1. $f(x) = a^2(x^2+1) + 2abx - 2b(a-b)$

$= a^2x^2 + 2abx + b^2 + a^2 - 2ab + b^2$ (5)

$= (ax+b)^2 + (a-b)^2 \geq 0$ (5) (5)

$[\because (a-b)^2 \geq 0,\ (ax+b)^2 \geq 0]$

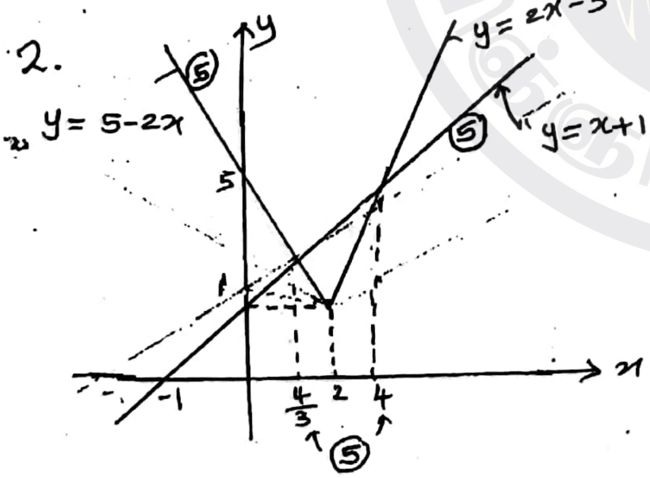
If $|a-b| = 2$, then $(a-b)^2 = 4$ (5)

$f(x) = (ax+b)^2 + 4$ $[\because (ax+b)^2 \geq 0]$

$f(x)_{min} = 4$ (5)

$\boxed{25}$

2.



$2|x-2| \leq x$ (5)

$2|x-2| + 1 \leq x+1$

$\frac{4}{3} \leq x \leq 4$ (5)

$\boxed{25}$

3. $\lim_{x\to 0} \frac{\sin 3x}{(8+x)^{\frac{1}{3}} - 2}$

$= \lim_{x\to 0} \frac{\frac{\sin 3x}{x}}{\frac{(8+x)^{\frac{1}{3}} - 2}{x}}$ (5)

$= \frac{3 \lim_{3x\to 0} \frac{\sin 3x}{3x}}{\lim_{(8+x)\to 8} \frac{(8+x)^{\frac{1}{3}} - 8^{\frac{1}{3}}}{(8+x) - 8}}$ (5)

$= \frac{3 \times 1}{\frac{1}{3} 8^{-\frac{2}{3}}}$ (5) (5)

$= 36$ (5)

$\boxed{25}$

4. $x = 2\cos\theta - \cos 2\theta$

$\frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$ (5)

$= 2(2)\cos\frac{3\theta}{2}\sin\frac{\theta}{2}$

$y = 2\sin\theta - \sin 2\theta$

$\frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$ (5)

$= 2(2)\sin\frac{3\theta}{2}\sin\frac{\theta}{2}$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{4\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}{4\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}$ (5)

$= \tan\frac{3\theta}{2}$

$\frac{dy}{dx} = 1 \Rightarrow \tan\frac{3\theta}{2} = 1 \Rightarrow \theta = \frac{\pi}{6}$ (5)

$P \equiv \left(\frac{2\sqrt{3}-1}{2}, \frac{2-\sqrt{3}}{2}\right)$ (5)

$\boxed{25}$

5. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$

$=\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$ (5)

$[\because \cos(\pi-\theta) = -\cos\theta]$

$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$

$= \sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$ (5)

$=\left(\frac{1-\cos\frac{\pi}{4}}{2}\right)\left(\frac{1-\cos\frac{3\pi}{4}}{2}\right)$ (5)

$=\frac{1}{4}\left(1-\frac{1}{\sqrt{2}}\right)\left(1+\frac{1}{\sqrt{2}}\right)$ (5)

$=\frac{1}{4}\left(1-\frac{1}{2}\right)$

$=\frac{1}{8}$ (5)

$\boxed{25}$

6.

$4g-T=4f$ (5)

$T-3g=3f$ (5)

$f=\frac{g}{7}$ (5)

$T=3g+3.\frac{g}{7}$

$=\frac{24}{7}g$ (5)

7.

