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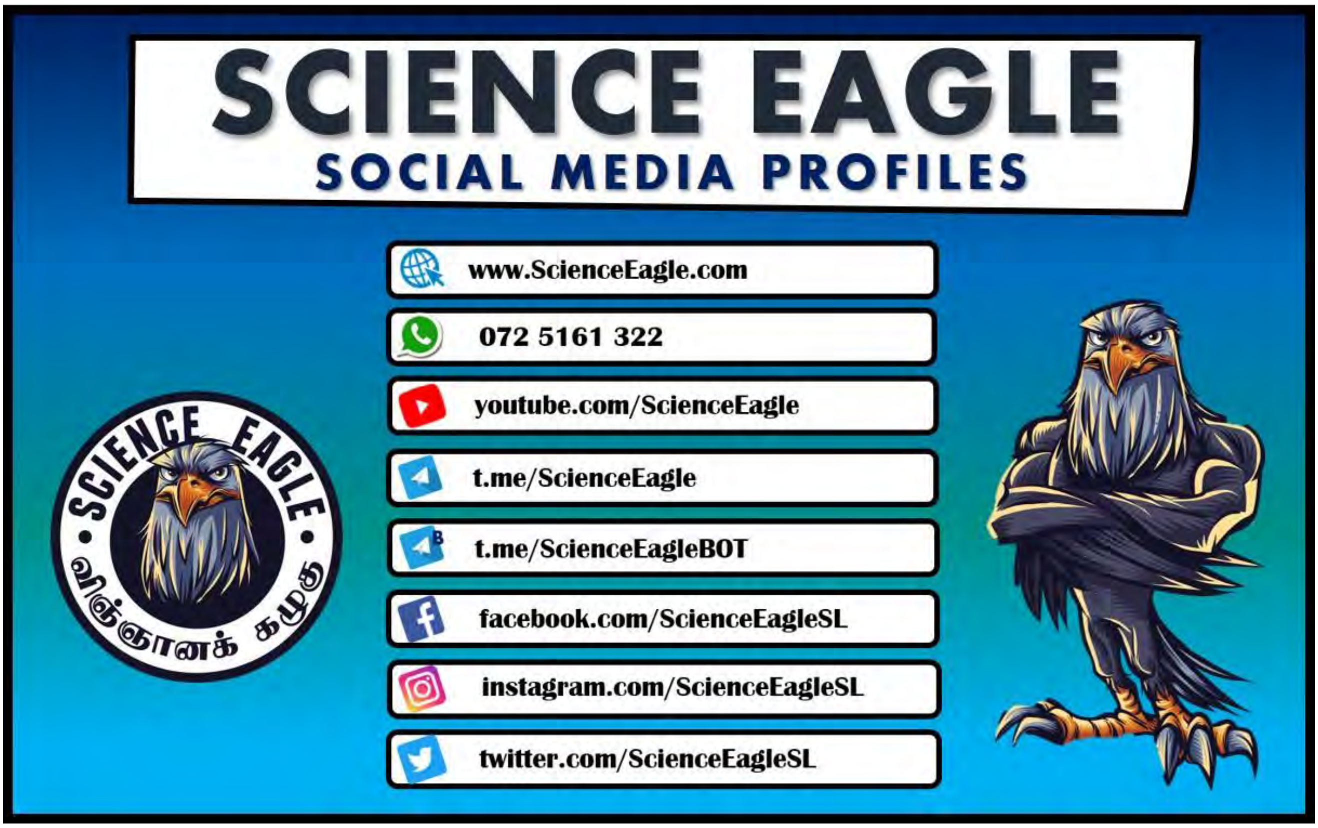
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தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்
ஐந்தாம் தவணைப் பரீட்சை-2022
Conducted by Field Work Centre, Thondaimanaru
Fifth Term Examination – 2022

Grade 13(2022) **Combined Mathematics I** **Marking Scheme**

1. $\sum_{r=1}^{n}(-1)^{r-1}r^2 = (-1)^{n-1}\frac{n(n+1)}{2}$

For $n=1$, L.H.S $= \sum_{r=1}^{1}(-1)^{r-1}r^2 = (-1)^0 1^2 = 1$

R.H.S $= (-1)^0\frac{1(2)}{2} = 1$

∴ the result is true for $n=1$ 5

Take any $p \in \mathbb{Z}^+$ and assume that the result is true for $n=p$

i.e., $\sum_{r=1}^{p}(-1)^{r-1}r^2 = (-1)^{p-1}\frac{p(p+1)}{2}$ 5

Now $\sum_{r=1}^{p+1}(-1)^{r-1}r^2 = \sum_{r=1}^{p}(-1)^{r-1}r^2 + (-1)^p(p+1)^2$ 5

$= (-1)^{p-1}\frac{p(p+1)}{2} + (-1)^p(p+1)^2$

$= (-1)^p\frac{(p+1)}{2}\left[-p+2(p+1)\right]$

$= (-1)^p\frac{(p+1)(p+2)}{2}$ 5

∴ the result is true for $n = p+1$

Hence, by the Principle of Mathematical Induction, the result is true for all $n \in \mathbb{Z}^+$ 5

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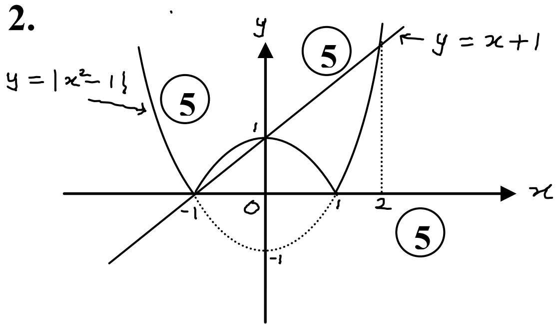
2.



$|x^2-1| \le x+1$

$\Leftrightarrow 0 \le x \le 2$ or $x = -1$ 10

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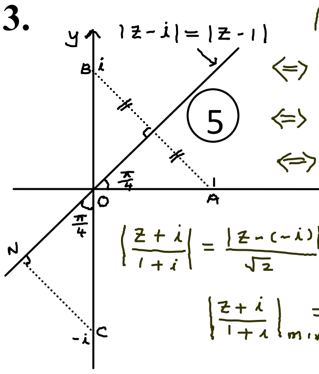
3.



$|iz-i| = |iz+1|$

$\Leftrightarrow |i(z-1)| = |iz-i^2|$ 5

$\Leftrightarrow |i||z-1| = |i||z-i|$ 5

$\Leftrightarrow |z-1| = |z-i|$ 5

$\left|\frac{z+i}{1+i}\right| = \frac{|z-(-i)|}{\sqrt{2}}$ 5

$\left|\frac{z+i}{1+i}\right|_{min} = \frac{1}{\sqrt{2}}CN = \frac{1}{\sqrt{2}}\sin\frac{\pi}{4} = \frac{1}{2}$ 5

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4. $\lim_{x\to\frac{\pi}{2}}\frac{1-\sqrt{\sin x}}{(2x-\pi)^2}$

$= \lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{(2x-\pi)^2(1+\sqrt{\sin x})}$ 5

$= \lim_{x\to\frac{\pi}{2}}\frac{1-\sin^2 x}{(2x-\pi)^2(1+\sqrt{\sin x})(1+\sin x)}$

$= \lim_{x\to\frac{\pi}{2}}\frac{\sin^2(\frac{\pi}{2}-x)}{4(x-\frac{\pi}{2})^2(1+\sqrt{\sin x})(1+\sin x)}$ 5

$= \left(\lim_{(\frac{\pi}{2}-x)\to 0}\frac{\sin(\frac{\pi}{2}-x)}{\frac{\pi}{2}-x}\right)^2 \cdot \lim_{x\to\frac{\pi}{2}}\frac{1}{4(1+\sqrt{\sin x})(1+\sin x)}$ 5

$= 1^2 \times \frac{1}{4\times 2\times 2} = \frac{1}{16}$.

5 5

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5. $\frac{d}{dx}\left\{x\ln(\sqrt{x-1}+\sqrt{x+1})\right\}$

$= x\frac{1}{\sqrt{x-1}+\sqrt{x+1}}\left\{\frac{1}{2\sqrt{x-1}}+\frac{1}{2\sqrt{x+1}}\right\} + \ln(\sqrt{x-1}+\sqrt{x+1})$ 5 5

$= \frac{x}{2\sqrt{x^2-1}} + \ln(\sqrt{x-1}+\sqrt{x+1})$

$\int\frac{x}{2\sqrt{x^2-1}}dx + \int\ln(\sqrt{x-1}+\sqrt{x+1})\,dx$

$= x\ln(\sqrt{x-1}+\sqrt{x+1}) + \text{constant}$ 5

$\Rightarrow \frac{1}{4}\frac{(x^2-1)^{\frac{1}{2}}}{\frac{1}{2}} + \int\ln(\sqrt{x-1}+\sqrt{x+1})\,dx$

$= x\ln(\sqrt{x-1}+\sqrt{x+1}) + \text{constant}$ 5

$\int\ln(\sqrt{x-1}+\sqrt{x+1})\,dx = x\ln(\sqrt{x-1}+\sqrt{x+1}) - \frac{1}{2}\sqrt{x^2-1} + c$ 5

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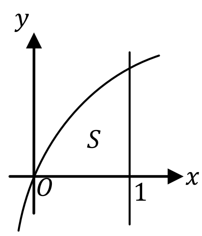
6. Area $= \int_0^1\frac{x}{\sqrt{x^2+1}}dx$ 5



$= \sqrt{x^2+1}\Big|_0^1$

$= \sqrt{2}-1$ square units

Volume $= \int_0^1\pi\left(\frac{x}{\sqrt{x^2+1}}\right)^2dx$ 5 5

$= \pi\int_0^1\frac{x^2}{x^2+1}dx$

$= \pi\int_0^1\frac{x^2+1-1}{x^2+1}dx$ 5

$= \pi\int_0^1\left(1-\frac{1}{x^2+1}\right)dx$

$= \pi(x-\tan^{-1}x)\Big|_0^1 = \pi(1-\frac{\pi}{4}) = \frac{\pi}{4}(4-\pi)$ cubic units. 5

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7. $x = \frac{c}{t}$ $\quad y = ct$ 5

$\frac{dx}{dt} = -\frac{c}{t^2}$ $\quad \frac{dy}{dt} = c$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{c}{-\frac{c}{t^2}} = -t^2$ 5

$\therefore$ the gradient of the normal at $(\frac{c}{t}, ct)$ is $\frac{1}{t^2}$

The equation of the normal at $(\frac{c}{t}, ct)$ is

$y - ct = \frac{1}{t^2}(x - \frac{c}{t})$ 5

$t^3 y - tx = c(t^4 - 1)$

$(\frac{c}{t}, ct) \equiv (\frac{c}{2}, 2c) \Rightarrow t = 2$

Equation of the normal at $P(\frac{c}{2}, 2c)$ is

$8y - 2x = 15c$ 5

Since this line passes through $(\frac{c}{T}, cT)$

$8cT - \frac{2c}{T} = 15c \Rightarrow 8T^2 - 15T - 2 = 0$ 5

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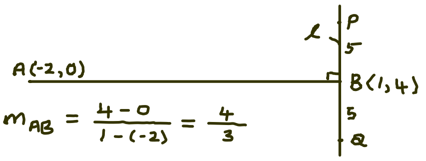
8.



