வடமாகாணக் கல்வித் திணைக்களத்துடன் இணைந்து
தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்
3ம் தவணைப் பரீட்சை - 2020
Conducted by Field Work Centre, Thondaimanaru
In Collaboration with Provincial Department of Education Northern Province
3rd Term Examination - 2020

Grade - 12 (2021) | **Combined Maths** | **Marking Scheme**

1. $f(x) = x^2 - 2ax - 2bx + a^2 + b^2$

$= x^2 - 2(a+b)x + (a+b)^2 - 2ab$

$= \{x-(a+b)\}^2 - 2ab$ ⑤

$a+b = 3$ — ① ⑤

$-2ab = -4 \Rightarrow ab = 2$ — ② ⑤

①, ② $\Rightarrow a + \frac{2}{a} = 3$

$\Rightarrow a^2 - 3a + 2 = 0$

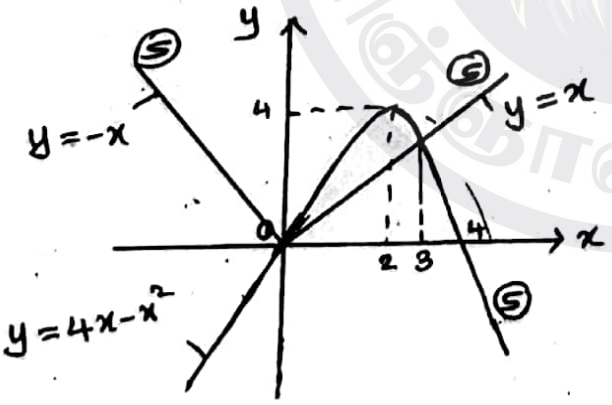
$\Rightarrow (a-1)(a-2) = 0$ ⑤

$\Rightarrow a = 1$ or $a = 2$

$a=1, b=2$ or $a=2, b=1$ ⑤ **25**

2. $y = 4x - x^2 = 4 - (x-2)^2$

$y = |x| = \begin{cases} x ; x \geq 0 \\ -x ; x < 0 \end{cases}$



⑤ ⑤ ⑤

$y = 4x - x^2$, $y = x$ $\Rightarrow 4x - x^2 = x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$ or $x = 3$ ⑤

$x(4-x) = |x|$

$4x - x^2 \geq |x|$

$0 \leq x \leq 3$ ⑤ **25**

3. $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$ ⑤

$x \equiv A(x+2) + B(x+1)$

x^1 : $1 = A + B$ — ① ⑤

x^0 : $0 = 2A + B$ — ②

①, ② $\Rightarrow A = -1, B = 2$

$\frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}$ ⑤

$\frac{2x}{(2x+1)(2x+2)} = \frac{-1}{(2x+1)} + \frac{2}{(2x+2)}$ ⑤

$\Rightarrow \frac{x}{(x+1)(2x+1)} = \frac{1}{x+1} - \frac{1}{2x+1}$ ⑤ **25**

4. $\lim_{x \to \frac{\pi}{2}} \frac{1-\sin x}{(2x-\pi)^2}$

$= \lim_{x \to \frac{\pi}{2}} \frac{1-\cos(\frac{\pi}{2}-x)}{4(\frac{\pi}{2}-x)^2}$ ⑤

$= \frac{1}{4} \lim_{x \to \frac{\pi}{2}} \frac{2\sin^2(\frac{\pi}{2}-x)}{(\frac{\pi}{2}-x)^2}$ ⑤

$= \frac{1}{8} \left\{ \lim_{x \to \frac{\pi}{2}} \frac{\sin(\frac{\pi}{2}-x)}{\frac{\pi}{2}-x} \right\}^2 = 8 \times 1^2$

$= 8$ ⑤ **25**

⑩

5. $\alpha = \tan^{-1}\left(\frac{1}{2x-1}\right)$, $\beta = \tan^{-1}\left(\frac{1}{2x+1}\right)$

$\tan\alpha = \frac{1}{2x-1}$, $\tan\beta = \frac{1}{2x+1}$

$\alpha + \beta = \tan^{-1} 2$ ⑤

$\Rightarrow \tan(\alpha+\beta) = 2$

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 2 \quad (5)$$

$$\frac{\frac{1}{2x-1} + \frac{1}{2x+1}}{1 - \frac{1}{2x-1}\cdot\frac{1}{2x+1}} = 2 \quad (5)$$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$x = -\frac{1}{2}$ or $x = 1$ (5)

