श्रीमद्भास्कराचार्यविरचिता लीलावती

व्याख्यानम् 12: प्रकीर्णकम् (गुणकर्म त्रैराशिकादिश्च)

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AICTE Sponsored QIP program

(Understanding Classical Scientific Texts of India in an Immersive Sanskrit Environment)

IIT Indore

September 14–October 2, 2020

प्रकीर्णकान्तर्गताः विषयाः — Topics in Miscellaneous operations

The following table presents the various topics discussed by Bhāskara to solve commonly encountered basic arithmetic problems, in the section titled *Prakīrṇaka*.

क्रमः	विषयः	अनुवादः
1	व्यस्तविधिः	Inverting mathematical processes
2	इष्टकर्म	Operations with assumed numbers
3	सङ्क्रमणं विषमकर्म च	Operations with sums and differences
4	वर्गकर्म	Operations with squares of numbers
5	गुणकर्म	Dealing with univariate quadratic equations
6	त्रैराशिकादिः	Rules of three etc.
7	भाण्डप्रतिभाण्डकम्	Barter of commodities

पार्थः कर्णवधाय मार्गणगणं[a] क्रुद्धो रणे सन्दधे
तस्यार्धेन निवार्य तच्छरगणं मूलैश्चतुर्भिर्हयान् ।
शल्यं षड्भिः अथेषुभिस्त्रिभिरपि छत्रं ध्वजं कार्मुकं
चिच्छेदास्य शिरः शरेण कति ते यानर्जुनः सन्दधे ॥७०॥

।शार्दूलविक्रीडितम् ।

[a]मार्गणः = मार्गयति लक्ष्यमिति ; शर इत्यर्थः।

An enraged Pārtha (Arjuna), in order to slay Karṇa in battle, shot a succession of arrows [on his bow]. Having countered his (Karṇa's) series of arrows with half of that (Arjuna's own), [he stopped] the horses by four times the square root [of his total arrows], [immobilised] Śalya by six arrows, then [destroyed] the umbrella, flag and bow by three arrows, [and] cut his head by one arrow. How many were those [arrows], which Arjuna nocked [on his bow]?

$$x^2 = \frac{x^2}{2} + 4x + 6 + 3 + 1.$$

Problem involving गुणकर्म – Quadratic equations

The following verse is also a beautiful piece of poetry in itself:

अलिकुलदलमूलं मालतीं यातमष्टौ
निखिलनवमभागाश्चालिनी भृङ्गमेकम् ।
निशि परिमललुब्धं पद्ममध्ये निरुद्धं
प्रतिरणति रणन्तं ब्रूहि कान्तेऽलिसङ्ख्याम् ॥७१॥ ।मालिनी ।

The square root of half of a swarm of bees and [also] eight-ninths of the total went to the *Mālatī* flower. A female bee is buzzing in response to the buzzing of a bee which attracted by the fragrance, got trapped in a lotus at night. O charming lady! Tell the number of bees.

$$2x^2 = x + \frac{2x^2}{9} \times 8 + 2 \qquad (1)$$

Problem involving गुणकर्म – Quadratic equations

Let the total number of bees be $2x^2$. Then according to the problem, x bees as well eight-ninths of $2x^2$ went to the *Mālatī* flower. We also have two more bees—the female bee, and the bee trapped in the lotus. Therefore,

$$2x^2 = x + \frac{8}{9} \times 2x^2 + 2$$

$$\text{or,} \qquad x^2 - \frac{9}{2}x = 9.$$

Now, applying the rule we have,

$$x^2 = \left[\sqrt{9 + \left(\frac{1}{2} \times \frac{9}{2}\right)^2} + \frac{1}{2} \times \frac{9}{2}\right]^2 = 36.$$

Therefore, the total number of bees is $2x^2 = 72$.

यो राशिरष्टादशभिः स्वमूलैः राशित्रिभागेन समन्वितश्च ।
जातं शतद्वादशकं तमाशु जानीहि पाट्यां पटुतास्ति ते चेत् ॥७२॥ ।इन्द्रवज्रा ।

Know quickly that *rāśi* which added by eighteen times its square root, and one-third of the *rāśi*, became twelve hundred, if you have expertise in arithmetic.

