1$,

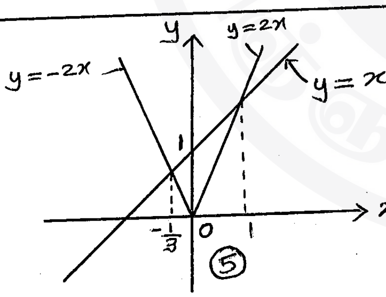
$$f(p+1) = (p+1)^3 - (p+1)$$
$$= p^3 + 3p^2 + 3p + 1 - p - 1$$
$$= p^3 - p + 3p(p+1)$$
(5) $$= 6k + 3(2k') \quad [\because \text{by } (*) \text{ and } p(p+1) - \text{even}]$$
$$= 6(k+k'),\ k+k' \in \mathbb{Z} \quad (5)$$

$\therefore$ the result is true for $n = p+1$

Hence, by the principle of mathematical induction, the result is true for all $n \in \mathbb{Z}^+$ (5)

$\boxed{25}$

2.



(5) (5)

$$\frac{x+1}{|x|} > 2$$

$\Leftrightarrow x+1 > 2|x|$ and $x \neq 0$ (5)

$\Leftrightarrow -\frac{1}{3} < x < 1$ and $x \neq 0$

$\Leftrightarrow -\frac{1}{3} < x < 0$ or $0 < x < 1$

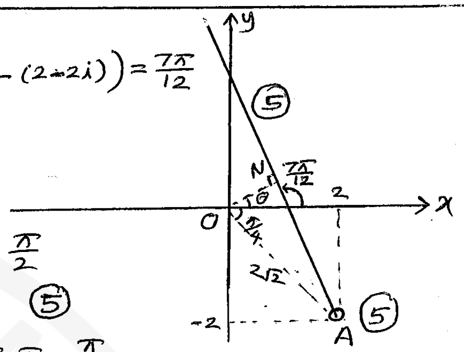
$$\frac{x+2}{|x|} > 2 \Leftrightarrow \frac{\frac{x}{2}+1}{\left|\frac{x}{2}\right|} > 2 \quad (5)$$

$\Leftrightarrow -\frac{1}{3} < \frac{x}{2} < 0$ or $0 < \frac{x}{2} < 1$

(5) $\Leftrightarrow -\frac{2}{3} < x < 0$ or $0 < x < 2$

$\boxed{25}$

3. $\operatorname{Arg}(z - (2+2i)) = \frac{7\pi}{12}$ (5)



(5)

$\theta = \frac{7\pi}{12} - \frac{\pi}{2}$

$= \frac{\pi}{12}$ (5)

$\theta + \frac{\pi}{4} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3}$

$ON = OA \cos\frac{\pi}{3} = 2\sqrt{2}\left(\frac{1}{2}\right) = \sqrt{2}$ (5)

$z_N = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ (5)

$\boxed{25}$

4. $f(x) = ax^3 + bx^2$

$f'(x) = 3ax^2 + 2bx$

$f''(x) = 6ax + 2b$ (5)

$= 2(3ax + b)$

$f''(1) = 0 \Rightarrow 2(3a+b) = 0$

(5) $\Rightarrow 3a + b = 0$ —— ①

$f(1) = -2 \Rightarrow a + b = -2$ —— ②

(5)

①, ② $\Rightarrow a = 1,\ b = -3$ (5)

$f''(x) = 6(x-1)$

$f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$

concave down : $(-\infty, 1)$

concave up : $(1, \infty)$ (5)

$\boxed{25}$

5. $$\lim_{x \to 0} \frac{(\sqrt{5} - \sqrt{4+\cos x})\sin x}{x^3}$$

$$= \lim_{x \to 0} \frac{(5 - 4 - \cos x)\sin x}{(\sqrt{5} + \sqrt{4+\cos x})\, x^3} \quad (5)$$

$$= \lim_{x \to 0} \frac{(1 - \cos x)\sin x}{(\sqrt{5} + \sqrt{4+\cos x})\, x^3}$$

$$= \lim_{x\to 0} \frac{(1-\cos^2 x)\sin x}{(1+\cos x)(\sqrt{5}+\sqrt{4+\cos x})\,x^3} \quad (5)$$

$$= \left(\lim_{x\to 0} \frac{\sin x}{x}\right)^3 \cdot \lim_{x\to 0} \frac{1}{(1+\cos x)(\sqrt{5}+\sqrt{4+\cos x})} \quad (5)\ (5)$$

$$= 1^3 \times \frac{1}{2(2\sqrt{5})} \quad (5)$$

$$= \frac{1}{4\sqrt{5}} \qquad \boxed{25}$$

6. $\text{Volume} = \int_0^{\ln 2} \pi (e^x + e^{-x})^2 \, dx$ (5)

$$= \pi \int_0^{\ln 2} \pi (e^{2x} + 2 + e^{-2x})\, dx \quad (5)$$

$$= \pi \left[\frac{e^{2x}}{2} + 2x + \frac{e^{-2x}}{-2} \right]_0^{\ln 2} \quad (5)$$

$$= \pi \left[\frac{4}{2} + 2\ln 2 - \frac{2^{-2}}{2} \right] \quad (5)$$

$$-\pi \left[\frac{1}{2} + 0 - \frac{1}{2} \right]$$

$$= \frac{\pi}{8}(15 + 16\ln 2) \quad (5) \qquad \boxed{25}$$

7. $x = 3\sec\theta$ $\quad y = 2\tan\theta$

$\frac{dx}{d\theta} = 3\sec\theta\tan\theta$ $\quad \frac{dy}{d\theta} = 2\sec^2\theta$ (5)

$$\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx} = 2\sec^2\theta \cdot \frac{1}{3\sec\theta\tan\theta}$$

$$= \frac{2\sec\theta}{3\tan\theta} \quad (5)$$

Gradient of the normal $= -\frac{3\tan\theta}{2\sec\theta}$

Eq^n of the normal is

$$y - 2\tan\theta = -\frac{3\tan\theta}{2\sec\theta}(x - 3\sec\theta) \quad (5)$$

$$(3\tan\theta)x + (2\sec\theta)y = 13\sec\theta\tan\theta$$

$(3\sec\theta, 2\tan\theta) \equiv (3\sqrt{2}, 2)$ (5)

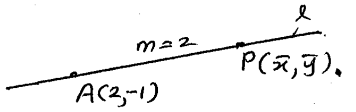
$\Rightarrow \theta = \pi/4$

$x = 0 \Rightarrow y = \frac{13}{2}$ (5)

The required y-intercept is $(0, \frac{13}{2})$

$\boxed{25}$

8.



Let $P \equiv (\bar{x}, \bar{y})$ be any point on l

$$\frac{\bar{y}+1}{\bar{x}-2} = 2 \Rightarrow \frac{\bar{y}+1}{2} = \frac{\bar{x}-2}{1} = t \text{ (say)} \quad (5)$$

$\bar{x} = t+2$ and $\bar{y} = 2t-1$ (5)

$P \equiv (t+2, 2t-1)$

$AP^2 = 5 \Rightarrow (\bar{x}-2)^2 + (\bar{y}+1)^2 = 5$ (5)

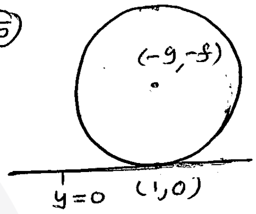
$\Rightarrow t^2 + 4t^2 = 5$

$\Rightarrow t^2 = 1 \Rightarrow t = \pm 1$

$t = 1 \Rightarrow B \equiv (3, 1)$ (5), $t = -1 \Rightarrow C \equiv (1, -3)$ (5) $\quad \boxed{25}$

9. $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$-g = 1 \Rightarrow g = -1$ (5)



radius $= -f$

$g^2 + f^2 - c = f^2$

$\Rightarrow c = g^2 = 1$ (5)

Equation of the common chord is

$S - S' = 0$

$2gx + 2fy + c + 9 = 0$ (5)

$x - fy - 5 = 0$

$(1, -f)$, $1 + f^2 - 5 = 0 \Rightarrow f = \pm 2$

$-f > 0 \Rightarrow f < 0$

$\therefore f = -2$ (5)

$S \equiv x^2 + y^2 - 2x - 4y + 1 = 0$ (5) $\quad \boxed{25}$

10. $\cos x + \sin 2x - \sin x = 1$

$\sin 2x - \sin x - (1 - \cos x) = 0$

$2\cos\frac{3x}{2}\sin\frac{x}{2} - 2\sin^2\frac{x}{2} = 0$ (5)

$2\sin\frac{x}{2}\left(\cos\frac{3x}{2} - \sin\frac{x}{2}\right) = 0$

$\sin\frac{x}{2} = 0$ or $\cos\frac{3x}{2} = \sin\frac{x}{2}$ (5)

$\sin\frac{x}{2} = \sin 0$ or $\cos\frac{3x}{2} = \cos\left(\frac{\pi}{2} - \frac{x}{2}\right)$ (5)

$\frac{x}{2} = n\pi$; $n \in \mathbb{Z}$ or $\frac{3x}{2} = 2n\pi \pm \left(\frac{\pi}{2} - \frac{x}{2}\right)$; $n \in \mathbb{Z}$

$x = 2n\pi$; $n \in \mathbb{Z}$ or $x = n\pi + \frac{\pi}{4}$; $n \in \mathbb{Z}$ (5)

or $x = 2n\pi - \frac{\pi}{2}$; $n \in \mathbb{Z}$ (5)