$x \neq -\frac{1}{2}$ (5)

$\therefore x = 1$ (5) [25]

---

b) $x + 2y = \text{Length}$

$\ddot{x} + 2\ddot{y} = 0$ ——— (1) (5)

for m

$\downarrow mg - T = m\ddot{x}$ (5)

M $\downarrow Mg - 2T = M\ddot{y}$ (5)

$(2m - M)g = 2m\ddot{x} - M\ddot{y}$

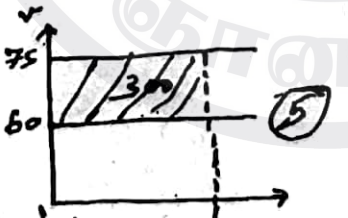
$= 2m\ddot{x} - M\left(-\frac{\ddot{x}}{2}\right)$

$\ddot{x} = \frac{2(2m-M)g}{4m+M}$ (5)

$M = 2m \Rightarrow$ system in rest (5)

---

(7)



(5)

distance travelled by A in t sec $= 75t$ (5)

" " B " $= 60t$ (5)

$75t - 60t = 300$ (5)

$t = \frac{300}{15}$

$= 20$ sec (5)

---

(8) $\underline{a}\cdot(\underline{a}+\underline{b}+\underline{c}) = 0$

$\underline{a}^2 + \underline{a}\cdot\underline{b} + \underline{a}\cdot\underline{c} = 0$ (5)

Similarly $\underline{a}\cdot\underline{b} + \underline{b}^2 + \underline{b}\cdot\underline{c} = 0$ (5)

$\underline{c}\cdot\underline{a} + \underline{c}\cdot\underline{b} + \underline{c}^2 = 0$ (5)

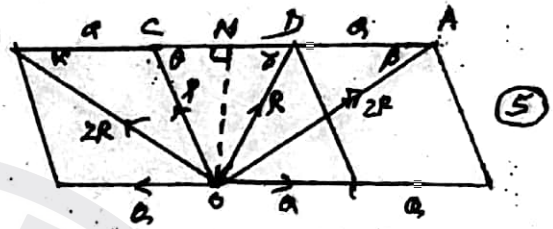
$a^2 + b^2 + c^2 + 2(\underline{a}\cdot\underline{b} + \underline{b}\cdot\underline{c} + \underline{c}\cdot\underline{a}) = 0$

$1^2 + 1^2 + 2^2 + 2(\underline{a}\cdot\underline{b} + \underline{b}\cdot\underline{c} + \underline{c}\cdot\underline{a}) = 0$ (5)

$\underline{a}\cdot\underline{b} + \underline{b}\cdot\underline{c} + \underline{c}\cdot\underline{a} = -\frac{21}{2}$ (5)

---

(9)



(5)

$OA = OB = 2R$ (5)

$\Rightarrow \alpha = \beta$

$CN = ND = \frac{a}{2}$ (5)

$\Rightarrow P = R$

$(2R)^2 - \left(\frac{3a}{2}\right)^2 = R^2 - \left(\frac{a}{2}\right)^2$ (5)

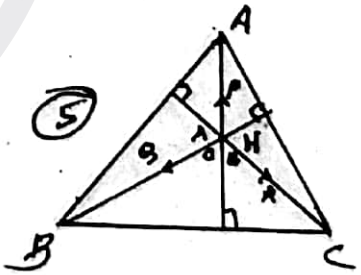
$4R^2 - R^2 = 2a^2$

$\sqrt{3}R = \sqrt{2}a$ (5)

$\Rightarrow \frac{R}{\sqrt{2}} = \frac{a}{\sqrt{3}} = \frac{P}{\sqrt{2}}$

---

(10)



(5)

In equilibrium

(5) $\frac{P}{\sin B\hat{A}C} = \frac{Q}{\sin A\hat{B}C} = \frac{R}{\sin A\hat{H}B}$

(5) $\frac{P}{\sin(B+C)} = \frac{Q}{\sin(A+C)} = \frac{R}{\sin(A+B)}$

(5) $\frac{P}{\sin A} = \frac{Q}{\sin B} = \frac{R}{\sin C}$

(5) $\frac{P}{a} = \frac{Q}{b} = \frac{R}{c}$

11] a) $x^2 - bx + c = 0$

$\alpha + \beta = b$
$\alpha\beta = c$ } (5)

