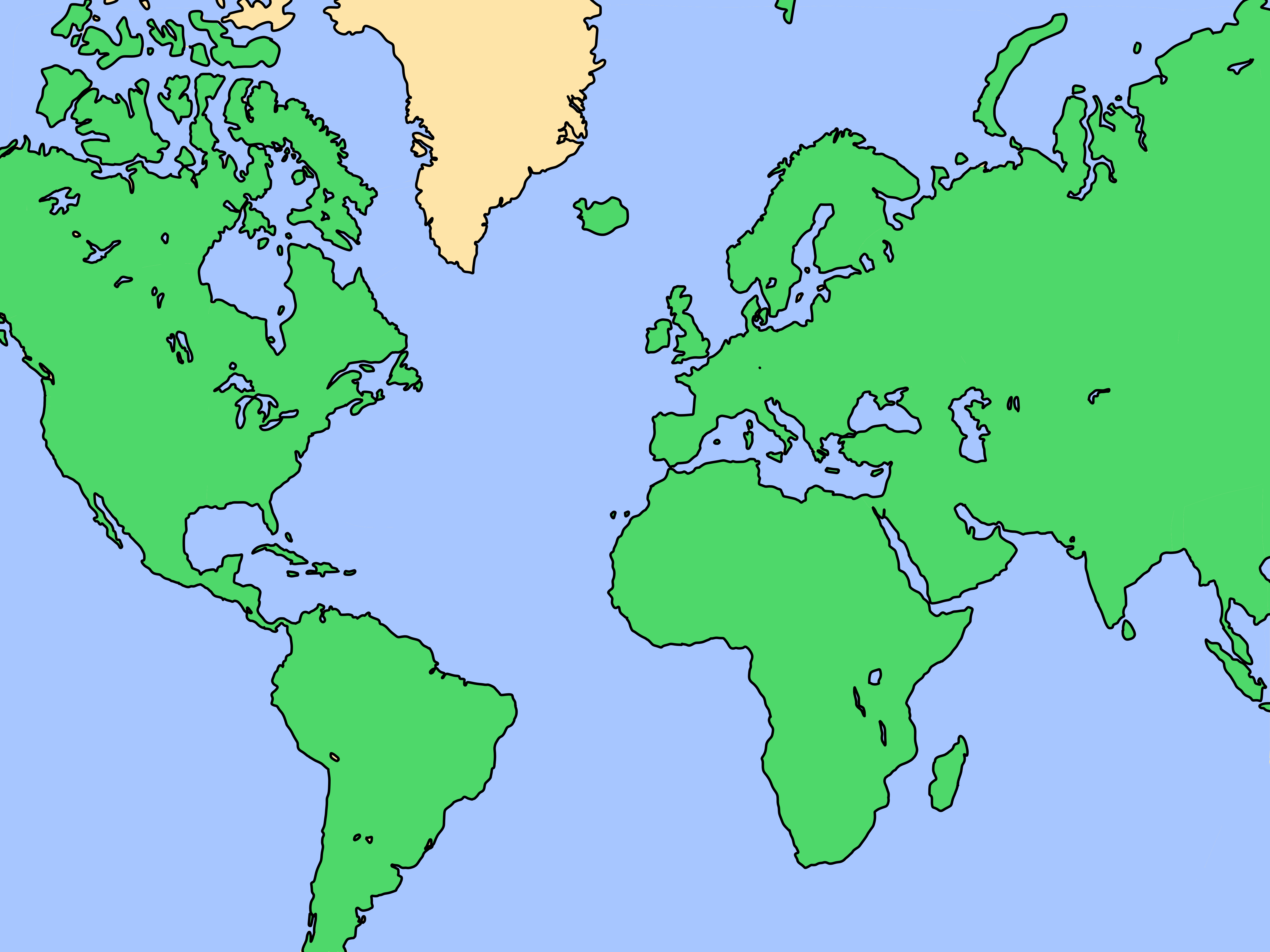
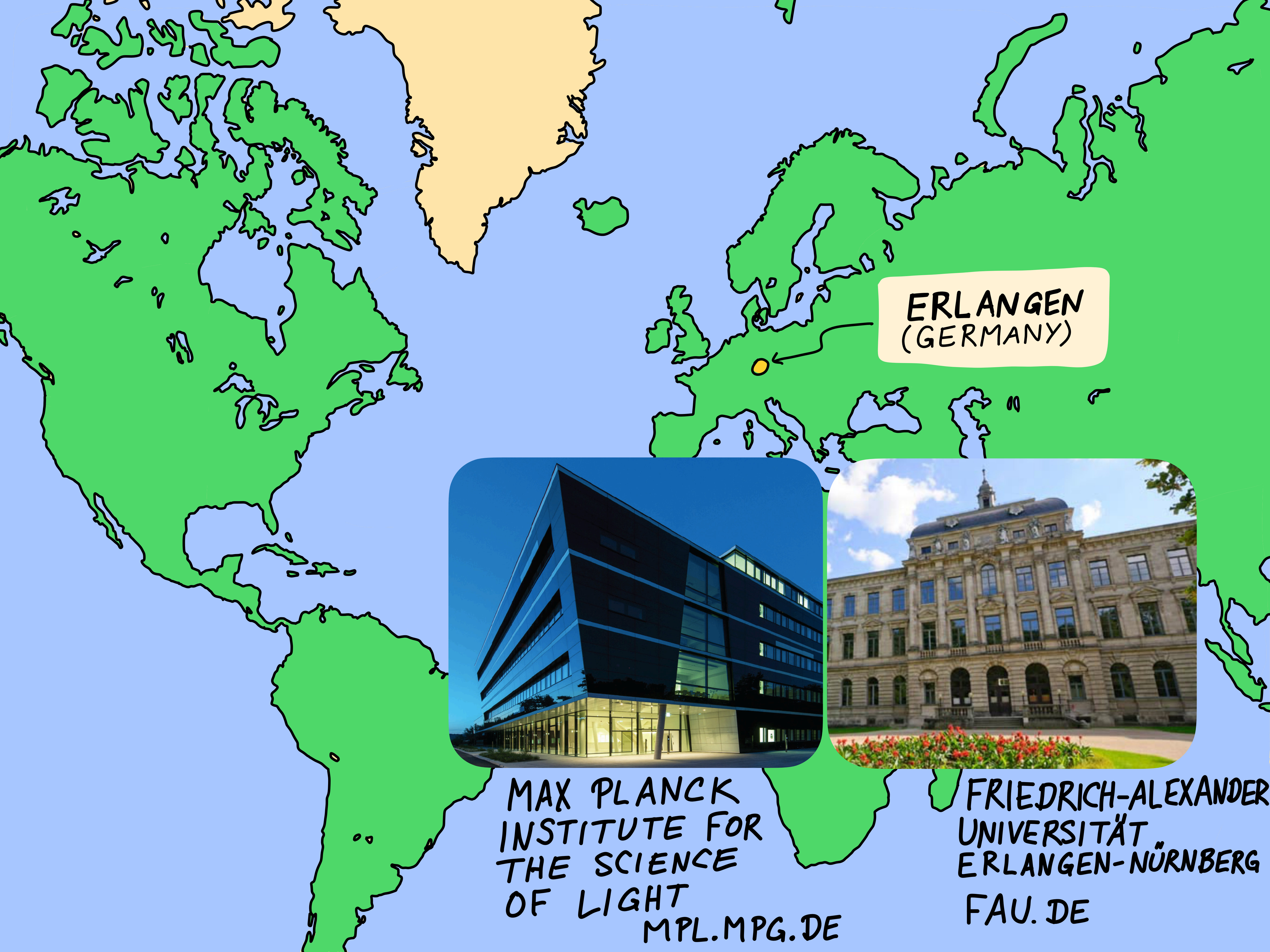
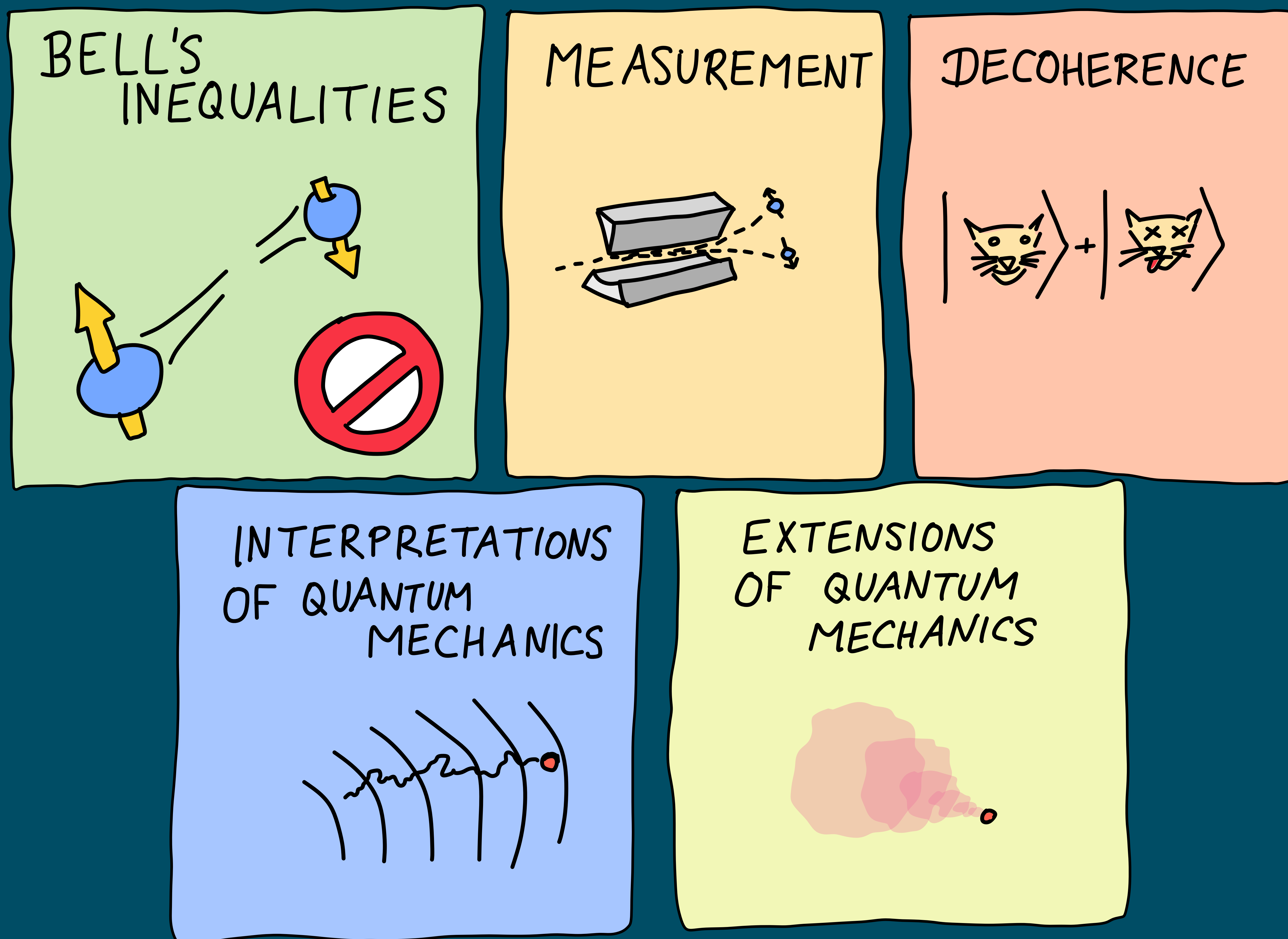
Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

