Grade - 13 (2020) | **Combined Maths I** | **Marking Scheme**

1)

For $n=1$

$L.H.S = \frac{1}{1.2} = \frac{1}{2}$ $\quad R.H.S = \frac{1}{1+1} = \frac{1}{2}$

$L.H.S = R.H.S$

The result is true for $n=1$ 5

Take any $p \in \mathbb{Z}^+$ and assume that the result is true for $n=p$

$$\sum_{r=1}^{p} \frac{1}{r(r+1)} = \frac{p}{p+1} \quad 5$$

Now $n=p+1$

$$\sum_{r=1}^{p+1} \frac{1}{r(r+1)} = \frac{p}{(p+1)} + \frac{1}{(p+1)(p+2)} \quad 5$$

$$= \frac{p(p+2)+1}{(p+1)(p+2)}$$

$$= \frac{(p+1)^2}{(p+1)(p+2)} = \frac{p+1}{p+2} \quad 5$$

hence if the result is true for $n=p$ then it is also true for $n=p+1$ 5 hence by the Principle of mathematical Induction the result is true for all $n \in \mathbb{Z}^+$

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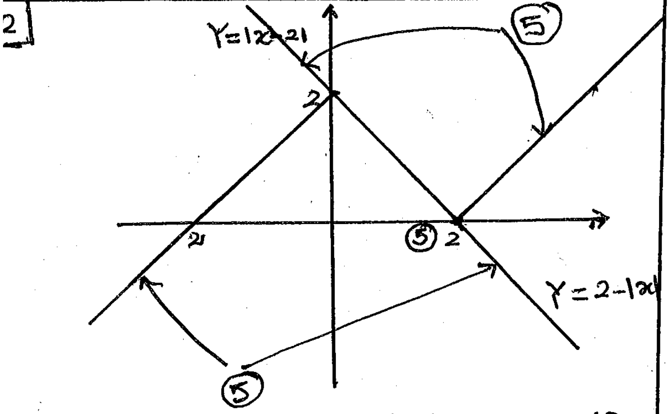
2)



$|x-2| = 2-|x| \iff 0 \leq x \leq 2$ 5

$|x-2|+|x| = 2 \iff 0 \leq x \leq 2$

$\therefore$ The solutions of x are $0 \leq x \leq 2$ 5

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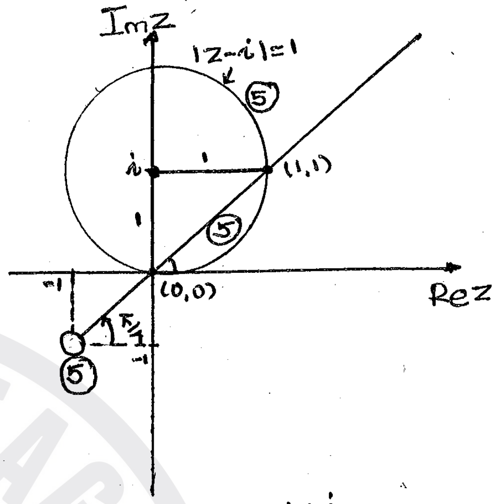
3)



$\therefore z = 0$ 5 or $z = 1+i$ 5

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4)

$$\lim_{x \to 2} \frac{\cos \frac{\pi x}{4}}{\sqrt{x+3} - \sqrt{5}} \quad 5$$

$$= \lim_{x \to 2} \frac{\sin\left(\frac{\pi}{2} - \frac{\pi x}{4}\right)\left(\sqrt{x+3} + \sqrt{5}\right)}{(x+3)-5} \quad 5$$

$$= \lim_{x \to 2} \frac{\sin \frac{\pi}{4}(2-x)\left(\sqrt{x+3}+\sqrt{5}\right)}{(x-2)}$$

$$= -\frac{\pi}{4} \lim_{x \to 2} \frac{\sin \frac{\pi}{4}(x-2)}{\frac{\pi}{4}(x-2)} \cdot \lim_{x \to 2}\left(\sqrt{x+3}+\sqrt{5}\right)$$

5 5

$$= -\frac{\pi}{4} \cdot 1 \cdot 2\sqrt{5}$$

$$= -\frac{\sqrt{5}\pi}{2} \quad 5$$

25

Page 1

5. $x = 2(1-\cos\theta)$

$$\frac{dx}{d\theta} = 2\sin\theta = 4\sin\theta/_2 \cos\theta/_2$$

$$y = 2(\theta + \sin\theta)$$

$$\frac{dy}{d\theta} = 2(1+\cos\theta) = 4\cos^2\theta/_2$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{4\cos^2\theta/_2}{4\sin\theta/_2 \cos\theta/_2} \quad 5$$

$$\frac{dy}{dx} = \cot\theta/_2 \quad 5$$

Point $(4, 2\pi)$ on $C \Rightarrow \theta = \pi$ 5

$$\left(\frac{dy}{dx}\right)_{\theta=\pi} = 0 \quad 5$$

Equation of the tangent is

$y = 2\pi$ 5

$\boxed{25}$

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6. $\frac{d}{dx}\left\{x\sqrt{1-x^2}\right\}$

$$= x\frac{1\,(-2x)}{2\sqrt{1-x^2}} + \sqrt{1-x^2} \quad 10$$

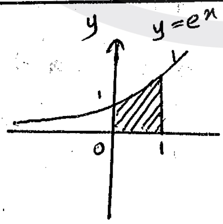
$$= \frac{1-2x^2}{\sqrt{1-x^2}} \quad 5$$

$$\int \frac{1+x^2}{\sqrt{1-x^2}}dx = -\frac{1}{2}\int\frac{1-2x^2}{\sqrt{1-x^2}}dx + \int\frac{\frac{3}{2}}{\sqrt{1-x^2}}dx \quad 5$$

$$= -\frac{1}{2}x\sqrt{1-x^2} + \frac{3}{2}\sin^{-1}x + C$$

5 $\boxed{25}$

7.



The volume generated

$$= \int_0^1 \pi(e^x)^2 dx \quad 10$$

$$= \int_0^1 \pi e^{2x} dx \quad 5$$

$$= \pi\left[\frac{e^{2x}}{2}\right]_0^1 \quad 5$$

$$= \frac{\pi}{2}(e^2 - 1) \quad 5$$

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page 2

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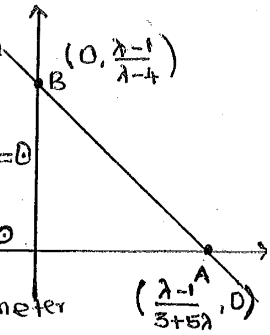
8) The required equation canbe writen



$(3x-4y+1) + \lambda(5x+y-1) = 0$ 5

$(3+5\lambda)x + (\lambda-4)y + (1-\lambda) = 0$

λ - parameter

But $\left|\frac{\lambda-1}{\lambda-4}\right| = \left|\frac{\lambda-1}{3+5\lambda}\right|$ 5

$\lambda - 1 = 0$ or $3+5\lambda = \pm(\lambda-4) \Rightarrow \lambda = \frac{-7}{4}$ or $\frac{1}{6}$

The equations 5 or λ=

$23x + 23y - 11 = 0$

$23x - 23y + 5 = 0$

$8x - 3y = 0$

10

$\boxed{25}$

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9) $S \equiv x^2 + y^2 + 2gx + 2fy + C = 0$

$(0,0) \Rightarrow C = 0$

$(0,2) \Rightarrow 0+4+0+4f+C = 0 \Rightarrow f = -1$ 5

$S \equiv x^2 + y^2 + 2gx - 2y = 0$

$S' \equiv x^2 + y^2 - 2x + 4y - 6 = 0$

Centre $O' \equiv (1, -2)$ 5

equation of common chord is

$S - S' = 0$

$(2g+2)x - 6y + 6 = 0$ 5

$(1,-2) \Rightarrow 2(g+1) + 12 + 6 = 0 \Rightarrow g = -10$ 5

$S \equiv x^2 + y^2 - 20x - 2y = 0$ 5 $\boxed{25}$

$\boxed{10}$ $\tan^{-1}\left(\frac{x}{x-1}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \tan^{-1}\left(\frac{2}{3}\right)$

$\Downarrow A \quad \Downarrow B \quad \Downarrow C$

$A + B = C$ 5 $\Rightarrow \tan(A+B) = \tan C$

$$\frac{\tan A + \tan B}{1 - \tan A \tan B} = \tan C \quad 5$$

$$\frac{\frac{x}{x-1} + \frac{x+2}{x+3}}{1 - \frac{x(x+2)}{(x-1)(x+3)}} = \frac{2}{3} \quad 5$$

$$\frac{2x^2 + 4x - 2}{-3} = \frac{2}{3} \quad 5$$

$x^2 + 2x - 1 = -1$

$x = 0$ or $x = -2$ 5

$\boxed{25}$

11 (a)

Suppose that 1 is a root of $(p+1)^2x^2+8x+2(p+1)=0$.