$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ (5)
$= b^3 - 3cb$ (5)

$\alpha^3\beta^3 = (\alpha\beta)^3 = c^3$ (5)

The equation whose roots are α^3, β^3 is

$x^2 - (\alpha^3+\beta^3)x + \alpha^3\beta^3 = 0$ (5)
$x^2 - (b^3 - 3bc)x + c^3 = 0$ (5) ——— (*)

[30]

Let $y = 1 + x$ (5)

$x = \alpha^3 \Rightarrow y = 1 + \alpha^3$

$x = \beta^3 \Rightarrow y = 1 + \beta^3$

Put $x = (y-1)$ in (*)

$(y-1)^2 - (b^3 - 3bc)(y-1) + c^3 = 0$ (5)

The required equation is

$(x-1)^2 - (b^3 - 3bc)(x-1) + c^3 = 0$ (5)

[15]

b) $f(x) = 0$

$x^2 + (\lambda+2)x + 2\lambda = 0$

$\Delta = (\lambda+2)^2 - 4(1)(2\lambda)$ (10)
$= \lambda^2 - 4\lambda + 4$ (5)
$= (\lambda-2)^2$ (5)
≥ 0 (5)

$\Delta \neq 0$, Hence $f(x) = 0$ has real roots (5)

[30]

Roots of $f(x) = 0$ are α, δ (real)

$\alpha + \delta = -(\lambda+2)$ } (5)
$\alpha\delta = 2\lambda$

$|\alpha - \delta| = 1$ (5)

$(\alpha-\delta)^2 = 1$

$(\alpha+\delta)^2 - 4\alpha\delta = 1$ (5)

$[-(\lambda+2)]^2 - 4(2\lambda) = 1$

$\lambda^2 - 4\lambda + 3 = 0$ (5)

$(\lambda-3)(\lambda-1) = 0$

$\lambda = 3$ or $\lambda = 1$ (5)

[25]

c) $g(x) = x^3 - 3x + 5$

$g(x) = (x-2)(x-1)\phi(x) + Ax + B$ (10)

$g(2) = 2A + B$
$7 = 2A + B$ ——— (1) (5)

$g(1) = A + B$
$3 = A + B$ ——— (2) (5)

(1), (2) $\Rightarrow$ $A = 4$, $B = -1$ (10)

Reminder $= Ax + B$
$= 4x - 1$

$x^3 - 3x + 5 = (x-2)(x-1)\phi(x) + 4x - 1$

$\Rightarrow x^3 - 7x + 6 = (x-2)(x-1)\phi(x)$ (5)
$= (x-2)(x-1)(x-k)$ (5)

x^0; $6 = -2k$
$k = -3$ (5)

$x^3 - 7x + 6 = (x-2)(x-1)(x+3)$ (5)

[50]

12. (a)

$$f(x) = \frac{3x^2-1}{x^3}$$

$$f'(x) = \frac{x^3(6x) - (3x^2-1)3x^2}{x^6} \quad (10)$$

$$= \frac{6x^2 - 9x^2 + 3}{x^4} \quad (5)$$

$$= -\frac{3(x^2-1)}{x^4} \quad [15]$$

There is no y intercept beacause $f(0)$ is undefined

$f(x) = 0 \Rightarrow \frac{3x^2-1}{x^3} = 0$

$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$

$\lim_{x\to 0^-} \frac{3x^2-1}{x^3} = -\infty$, $\lim_{x\to 0^+} \frac{3x^2-1}{x^3} = \infty$

$\therefore x = 0$ is a vertical asymptote. (5)

$\lim_{x\to\pm\infty} \frac{3x^2-1}{x^3} = \lim_{x\to\pm\infty}\left\{\frac{3}{x} - \frac{1}{x^3}\right\} = 0$

$\therefore y = 0$ is a horizontal asymptote. (5)

$f'(x) = 0 \Leftrightarrow x = \pm 1$ (5)

	$x<-1$	$-1<x<0$	$0<x<1$	$x>1$
sign of $f'(x)$	(−)	(+)	(+)	(−)
	decreases	increases	increases	decreases
	(5)	(5)	(5)	(5)

$(-1, -2)$ is a local minimum and $(1, 2)$ is a local maximum (5)

