श्रीमद्भास्कराचार्यविरचिता लीलावती

व्याख्यानम् 10: प्रकीर्णकम्

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Question related to previous lecture

In the *Bījagaṇita*, Bhaskara has used the *khahara* in the following verse

अस्मिन् विकारः खहरे न राशावपि प्रविष्टेष्वपि निःसृतेषु।
बहुष्वपि स्याल्लयसृष्टिकालेऽनन्तेऽच्युते भूतगणेषु यद्वत् ॥६॥

And *khahara* used in the following verse 45 of *Līlāvatī* is different meaning or same?

योगे खं क्षेपसमं वर्गादौ खं खभाजितो राशिः ।
खहरः स्यात् खगुणः खं खगुणश्चिन्त्यश्च शेषविधौ ॥

Answer: Yes and No!

Yes: When you derive the term खभाजित as खेन भाजितः।

No: When you derive the term the other way round as explained in the previous lecture.

प्रकीर्णकम्
Miscellaneous operations

Verses 48 – 89

The following table presents the various topics discussed by Bhāskara to solve commonly encountered basic arithmetic problems, in the section titled *Prakīrṇaka*.

क्रमः	विषयः	अनुवादः
1	व्यस्तविधिः	Inverting mathematical processes
2	इष्टकर्म	Operations with assumed numbers
3	सङ्क्रमणं विषमकर्म च	Operations with sums and differences
4	वर्गकर्म	Operations with squares of numbers
5	गुणकर्म	Operations with squares
6	त्रैराशिकादिः	Rules of three etc.
7	भाण्डप्रतिभाण्डकम्	Barter of commodities

व्यस्तविधिः — Inverting a mathematical process

When the result of performing a series of arithmetical operations on an unknown quantity *rāśi* is known, the series of operations that have to be performed to find that unknown *rāśi* is given in the verse below –

> छेदं गुणं गुणं छेदं वर्गं मूलं पदं कृतिम् ।
> ऋणं स्वं स्वमृणं कुर्यात् दृश्ये राशिप्रसिद्धये ॥४८॥ ।अनुष्टुभ्।

> To obtain the *rāśi*, one should make the divisor the multiplier, the multiplier the divisor, the square as square root, the square root as square, subtraction as addition, and addition as subtraction in the *dṛśya*.

Symbolically the content of the above verse can be denoted as:

Process		Its inverse
$\div$	$\longrightarrow$	$\times$
$(\;)^2$	$\longrightarrow$	$\sqrt{(\;)}$
$-$	$\longrightarrow$	$+$

Process		Its inverse
$\times$	$\longrightarrow$	$\div$
$\sqrt{(\;)}$	$\longrightarrow$	$(\;)^2$
$+$	$\longrightarrow$	$-$

स्वांशाधिकोने व्यस्तविधिः — A particular case of inverting a process

अथ स्वांशाधिकोने तु लवाढ्योनो हरो हरः ।
अंशस्त्वविकृतस्तत्र विलोमे शेषमुक्तवत् ॥ ४९॥ ॥ अनुष्टुभ् ॥

Now, in the case where the *rāśi* has been increased or decreased by a part of itself, the denominator [of the fraction by which the *dṛśya* has to be modified] is the denominator [of the fraction of the *rāśi*] increased or decreased by the numerator [of the fraction of the *rāśi*]. The numerator [of the fraction by which the *dṛśya* has to be modified] remains unchanged [from the numerator of the fraction of the *rāśi*] there. Rest [of the operations] are [carried out] as stated earlier in the reverse process.

