

Phynance

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*Dedicated to the memory of my father
Jemal Kakushadze, Ph.D. (1940-2005)*

Abstract

These are the lecture notes for an advanced Ph.D. level course I taught in Spring'02 at the C.N. Yang Institute for Theoretical Physics at Stony Brook. The course primarily focused on an introduction to stochastic calculus and derivative pricing with various stochastic computations recast in the language of path integral, which is used in theoretical physics, hence “Phynance”. I also included several “quiz” problems (with solutions) comprised of (pre-)interview questions quantitative finance job candidates were sometimes asked back in those days. The course to a certain extent follows an excellent book “Financial Calculus: An Introduction to Derivative Pricing” by M. Baxter and A. Rennie.

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