$A \to C \uparrow$ $v^2=u^2+2as$

$0=(u\sin\alpha)^2-2gH$

$H=\frac{u^2\sin^2\alpha}{2g}=\frac{u^2}{8g}$ (10)

$\sin\alpha=\frac{1}{2}$ $(\alpha<\frac{\pi}{2})$

$\alpha=\frac{\pi}{6}$ (5)

$A \to B$ $\uparrow$ $s=ut+\frac{1}{2}at^2$

$0=u\sin\alpha\, t-\frac{1}{2}gt^2$

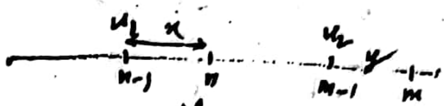
$t=\frac{2u\sin\alpha}{g}$ (5)

$\rightarrow$ $R=u\cos\alpha.t$

$=\frac{u^2\sin 2\alpha}{g}$

$=\frac{\sqrt{3}u^2}{2g}$ (5)

8.



$s=ut+\frac{1}{2}at^2$

$x=u_1.1+\frac{1}{2}a.1^2$ (5)

$y=u_2.1+\frac{1}{2}a.1^2$ (5)

$y-x=u_2-u_1$ (5)

$v=u+at$

$u_2=u_1+a(m-n)$ (5)

$u_2-u_1=a(m-n)$

$\therefore y-x=a(m-n)$

$a=\frac{y-x}{m-n}$ (5)

9. $(\underline{a}-\underline{b})\perp\underline{b}$

$\Rightarrow \underline{b}.(\underline{a}-\underline{b})=0$ (10)

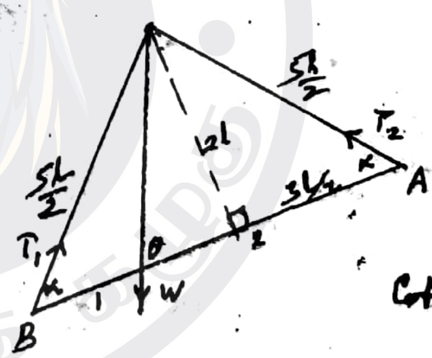
$\underline{b}.\underline{a}-b^2=0$ (5)

$|\underline{b}||\underline{a}|\cos\theta - b^2=0$ (5)

$\cos\theta=\frac{|b|}{|a|}$ (5)

$\theta=\cos^{-1}\left(\frac{|b|}{|a|}\right)$

10.



$\cot\alpha=\frac{3}{4}$ (5)

$(2+1)\cot\theta=2\cot\alpha-1\cot\alpha$ (5)

$3\cot\theta=\frac{3}{4}$

$\cot\theta=\frac{1}{4}$

$\tan\theta=4$ (5)

$A \curvearrowleft$ $T_1.3l\sin\alpha-W.2l\sin\theta=0$ (5)

$T_1=\frac{2W}{3}\frac{\sin\theta}{\sin\alpha}$

$=\frac{2}{3}W.\frac{4}{\sqrt{17}}.\frac{5}{4}$

$=\frac{10}{3\sqrt{17}}W$

$B \curvearrowright$ $T_2.3l\sin\alpha-Wl\sin\theta=0$

$T_2=\frac{5}{3\sqrt{17}}W$ (5)

11) let $\alpha, \beta (\in \mathbb{R}^+)$ be the roots of equation $f(x) = x^2 - x + P = 0$

then $\alpha + \beta = +1$ ⑤

$\alpha\beta = P$ ⑤

Now $\frac{1}{(\alpha^2+1)} + \frac{1}{(\beta^2+1)} = \frac{(\beta^2+1)+(\alpha^2+1)}{(\alpha^2+1)(\beta^2+1)}$

$= \frac{(\alpha^2+\beta^2)+2}{\alpha^2\beta^2+(\alpha^2+\beta^2)+1}$ ⑤ $= \frac{1-2P+2}{P^2+1-2P+1}$ ⑤

$= \frac{3-2P}{(P-1)^2+1}$ ⑤

$\frac{1}{(\alpha^2+1)(\beta^2+1)} = \frac{1}{\alpha^2\beta^2+\alpha^2+\beta^2+1}$ ⑤

$= \frac{1}{P^2+1-2P+1}$

$= \frac{1}{(P-1)^2+1}$ ⑤

The required equation is

$x^2 - \left(\frac{1}{\alpha^2+1} + \frac{1}{\beta^2+1}\right)x + \frac{1}{(\alpha^2+1)(\beta^2+1)} = 0$