$$x \pm \frac{a}{b} \times x = k,$$

In those instances where we add a part of it to itself, General solution:

$$\frac{x \times (b \pm a)}{b} = k$$

$$x = \frac{b}{b \pm a} \times k.$$

उदाहरणप्रश्नः

यस्त्रिघ्नस्त्रिभिरन्वितः स्वचरणैः भक्तस्ततः सप्तभिः
स्वत्र्यंशेन विवर्जितस्स्वगुणितो हीनो द्विपञ्चाशता ।
तन्मूलेऽष्टयुते हृते च दशभिः जातं द्वयं ब्रूहि तं
राशिं वेत्सि हि चञ्चलाक्षि विमलां बाले विलोमक्रियाम् ॥ ५० ॥ । शार्दूलविक्रीडितम् ।

$$\frac{\sqrt{\left[\frac{3x + \frac{3}{4} \times 3x}{7} - \left(\frac{1}{3} \times \frac{3x + \frac{3}{4} \times 3x}{7}\right)\right]^2 - 52} + 8}{10} = 2.$$

न्यासः — गु ३। युतः $\boxed{\begin{matrix} ३ \\ ४ \end{matrix}}$ । भागः ७। स्वत्र्यंशरहितः $\boxed{\begin{matrix} १ \\ ३ \end{matrix}}$ । वर्गः। हीनः ५२। तन्मूलयुतः ८। भागः १०। जातं दृश्यम् २। लब्धो राशिः २८।

	उत्क्रमविधिः		व्यस्तविधिः			
No.	Operation	Notation/quantity	Operation	Notation/quantity		
1.	Division	$\div 10$	Multiplication	2×10	$=$	20
2.	Addition	$+8$	Subtraction	$20 - 8$	$=$	12
3.	Square root	$\surd$	Squaring	12^2	$=$	144
4.	Subtraction	-52	Addition	$144 + 52$	$=$	196
5.	Squaring	^{2}	Square root	$\sqrt{196}$	$=$	14
6.	*Bhāgāpavāha*	$\times(1 - \frac{1}{3})$	*Bhāgānubandha*	$14 + \frac{1}{3-1} \times 14$	$=$	21
7.	Division	$\div 7$	Multiplication	21×7	$=$	147
8.	*Bhāgānubandha*	$\times(1 + \frac{3}{4})$	*Bhāgāpavāha*	$147 - \frac{3}{4+3} \times 147$	$=$	84
9.	Multiplication	$\times 3$	Division	$84 \div 3$	$=$	$\mathbf{28}$

इष्टराशिम् आदाय क्रियमाणं कर्म इष्टकर्म।

उद्देशकालापवदिष्टराशिः क्षुण्णो हृतोंशै रहितो युतो वा ।
इष्टाहतं दृष्टमनेन भक्तं राशिर्भवेत् प्रोक्तमितीष्टकर्म ॥५१॥ । इन्द्रवज्रा ।

An assumed number (*iṣṭa*) is multiplied, divided, subtracted or added by [its] parts as per the statement of the questioner. The given quantity (*dṛṣṭa*) multiplied by the assumed number, and divided by this [result] would be the *rāśi*. Thus stated, the method of assumption.

If $x \times () = k$ and $i \times () = k'$, then $\dfrac{x}{i} = \dfrac{k}{k'}$

$$x = \frac{i \times k}{k'}$$

$$\text{i.e.,} \qquad \text{राशिः} = \frac{\text{इष्टम्} \times \text{दृष्टम्}}{\text{फलम्}}$$

- अत्र आलापः = कथनम्। अपि च "वत्" इति न मतुबर्थे। नापि उपमार्थे। अपि तु प्रक्रियासाम्यद्योतनार्थम्। अनुरूपार्थे।

पञ्चघ्नः स्वत्रिभागोनो दशभक्तः समन्वितः ।
राशित्र्यंशार्धपादैः स्यात् को राशिः द्व्यूनसप्ततिः ॥ ५२॥ । अनुष्टुभ् ।

What number would be equal to seventy less two (sixty-eight), being multiplied by five, deducted by one-third of itself, divided by ten, and added with one-third, half, and one-fourth of the *rāśi*?

$$\frac{5x - \left(\frac{1}{3} \times 5x\right)}{10} + \frac{1}{3}x + \frac{1}{2}x + \frac{1}{4}x = 68.$$