By substituting $x=1$, we must have $(p+1)^2+8+2(p+1)=0$ (5)

This is impossible, as $p>-1$ implies that $(p+1)^2+8+2(p+1)>0$ (5)

$\therefore$ 1 is not a root of $(p+1)^2x^2+8x+2(p+1)=0$ $\boxed{10}$

The discriminant

$$\Delta = 8^2-4(p+1)^2\,2(p+1) \quad (10)$$

$$= 8\{8-(p+1)^3\} \geq 0 \quad (\because -1<p\leq 1) \quad (5)$$

$\therefore$ α and β are both real (5) $\boxed{20}$

$\alpha+\beta = -\dfrac{8}{(p+1)^2}$ (5), and $\alpha\beta = \dfrac{2}{p+1}$ (5)

$$\frac{1}{(\alpha-1)(\beta-1)} = \frac{1}{\alpha\beta-(\alpha+\beta)+1} \quad (5)$$

$$= \frac{1}{\frac{2}{p+1}+\frac{8}{(p+1)^2}+1}$$

$$= \frac{(p+1)^2}{p^2+4p+11} \quad (5) \quad \boxed{20}$$

$$\frac{\alpha}{\alpha-1}+\frac{\beta}{\beta-1} = \frac{2\alpha\beta-(\alpha+\beta)}{(\alpha-1)(\beta-1)} \quad (5)$$

$$= \left(\frac{4}{p+1}+\frac{8}{(p+1)^2}\right)\cdot\frac{(p+1)^2}{p^2+4p+11}$$

$$= \frac{4(p+3)}{p^2+4p+11} \quad (5)$$

(5)

$$\frac{\alpha}{\alpha-1}\cdot\frac{\beta}{\beta-1}$$

$$= \frac{\alpha\beta}{(\alpha-1)(\beta-1)}$$

$$= \frac{2}{p+1}\cdot\frac{(p+1)^2}{p^2+4p+11} \quad (5)$$

$$= \frac{2(p+1)}{p^2+4p+11} \quad (5)$$

Hence, the required quadratic equation is given by

$$x^2-\frac{4(p+3)}{p^2+4p+11}x+\frac{2(p+1)}{p^2+4p+11}=0 \quad (5)$$

$$(p^2+4p+11)x^2-4(p+3)x+2(p+1)=0 \quad \text{---} (*) \quad \boxed{30}$$

$$\frac{\alpha}{\alpha-1}+\frac{\beta}{\beta-1} = \frac{4(p+3)}{(p+2)^2+7} > 0 \quad (5) \quad (\because p>-1)$$

$$\frac{\alpha}{\alpha-1}\cdot\frac{\beta}{\beta-1} = \frac{2(p+1)}{(p+2)^2+7} > 0 \quad (5) \quad (\because p>-1)$$

Hence, both of these roots are positive $\boxed{10}$

(b) $ax^n+b = (x^2-1)\phi(x)+x+2$ (10)

$x=1 \Rightarrow a+b=3$ (5)

$x=-1 \Rightarrow -a+b=1$ (5)

$a=1,\ b=2$ (5)

$x^n+2 \equiv (x^2-1)\phi(x)+x+2$

$n=7 \Rightarrow x^7+2 \equiv (x^2-1)\phi_1(x)+x+2$ (5) — ①

$n=5 \Rightarrow x^5+2 \equiv (x^2-1)\phi_2(x)+x+2$ (5) — ②

$n=3 \Rightarrow x^3+2 \equiv (x^2-1)\phi_3(x)+x+2$ (5) — ③

① + ② + ③ $\Rightarrow$

$$x^7+x^5+x^3+6 \equiv (x^2-1)[\phi_1(x)+\phi_2(x)+\phi_3(x)] + 3x+6 \quad (5)$$

$\therefore$ Remainder $= 3x+6$ (5) $\boxed{50}$

$1-\dfrac{1}{\alpha} = \dfrac{\alpha-1}{\alpha}$, $1-\dfrac{1}{\beta} = \dfrac{\beta-1}{\beta}$

Replacing x by $\dfrac{1}{x}$ in (*), we get (5)

$$(p^2+4p+11)\left(\tfrac{1}{x}\right)^2-4(p+3)\tfrac{1}{x}+2(p+1)=0$$

$$2(p+1)x^2-4(p+3)x+p^2+4p+11=0 \quad (5) \quad \boxed{10}$$

12

(a)

(i) $^9C_3 = \frac{9!}{6! \times 3!}$ (10)

$= 84$ (5)

(ii) $^{18}C_3 - {}^9C_3$ (10)

$= 816 - 84$

$= 732$ (5)

(iii) $^6C_1 \, ^6C_1 \, ^6C_1 = 216$ (5) (10)

(iv) $^3C_1 \, ^2C_1 \, ^1C_1 \times 3 = 18$ (15) (5) (5)

$\boxed{70}$

(b)

$U_r = \frac{r+3}{r(r+1)(r+2)}, \quad V_r = \frac{2r+3}{r(r+1)}$

$V_r - V_{r+1}$

$= \frac{2r+3}{r(r+1)} - \frac{2r+5}{(r+1)(r+2)}$ (5)

$= \frac{(2r+3)(r+2)-(2r+5)r}{r(r+1)(r+2)}$ (5)

$= \frac{2r^2+7r+6-2r^2-5r}{r(r+1)(r+2)}$

$= \frac{2(r+3)}{r(r+1)(r+2)}$ (5)

$= 2U_r$ (5)

$\boxed{20}$

page 4

$2U_r = V_r - V_{r+1}$

$r=1; \quad 2U_1 = V_1 - \cancel{V_2}$ (5)

$r=2; \quad 2U_2 = \cancel{V_2} - \cancel{V_3}$

$\vdots$

$r=n-1; \quad 2U_{n-1} = \cancel{V_{n-1}} - \cancel{V_n}$ (5)

$r=n; \quad 2U_n = \cancel{V_n} - V_{n+1}$

$2\sum_{r=1}^{n} U_r = V_1 - V_{n+1}$ (5)

$= \frac{5}{2} - \frac{2n+5}{(n+1)(n+2)}$ (5)

$\Rightarrow \sum_{r=1}^{n} U_r = \frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)}$ (5)

$\boxed{25}$

$\lim_{n\to\infty} \sum_{r=1}^{n} U_r = \lim_{n\to\infty} \left\{ \frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)} \right\}$ (5)

$= \lim_{n\to\infty} \left\{ \frac{5}{4} - \frac{\frac{2}{n}+\frac{5}{n^2}}{2(1+\frac{1}{n})(1+\frac{2}{n})} \right\}$ (5)

$= \frac{5}{4} - 0$

$= \frac{5}{4}$ (5)

$\therefore \sum_{r=1}^{\infty} U_r$ is convergent and the sum is $\frac{5}{4}$ (5)

$\boxed{20}$

$\sum_{r=3}^{\infty} 3U_r$

$= 3\sum_{r=1}^{\infty} U_r - 3U_1 - 3U_2$ (5)

$= 3 \times \frac{5}{4} - 3\left(\frac{2}{3}\right) - 3\left(\frac{5}{24}\right)$ (5)

$= \frac{9}{8}$ // (5)

$\boxed{15}$

13

$z \in \mathbb{C} \quad z = x + iy \quad x, y \in \mathbb{R}$

a) $|z| = \sqrt{x^2+y^2}$ (5) $\quad \bar{z} = x - iy$ (5)

1. $z.\bar{z} = (x+iy)(x-iy) = x^2 - (iy)^2$ (5)
$= x^2 + y^2 = |z|^2$ (5)

$z + \bar{z} = x + iy + x - iy = 2x = 2\,\mathrm{Re}\,z$ (5)

$z - \bar{z} = x + iy - x + iy = 2iy = 2i\,\mathrm{Im}\,z$ (5)

$\boxed{30}$

b) $z = x + iy = \sqrt{x^2+y^2}\left(\dfrac{x}{\sqrt{x^2+y^2}} + \dfrac{iy}{\sqrt{x^2+y^2}}\right)$ (5)

