வடமாகாணக் கல்வித் திணைக்களத்தின் அனுசரணையுடன்
தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்

Field Work Centre

தவணைப் பரீட்சை, யூலை - 2017

FWC

Term Examination, July - 2017

தரம் :- 12 (2018)	கணிதம்	புள்ளித்திட்டம்

① $4>0$, $\Delta<0$ (5)

$\Rightarrow [2(k+4)]^2-4\cdot4\cdot9<0$ (10)

$\Rightarrow (k+10)(k-2)<0$ (5)

$\Rightarrow -10<k<2$ (5)

② $\frac{x}{2x-1}\le -2$ $\quad x\ne\frac{1}{2}$ (5)

(5)

$\Rightarrow \frac{x}{2x-1}+2\le 0$

$\Rightarrow \frac{5x-2}{2x-1}\le 0$ (5)

$\Rightarrow \frac{(5x-2)(2x-1)}{(2x-1)^2}\le 0$ (5)

$\Rightarrow (5x-2)(2x-1)\le 0$

$\frac{2}{5}\le x<\frac{1}{2}$ (5)

③ $\lim\limits_{x\to0}\frac{1+2x^2-\cos3x}{x\sin x}$

$=\lim\limits_{x\to0}\frac{1-\cos3x}{x\sin x}+\lim\limits_{x\to0}\frac{2x}{\sin x}$ (5)

$=\lim\limits_{x\to0}\frac{\sin^2 3x}{x\sin x(1+\cos3x)}+\frac{1}{\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)}$ (5)

$=\lim\frac{\frac{\sin^2 3x}{x^2}}{\frac{\sin x}{x}(1+\cos3x)}+\frac{2}{1}$ (5)

$=\frac{9\lim\left(\frac{\sin3x}{3x}\right)^2}{\lim\left(\frac{\sin x}{x}\right)\lim(1+\cos3x)}+2$ (5)

$=\frac{9}{2}+2=\frac{13}{2}$ (5)

④ $2x-y+3+\lambda(x+7y-11)=0$ (5)

this line through the origin (10)

$\Rightarrow \lambda=\frac{3}{11}$

$\therefore$ eqn of the line is $5x+2y=0$ (10)

⑤ $2\alpha=(\alpha+\beta)+(\alpha-\beta)$ (5)

$\tan2\alpha=\frac{\tan\overline{\alpha+\beta}+\tan\overline{\alpha-\beta}}{1-\tan\overline{\alpha+\beta}\cdot\tan\overline{\alpha-\beta}}$ (5)

$\tan\overline{\alpha-\beta}=\frac{5}{12}$ (5) $\quad \tan\overline{\alpha+\beta}=\frac{3}{4}$ (5)

$\tan2\alpha=\frac{56}{33}$ (5)

⑥ $\uparrow v^2=u^2+2as$

$16^2=56^2-2gh$ (5)

$h=144$ m (10)

let the velocity w to just reach the top of the tower

(5) $0=w^2-2g\cdot144$

(5) $w=24\sqrt{5}\ ms^{-1}$

⑦  

$\frac{OA}{OB}=\frac{Q}{P}$ (5) $\quad \frac{AC}{CB}=\frac{Q}{P}$

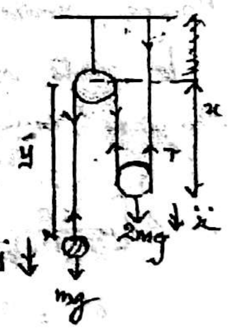
$\frac{OA}{AB}=\frac{Q}{P-Q}$ (5) $\quad \frac{AC}{AB}=\frac{Q}{P+Q}$ (5)

$OC=OA+AC$ (5)

$=\frac{Q}{P-Q}\cdot AB+\frac{Q}{P+Q}\cdot AB$

$=AB\left[\frac{2PQ}{P^2-Q^2}\right]$ (5)

⑧ $2x+y=$ Const.