$x^2 - \frac{(3-2P)}{(P-1)^2+1}x + \frac{1}{(P-1)^2+1} = 0$ ⑩

$g(x) = ((P-1)^2+1)x^2 + (2P-3)x + 1 = 0$ ⑤ $\boxed{45}$

The equation $g(x) = 0$ be the roots are different real roots

then $\Delta > 0$ ⑤

$(2P-3)^2 - 4.((P-1)^2+1).1 > 0$ ⑤ ⑤

$4P^2 - 12P + 9 - 4P^2 + 8P - 8 > 0$

⑤ $1 - 4P > 0$

$0 < P < 1/4$ —①

but $\alpha\beta = P > 0$ ⑤ —② $(\alpha, \beta \in \mathbb{R}^+)$

①,② $0 < P < 1/4$ —⑤

$\boxed{30}$

$g(x) = 0$ has same real roots ⑤

$\Leftrightarrow \frac{1}{\alpha^2+1} = \frac{1}{\beta^2+1}$

$\Leftrightarrow \alpha^2+1 = \beta^2+1$ ⑤

$\Leftrightarrow \alpha^2 = \beta^2$ ⑤

$\Leftrightarrow \alpha = \pm\beta$ $(\alpha, \beta \in \mathbb{R}^+)$ ⑤

$f(x) = 0$ has same real root

$\boxed{20}$

b) $(x+1)$ is a factor of $g(x)$

$g(-1) = 0$ ⑤ & $g(x) = x^3 + px^2 + qx - 2$

$g(-1) = -1 + p - q - 2 = 0$ ⑤

$p - q = 3$ —① ⑤

$g(1)$ is a remainder when $g(x)$ is divided by $(x-1)$

$g(2)$ is a remainder when $g(x)$ is dividede by $(x-2)$

$g(2) = 2g(1)$ ⑤

$8 + 4p + 2q - 2 = 2 + 2p + 2q - 4$ ⑤ ⑤

$2p = -8$

$p = -4$

$① \Rightarrow q = -7$ ⑤ ⑤

$g(x) = x^3 - 4x^2 - 7x - 2$ $\boxed{40}$

$f(x) = (x+2)g(x) + 5$ ⑤

$f(x) = (x+2)(x^3 - 4x^2 - 7x - 2) + 5$ ⑤

$= x^4 - 2x^3 - 15x^2 - 16x + 1$ ⑤

$\boxed{15}$

12.

(a) $f(x) = \dfrac{x+1}{(x-2)^2}$

$$f'(x) = \frac{(x-2)^2(1) - (x+1)2(x-2)}{(x-2)^4} \quad 10$$

$$= \frac{x-2-2x-2}{(x-2)^3} \quad 5$$

$$= -\frac{x+4}{(x-2)^3}$$

$$f''(x) = -\left\{\frac{(x-2)^3(1) - (x+4)3(x-2)^2}{(x-2)^6}\right\} \quad 10$$

$$= -\left\{\frac{x-2-3x-12}{(x-2)^4}\right\} \quad 5$$

$$= \frac{2(x+7)}{(x-2)^4}$$

When $x=0$, $y = \frac{1}{4}$

When $y=0$, $x=-1$

$\lim_{x \to 2} f(x) = \infty$

Vertical asymptote: $x=2$ 5

$$\lim_{x \to \pm\infty} \frac{x+1}{(x-2)^2} = \lim_{x \to \pm\infty} \frac{\frac{1}{x}+\frac{1}{x^2}}{(1-\frac{2}{x})^2} = 0$$

Horizontal asymptote: $y=0$ 5

When $f'(x)=0$, $x=-4$ 5

$x<-4$	$-4<x<2$	$x>2$
$f'(x)<0$	$f'(x)>0$	$f'(x)<0$
Decreasing	increasing	decreasing
5	5	5

$(-4, -\frac{1}{12})$ is a local minimum 5

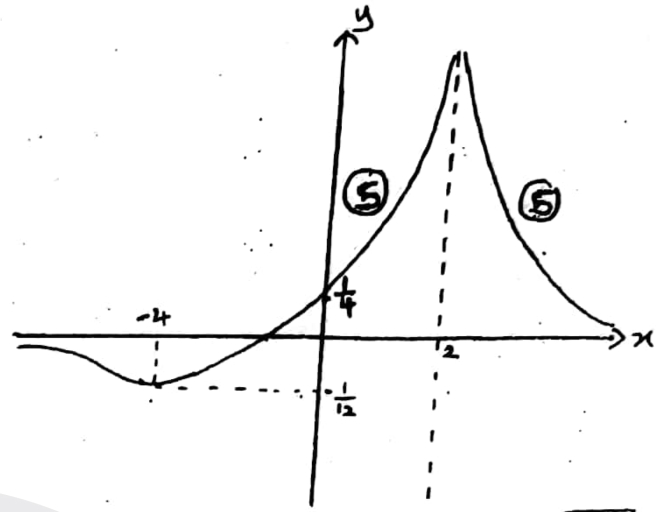
When $f''(x)=0$, $x=-7$ 5

$x<-7$	$-7<x<2$	$x>2$
$f''(x)<0$	$f''(x)>0$	$f''(x)>0$
concave down	concave up	concave up
5	5	5