$\boxed{25}$

11. (a)

(i) $\Delta_1 = a^2 - 4b$, (5) $\Delta_2 = a^2c^2 - 4bc^2$
$= c^2(a^2-4b)$ (5)

α and β are real

$\Leftrightarrow \Delta_1 = a^2-4b \geq 0$ (5)

$\Leftrightarrow \Delta_2 = c^2(a^2-4b) \geq 0$ (5)

$\Leftrightarrow \gamma$ and δ are real

$\boxed{20}$

(ii) $\alpha+\beta = -a$ (5) $\gamma+\delta = -ac$ (5)

$\alpha\beta = b$ $\gamma\delta = bc^2$

$(\alpha\gamma+\beta\delta)+(\alpha\delta+\beta\gamma)$

$= \alpha(\gamma+\delta) + \beta(\gamma+\delta)$

$= (\alpha+\beta)(\gamma+\delta)$ (5)

$= (-a)(-ac)$

$= a^2c$ (5)

$(\alpha\gamma+\beta\delta)(\alpha\delta+\beta\gamma)$ (5)

$= \alpha^2\gamma\delta + \alpha\beta\gamma^2 + \alpha\beta\delta^2 + \beta^2\gamma\delta$

$= \gamma\delta(\alpha^2+\beta^2) + \alpha\beta(\gamma^2+\delta^2)$

$= \gamma\delta\{(\alpha+\beta)^2 - 2\alpha\beta\} + \alpha\beta\{(\gamma+\delta)^2 - 2\gamma\delta\}$ (5)

$= bc^2\{a^2-2b\} + b\{a^2c^2 - 2bc^2\}$ (5)

$= bc^2(a^2-2b) + bc^2(a^2-2b)$

$= 2bc^2(a^2-2b)$ (5)

The required equation is

$(x-(\alpha\gamma+\beta\delta))(x-(\alpha\delta+\beta\gamma)) = 0$ (5)

$x^2 - a^2c\,x + 2c^2b(a^2-2b) = 0$

$\boxed{45}$

(iii) $\alpha = \gamma$

$\alpha^2 + a\alpha + b = 0$ —— ① (5)

$\alpha^2 + ac\alpha + bc^2 = 0$ —— ②

① − ② $\Rightarrow a(1-c)\alpha + b(1-c^2) = 0$

$\alpha = -\dfrac{b(1+c)}{a}$ —— ③ (5)

② − c × ① $\Rightarrow (1-c)\alpha^2 + bc^2 - bc = 0$

$\alpha^2 = \dfrac{bc(1-c)}{1-c} = bc$ —— ④ (5)

③, ④ $\Rightarrow \dfrac{b^2(1+c)^2}{a^2} = bc$ (5)

$\Rightarrow b(1+c)^2 = ca^2$

$\boxed{20}$

(b) $h(x) \equiv (x-k)^2\phi(x)$ (5)

when $x=k$, $h(k) = (k-k)^2\phi(k)$
$= 0$ (5)

$h'(x) = (x-k)^2\phi'(x) + \phi(x)2(x-k)$ (5)

$h'(k) = (k-k)^2\phi'(k) + \phi(k)2(k-k)$
$= 0$ (5)

$\therefore h(k) = h'(k) = 0$

$\boxed{20}$

Since $(x-1)^2$ is a factor of $h(x)$

$h(1)=0 \Rightarrow 1+a+b+c+2=0$ (5)

$\Rightarrow a+b+c=-3$ —— ①

$h'(x) = 4x^3+3ax^2+2bx+c$

$h'(1)=0 \Rightarrow 4+3a+2b+c=0$ (5)

$\Rightarrow 3a+2b+c=-4$ —— ②

$h(x) = x(x+2)\psi(x) + x+\lambda$ (5)

$h(0)=\lambda \Rightarrow 2=\lambda$ (5)

$h(-2) = -2+\lambda$

$16-8a+4b-2c+2 = -2+2$ (5)

$\Rightarrow 4a-2b+c=9$ —— ③

①,②,③ $\Rightarrow a=1, b=-3, c=-1$ (5)

$h(x) = x^4+x^3-3x^2-x+2$

$= (x-1)(x^3+2x^2-x-2)$ (5)

$= (x-1)(x-1)(x^2+3x+2)$ (5)

$= (x-1)^2(x+1)(x+2)$ (5)

$\boxed{45}$

12. (i) ${}^5C_2\,{}^5C_2 = 100$ 5 5 $\boxed{10}$

(ii) ${}^{10}C_4 - {}^5C_4 = 210 - 5$ 5 5

$= 205.$ 5 $\boxed{15}$

(iii) ${}^5C_1\,{}^2C_2\,{}^4C_2\,{}^2C_1\,{}^2C_1$ 15

$= 120$ 5 $\boxed{20}$

(iv)

Boys	Girls	Number of ways
4	–	${}^5C_4 = 5$ 5
3	1	${}^5C_3\,{}^2C_1 = 20$ 5
2	2	${}^5C_2\,{}^3C_2 = 30$ 5
1	3	${}^5C_1\,{}^4C_3 = 20$ 5
–	4	${}^5C_4 = 5$ 5

The required number of ways $= 5+20+30+20+5$

$= 80.$ 5 $\boxed{30}$

(b) $U_r = V_r - V_{r+1}$

$$\frac{1}{r^2(r+1)(r+2)^2} = \frac{A}{r^2(r+1)^2} - \frac{A}{(r+1)^2(r+2)^2}$$ 5

$r+1 = A(r+2)^2 - Ar^2$ 5

$r+1 = A(4r+4)$

$A = \frac{1}{4}$ 5 $\boxed{15}$

$U_r = V_r - V_{r+1}$ 5

$r=1;\ U_1 = V_1 - \cancel{V_2}$

$r=2;\ U_2 = \cancel{V_2} - \cancel{V_3}$

$r=3;\ U_3 = \cancel{V_3} - \cancel{V_4}$

$\vdots$

$r=n-1;\ U_{n-1} = \cancel{V_{n-1}} - \cancel{V_n}$ 5

$r=n;\ U_n = \cancel{V_n} - V_{n+1}$

5

$$\sum_{r=1}^{n} U_r = V_1 - V_{n+1}$$

$$= \frac{1}{16} - \frac{1}{4(n+1)^2(n+2)^2}$$ 5 $\boxed{20}$

$$\lim_{n\to\infty} \sum_{r=1}^{n} U_r = \lim_{n\to\infty}\left\{\frac{1}{16} - \frac{1}{4(n+1)^2(n+2)^2}\right\}$$ 5

$= \frac{1}{16} - 0$

$= \frac{1}{16}$ 5

$\therefore$ the infinite series $\sum_{r=1}^{\infty} U_r$ is convergent and the sum is $\frac{1}{16}$. 5 $\boxed{15}$

$W_r = U_r + U_{r+1}$

$\sum_{r=1}^{n} W_r = \sum_{r=1}^{n} U_r + \sum_{r=1}^{n} U_{r+1}$ 5

5

$= \sum_{r=1}^{n} U_r + \sum_{r=1}^{n} U_r - U_1 + U_{n+1}$

$= 2\sum_{r=1}^{n} U_r - \frac{1}{18} + \frac{1}{(n+1)^2(n+2)(n+3)^2}$ 5

$\lim_{n\to\infty} \sum_{r=1}^{n} W_r = 2\left(\frac{1}{16}\right) - \frac{1}{18} + 0$ 5

$= \frac{5}{72}$

$\therefore \sum_{r=1}^{\infty} W_r$ is convergent and the sum is $\frac{5}{72}$. 5 $\boxed{25}$

13. (a) $|z| = \sqrt{a^2+b^2}$ 5

$\bar{z} = a - ib$ 5 $\boxed{10}$

(i) $z\bar{z}$

$= (a+ib)(a-ib)$

$= a^2 - i^2b^2$

$= a^2+b^2$ 5 $\boxed{5}$

$= |z|^2$

(ii) $z + \bar{z}$

$= (a+ib) + (a-ib)$ $\boxed{5}$

$= 2a = 2\,\mathrm{Re}\, z$ 5

$|z_1 - z_2|^2$

$= (z_1 - z_2)(\overline{z_1 - z_2})$ 5

$= (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$

$= z_1\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_1 + z_2\bar{z}_2$

$= |z_1|^2 - (z_1\bar{z}_2 + \overline{z_1\bar{z}_2}) + |z_2|^2$ 5

$= |z_1|^2 - 2\,\mathrm{Re}(z_1\bar{z}_2) + |z_2|^2$ 5 $\boxed{15}$

$|1 - z_1\bar{z}_2| < |z_1 - z_2|$

$\Leftrightarrow |1 - z_1\bar{z}_2|^2 < |z_1 - z_2|^2$ (5)

$\Leftrightarrow 1 - 2\text{Re}\,\overline{\bar{z}_1 z_2} + |z_1\bar{z}_2|^2$ (5)

$< |z_1|^2 - 2\text{Re}\, z_1\bar{z}_2 + |z_2|^2$

$\Leftrightarrow 1 - 2\text{Re}\,\overline{z_1\bar{z}_2} + |z_1|^2|z_2|^2 < |z_1|^2 - 2\text{Re}\, z_1\bar{z}_2 + |z_2|^2$ (5)