The given problem can be represented as:

$$x^2 + \frac{1}{3}x^2 + 18x = 1200$$

$$\text{or,} \qquad x^2 + \frac{27}{2}x = 900.$$

$$x^2 = \left[\sqrt{900 + \left(\frac{1}{2} \times \frac{27}{2}\right)^2} - \frac{1}{2} \times \frac{27}{2}\right]^2 = 576.$$

त्रैराशिकम् – Rule of three

- The term *trairāśika* refers to the mathematical process of determining an unknown quantity from three known quantities, based on the rule of simple proportions.

$$\frac{icch\bar{a}phala}{icch\bar{a}} = \frac{pram\bar{a}\d{n}aphala}{pram\bar{a}\d{n}a}.$$

- Of these, the *pramāṇa*, *pramāṇaphala*, and the *icchā* are known quantities, while the *icchāphala* has to be determined.
- In Indian philosophy, particularly in logic, the term *pramāṇa* has been extensively analysed. It used to refer to the means of cognition.
- Here, it seems to have been used in the sense of a known quantity which helps in determining the unknown quantity. One of the well known definitions of *pramāṇa*:

  अनधिगत-अबाधितार्थबोधकं प्रमाणम्।

A note on *pramāṇa*

- A simple definition: प्रमाकरणं प्रमाणम्। (प्र + मा–भावे करणे वा ल्युट्)
- When *pramāṇa* is used to refer to means of knowledge the suffix is only in करणार्थ।
- How many are accepted and by whom?
  - प्रत्यक्षमेकमेव प्रमाणमिति चार्वाकाः सङ्गिरते ।
  - प्रत्यक्षमनुमानं चेति द्वेप्रमाणे इति वैशेषिका बौद्धा आर्हताश्च।
  - प्रत्यक्षमनुभानमाप्तवचनमिति त्रीणि प्रमाणानीति सांख्या वेदान्तिनश्च (विशिष्टाद्वैतिनः)।
  - प्रत्यक्षानुमानोपमानशब्दाः इति नैयायिकाः।
  - प्रत्यक्षानुमानोपमानशब्दा अर्थापत्तिश्चेति प्राभाकराः।
  - प्रत्यक्षानुमानोपमानशब्दा अर्थापत्तिरनुपलब्धिश्चेति षड्प्रमाणानीत्यपरे भाट्टावेदान्तिनः।
  - सम्भवैतिह्ये अप्यतिरिक्ते प्रमाणे इति पौराणिकाः ।
- In *Mānameyodaya*, Nārāyaṇa Bhaṭṭatiri brilliantly summarises:

  चार्वाकास्तावदेकं द्वितयमपि पुनर्बौद्धवैशेषिकौ द्वौ
  भासर्वज्ञश्च सांख्यस्त्रितयमुदयनाद्याश्चतुष्कं वदन्ति ।
  प्राहुः प्राभाकराः पञ्चकमपि च वयं तेऽपि वेदान्तविज्ञाः
  षट्कं पौराणिकास्त्वष्टकमभिदधिरे संभवैतिह्ययोगात् ॥

प्रमाणमिच्छा च समानजाती आद्यन्तयोस्स्तः फलमन्यजातिः ।
मध्ये तदिच्छाहतमाद्यहृत् स्यात् इच्छाफलं व्यस्तविधिर्विलोमे ॥७३॥ ।उपजातिः ।

Pramāṇa and *icchā* [which] are of the same kind (units), are [placed] in the first and last positions. The result (*pramāṇaphala*) [which is] of another kind is [placed] in the middle. That (*pramāṇaphala*), multiplied by the *icchā*, and divided by the *ādya* (*pramāṇa*) would be *icchāphala*. In [case of] inverse proportion, reverse process [is employed].

