

**Name:** Hari Thapliyal

**Assignment:** Hypothesis Testing

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**Question 1:**

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not.

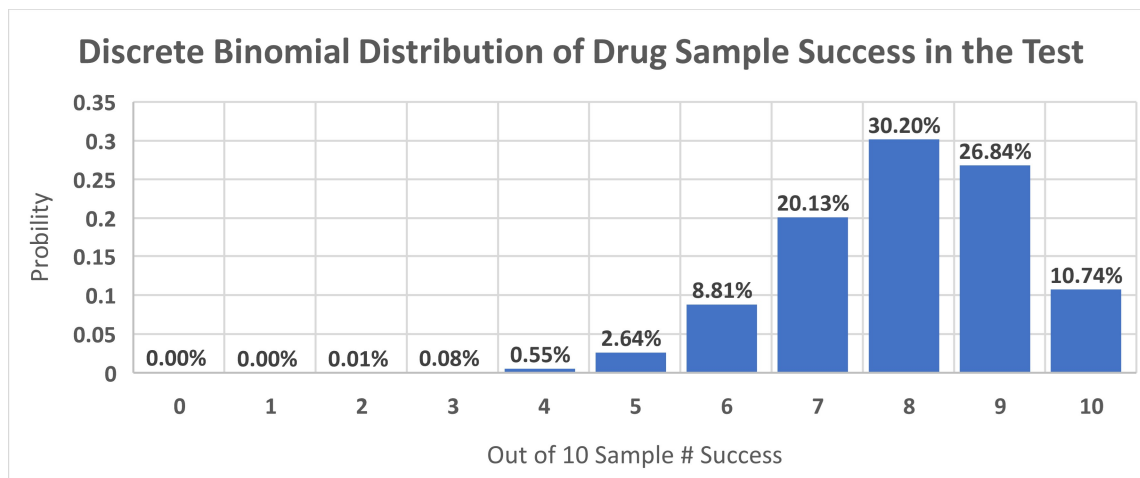
Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows.

b.) Calculate the required probability.

**Answer**

- a. This is Discrete Binomial Distribution. Discrete because number of drugs going for trial are either 1 or 2 or 3 etc. It can never be in decimals like 1.2, 1.5 etc. Binomial distribution because outcome of each test is either fail or pass.



**Conditions of the binomial distribution**

1. Number of observations are fixed. In our case this is 10
2. Each observation is independent. In our case success of drug 1 has no relationship with drug 2 etc.
3. Each observation represents one of the two outcomes fail or pass, success or failure, head or tail, has my chosen number or not etc. In our case a particular drug can pass the test or fail.
4. Probability of the success of each outcome is same. In our case it is .8 for each drug test.

**b.) Calculate the required probability.**

Random Variable  $X$  = A sample fail the test

$p$  = Probability of fail

We know from the question that Probability of success is 4 times more than fail i.e.  $p + 4p = 1 \Rightarrow p = .2$  or 20%

Probability of at most 3 sample fail or maximum three sample fail.

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$P(X=0) = {}^{nC_r} \cdot p^r \cdot (1-p)^{(n-r)} = {}^{10C_0} \cdot .2^0 \cdot .8^{(10-0)} = .1074$$

$$P(X=1) = {}^{nC_r} \cdot p^r \cdot (1-p)^{(n-r)} = {}^{10C_1} \cdot .2^1 \cdot .8^{(10-1)} = .2684$$

$$P(X=2) = {}^{nC_r} \cdot p^r \cdot (1-p)^{(n-r)} = {}^{10C_2} \cdot .2^2 \cdot .8^{(10-2)} = .3020$$

$$P(X=3) = {}^{nC_r} \cdot p^r \cdot (1-p)^{(n-r)} = {}^{10C_3} \cdot .2^3 \cdot .8^{(10-3)} = .2013$$

$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = 0.1074 + 0.2684 + 0.3020 + 0.2013 = .8791$$

**88% probability that at most 3 drugs can fail the test in the sample of 10 drugs**

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### Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

b.) Find the required range.