$m_{AB} = \frac{4-0}{1-(-2)} = \frac{4}{3}$

Equation of l is $y - 4 = -\frac{3}{4}(x-1)$ 5

$\frac{y-4}{3} = \frac{x-1}{-4} = t$ (say)

$x = 1 - 4t, \quad y = 4 + 3t$ 5

$(1-4t, 4+3t)$

$\sqrt{(1-4t-1)^2 + (4+3t-4)^2} = 5$ 5

$5|t| = 5$

$t = \pm 1$

$t = 1 \Rightarrow P \equiv (-3, 7)$ 5

$t = -1 \Rightarrow Q \equiv (5, 1)$ 5

25

9. Centre $C \equiv (1, 2)$

radius $r = \sqrt{1+4+4} = 3$ 5

Perpendicular distance $= \frac{|1+3(2)-6|}{\sqrt{1+9}} = \frac{1}{\sqrt{10}} < 3$ 5

$\therefore$ the line $x + 3y - 6 = 0$ intersects the circle at two distinct points.

The equation of the required circle can be written as

$x^2 + y^2 - 2x - 4y - 4 + \lambda(x + 3y - 6) = 0$ 5

Centre $(-\frac{1}{2}(\lambda-2), -\frac{1}{2}(3\lambda-4))$ lies on $x + 3y - 6 = 0$

$-\frac{1}{2}(\lambda - 2) - \frac{3}{2}(3\lambda - 4) - 6 = 0$

$\lambda = \frac{1}{5}$ 5

$5x^2 + 5y^2 - 9x - 17y - 26 = 0$ 5

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10. $2\sin x \cos x + 2\sin^2 x - 1$

$= \sin 2x - \cos 2x$ 5

$= \sqrt{2}(\frac{1}{\sqrt{2}} \sin 2x - \frac{1}{\sqrt{2}} \cos 2x)$

$= \sqrt{2}(\cos\frac{\pi}{4} \sin 2x - \sin\frac{\pi}{4} \cos 2x)$

$= \sqrt{2} \sin(2x - \frac{\pi}{4})$ 5

$\sin x \cos x + \sin^2 x = 1$

$2\sin x \cos x + 2\sin^2 x - 1 = 1$

$\sqrt{2} \sin(2x - \frac{\pi}{4}) = 1$ 5

$\sin(2x - \frac{\pi}{4}) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$ 5

$2x - \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}$; $n \in \mathbb{Z}$

$x = \frac{n\pi}{2} + (-1)^n \frac{\pi}{8} + \frac{\pi}{8}$; $n \in \mathbb{Z}$ 5

25

11.(a) $f(x) = x^2 - (\lambda+2)x + (2\lambda - 1) = 0$

Discriminant $\Delta = (\lambda+2)^2 - 4(2\lambda - 1)$ 5

$= \lambda^2 - 4\lambda + 8$

$= (\lambda - 2)^2 + 4 > 0 \; [\because (\lambda-2)^2 \geq 0]$

5 5

$\therefore$ $f(x) = 0$ has two distinct real roots. 5

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$\alpha + \beta = \lambda + 2$ 5

$\alpha\beta = 2\lambda - 1$

$\alpha > 0$ and $\beta > 0$

$\Longleftrightarrow \alpha + \beta > 0$ and $\alpha\beta > 0$ 5

$\Longleftrightarrow \lambda + 2 > 0$ and $2\lambda - 1 > 0$ 5

$\Longleftrightarrow \lambda > -2$ and $\lambda > \frac{1}{2}$ 5

$\Longleftrightarrow \lambda > \frac{1}{2}$ 5

25

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ 5

$= (\lambda+2)^2 - 2(2\lambda - 1)$

$= \lambda^2 + 6$ 5

$\alpha^2\beta^2 = (\alpha\beta)^2 = (2\lambda - 1)^2$ 5

The required equation is

$(x - \alpha^2)(x - \beta^2) = 0$ 5

$\Rightarrow x^2 - (\alpha^2 + \beta^2)x + \alpha^2\beta^2 = 0$

$\Rightarrow x^2 - (\lambda^2 + 6)x + (2\lambda - 1)^2 = 0.$ 5

25

Let $y = 1 + x$. 5

When $x = \alpha^2$, $y = 1 + \alpha^2$ and

when $x = \beta^2$, $y = 1 + \beta^2$.

Putting $x = y - 1$ in $x^2 - (\lambda^2+6)x + (2\lambda - 1)^2 = 0$,

we have $(y-1)^2 - (\lambda^2+6)(y-1) + (2\lambda-1)^2 = 0$

$y^2 - 2y + 1 - (\lambda^2+6)(y-1) + 4\lambda^2 - 4\lambda + 1 = 0$ 5

5

i.e., $y^2 - (\lambda^2 + 8)y + 5\lambda^2 - 4\lambda + 8 = 0$ 5

20

Replacing y by x we have

$x^2 - (\lambda^2 + 8)x + 5\lambda^2 - 4\lambda + 8 = 0$

(b) Since $x-1$ and $x+2$ are factors of $f(x)$.

$f(1)=0$ and $f(-2)=0$

$4+5+a+b=0$ and $4(-2)^3+5(-2)^2-2a+b=0$

5 5

$a+b=-9 \quad (1)$ and $2a-b=-12 \quad (2)$

$(1),(2) \Rightarrow a=-7,\ b=-2$

5 5

$g(x)=(x-1)^2\phi(x)$, $g(x)=x^3+cx+2$

$\Rightarrow g(1)=0$ 5

$\Rightarrow 1+c+2=0$

$\Rightarrow c=-3$ 5

$\boxed{30}$

$f(x)=4x^3+5x^2-7x-2$, $g(x)=x^3-3x+2$

$f(x)-4g(x)$

$=4x^3+5x^2-7x-2-4(x^3-3x+2)$ 5

$=5x^2+5x-10$ 5

$=5(x^2+x-2)$

$=5\left(x+\frac{1}{2}\right)^2-\frac{45}{4} \geq -\frac{45}{4} \quad \left[\because \left(x+\frac{1}{2}\right)^2 \geq 0\right]$

5 5

$f(x)-4g(x)=5\{(x+2)^2-3x-6\}$

$=5(x+2)^2-15(x+2)$ 5

Hence the remainder is $-15(x+2)$. 5

$\boxed{30}$

12. (a)

(i) Number of 4 different digits

$\begin{matrix} 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 3 & 4 & 5 \end{matrix}$

$= {}^5P_4 = 5! = 120$ 5

10

$\boxed{15}$

(ii) Number of distinct 4-digi numbers excluding 3 $= {}^4P_4$ 10

$= 4!$

$= 24$ 5

$\boxed{15}$

Cases	Number of different 4 di it numbers
Four alike	${}^2C_1 = 2$ 5
Three alike, one different	${}^3C_1\,{}^4C_1\,\frac{4!}{3!} = 48$ 5
Two alike, two others alike	${}^4C\ \frac{4!}{2!\,2!} = 36$ 5
Two alike, the other two different	${}^4C_1\ C_2\ \frac{4!}{2!} = 144$ 5
All four different	${}^5C_4 \cdot 4! = 120$ 5

The required number of ways

$= 2+48+36+144+120$ 5

$= 350.$ 5

$\boxed{35}$

(b) $U_r = \frac{9r^3+21r^2+13r-1}{(3r-1)^2(3r+2)^2}$, $U_r = \frac{Ar}{(3r-1)^2} - \frac{r+B}{(3r+2)^2}$

$\frac{9r^3+21r^2+13r-1}{(3r-1)^2(3r+2)^2} = \frac{Ar}{(3r-1)^2} - \frac{r+B}{(3r+2)^2}$

$9r^3+21r^2+13r-1 = Ar(3r+2)^2-(r+B)(3r-1)^2$ 5

comparing coefficient of power of r:

r^3: $9=9A-9$

r^2: $21=12A+6-9B$

r^1: $13=4A-1+6B$ 15

r^0: $-1=-B$

$A=2$ 5

$B=1$ 5

$\boxed{30}$

$U_r = \frac{2r}{(3r-1)^2} - \frac{r+1}{(3r+2)^2}$

$\left(\frac{1}{2}\right)^{r+1} U_r = \frac{r}{(3r-1)^2}\left(\frac{1}{2}\right)^r - \frac{r+1}{(3r+2)^2}\left(\frac{1}{2}\right)^{r+1}$ 5

$\left(\frac{1}{2}\right)^{r+1} U_r = f(r)-f(r+1)$; where $f(r)=\frac{r}{(3r-1)^2}\left(\frac{1}{2}\right)^r$ 5

$r=1,\ \left(\frac{1}{2}\right)^2 u_1 = f(1) - \cancel{f(2)}$ 5

$r=2;\ \left(\frac{1}{2}\right)^3 u_2 = \cancel{f(2)} - \cancel{f(3)}$

$\vdots$

$r=n-1;\ \left(\frac{1}{2}\right)^n u_{n-1} = \cancel{f(n-1)} - \cancel{f(n)}$ 5

$r=n;\ \left(\frac{1}{2}\right)^{n+1} u_n = \cancel{f(n)} - f(n+1)$

$\sum_{r=1}^{n}\left(\frac{1}{2}\right)^{r+1} U_r = f(1)-f(n+1) = \frac{1}{8} - \frac{(n+1)}{(3n+2)^2}\left(\frac{1}{2}\right)^{n+1}$

5 5

$\boxed{30}$

$\lim_{n\to\infty} \sum_{r=1}^{n}\left(\frac{1}{2}\right)^{r+1} U_r = \lim_{n\to\infty}\left\{\frac{1}{8} - \frac{\left(\frac{1}{n}+\frac{1}{n^2}\right)}{\left(3+\frac{2}{n}\right)^2}\left(\frac{1}{2}\right)^{n+1}\right\}$

$= \frac{1}{8} - 0 = \frac{1}{8}$ 5

$\therefore \sum_{r=1}^{\infty}\left(\frac{1}{2}\right)^{r+1} U_r$ is convergent and the sum is $\frac{1}{8}$.