Start at 6pm CET




ERLANGEN
(GERMANY)
MAX PLANCK
INSTITUTE FOR
THE SCIENCE
OF LIGHT
MPL.MPG.DE
FRIEDRICH-ALEXANDER
UNIVERSITÄT
ERLANGEN-NÜRNBERG
FAU.DE


BELL'S INEQUALITIES
MEASUREMENT
DECOHERENCE
INTERPRETATIONS OF QUANTUM MECHANICS
EXTENSIONS OF QUANTUM MECHANICS

pad.gwdg.de/s/Foundations_Of_Quantum_Mechanics

DISCUSSION GROUP

ALSO: DISCUSS MATTERS FOR FAU STUDENTS (EXAM ETC)

VIDEOS / SLIDES / LECTURE NOTES

LINKS TO ORIGINAL ARTICLES

Foundations of Quantum Mechanics

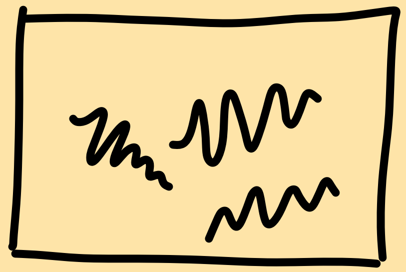
Winter term 2020/21 Florian Marquardt

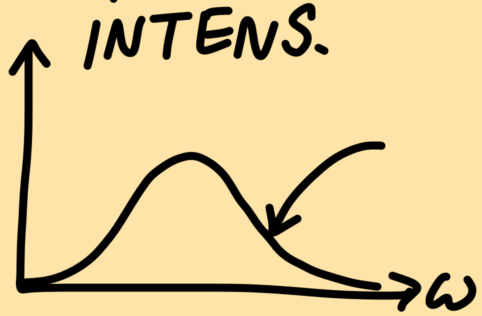
1 Introduction

$$i\hbar\partial_t\psi = \hat{H}\psi$$

SCHRÖDINGER EQUATION

1.1 Schrödinger equation





$$\boxed{E = \hbar\omega}$$

PLANCK 1900
EINSTEIN 1905

$$\frac{h}{2\pi} \sim 10^{-34}\,\mathrm{Js}$$

$$E = cp \qquad \omega = ck \quad \left(k = \frac{2\pi}{\lambda}\right)$$

$$\boxed{\vec{p} = \hbar\vec{k}}$$

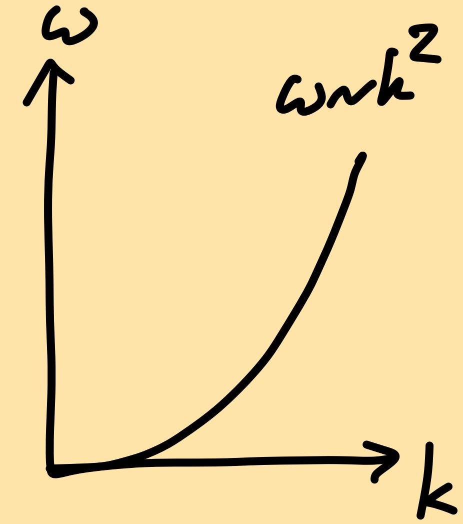
EINSTEIN

RELATIVITY

$$(E, c\vec{p})$$
$$(\omega, c\vec{k})$$

de Broglie (1924) $\longrightarrow$ MATTER WAVES

$$\boxed{E = \hbar\omega} \qquad \boxed{\vec{p} = \hbar\vec{k}}$$



BUT: $E = \dfrac{\vec{p}^{\,2}}{2m}$

$$\underline{\hbar\omega = \frac{\hbar^2\vec{k}^2}{2m}} \Rightarrow \underline{\omega = \omega(\vec{k})}$$

$\Rightarrow$ WAVE EQUATION? MAYBE LINEAR?

IN FREE SPACE: TRANSLATIONAL INVARIANCE

$$\Rightarrow \psi(\vec{x},t) \sim e^{i(\underline{\vec{k}}\vec{x} - \underline{\underline{\omega}}t)}$$

$$i\hbar\partial_t\psi = \overbrace{\hbar\omega}^{E}\psi$$

(1D) $$-i\hbar\partial_x\psi = \underbrace{\hbar k}_{p}\psi$$

$$\boxed{\underline{\underline{i\hbar\partial_t\psi}} = \frac{(-i\hbar\partial_x)^2}{2m}\psi + V(x)\psi}$$

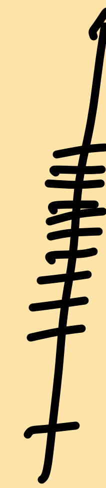
SCHRÖDINGER EQUATIONS 1926

$$\left(E\psi = \frac{p^2}{2m}\psi + V\psi \ \checkmark\right)$$

(& keeps $\int|\psi|^2dx$ conserved)

[MATRIX MECHANICS (HEISENBERG 1925)]

$E_n = \hbar \omega_n$



"STANDING WAVES" $\psi_n(x,t) = \phi_n(x)\, e^{-i\omega_n t}$

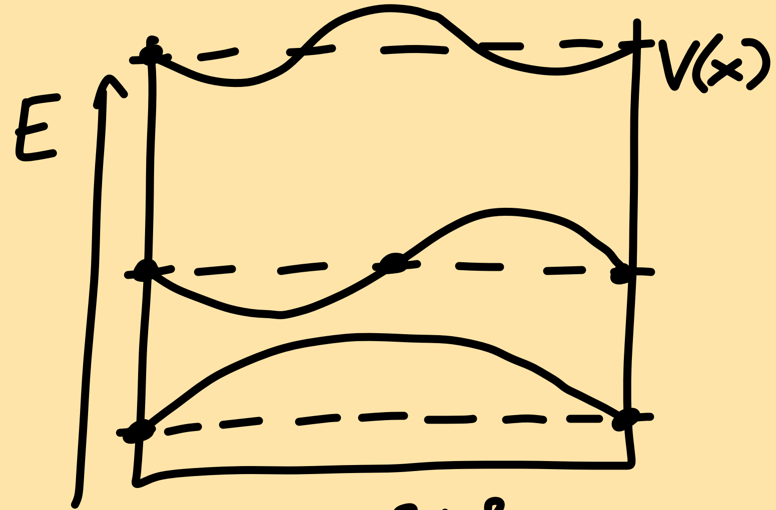
$$\hat{H}\phi_n = \underbrace{E_n}_{\text{energy eigenvalues}}\phi_n$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x) \qquad \hat{p} = -i\hbar\,\partial_x$$

MOMENTUM OPERATOR

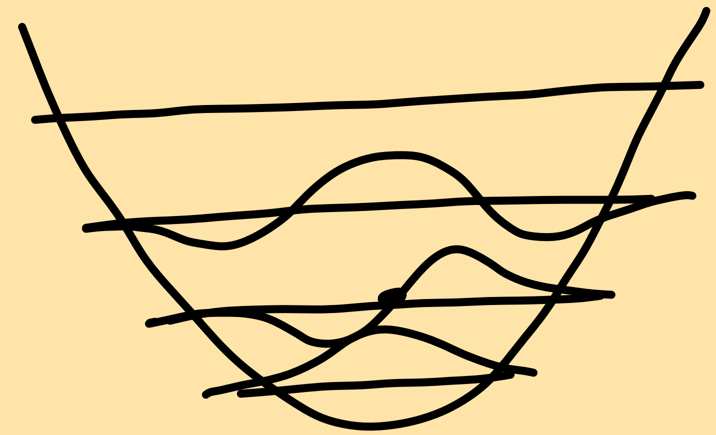
ϕ_n ORTHONORMAL BASIS

$$\underbrace{\langle \phi_n | \phi_m \rangle}_{\text{SCALAR PRODUCT}} = \int \phi_n^*(x)\,\phi_m(x)\,dx = \delta_{n,m} = \begin{cases} 1 & n=m \\ 0 & n\neq m \end{cases}$$