$= |z|\,(\cos\theta + i\sin\theta)$

$|z| = \sqrt{x^2+y^2}$ (5) $\mathrm{Arg}(z) = \theta$

$\cos\theta = \dfrac{x}{\sqrt{x^2+y^2}}$, $\sin\theta = \dfrac{y}{\sqrt{x^2+y^2}}$

$z_1 = r_1(\cos\theta_1 + i\sin\theta_1) \quad |z_1| = r_1$
$\mathrm{Arg}(z_1) = \theta_1$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2) \quad |z_2| = r_2$
$\mathrm{Arg}(z_2) = \theta_2$

$z_1.z_2 = r_1.r_2\,(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$

$= r_1 r_2\left((\cos\theta_1\cos\theta_2 + i^2\sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)\right)$ (5)

$= r_1 r_2\,(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$ (5)

$|z_1.z_2| = r_1 r_2 = |z_1||z_2|$ (5)

$\arg(z_1 z_2) = \theta_1 + \theta_2 = \arg(z_1) + \arg(z_2)$ (5)

$\boxed{30}$

c) $z_1 = \dfrac{4}{1 - i\sqrt{3}} \qquad z_2 = \dfrac{2}{1+i}$

$z_1 = \dfrac{4(1+i\sqrt{3})}{(1 - i^2 3)}$ (5) $\qquad z_2 = \dfrac{2(1-i)}{1-i^2}$ (5)

$z_1 = (1 + i\sqrt{3}) \qquad z_2 = 1 - i$

$z_1 = 2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$ (5) $\qquad z_2 = \sqrt{2}\left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)$ (5)

$z_1 = 2(\cos\pi/3 + i\sin\pi/3)$ (5) $\qquad z_2 = \sqrt{2}(\cos(-\pi/4) + i\sin(-\pi/4))$ (5)

$|z_1| = 2$ (5) $\qquad |z_2| = \sqrt{2}$ (5)
$\mathrm{Arg}(z_1) = \pi/3 \qquad \mathrm{Arg}(z_2) = -\pi/4$

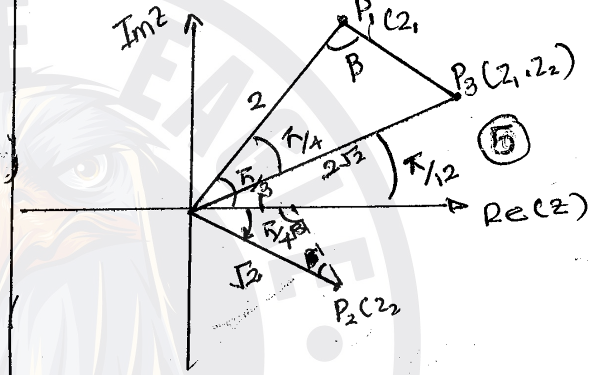
$|z_1.z_2| = |z_1|.|z_2| = 2\sqrt{2}$ (5)

$\arg(z_1.z_2) = \arg(z_1) + \arg(z_2)$
$= \pi/3 - \pi/4 = \dfrac{\pi}{12}$ (5)

$\mathrm{Arg}(z_1 z_2) = \pi/12$

$z_1.z_2 = |z_1 z_2|\,(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

$= 2\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ (5)



(5)

$\boxed{60}$

c) $z_1^3 + 2z_2^4 = 2^3(\cos 3\times\pi/3 + i\sin 3\times\pi/3)$
$+ 2\times(\sqrt{2})^4(\cos(4\cdot -\pi/4) + i\sin 4\cdot -\pi/4)$ (5)

$= 2^3\left[\cos\pi + i\sin\pi + \cos\pi - i\sin\pi\right]$ (5)

$= 2^3 \times 2.\cos\pi$ (5)

$= -16$ (5)

$\boxed{30}$

Page 5

14. $f(x) = \dfrac{x+1}{(x+3)^2}$

$f'(x) = \dfrac{(x+3)^2(1) - (x+1)2(x+3)}{(x+3)^4}$ (10)

$= \dfrac{(x+3)[x+3-2x-2]}{(x+3)^4}$

$= -\dfrac{x-1}{(x+3)^3}$

$f''(x) = -\left\{\dfrac{(x+3)^3(1) - (x-1)3(x+3)^2}{(x+3)^6}\right\}$ (10)

$= -\left\{\dfrac{x+3-3x+3}{(x+3)^4}\right\}$

$= \dfrac{2(x-3)}{(x+3)^4}$ $\boxed{20}$

When $x=0$, $y=\frac{1}{9}$

when $y=0$, $x=-1$

$\lim_{x\to -3} f(x) = \infty$

Vertical asymptote : $x=-3$ (5)

$\lim_{x\to \pm\infty} \dfrac{x+1}{(x+3)^2} = \lim_{x\to\pm\infty} \dfrac{\frac{1}{x}+\frac{1}{x^2}}{(1+\frac{3}{x})^2} = 0$

Horizontal asymptote : $y=0$ (5)

when $f'(x)=0$, $x=1$ (5)

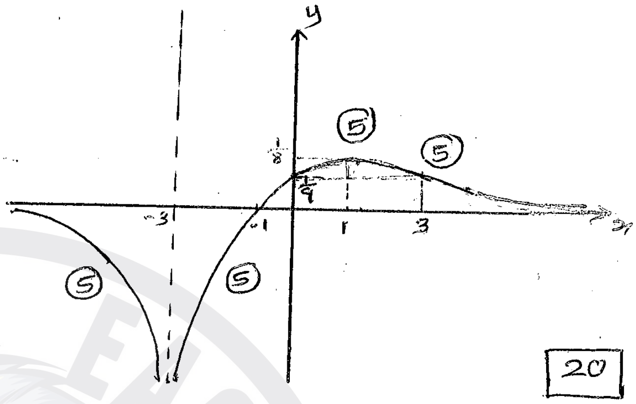
$x<-3$	$-3<x<1$	$x>1$
$f'(x)<0$	$f'(x)>0$	$f'(x)<0$
decreasing	increasing	decreasing
(5)	(5)	(5)

$(1, \frac{1}{8})$ is a local maximum (5)

$f''(x) = 0 \Leftrightarrow x = 3$ (5)

$x<-3$	$-3<x<3$	$x>3$
$f''(x)<0$	$f''(x)<0$	$f''(x)>0$
concave down (5)	concave down (5)	concave up (5)

$(3, \frac{1}{9})$ is a point of inflection (5) $\boxed{60}$



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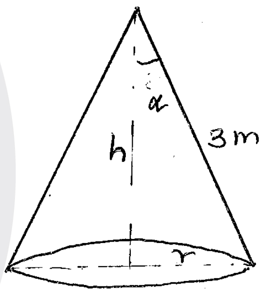


$h^2 + r^2 = 3^2$ (5)

$\Rightarrow r^2 = 9 - h^2$

Volume $V = \frac{1}{3}\pi r^2 h$ (5)

$= \frac{1}{3}\pi h(9-h^2)$ (5)

$\dfrac{dV}{dh} = \frac{1}{3}\pi(9-3h^2)$ (10)

$= -\pi(h^2-3)$

$\dfrac{dV}{dh} = 0 \Leftrightarrow h = \sqrt{3}$ $[\because h>0]$ (5)

For $0<h<\sqrt{3}$, $\dfrac{dV}{dh}>0$ (5) and $h>\sqrt{3}$, $\dfrac{dV}{dh}<0$ (5)

$\therefore$ V is maximum when $h=\sqrt{3}$ (5)

$h=\sqrt{3} \Rightarrow r=\sqrt{6}$

$\tan\alpha = \dfrac{r}{h} = \dfrac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$

$\therefore$ V is maximum when (5) $\alpha = \tan^{-1}(\sqrt{2})$

$\boxed{50}$

Page 6

15

a) $\frac{d \ln(x^2+1)}{dx} = \frac{1}{(x^2+1)} \times 2x = \frac{2x}{(x^2+1)}$ (5)

$\int$ing both side w.r.t x

$\int \frac{2x}{x^2+1} = \ln(x^2+1) + c'$ (5) (5)

$\int \frac{x}{x^2+1} = \frac{\ln(x^2+1)}{2} + C$

C - arbitary constant

$\boxed{15}$

II $\frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x-2)} + \frac{D}{(x+2)}$ (10)