$2\ddot{x}+\ddot{y}=0$ —— ①

$\downarrow\ 2mg-2T=2m\ddot{x}$ —— ②

$mg-T=m\ddot{x}$

$\downarrow\ mg-T=m\ddot{y}$ —— ③

$\ddot{x}=\ddot{y}=0$

$T=mg$

⑨ $\underline{a}=2\underline{i}+3\underline{j}$ $\quad \underline{b}=\underline{i}+\underline{j}$ $\quad \underline{c}=-2\underline{i}+\underline{j}$

$\underline{a}\cdot\underline{b}=ab\cos A\hat{O}B$ (5)

$2+3=\sqrt{13}\cdot\sqrt{2}\cos A\hat{O}B$ (5)

$\cos A\hat{O}B=\frac{5}{\sqrt{26}}$

$A\hat{O}B=\cos^{-1}\frac{5}{\sqrt{26}}$ (5)

$\underline{c}\cdot(\underline{b}-\underline{a})$ (5)

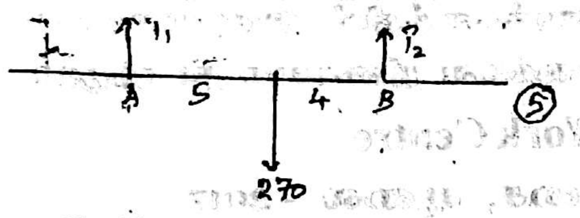
$=(-2\underline{i}+\underline{j})\cdot(-\underline{i}-2\underline{j})$

$=2-2$

$=0$ (5)

$\therefore OC\perp AB$

(10)



(5)

$\uparrow \quad T_1 + T_2 = 270$ (5)

$A \circlearrowright \quad T_2 \cdot 9 = 5 \cdot 270$ (5)

$T_2 = 150$ (5)

$T_1 = 120$ (5)

(11) (a) $\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$ (5)

$$\left.\begin{aligned}(\alpha+k)+(\beta+k) &= -\frac{q}{p}\\ (\alpha+k)(\beta+k) &= \frac{r}{p}\end{aligned}\right\}$$ (5)

$\alpha - \beta = (\alpha+k) - (\beta+k)$ (5)

$(\alpha-\beta)^2 = [(\alpha+k) - (\beta+k)]^2$

$(\alpha+\beta)^2 - 4\alpha\beta = [\overline{\alpha+k} + \overline{\beta+k}]^2 - 4(\alpha+k)(\beta+k)$ (10)

$\left(-\frac{b}{a}\right)^2 - 4\frac{c}{a} = \left(-\frac{q}{p}\right)^2 - 4\frac{r}{p}$ (5)

$\therefore \frac{b^2-4ac}{a^2} = \frac{q^2-4pr}{p^2}$ $\boxed{30}$

$\Delta = a^2 - 4(a-b-c)(b+c)$ (10)

$= a^2 - 4(a - \overline{b+c})(b+c)$

$= a^2 - 4a(b+c) + 4(b+c)^2$ (10)

$= [a - 2\overline{b+c}]^2$ (10)

≥ 0 (5)

Hence roots are real (5)

let roots are α and 2α

$\therefore 3\alpha = \frac{-a}{a-b-c}$ (5), $2\alpha^2 = \frac{b+c}{a-b-c}$ (5)

$\therefore 2\left[\frac{-a}{3(a-b-c)}\right]^2 = \frac{b+c}{a-b-c}$ (5)

$2a^2 = 9(b+c)[a - \overline{b+c}]$

$9(b+c)^2 - 9a(b+c) + 2a^2 = 0$ (10)

$[3\overline{b+c} - a][3\overline{b+c} - 2a] = 0$ (10)

$\therefore b+c = \frac{a}{3}$ or $\frac{2a}{3}$ (5)

$\boxed{80}$

$g(x) = x^4 + 4x^3 + 4x^2 - 4x - 5$

$g(1) = 1 + 4 + 4 - 4 - 5 = 0$

$\therefore (x-1)$ is a factor of $g(x)$ (5)

IIIly $(x+1)$ also a factor. (5)

$g(x) = (x-1)(x+1)(x^2+Bx+C)$ (10)

$B = 4$ (5)

$C = 5$ (5)

$g(x) = (x+1)(x-1)(x^2+4x+5)$

$g(x) = 0 \Rightarrow x = 1$ or -1 (5)

or $x^2+4x+5=0$, $\Delta < 0 \Rightarrow$ imaginary roots (5)

Hence, only one pair of real roots. $\boxed{45}$

12(a) $x^2 + y^2 = 7xy$

$(x+y)^2 = 9xy$ (10)

$\log(x+y)^2 = \log 9xy$ (10)