Considering 12 as *iṣṭa*,

$$\frac{5 \times 12 - \left(\frac{1}{3} \times 5 \times 12\right)}{10} + \frac{1}{3} \times 12 + \frac{1}{2} \times 12 + \frac{1}{4} \times 12 = 17.$$

Now applying the principle of proportionality, we have

$$x = \frac{68 \times 12}{17} = \mathbf{48}.$$

न्यासः — गुणः ५। ऊनः $\begin{array}{|c|}\text{१}\\ \text{३}\end{array}$। भागः १०। इष्टराशि $\begin{array}{|c|c|c|}\text{१} & \text{१} & \text{१}\\ \text{३} & \text{२} & \text{४}\end{array}$ युतः। दृश्यः ६८।

$$\frac{5x - \left(\frac{1}{3}\times 5x\right)}{10} + \frac{1}{3}x + \frac{1}{2}x + \frac{1}{4}x = 68.$$

अत्र किल इष्टराशिः ३। अयं पञ्चघ्नः १५। अयं स्वत्रिभागो(५)नः १०। अयं दशभक्तः १।

अत्र कल्पितराशित्रये राशित्र्यंशार्धपादाः $\begin{array}{|c|c|c|}\text{३} & \text{३} & \text{३}\\ \text{३} & \text{२} & \text{४}\end{array}$। एतैः समन्वितो जातः $\begin{array}{|c|}\text{१७}\\ \text{४}\end{array}$।

अनेन दृष्टम् ६८ इष्टाहतं भक्तं जातो राशिः ४८।

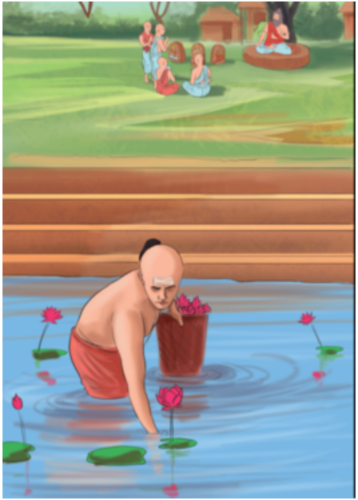
$$\frac{5\times 3 - \left(\frac{1}{3}\times 5\times 3\right)}{10} + \frac{1}{3}\times 3 + \frac{1}{2}\times 3 + \frac{1}{4}\times 3 = \frac{17}{4}.$$

$$x = \frac{68\times 3}{\frac{17}{4}} = \mathbf{48}.$$

अमलकमलराशेः त्र्यंशपञ्चांशषष्ठैः
त्रिनयनहरिसूर्या येन तुर्येण चार्या ।
गुरुपदमथ षड्भिः पूजितं शेषपद्मैः
सकलकमलसङ्ख्यां क्षिप्रमाख्याहि तस्य ॥५३॥ ।मालिनी।

Quickly tell the total number of lotuses with that person by whom Trinayana (Śiva), Hari (Viṣṇu), Sūrya, and Āryā (Durgā) were worshipped with one-third, one-fifth, and one-sixth, and one-fourth of a certain number of pristine lotuses, and then the feet of the *guru* were worshipped by the remaining six lotuses.

$$x - \frac{1}{3}x - \frac{1}{5}x - \frac{1}{6}x - \frac{1}{4}x = 6$$



$$x - \frac{1}{3}x - \frac{1}{5}x - \frac{1}{6}x - \frac{1}{4}x = 6$$

न्यासः –

१	१	१	१
३	४	५	६

इष्टम् $= 30$ प्रकल्प्य,

$$30 - \left(\frac{1}{3} \times 30\right) - \left(\frac{1}{5} \times 30\right) - \left(\frac{1}{6} \times 30\right) - \left(\frac{1}{4} \times 30\right) = 1.$$

$$30 - 10 - 12 - 5 - \frac{30}{4} = \frac{6}{4}.$$

इष्टाहतम् इति कर्मणा –

$$\text{कमलराशिः} = \frac{6 \times 30}{\frac{6}{4}} = \mathbf{120}.$$

स्वार्धं प्रादात् प्रयागे नवलवयुगलं योऽवशेषाच्च काश्यां
शेषाङ्घ्रिं शुल्कहेतोः पथि दशमलवान् षट् च शेषाद्गयायाम् ।
शिष्टा निष्कत्रिषष्टिः निजगृहमनया तीर्थपान्थः प्रयातः
तस्य द्रव्यप्रमाणं वद यदि भवता शेषजातिः श्रुता स्यात् ॥५४॥ ।स्रग्धरा ।