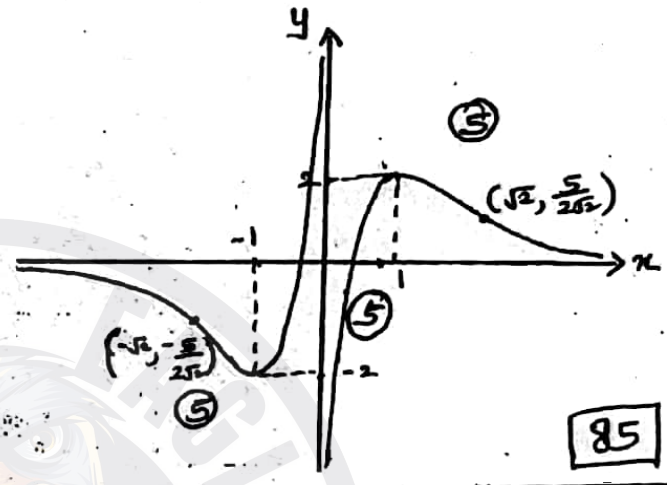
$f''(x) = \frac{6(x^2-2)}{x^5} = \frac{6(x-\sqrt{2})(x+\sqrt{2})}{x^5}$

$f''(x) = 0 \Leftrightarrow x = \pm\sqrt{2}$ (5)

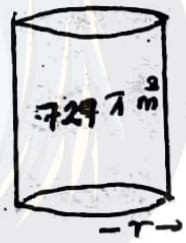
$f(\sqrt{2}) = \frac{5}{2\sqrt{2}}$, $f(-\sqrt{2}) = -\frac{5}{2\sqrt{2}}$

	$x<-\sqrt{2}$	$-\sqrt{2}<x<0$	$0<x<\sqrt{2}$	$x>\sqrt{2}$
sign of $f''(x)$	(−)	(+)	(−)	(+)
	concave down	concave up	concave down	concave up
	(5)	(5)	(5)	(5)

$(-\sqrt{2}, -\frac{5}{2\sqrt{2}})$, $(\sqrt{2}, \frac{5}{2\sqrt{2}})$ are inflection points (5)



(5) (5) (5) [85]



Volume $= 729\pi$

$\pi r^2 h = 729\pi$

$h = \frac{729}{r^2}$ —(1) (5)

Total surface area $A = 2\pi rh + \pi r^2$ (5)

$A = 2\pi\left(\frac{729}{r}\right) + \pi r^2$ (5)

$\frac{dA}{dr} = 2\pi(729)\left(-\frac{1}{r^2}\right) + \pi(2r)$ (10)

$= 2\pi\left(-\frac{729}{r^2} + r\right)$

$= 2\pi\left(\frac{r^3 - 9^3}{r^2}\right)$ (5)

$\frac{dA}{dr} = 0 \Leftrightarrow r = 9$ (5)

$r < 9$ $\quad r > 9$ (10)

$\frac{dA}{dr} < 0$ $\quad \frac{dA}{dr} > 0$

$\therefore$ A is minimum when $r = 9$m

$r = 9 \Rightarrow h = 9$m (5)

[50]

Q4 (i) $\cos 2\theta = \cos(\theta+\theta)$

$= \cos\theta.\cos\theta - \sin\theta.\sin\theta$ ⑤

$= (1-\sin^2\theta) - \sin^2\theta$ ⑤

$= 1 - 2\sin^2\theta$.

$\sin 3\theta = \sin(2\theta+\theta)$ ⑤

$= \sin 2\theta.\cos\theta + \cos 2\theta.\sin\theta$

$= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$ ⑤

$= 2\sin\theta(1-\sin^2\theta) + (1-2\sin^2\theta)\sin\theta$ ⑤

$= 3\sin\theta - 4\sin^3\theta$.