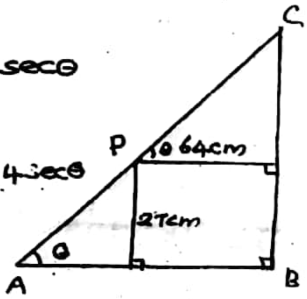
When $x=-7$, $y=-\frac{2}{27}$

$(-7, -\frac{2}{27})$ is a point of inflection 5 $\boxed{90}$

(b)



$\boxed{10}$



$AC = 27\operatorname{cosec}\theta + 64\sec\theta$

Let $AC = l$

Then $l = 27\operatorname{cosec}\theta + 64\sec\theta$ 10

$$\frac{dl}{d\theta} = -27\operatorname{cosec}\theta\cot\theta + 64\sec\theta\tan\theta \quad 10$$

$$= 64\operatorname{cosec}\theta\cot\theta\left(\tan^3\theta - \left(\tfrac{3}{4}\right)^3\right)$$

$$\frac{dl}{d\theta} = 0 \Rightarrow \tan^3\theta = \left(\tfrac{3}{4}\right)^3 \quad 5$$

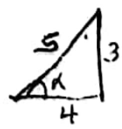
$$\Rightarrow \tan\theta = \tfrac{3}{4}$$

$$\Rightarrow \theta = \tan^{-1}\left(\tfrac{3}{4}\right) = \alpha \text{ (say)} \quad 5$$

$0<\theta<\alpha$	$\theta=\alpha$	$\alpha<\theta<\frac{\pi}{2}$
$\frac{dl}{d\theta}<0$	$\frac{dl}{d\theta}=0$	$\frac{dl}{d\theta}>0$

10

$\therefore$ l is minimum when $\theta = \tan^{-1}(\frac{3}{4})$ 5



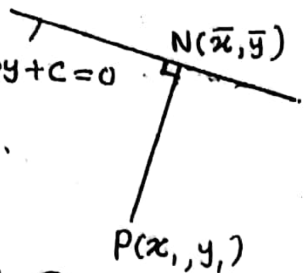
$$(l)_{min} = 27\operatorname{cosec}\alpha + 64\sec\alpha$$

$$= 27\left(\tfrac{5}{3}\right) + 64\left(\tfrac{5}{4}\right)$$

$$= 125 \text{ cm} \quad 5$$

$\boxed{50}$

13.



Let $N \equiv (\bar{x},\bar{y})$.

Then

$$\frac{\bar{y}-y_1}{\bar{x}-x_1} \cdot x -\frac{a}{b} = -1 \quad (5)$$

$$\frac{\bar{y}-y_1}{b} = \frac{\bar{x}-x_1}{a} = t \text{ (say)} \quad (5)$$

$$\bar{x} = x_1+at, \quad \bar{y} = y_1+bt \quad (5)$$

$N(\bar{x},\bar{y})$ lies on $ax+by+c=0$

$$a(x_1+at)+b(y_1+bt)+c=0 \quad (5)$$

$$t = -\frac{ax_1+by_1+c}{a^2+b^2} \quad (5)$$

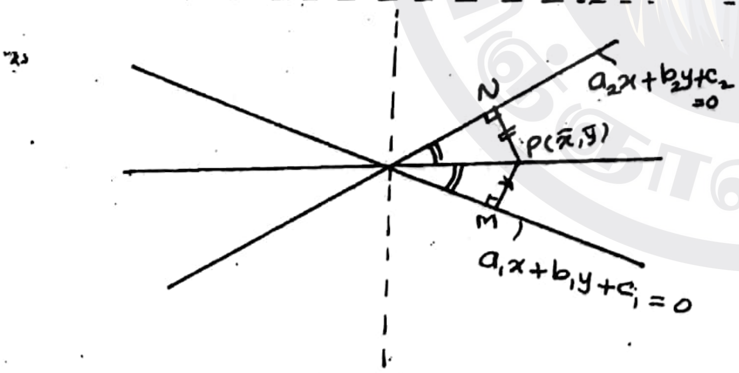
$$PN = \sqrt{(\bar{x}-x_1)^2+(\bar{y}-y_1)^2} \quad (5)$$

$$= \sqrt{a^2t^2+b^2t^2}$$

$$= |t|\sqrt{a^2+b^2} \quad (5)$$

$$= \frac{|ax_1+by_1+c|}{a^2+b^2}\sqrt{a^2+b^2}$$

$$= \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}} \quad (5) \quad \boxed{40}$$