$2\log(x+y) = \log 9 + \log x + \log y$ (10)

$2\log(x+y) = 2\log 3 + \log x + \log y$ (10)

II $\log_{mn} x = \frac{\log_n x}{\log_n mn}$ (10)

$= \frac{\log_n x}{\log_n m + \log_n n}$ (10)

$= \frac{\log_n x}{\log_n m + 1}$ (10)

(b) $(a+b+c)^2 \geq 0$ (5)

$a^2+b^2+c^2 + 2(ab+bc+ca) \geq 0$ (5)

$\Rightarrow ab+bc+ca \geq -\frac{1}{2}$ (5)

$(a-b)^2 + (b-c)^2 + (c-a)^2 \geq 0$ (10)

$2(a^2+b^2+c^2) - 2(ab+bc+ca) \geq 0$ (5)

$ab+bc+ca \leq 1$ (5)

$\therefore -\frac{1}{2} \leq ab+bc+ca \leq 1$ (5)

II $|x-3| \leq 2|x-2|$

$x < 2 \Rightarrow -x+3 \leq -2x+4$, $x \leq 1$ (10)

$2 \leq x < 3 \quad -x+3 \leq 2x-4$, $x \geq \frac{7}{3}$ (10)

$x \geq 3 \quad x-3 \leq 2x-4$, $x \geq 1$ (10)

$x \leq 1$ or $x \geq \frac{7}{3}$ (10)

$y = \ln(1+\sin x) \Rightarrow 1+\sin x = e^{y}$ (5)

$\frac{dy}{dx} = \frac{\cos x}{1+\sin x}$ (5)

$e^{y}\frac{dy}{dx} = \cos x$ (5)

$e^{y}\frac{d^2y}{dx^2} + \frac{dy}{dx}e^{y}\frac{dy}{dx} = -\sin x$ (5)

$e^{y}\frac{d^2y}{dx^2} + e^{y}\left(\frac{dy}{dx}\right)^2 = 1-e^{y}$ (5)

$e^{y}\frac{d^2y}{dx^2} + e^{y}\left[1+\left(\frac{dy}{dx}\right)^2\right] = 1$ $\boxed{30}$

(b) $x = e^{t}$ $y = \sin t$

$\frac{dx}{dt} = e^{t}$ (5) $\frac{dy}{dt} = \cos t$ (5)

$\frac{dy}{dx} = \frac{\cos t}{e^{t}}$ (5) $\Rightarrow x\frac{dy}{dx} = \cos t$ (5)

$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\sin t \frac{dt}{dx}$ (10)

$= -\frac{y}{x}$ (5)

$\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ $\boxed{40}$

(c) $f(x) = \frac{x^2+1}{x^2-1}$

$f'(x) = \frac{(x^2-1)2x-(x^2+1)2x}{(x^2-1)^2}$ (10)

$= -\frac{4x}{(x^2-1)^2}$

$f'(x) = 0 \Rightarrow x = 0$ (05)

$x=1,\ x=-1$ are asymtotes. (5)

$\left(\frac{dy}{dx}\right)_{x<-1} > 0$ $\left(\frac{dy}{dx}\right)_{x>-1} > 0 \Rightarrow y \nearrow$ (10)

$\left(\frac{dy}{dx}\right)_{x<0} > 0$ $\left(\frac{dy}{dx}\right)_{x>0} < 0$ y_{max} at $x=0$ (10)

$\left(\frac{dy}{dx}\right)_{x<1} < 0$ $\left(\frac{dy}{dx}\right)_{x>1} < 0 \Rightarrow y \searrow$ (10)