$x^3+4x^2-4x+4 = (Ax+B)(x^2-4) + C(x^2+1)(x+2) + D(x^2+1)(x-2)$

$x=2 \quad 20C = 20 \Rightarrow C=1$

$x=-2 \quad -20D = 20 \Rightarrow D=-1$ 4 eq^n / 4 ans (15)

con $\quad 2C-2D-4B = 4 \Rightarrow B=0$

$x^3 \quad A+C+D = 1 \Rightarrow A=1$

$\frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)} = \frac{x}{x^2+1} + \frac{1}{x-2} + \frac{-1}{x+2}$

$\therefore \int$ing both side w.r.t x

$\int \frac{x^3+4x^2-4x+4}{(x^2+1)(x^2-4)} dx = \int \frac{x\,dx}{x^2+1} + \int \frac{dx}{x-2} - \int \frac{dx}{x+2}$

(5) (5) (5) (5)

$= \frac{\ln(x^2+1)}{2} + \ln|x-2| - \ln|x+2| + K$

K - arbitary constant

$\boxed{45}$

b) $\int x \sin^2 x\,dx = \int \frac{x(1-\cos 2x)}{2} dx$ (5)

$= \int \frac{x}{2} dx - \int \frac{x \cos 2x}{2} dx$

$= \frac{x^2}{4}$ (5) $- \frac{x \sin 2x}{4}$ (5) $+ \int \frac{\sin 2x}{4} dx + c'$ (5)

$= \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}$ (5) $+ C$ (5)

C - arbitary con

$\int x \cos^2 x\,dx + \int x \sin^2 x\,dx = \int x(\sin^2 x + \cos^2 x)$ (5)

$\Rightarrow \int x \cos^2 x\,dx = \int x\,dx - \int x \sin^2 x\,dx.$ (5)

$\Rightarrow \int x \cos^2 x\,dx = \frac{x^2}{2} - \left(\frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{\cos 2x}{8}\right) + c$ (5)

$= \frac{x^2}{4} + \frac{x \sin 2x}{4} + \frac{\cos 2x}{8} + C$ (5)

$\boxed{50}$

C) $x = 1 + 3\sin^2\theta$

$x: 1 \to 4 \quad x = 1 \Leftrightarrow \sin^2\theta = 0$, $\theta = 0$ (5)

$\theta: 0 \to \pi/2 \quad x = 4 \Leftrightarrow \sin^2\theta = 1$, $\theta = \pi/2$ (5)

$dx = 6 \sin\theta \cos\theta\, d\theta \; (0 \le \theta \le \pi/2)$ (5)

$\int_1^4 \frac{dx}{\sqrt{(x-1)(4-x)}} = \int_0^{\pi/2} \frac{6 \sin\theta \cos\theta\, d\theta}{\sqrt{3\sin^2\theta \cdot 3\cos^2\theta}}$ (5)

$= \int_0^{\pi/2} \frac{6 \sin\theta \cos\theta\, d\theta}{3 \sin\theta \cos\theta}$ (5) $= 2\int_0^{\pi/2} d\theta$ (5)

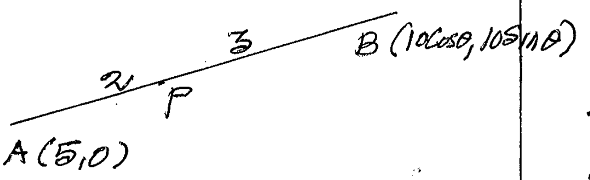
$= 2\,\theta \Big|_0^{\pi/2}$ (5)

$= 2(\pi/2 - 0) = \pi$ (5)

$\boxed{40}$

Page 7

Q16]



$P \equiv \left(\frac{20\cos\theta + 15}{5}, \frac{20\sin\theta + 0}{5}\right)$ (10)

$P \equiv (4\cos\theta + 3, 4\sin\theta)$

$P \equiv (\bar{x}, \bar{y})$ (say)

$\bar{x} = 4\cos\theta + 3 \quad \bar{y} = 4\sin\theta$ (5)

but $\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{\bar{x}-3}{4}\right)^2 + \left(\frac{\bar{y}}{4}\right)^2 = 1$ (5)

$\bar{x}^2 + \bar{y}^2 - 6\bar{x} - 7 = 0$ (5)

put $\bar{x} \to x$, $\bar{y} \to y$

$S \equiv x^2 + y^2 - 6x - 7 = 0$ (5)

$(x-3)^2 + (y-0)^2 = 4^2$ the path is circle (5)

centre $C \equiv (3, 0)$

radius $= 4$ (5) $\boxed{40}$

$Q \equiv (2, 5)$

$CQ = \sqrt{(3-2)^2 + (0-5)^2}$ (5)

$= \sqrt{26} > 4$ (5)

The point Q lies outside the circle

The equation of tangent is given by

$y - 5 = m(x - 2)$ (5)

$mx - y + (5 - 2m) = 0$

$4 = \left|\frac{m \times 3 - 0 + 5 - 2m}{\sqrt{m^2 + (-1)^2}}\right|$ (10)

$16(m^2 + 1) = (m + 5)^2$

$15m^2 - 10m + 9 = 0$ (5)

$m = \frac{10 \pm \sqrt{100 - 4(15)(-9)}}{2 \times 15}$ (10)

$= \frac{5 \pm 4\sqrt{10}}{15}$ (5)

Tangents are

$y - 5 = \left(\frac{5 \pm 4\sqrt{10}}{15}\right)(x - 2)$ (10) $\boxed{55}$

CD

$2x + 5y - 3(2 + x) - 7 = 0$

$x - 5y + 13 = 0$ (10)

S' can be write

$x^2 + y^2 - 6x - 7 + \lambda(x - 5y + 13) = 0$ (5)

$(1, 2) \Rightarrow$

$1 + 4 - 6 - 7 + \lambda(1 - 10 + 13) = 0$

$\lambda = 2$ (5)

$S' \equiv x^2 + y^2 - 4x - 10y + 19 = 0$ (5) $\boxed{25}$

$S'' \equiv x^2 + y^2 + 2g''x + 2f''y + c'' = 0$ (say)

$(0, 6) \Rightarrow 0 + 36 + 12f'' + c'' = 0$ (5)

$c'' = -36 - 12f''$ (1)

The circles $S' = 0$, $S'' = 0$ are interset orthogonal

$2\{(g'')(-2) + (f'')(-5)\} = c'' + 19$ (10)

$-4g'' - 10f'' = -36 - 12f'' + 19$ (by (1))

$4g'' - 2f'' - 17 = 0$ (5)

put $-g'' = x$, $-f'' = y$ (5)

$4x - 2y + 17 = 0$ (5)

$\boxed{30}$

page 8

Q17]

a) $\sin(A+B) = \sin A\cos B + \cos A\sin B$ —— (1) (5)

$\sin(A-B) = \sin\{A+(-B)\}$

$= \sin A\cos(-B) + \cos A\sin(-B)$

$= \sin A\cos B - \cos A\sin B$ (5) —— (2)

(1)+(2) ⇒

$\sin(A+B)+\sin(A-B) = 2\sin A\cos B$ (5)

put $B \to A$

$\sin(2A)+\sin(A-A) = 2\sin A\cos A$

$\sin 2A = 2\sin A\cos A$ (5) $\boxed{20}$

$\sin\theta\{8\cos\theta\cos 2\theta\cos 3\theta - 1\}$

$= 8\sin\theta\cos\theta\cos 2\theta\cos 3\theta - \sin\theta$ (5)

$= 4\sin 2\theta\cos 2\theta\cos 3\theta - \sin\theta$ (5)

$= 2\sin 4\theta\cos 3\theta - \sin\theta$ (5)

$= \sin 7\theta + \sin\theta - \sin\theta$ (5)

$= \sin 7\theta$

$\cos\theta\cos 2\theta\cos 3\theta = \frac{1}{4}$

$\frac{1}{8}\left\{\frac{\sin 7\theta}{\sin\theta} + 1\right\} = \frac{1}{4}$ (5)

$\sin 7\theta = \sin\theta$

$7\theta = n\pi + (-1)^n\theta$ (5) ; $n \in \mathbb{Z}$

$n=0 \Rightarrow \theta = 0$

$n=1 \Rightarrow \theta = \frac{\pi}{8}$

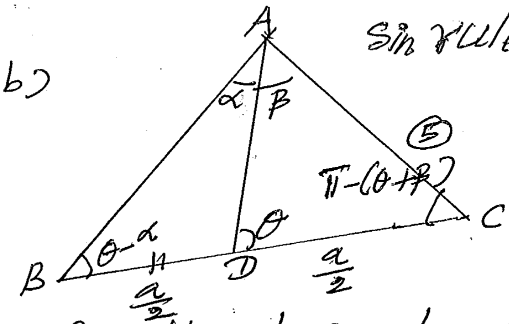
$n=2 \Rightarrow \theta = \frac{\pi}{4}$