$\cos 2\theta - \sin 3\theta$

$= 1 - 2\sin^2\theta - 3\sin\theta + 4\sin^3\theta$ ⑤ ⑤

$\sin\theta = t$

$= 4t^3 - 2t^2 - 3t + 1$ 35

$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$ $\cos 2\theta - \sin 3\theta = 0$

$\cos 2\theta = \sin 3\theta = \cos(\pi/2 - 3\theta)$ ⑤

$2\theta = 2n\pi \pm (\pi/2 - 3\theta)$ $n \in \mathbb{Z}$

(+) $5\theta = 2n\pi + \pi/2$ ⑤ (−) $-\theta = 2n\pi - \pi/2$ ⑤

$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$ $\theta = \frac{9\pi}{10}, \frac{\pi}{2}, \frac{13\pi}{10}$ are solutions

⑤ ⑤ ⑤

The roots of $4t^3 - 2t^2 - 3t + 1 = 0$ are

$\sin 13\pi/10$, $\sin \pi/2$ & $\sin \frac{9\pi}{10}$ ⑤

$\sin \pi/2 = 1 \Rightarrow t-1$ is a factor of $4t^3 - 2t^2 - 3t + 1$ ⑤

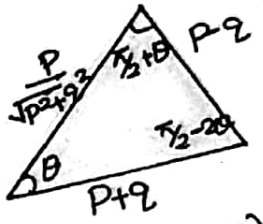
$\Rightarrow 4t^3 - 2t^2 - 3t + 1 = (t-1)(4t^2 + 2t - 1) = 0$ ⑤

$\sin\frac{13\pi}{10}$, $\sin\frac{9\pi}{10}$ are roots of $4t^2 + 2t - 1 = 0$ ③

$\Rightarrow t = \frac{-2 \pm \sqrt{4+16}}{8} = \frac{\pm\sqrt{5}-1}{4}$ ⑤

$t = \frac{\sqrt{5}-1}{4}$ $t = -\frac{\sqrt{5}-1}{4}$

$\sin\frac{9\pi}{10} = \frac{\sqrt{5}-1}{4}$ ⑤ $\sin\frac{9\pi}{10} > 0$ 60



$\frac{\sin\theta}{p-q} = \frac{\sin(\pi-2\theta)}{p/\sqrt{p^2+q^2}} = \frac{\sin(\pi/2+\theta)}{p+q}$ ⑩

$\frac{\sin\theta}{p-q} = \frac{\cos 2\theta}{p}.\sqrt{p^2+q^2} = \frac{\cos\theta}{p+q} = k$ ⑤

$\sin^2\theta + \cos^2\theta = 1$ ⑤

$k^2(p-q)^2 + k^2(p+q)^2 = 1$ ⑤

$2k^2(p^2+q^2) = 1$

$k = \frac{1}{\sqrt{2(p^2+q^2)}}$ ⑤

$\cos\theta = k(p+q) = \frac{(p+q)}{\sqrt{2(p^2+q^2)}}$ ⑤

$\sin\theta = k(p-q) = \frac{(p-q)}{\sqrt{2(p^2+q^2)}}$ ⑤

$\cos 2\theta = \frac{kp}{\sqrt{p^2+q^2}} = \frac{p}{\sqrt{2(p^2+q^2)}}$ —① ⑤

$\cos 2\theta = 2\cos^2\theta - 1$

$= \frac{2(p+q)^2}{2(p^2+q^2)} - 1 = \frac{4pq}{2(p^2+q^2)}$ —② ⑤

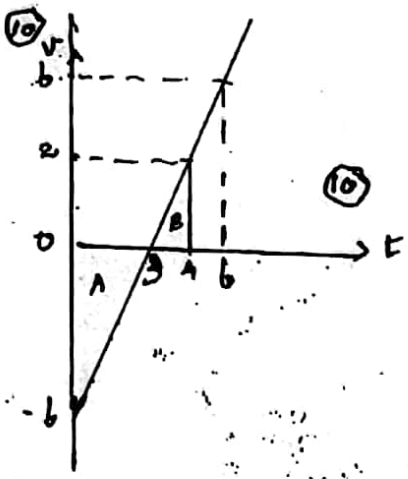
①&② $\frac{p}{\sqrt{2}(p^2+q^2)} = \frac{4pq}{2(p^2+q^2)}$ ⑤

$q = \frac{1}{2\sqrt{2}}$ 55

(15) (a)

$x = t^2 - 6t + 5$

$\dot{x} = 2t - 6$ (10)



(10)