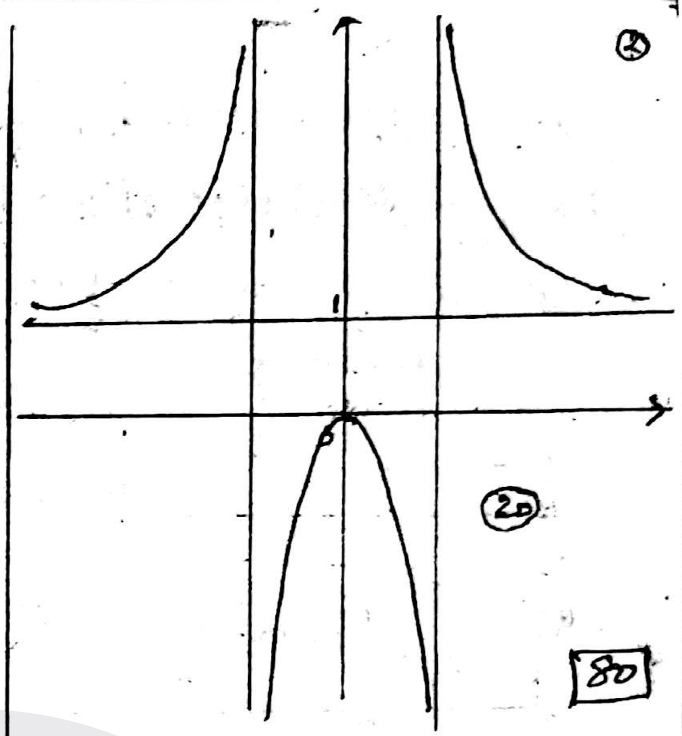
max point $(0,-1)$: (5)

$x=0 \Rightarrow y=-1$
$y=0 \Rightarrow x^2+1=0 \Rightarrow$ impossible.

$f(x) = \frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$ (5)

$x \to \pm\infty,\ y \to 1$

(3)



(20)

$\boxed{80}$

4(a) $m\tan(\theta-30) = n\tan(\theta+120)$

$\frac{m}{n} = \frac{\tan(\theta+120)}{\tan(\theta-30)}$ (10)

$\frac{m+n}{m-n} = \frac{\tan(\theta+120)+\tan(\theta-30)}{\tan(\theta+120)-\tan(\theta-30)}$ (10)

$= \frac{\sin(\theta+120)\cos(\theta-30)+\cos(\theta+120)\sin(\theta-30)}{\sin(\theta+120)\cos(\theta-30)-\cos(\theta+120)\sin(\theta-30)}$ (10)

$= \frac{\sin(2\theta+90)}{\sin(150)}$ (10)

$= 2\cos 2\theta$ (5)

(b) $\sin^{-1}x = \alpha$, $\cos^{-1}x = \beta$

$\sin\alpha = x$ $\cos\beta = x$ (10)

$\sin\alpha = x = \cos\beta = \sin\left(\frac{\pi}{2}-\beta\right)$ (10)

$\sin\alpha = \sin\left(\frac{\pi}{2}-\beta\right)$

$\alpha = \frac{\pi}{2}-\beta$ (5) $\left(-\frac{\pi}{2}\le\alpha\le\frac{\pi}{2},\ 0\le\beta\le\pi\right)$ (5)

$\alpha+\beta = \frac{\pi}{2}$ $\boxed{30}$

$\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$ — (1)

$\cos^{-1}x - \cos^{-1}y = \frac{\pi}{3}$ — (2)

$\frac{\pi}{2} - \sin^{-1}x - \left(\frac{\pi}{2}-\sin^{-1}y\right) = \frac{\pi}{3}$ (10)

$\sin^{-1}y - \sin^{-1}x = \frac{\pi}{3}$ — (3) (5)

(1)+(3) $\Rightarrow$ $2\sin^{-1}y = \pi \Rightarrow y = 1$ (10)

(1)−(3) $\Rightarrow$ $2\sin^{-1}x = \frac{\pi}{3} \Rightarrow x = \frac{1}{2}$ (10)

$\boxed{35}$

(c) $\cos A \sin^2\frac{A}{2} + \cos B \sin^2\frac{B}{2} + \cos C \sin^2\frac{C}{2} = \frac{3}{8}$ (10)

$\cos A(1-\cos A) + \cos B(1-\cos B) + \cos C(1-\cos C) = \frac{3}{4}$ (10)

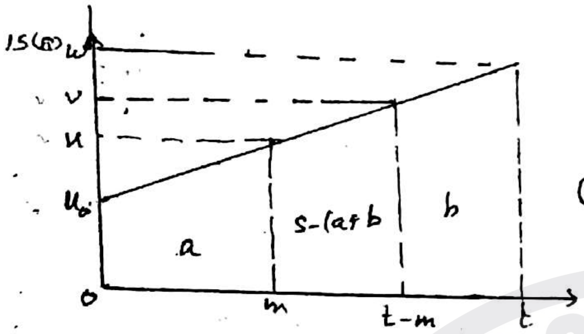
$\left(\cos A - \frac{1}{2}\right)^2 + \left(\cos B - \frac{1}{2}\right)^2 + \left(\cos C - \frac{1}{2}\right)^2 = 0$ (10)