5

$\boxed{10}$

$\sum_{r=2}^{\infty}\left(\frac{1}{2}\right)^r U_r = 2\sum_{r=1}^{\infty}\left(\frac{1}{2}\right)^{r+1} U_r - \frac{1}{2}u_1 = 2\left(\frac{1}{8}\right) - \frac{21}{100} = \frac{1}{25}.$ 5

5 5

$\boxed{15}$

13. (a) $Z_1 = 2(\sqrt{3}+i)$, $Z_2 = 2(1-i)$

$\frac{Z_1}{Z_2} = \frac{2(\sqrt{3}+i)}{2(1-i)} = \frac{(\sqrt{3}+i)(1+i)}{1^2-i^2} = \frac{(\sqrt{3}-1)+i(\sqrt{3}+1)}{2}$

5

$\frac{Z_1}{Z_2} = \left(\frac{\sqrt{3}-1}{2}\right) + i\left(\frac{\sqrt{3}+1}{2}\right)$ 5

$\boxed{10}$

$Z_1 = 2(\sqrt{3}+i) = 4\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right) = 4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

5 5

$Z_2 = 2(1-i) = 2\sqrt{2}\left(\frac{1}{\sqrt{2}}+i\left(-\frac{1}{\sqrt{2}}\right)\right)$ 5

$= 2\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$ 5

$\boxed{20}$

$\frac{Z_1}{Z_2} = \frac{4\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)}{2\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)}$

$= \frac{\sqrt{2}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}{\cos^2\frac{\pi}{4}-i^2\sin^2\frac{\pi}{4}}$ 5

$= \sqrt{2}\left[\left(\cos\frac{\pi}{6}\cos\frac{\pi}{4}-\sin\frac{\pi}{6}\sin\frac{\pi}{4}\right)+i\left(\sin\frac{\pi}{6}\cos\frac{\pi}{4}+\cos\frac{\pi}{6}\sin\frac{\pi}{4}\right)\right]$

5

$= \sqrt{2}\left(\cos\frac{5\pi}{12}+i\sin\frac{5\pi}{12}\right)$ 5

$\boxed{15}$

$\text{Im}\left(\frac{z_1}{z_2}\right) = \sqrt{2}\sin\frac{5\pi}{12} = \frac{\sqrt{3}+1}{2}$ 5

$\sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}$ $\boxed{5}$

(b) Let $z = a+ib$.

$z = \bar{z}$

$\Leftrightarrow a+ib = a-ib$ 5

$\Leftrightarrow 2ib = 0$

$\Leftrightarrow b = 0$ 5

$\Leftrightarrow z$ is a real number. 5 $\boxed{15}$

$\frac{z}{1+z^2}$ is a real number

$\Rightarrow \frac{z}{1+z^2} = \overline{\left(\frac{z}{1+z^2}\right)}$ 5

$\Rightarrow \frac{z}{1+z^2} = \frac{\bar{z}}{1+\bar{z}^2}$

$\Rightarrow z + z\bar{z}^2 = \bar{z} + z^2\bar{z}$ 5

$\Rightarrow z - \bar{z} - z\bar{z}(z-\bar{z}) = 0$

$\Rightarrow (z-\bar{z})(1-|z|^2) = 0$ 5

$\Rightarrow |z| = 1$ $[\because z \notin \mathbb{R}]$ 5 $\boxed{20}$

(c) $z = \cos\theta + i\sin\theta$ — (1)

$\frac{1}{z} = \frac{1}{\cos\theta + i\sin\theta} = \frac{\cos\theta - i\sin\theta}{\cos^2\theta - i^2\sin^2\theta}$ 5

$= \cos\theta - i\sin\theta$ — (2)

(1)+(2) $\Rightarrow z + \frac{1}{z} = 2\cos\theta$ 5 $\boxed{10}$

$x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow x + \frac{1}{x} = z + \frac{1}{z}$; where $z = \cos\theta + i\sin\theta$ 5

$\Rightarrow x^2 - (z + \frac{1}{z})x + 1 = 0$

$\Rightarrow (x-z)(x-\frac{1}{z}) = 0$

$\Rightarrow x = z$ or $x = \frac{1}{z}$ 5

$\Rightarrow x = \cos\theta + i\sin\theta$ or $x = \cos\theta - i\sin\theta$ 5 $\boxed{15}$

$x + \frac{1}{x} = 2\cos\theta$

$\Rightarrow x = z$ or $x = \frac{1}{z}$; where $z = \cos\theta + i\sin\theta$ 5

$\Rightarrow x^n + \frac{1}{x^n} = z^n + \frac{1}{z^n}$ 5

$= (\cos\theta + i\sin\theta)^n + \frac{1}{(\cos\theta + i\sin\theta)^n}$

$= \cos n\theta + i\sin n\theta + \frac{1}{\cos n\theta + i\sin n\theta}$ 5

$= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$ 5

$= 2\cos n\theta$ 5 $\boxed{25}$

$\frac{x^{2n}+1}{x^{2n-1}+x} = \frac{x^n + \frac{1}{x^n}}{x^{n-1} + \frac{1}{x^{n-1}}} = \frac{2\cos n\theta}{2\cos(n-1)\theta} = \frac{\cos n\theta}{\cos(n-1)\theta}$

5 5 5 $\boxed{15}$

14.(a) $f(x) = \frac{(x+2)^2}{(x+3)^3}$

$f'(x) = \frac{(x+3)^3\, 2(x+2) - (x+2)^2\, 3(x+3)^2}{(x+3)^6}$ 20

$= \frac{2(x+3)(x+2) - 3(x+2)^2}{(x+3)^4}$

$= \frac{2(x^2+5x+6) - 3(x^2+4x+4)}{(x+3)^4}$ 5

$= -\frac{x(x+2)}{(x+3)^4}$ $\boxed{25}$

$f'(x) = 0 \Leftrightarrow x = 0$ or $x = -2$ 5

	$-\infty < x < -3$	$-3 < x < -2$	$-2 < x < 0$	$0 < x < \infty$
Sign of $f'(x)$	$(-)$	$(-)$	$(+)$	$(-)$
$f(x)$ is	decreasing	decreasing	increasing	decreasing

5 5 5 5

$\therefore f(x)$ is increasing on $[-2, 0]$ and decreasing on $(-\infty, -3)$, $(-3, -2]$ and $[0, \infty)$ 5 $\boxed{30}$

Turning points:

$(-2, 0)$ is a local minimum and 5

$(0, \frac{4}{27})$ is a local maximum 5

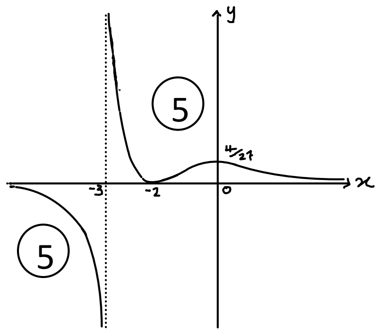
y-intercept: $(0, \frac{4}{27})$ 5

Vertical asymptote: $x = -3$ 5

Horizontal asymptote:

$\lim_{x\to\pm\infty} \frac{(x+2)^2}{(x+3)^3} = \lim_{x\to\pm\infty} \frac{1}{x} \frac{(1+\frac{2}{x})^2}{(1+\frac{3}{x})^3} = 0$ 5

$\therefore y = 0$ 5 $\boxed{30}$



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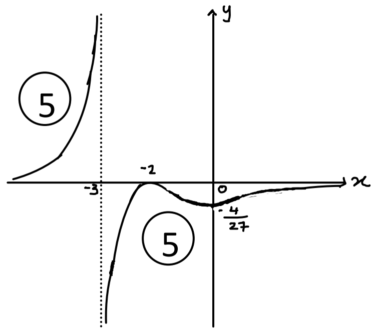
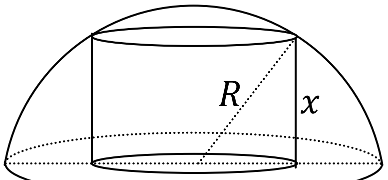
Graph of $y = -f(x)$



$\boxed{10}$

(b)



$$\text{Volume } V = \pi\left(\sqrt{R^2-x^2}\right)^2 x \qquad 5$$
$$= \pi(R^2x - x^3)$$
$$\frac{dV}{dx} = \pi(R^2 - 3x^2) \qquad 5$$
$$= -3\pi\left(x^2 - \frac{R^2}{3}\right)$$
$$\frac{dV}{dx} = 0 \Leftrightarrow x = \frac{R}{\sqrt{3}} \quad [\because x > 0] \qquad 5$$

For $0 < x < \frac{R}{\sqrt{3}}$, $\frac{dV}{dx} > 0$ and 10

for $x > \frac{R}{\sqrt{3}}$, $\frac{dV}{dx} < 0$.

$\therefore V$ is maximum when $x = \frac{R}{\sqrt{3}}$. 5

$$V_{max} = \pi\left(R^2\frac{R}{\sqrt{3}} - \frac{R^3}{3\sqrt{3}}\right) = \frac{2\pi R^3}{3\sqrt{3}} \qquad 5$$

$V_{max} = \frac{1}{\sqrt{3}}\left(\frac{2}{3}\pi R^3\right) = \frac{1}{\sqrt{3}}$(Volume of the solid hemisphere)

5

$\therefore$ The volume of the cylinder cannot exceed $\frac{1}{\sqrt{3}}$ times the volume of the hemisphere. 5

$\boxed{45}$

15. (a) $16x^4 + 4x^3 + 16x^2 + x + 1 \equiv A(4x^2+1)^2 + Bx(4x^2+1) + Cx^2$

comparing coefficients of powers of x:

x^4 : $16 = 16A$

x^3 : $4 = 4B$ 10

x^2 : $16 = 8A + C$

x^1 : $1 = B$

x^0 : $1 = A$

$A = 1$, $B = 1$, $C = 8$ 5

$\boxed{15}$

$$\frac{16x^4+4x^3+16x^2+x+1}{x(4x^2+1)^2} = \frac{(4x^2+1)^2 + x(4x^2+1) + 8x^2}{x(4x^2+1)^2}$$
$$= \frac{1}{x} + \frac{1}{4x^2+1} + \frac{8x}{(4x^2+1)^2} \qquad 5$$

$$\int \frac{16x^4+4x^3+16x^2+x+1}{x(4x^2+1)^2}\,dx$$
$$= \int \frac{1}{x}\,dx + \int \frac{1}{4x^2+1}\,dx + \int \frac{8x}{(4x^2+1)^2}\,dx \qquad 5$$

$= \ln|x| + \frac{1}{2}\tan^{-1}(2x) - \frac{1}{4x^2+1} + c$; where c is an arbitrary constant.