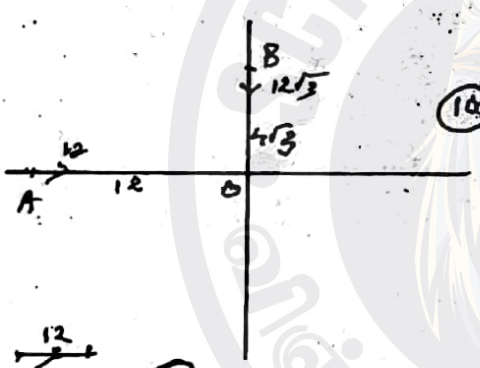
$A = -\frac{1}{2} \times 6 \cdot 3 = 9$ (10)

$B = \frac{1}{2} \cdot 1 \cdot 2 = 1$

displacement in 4 sec $= -9 + 1 = -8$ (10)

Average velocity $= -\frac{8}{4} = -2$ (10)

(b)

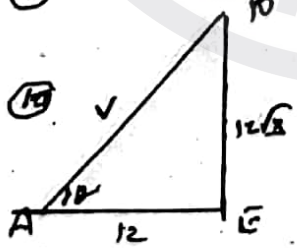


(10)

V_{AE} $\xrightarrow{12}$ (10)

V_{BE} $\downarrow 12\sqrt{3}$

$V_{AB} = V_{AE} + V_{EB}$ (10)



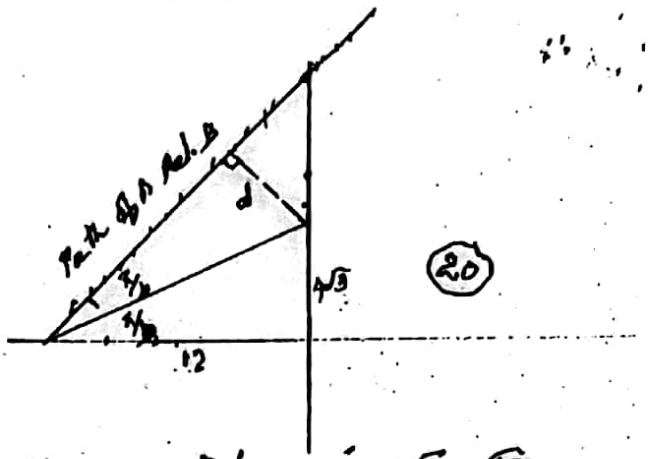
(10)

$V_{AB}^2 = 12^2 + (12\sqrt{3})^2$ (10)

$V_{AB} = 24$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$ (10)



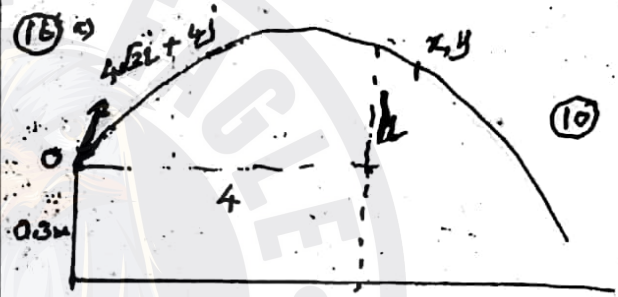
(20)

Minimum distance is $4\sqrt{3}$ (10)

time taken to this $= \frac{12}{24}$

$= 30$ min (10)

(16) (a)



(10)

$\rightarrow \quad x = 4\sqrt{3}t$ (10)

$\uparrow \quad y = 4t - \frac{1}{2}gt^2$ (10)

$= \frac{x}{\sqrt{3}} - \frac{1}{2} \cdot g \cdot \frac{x^2}{48}$

$y = \frac{x}{\sqrt{3}} - \frac{49x^2}{480}$

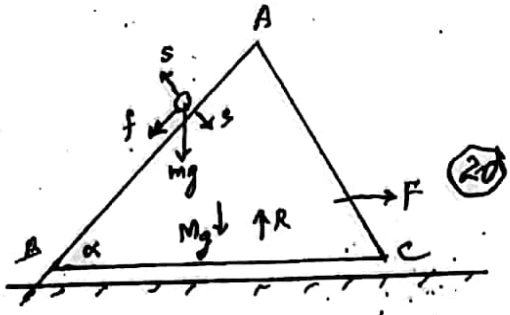
$x = 4, \quad y = h$

$h = \frac{4}{\sqrt{3}} - \frac{49 \times 16}{480}$ (10)

$\therefore H = \frac{4\sqrt{3}}{3} - \frac{49}{30} + 0.3$ (10)

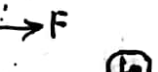
$\simeq 4.2$ m.