$\cos A - \frac{1}{2} = 0, \quad \cos B - \frac{1}{2} = 0, \quad \cos C - \frac{1}{2} = 0$ (10)

$A = B = C = \frac{\pi}{3}$

$\boxed{40}$

15(b)



(10)

$\frac{u-u_0}{m} = \frac{w-v}{m}$ (10)

$u+v = w+u_0$ (5)

$S = \left(\frac{w+u_0}{2}\right)t$ (10)

$S-(a+b) = \left(\frac{u+v}{2}\right)(t-2m)$ (10)

$\frac{u+v}{2} = \frac{w+u_0}{2} = \frac{s}{t}$ (10)

$\frac{s}{s-(a+b)} = \left(\frac{w+u_0}{u+v}\right)\cdot\frac{t}{t-2m} = \frac{t}{t-2m}$ (10)

$\frac{s}{t} = \frac{s-(a+b)}{t-2m} = \frac{a+b}{2m}$ (10)

$\Rightarrow s = \frac{(a+b)}{2m}t.$

(iv) Average speed $= \frac{s-(a+b)}{t-2m} = \frac{a+b}{2m}$ (10) $\boxed{85}$

(b)

V_{XE} ↓ $\quad V_{XW} = 30$ ↓ $\quad V_{WE}$ $60°$ $10\sqrt{3}$

(5)

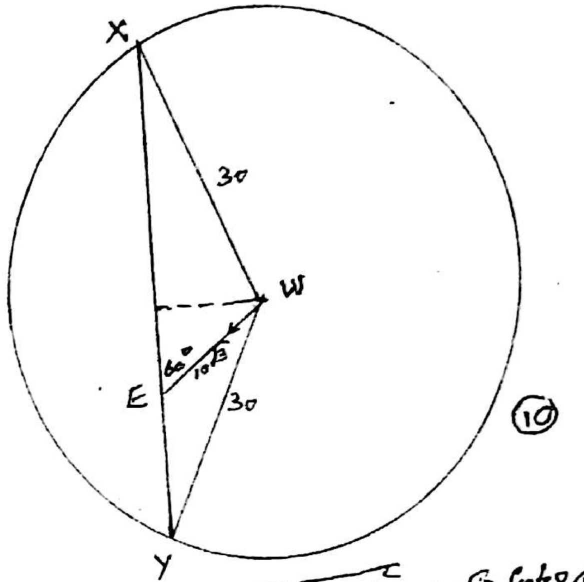
V_{YE} ↑ $\quad V_{YW} = 30$ ↑ $\quad V_{WE}$ $60°$ $10\sqrt{3}$

$V_{XE} = V_{XW} + V_{WE}$ (10)

↓ $\quad 30 \quad + \quad 60°\ 10\sqrt{3}$

$V_{YE} = V_{YW} + V_{WE}$ (10)

↑ $\quad 30 \quad\quad 60°\ 10\sqrt{3}$



(10)

$V_{XE} = w_1 = \sqrt{30^2 - (10\sqrt{3}\sin 60)^2} + 10\sqrt{3}\cos 60$ (10)

$= \sqrt{30^2 - 15^2} + 5\sqrt{3}$

$= 15\sqrt{3} + 5\sqrt{3} = 20\sqrt{3}$

$V_{YE} = 15\sqrt{3} - 5\sqrt{3} = 10\sqrt{3}$ (10)

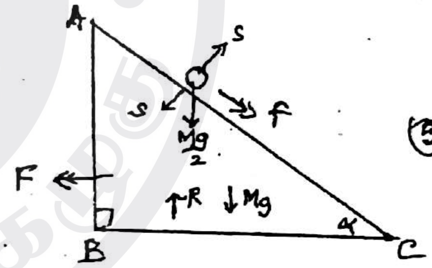
Let time taken to meet is T

$60 = 20\sqrt{3}T + 10\sqrt{3}T$ (10)

$T = \frac{2\sqrt{3}}{3}$

$\boxed{65}$

16(a)



(5)