The relation given in the verse to determine the *icchāphala* in terms of *ādi* and *anta* can be interpreted as follows:

$$\textit{icchāphala} = \frac{\textit{pramāṇaphala} \times \textit{icchā}}{\textit{pramāṇa}}$$

कुङ्कुमस्य सदलं पलद्वयं निष्कसप्तमलवैस्त्रिभिर्यदि ।
प्राप्यते सपदि मे वणिग्वर ब्रूहि निष्कनवकेन तत्कियत् ॥७४॥ ।रथोद्धता ।

If two and half *palas* of saffron are obtained by three-seventh *niṣkas*, then O great merchant! Tell me quickly how much of that can be obtained with nine *niṣkas*?

In this problem, we have:

$$\textit{icchāphala} = \frac{\frac{5}{2} \times 9}{\frac{3}{7}} = \frac{105}{2} = 52.5 \; \textit{palas}.$$

As 1 *pala* = 4 *karṣas*, we thus obtain 52 *palas* and 2 *karṣas* of saffron for 9 *niṣkas*. The word कुङ्कुम (कुक्यते आदीयते) is used in multiple senses. The following verse would also be instructive.

काश्मीरदेशजे क्षेत्रे कुङ्कुमं यद्भवेद्धि तत् । सूक्ष्मकेशरमारक्तं पद्मगन्धि तदुत्तमम् ॥

Problem related to त्रैराशिकम् – Rule of three

Having provided an example wherein we find the quantity that can be procured with given amount of money, Bhāskara constructs another example, vice versa:

प्रकृष्टकर्पूरपलत्रिषष्ट्या चेल्लभ्यते निष्कचतुष्कयुक्तम् ।
शतं तदा द्वादशभिः सपादैः पलैः किमाचक्ष्व सखे विचिन्त्य ॥७५॥ ।उपजातिः ।

If a hundred and four *niśkas* are earned by [selling] sixty-three *palas* of superior camphor, then O friend! Having thought, tell what [will be earned] by [selling] twelve and a quarter *palas* [of camphor].

In this problem, we have:

$$\textit{icchāphala} = \frac{104 \times \frac{49}{4}}{63} = \frac{182}{9} \; \textit{niṣkas}.$$

Converting the currency, the above amount corresponds to 20 *niṣkas*, 3 *drammas*, 8 *paṇas*, 3 *kākiṇīs*, and $11\frac{1}{9}$ *varāṭakas*.

The following problem is different from the earlier ones in the sense that it involves conversion from one unit to other.

द्रम्मद्वयेन साष्टांशा शालितण्डुलखारिका ।
लभ्या चेत् पणसप्तत्या तत्किं सपदि कथ्यताम् ॥७६॥ ।अनुष्टुभ् ।

If one and one-eighth of a *khārikā* of rice can be purchased by two *drammas*, then quickly tell how much of that [can be purchased] by seventy *paṇas*.

In this problem, we have:

$$\textit{icchāphala} = \frac{\frac{9}{8} \times 70}{32} = \frac{315}{128}\ \textit{khārikās}.$$

Converting the currency, the obtained weight amounts to 2 *khārikās*, 7 *droṇas*, 1 *āḍhaka*, and 2 *prasthas*.

Note on शालिः in राजनिर्घण्टुः

शालयो मधुराः शीता लघुपाका बलावहाः । पित्तघ्नाश्चानिलकफाः स्निग्धा बद्धाल्पवर्चसः ॥

इच्छावृद्धौ फले ह्रासः ह्रासे वृद्धिः फलस्य तु ।
व्यस्तं त्रैराशिकं तत्र ज्ञेयं गणितकोविदैः ॥७७॥ ।अनुष्टुभ् ।

Wherein reduction happens in [*icchā*] *phala* when *icchā* increases and increment happens [in *phala*] when *icchā* decreases, there inverse proportion should be considered by experts of mathematics.

$$icch\bar{a}phala \propto \frac{1}{icch\bar{a}} \qquad pram\bar{a}\d{n}aphala \propto \frac{1}{pram\bar{a}\d{n}a}$$

And so,

$$icch\bar{a}phala \times icch\bar{a} = pram\bar{a}\d{n}aphala \times pram\bar{a}\d{n}a$$

$$\text{or,} \qquad icch\bar{a}phala = \frac{pram\bar{a}\d{n}aphala \times pram\bar{a}\d{n}a}{icch\bar{a}}$$

Thanks!

धन्यवादाः!