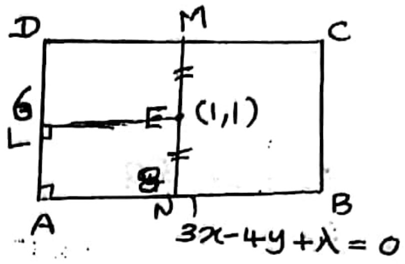
$$PM = PN$$

$$\frac{|a_1\bar{x}+b_1\bar{y}+c_1|}{\sqrt{a_1^2+b_1^2}} = \frac{|a_2\bar{x}+b_2\bar{y}+c_2|}{\sqrt{a_2^2+b_2^2}} \quad (10)$$

$$\frac{a_1\bar{x}+b_1\bar{y}+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2\bar{x}+b_2\bar{y}+c_2}{\sqrt{a_2^2+b_2^2}} \quad (5)$$

Let $\bar{x} \equiv x$, $\bar{y} \equiv y$ (5)

$$\frac{a_1x+b_1y+c_1}{\sqrt{a_1^2+b_1^2}} = \pm\frac{a_2x+b_2y+c_2}{\sqrt{a_2^2+b_2^2}} \quad \boxed{20}$$



(i)

AB: $3x-4y+\lambda=0$

$EN = 3$

$$\frac{|3(1)-4(1)+\lambda|}{5} = 3 \quad (10)$$

$$|\lambda-1| = 15 \quad (5)$$

$$\lambda = 16 \text{ or } \lambda = -14 \quad (10)$$

$$\lambda>0 \Rightarrow \lambda = 16 \quad (5) \quad \boxed{30}$$

AB: $3x-4y+16=0$

(ii) DC: $3x-4y-14=0$ (5) $\boxed{05}$

(iii) AD: $4x+3y+\mu=0$

$EL = 4$

$$\frac{|4(1)+3(1)+\mu|}{5} = 4 \quad (10)$$

$$|\mu+7| = 20 \quad (5)$$

$$\mu = 13 \text{ or } \mu = -27 \quad (10)$$

$$\mu>0 \Rightarrow \mu = 13 \quad (5)$$

AD: $4x+3y+13=0$ $\boxed{30}$

(iv) $$\frac{3x-4y+16}{5} = \pm\frac{4x+3y+13}{5} \quad (10)$$

$$3x-4y+16 = \pm(4x+3y+13) \quad (5)$$

$$(+) \Rightarrow x+7y-3=0 \quad (5)$$

$$(-) \Rightarrow 7x-y+29=0 \quad (5) \quad \boxed{25}$$

14. a) (i) $\cos\theta + \cos 3\theta + \cos 5\theta = 0$

$\cos\theta + \cos 5\theta + \cos 3\theta = 0$

$2\cos 3\theta \cos 2\theta + \cos 3\theta = 0$ (5)

$\cos 3\theta \{2\cos 2\theta + 1\} = 0$ (5)

$\cos 3\theta = 0$ or $2\cos 2\theta + 1 = 0$

$\cos 3\theta = \cos\frac{\pi}{2}$ (5) $\qquad \cos 2\theta = \cos\frac{2\pi}{3}$ (5)

$3\theta = 2n_1\pi \pm \frac{\pi}{2}$ $\qquad 2\theta = 2n_2\pi \pm \frac{2\pi}{3}$

$\theta = \frac{2n_1\pi}{3} \pm \frac{\pi}{6}$ (5), $n_1 \in \mathbb{Z}$ $\qquad \theta = n_2\pi \pm \frac{\pi}{3}$ (5) $n_2 \in \mathbb{Z}$

$\boxed{30}$

(ii) $\sqrt{3}(\sin\theta + \cos\theta)^2 = \cos 2\theta$

$\sqrt{3}(1 + \sin 2\theta) = \cos 2\theta$ (5)

$\cos 2\theta - \sqrt{3}\sin 2\theta = \sqrt{3}$

$\frac{1}{2}\cos 2\theta - \frac{\sqrt{3}}{2}\sin 2\theta = \frac{\sqrt{3}}{2}$ (5)

$\cos\frac{\pi}{3}\cos 2\theta - \sin\frac{\pi}{3}\sin 2\theta = \cos\frac{\pi}{6}$ (5)

$\cos(2\theta + \frac{\pi}{3}) = \cos\frac{\pi}{6}$ (5)

$2\theta + \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{6}$ (5) $n \in \mathbb{Z}$

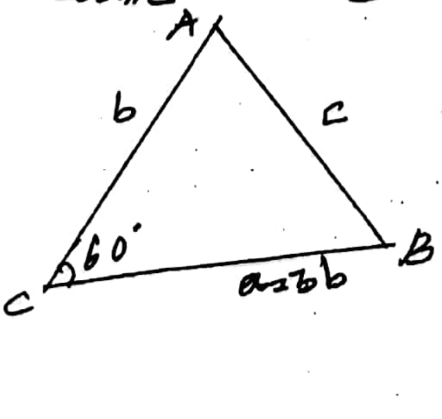
$2\theta = 2n\pi - \frac{\pi}{3} \pm \frac{\pi}{6}$