$f_{mE} = f_{mM} + f_{ME}$

$= \searrow f \quad \overleftarrow{F} \quad = \quad F \leftarrow \bullet \searrow f$ (10)

for the particle $\searrow F = ma$

$\frac{M}{2} g \sin\alpha = \frac{M}{2}(f - F\cos\alpha)$ (10)

$g\sin\alpha = f - F\cos\alpha$ —— (1)

for the Wedge and the particle

$\leftarrow \quad 0 = MF + \frac{M}{2}(F - f\cos\alpha)$ (10)

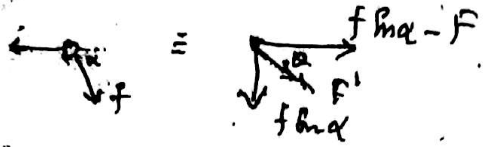
$3F = f\cos\alpha$ —— (2)

(1)+(2) $\Rightarrow 3F = \cos\alpha(g\sin\alpha + F\cos\alpha)$

$F(3 - \cos^2\alpha) = g\sin\alpha\cos\alpha$

$F = \frac{g\sin\alpha\cos\alpha}{2+\sin^2\alpha}$ (10)

$f = \frac{3g\sin\alpha}{2+\sin^2\alpha}$ (10)



$$\tan\theta = \frac{f\sin\alpha}{f\cos\alpha - F} = \frac{f\sin\alpha}{f\cos\alpha - \frac{1}{3}f\cos\alpha} \quad (10)$$

$$= \frac{3\sin\alpha}{2\cos\alpha}$$

$$= \frac{3}{2}\tan\alpha$$

$$\theta = \tan^{-1}\left(\frac{3}{2}\tan\alpha\right) \quad (5)$$

$$(F^1)^2 = (f\sin\alpha)^2 + (f\cos\alpha - F)^2 \quad (10)$$

$$= f^2\sin^2\alpha + \left(\frac{2}{3}f\cos\alpha\right)^2$$

$$= f^2\left(\sin^2\alpha + \frac{4}{9}\cos^2\alpha\right)$$

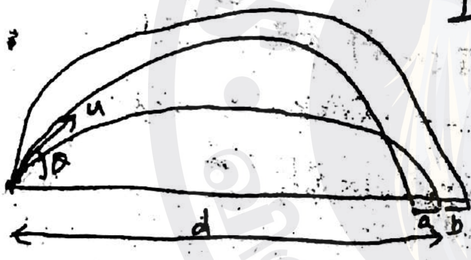
$$= f^2\left(1 - \frac{5}{9}\cos^2\alpha\right)$$

$$F^1 = \frac{f}{3}\left(9 - 5\cos^2\alpha\right)^{\frac{1}{2}}$$

$$= \frac{g\sin\alpha}{2+\sin^2\alpha}\sqrt{9 - 5\cos^2\alpha} \quad (5)$$

$\boxed{85}$

b)



$\rightarrow s = ut + \frac{1}{2}at^2$

$d = u\cos\theta \cdot t$ (10)

$\uparrow 0 = u\sin\theta\, t - \frac{1}{2}gt^2$

$t = \frac{2u\sin\theta}{g}$ (10)

$d = \frac{u^2}{g}\sin 2\theta$ (10)

$d - a = \frac{u^2}{g}\sin\frac{\pi}{6} = \frac{u^2}{g}\cdot\frac{1}{2}$ (10)

$d + b = \frac{u^2}{g}\sin\frac{\pi}{4} = \frac{u^2}{g}\cdot\frac{1}{\sqrt{2}}$ (5)

$b + a = \frac{u^2}{g}\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$ (5)

$a = \frac{u^2}{g}\left(\sin 2\theta - \frac{1}{2}\right)$ (5)

$\frac{b+a}{a} = \frac{\sqrt{2}-1}{2\sin 2\theta - 1}$ (5)

$2\sin 2\theta - 1 = \frac{a(\sqrt{2}-1)}{b+a}$

$\sin 2\theta = \frac{\sqrt{2}a + b}{2(a+b)}$ (5)

17(a) $\underline{p} = -8\underline{i} - 6\underline{j}$

$\underline{q} = 2\underline{i} + 4\underline{j}$

$\underline{r} = h\underline{i} - 3\underline{j}$

$\overrightarrow{PQ} = \underline{q} - \underline{p}$

$= 10\underline{i} + 10\underline{j}$ (10)

If P, Q, R. in a st. line

$\overrightarrow{PQ} = \lambda\overrightarrow{QR}$ (10)