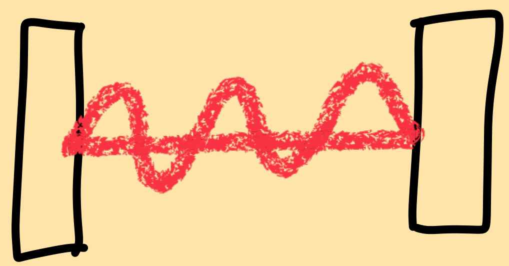
$$E_n = \frac{\hbar^2 k_n^2}{2m}$$

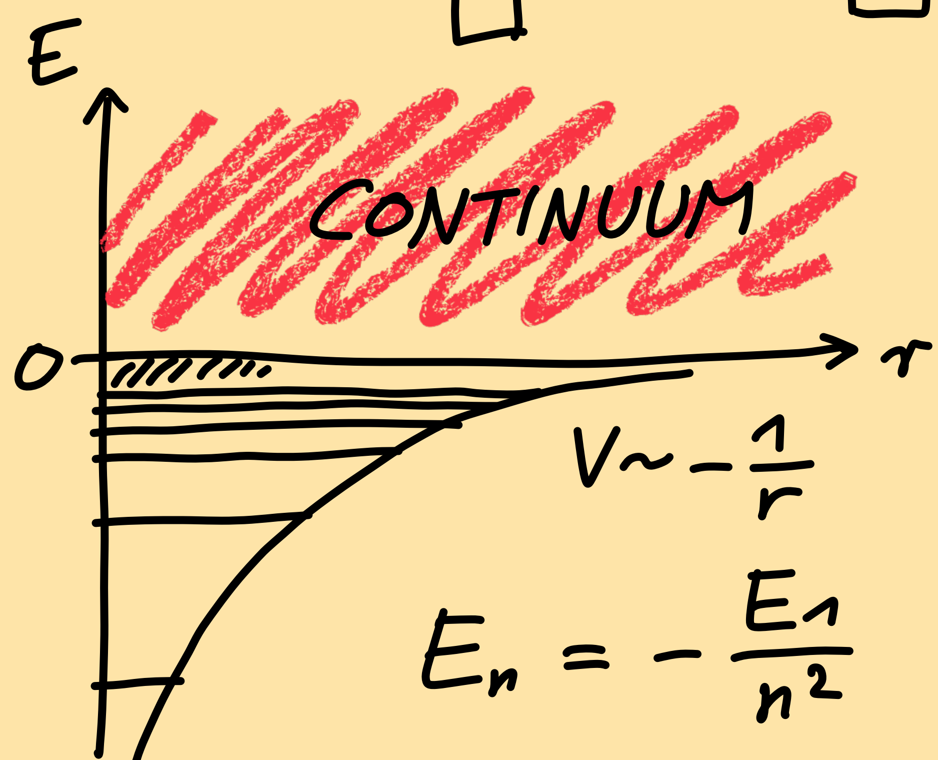
$$k_n = n\frac{\pi}{L}$$



$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

$$n = 0, 1, 2, \ldots$$





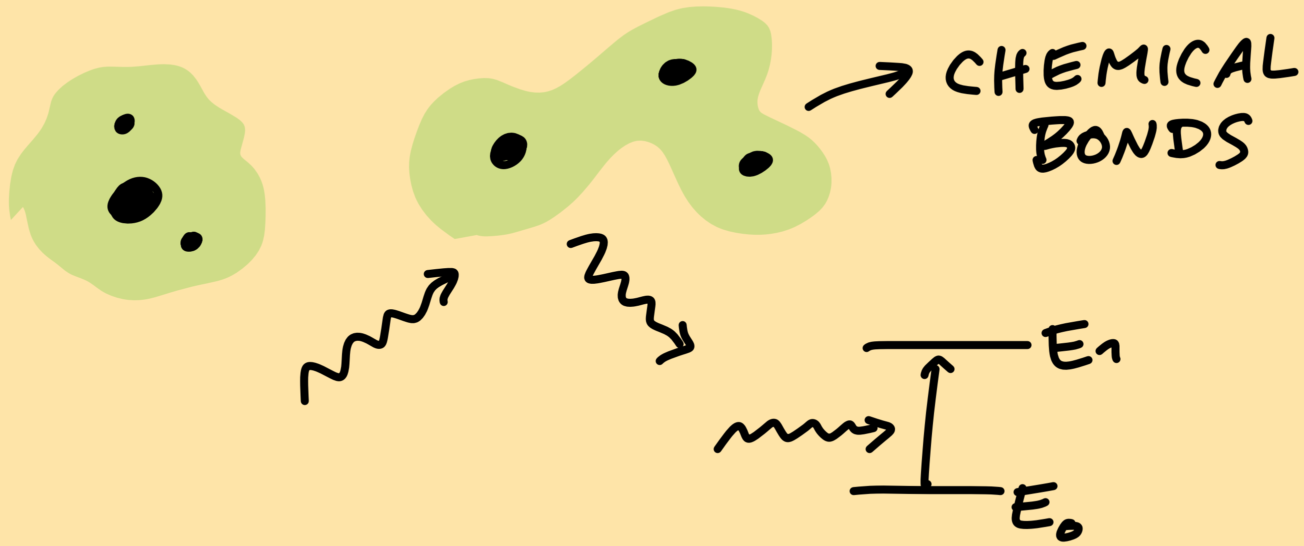
$$E_n = -\frac{E_1}{n^2} \qquad E_1 = 13.6\,eV$$

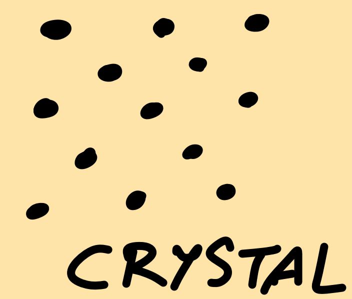
$$n = 1, 2, 3, \ldots$$

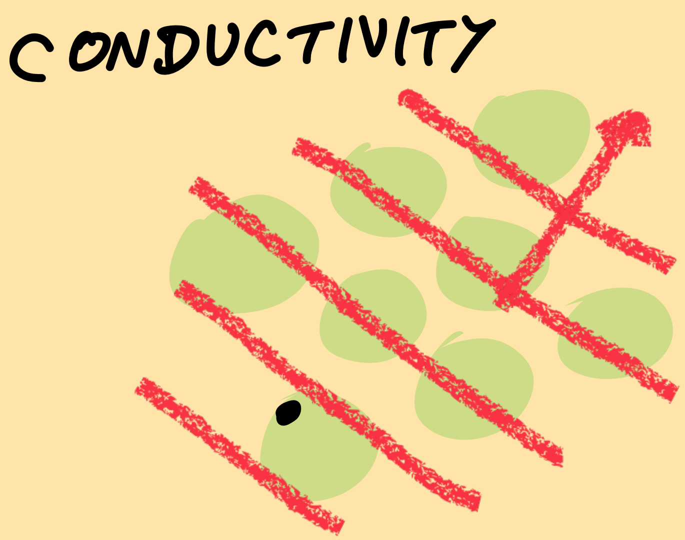
MANY PARTICLES?

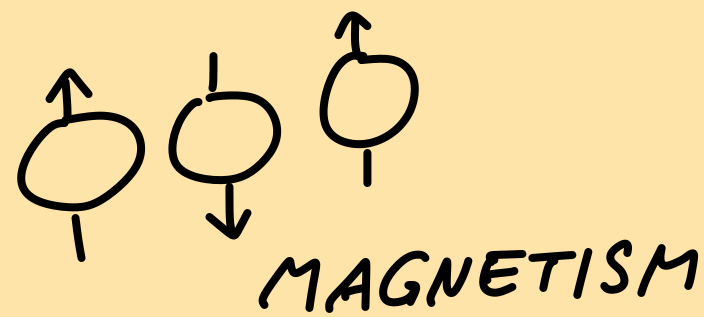
$$\hat{H} = \frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + V(x_1, x_2)$$

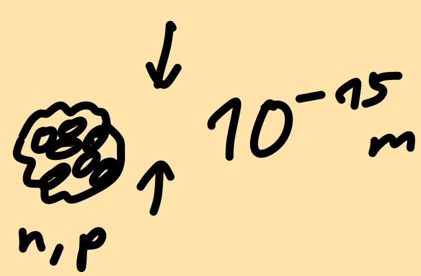
$$\psi(x_1, x_2)$$


CHEMICAL BONDS
E1
E0

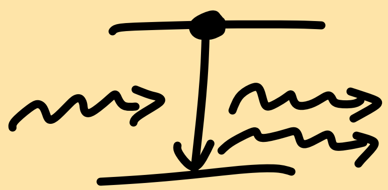

CRYSTAL


CONDUCTIVITY


MAGNETISM


10-15 m
n, p

APPLICATIONS

LASERS 

SUPERCONDUCTORS

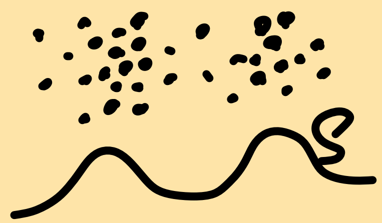
NMR/ESR

SEMICONDUCTORS

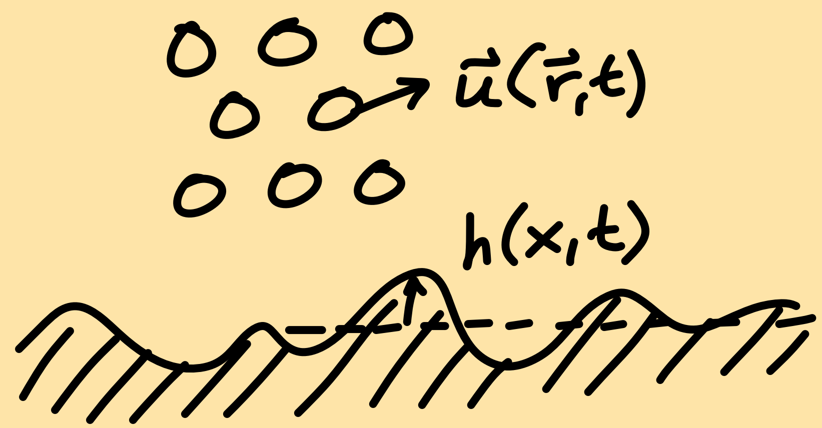
1.2 THE "MEANING" OF ψ?

COMPARE

SOUND WAVES

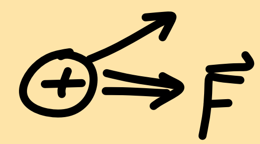


ELASTIC WAVES



EL. MAGN. WAVES

$\vec{E}$ $\vec{B}$



(a) Conserved density

$$\underbrace{\partial_t \rho}_{?} + \operatorname{div} \underbrace{\vec{j}}_{?} = 0$$

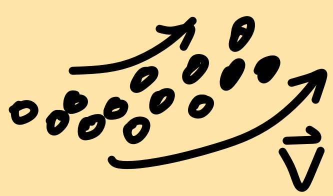
EQ. OF CONTINUITY

CLAIM: FOR SCHRÖD. EQ.