$\theta = n\pi - \frac{\pi}{6} \pm \frac{\pi}{12}$ (5)

$\theta = n\pi - \frac{\pi}{12}$ or $\theta = n\pi - \frac{\pi}{4}$ $n \in \mathbb{Z}$ $\boxed{30}$

b) Sine rule (5)

Cosine rule (5)



In triangle by cosine rule (15)

$c^2 = b^2 + (3b)^2 - 2b(3b)\cos 60°$ (10)

$c^2 = b^2 + 9b^2 - 3b^2$

$c^2 = 7b^2$

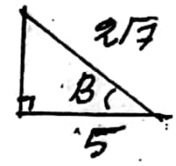
$c = \sqrt{7}b$ (5)

by sine rule

$\frac{\sin B}{b} = \frac{\sin C}{c} = \frac{\sin 60°}{\sqrt{7}b}$ (10)

$\sin B = \frac{\sqrt{3}}{2\sqrt{7}}$ (5)

$\tan B = \frac{\sqrt{3}}{5}$ $(\because 0 < B < \frac{\pi}{2})$ (5)



$A + B + C = 180°$

$A + B = 120°$ $\quad C = 60°$

$\tan(A+B) = \tan 120°$ (5)

$\frac{\tan A + \frac{\sqrt{3}}{5}}{1 - \tan A(\frac{\sqrt{3}}{5})} = -\sqrt{3}$ (5)

$\tan A = -3\sqrt{3}$ (5) $\boxed{60}$

c) $\cos 2(\tan^{-1}\frac{1}{7}) = \sin 4(\tan^{-1}\frac{1}{3})$

$\alpha = \tan^{-1}(\frac{1}{7})$
$\beta = \tan^{-1}(\frac{1}{3})$ (say)

$\cos 2\alpha = \frac{1 - \tan^2\alpha}{1 + \tan^2\alpha}$ (5)

$= \frac{1 - \frac{1}{49}}{1 + \frac{1}{49}} = \frac{24}{25}$ (5)

$\sin 4\beta = 2\sin 2\beta \cos 2\beta$

$= 2\left\{\frac{2\tan\beta}{1 + \tan^2\beta}\right\}\left\{\frac{1 - \tan^2\beta}{1 + \tan^2\beta}\right\}$ (5)

$= 2\left\{\frac{\frac{2}{3}}{1 + \frac{1}{9}}\right\}\left\{\frac{1 - \frac{1}{9}}{1 + \frac{1}{9}}\right\}$ (5)

$= 2 \times \frac{6}{10} \times \frac{8}{10}$

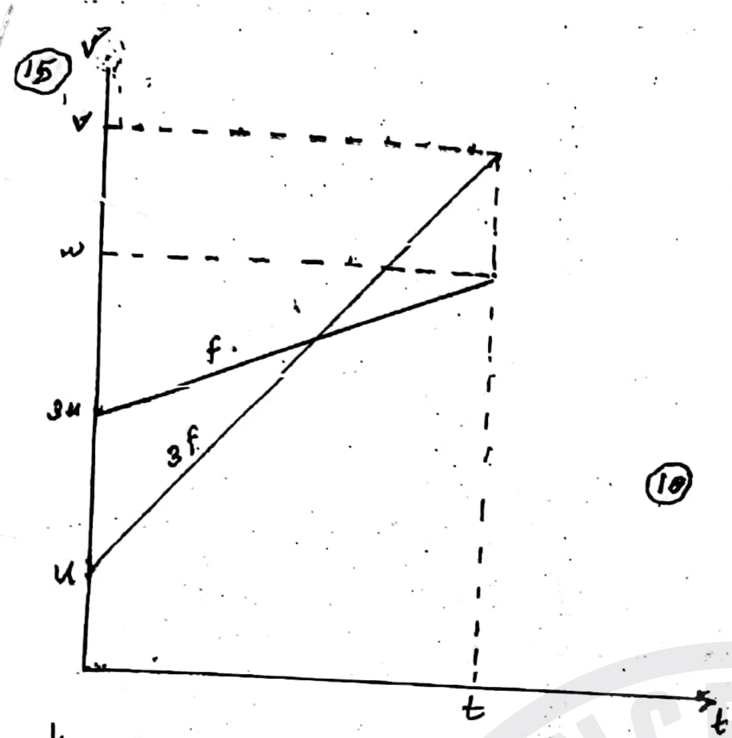
$= \frac{24}{25}$ (5)

$\therefore \cos 2\alpha = \sin 4\beta$

$\cos 2(\tan^{-1}\frac{1}{7}) = \sin 4(\tan^{-1}\frac{1}{3})$ (5)