5 5 5 5

$\boxed{30}$

(b) $t = \sqrt{x} \Rightarrow t^2 = x$

$$2t\frac{dt}{dx} = 1 \qquad 5$$

$x = 0 \Rightarrow t = 0$, $x = 1 \Rightarrow t = 1$ 5

$$\int_0^1 \frac{x^{3/2}}{1+x}\,dx = \int_0^1 \frac{t^3\,2t\,dt}{1+t^2} = 2\int_0^1 \frac{t^4}{1+t^2}\,dt \qquad 5$$

$$\Rightarrow \int_0^1 \frac{x^{3/2}}{1+x}\,dx = 2\int_0^1 \frac{t^4 - 1 + 1}{1+t^2}\,dt \qquad 5$$
$$= 2\int_0^1 \left(t^2 - 1 + \frac{1}{1+t^2}\right)dt \qquad 5$$
$$= 2\left(\frac{t^3}{3} - t + \tan^{-1}t\right)\Big|_0^1 \qquad 5$$
$$= 2\left(\frac{1}{3} - 1 + \frac{\pi}{4}\right) = \frac{1}{6}(3\pi - 8). \qquad 5$$

$\boxed{35}$

$$\int_0^1 x\tan^{-1}\sqrt{x}\,dx$$
$$= \left(\tan^{-1}\sqrt{x}\,\frac{x^2}{2}\right)\Big|_0^1 - \int_0^1 \frac{x^2}{2}\,\frac{1}{1+x}\,\frac{1}{2\sqrt{x}}\,dx \qquad 10$$
$$= \frac{\pi}{8} - \frac{1}{4}\int_0^1 \frac{x^{3/2}}{1+x}\,dx = \frac{\pi}{8} - \frac{1}{24}(3\pi - 8) = \frac{1}{3}. \qquad 5$$

5 5

$\boxed{25}$

(c) $$I = \int_{-1}^{1} \frac{x^{2022}}{1+e^x}\,dx = \int_{-1}^{1} \frac{(-x)^{2022}}{1+e^{-x}}\,dx = \int_{-1}^{1} \frac{e^x x^{2022}}{1+e^x}\,dx \qquad 5$$

5

$$2I = I + I$$
$$= \int_{-1}^{1} \frac{x^{2022}}{1+e^x}\,dx + \int_{-1}^{1} \frac{e^x x^{2022}}{1+e^x}\,dx = \int_{-1}^{1} \frac{(1+e^x)x^{2022}}{1+e^x}\,dx \qquad 5$$

5

$$2I = \int_{-1}^{1} x^{2022}\,dx = \frac{x^{2023}}{2023}\Big|_{-1}^{1} = \frac{1}{2023} + \frac{1}{2023} = \frac{2}{2023}$$

5 $\Rightarrow I = \frac{1}{2023}$. 5

$\boxed{30}$

Put $y = 2x$

$$\frac{dy}{dx} = 2 \qquad 5$$

$x = -\frac{1}{2} \Rightarrow y = -1$

$x = \frac{1}{2} \Rightarrow y = 1$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x^{2022}}{1+e^{2x}}\,dx = \int_{-1}^{1} \frac{\left(\frac{y}{2}\right)^{2022}}{1+e^y}\,\frac{1}{2}\,dy = \frac{1}{2^{2023}}\int_{-1}^{1} \frac{y^{2022}}{1+e^y}\,dy$$

5

$$= \frac{1}{2^{2023}} \times \frac{1}{2023}. \qquad 5$$

$\boxed{15}$

16. $N(\bar{x}, \bar{y})$

$ax + by + c = 0$

$P(x_1, y_1)$

If $a \neq 0$ and $b \neq 0$, $\frac{\bar{y} - y_1}{\bar{x} - x_1} \times -\frac{a}{b} = -1$. 5

$$\frac{\bar{y} - y_1}{b} = \frac{\bar{x} - x_1}{a} = t_1 \text{ (say)} \qquad 5$$

$\bar{x} = x_1 + at_1$, $\bar{y} = y_1 + bt_1$

This result is also true when $a = 0$ and $b \neq 0$ or when $a \neq 0$ and $b = 0$.

since $N(x_1, y_1)$ lies on $ax + by + c = 0$, we have

$$a(x_1 + at_1) + b(y_1 + bt_1) + c = 0 \qquad 5$$
$$t_1 = -\frac{ax_1 + by_1 + c}{a^2 + b^2} \qquad 5$$

Perpendicular distance $PN = \sqrt{(\bar{x} - x_1)^2 + (\bar{y} - y_1)^2}$ 5

$$= \sqrt{a^2t_1^2 + b_2t_1^2}$$
$$= \sqrt{a^2 + b^2}\,|t_1| \qquad 5$$
$$= \sqrt{a^2 + b^2}\,\frac{|ax_1 + by_1 + c|}{a^2 + b^2}$$
$$= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \qquad 5$$

$\boxed{35}$

$l \equiv x-2y+5=0$

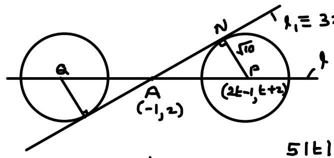
L.H.S $= -1-2(2)+5=0=$ R.H.S

$\therefore$ the point $A(-1,2)$ lies on l 5 5 **10**

$x-2y+5=0 \Rightarrow x+1=2(y-2)$ 5

$\Rightarrow \frac{x+1}{2}=\frac{y-2}{1}=t$ (say) 5

$\Rightarrow x=2t-1$ and $y=t+2$ 5 **15**



$l_1 \equiv 3x-y+5=0$

$PN=\sqrt{10}$

$\frac{|3(2t-1)-(t+2)+5|}{\sqrt{10}}=\sqrt{10}$ 5

$5|t|=10 \Rightarrow |t|=2 \Rightarrow t=\pm 2$ 5

Centre : $(3,4)$ or $(-5,0)$ 5 5

$S_1:(x-3)^2+(y-4)^2=10$ or $S_2:(x+5)^2+y^2=10$ 5 5

Let the gradient of the tangent be m.

Equation of the tangent is $y-2=m(x+1)$ 5

ie, $mx-y+(2+m)=0$

$\frac{|m(-5)-0+(2+m)|}{\sqrt{m^2+1}}=\sqrt{10}$ 5

$(2-4m)^2=10(m^2+1)$

$\Rightarrow 3m^2-8m-3=0$ 5

$\Rightarrow (3m+1)(m-3)=0$

$\Rightarrow m=-\frac{1}{3}$ or $m=3$ 5

$\therefore$ the gradient of the other common tangent is $-1/3$ 5

The equation of the other common tangent is

$y-2=-\frac{1}{3}(x+1)$ 5

$x+3y-5=0$ **60**

Let $S \equiv x^2+y^2+2gx+2fy+c=0$

$(-g,-f)\equiv(-1,2) \Rightarrow g=1,\ f=-2$ 10

$\therefore S \equiv x^2+y^2+2x-4y+c=0$

$S_1 \equiv x^2+y^2-6x-8y+15=0$

Equation of the common chord of S and S_1 is

$S-S_1=0$

$8x+4y+c-15=0$ 5

since centre (3,4) lies on $8x+4y+c-15=0$

5 $8(3)+4(4)+c-15=0 \Rightarrow c=-25$ 5

$\therefore$ the required equation is

$S \equiv x^2+y^2+2x-4y-25=0$ 5 **30**

---

17. (a) $\sin(A+B)=\sin A\cos B+\cos A\sin B$ 5

$\sin 2A=\sin(A+A)$

$=\sin A\cos A+\cos A\sin A$ 5

$=2\sin A\cos A$

$\cos 2A=\sin(\frac{\pi}{2}+2A)$ 5

$=\sin[(\frac{\pi}{2}+A)+A]$

$=\sin(\frac{\pi}{2}+A)\cos A+\cos(\pi/2+A)\sin A$ 5

$=\cos A\cos A-\sin A\sin A$

$=1-\sin^2A-\sin^2A$ 5

$=1-2\sin^2A$ **25**

$\sin(3A)=\sin(2A+A)$

$=\sin 2A\cos A+\cos 2A\sin A$ 5

$=2\sin A\cos A\cos A+(1-2\sin^2A)\sin A$ 5

$=2\sin A(1-\sin^2A)+\sin A-2\sin^3A$ 5

$=3\sin A-4\sin^3A$ **15**

$\cos 3A=-\sin(\frac{3\pi}{2}-3A)$ 5

$=-\sin 3(\frac{\pi}{2}-A)$

$=-[3\sin(\frac{\pi}{2}-A)-4\sin^3(\frac{\pi}{2}-A)]$ 5

$=-3\cos A+4\cos^3A$ 5

$=4\cos^3A-3\cos A$ **15**

$\cos 3x-\sin 3x-3(\sin x+\cos x)=0$

$4\cos^3x-3\cos x-(3\sin x-4\sin^3x)-3(\sin x+\cos x)=0$

$4(\cos^3x+\sin^3x)-6(\cos x+\sin x)=0$ 5

$2(\cos x+\sin x)(\cos^2x-\cos x\sin x+\sin^2x)-3(\cos x+\sin x)=0$

$(\cos x+\sin x)(\sin 2x+1)=0$ 5 5

$\cos x+\sin x=0$ or $\sin 2x=-1$

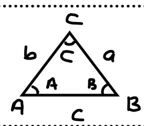
$\tan x=-1$ or $\sin 2x=-1$ 5

$\tan x=\tan(-\pi/4)$ or $\sin 2x=\sin(-\pi/2)$

$x=n\pi-\frac{\pi}{4}$; $n\in\mathbb{Z}$ or $2x=n\pi+(-1)^n(-\pi/2)$; $n\in\mathbb{Z}$

5 $x=\frac{n\pi}{2}-(-1)^n\frac{\pi}{4}$, $n\in\mathbb{Z}$ 5 **30**

(b) $\cos A=\frac{b^2+c^2-a^2}{2bc}$



5 **5**

$\frac{\cos A}{a}+\frac{\cos B}{b}=\frac{b^2+c^2-a^2}{a2bc}+\frac{c^2+a^2-b^2}{b2ca}=\frac{2c^2}{2abc}=\frac{c}{ab}$ —(1) 5 5

Similarlly, $\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a}{bc}$ —(2), $\frac{\cos C}{c}+\frac{\cos A}{a}=\frac{b}{ca}$ —(3) 5 5

(1)+(2)+(3) $\Rightarrow 2\left(\frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}\right)=\frac{c}{ab}+\frac{a}{bc}+\frac{b}{ca}$ 5

$\Rightarrow \frac{\cos A}{a}+\frac{\cos B}{b}+\frac{\cos C}{c}=\frac{a^2+b^2+c^2}{2abc}$ 5 **30**

(b) $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$

Let $\alpha=\cos^{-1}x$, $\beta=\cos^{-1}y$ and $\gamma=\cos^{-1}z$.