$\Leftrightarrow 1 - |z_1|^2 - |z_2|^2 + |z_1|^2|z_2|^2 < 0$ (5)

$[\because \text{Re}\, z = \text{Re}\, \bar{z}]$

$\Leftrightarrow (1 - |z_1|^2)(1 - |z_2|^2) < 0$ (5)

$\Leftrightarrow (1 + |z_1|)(1 - |z_1|)(1 - |z_2|)(1 + |z_2|) < 0$

$\Leftrightarrow (1 - |z_1|)(1 - |z_2|) < 0$ (5) $\boxed{30}$

$\left|\dfrac{1 + 2iz}{z - 2i}\right| < 1$

$\Leftrightarrow |1 + 2iz| < |z - 2i|$ (5)

$\Leftrightarrow |1 - z(\overline{2i})| < |z - 2i|$ (5)

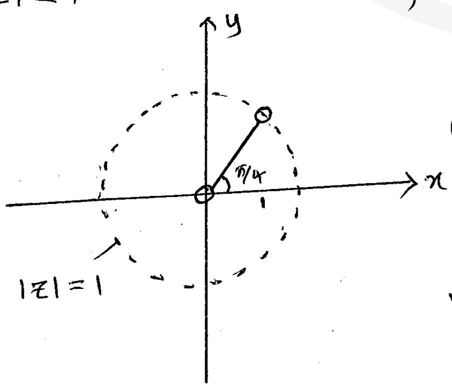
$\Leftrightarrow (1 - |z|)(1 - |2i|) < 0$ (5)

$\Leftrightarrow -(1 - |z|) < 0$ $\boxed{20}$

$\Leftrightarrow |z| < 1$ (5)

$\left|\dfrac{1 + 2iz}{z - 2i}\right| < 1$ and $\text{Arg}(z) = \dfrac{\pi}{4}$

$\Leftrightarrow |z| < 1$ and $\text{Arg}(z) = \dfrac{\pi}{4}$ (5)



(15)

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(b) $\dfrac{\cos\alpha + i\sin\alpha}{\cos\beta + i\sin\beta}$

$= \dfrac{(\cos\alpha + i\sin\alpha)(\cos\beta - i\sin\beta)}{\cos^2\beta - i^2\sin^2\beta}$ (5)

$= \dfrac{(\cos\alpha\cos\beta + \sin\alpha\sin\beta) + i(\sin\alpha\cos\beta - \cos\alpha\sin\beta)}{\cos^2\beta + \sin^2\beta}$ (5)

$= \cos(\alpha - \beta) + i\sin(\alpha - \beta)$ (5)

$\boxed{15}$

$\dfrac{\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)^n}{\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^m} = i$

$\dfrac{\cos\frac{n\pi}{9} + i\sin\frac{n\pi}{9}}{\left(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\right)^m} = i$ (5)

$\dfrac{\cos\frac{n\pi}{9} + i\sin\frac{n\pi}{9}}{\cos(-\frac{m\pi}{4}) + i\sin(-\frac{m\pi}{4})} = i$ (5)

$\Rightarrow \cos\left(4n + 9m\right)\frac{\pi}{36} + i\sin(4n + 9m)\frac{\pi}{36}$ (5)

$= \cos\left(2k\pi + \frac{\pi}{2}\right) + i\sin\left(2k\pi + \frac{\pi}{2}\right)$

$(4n + 9m)\frac{\pi}{36} = 2k\pi + \frac{\pi}{2}$ (5)

$\Rightarrow 4n + 9m = 72k + 18,\ k \in \mathbb{Z}$

$1 \le m \le 9,\ 1 \le n \le 9$

$\Rightarrow 13 < 4n + 9m < 117$

$4n + 9m = 18$ or $4n + 9m = 90$ (5)

which is impossible (5)

$4n = 9(10 - m)$

$n = 9,\ m = 6$

$\boxed{30}$

14] a) $f(x) = \dfrac{x}{(x+1)^3}$, $x \neq -1$

$f'(x) = \dfrac{(x+1)^3 \cdot 1 - x \cdot 3(x+1)^2}{(x+1)^6}$ (15)

$= \dfrac{(x+1) - 3x}{(x+1)^4}$

$= \dfrac{1-2x}{(x+1)^4}$ (5)

$= \dfrac{-2(x-\frac{1}{2})}{(x+1)^4}$

$\boxed{20}$

$f(x) = \dfrac{x}{(x+1)^3}$

$= \dfrac{\frac{1}{x^2}}{(1+\frac{1}{x})^3}$

$x \to \pm\infty \quad f(x) \to 0$

$y=0$ is a horizontal asymptote (5)

$x=-1$ is a vertical asymptote (5)

$f'(x) = 0 \iff x = \frac{1}{2}$ (5)

	$-\infty<x<-1$	$-1<x<\frac{1}{2}$	$\frac{1}{2}<x<\infty$
Sign of $f'(x)$	(+) (5)	(+) (5)	(−) (5)
$f(x)$ is	Increasing	Increasing	Decreasing

Turning point $(\frac{1}{2}, \frac{2}{9})$ (5) is a local maximum

$f''(x) = \dfrac{6(x-1)}{(x+1)^5}$, $x \neq -1$

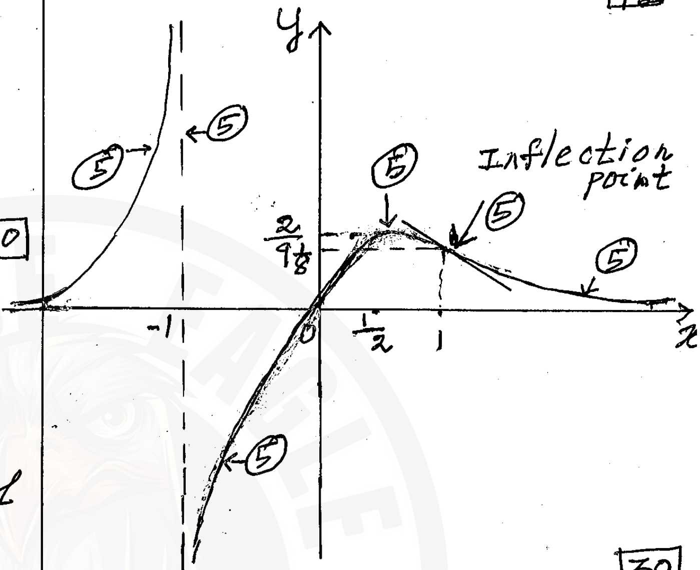
$f''(x) = 0 \iff x = 1$ (5)

$\boxed{40}$

	$-\infty<x<-1$	$-1<x<1$	$1<x<\infty$
Sign of $f''(x)$	(+)	(−)	(+) (10)
Concavity	Concave up	Concave down	Concave up

$(1, \frac{1}{8})$ is a inflection point (5)

$\boxed{15}$



$\boxed{30}$

b) $V = 1500 cm^3$

$\{\frac{1}{2} \times 3x \times 4x\} y = 1500$ (5)

$x^2 y = 250$ (5)

$S = \{\frac{1}{2} \times 4x \times 3x\}2 + y\{3x+4x+5x\}$ (5)

$= 12x^2 + 12xy$

$= 12x^2 + 12x \times \frac{250}{x^2}$ (5)

$= 12\{x^2 + \frac{250}{x}\}$ (5)

$\frac{ds}{dx} = 12\{2x + 250(-1)x^{-2}\}$ (5)

$= \frac{24}{x^2}\{x^3 - 5^3\}$

$\frac{ds}{dx} = 0 \iff x = 5$ (5)

for $0<x<5 \quad \frac{ds}{dx} < 0$

$5<x \quad \frac{ds}{dx} > 0$ (5)

$\therefore$ S is minimum when $x = 5 cm$ (5)

$\boxed{45}$

4

15]
a)

$$8x^3+x^2+18x-1 = A(x+1)(4x^2+9) + B(x-1)(4x^2+9) + (x^2-1)$$

Comparing Coefficients of powers of x

$x^3;\ 8 = 4A+4B$ (5)
$2 = A+B$
$x^0;\ -1 = 9A-9B+1$ (5)

$\Rightarrow A=1,\ B=+1$ (5)

$$\frac{8x^3+x^2+18x-1}{(x^2-1)(4x^2+9)} = \frac{(x+1)(4x^2+9)+(x-1)(4x^2+9)+(x^2-1)}{(x^2-1)(4x^2+9)}$$

$$= \frac{1}{x-1}+\frac{1}{x+1}+\frac{1}{4x^2+9}$$ (15)

$$\int \frac{8x^3+x^2+18x-1}{(x^2-1)(4x^2+9)}dx = \int\frac{1}{x-1}dx+\int\frac{1}{x+1}dx+\int\frac{1}{(2x)^2+3^2}dx$$

$$= \ln|x-1| + \ln|x+1| + \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)+C$$ (5) (5) (5) (5)

C - arbitary Constant

$\boxed{45}$

b) $y = 2^x$

$\ln y = \ln 2^x = x\ln 2$ (5)

$\frac{1}{y}\frac{dy}{dx} = \ln 2$ (5)

$\frac{dy}{dx} = (\ln 2)2^x$ (5)

$\boxed{15}$

$I = \int 2^x \sin x\,dx$ (say)

$= 2^x(-\cos x) - \int(-\cos x)(\ln 2)2^x dx$ (15)