$\boxed{30}$

(15)



(10)

1. $t = \frac{V-u}{3f} = \frac{w-3u}{f}$ (10)

$3w - v = 8u$ —— ① (5)

$\frac{1}{2}(V+u).t = \frac{1}{2}(w+3u).t$ (10)

$\Rightarrow\ V - w = 2u$ —— ② (5)

$w = 5u$ (5)

$v = 7u$ (5)

ii. $t = \frac{7u-u}{3f} = \frac{2u}{f}$ (10)

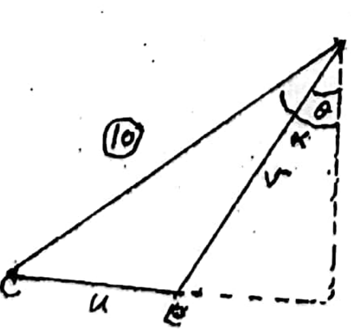
iii. $AB = \frac{1}{2}(v+u)t$ (10)

$= \frac{1}{2}\cdot 8u \cdot \frac{2u}{f}$

$= \frac{8u^2}{f}$ (10)

b) V_{WE} $V_{CE} \xrightarrow{u}$ V_{WC} (10)

V_{CE} u V_{WC}

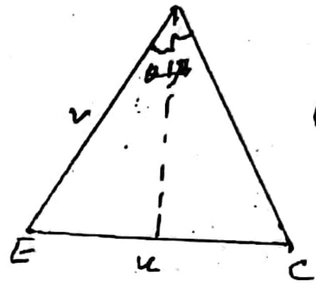


(10)

$V_{WC} = V_{WE} + V_{EC}$ (10)

$\tan\alpha = \frac{u + v\sin\theta}{v\cos\theta}$ (10)

$= \frac{u}{v\cos\theta} + \tan\theta$ (10)



(10)

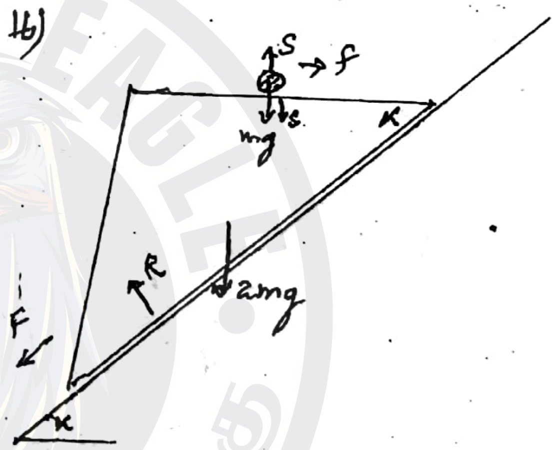
$\tan\beta = \frac{u - v\sin\theta}{v\cos\theta}$ (10)

$= \frac{u}{v\cos\theta} - \tan\theta$

$= \tan\alpha - \tan\theta - \tan\theta$

$2\tan\theta = \tan\alpha - \tan\beta$

16)



for the system ↙ $F = ma$

$2mg\sin\alpha + mg\sin\alpha = 2mF + m(F - f\cos\alpha)$ (15)

$3g\sin\alpha = 3F - f\cos\alpha$ —— ①

for m → $0 = m(f - F\cos\alpha)$ (15)

$f = F\cos\alpha$ —— ②

$F = \frac{3g\sin\alpha}{3-\cos^2\alpha} = \frac{3g\sin\alpha}{2+\sin^2\alpha}$ (10)

$f = \frac{3g\sin\alpha\cos\alpha}{2+\sin^2\alpha}$

$= \frac{3g\tan\alpha}{2+3\tan^2\alpha}$ (10)

for m ↑ $S - mg = m(-f\sin\alpha)$ (10)

$S = mg - \frac{3mg\sin^2\alpha}{2+\sin^2\alpha}$

$= \frac{2mg\cos^2\alpha}{2+\sin^2\alpha}$

$= \frac{2mg}{2+3\tan^2\alpha}$ (5)

for m related to wedge $s = ut + \frac{1}{2}at^2$

$d = 0 + \frac{1}{2}ft^2$

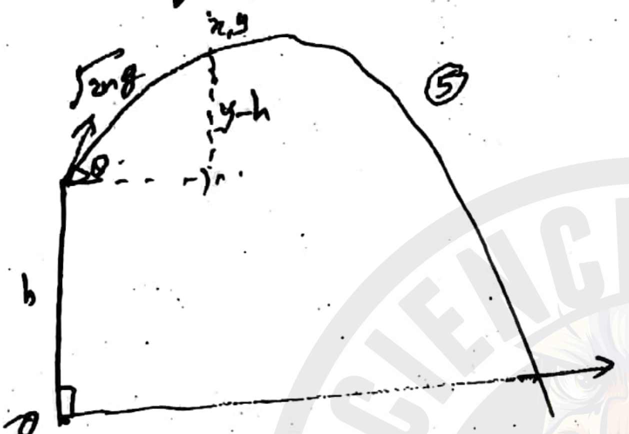
$t^2 = \frac{2d}{f}$ (10)