$\rho = |\psi|^2$ IS CONSERVED DENSITY

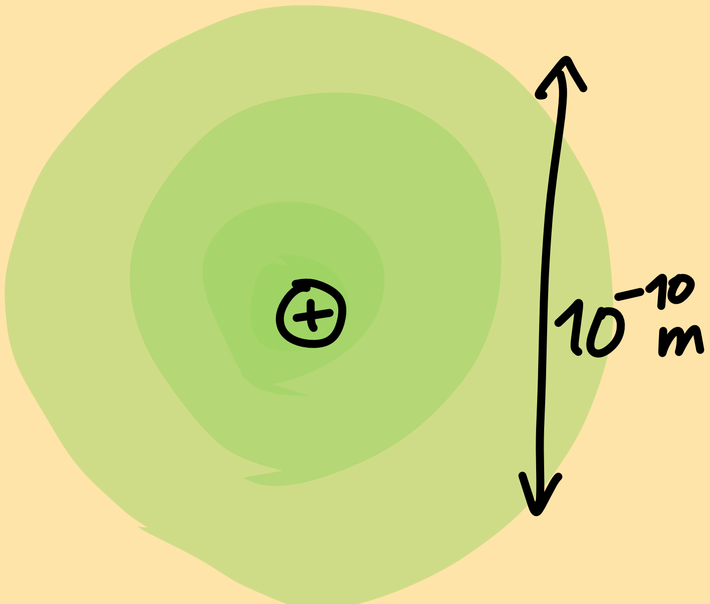
WITH $\vec{j}(\vec{x}) = \mathrm{Re}\left[\psi^*(\vec{x}) \underbrace{\frac{-i\hbar\vec{\nabla}}{m}}_{\text{VELOCITY OP.}} \psi(\vec{x})\right]$

ASIDE: HYDRODYNAMICS



$\vec{j} = \rho \cdot \vec{v}$

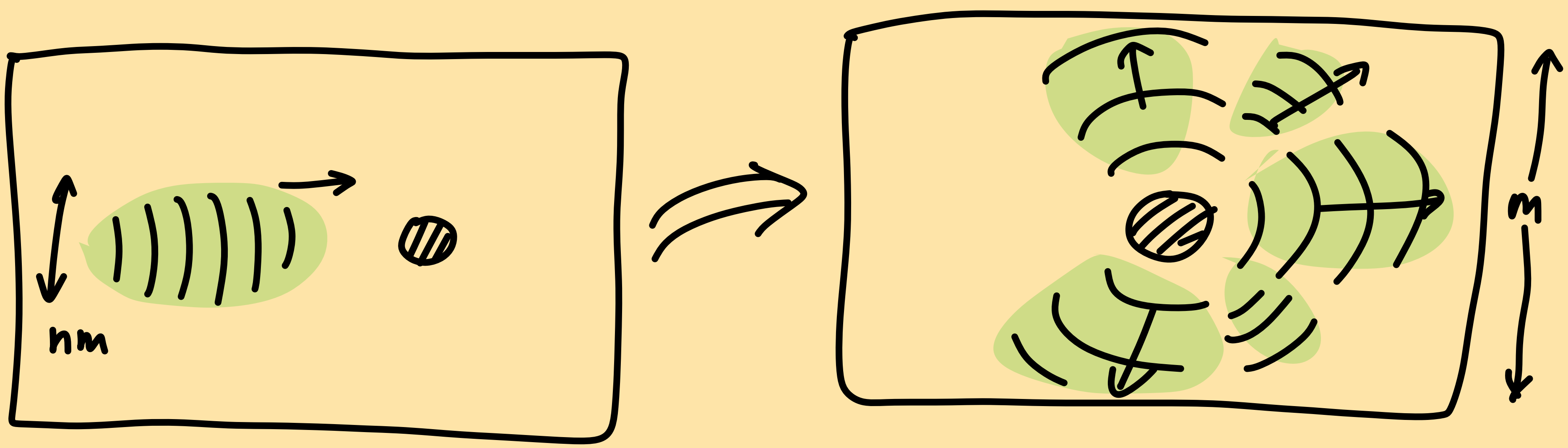
MEANING OF $|\psi|^2$?

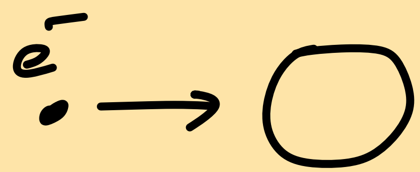


CHARGE DENSITY $q_e|\psi|^2 = \rho_{CHARGE}$

e^- SMEARED OUT?

SCHRÖDINGER





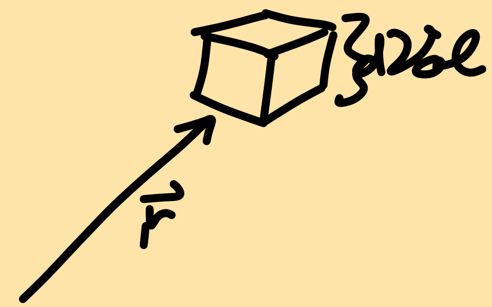
ATOM

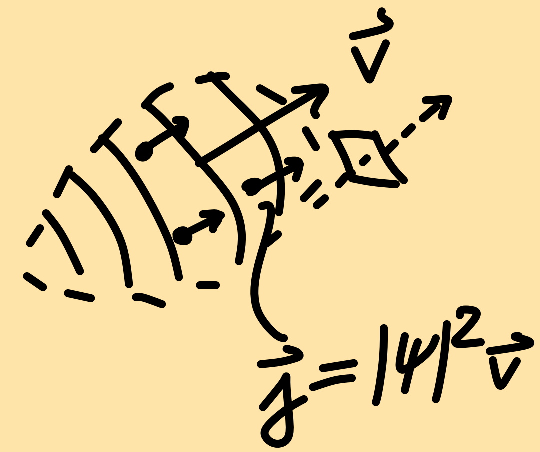
FRANCK-HERTZ
EXPERIMENT

MAX BORN (1926)

$$\boxed{|\psi|^2 = \text{PROBABILITY DENSITY}}$$

$$\underbrace{|\psi(\vec{r})|^2 d\text{Vol}}_{P(\text{"find } e^- \text{ inside Vol"})}$$





$$\Rightarrow \quad \partial_t \rho + \operatorname{div} \vec{j} = 0$$

BECOMES CONSERVATION
OF PROBABILITY

$$P = \int \rho \, d\text{Vol} \underset{\text{all times}}{\equiv} 1$$

$$\frac{d}{dt} P = \int \partial_t \rho \, d\text{Vol} = \int - \operatorname{div} \vec{j} \, d\text{Vol} \underset{\text{(GAUSS)}}{=} 0 \, .$$

$$|\psi(t)\rangle = \hat{U}(t)\,|\psi(0)\rangle$$

$$\hat{U}(t)^{+} = \hat{U}^{-1}(t)$$

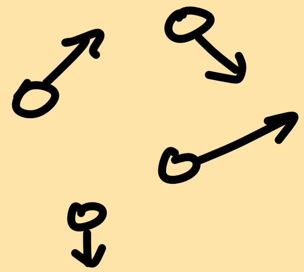
$$\underbrace{\langle \psi(t)|\psi(t)\rangle}_{\displaystyle = \int |\psi(\vec{x},t)|^2 d\vec{x}} = \langle \hat{U}(t)\psi(0)|\,\hat{U}(t)\psi(0)\rangle$$

$$= \langle \psi(0)|\,\underbrace{\hat{U}^{+}(t)\hat{U}(t)}_{= \mathbb{1}}\,\psi(0)\rangle$$

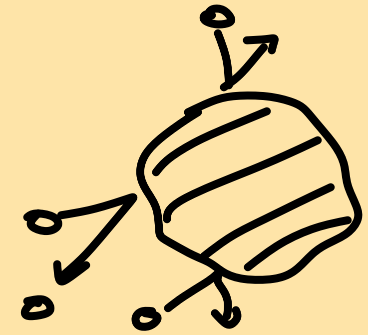
$$= \langle \psi(0)|\psi(0)\rangle$$

STATISTICAL IDEAS BEFORE 1926?

BOLTZMANN

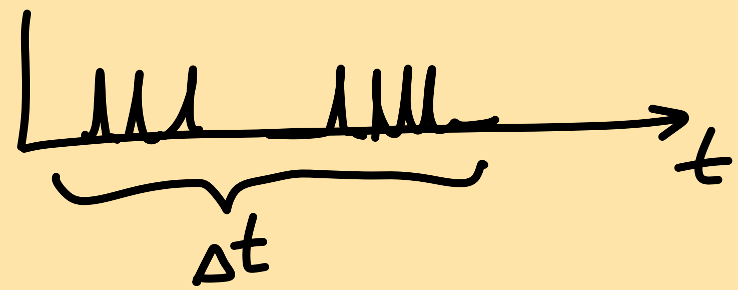


$$\rho(\vec{x}) \sim e^{-V(\vec{x})/k_B T}$$



BROWNIAN MOTION

RADIOACTIVE DECAY



TODAY:

- PHOTON DET.
- ELECTRON DET.
- SINGLE ATOMS/IONS

Lecture 2

Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

Start at 6pm CET

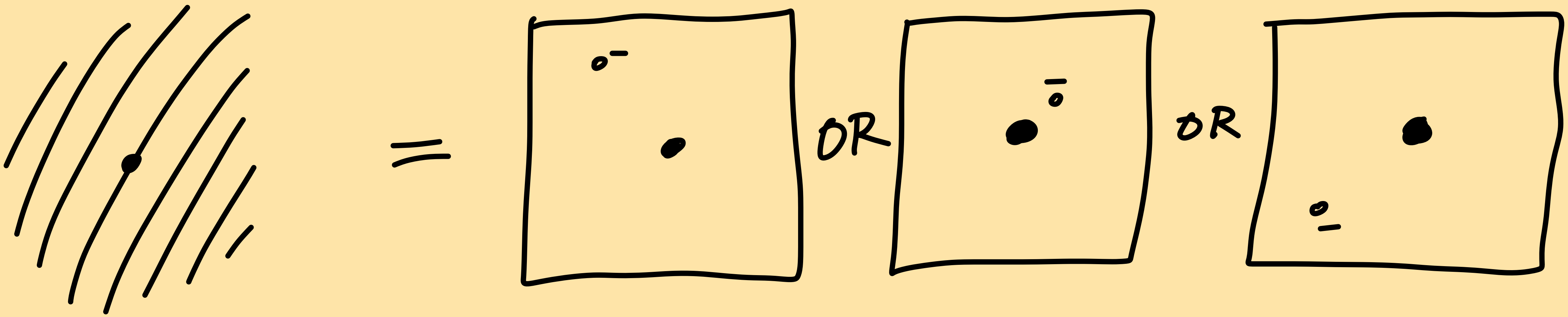
EXAM FOR FAU STUDENTS

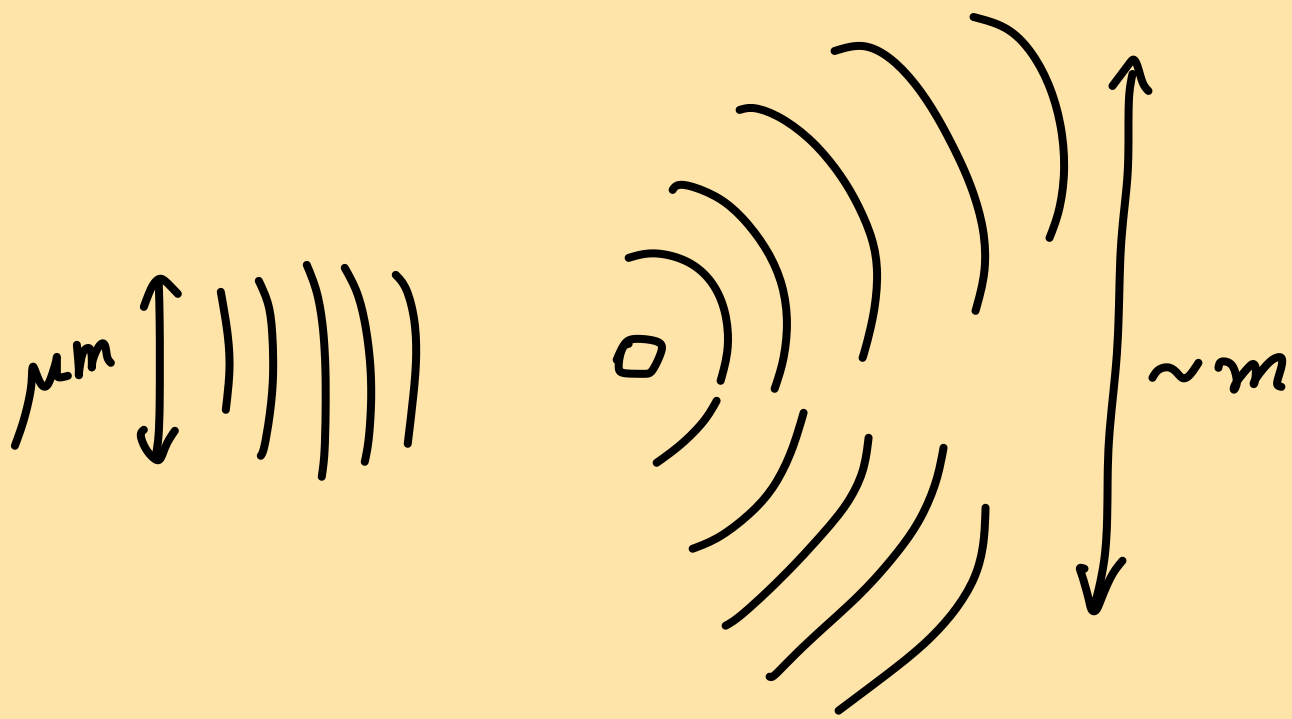
FEBRUARY 17 (WED)

10-12

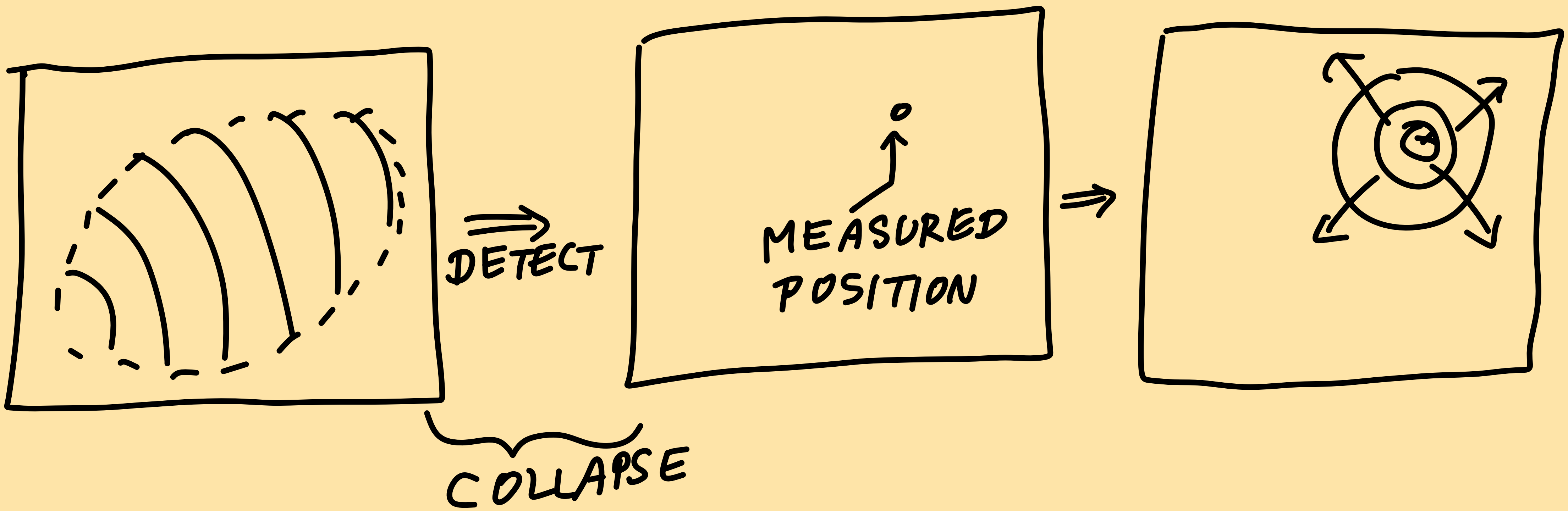
(SEE DISCUSSION GROUP)

BORN $|\psi|^2$ = PROB. DENSITY





(b) "THE COLLAPSE OF THE WAVEFUNCTION"



GENERAL RULE (VON NEUMANN)

OBSERVABLE $\hat{A}$

$$\hat{A}\,|\phi_n\rangle = A_n\,|\phi_n\rangle$$

OBTAIN VALUE A_n WITH PROBABILITY $|\langle\phi_n|\psi\rangle|^2$

$$\Rightarrow \quad |\psi\rangle \longmapsto |\psi\rangle^{\text{AFTER MSMT}} = |\phi_n\rangle$$

"PROBLEMS"

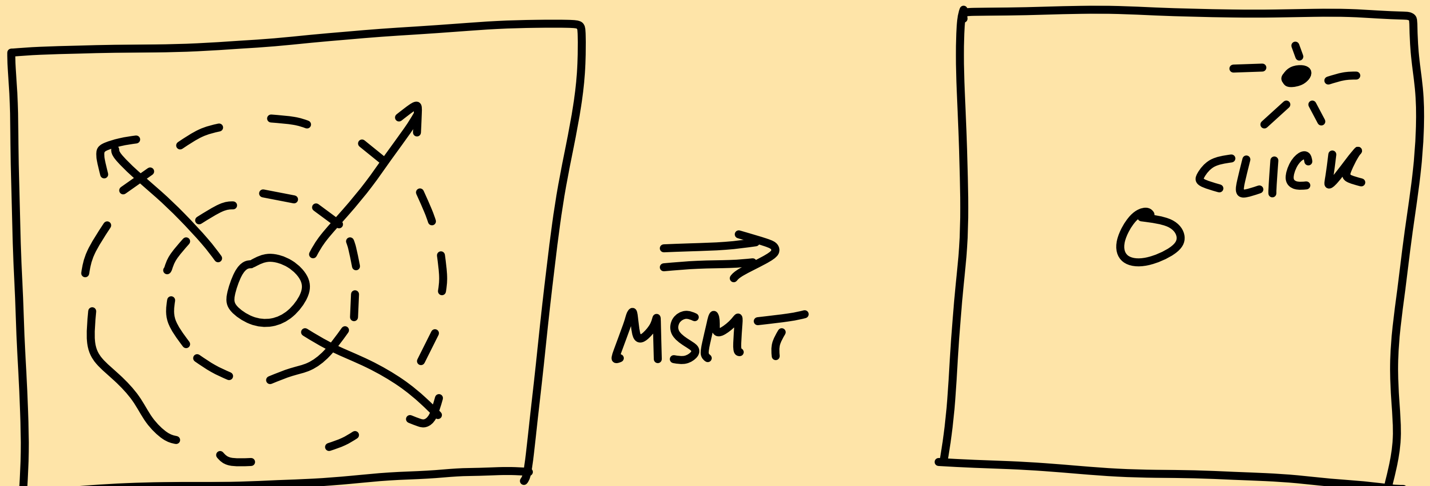
- "AD HOC" PRESCRIPTION
  OUTSIDE UNITARY
  EVOLUTION IN S.Eq.
- ARTIFICIAL DISTINCTION
  Q. SYSTEM $\longleftrightarrow$ MSMT
  APPARATUS

$\longrightarrow$ MSMTs COMPLETELY WITHIN SEq.?

$\longrightarrow$ "WEAK MSMTS"

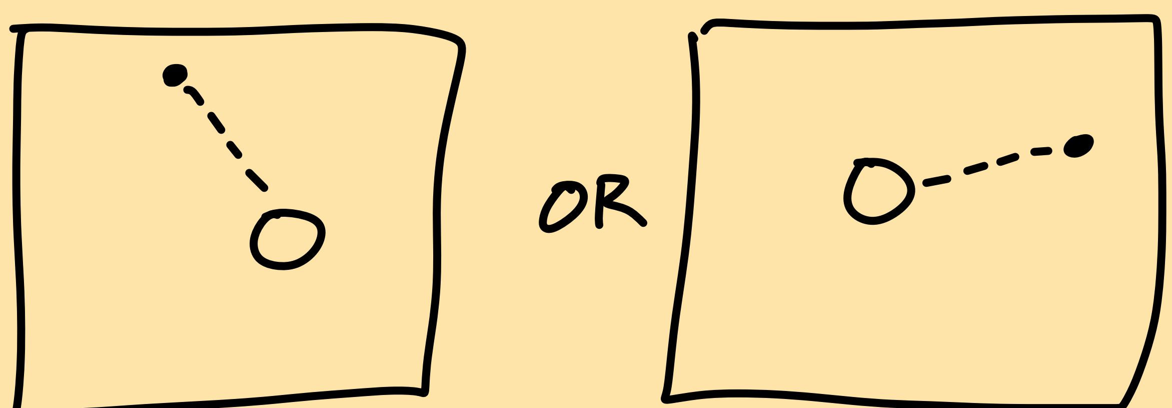
(c) "WHAT HAPPENS AT THE LEVEL OF SINGLE PARTICLES?"