$\alpha+\beta+\gamma=\pi$

$\alpha+\beta=\pi-\gamma$

$\cos(\alpha+\beta)=\cos(\pi-\gamma)$ 5

$\cos\alpha\cos\beta-\sin\alpha\sin\beta=-\cos\gamma$ 5

$xy-\sqrt{1-x^2}\sqrt{1-y^2}=-z$ 5

$(xy+z)^2=(1-x^2)(1-y^2)$

$x^2y^2+2xyz+z^2=1-y^2-x^2+x^2y^2$

$x^2+y^2+z^2+2xyz=1$ —(*) 5

$A+B+C=\frac{\pi}{2} \Rightarrow (\frac{\pi}{2}-A)+(\frac{\pi}{2}-B)+(\frac{\pi}{2}-C)=\pi$

5

Put $\cos^{-1}x=\frac{\pi}{2}-A$, $\cos^{-1}y=\frac{\pi}{2}-B$ and $\cos^{-1}z=\frac{\pi}{2}-C$ in equation (*), we get

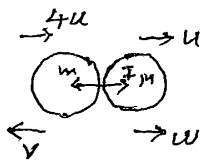
$\sin^2A+\sin^2B+\sin^2C+2\sin A.\sin B.\sin C=1$ 5

30

Grade - 13 (2022) இணைந்த கணிதம் - II Marking Scheme

Part II

(1)



for m. $I = \Delta mv$

$I = mv + m.4u$

$5mu = mv + 4mu$

$v = u$ → (5)

for M → $I = Mw - Mu$

$5mu = Mw - Mu$ → (5)

NLR $v + w = e(4u - u)$ → (5)

$u + w = \frac{4}{5} \cdot 3u$

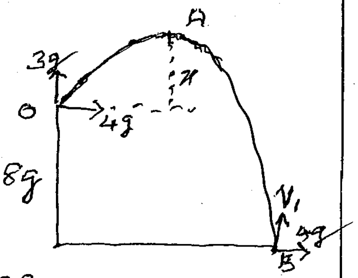
$w = \frac{7u}{5}$ → (5)

$Mw = 5mu + Mu$

$M\left(\frac{7u}{5} - u\right) = 5mu$

$M = \frac{25}{2}m$ → (5)

(2)



O → A $v^2 = u^2 + 2as$

$0 = (3g)^2 - 2gx$ → (5)

$x = 9g/2$

max. Height $8g + \frac{9g}{2} = \frac{25g}{2}$ → (5)

↑ $-8g = 3gt - \frac{1}{2}g.t^2$ → (5)

$t^2 - 6t - 16 = 0$

$(t-8)(t+2) = 0$

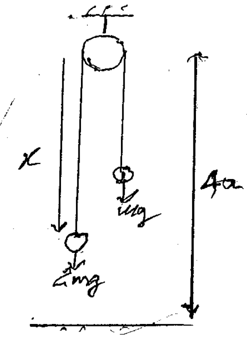
$t = 8$ → (5)

↑ $V_1 = 3g - g.8$

$V_1 = -5g$

$w = \sqrt{41}\,g$ → (5)

(3)



$\frac{1}{2}2m\dot{x}^2 - 2mgx + \frac{1}{2}m\dot{x}^2 - mg(4a - x) = -4mga$

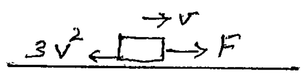
$2\dot{x}\ddot{x} - 2g\dot{x} + \dot{x}\ddot{x} + g\dot{x} = 0$ (5)+(5)+(5)

$\ddot{x} = g/3$ → (5)

$x = 4a \Rightarrow \dot{x}^2 = \frac{4ag}{3}$

$\dot{x} = \sqrt{\frac{4ag}{3}}$ → (5)

(4)



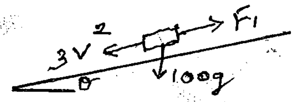
$P = F.v$

$\frac{3000}{v} = F.v$ → (5)

$F = \frac{3000}{v^2} = 120$

→ $F - 3v^2 = 100f$

$f = 0.45\ ms^{-2}$ → (5)



$F.v = P$

$F_1 = \frac{3000}{v^2}$ → (5)

↗ $F_1 - 3v^2 - 100g\sin\theta = 0$ → (5)

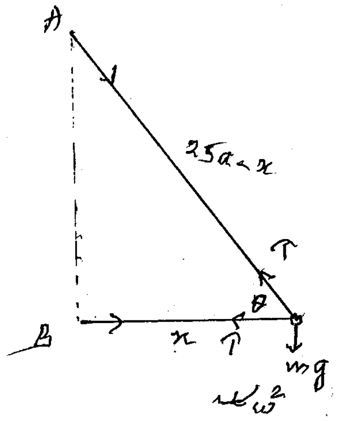
$\frac{3000}{v^2} - 3v^2 - 10 = 0$

$3v^4 + 10v^2 - 3000 = 0$

$(3v^2 + 100)(v^2 - 30) = 0$

$v = \sqrt{30}$ → (5)

(5)



$\tan\theta = \frac{5}{12}$, $x = 12a$ → (5)

$\uparrow$ $T\sin\theta = mg$ → (5)

$\leftarrow$ $T + T\cos\theta = m\omega^2 x$ → (5)

$T = \frac{13}{5}mg$ → (5)

$\frac{13}{5}mg\left(1+\frac{12}{13}\right) = m\cdot 12a\cdot\omega^2$

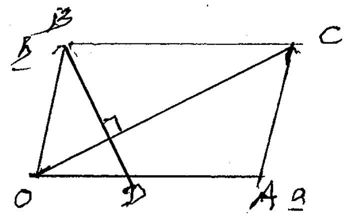
$5g = 12a\omega^2$

$\omega^2 = \frac{5g}{12a}$

$v = x\omega$

$v = \sqrt{60ag}$ → (5)

(7)



$\underline{a} = 2\sqrt{3}\,\underline{i} + 2\underline{j}$

$\underline{b} = \sqrt{3}\,\underline{i} - \underline{j}$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \underline{a} + \underline{b}$

$= 2\sqrt{3}\,\underline{i} + 2\underline{j} + \sqrt{3}\,\underline{i} - \underline{j}$

$= 3\sqrt{3}\,\underline{i} + \underline{j}$ → (5)

Let $\overrightarrow{OD} = \lambda\underline{a}$

$\therefore$ $\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$

$= -\underline{b} + \lambda\underline{a}$ → (5)

$= \lambda\underline{a} - \underline{b}$

$\overrightarrow{OC}\cdot\overrightarrow{BD} = 0$ → (5)

$(\underline{a}+\underline{b})\cdot(\lambda\underline{a}-\underline{b}) = 0$

$\lambda a^2 - b^2 + (\lambda-1)\,\underline{a}\cdot\underline{b} = 0$

$\lambda\cdot 4^2 - 2^2 + (\lambda-1)[6-2] = 0$ → (5)

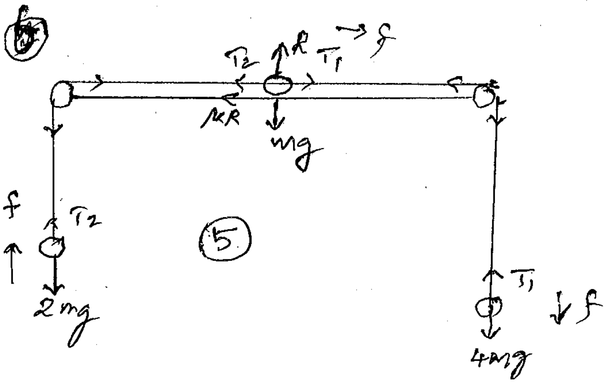
$16\lambda - 4 + 4\lambda - 4 = 0$

$20\lambda = 8$

$\lambda = \frac{2}{5}$ → (5)

$\therefore$ $\overrightarrow{OD} = \frac{2}{5}(2\sqrt{3}\,\underline{i} + 2\underline{j})$

(6)



$R = mg$

$\downarrow$ $4mg - T_1 = 4mf$ } → (5)

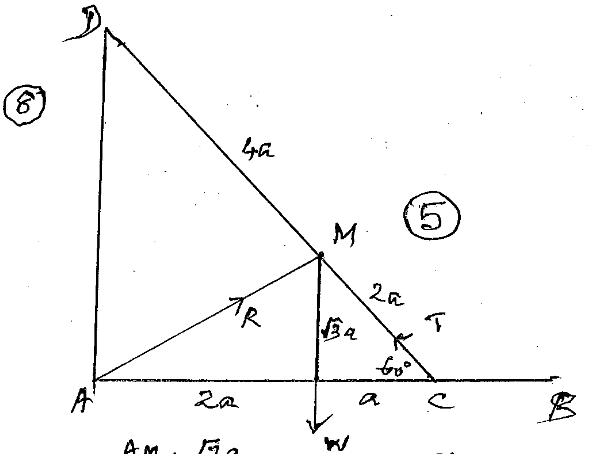
$\rightarrow$ $T_1 - T_2 - \mu mg = mf$ → (5)

$\uparrow$ $T_2 - 2mg = 2mf$ → (5)

$(2-\mu)mg = 7mf = \frac{7mg}{4}$

$\mu = \frac{1}{2}$ → (5)

(8)



$AM = \sqrt{7}a$, $AD = 3\sqrt{3}a$

$AM \parallel R$, $MD \parallel T$, $DA \parallel W$

$\therefore$ AMD Triangle of Force → (5)