A pilgrim who offered half of his money in Prayāga, two-ninths from the remaining in Kāśī, one-fourth of the rest for toll on the way, and six times one-tenths from the remaining in Gayā, went to his home with the remaining sixty-three *niṣkas*. Tell his amount of money, if the *śeṣajati* method is known by you.

$$\left[\left(\left(x-\frac{1}{2}x\right)-\frac{2}{9}\times\left(x-\frac{1}{2}x\right)\right)-\frac{1}{4}\times\left(\left(x-\frac{1}{2}x\right)-\frac{2}{9}\times\left(x-\frac{1}{2}x\right)\right)\right]-$$
$$\frac{6}{10}\times\left[\left(\left(x-\frac{1}{2}x\right)-\frac{2}{9}\times\left(x-\frac{1}{2}x\right)\right)-\frac{1}{4}\times\left(\left(x-\frac{1}{2}x\right)-\frac{2}{9}\times\left(x-\frac{1}{2}x\right)\right)\right]=63.$$

शेषजात्युदाहरणम्

न्यासः — $\begin{array}{|c|c|c|c|} १ & २ & १ & ६ \\ २ & ९ & ४ & १० \end{array}$ । दृश्यम् ६३।

Considering *iṣṭa*=18, we have

क्रमः	प्रक्रिया	उत्तरम्
1	$18 \times (1 - \frac{1}{2})$	$= 9$
2	$9 \times (1 - \frac{2}{9})$	$= 7$
3	$7 \times (1 - \frac{1}{4})$	$= \frac{21}{4}$
4	$\frac{21}{4} \times (1 - \frac{6}{10})$	$= \frac{84}{40} = \frac{21}{10}$

$$\text{द्रव्यराशिः} = \frac{63 \times 18}{\frac{21}{10}} = \mathbf{540} \text{ निष्काः}.$$

पञ्चांशोऽलिकुलात् कदम्बमगमत् त्र्यंशः शिलीन्ध्रं तयोः
विश्लेषस्त्रिगुणो मृगाक्षि कुटजं दोलायमानोऽपरः ।
कान्ते केतकमालतीपरिमलप्राप्तैककालप्रिया-
दूताहूत इतस्ततो भ्रमति खे भृङ्गोऽलिसङ्ख्यां वद ॥५५॥ ।शार्दूलविक्रीडितम् ।

O doe-eyed [girl]! One-fifth from a group of bees went to the *Kadamba* flower, one-third to the *Śilīndhra*, [and] three times their difference to the *Kuṭaja* flower. Another wavering bee is wandering here and there in the sky, enticed by the fragrances of the *Ketaka* and *Mālatī* flowers, like the one who has been approached by the messengers of two beloveds at the same time. Dear! Tell the number of bees.

$$x - \frac{1}{5}x - \frac{1}{3}x - 3 \times \left(\frac{1}{3} - \frac{1}{5}\right)x = 1.$$



विश्लेषजात्युदाहरणम्

$$x - \frac{1}{5}x - \frac{1}{3}x - 3 \times \left(\frac{1}{3} - \frac{1}{5}\right) x = 1.$$

iṣṭa=30

क्रमः	अंशः	उत्तरम्
1	$\frac{1}{5} \times 30 = 6$	24
2	$\frac{1}{3} \times 30 = 10$	14
3	$(10-6) \times 3 = 12$	2

$$\text{अलिकुलम्} = \frac{1 \times 30}{2} = \mathbf{15}.$$

iṣṭa=15

क्रमः	अंशः	उत्तरम्
1	$\frac{1}{5} \times 15 = 3$	12
2	$\frac{1}{3} \times 15 = 5$	7
3	$(5-3) \times 3 = 6$	1