$= -2^x\cos x + (\ln 2)\int 2^x \cos x\,dx$

$= -2^x\cos x + (\ln 2)\left\{2^x\sin x - \int \sin x(\ln 2)2^x dx\right\}$ (15)

$= -2^x\cos x + (\ln 2)2^x\sin x - (\ln 2)^2 I$ (5)

$\Rightarrow \{1+(\ln 2)^2\}I = 2^x\{(\ln 2)\sin x - \cos x\}$ (5)

$$I = \frac{2^x\{(\ln 2)\sin x - \cos x\}}{\{1+(\ln 2)^2\}} + C_1$$ (5)

$\boxed{45}$

c)

$$\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx = \int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)dx}{1+\cos^2(\pi-x)}$$ (5)

$$= \int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx$$ (5)

$$= \pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx - \int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$$ (5)

$$\Rightarrow \int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx = \frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$ (5)

$$= -\frac{\pi}{2}\int_0^{\pi}\frac{1}{1+(\cos x)^2}d(\cos x)$$ (5)

$$= -\frac{\pi}{2}\left\{\tan^{-1}(\cos x)\right\}_0^{\pi}$$ (5)

$$= -\frac{\pi}{2}\left\{\tan^{-1}(-1) - \tan^{-1}1\right\}$$ (5)

$$= \pi\tan^{-1}1$$ (5)

$$= \pi\times\frac{\pi}{4} = \frac{\pi^2}{4}$$ (5)

$\boxed{45}$

16. (i) Equation of l_1 is

$y+4 = \frac{-6}{2}(x-2)$

$3x+y-2=0$ (10)

Equation of l_2 is

$y-0 = \frac{4}{12}(x-14)$ (10)

$x-3y-14=0$ $\boxed{20}$

(ii) Equations of the bisectors are

$\frac{3x+y-2}{\sqrt{10}} = \pm \frac{x-3y-14}{\sqrt{10}}$ (15)

$\oplus \Rightarrow x+2y+6=0$ (5)

$\ominus \Rightarrow 2x-y-8=0$ (5)

$2x-y-8=0$, $B(0,2)$, $C(14,0)$

$[2(0)-2-8][2(14)-0-8]$ (10)

$=(-10)(20)$

$=-200<0$

$\therefore$ B and C are on opposite side of

$2x-y-8=0$

$\therefore$ Equation of l is (5)

$2x-y-8=0$ $\boxed{40}$

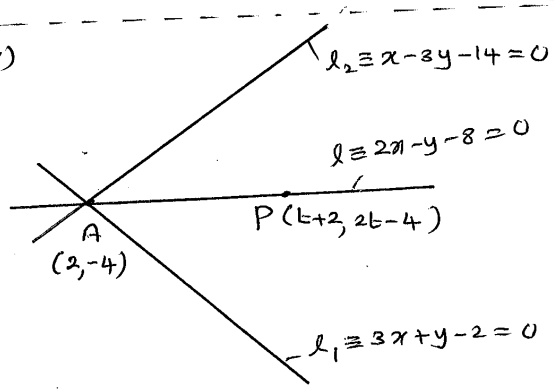
(iii) $2(x-2)=y+4$

$\frac{x-2}{1} = \frac{y+4}{2} = t$ (say) (10)

$x=t+2$, $y=2t-4$ (5)

$P(t+2, 2t-4)$ $\boxed{15}$

(iv)



$\frac{|3(t+2)+(2t-4)-2|}{\sqrt{10}} = \frac{3\sqrt{10}}{2}$ (15)

$5|t|=15$

$|t|=3$

$t=\pm 3$ (5)

$t=3 \Rightarrow P(5,2)$ (5)

$t=-3 \Rightarrow P(-1,-10)$ (5)

Since P lies on the 1st quadrant

centre $P \equiv (5,2)$ (5)

Equation of C_1 is

$(x-5)^2+(y-2)^2=\left(\frac{3\sqrt{10}}{2}\right)^2$ (5)

$x^2+y^2-10x-4y+\frac{13}{2}=0$ (5) $\boxed{45}$

$C_2 \equiv x^2+y^2+2gx+2fy+c=0$

$(-g,-f) \equiv (0,2)$

(5) $g=0$, $f=-2$ (5)

$2(0)(-5)+2(-2)(-2) = c+\frac{13}{2}$ (10)

$\Rightarrow c=\frac{3}{2}$ (5)

$C_2 \equiv x^2+y^2-4y+\frac{3}{2}=0$ (5) $\boxed{30}$

17] a) $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ (5)

$\alpha=\beta=\theta$ ஆக

$$\tan 2\theta=\frac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta} \quad (5)$$
$$=\frac{2\tan\theta}{1-\tan^2\theta} \quad (5)$$

$$\tan 3\theta=\tan(2\theta+\theta)$$
$$=\frac{\tan2\theta+\tan\theta}{1-\tan2\theta\tan\theta} \quad (5)$$
$$=\frac{\frac{2\tan\theta}{1-\tan^2\theta}+\tan\theta}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\tan\theta} \quad (5)$$
$$=\frac{2\tan\theta+\tan\theta(1-\tan^2\theta)}{1-\tan^2\theta-2\tan^2\theta} \quad (5)$$
$$=\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} \quad (5) \quad \boxed{35} \quad \text{—(A)}$$

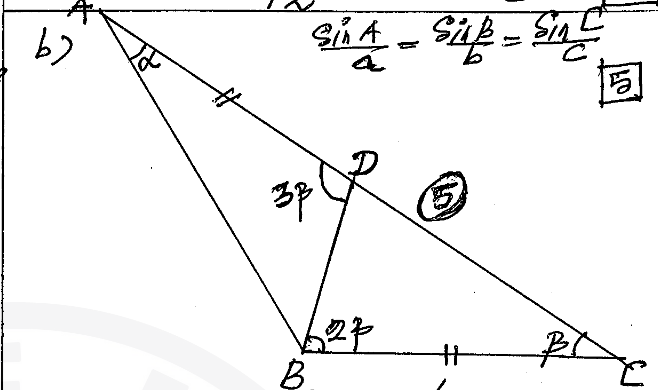
(A)இல் $\theta=\frac{\pi}{12}$ ஆக

$$\tan\frac{\pi}{4}=\frac{3\tan\frac{\pi}{12}-\tan^3\frac{\pi}{12}}{1-3\tan^2\frac{\pi}{12}} \quad (5)$$
$$1-3\tan^2\frac{\pi}{12}=3\tan\frac{\pi}{12}-\tan^3\frac{\pi}{12}$$
$$\tan^3\frac{\pi}{12}-3\tan^2\frac{\pi}{12}-3\tan\frac{\pi}{12}+1=0 \quad (5)$$
$$x^3-3x^2-3x+1=0 \quad (5) \text{ where } x=\tan\frac{\pi}{12}$$

$\therefore \tan\frac{\pi}{12}$ is a root of $x^3-3x^2-3x+1=0$ (5)

$$(x+1)(x^2-4x+1)=0$$
$$\Rightarrow x^2-4x+1=0 \quad (\because \tan\tfrac{\pi}{12}>0) \quad (5)$$
$$x=\frac{4\pm\sqrt{(-4)^2-4(1)(1)}}{2\times1} \quad (5)$$
$$x=2\pm\sqrt{3}$$
$$0<\tan\frac{\pi}{12}<1 \Rightarrow \quad (5)$$
$$\therefore \tan\frac{\pi}{12}=2-\sqrt{3} \quad (5) \quad \boxed{40}$$

b) $\dfrac{\sin A}{a}=\dfrac{\sin B}{b}=\dfrac{\sin C}{c}$ $\boxed{5}$



In ΔBDC by sine rule

$$\frac{BD}{\sin\beta}=\frac{BC}{\sin(\pi-3\beta)} \quad (10) \quad =\frac{BC}{\sin3\beta} \quad \text{—(1)}$$

ΔADC

$$\frac{BD}{\sin\alpha}=\frac{AD}{\sin\{\pi-(\alpha+3\beta)\}} \quad (10)$$
$$=\frac{AD}{\sin(\alpha+3\beta)} \quad \text{—(2)}$$

(1), (2) $\Rightarrow$
$$\frac{\sin\beta}{\sin3\beta}=\frac{\sin\alpha}{\sin(\alpha+3\beta)} \quad (5) \quad (\because BC=AD)$$
$$\frac{\sin\beta}{3\sin\beta-4\sin^3\beta}=\frac{\sin\alpha}{\sin(\alpha+3\beta)} \quad (5)$$
$$\sin(\alpha+3\beta)=\sin\alpha(3-4\sin^2\beta) \quad (5) \quad \boxed{45}$$

c) $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

$\alpha=\tan^{-1}\left(\frac{1}{4}\right)$ $\quad 0<\alpha<\frac{\pi}{4}$

$\beta=\tan^{-1}\left(\frac{2}{9}\right)$ $\quad 0<\beta<\frac{\pi}{4}$ (5)