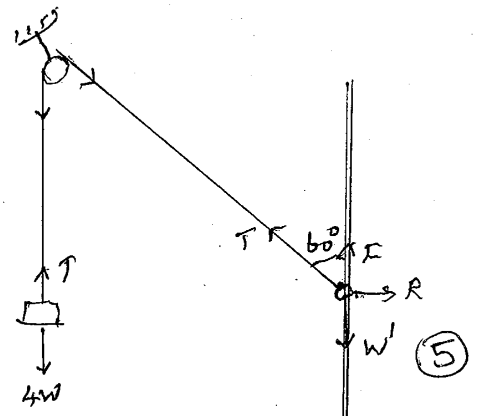
$\therefore$ $\frac{R}{AM} = \frac{T}{MD} = \frac{W}{DA}$ → (5)

$\frac{R}{\sqrt{7}a} = \frac{T}{4a} = \frac{W}{3\sqrt{3}a}$

$R = \frac{1}{3}\sqrt{\frac{7}{3}}\,W$, $T = \frac{4W}{3\sqrt{3}}$

(5) (5)

(9)



(5)

$T = 4W$

for Ring ↑ $F + T\cos 60 - W' = 0 \rightarrow (5)$

→ $R - T\sin 60 = 0 \rightarrow (5)$

$R = 2\sqrt{3}\,W$

$F = W' - 2W$

$\frac{F}{R} = \mu \rightarrow (5)$

$\frac{W' - 2W}{2\sqrt{3}\,W} = \frac{\sqrt{3}}{2}$

$W' = 5W \rightarrow (5)$

(10) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cap B) = \frac{1}{4}$

(i) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4} \rightarrow (5)$ (5)

(ii) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2} \rightarrow (5)$

(iii) $P(A'/B) = \frac{P(A' \cap B)}{P(B)}$

$= \frac{P(B) - P(A \cap B)}{P(B)} \rightarrow (5)$

$= \frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3}} = \frac{\frac{1}{12}}{\frac{1}{3}} = \frac{1}{4} \rightarrow (5)$

All for

$P(A'/B) = 1 - P(A/B)$

$= 1 - \frac{3}{4}$

$= \frac{1}{4}$

17 a)

I $(A \cap B) \cap (A \cap B') = \phi$ (5)

$(A \cap B) \cup (A \cap B') = A$ (5)

$\therefore P(A) = P(A \cap B) + P(A \cap B')$ (5)

II $(A \cap B') \cap B = \phi \rightarrow (5)$

$(A \cap B') \cup B = A \cup B \rightarrow (5)$

$\therefore P(A \cup B) = P(A \cap B') + P(B) \rightarrow (5)$

$= P(A) - P(A \cap B) + P(B) \rightarrow (5)$

$= P(A) + P(B) - P(A \cap B)$

III $P(A'/B) = \frac{P(A' \cap B)}{P(B)} \rightarrow (5)$

$= \frac{P(B) - P(A \cap B)}{P(B)}$

$= 1 - \frac{P(A \cap B)}{P(B)} \rightarrow (5)$

$= 1 - P(A/B)$ $\boxed{40}$

b) I $P(R_1 \cap R_2 \cap R_3)$

$= P(R_1) \cdot P(R_2/R_1) \cdot P(R_3/R_1 \cap R_2)$ (5) (5) (5)

$= \frac{4}{12} \cdot \frac{6}{14} \cdot \frac{8}{16}$ (5) (5) (5)

$= \frac{1}{14}$ (5) $\boxed{35}$

II $P(R_1 \cap G_2 \cap R_3)$

$= P(R_1) \cdot P(G_2/R_1)\, P(R_3/R_1 \cap G_2)$

$= \frac{4}{12} \cdot \frac{8}{14} \cdot \frac{6}{16}$ $\boxed{35}$

$= \frac{1}{14}$

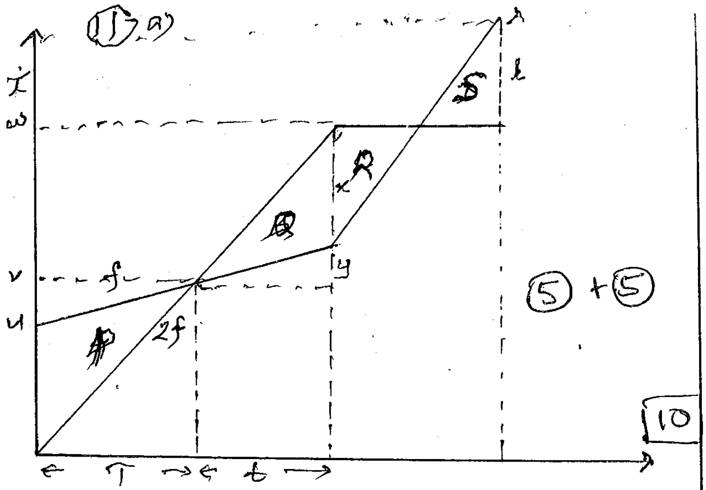
III $P(G_3/R_1) = \frac{P(R_1 \cap G_3)}{P(R_1)}$ (5)

$= \frac{P(R_1 \cap R_2 \cap G_3) + P(R_1 \cap G_2 \cap G_3)}{P(R_1)}$ (5) (5)

$= \frac{(10)\ \frac{4}{12} \cdot \frac{6}{14} \cdot \frac{8}{16} + \frac{4}{12} \cdot \frac{8}{14} \cdot \frac{10}{16}\ (10)}{\frac{4}{12}}$

$= \frac{4}{7}$ (5) $\boxed{40}$

11) a)



⑤ + ⑤ $\boxed{10}$

ii $\frac{v-u}{T} = f \longrightarrow$ ⑤

$\frac{v}{T} = 2f \longrightarrow$ ⑤

$T = \frac{u}{f} \Rightarrow v = 2u \longrightarrow$ ⑤ $\boxed{15}$

iii Triangles A and Q are similar and equal

$\therefore x = u \longrightarrow$ ⑤

$\left.\begin{array}{l}\frac{y}{t} = f \\ \frac{x+y}{t} = 2f\end{array}\right\} \longrightarrow$ ⑤

$\therefore y = x = u.$

$t = \frac{u}{f} = T \longrightarrow$ ⑤

time is $2T \longrightarrow$ ⑤ $\boxed{20}$

iv $\omega = x + y + v$

$= u + u + 2u$

$= 4u \rightarrow$ ⑤

$V_B = 4u, \rightarrow$ ⑤ $V_A = 3u \rightarrow$ ⑤ $\boxed{15}$

v Triangles R and S are similar and equal

$l = x = u \longrightarrow$ ⑤

$\therefore \lambda = \omega + l$

$= 4u + u$

$= 5u. \longrightarrow$ ⑤ $\boxed{10}$

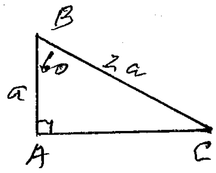
11 b)

V_{AE} $30°$

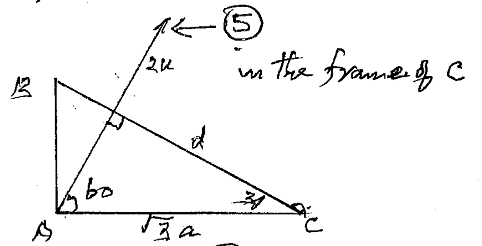
V_{BE} $30°$ $\sqrt{3}u$

V_{CE} u

V_{AC} $60°$ $2u$



I



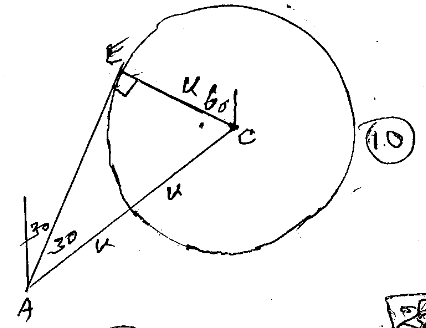
⑤ in the frame of C

II $d = \sqrt{3}a \sin 60° \longrightarrow$ ⑤

$= \frac{3a}{2} \longrightarrow$ ⑤ $\boxed{15}$

III $V_{AE} = V_{AC} + V_{CE}$ ⑤

$30°$ $\quad$ $60°\ 2u + u$ ⑤



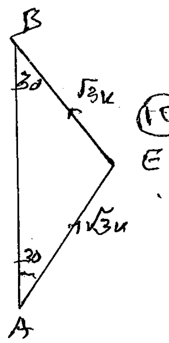
$\boxed{10}$

$\boxed{25}$

$V_{AE} = \sqrt{3}u \longrightarrow$ ⑤

IV $V_{AB} = V_{AE} + V_{EB}$ ⑤

$\sqrt{3}u$ $\quad$ $\sqrt{3}u$



$V_{AB} = 2\sqrt{3}u \cos 30 \longrightarrow$ ⑤

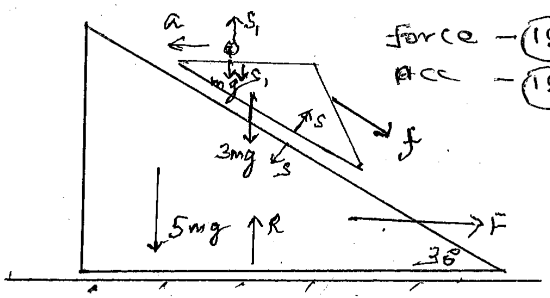
$= 3u \uparrow N \longrightarrow$ ⑤ $\boxed{10}$

$\therefore$ A meet B $\longrightarrow$ ⑤

v $T = \frac{a}{3u} \longrightarrow$ ⑤

vi Direction of c is $N 60° W$ $\longrightarrow$ ⑤ $\boxed{40}$

12 a)



force — (15)

acc — (15)

for $m \rightarrow 0 = m(F - a + f\cos 30) \rightarrow$ (10)

for $m, 3m$ ↘

$4mg\sin 30 = m(f + F\cos 30 - a\cos 30) + 3m(f + F\cos 30) \rightarrow$ (15)

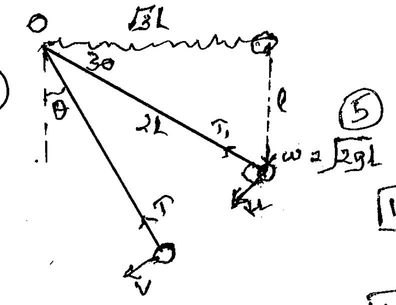
for $m, 3m, 5m \rightarrow$

$0 = m(F - a + f\cos 30) + 3m(F + f\cos 30) + 5mF \rightarrow$ (20)

for m ↑ $S_1 - mg = m(-f\sin 30) \rightarrow$ (10)