$n=3 \Rightarrow \theta = \frac{3\pi}{8}$

$n=4 \Rightarrow \theta = \frac{2\pi}{3} > \frac{\pi}{2}$

sol^n :- $\left\{\frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}\right\}$ (10)

Page-9 $\boxed{40}$

b) 

Sin rule — (5)

for the triangle ABD

$\frac{BD}{\sin\alpha} = \frac{AD}{\sin(\theta-\alpha)}$ (5)

$\frac{\frac{a}{2}}{\sin\alpha} = \frac{AD}{\sin(\theta-\alpha)}$

$\frac{a}{2\sin\alpha} = \frac{AD}{\sin(\theta-\alpha)}$ (5) (1)

In ΔADC

$\frac{AD}{\sin\{\pi-(\theta+\beta)\}} = \frac{\frac{a}{2}}{\sin\beta}$ (10)

$\frac{AD}{\sin(\theta+\beta)} = \frac{a}{2\sin\beta}$ —— (2)

(1),(2) ⇒

$\frac{a\sin(\theta-\alpha)}{2\sin\alpha} = \frac{a\sin(\theta+\beta)}{2\sin\beta}$ (5)

$\sin\beta\{\sin\theta\cos\alpha - \cos\theta\sin\alpha\} = \sin\alpha\{\sin\theta\cos\beta + \cos\theta\sin\beta\}$ (10)

divide by $\sin\alpha\sin\beta\sin\theta$ both side

$\cot\alpha - \cot\theta = \cot\beta + \cot\theta$ (5)

$\cot\alpha - \cot\beta = 2\cot\theta$ (5) $\boxed{55}$

---

c) $2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{6}{5}\right) = \frac{\pi}{2}$ —— (1)

$\alpha = \tan^{-1}\left(\frac{1}{5}\right)$ $\beta = \tan^{-1}\left(\frac{6}{5}\right)$

$\Leftrightarrow 2\alpha + \beta = \frac{\pi}{2}$ (5)

$\Leftrightarrow 2\alpha = \frac{\pi}{2} - \beta$

$\Leftrightarrow \tan 2\alpha = \tan\left(\frac{\pi}{2} - \beta\right)$ (5) ($2\alpha, \left(\frac{\pi}{2}-\beta\right)$ are acute)

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$ (5)

$= \frac{2\left(\frac{1}{5}\right)}{1-\frac{1}{25}}$

$= \frac{\frac{10}{24}}{}$

$= \frac{5}{6}$ (5)

$\tan\left(\frac{\pi}{2}-\beta\right) = \cot\beta$ (5)

$= \frac{5}{6}$

$\therefore 2\alpha + \beta = \frac{\pi}{2}$

$2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{6}{5}\right) = \frac{\pi}{2}$

$\tan^{-1}\left(\frac{1}{5}\right) = \frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{6}{5}\right)$ (5)

$\sin^{-1}\left(\frac{1}{\sqrt{26}}\right) = \frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{6}{5}\right)$ (5)

$\frac{1}{\sqrt{26}} = \sin\left\{\frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{6}{5}\right)\right\}$ $\boxed{35}$

வடமாகாணக் கல்வித் திணைக்களத்துடன் இணைந்து தொண்டைமானாறு

வெளிக்கள நிலையம் நடாத்தும் தவணைப் பரீட்சை, மார்ச் - 2020

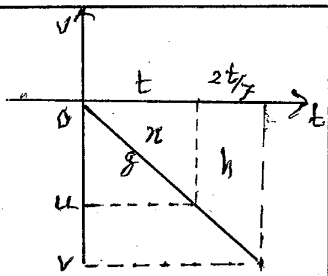
Conducted by Field Work Centre, Thondaimanaru

In Collaboration with Provincial Department of Education Northern Province

Term Examination, March - 2020

Grade - 13 (2020) | **Combined Maths II** | **Marking Scheme**

(1)



$t = \frac{u}{g}$

$t + \frac{2t}{7} = \frac{v}{g}$

$h = \frac{1}{2}(u+v) \cdot \frac{2t}{7}$ (5)

$= \frac{t}{7}\left[gt + \frac{9gt}{7}\right]$

$= \frac{16}{49} g t^2$

$t = \frac{7}{4}\sqrt{\frac{h}{g}}$ (5)

$AC = h + x$ (5)

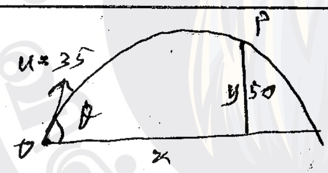
$= h + \frac{1}{2} g t^2$

$= h + \frac{1}{2} g \frac{49}{16} \cdot \frac{h}{g}$

$= \frac{81h}{32}$ (5)

$\boxed{25}$

(2)



$O \to P$ $\quad s = ut + \frac{1}{2}at^2$ $\qquad \tan\theta = 2$

$\rightarrow \quad x = u\cos\theta \cdot t$ —— (1)

$\uparrow \quad y = u\sin\theta t - \frac{1}{2}gt^2$ —— (2)

$(1)(2) \Rightarrow y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2}(\sec^2\theta)$ —— (3) (5)

$50 = x \cdot 2 - \frac{9.8\, x^2 \cdot 5}{2 \cdot 35^2}$ (5)

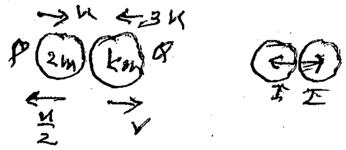
$x^2 - 100x + 2500 = 0$ (5)

$(x-50)^2 = 0$

$x = 50$ (5)

$(1) \Rightarrow t = \frac{10\sqrt{5}}{7}$ (5)

(3)



for $P \rightarrow \underline{I} = \Delta m\underline{v}$

$-I = 2m\left(-\frac{u}{2}\right) - 2mu$

$I = 3mu$

for $Q \rightarrow \underline{I} = kmv - km(-3u)$

$3mu = kmv + 3kmu$ (5)

$v = \frac{3u}{k}(1-k)$ (5)

$v > 0 \Rightarrow \frac{1}{k}(1-k) > 0 \qquad k > 0$

$k < 1 \qquad + k > 0$

$\Rightarrow \quad 0 < k < 1.$ (5)

N.R.L $\quad v + \frac{u}{2} = e(u + 3u)$ (5)

$k = \frac{1}{2} \Rightarrow v = 3u$

$\therefore e = \frac{7}{8}$ (5)

$\boxed{25}$

(4)

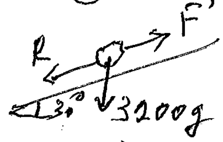
$36 \times 1000 = F \times 20$ (5)

$F = 1800$ N

$\rightarrow F - R = 3200\, a$ (5)

$1800 - R = 3200 \times 0.2$

$R = 1160$ N (5)



$\nearrow \quad F' - R - 3200g \sin 30^\circ = 0$ (5)

$F' = 17160$ N

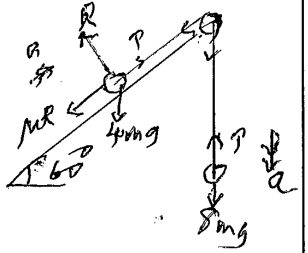
$P = 17160 \times 30$

$= 514800$ W (5)

$= 514.8$ kW

$\boxed{25}$

(5)



P ↗ $R = 4mg\cos 60 = 2mg$ (5)

↗ $T - \mu R - 4mg\sin 60 = 4ma$ (5)

Q ↓ $8mg - T = 8ma$ (5)

$a = \frac{g}{24}(15 - 4\sqrt{3})$ (5)

$T = \frac{g}{3}(9 + 4\sqrt{3})$ (5)

$\boxed{25}$

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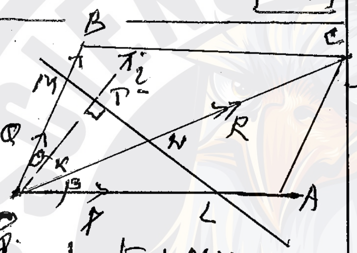
(6)

(5)



OT is a perpendicular to LMN

Let $\underline{i}$ is a unit vector in the direction $\overrightarrow{OT}$

$\underline{P} + \underline{Q} = \underline{R}$

$\underline{P}\cdot\underline{i} + \underline{Q}\cdot\underline{i} = \underline{R}\cdot\underline{i}$