EXAMPLE:



EMISSION
(OF e^- OR γ)

POSSIBLE INTERPRETATION:



OR ....

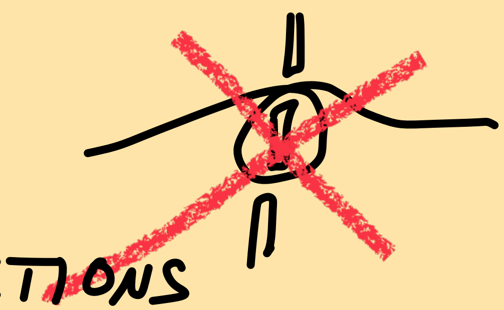
RANDOM CLASSICAL TRAJECTORIES?

EXAMPLE: DOUBLE-SLIT SETUP

ASSUME:

(1) EACH e^- HAS DEFINITE TRAJECTORY

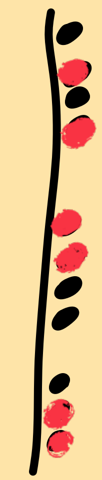
(2) EACH TRAJ. ONLY GOES THROUGH <u>ONE</u> SLIT



(3) NO LONG-RANGE INTERACTIONS

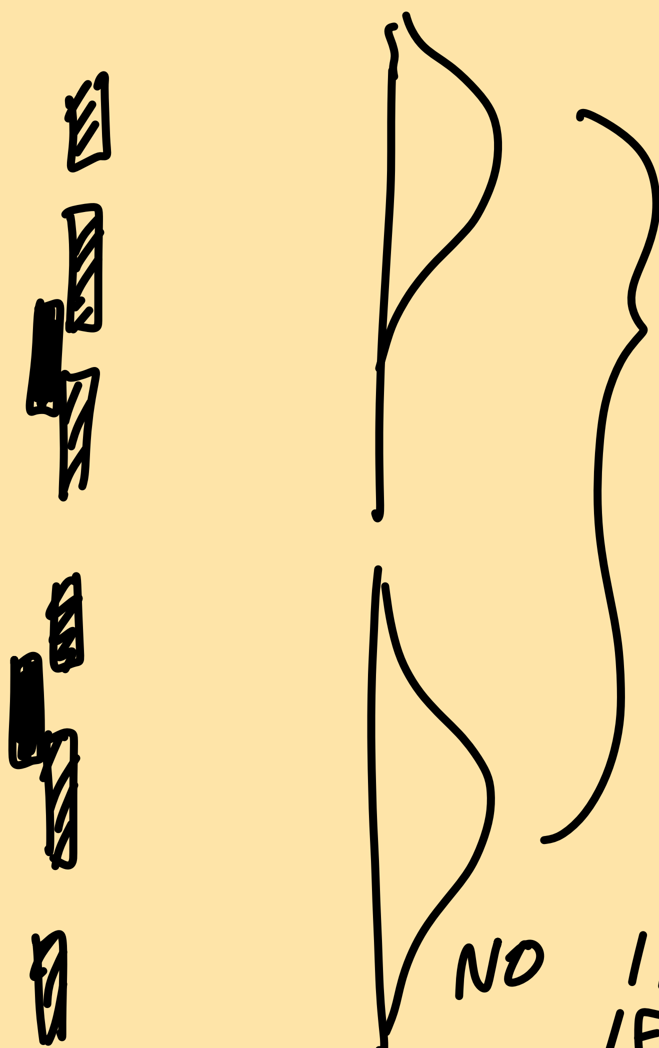
(4) e^- DO NOT INTERACT WITH EACH OTHER

(1)



PATTERN
COMES FROM
(e.g.) 50% FROM
UPPER SLIT
50% LOWER

(2) PATTERN FROM 'UPPER'
TRAJECTORIES COULD
BE OBSERVED BY
CLOSING LOWER SLIT (ETC.)



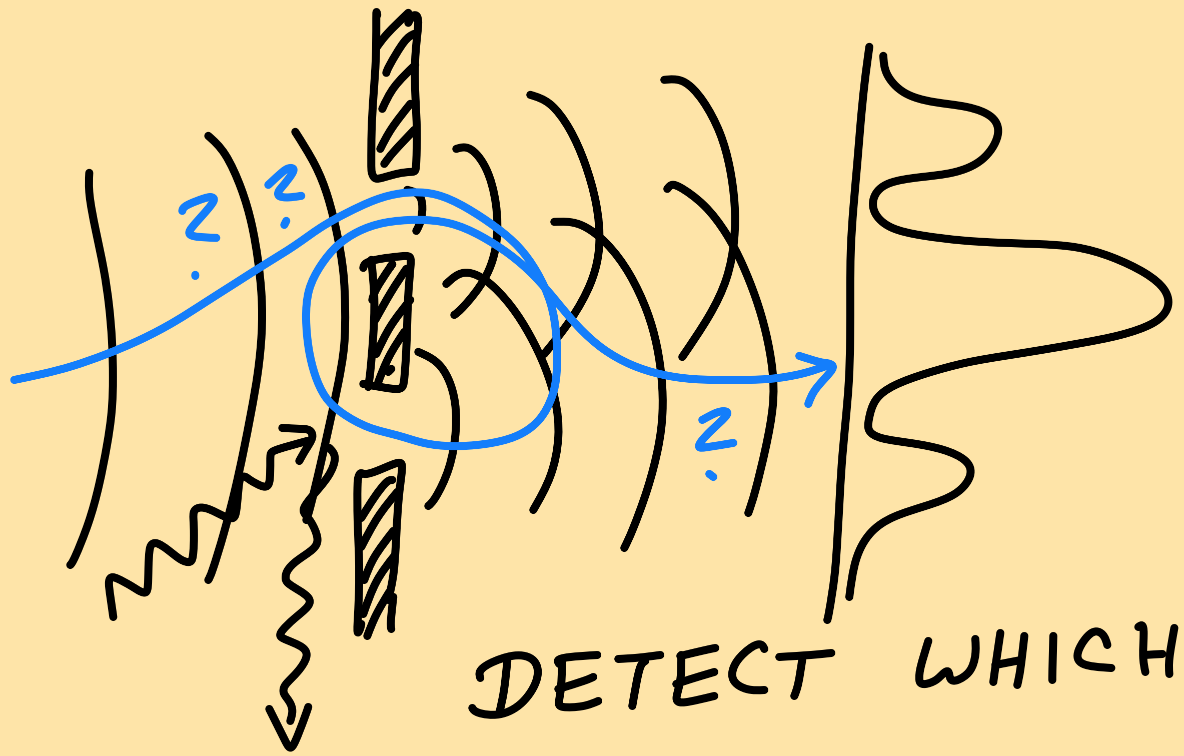
SUM OF THESE
IS <u>NOT</u> EQUAL
TO THE ACTUAL
PATTERN

NO INTERFERENCE
IF ONE SLIT IS
CLOSED!

$$|\psi_u + \psi_L|^2 = |\psi_u|^2 + |\psi_L|^2 + \underbrace{\psi_u^* \psi_L + \psi_L^* \psi_u}_{\text{INTERFERENCE TERMS}}$$

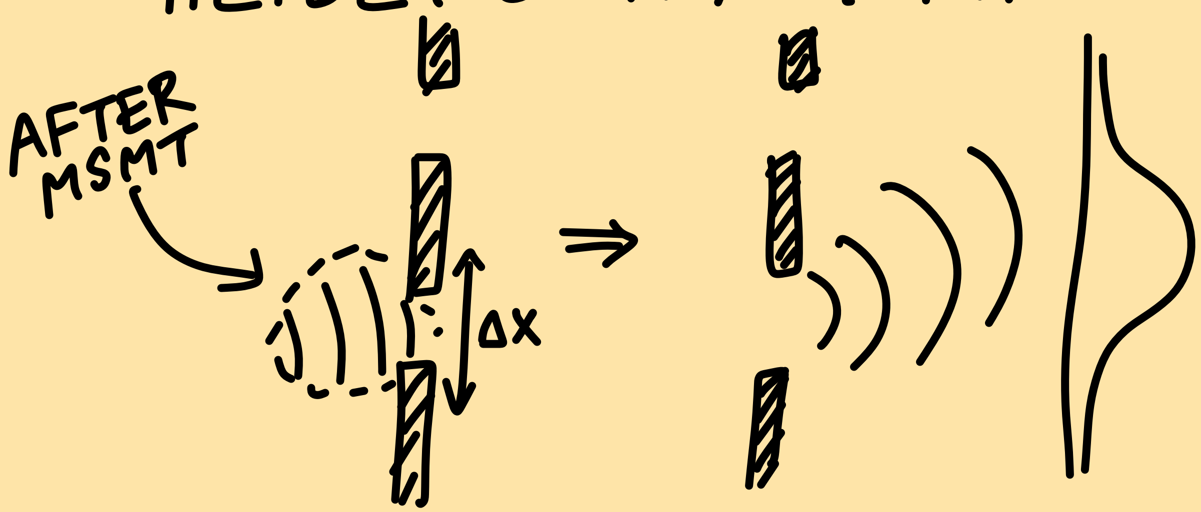
NAIVE IDEAS ONLY YIELD $|\psi_u|^2 + |\psi_L|^2$

TRY TO BE LESS DISRUPTIVE



DETECT WHICH SLIT!

"HEISENBERG MICROSCOPE"



$$\Delta x \longmapsto \Delta p \gtrsim \frac{\hbar}{2\Delta x} \Rightarrow \text{IF } \Delta x < \text{SLIT DISTANCE}$$

$\Delta x \Delta p \geq \frac{\hbar}{2}$

$\Rightarrow$ DESTROY INTERFERENCE!