$\boxed{185}$

12 b)



(5) (5) $\boxed{10}$

$u = \omega\cos 30^\circ$

$= \sqrt{2gl}\,\frac{\sqrt{3}}{2}$

$= \sqrt{\frac{3}{2}gl} \rightarrow$ (5)

$\boxed{05}$

$\frac{1}{2}mv^2 - mg2l\cos\theta = \frac{1}{2}mu^2 - mgl$ (10)

$v^2 = u^2 - 2gl + 4gl\cos\theta$

$= \frac{3}{2}gl - 2gl + 4gl\cos\theta$

$= gl(8\cos\theta - 1)$ (5)

↖ $T - mg\cos\theta = m\frac{v^2}{2l}$ (10)

$T = \frac{m}{2l}[u^2 - 2gl + 6gl\cos\theta]$

$= \frac{mg}{4}[12\cos\theta - 1]$ (5)

$\boxed{30}$

$\theta = \frac{\pi}{3} \rightarrow 0$, v increase (5)

$\theta = 0 \rightarrow (-\alpha)$ $\cos\theta$ decrease (5)

$\Rightarrow v$ decrease

$\theta = -\alpha \Rightarrow v = 0$ (say) (5)

$\cos(-\alpha) = \frac{1}{8} = \cos\alpha$

$\therefore h = 2l\cos(-\alpha)$

$= 2l \cdot \frac{1}{8}$

$= \frac{l}{4}$ (5)

Particle rest in the depth $\frac{l}{4}$ $\boxed{20}$

13) I

$T_1 = \frac{\lambda}{2a}[6a - 2a]$

$= 2\lambda \rightarrow$ (5)

$T_2 = \frac{\lambda}{a}[2a - a]$

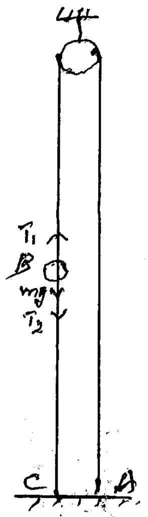
$= \lambda \rightarrow$ (5)

in Equilibrium

$T_1 = T_2 + mg \rightarrow$ (10)

$2\lambda = \lambda + mg$

$\lambda = mg \rightarrow$ (5)



$\boxed{25}$

II

$T_3 = \frac{mg}{2a}[4a + x - 2a]$

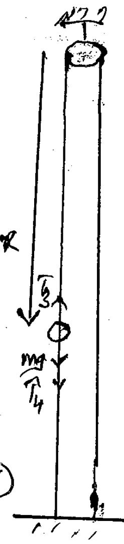
$= \frac{mg}{2a}[2a + x] \rightarrow$ (5)

$T_4 = \frac{mg}{a}[4a - x - a]$

$= \frac{mg}{a}(3a - x) \rightarrow$ (5)

$T_4 + mg - T_3 = m\ddot{x} \rightarrow$ (10)

$\frac{mg}{a}(3a - x) + mg - \frac{mg}{2a}(2a + x) = m\ddot{x}$ (5)



$\boxed{30}$

$\ddot{x} = -\frac{3g}{2a}(x - 2a) \rightarrow$ (5)

let $X = x - 2a \Rightarrow \ddot{x} = \ddot{X} \rightarrow$ (5)

III $\therefore \ddot{X} = -\frac{3g}{2a}X$ S.H.M $\rightarrow$ (5)

centre $X = 0$, $x = 2a$ (B) $\rightarrow$ (5)

$\boxed{15}$

iv $\dot{x}^2 = \omega^2(b^2-x^2)$

$2\dot{x}\ddot{x} = \omega^2(-2x\dot{x})$

$\ddot{x} = -\omega^2 x \rightarrow (5)$

$\therefore \omega = \sqrt{\frac{3g}{2a}} \rightarrow (5)$

$x=a,\ x=-a,\ \dot{x}=0 \rightarrow (5)$

$\Rightarrow \therefore b=a \rightarrow (5)$ $\boxed{20}$

v $x=3a,\ x=a,\ \dot{x}=V_1 \rightarrow (5)$

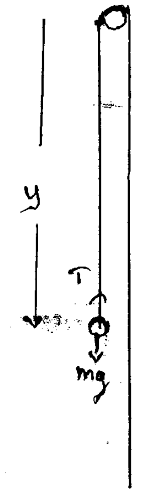
$V_1^2 = \omega^2(a^2-a^2)$

$= 0 \rightarrow (5)$ $\boxed{10}$

vi $mg - T = m\ddot{y} \rightarrow (10)$

$mg - \frac{mg}{2a}(4a+y-2a) = m\ddot{y} \rightarrow (5)$

$\ddot{y} = -\frac{g}{2a}y \rightarrow (5)$



SHM

centre $y=0$
Amplitude is $3a$ $\} \rightarrow (5)$ $\boxed{25}$

vii $T_1 = \frac{\pi}{\omega_1},\ T_2 = \frac{\pi}{2\omega_2} \leftarrow (5)$ (5)

$T = T_1 + T_2$

$= \pi\left(\frac{1}{\omega_1} + \frac{1}{2\omega_2}\right)$

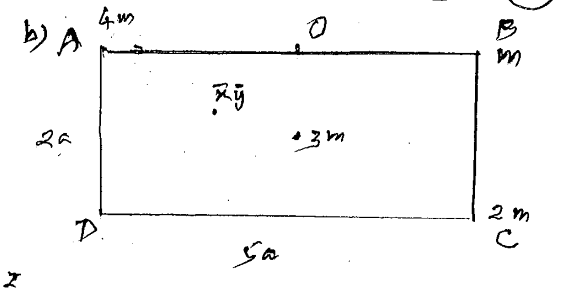
$= \pi\left[\sqrt{\frac{2a}{3g}} + \frac{1}{2}\sqrt{\frac{2a}{g}}\right]$ (5) + படம் (10).

$= \pi\sqrt{\frac{2a}{3g}}\left[1 + \frac{\sqrt{3}}{2}\right]$

$= \pi\sqrt{\frac{a}{6g}}\left[2+\sqrt{3}\right]$ $\boxed{25}$

16 a) Theory (20) + (30)

b)



i

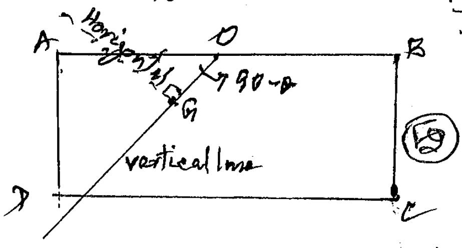
AD: $10mg.\bar{x} = m.5a + 2m.5a + 3m.\frac{5a}{2}$ (25)

$\bar{x} = \frac{9a}{4}$ (5)

AB: $10mg\bar{y} = 2m.2a + 3m.a$ (20)

$\bar{y} = \frac{7a}{10}$ (5) $\boxed{55}$

ii



(15)

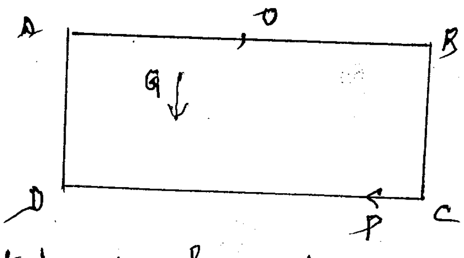
Angle between AB and horizontal line be θ

$\tan(90-\theta) = \frac{\bar{y}}{\frac{5a}{2}-\bar{x}}$ (5) (5)

$\cot\theta = \frac{14}{5}$

$\tan\theta = \frac{5}{14}$ (5) $\boxed{20}$

iii



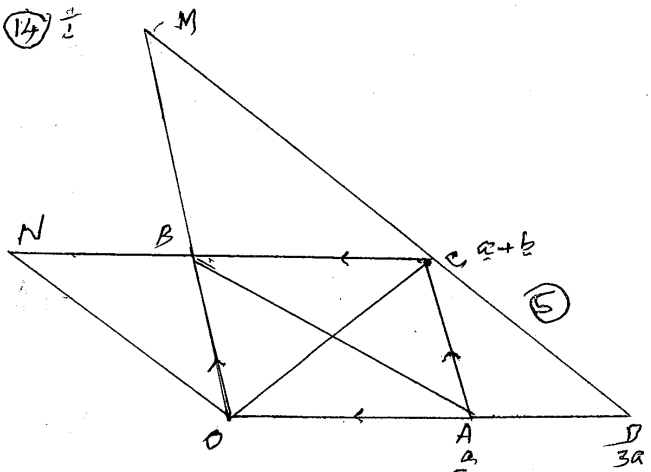
(5)

to be AB horizontal (5)

O↺ $P.2a - 10mg\left(\frac{5a}{2} - \bar{x}\right) = 0$ (10)

$P = \frac{5mg}{4}$ (5) $\boxed{25}$

(14) i



(5)

I $\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$ (5) (OACB ▱) (5)

$= \underline{a}+\underline{b} - \underline{a}$

$= \underline{b}$ (5) $\boxed{20}$

ii $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\underline{a} + \underline{b}$ (5)

OABN parallelogram (5) $\Rightarrow \overrightarrow{ON} = \overrightarrow{AB} = \underline{b} - \underline{a}$ (5) $\boxed{15}$

iii $\overrightarrow{DC} = \overrightarrow{DO} + \overrightarrow{OC}$

$= -3\underline{a} + \underline{a} + \underline{b}$ $\boxed{05}$

$= \underline{b} - 2\underline{a} \rightarrow$ (5)

iv $\overrightarrow{DM} = \lambda \overrightarrow{DC}$.

$= \lambda(\underline{b} - 2\underline{a}) \rightarrow$ (5)

$\overrightarrow{OM} = \mu \overrightarrow{OB}$.