$|\underline{P}|.1.\cos(\alpha+\beta) + |\underline{Q}|.1.\cos\theta = |\underline{R}|.1\cos\alpha$ (10)

$P.\frac{OT}{OL} + Q.\frac{OT}{OM} = R.\frac{OT}{ON}$ (5)

$\Rightarrow \frac{P}{OL} + \frac{Q}{OM} = \frac{R}{ON}$ (5)

$\boxed{25}$

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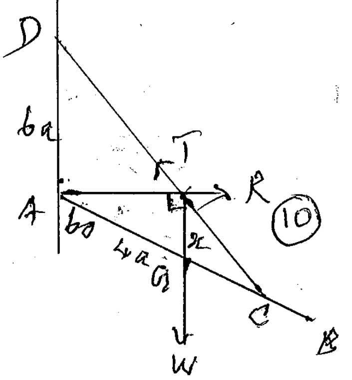
(7)

$x = 2a$

$\frac{AC}{CG} = \frac{AD}{x} = \frac{6a}{2a}$ (10)



(10)

$\therefore CG = 2a$

$\therefore BC = 2a$ (5)

$\boxed{25}$

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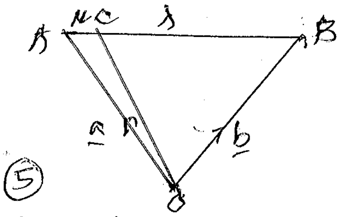
(8)



$\frac{AC}{CB} = \frac{\mu}{\lambda}$

$\lambda\overrightarrow{AC} = \mu\overrightarrow{CB}$ (5)

$\lambda(\underline{c} - \underline{a}) = \mu(\underline{b} - \underline{c})$ (5)

$(\lambda+\mu)\underline{c} = \lambda\underline{a} + \mu\underline{b}$

$\underline{c} = \frac{\lambda\underline{a} + \mu\underline{b}}{\lambda+\mu}$ (5)

$\alpha\underline{a} + \beta\underline{b} + \gamma\underline{c} = \underline{0}$ & $\alpha+\beta+\gamma = 0$

$\alpha\underline{a} + \beta\underline{b} = -\gamma\underline{c}$ $\quad \alpha+\beta = -\gamma$

$\therefore \frac{\alpha\underline{a}+\beta\underline{b}}{\alpha+\beta} = \frac{-\gamma\underline{c}}{-\gamma}$ (5)

$\Rightarrow \frac{\alpha\underline{a}+\beta\underline{b}}{\alpha+\beta} = \underline{c}$ (5)

$\boxed{25}$

from the 1st part A,B,C are collinear.

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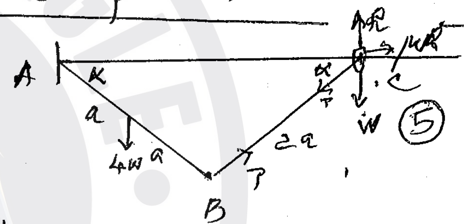
(9)



(5)

for Ring ← $T\cos\alpha - \mu R = 0$ (5)

↑ $R - W - T\sin\alpha = 0$ (5)

for Rod & Ring A↺

$4a\cos\alpha(R - W) - 4wa\cos\alpha = 0$ (5)

$R = 2W$

$\frac{(2)}{(1)} \Rightarrow \tan\alpha = \frac{R-W}{\mu R} = \frac{1}{2\mu}$ (5)

$\tan\alpha = \frac{1}{2\mu}$

$\boxed{25}$

---

(10) $P(A\cap B') = \frac{8}{25}$, $P(A'\cap B) = \frac{11}{100}$

$P(A\cup B) = \frac{13}{20}$

I $P(A\cup B) = P(A\cap B') + P(A'\cap B) + P(A\cap B)$ (5)

$\frac{13}{20} = \frac{8}{25} + \frac{11}{100} + P(A\cap B)$

$P(A\cap B) = \frac{22}{100}$ (5)

II $P(A) = P(A\cap B) + P(A\cap B')$

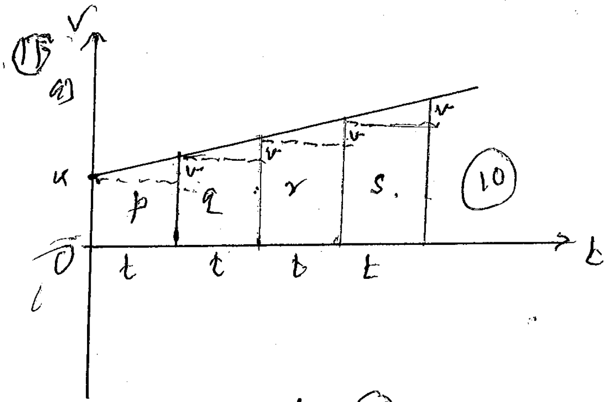
$= \frac{22}{100} + \frac{8}{25} = \frac{54}{100}$ (5)

III $P(B) = P(A\cap B) + P(A'\cap B) = \frac{33}{100}$ (5)

IV $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{2}{3}$ (5)

$\boxed{25}$

(11) a)



$p = \frac{1}{2}(2u+v)t$ (10)

$q = \frac{1}{2}(2u+3v)t$ (10)

$r = \frac{1}{2}(2u+5v)t$ (10)

$\Rightarrow p + r = 2q$

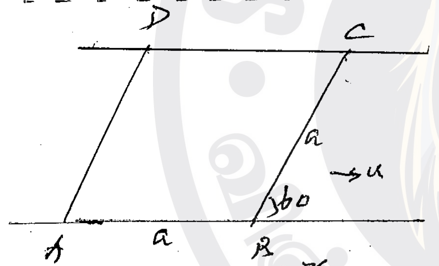
$s = \frac{t}{2}(2u+7v)$. (5)

$2r - q = \frac{t}{2}\left[2(2u+5v) - (2u+3v)\right]$ (10)

$= \frac{t}{2}\left[2u+7v\right]$

$\Rightarrow s = 2r - q$ (5) $\boxed{60}$

b)

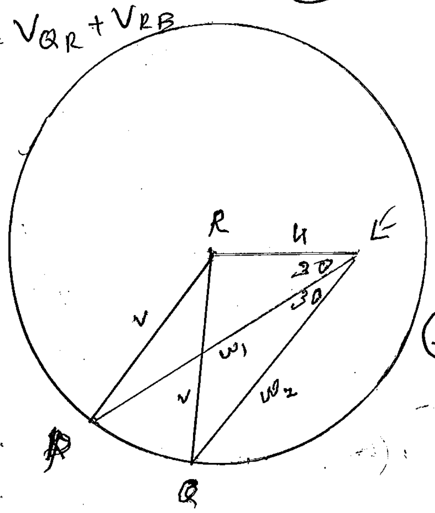


$V_{RE} \xrightarrow{u}$ $\quad V_{PE}$ 30 $\quad V_{PR}$ (v)

V_{QE} 60 (5) V_{QR} (v)

$V_{PE} = V_{PR} + V_{RE}$ (5)

$V_{QE} = V_{QR} + V_{RE}$



$V_{PE} = w_1 = u\cos 30 + \sqrt{v^2 - u^2\sin^2 30}$ (10)

$= \frac{u\sqrt{3}}{2} + \frac{\sqrt{4v^2 - u^2}}{2}$

$= \frac{1}{2}\left[\sqrt{3}u + \sqrt{4v^2 - u^2}\right]$ (5)

$V_{QE} = w_2 = u\cos 60 + \sqrt{v^2 - u^2\sin^2 60}$ (10)

$= \frac{u}{2} + \frac{\sqrt{4v^2 - 3u^2}}{2}$

$= \frac{1}{2}\left[u + \sqrt{4v^2 - 3u^2}\right]$ (5)

$t_{AC} = t_{AD}$

$\frac{4a\cos 30}{w_1} = \frac{2a}{w_2}$ (10)