16)



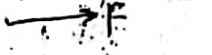
(20)

for m acceleration



(10)

for M



for m Force

$mg\sin\alpha = m(f - F\cos\alpha)$ (20)

For M+m →

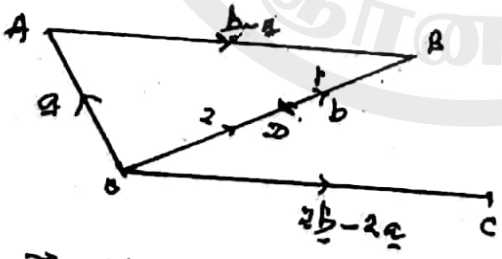
$0 = Mf + m(F - f\cos\alpha)$ (20)

$0 = (M+m)F - mf\cos\alpha$

$F = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}$ (10)

$f = \frac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}$ (10)

17) Theory - (10)



$\vec{AD} = \vec{AO} + \vec{OD}$

$= -\underline{a} + \frac{2}{3}\underline{b}$

$= (2\underline{b} - 3\underline{a})\frac{1}{3}$ (10)

$\vec{AC} = \vec{AO} + \vec{OC}$

$= -\underline{a} + 2\underline{b} - \underline{a}$ (10)

$= 2\underline{b} - 3\underline{a}$

$= 3\vec{AD}$ (5)

⇒ A, C, D Collinear. (5)

AC, OB intersect at D (5)

from the first part

any point on OA can be written in the form $\underline{r} = \mu\underline{a}$ (5)

& the point on CB is

$\underline{r} = \lambda(2\underline{b} - 2\underline{a}) + (1-\lambda)\underline{b}$ (5)

∴ intersecting point is given by

$\mu\underline{a} = \lambda(2\underline{b} - 2\underline{a}) + (1-\lambda)\underline{b}$ (5)

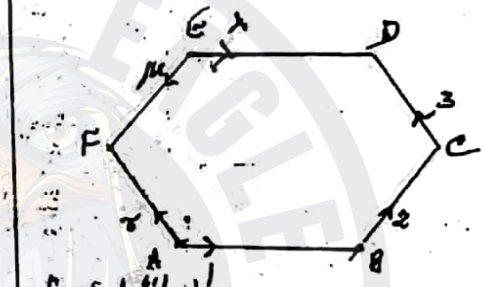
$(\mu + 2\lambda)\underline{a} + (\lambda - 1 - 2\lambda)\underline{b} = \underline{0}$

$\Rightarrow \mu + 2\lambda = 0 \quad + \quad -\lambda - 1 = 0$

$\lambda = -1$ (10)

$\mu = 2$

∴ Intersecting point is $2\underline{a}$



(5)

for Equilibrium.

$\rightarrow X = 1 + \lambda + 2\cos\theta - 3\cos\theta - \mu\cos\theta - \gamma\cos\theta$ (10)

$\lambda - \frac{\mu}{2} - \frac{\gamma}{2} + \frac{1}{2} = 0$

$\uparrow (2 + 3 - \mu + \gamma)\sin\theta = 0$ (10)

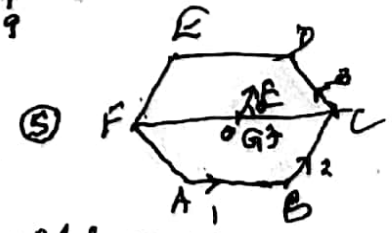
$\gamma - \mu + 5 = 0$

$\rightarrow -\lambda a\sin\theta + 3.2a\sin\theta + 2.2a\sin\theta + 1.a\cos\theta = 0$ (10)

$\lambda = 11$

$\gamma = 14$ (15)

$\gamma = 9$



(5)

$\rightarrow F\cos\alpha - 3\cos\theta + 2\cos 2\theta + 1 = 0$

$\cos\alpha = -\frac{1}{2}$ (5)

$\uparrow F\sin\alpha + 2\sin\theta + 2\sin 2\theta = 0$ (5)

$F\sin\alpha = -\frac{5\sqrt{3}}{2}$

$\tan\alpha = 5\sqrt{3}$ (5)

$F = \sqrt{19}$ N (5)

(8) $Q + (2+1+1)R\sin\theta = 0$

$Q = 3\sqrt{3}a$ N (5)