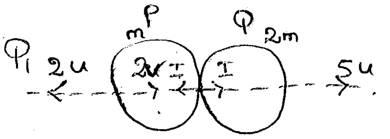
$\gamma=\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ $\quad 0<\gamma<\frac{\pi}{6}$

$$\tan(\alpha+\beta)=\frac{\frac{1}{4}+\frac{2}{9}}{1-\left(\frac{1}{4}\right)\left(\frac{2}{9}\right)}=\frac{1}{2} \quad (5)$$
$$0<\alpha+\beta<\frac{\pi}{2}$$
$$\Rightarrow \alpha+\beta=\tan^{-1}\left(\frac{1}{2}\right) \quad (5)$$
$$\cos2\gamma=\frac{3}{5}$$
$$\frac{1-\tan^2\gamma}{1+\tan^2\gamma}=\frac{3}{5} \quad (5)$$
$$\tan^2\gamma=\frac{1}{4} \quad (5)$$
$$\tan\gamma=\frac{1}{2} \quad (5)$$
$$\gamma=\tan^{-1}\left(\frac{1}{2}\right) \quad \boxed{30}$$

6

Grade - 13 (2021) **இணைந்த கணிதம் II** Marking Scheme

1)



$I = \Delta mv$

P $\rightarrow$ $-I = -2mu - 2mv$

$I = 2mu + 2mv \rightarrow (1)$ (5)

Q $\rightarrow$ $I = 10mu \rightarrow (2)$ (5)

(2)-(1) $4u = v$

$u = \frac{v}{4}$ (5)

நியூ.வி. $7u = e \times 2v$

$\frac{7v}{4} = e \times 2v$

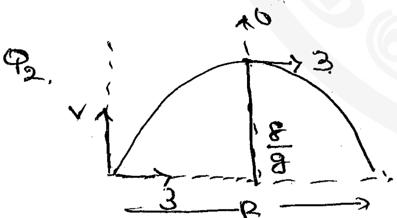
$e = \frac{7}{8}$ // (5)

$V_P = \overleftarrow{2u}$ $\quad V_Q = \rightarrow 5u$

$= \frac{v}{2}$ // (5) $= \frac{5v}{4}$ //

$\boxed{25}$

Q2.



$\uparrow$ $v^2 = u^2 + 2as$

$0 = v^2 - 2g \times \frac{8}{g}$ (5)

$v^2 = 16$

$v = 4$ // (5)

$\uparrow$ ~~$v^2 = u^2 + 2as$~~ $s = ut + \frac{1}{2}at^2$

$0 = vT - \frac{1}{2}gT^2$ (5)

$T = \frac{2v}{g}$ (5)

$\frac{2 \cdot 8}{g}$ (5)

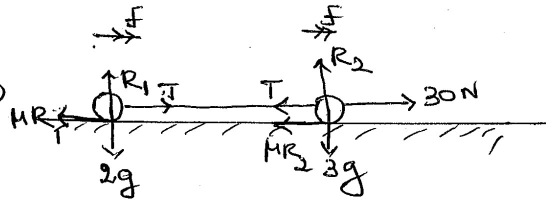
$\rightarrow s = ut$

$R = 3 \times T$

$= \frac{24}{g}$ m. // (5)

$\boxed{25}$

3)



P $\uparrow$ $R_1 = 2g \rightarrow (1)$ (5)

$\rightarrow$ $T - \mu R_1 = 2f \rightarrow (2)$ (5)

Q $\uparrow$ $R_2 = 3g \rightarrow (3)$ (5)

$\rightarrow$ $30 - T - \mu R_2 = 3f \rightarrow (4)$ (5)

Q க்கு $s = ut + \frac{1}{2}at^2$

$6 = 0 + \frac{1}{2}f \times 9 \rightarrow$ (5)

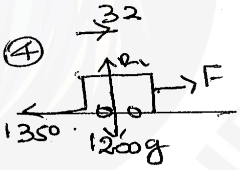
(5)

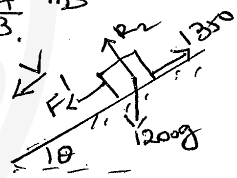
$f = \frac{12}{9}$

$= \frac{4}{3}$ ms^{-2}.

25

4)





$\rightarrow$ $F = 1350N$ (5)

$P = Fv$

$= 1350 \times 32$ W

$= 43200$ W (5)

$\swarrow$ $F' + 1200g \sin\theta$

$= 1350$ (5)

$F' = 1350 - 1200g \frac{1}{20}$

$= 1350 - 600$

$= 750N$ (5)

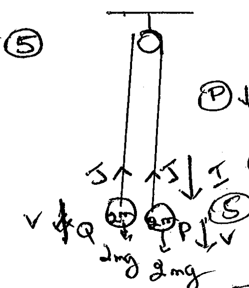
$v = \frac{P}{F'}$

$= \frac{31.5 \times 10^3}{750}$

$= 10.5 \times 4$

$= 42 ms^{-1}$ (5)

$\boxed{25}$



P $\downarrow$ $I = \Delta mv$.

$I - J = 2mv$ (5)

Q $\uparrow$ $J = 2mv$ (5)

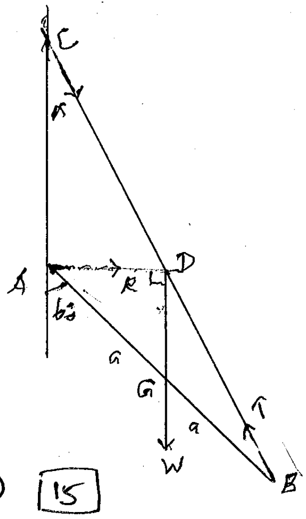
(5)

$v = \frac{I}{4m}$ // (5)

$J = \frac{I}{2}$ // (5)

$\boxed{25}$

(6)



$GD = a\cos 60$

$= \frac{a}{2}$

$\therefore AC = a$

$AD = a\sin 60$

$= \frac{a\sqrt{3}}{2}$ (5)

$\tan\alpha = \frac{AD}{AC}$

$= \frac{\sqrt{3}}{2}$ (5)

$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (5) $\boxed{15}$

$\uparrow\ T\cos\alpha = W$ (5)

$T = \frac{W\sqrt{7}}{2}$ (5) $\boxed{10}$ $\boxed{25}$

(7) $|\underline{a}| = |\underline{b}| = |\underline{a}+\underline{b}| = \lambda$

$(\underline{a}+\underline{b}).(\underline{a}+\underline{b}) = |\underline{a}+\underline{b}|^2$ (5)

$|\underline{a}|^2 + |\underline{b}|^2 + 2\underline{a}.\underline{b} = |\underline{a}+\underline{b}|^2$ (5)

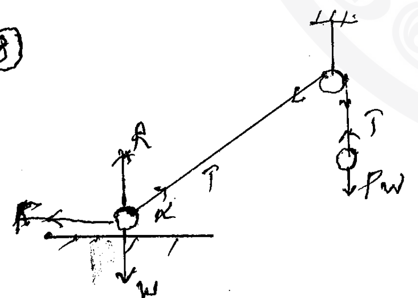
$2\lambda^2 + 2|\underline{a}||\underline{b}|\cos\theta = \lambda^2$ (5)

$2\lambda^2\cos\theta = -\lambda^2$

$\cos\theta = -\frac{1}{2}$ (5)

$\theta = \frac{2\pi}{3}$ (5) $\boxed{25}$

(8)



$\uparrow\ T = PW$ (5)

$\leftarrow\ F - T\cos\alpha = 0$

$F = T\cos\alpha$ (5)

$\uparrow\ R - W + T\sin\alpha = 0$

$R = W - T\sin\alpha$ (5)

$\frac{F}{R} = \tan\lambda$ (5)

$\frac{T\cos\alpha}{W - T\sin\alpha} = \frac{\sin\lambda}{\cos\lambda}$ (5)

$P(\cos\alpha\cos\lambda + \sin\alpha\sin\lambda) = W\sin\lambda$

$P\cos(\alpha - \lambda) = W\sin\lambda.$ $\boxed{25}$

(9) $P(A) = \frac{1}{2}$, $P(A\cup B) = \frac{2}{3}$

A, B independant $\Rightarrow P(A\cap B) = P(A)P(B)$ (5)

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$ (5)

$\frac{2}{3} = \frac{1}{2} + P(B) - \frac{1}{2}P(B)$

$\frac{1}{6} = \frac{1}{2}P(B)$

$P(B) = \frac{1}{3}$ (5) $\boxed{15}$

$P(A'/B) = \frac{P(A'\cap B)}{P(B)}$ (5)

$= \frac{P(B) - P(A\cap B)}{P(B)}$

$= \frac{\frac{1}{3} - \frac{1}{2}\cdot\frac{1}{3}}{\frac{1}{3}}$

$= \frac{1}{2}$ (5) $\boxed{10}$ $\boxed{10}$ $\boxed{25}$

(10) i $A\cap B = \phi$ $P(B/A) = \frac{P(A\cap B)}{P(A)} = 0$ (5)

ii $A\subset B \Rightarrow P(A\cap B) = P(A)$ (5) (5)

$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$ (5) $\boxed{10}$

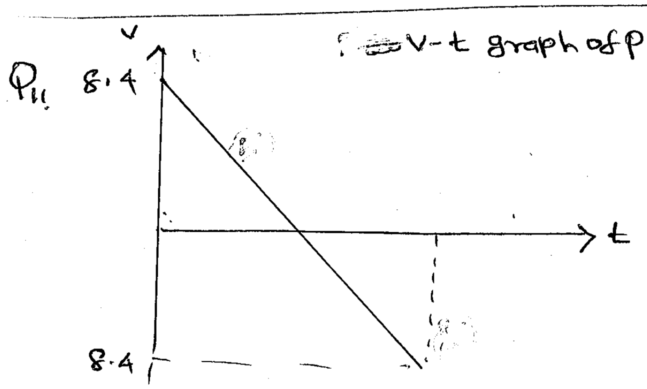
iii $P(A/B) + P(A'/B)$.