$10\underline{i} + 10\underline{j} = \lambda[(h-2)\underline{i} - 7\underline{j}]$ (5)

$\frac{10}{h-2} = \frac{10}{-7}$ (5)

$h - 2 = -7$

$h = -5$ (5)

$\overrightarrow{OP} - m\overrightarrow{OQ} - n\overrightarrow{OR} = 0$

$\underline{p} - m\underline{q} - n\underline{r} = 0$

$-8\underline{i} - 6\underline{j} - m(2\underline{i} + 4\underline{j}) - n(\underline{i} - 3\underline{j}) = 0$ (10)

$2m + 8 + n = 0$ —— ①

$4m + 6 - 3n = 0$ —— ② (5)

$2m + n = -8$

$4m - 3n = -6$

$6m + 3n = -24$

$10m = -30$

$m = -3$

$n = -2$ (5)

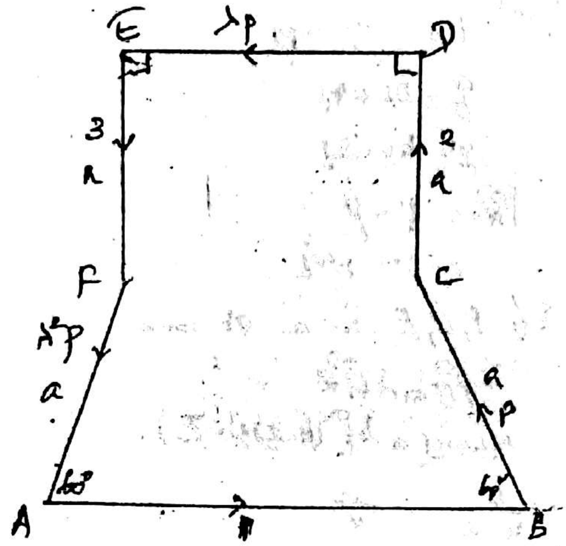
P,Q,R,S Parallelogram

$\Rightarrow \overrightarrow{PQ} = \overrightarrow{SR}$ (5)

$10(\underline{i} + \underline{j}) = -\underline{s} + (\underline{i} - 3\underline{j})$

$\underline{s} = -9\underline{i} + 13\underline{j}$ (10)

$\boxed{70}$



$\rightarrow X = 1 - p\cos 60 - \lambda p - \lambda^2 p \sin 60$ (10)

$= 1 - \frac{p}{2} - \lambda p - \frac{\lambda^2 p}{2}$

$\uparrow Y = p\sin 60 + 2 - 3 - \lambda^2 p \sin 60$ (10)

$= p\frac{\sqrt{3}}{2} - 1 - \lambda^2 p \frac{\sqrt{3}}{2}$

System reduce to couple (10)

$X = 0$ and $Y = 0$ (10)

$2 - p - 2\lambda p - \lambda^2 p = 0$ + $p\sqrt{3} - 2 - \lambda^2\sqrt{3}p = 0$

$p(1 + 2\lambda + \lambda^2) = 2$ $\quad p\sqrt{3}(1-\lambda^2) = 2$

$\therefore 1 + 2\lambda + \lambda^2 = \sqrt{3}(1-\lambda^2)$

$(1+\sqrt{3})\lambda^2 + 2\lambda + 1 - \sqrt{3} = 0$

$(\lambda \neq 0)$

$\lambda = 2 - \sqrt{3}$

System reduce to a single force along AC (10)

Then $\tan 30 = \frac{Y}{X}$

$\frac{1}{\sqrt{3}} = \frac{p\frac{\sqrt{3}}{2} - 1 - \lambda^2 p\frac{\sqrt{3}}{2}}{1 - \frac{p}{2} - \lambda p - \lambda^2\frac{p}{2}}$

$p = \frac{1+\sqrt{3}}{2+\lambda-\lambda^2}$ (10)

$= \frac{1+\sqrt{3}}{(1+\lambda)(2-\lambda)}$ (5)

$p > 0 \Rightarrow (1+\lambda)(2-\lambda) > 0$

$(\lambda+1)(\lambda-2) < 0$ $\quad + \lambda > 0$

$\therefore -1 < \lambda < 2$ (5) $\boxed{80}$

$\Rightarrow 0 < \lambda < 2$