$= \mu \underline{b} \rightarrow$ (5) $\boxed{10}$

v $\overrightarrow{OM} = \overrightarrow{OD} + \overrightarrow{DM} \rightarrow$ (5)

$\mu \underline{b} = 3\underline{a} + \lambda(\underline{b} - 2\underline{a})$

$(3 - 2\lambda)\underline{a} + (\lambda - \mu)\underline{b} = 0$

$3 - 2\lambda = 0 \quad \lambda - \mu = 0 \rightarrow$ (5) $\boxed{15}$

$\lambda = \frac{3}{2} = \mu \rightarrow$ (5)

vi $\overrightarrow{OM} = \mu \underline{b}$

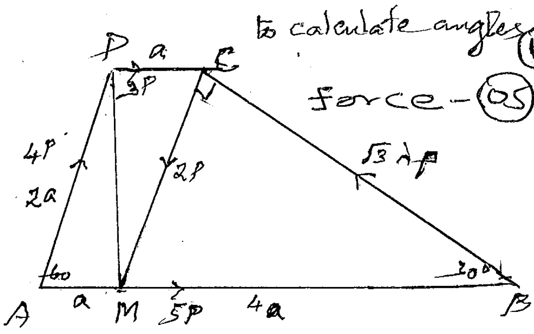
$= \frac{3}{2}\underline{b}$.

$\frac{OM}{OB} = \frac{3}{2}$ $\boxed{05}$

$\rightarrow$ (5)

$OB : BM = 2 : 1$

II



to calculate angles (10)

force - (05)

$\rightarrow X = 5P + 3P - 2P\cos 60 + 4P\cos 60 - \sqrt{3}\lambda P\cos 30$ (5)

$= 9P - \frac{3\lambda}{2}P$ (5)

$\uparrow Y = 4P\sin 60 - 2P\sin 60 + \sqrt{3}\lambda P\sin 30$ (5)

$= \sqrt{3}P + \frac{\sqrt{3}}{2}\lambda P$ (5)

R // MD $\perp$ AB $\Rightarrow X = 0$ (5)
(5)

$\Rightarrow \lambda = 6$ (5)

$\therefore Y = 4\sqrt{3}P$

$\therefore R = 4\sqrt{3}P$ (5) $\boxed{50}$

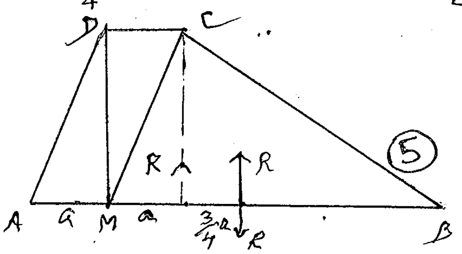
M↻

$R.x = 6\sqrt{3}P.2a - 3P.2a\sin 60$

$-4P.a\sin 60 \rightarrow$ (10)

$4\sqrt{3}Px = 12\sqrt{3}Pa - 5\sqrt{3}Pa$

$x = \frac{7}{4}a \rightarrow$ (5) $\boxed{15}$



(5)

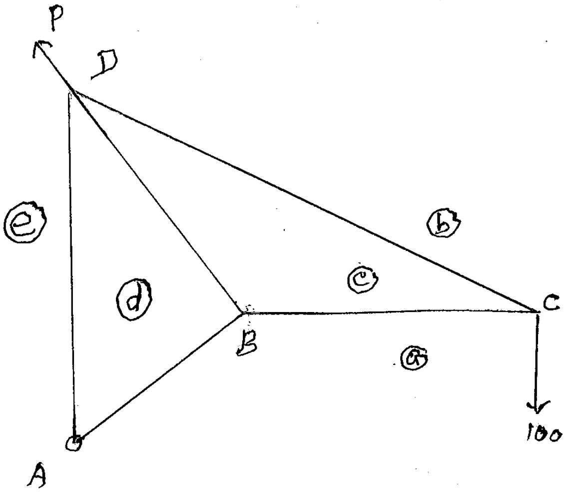
Let G be the couple to be added.

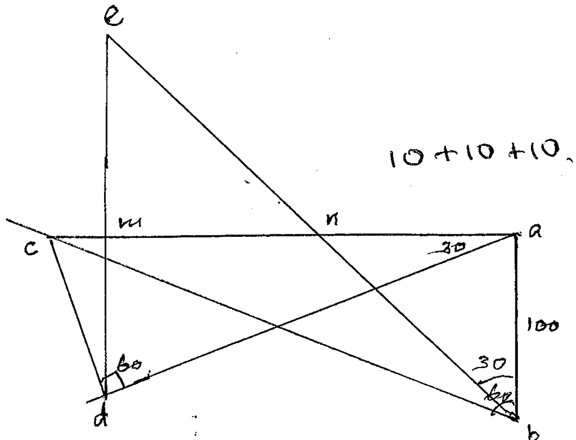
G↻ $= R.\frac{3}{4}a$ (5)

$= 4\sqrt{3}.P.\frac{3}{4}a$

$= 3\sqrt{3}Pa$. (5) $\boxed{15}$

15) II



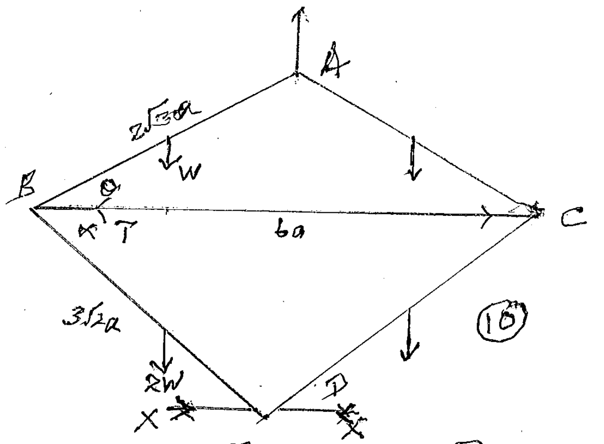


10 + 10 + 10.

Rod	Notation	Tension	Thrust	
AB	ad	-	150	5+5
BC	ac	-	$100\sqrt{3}$	5+5
CD	bc	200	-	5+5
BD	cd	-	$50\sqrt{3}$	5+5
AD	de	200	-	5+5
P	be	$150\sqrt{3}$	-	05
R	ae	$30\sqrt{13}$	-	05

100

15 I



$\cos\theta = \frac{3a}{2\sqrt{3}a} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$ 5

$\cos\alpha = \frac{3a}{3\sqrt{2}a} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{\pi}{4}$ 5

By Symmetry vertical component of the reaction at C is zero 5

for BC

$B\circlearrowleft \; X \cdot 3\sqrt{2}\,a\sin\alpha - 2W \cdot \frac{3\sqrt{2}a}{2}\cos\alpha = 0$ 10

$X = W$ 5

for AB, BC

$A\circlearrowleft \; T \cdot 2\sqrt{3}\,a\sin\frac{\pi}{6} - (W+2W)\frac{3a}{2} - X\cdot\left(2\sqrt{3}a\sin\frac{\pi}{6} + 3\sqrt{2}a\sin\frac{\pi}{4}\right) = 0$

$\sqrt{3}T - \frac{9W}{2} - W(\sqrt{3} + 3) = 0$ 15

$\sqrt{3}T - \left(\frac{15+2\sqrt{3}}{2}\right)W = 0$

$T = \frac{W}{2}(5\sqrt{3}+2)$ 5

60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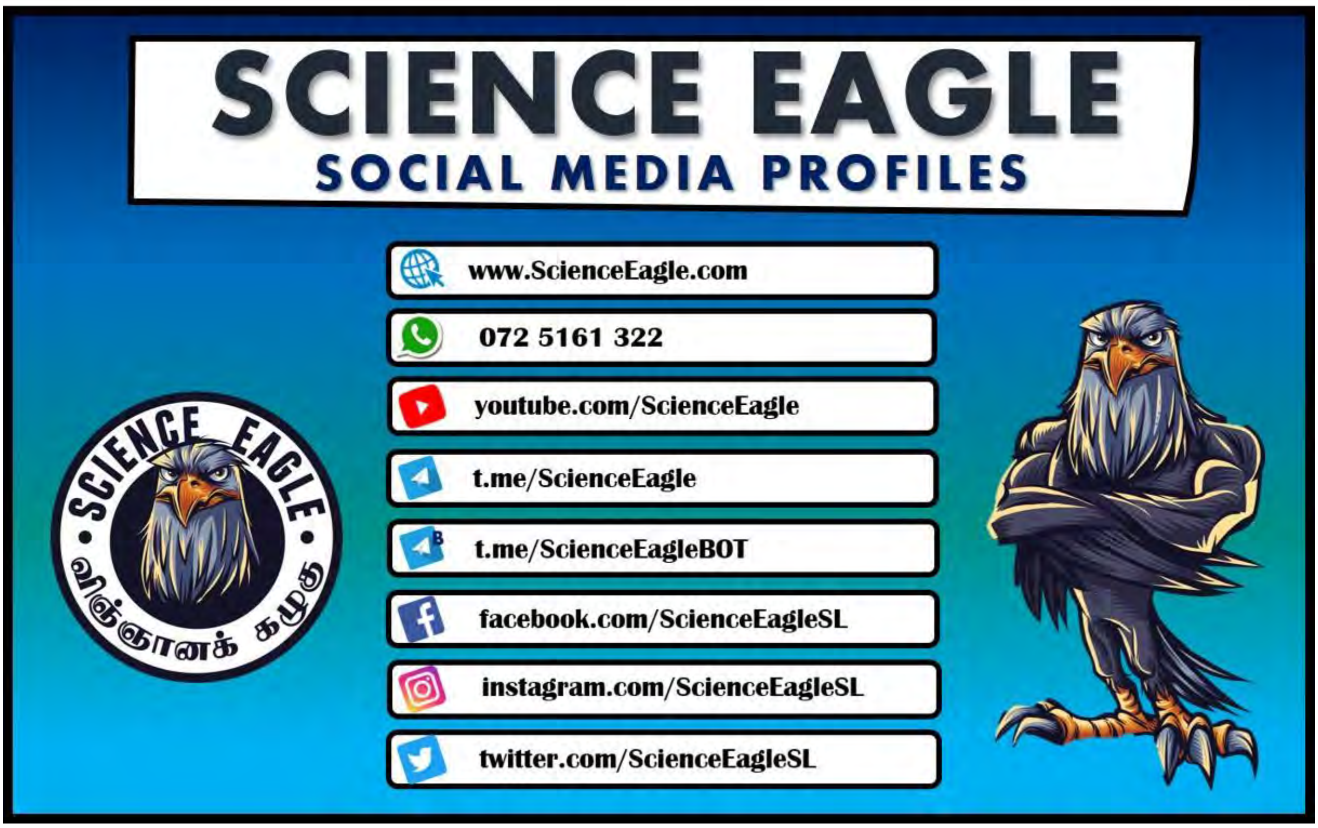
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