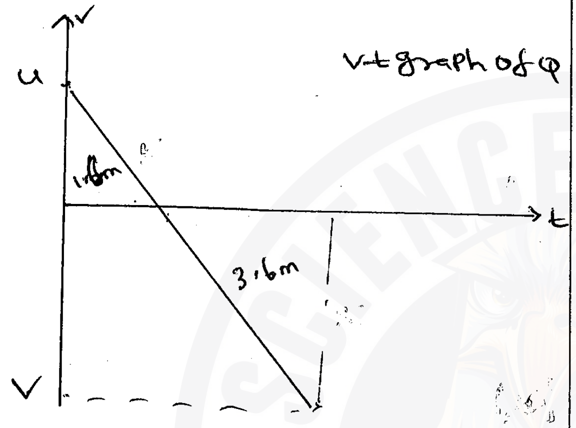
$= \frac{P(A\cap B)}{P(B)} + \frac{P(A'\cap B)}{P(B)}$ (5)

$= \frac{P(B)}{P(B)}$ (5) $\boxed{10}$

$= 1$ $\boxed{25}$

Q_{11}





(ii) hight of P when velocity zero $= \frac{1}{2} \times 8.4 \times \frac{8.4}{9.8}$

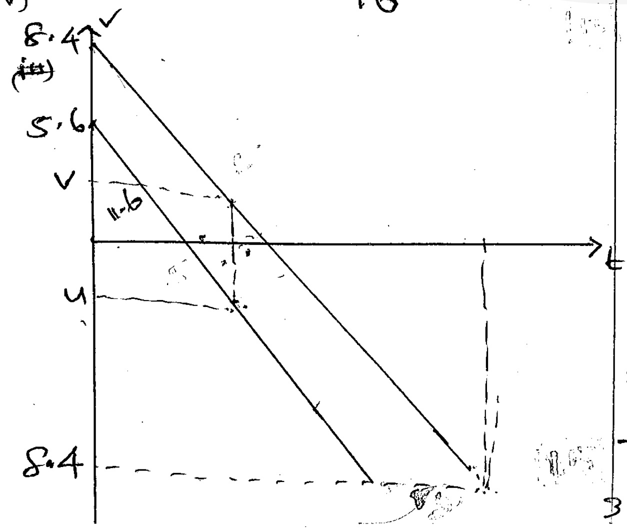
$= 3.6\text{ m}$

(iii) $1.6 = \frac{1}{2} \times u \times \frac{u}{9.8}$

$u^2 = 1.6 \times 19.6$

$u = \frac{4 \times 14}{10} = 5.6\text{ ms}^{-1}$

(iv)



From the graph when they are moving opposite direction they are in same hight

$v + u = 8.4 - 5.6$

$v + u = 2.8 \rightarrow (1)$

$3.6 - \frac{v^2}{2g} = 3.6 - \frac{u^2}{2g}$

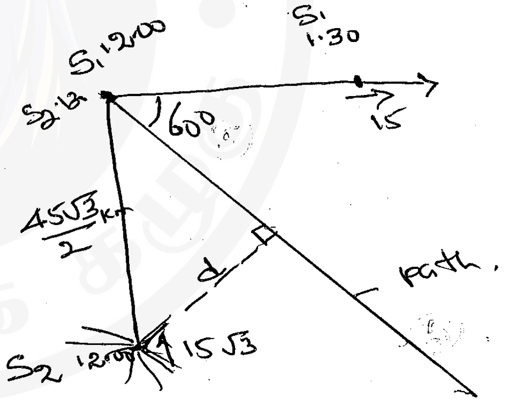
$v^2 = u^2$

$v = u \rightarrow (2)$

$v = u = 1.4\text{ ms}^{-1}$

(b) $V_{S_1,E} = \xrightarrow{15\text{ km/h}}$

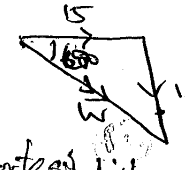
$V_{S_2,E} = \uparrow 15\sqrt{3}\text{ km/h}$



$V_{S_1,S_2} = V_{S_1,E} + V_{E,S_2}$

$= \uparrow 15\sqrt{3} + \xleftarrow{} 15$

$= \xrightarrow[15]{} + \downarrow 15\sqrt{3}$



$W = \sqrt{15^2 + (15\sqrt{3})^2}$

$= 30\text{ km/h}$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

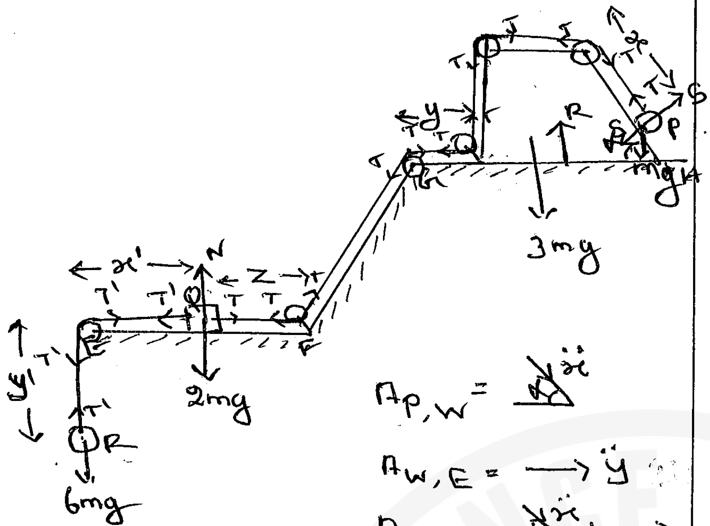
Shortest distance $= \frac{45\sqrt{3}}{2} \sin 30$

Shortest distance $= \frac{45\sqrt{3}}{4}$

$T = \frac{\frac{45\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}}{30} = \frac{9}{8}\text{ h}$

$= \frac{45}{2}$

(12) (a)



$A_{P,W} = \overset{\ddot{x}}{\searrow}$

$A_{W,E} = \overset{\ddot{y}}{\rightarrow}$

$A_{P,E} = \overset{\ddot{x}}{\searrow} + \overset{\ddot{y}}{\rightarrow}$

$x + y + z = \text{const}$

$\ddot{x} + \ddot{y} + \ddot{z} = 0$

$x' + y' = \text{consta.}$

$\ddot{x}' + \ddot{y}' = 0$

$x' + z = \text{consta}$

$\ddot{x}' + \ddot{z} = 0$

$A_{Q,E} = \overset{\ddot{z}}{\leftarrow}$

$= \overset{-\ddot{x}-\ddot{y}}{\leftarrow}$

$A_{R,E} = \downarrow \ddot{y}'$

$= \downarrow -\ddot{x}'$

$= \downarrow \ddot{z}$

$\boxed{} = \downarrow -\ddot{x} - \ddot{y}$

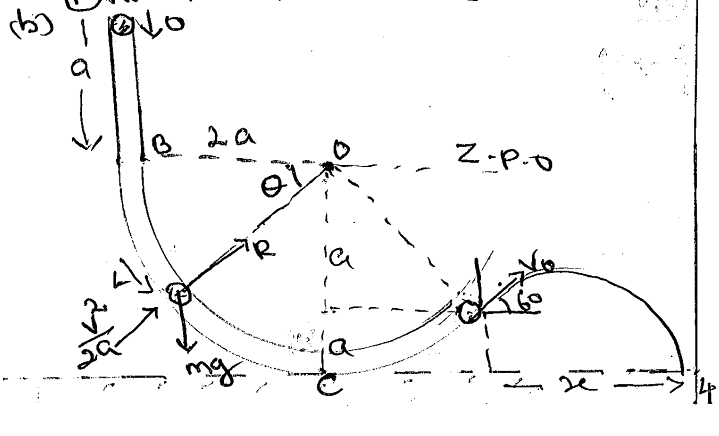
R $\downarrow$ $6mg - T' = 6m(-\ddot{x} - \ddot{y})$

Q $\leftarrow$ $T' - T = 2m(-\ddot{x} - \ddot{y})$

P $\searrow$ $mg\sin\alpha - T = m(\ddot{x} + \ddot{y}\cos\alpha)$

P+W $\rightarrow$ $-T = 3m\ddot{y} + m(\ddot{y} + \ddot{x}\cos\alpha)$

(b)



By the law of conservation

$mga = \frac{1}{2}mv^2 - mg \times 2a\sin\theta$

$v^2 = 2ga(1 + 2\sin\theta)$

$\nearrow$ $\underline{F} = m\underline{a}$

$R - mg\sin\theta = m\frac{v^2}{2a}$

$R - mg\sin\theta = mg(1 + 2\sin\theta)$

$R = mg(1 + 3\sin\theta)$

velocity of particle when $\theta = \frac{5\pi}{6}$.