$$\text{अलिकुलम्} = \frac{1 \times 15}{1} = \mathbf{15}.$$

योगोऽन्तरेणोनयुतोऽर्धितः तौ राशी स्मृतौ सङ्क्रमणाख्यमेतत् ॥५६॥ । इन्द्रवज्रा ।

The [given] sum [separately] subtracted by and added with the [given] difference, and halved. Those are known to be the two numbers [which make the given sum and difference]. This [method] is named *saṅkramaṇa*.

- योगः अन्तरेण ऊनयुतः (ऊनः युतः च)। अर्धितः।
- तौ राशी स्मृतौ। एतत् [कर्म] सङ्क्रमणाख्यम् [अस्ति]।

$$\text{यो}\,१ = \text{का}\,१ + \text{नी}\,१, \qquad \text{अं}\,१ = \text{का}\,१ - \text{नी}\,१$$

$$\text{Sum,} \quad s = a + b, \qquad \text{diff.,} \quad d = a - b.$$

$$\text{का}\,१ = \frac{\text{यो}\,१ + \text{अं}\,१}{२}, \qquad \text{नी}\,१ = \frac{\text{यो}\,१ - \text{अं}\,१}{२}.$$

$$a = \frac{s+d}{2}, \qquad b = \frac{s-d}{2}.$$

ययोर्योगः शतं सैकं वियोगः पञ्चविंशतिः ।
तौ राशी वद मे वत्स वेत्सि सङ्क्रमणं यदि ॥५७॥ । अनुष्टुभ् ।

O son! Tell me those two numbers whose sum is a hundred with one and difference is twenty-five, if you know *saṅkramaṇa*.

योगः $s = 101$, वियोगः $d = 25$.

सङ्क्रमणविधिना

$$a = \frac{101 + 25}{2} = 63$$

$$b = \frac{101 - 25}{2} = 38.$$

The process to determine the values of two unknown quantities when their difference, and the difference of their squares are known.

वर्गान्तरं राशिवियोगभक्तं योगस्ततः प्रोक्तवदेव राशी ॥५८॥ । इन्द्रवज्रा ।

The [given] difference of squares divided by the [given] difference of the numbers is the sum. Therefrom, the two numbers [should be found] as stated [before].

$$\frac{\text{वर्गान्तरम्}}{\text{राशिवियोगः}} = \text{राशियोगः}$$

$$\frac{a^2 - b^2}{a - b} = a + b.$$

- वर्गान्तरं राशिवियोगभक्तं योगः।
- ततः प्रोक्तवदेव राशी [भवतः]।

राश्योर्ययोः वियोगोऽष्टौ तत्कृत्योश्च चतुःशती ।
विवरं ब्रूहि तौ राशी शीघ्रं गणितकोविद ॥५९॥ *[अनुष्टुभ्]*

O expert in mathematics! Tell quickly those two numbers whose difference is eight, [given] the difference of their squares is four hundred.

वर्गान्तरम्, $d_s = a^2 - b^2 = 400$ राशिवियोगः, $d = a - b = 8$.

$$a + b = \frac{400}{8} = 50.$$

अधुना, सङ्क्रमणकर्मणा

$$a = \frac{50+8}{2} = 29$$
$$b = \frac{50-8}{2} = 21.$$

- The determination of two unknown *rāśis*, the sum as well as difference of whose squares when reduced by one, also result in squares.
- ययोः द्वयोः अज्ञातराश्योः वर्गयोः योगः अन्तरं च रूपविहीनं वर्गौ एव भवतः तयोः साधनं वर्गकर्म भवति।
- In other words, Bhāskara seeks to determine two quantities a and b such that

$$a^2 + b^2 - 1 = S_1 \tag{1}$$

$$b^2 - a^2 - 1 = S_2, \tag{2}$$

where S_1 and S_2 are squares, and $b > a$.

Thanks!

धन्यवादाः!