### Answer

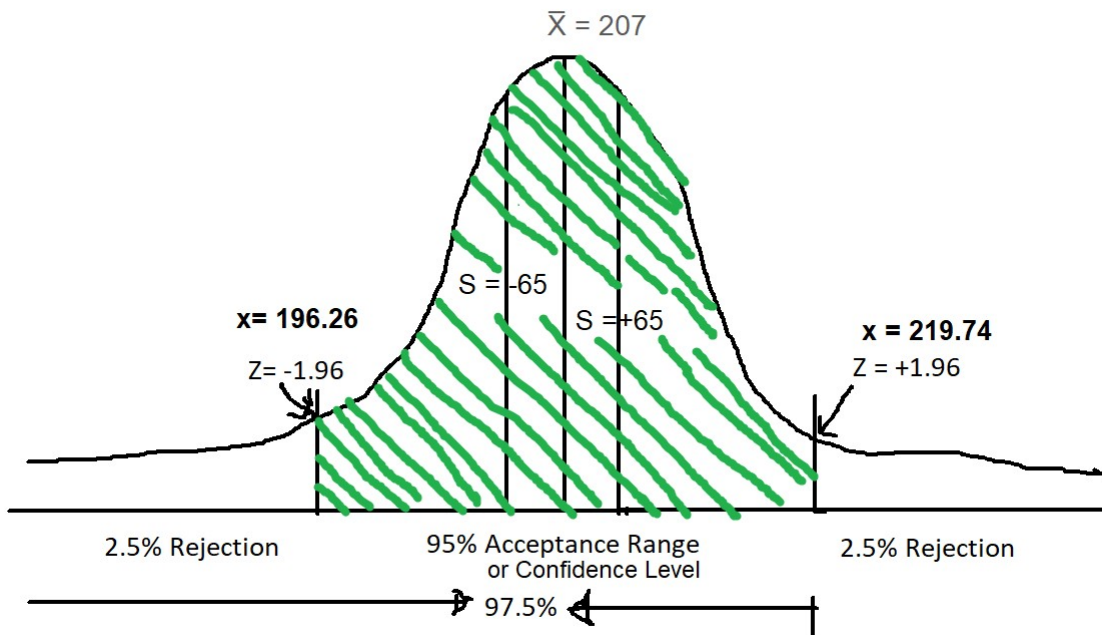
**a.) Discuss the main methodology using which you will approach this problem.**

We can use binomial discrete probability distribution curve to analyse this situation and get to know the range required for 95% confidence level. If drug can create an effect between 196 and 219 seconds then we can say with 95% confidence that drug is effective.

$$H_0 (\bar{X} = 207)$$

$$H_1 (\bar{X} \neq 207)$$

It will be 2 tail normal distribution.



**b.) Find the required range.**

We know  $\bar{X} - (Z^*(S/\sqrt{n})) < \mu < \bar{X} + (Z^*(S/\sqrt{n}))$

For 95% confidence level Z score is 1.96

$n=100$ ,  $\bar{X} = 207$ ,  $S = 65$

Putting all these values in above formula we get following range

**$194.26 < \mu < 219.74$**

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**Question 3:**

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

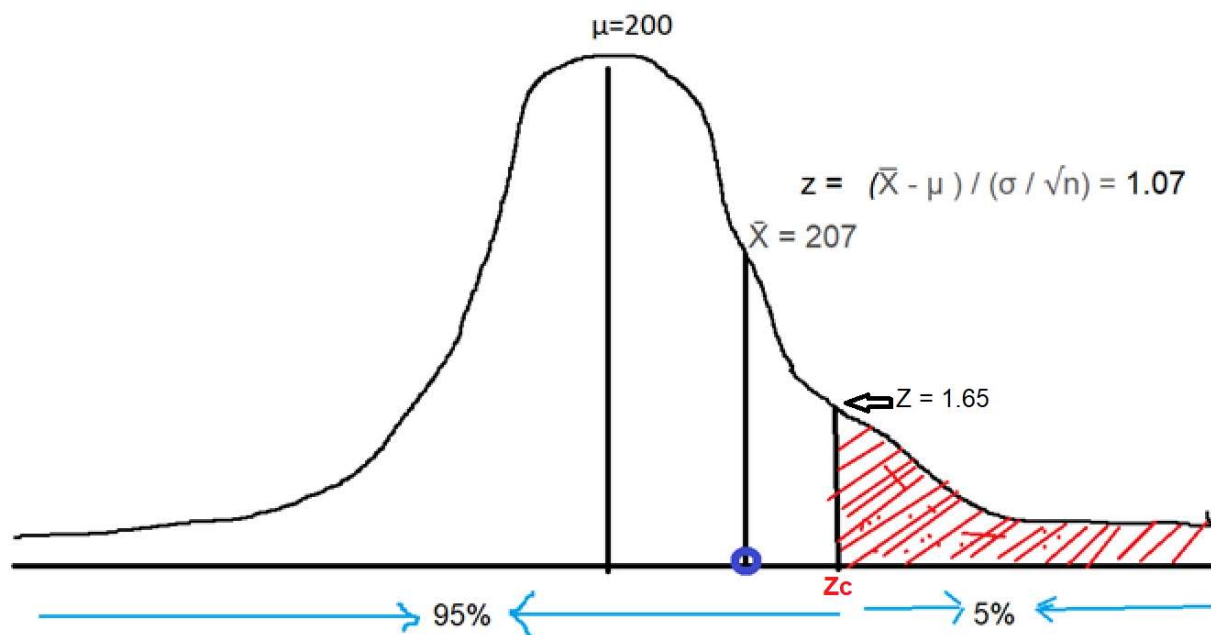
b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by  $\alpha$  and  $\beta$  respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of  $\alpha$  and  $\beta$  come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure (with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of  $\alpha$  and  $\beta$  are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having  $\alpha$  and  $\beta$  as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both  $\alpha$  and  $\beta$  values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your choice (Assume that only the values of  $\alpha$  and  $\beta$  as mentioned above are provided to you and no other information is available).