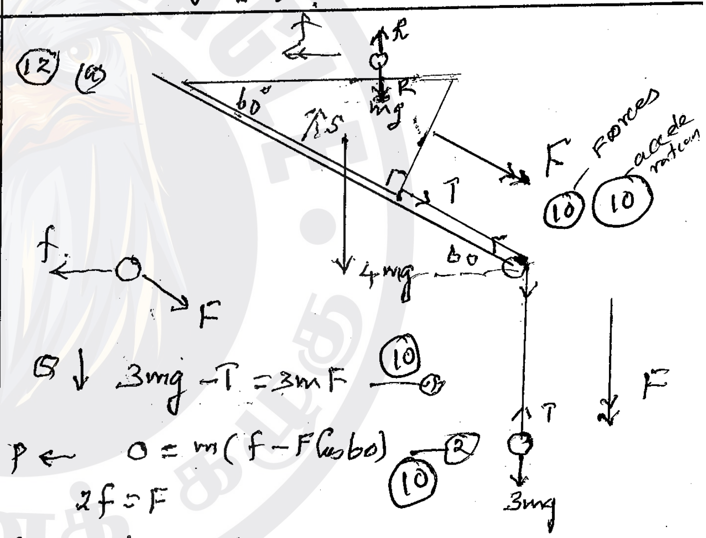
$\sqrt{3}w_2 = w_1$

$\frac{\sqrt{3}}{2}(u + \sqrt{4v^2 - 3u^2}) = (\sqrt{3}u + \sqrt{4v^2 - u^2})\frac{1}{2}$ (5)

$\sqrt{3}\sqrt{4v^2 - 3u^2} = \sqrt{3}u + \sqrt{4v^2 - u^2}$ (5) $\boxed{90}$

$v = u$

(12) (a)



③ $\downarrow$ $3mg - T = 3mF$ (10) ①

$P \leftarrow$ $0 = m(f - F\cos 60)$ (10) ②

$2f = F$

for Wedge + P

$\rightarrow$ $T + 5mg\sin 60^\circ = 4mF + m(F - f\cos 60)$ (15) ③

①+③ $\Rightarrow$ $\left(3 + 5\frac{\sqrt{3}}{2}\right)g = 8F - \frac{f}{2}$

$\frac{6+5\sqrt{3}}{2}g = \frac{31}{2}f$

$f = \frac{6+5\sqrt{3}}{31}g$ (5)

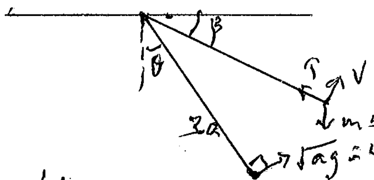
for P $\leftarrow$ $s = ut + \frac{1}{2}at^2$ (10)

$2l = 0 + \frac{1}{2}\left(\frac{6+5\sqrt{3}}{31}\right)g t^2$

$t = \sqrt{\frac{124 l}{(6+5\sqrt{3})g}}$ (5)

$\boxed{75}$

12(b)



(10)

using conservation of Energy.

$\frac{1}{2}mu^2 - mg\,3a\cos\alpha = \frac{1}{2}mV^2 - mg\,3a\sin\beta$ (15)

$V^2 = 3ag(2\sin\beta - 1)$ (5)

↖ $T - mg\sin\beta = m\frac{V^2}{3a}$ (10)

$T = mg[3\sin\beta - 1]$ (10)

$V = 0 \Rightarrow \beta = \pi/6$ (5)

$\Rightarrow T > 0$

$\therefore T_{min} = \frac{mg}{2}$ (10) $\beta = \frac{\pi}{6}$

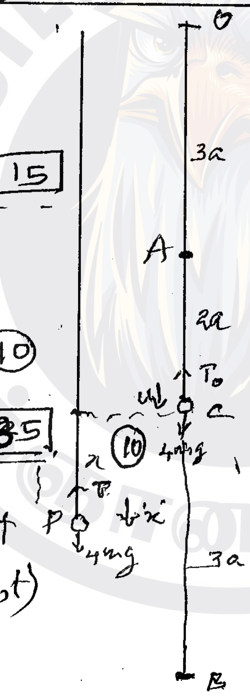
$T_{max} = 2mg$ (10) $\beta = \frac{\pi}{2}$ $\boxed{75}$

---

13 (i) $T_0 = 4mg$ (5)

$\frac{\lambda}{3a} \times 2a = 4mg$ (5)

$\lambda = 6mg$ (5) $\boxed{15}$



(ii)

at P

$4mg - T = 4m\ddot{x}$ (10)

$4mg - \frac{6mg}{3a}(2a+x) = 4m\ddot{x}$ (10)

$\ddot{x} = -\frac{g}{2a}x$ (5) $\boxed{35}$

(iii) $x = 0$ is centre (10)

$x = A\cos\omega t + B\sin\omega t$

(5) $\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$

(5) $\ddot{x} = -\omega^2(A\cos\omega t + B\sin\omega t)$

$= -\omega^2 x$

$\therefore \omega = \sqrt{\frac{g}{2a}}$ (5)

$t = 0,\ x = 0\ \ \dot{x} = u$ (5)

$\Rightarrow$ (5) $A = 0$ (5)

$u = B\omega \Rightarrow B = \frac{u}{\omega} = u\sqrt{\frac{2a}{g}}$ (5) $\boxed{35}$

(iv)

(5) $\therefore x = \frac{u}{\omega}\sin\omega t \Rightarrow \dot{x} = u\cos\omega t$ (5)

$x = 3a$, (5) $\dot{x} = 0$, at $P \Rightarrow 0 = u\cos\omega t$ (5)

$\omega t = \frac{\pi}{2}$ (5)

$\therefore 3a = \frac{u}{\omega}\sin\frac{\pi}{2}$ (5)

$u = 3a\omega = 3\sqrt{\frac{ag}{2}}$ (5)

amplitude is $3a$ (5) $\boxed{40}$

$t_{BA} = \frac{\pi - \theta}{\omega}$ (5) $\cos\theta = \frac{2}{3}$

$t_{CB} = \frac{\pi}{2\omega}$ (5)

Total time $= \frac{\pi}{2\omega} + \frac{\pi - \theta}{\omega}$ (5)

$= \frac{1}{\omega}\left(\frac{\pi}{2} + \pi - \theta\right)$

$= \frac{1}{\omega}\left(\frac{3\pi}{2} - \theta\right)$

$= \sqrt{\frac{2a}{g}}\left[\frac{3\pi}{2} - \cos^{-1}\frac{2}{3}\right]$ (10) $\boxed{25}$

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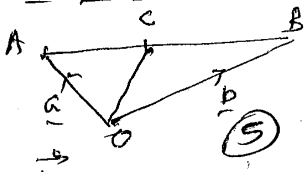
14 (a)



(5)

$\vec{OC} = \vec{OA} + \vec{AC}$

$\underline{c} = \underline{a} + \lambda(\vec{AB})$ (10)

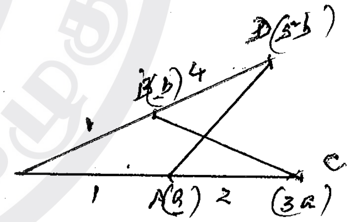
$= \underline{a} + \lambda(\underline{b} - \underline{a})$ (5)

$= (1-\lambda)\underline{a} + \lambda\underline{b}$

let $1 - \lambda = \alpha$

$\therefore \lambda = 1 - \alpha$

$\underline{c} = \alpha\underline{a} + (1-\alpha)\underline{b}$ (5) $\boxed{25}$



Let M is on AD and N is on BC

from the first part $\underline{m} = \lambda\underline{a} + (1-\lambda)5\underline{b}$ (10)

$\underline{n} = \mu 3\underline{a} + (1-\mu)\underline{b}$

where λ, μ are parameters (10)

to find the intersecting point

$\underline{m} = \underline{n}$ (5)

$\Rightarrow \lambda\underline{a} + (1-\lambda)5\underline{b} = \mu 3\underline{a} + (1-\mu)\underline{b}$ (5)

$(\lambda - 3\mu)\underline{a} + [5(1-\lambda) - (1-\mu)]\underline{b} = \underline{0}$

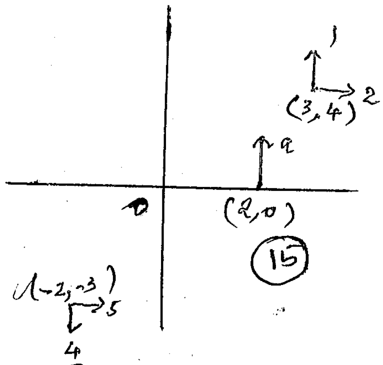
$\Rightarrow \lambda - 3\mu = 0$ (5) $+ 5\lambda - \mu = 4$ (5)

$\mu = \frac{2}{7},\ \lambda = \frac{6}{7}$ (5)

$\underline{m} = \frac{6}{7}\underline{a} + \frac{5}{7}\underline{b}$ (5) $\boxed{50}$

M divides AD in $1:6$

14 b)



(15)

$\rightarrow \quad X = 7$ (5)

$\uparrow \quad Y = a - 3$ (5)