LESSONS:

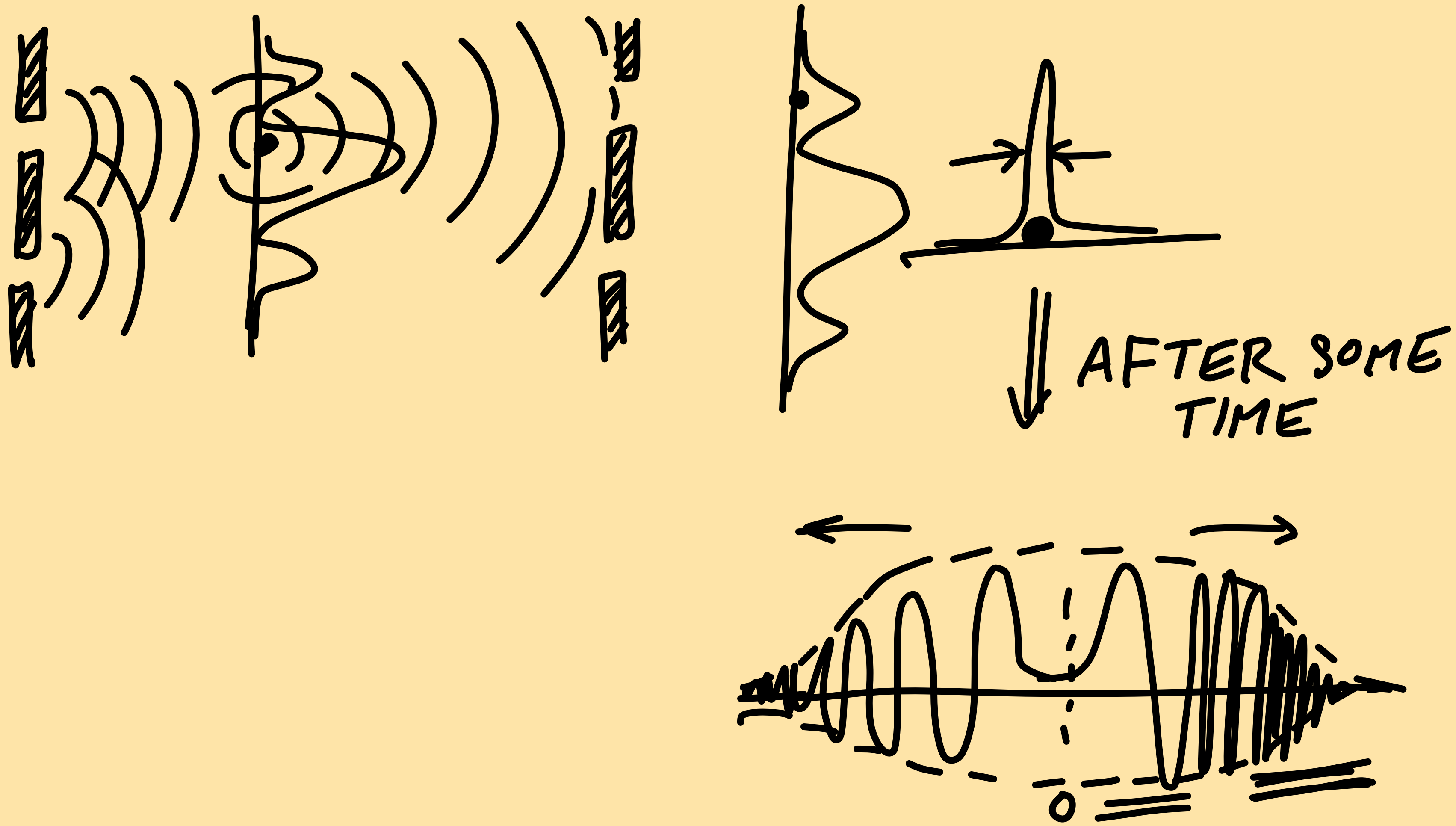
- OBSERVATION → PERTURBATION!
- IN QM: PERTURBATION IS:

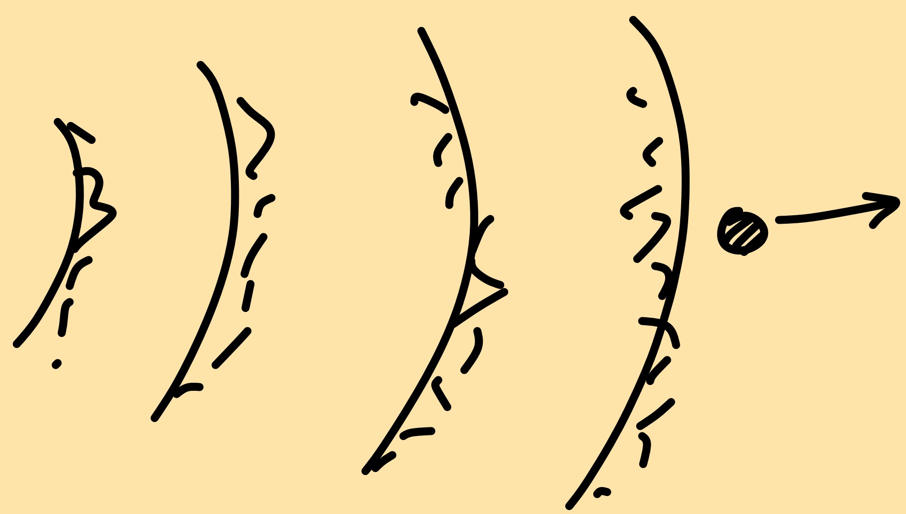
  <u>STOCHASTIC</u> STRONG

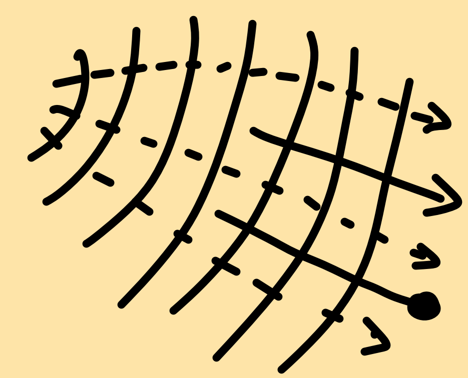
  ⇒ PERTURBATION SO STRONG THAT IT DESTROYS THE TRAJECTORY!

⇒ 'COPENHAGEN INTERPRETATION' (BOHR ETC.)

- <u>NO</u> TRAJECTORIES IN QM
- POSITION BECOMES REAL <u>ONLY</u> WHEN MSMT IS PERFORMED!

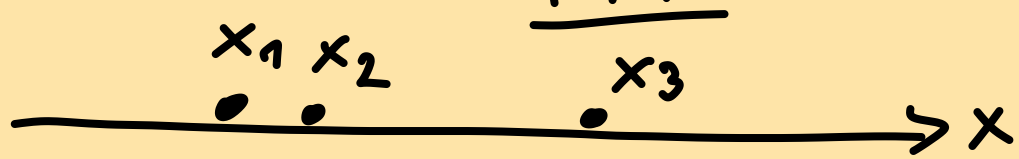

AFTER SOME TIME
0



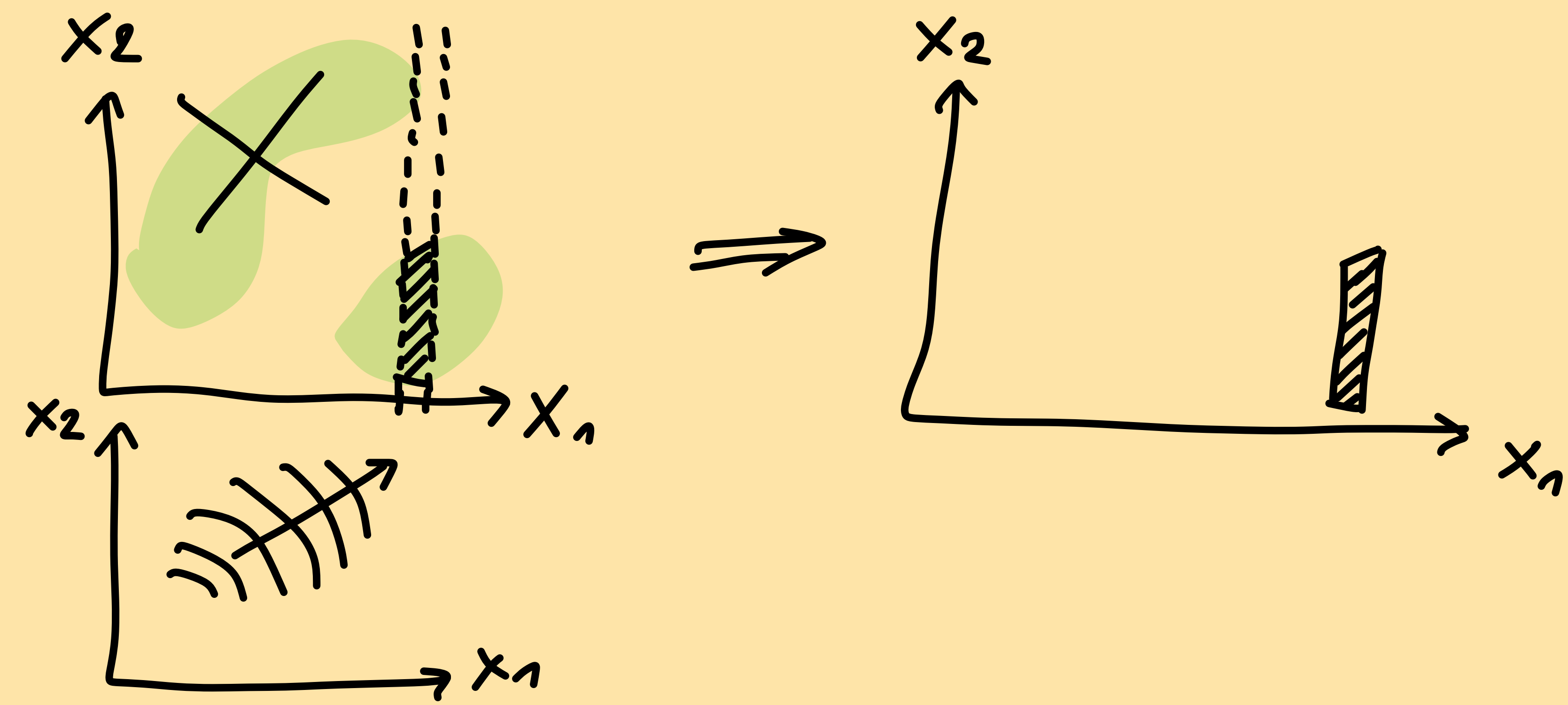


(d) MANY-PARTICLE WAVE FUNCTIONS

$$\underbrace{|\Psi(\underline{x}_1, \underline{x}_2, \dots, \underline{x}_N)|^2 \, dx_1 \, dx_2 \dots dx_N}_{\text{PROB. TO FIND } \underline{\text{THIS}} \text{ CONFIGURATION}}$$



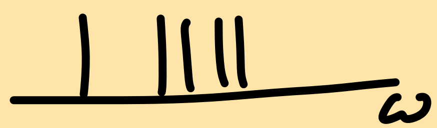
CHALLENGES:

- WAVES IN CONFIGURATION SPACE?
- 

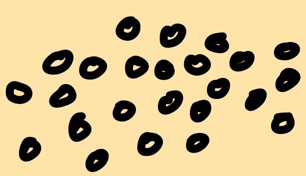
1.3 EXPERIMENTAL PROGRESS

1925

ATOMS



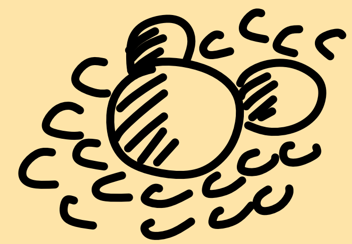
WEAK EXCITATION



ENSEMBLES!