$v_0^2 = 2ga(1 + 2 \times \frac{1}{2})$

$= 4ag$

$s = ut + \frac{1}{2}at^2$

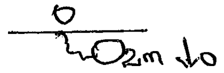
$\uparrow$ $-a = v_0 \sin 60\, t - \frac{1}{2}gt^2 \rightarrow *$

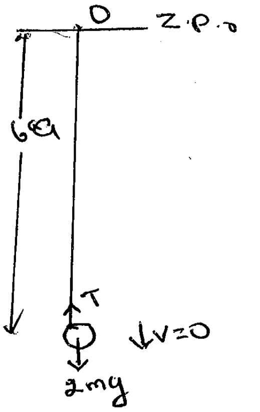
$\rightarrow$ $x = v_0 \cos 60\, t \rightarrow **$

$*\&**$ $-a = x\tan 60 - \frac{gx^2}{2v_0^2\cos^2 60}$

$-2a^2 = 2\sqrt{3}ax - x^2$

$x^2 - 2\sqrt{3}ax - 2a^2 = 0$

13) 

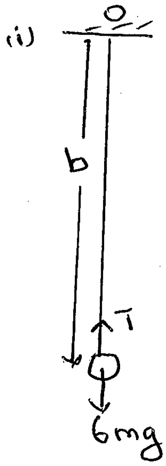


by the law of conservation of energy

$$0 = -2mg\cdot 6a + \frac{1}{2}\frac{\lambda(4a)^2}{2a}$$

$$4\lambda a = 12mga$$

$$\lambda = 3mg$$

(i) 

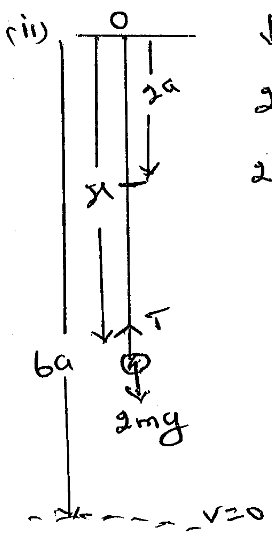
For the equilibrium

$$T = 6mg$$

$$\frac{3mg(b-2a)}{2a} = 6mg$$

$$b - 2a = 4a$$

$$b = 6a$$

(ii) 

$\downarrow F = ma$

$$2mg - T = 2m\ddot{x}$$

$$2mg - \frac{3mg(x-2a)}{2a} = 2m\ddot{x}$$

$$\ddot{x} = g - \frac{3g}{4a}(x-2a)$$

$$= -\frac{3g}{4a}\left(x-2a-\frac{4a}{3}\right)$$

$$= -\frac{3g}{4a}\left(x-\frac{10a}{3}\right) \to *$$

$$y = x - \frac{10a}{3}$$

$$\ddot{y} = \ddot{x}$$

$$* \Rightarrow \ddot{y} = -\frac{3g}{4a}y \to **$$

(iii) $x = \frac{10a}{3} + A\cos\omega t + B\sin\omega t$

$$x - \frac{10a}{3} = A\cos\omega t + B\sin\omega t \to (1)$$

$$\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t \to (2)$$

$$\ddot{x} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$$

$$= -\omega^2(A\cos\omega t + B\sin\omega t)$$

$$= -\omega^2\left(x - \frac{10a}{3}\right)$$

$$\ddot{y} = -\omega^2 y \to (3)$$

$** \,\&\, (3) \Rightarrow \omega^2 = +\frac{3g}{4a}$

$$\omega = \sqrt{\frac{3g}{4a}}$$

$t=0 \quad x = 6a, \quad \dot{x} = 0$

$(1) \Rightarrow 6a - \frac{10a}{3} = A + 0$

$$A = \frac{8a}{3}$$

$(2) \Rightarrow 0 = 0 + B\omega$

$$B = 0$$

$A = \frac{8a}{3}, \; B = 0, \; \omega = \sqrt{\frac{3g}{4a}}$

(iv) $x - \frac{10a}{3} = \frac{8a}{3}\cos\omega t$

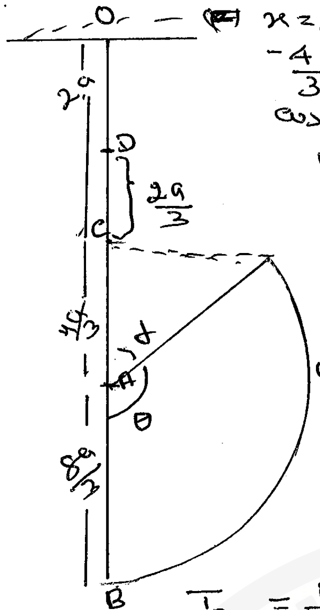
$$y = \frac{8a}{3}\cos\omega t$$

$y_{max} = \frac{8a}{3} \Rightarrow$ வீச்சம் $= \frac{8a}{3}$

$\ddot{y} = 0 \Rightarrow y = 0 \Rightarrow x = \frac{10a}{3}$

center $\frac{10a}{3}$ from the point O

(v) 5



$x = 2a$

$-\frac{4a}{3} = \frac{8a}{3}\cos wt$

$\cos wt = -\frac{1}{2}$

$wt = \frac{2\pi}{3}$

$T_{BC} = \frac{2\pi}{3w}$

$\theta = \pi - \alpha$

$= \pi - \cos^{-1}\frac{1}{2}$

$= \pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$

$T_{BC} = \frac{\theta}{w}$

$= \frac{2\pi}{3} \times \sqrt{\frac{4a}{3g}}$

$= \frac{4\pi}{3}\sqrt{\frac{a}{3g}}$ //

$\dot{y} = -\frac{8a}{3} w \sin wt.$

$wt = \frac{2\pi}{3}$ ஆக.

$\dot{y} = -\frac{8a}{3} w \times \sin\frac{2\pi}{3}$

$= -\frac{8a}{3} w \times \frac{\sqrt{3}}{2}$

$= -\frac{4}{3} aw$

$v^2 = u^2 + 2as$

$0 = \frac{16}{9}a^2w^2 - 2gh.$

$h = \frac{16a^2}{9} \times \frac{3g}{4a} \times \frac{1}{2g} = \frac{2a}{3}$

$v = u + at.$

$0 = \frac{4aw}{3} - g\,T_{CO}$

$T_{CO} = \frac{4aw}{3g} = \frac{1}{w} = 2\sqrt{\frac{a}{3g}}$

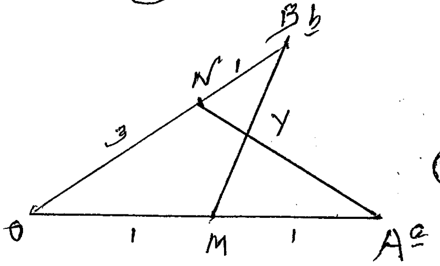
$T = T_{BC} + T_{CO}$

$= \frac{2\pi}{3w} + \frac{1}{w} = \left(\frac{2\pi}{3} + 1\right) 2\sqrt{\frac{a}{3g}}$ //

$OD = 2 \cdot a - \frac{2a}{3} = \frac{4a}{3}$ //

6

14 a) I Theory. (15) 15



(10)

$\triangle$ $\underline{y} = \lambda \underline{a} + (1-\lambda)\frac{3}{4}\underline{b}$ (10)

$\triangle$ $\underline{y} = \mu \frac{\underline{a}}{2} + (1-\mu)\underline{b}$ (5) 25

$\therefore \lambda \underline{a} + (1-\lambda)\frac{3}{4}\underline{b} = \mu\frac{\underline{a}}{2} + (1-\mu)\underline{b}$ (5)

$(\lambda - \frac{\mu}{2})\underline{a} + \left[\frac{3}{4}(1-\lambda) - 1 + \mu\right]\underline{b} = \underline{0}$

$\lambda - \frac{\mu}{2} = 0 \quad + \quad \frac{3}{4}(1-\lambda) - 1 + \mu = 0$

(5)

$\mu - \frac{3}{4}\lambda - \frac{1}{4} = 0$

$\mu - \frac{3}{8}\mu = \frac{1}{4}$

$\frac{5}{8}\mu = \frac{1}{4}$

$\mu = \frac{2}{5}$ (5)

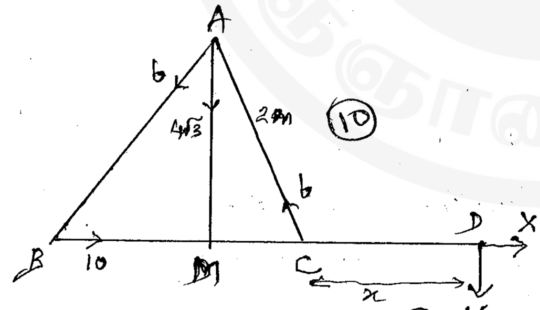
20

$\lambda = \frac{1}{5}$ (5)

$\frac{AY}{YN} : = \frac{1-\lambda}{\lambda}$ (5) $= \frac{\frac{4}{5}}{\frac{1}{5}} = \frac{4}{1}$ (5) 10

70

b)



(10)

$\rightarrow X = 10 - b\cos 60 - b\cos 60$ (5)

$= 4$ (5)

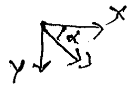
$\downarrow Y = 4\sqrt{3} + b\sin 60 - b\sin 60$ (5)

$= 4\sqrt{3}$ (5)

$R^2 = 4^2 + (4\sqrt{3})^2$

$= 8$ (5)

7



$\tan\alpha = \frac{4\sqrt{3}}{4}$

$= \sqrt{3}$

$\alpha = \frac{\pi}{3}$ (5)

$\boxed{40}$

$A \circlearrowleft$ $2\sin 60 \cdot 4 - (1+x)4\sqrt{3} = 2\sin 60 \cdot 10$ (10)