Answer

a)

- $H_0 (\mu \leq 200)$
- $H_1 (\mu > 200)$
- Based on  $H_1$  we can say it is one tailed, right sided probability distribution curve
- Calculate Z score for 207. It is 1.07
- Get z score for 95%. It is 1.65. It is  $Z_c$  (Z critical)
- Z score of sample average is within the acceptance range ( $Z_{\bar{x}} (1.07) < Z_c (1.65)$ ). Red marked is critical range.
- Hence null hypothesis cannot be rejected. It means the claim by the company that the newer batch produces a satisfactory result and passes the quality assurance test is true



b)

In this situation Type I error ( $\alpha$ ) means rejecting an effective medicine. It means even a good medicine does not reach to the patient.

Type II error ( $\beta$ ) means allowing an ineffective or less effective medicine to be consumed by the patient. It means patient do not see any effect or see late effect after taking the medicine.

We know there is tradeoff between Type I and Type II errors. If you try to reduce Type I error, Type II will increase and vice versa.

**For current sampling procedure:  $\alpha = 0.05$ ,  $\beta = 0.45$**

- Compare to second sampling procedure Type I error is less in this method. It means less rejection of good medicine.
- This sampling method is less stringent compare to second method.
- This method reduces the wastes. But because of tradeoff between Type I and Type II error, this reduction of waste happens at the cost of high Type II errors.
- This method allows more customer complains compare to the medicine which goes to market using second sampling procedure.
- You can afford to do this if by taking less effective there is no side effect on the patient's body. Otherwise it can create other kind of legal, business, market reputation issues.

**For second sampling procedure:  $\alpha = 0.15$ ,  $\beta = 0.15$**

- In this situation Type I error is more than compare to previous situation. It means more wastage of money. You produce good drug but not able to sell those because quality process is rejecting those.
- This sampling method is more stringent compare to the first one.
- Type II error in this method is less compare to first sampling method. It means lessor complain from customer and higher customer satisfaction.

If cost of producing a medicine is less compare to the loss due to lessor effective medicine we can allow more waste of good medicine and go for second sampling procedure. But if cost of production is high and cost of complain is not that high because you are solo supplier of that medicine in the market or some other business reasons then you can go for first sampling procedure.

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#### **Question 4:**

*Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.*

*Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.*

**Answer**

- Create a list of people with whom you want to experiment this drug (it may include, doctors, patients, relative/friends of patients, marketing agencies who sell the drug)
- Create 2 group from the above list. Make sure that any group is not having more doctors or patients etc. Also make sure that people are randomly put in these two groups.
- Create 2 set of web pages (dynamic), 2 set of brochures using the selected two taglines.
- Decide call to action. It may be (buy, enquiry, etc)
- Connect to customers
- Collect the call to action data for these two taglines.
- Mark all the success data (bought or enquired) to 1 and other as 0 for the data related to both taglines
- Prepare a hypothesis. Let's say Tagline 1 is better than Tagline 2. So  $H_1(T_1 > T_2)$  and  $H_0(T_1 \leq T_2)$
- Perform A/B test. There are many online tools or excel plugin. In our case we can use XLSTAT -> Test a Hypothesis -> Parametric Test -> Test for 2 Proportions
- Enter the total sample size and success rate of both the taglines in the excel
- Check p-value
- Decide significance level
- If p-value is greater than significance level value then null hypothesis cannot be rejected It means Tagline two is more effective or equally effective than Tagline one.
- If p-value is lessor than significance level value then null hypothesis is rejected or alternative hypothesis is accepted it means Tagline one is more effective