$\circlearrowleft \quad a \cdot 2 + 1 \cdot 3 - 2 \cdot 4 + 5 \cdot 3 + 4 \cdot 2 = \pm 24$ (10)

$2a + 18 = \pm 24$

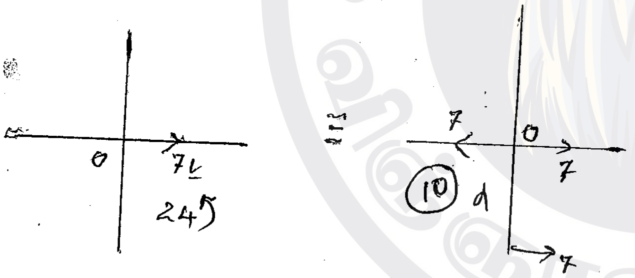
$2a = 6, -42$

$a = 3, -21$ (5) (5)

$a = 3 \Rightarrow \underline{F} = 7\underline{i}$ (5)

$a = -21 \Rightarrow \underline{F} = 7\underline{i} - 24\underline{j}$ (5)

$a = 3 \Rightarrow G = 24$ (5)



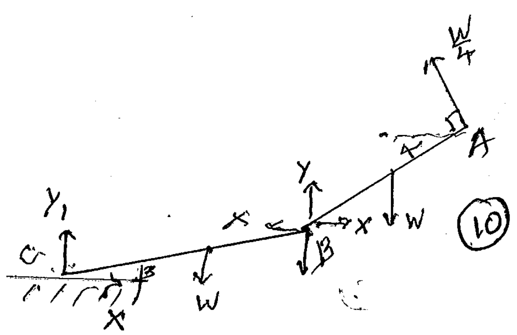
(10)

$d = \frac{G}{7} = \frac{24}{7}$

Line Eqⁿ of the line of action of the equivalent force is $y = -\frac{24}{7}$ (5)

$\boxed{75}$

15 (a)



(10)

for AB $\circlearrowleft \quad \frac{W}{4} \cdot 2a - W \cdot a\cos\alpha = 0$ (10)

$\cos\alpha = \frac{1}{2}$

$\alpha = \frac{\pi}{3}$ (5)

$\rightarrow \quad X = \frac{W}{4}\cos 30 = \frac{\sqrt{3}W}{8}$ (5)

$\uparrow \quad Y = W - T\sin 30$ (5)

$= W - \frac{W}{4} \cdot \frac{1}{2}$

$= \frac{7W}{8}$ (5)

for BC $\circlearrowleft$

$X \cdot 2a\sin\beta + Y \cdot 2a\cos\beta - W a\cos\beta = 0$ (10)

$\frac{\sqrt{3}}{8}W \cdot 2\sin\beta - \frac{7W}{8} \cdot 2\cos\beta - W\cos\beta = 0$ (5)

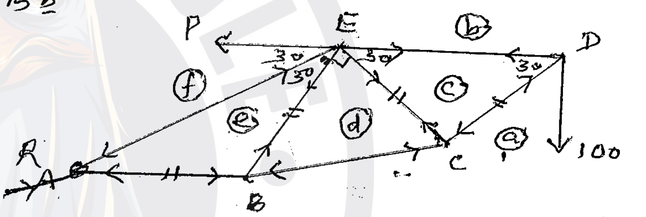
$\sqrt{3}\tan\beta = 11$

$\tan\beta = \frac{11}{\sqrt{3}}$

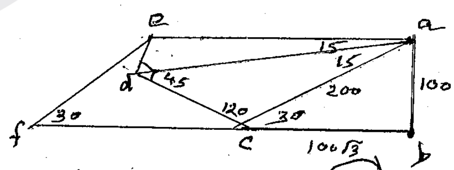
$\beta = \tan^{-1}\left(\frac{11}{\sqrt{3}}\right)$ (5)

$\boxed{60}$

15 b



Reaction on A is in the direction $\overrightarrow{AB}$ for the equilibrium of System, three forces 100, P, R. Their line of action of these forces meet at D. (10)



(30)

feae is a Rhombus
$\Rightarrow ef = 200$

$\frac{de}{\sin 15} = \frac{ad}{\sin 120} = \frac{200}{\sin 45}$ (10)

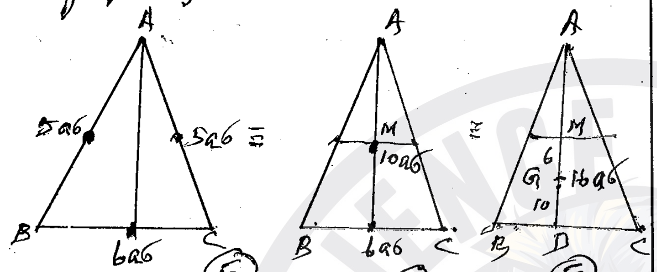
$ad = 100\sqrt{6}$, $de = 100(\sqrt{3}-1) = de$

$ae = 200 = cf$

Rod	Notation	Thrust	Tension	
AB	ae	200	-	5
BC	da	$100\sqrt{6}$	-	5
CD	ca	200	-	5
DE	bc	-	$100\sqrt{3}$	5
EC	cd	-	$100(\sqrt{3}-1)$	5
BE	$e\cancel{d}$	-	$100(\sqrt{3}-1)$	5
AE	fe	200	-	5

$P = 200+100\sqrt{3} = 100(2+\sqrt{3})$ 5 $\boxed{90}$

16) a) by symmetry c.m lies on median through A



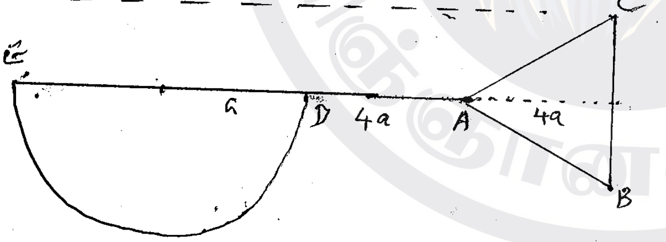
5 5 5

$\frac{MG}{GD} = \frac{6}{10} = \frac{3}{5}$ 5

$MG = \frac{3}{8}\cdot 2a = \frac{3}{4}a$ 5

$AG = 2a+\frac{3}{4}a = \frac{11a}{4}$ 5 $\boxed{30}$

b) Theory $\boxed{40}$



	Mass	C.m from BC	C.m from AD
Hemisphere	$2\pi a^2\rho$ 5	$9a$ 5	$\frac{a}{2}$ 5
AD	$4a\rho$ 5	$6a$ 5	0 5
ABC	$16a\rho$ 5	$\frac{5a}{4}$ 5	0 5
System	$(2\pi a^2+20a)\rho$ 5	$\bar{x}$	$\bar{y}$

$(2\pi a^2+20a)\rho\,\bar{x} = 2\pi a^2\rho\cdot 9a + 4a\rho\cdot 6a + 16a\rho\cdot\frac{5a}{4}$ 10

$\bar{x} = \frac{9\pi a^2+22a}{\pi a+10}$ 5

$(2\pi a^2+20a)\rho\,\bar{y} = 2\pi a^2\rho\cdot\frac{a}{2}$ 10

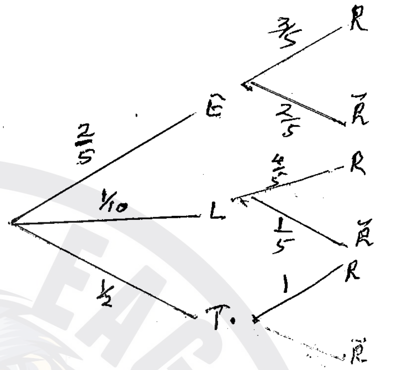
$\bar{y} = \frac{\pi a^2}{2(\pi a+10)}$ 5 $\boxed{80}$

17) a) $P(A/B) = \frac{P(A\cap B)}{P(B)}$ 10 $P(B)>0$

b) A_i are partitions in sample space S. & B is a event.

$P(B) = \sum_i P(A_i)\cdot P(B/A_i)$ 20

c) $R = \{$Student receiving the message$\}$



40

$P(R) = \frac{2}{5}\cdot\frac{3}{5}+\frac{1}{10}\cdot\frac{4}{5}+\frac{1}{2}\cdot 1$ 20

$= \frac{41}{50}$ 10

$P(E/R) = \frac{P(E)\cdot P(R/E)}{P(R)}$ 30

$= \frac{\frac{2}{5}\cdot\frac{3}{5}}{\frac{41}{50}}$ 10

$= \frac{12}{41}$ 10

$\boxed{150}$