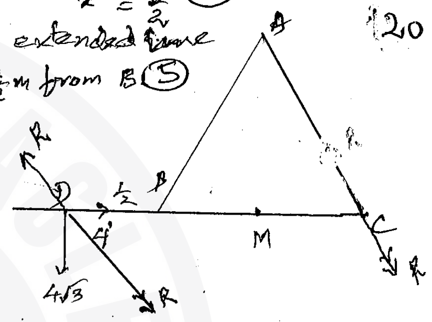
$4\sqrt{3}(1+x) = -12 \cdot \frac{\sqrt{3}}{2}$

$1 + x = -\frac{3}{2}$

$x = -\frac{5}{2}$ (5)

20

D lies at extended line CB. 4 $\frac{1}{2}$m from B (5)



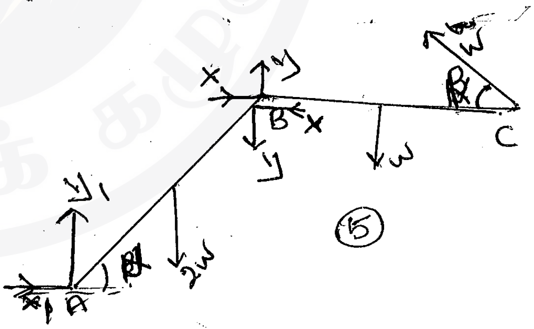
(5)

$\circlearrowleft G = 8 \cdot \frac{5}{2}\sin 60$ (10)

$= 10\sqrt{3}$ (5)

$\boxed{20}$

150



(5)

BC.

$B \circlearrowleft$ $W\sin\beta \times 2a = W \times a$ (5)

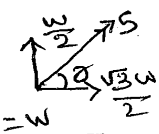
$\sin\beta = \frac{1}{2}$

$\beta = \frac{\pi}{6}$ // (5)

(BC) $\rightarrow X = W\cos\frac{\pi}{6} = \frac{W\sqrt{3}}{2}$ (5)

$\uparrow Y + W\sin\frac{\pi}{6} = W$ (5)

$Y = \frac{W}{2}$



(5) $S = W$

(5) $\theta = \frac{\pi}{6}$ //

AB.

A) $\frac{\sqrt{3}w}{2}\sin\alpha\, 2a = 2w\cos\alpha\, a + \frac{w}{2}\cos\alpha\, 2a$ (10)

$\sqrt{3}\tan\alpha = 3$

$\tan\alpha = \sqrt{3}$

$\alpha = \frac{\pi}{3}$ (5)

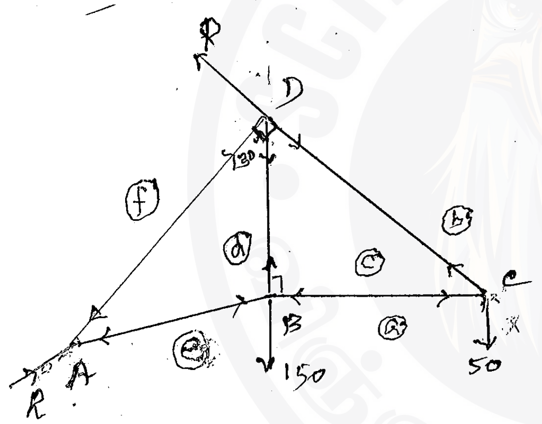
(AB) $\rightarrow X_1 = X = \frac{\sqrt{3}w}{2}$ (5)

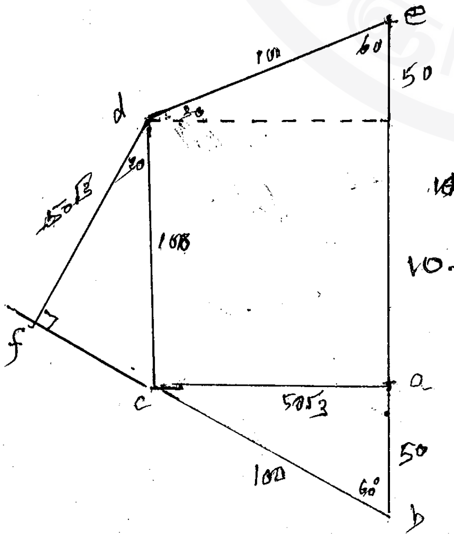
$\uparrow Y_1 = 2w + \frac{w}{2}$

$= \frac{5w}{2}$ (5)

$\boxed{60}$

b)





10+10+10

Rod	Notation	Tension	Thrust
AB	de	–	100 10
BC	ca	–	$50\sqrt{3}$ 10
CD	bc	100	– 10
DA	fA	–	$50\sqrt{3}$ 10
DB	dc	100	– 10

$P = 150$ 5

$R = 50\sqrt{3}$ 5

$\boxed{90}$

150

16) Theory.

30 + 30

$\boxed{60}$

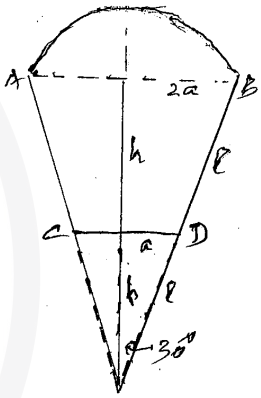


figure	Mass	c.o.m from AB
Large Cone	$\pi\, 2a . 2l\rho$ (5)	$\frac{1}{3}.2h$ (5)
Small Cone	$\pi a l \rho$ (5)	$h + \frac{1}{3}h = \frac{4h}{3}$ (5)
Disc CD	$\pi a^2 \sigma$ (5)	h (5)
Hemisphere	$2\pi.(2a)^2\sigma$ (5)	$(-a)$ (5)
Vessel	$3\pi a l\rho + 9\pi a^2\sigma$ (5)	$\bar{y}$

$(3\pi a l\rho + 9\pi a^2\sigma)\bar{y} = 4\pi a l\rho.\frac{2}{3}h - \pi a l\rho.\frac{4h}{3}$ (5) (5) (5)

$+ \pi a^2\sigma.h + 8\pi a^2\sigma(-a)$ (5) (5)

$(3l\rho + 9a\sigma)\bar{y} = \frac{4}{3}lh\rho + ah\sigma - 8a^2\sigma$

$l = 2a,\ h = \sqrt{3}a \Rightarrow$

$(6a\rho + 9a\sigma)\bar{y} = \frac{8\sqrt{3}}{3}a^2\rho + (\sqrt{3}-8)a^2\sigma$

$\bar{y} = \left[\frac{8\sqrt{3}}{3}\rho + (\sqrt{3}-8)\sigma\right]\frac{a}{6\rho + 9\sigma}$ (5)

$\boxed{70}$

for Equilibrium

$\bar{y} = 0$ (10)

$\Rightarrow \frac{8\sqrt{3}}{3}p + (\sqrt{3} - 8)\sigma = 0$ (5)

$8p = (8\sqrt{3} - 3)\sigma$ (5) $\boxed{20}$

150

17) a)

(5) I $S = \{ggg, ggb, gbb, gbg, bbg, bgg, bgb, bbb\}$

(5) $A = \{ggb, gbb, gbg, bbg, bgg, bgb\}$

(5) $B = \{ggb, gbg, bgg, ggg\}$

(5) $A \cap B = \{ggb, gbg, bgg\}$

(5) $P(A) = \frac{6}{8}$, $P(B) = \frac{4}{8}$ (5)

(5) $P(A \cap B) = \frac{3}{8}$

$P(A).P(B) = \frac{6}{8} . \frac{4}{8}$ (5)

$= \frac{3}{8}$ (5) $= P(A \cap B)$

$\Rightarrow$ A, B are independant (5) $\boxed{50}$

II. $S = \{gg, gb, bg, bb\}$ 5

$A = \{gb, bg\}$ 5

$B = \{gg, gb, bg\}$ 5

$A \cap B = \{gb, bg\}$ 5

$P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$

$P(A \cap B) = \frac{1}{2}$ (5)

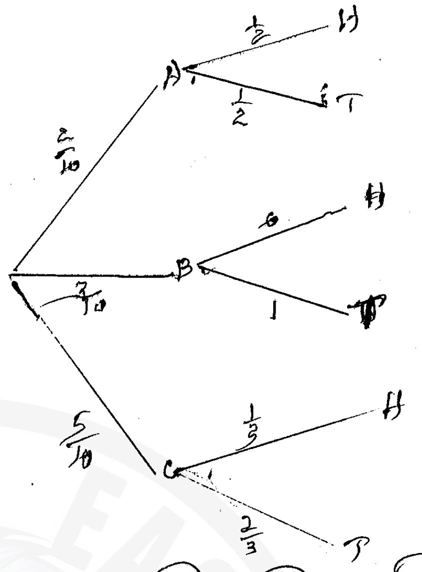
$P(A).P(B) = \frac{1}{2} . \frac{3}{4}$ (5)

$= \frac{3}{8}$ (5) $\neq P(A \cap B)$

$\Rightarrow$ A, B dependant 5

$\boxed{40}$

b)



(10) (10) (10)

I $P(H) = \frac{2}{10} . \frac{1}{2} + \frac{3}{10} . 0 + \frac{5}{10} \times \frac{1}{3}$

$= \frac{4}{15}$ (5)

II $P(A/H) = \frac{P(A \cap H)}{P(H)}$ (5) (5)

$= \frac{\frac{2}{10} . \frac{1}{2}}{\frac{4}{15}}$ (5) (5)

$= \frac{3}{8}$ (5)

$\boxed{60}$

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