$= \frac{2d(2+3\tan^2\alpha)}{3g\tan\alpha}$

$= \frac{2d}{3g}\left[2\cot\alpha + 3\tan\alpha\right]$ (5)

$t = \left[\frac{2d}{3g}(2\cot\alpha + 3\tan\alpha)\right]^{\frac{1}{2}}$

b)



(5)

$\rightarrow x = \sqrt{2ng}\cos\theta \cdot t$ —— ① (10)

$\uparrow y - h = \sqrt{2ng}\sin\theta \cdot t - \frac{1}{2}gt^2$ (10)

$= x\tan\theta - \frac{gx^2}{2\times 2ng\cos^2\theta}$

$y - h = x\tan\theta - \frac{x^2}{4n}(1+\tan^2\theta)$ (10)

$x^2\tan^2\theta - 4nx\tan\theta + 4n(y-h) + x^2 = 0$

ii $x = 2h$, $y = 0$ (5)

$4h^2\tan^2\theta - 8hn\tan\theta - 4nh + 4h^2 = 0$ (10)

for a single elevation $\Delta = 0$ } (10)

$\Rightarrow \tan\theta = \frac{n}{h}$

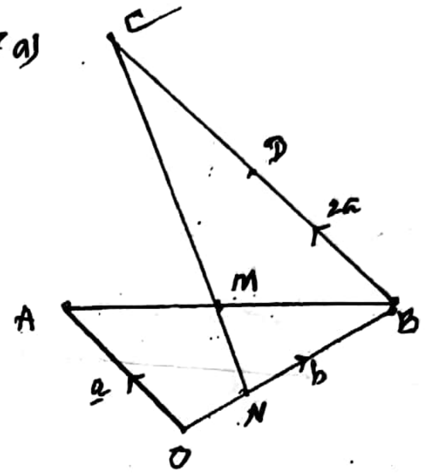
$+ 64h^2n^2 - 4\cdot 4h^2(4h^2 - 4nh) = 0$ (10)

$n^2 + nh - h^2 = 0$

$n = \frac{-h \pm \sqrt{h^2+4h^2}}{2}$

$= \left(\frac{\sqrt{5}-1}{2}\right)h$ $(n > 0)$

17 a)



$\vec{OM} = \vec{OA} + \vec{AM}$

$= a + \frac{1}{2}(b - a)$

$= \frac{1}{2}(a+b)$ (10)

$\vec{OD} = \vec{OB} + \vec{BD}$

$= b + a$ (10)

$= 2\vec{OM}$ (10)

$\Rightarrow$ O, M, D collinear.

$\vec{ON} = \lambda b$

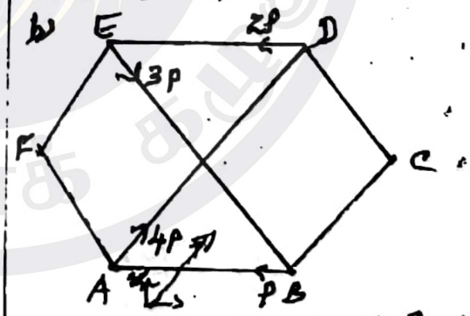
$\vec{NM} = -\lambda b + \frac{a+b}{2} = \frac{1}{2}a + \left(\frac{1}{2} - \lambda\right)b$ (10)

$\vec{MC} = \frac{1}{2}(b-a) + 2c = \frac{3}{2}a + \frac{1}{2}b$ (10)

$NM \parallel MC \Rightarrow \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{\frac{1}{2}-\lambda}{\frac{1}{2}} \Rightarrow \lambda = \frac{1}{3}$ (10)

ON : NB = 1 : 2.

b)



$\rightarrow X = -P - 2P + (4P + 3P)\cos 60$ (10)

$= \frac{P}{2}$ (5)

$\uparrow Y = (4P - 3P)\sin 60$ (10)

$= \frac{P\sqrt{3}}{2}$ (5)

$R = P$, $\tan\theta = \sqrt{3}$, $\theta = 60^\circ$ (15)

A↺ $2P \cdot 2a\cos 30 - 3P a\sin 60 = x \frac{P\sqrt{3}}{2}$ (10)

$x = a$ (5)

Resultant acts along BC. (10)

$G' = P \cdot a\sin 60$

$= \frac{\sqrt{3}}{2}aP$ (10)

(10)