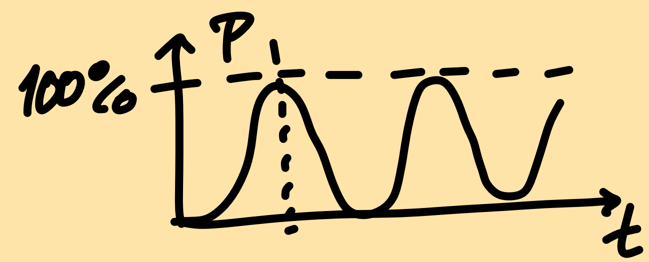
NATURAL SYSTEMS

TODAY

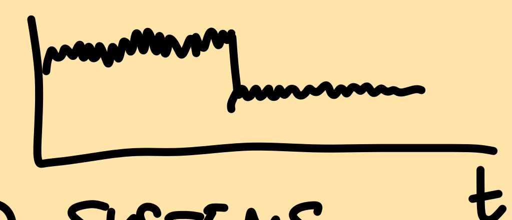


STM / AFM
INDIVIDUAL ATOMS & ORBITALS!

STRONG EXCITATION



QUANTUM JUMPS



INDIVIDUAL Q. SYSTEMS
(DETECT, CONTROL)

ARTIFICIAL Q. SYSTEMS

(2) BELL'S INEQUALITIES (AND ENTANGLEMENT)

Q: ANY UNDERLYING 'MICROSCOPIC' THEORY BEHIND QM?

COMPARE:

THERMODYNAMICS

$\uparrow$

CLASSICAL STATISTICAL PHYSICS

$\uparrow$

CLASSICAL MECHANICS

Lecture 3

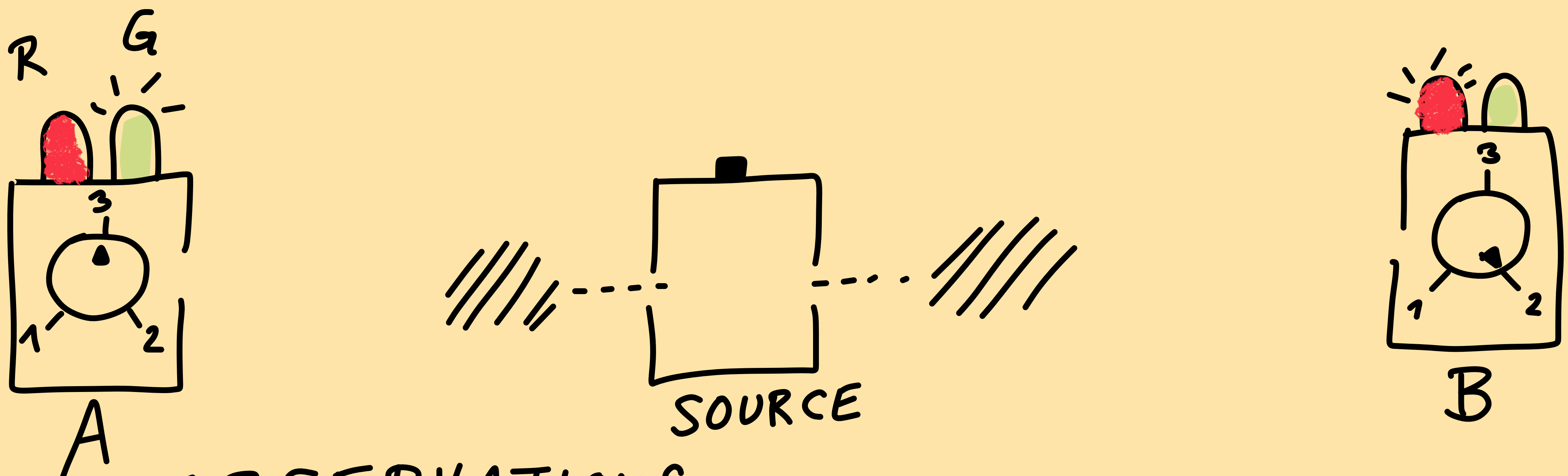
Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

Start at 6pm CET

2.1 STRANGE CORRELATIONS

(MERMIN 1985)



OBSERVATIONS

(1) SAME SETTINGS → SAME COLOR

(2) SETTINGS RANDOM $[P(11)=P(12)=\ldots=\frac{1}{9}]$

→ COLORS UNCORRELATED

$[P(RR)=P(GG)$

$=P(RG)=P(GR)=\frac{1}{4}]$

SHOULD YOU WORRY?

OBSERVE STATISTICS

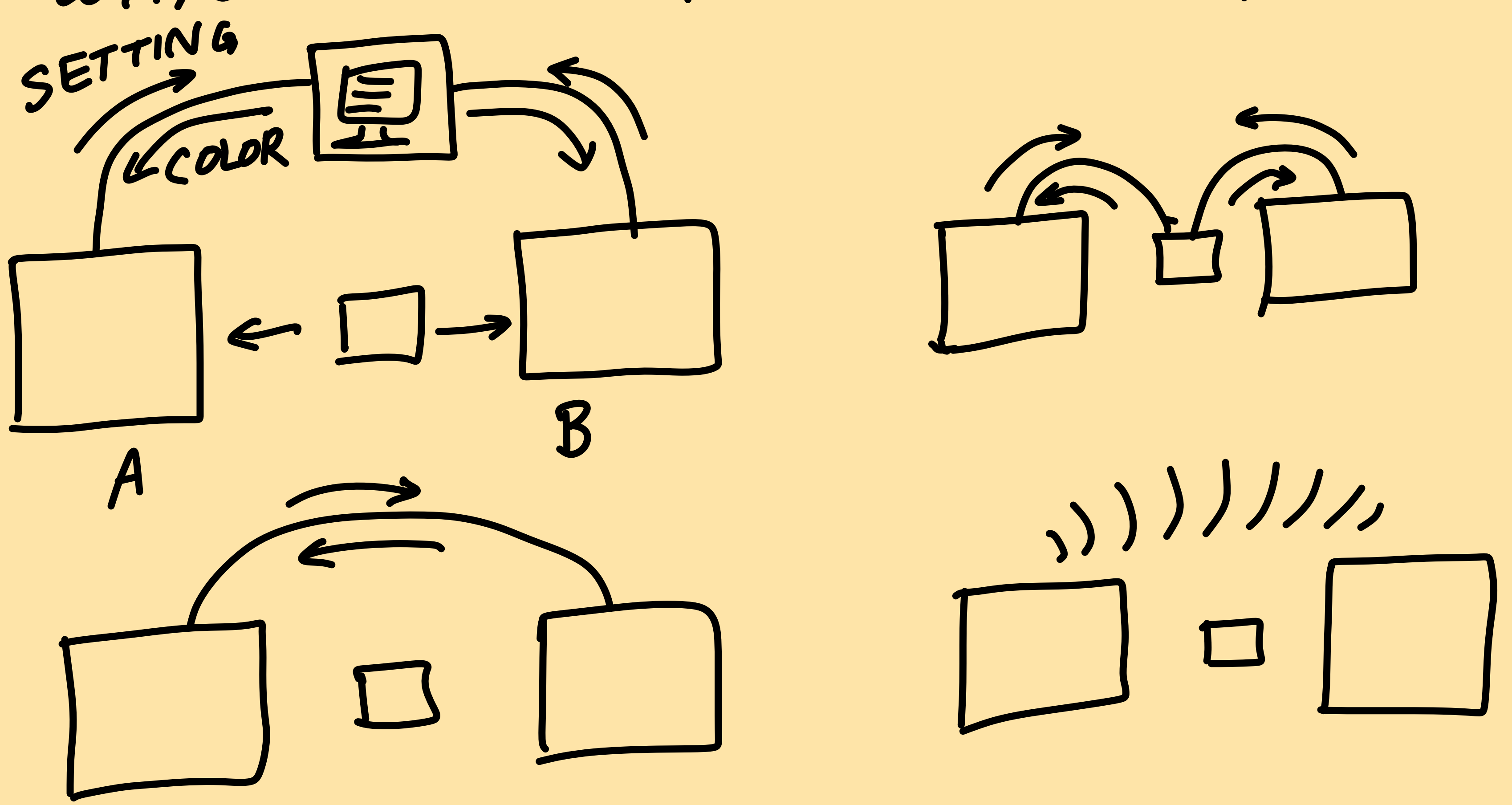
(EXAMPLE)

1	1	
R	R	20%
R	G	0%
G	R	0%
G	G	80%

1	2	
R	R	10%
R	G	40%
G	R	30%
G	G	20%

...

SIMPLE WAYS TO GET ARBITRARY STATISTICS

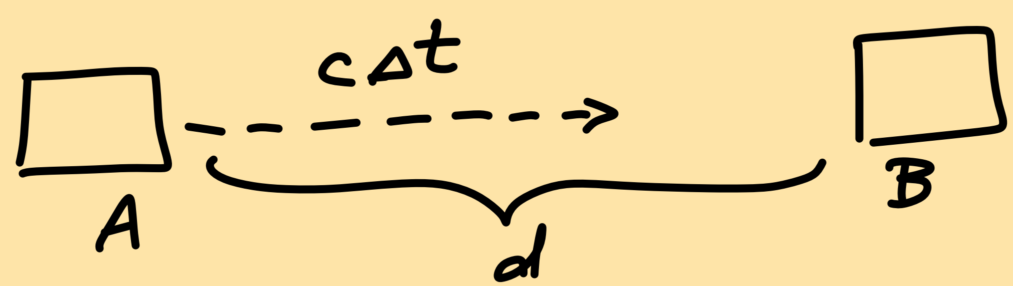


OBSERVE: – NO CABLES
– NO RADIO

BUT: NEUTRINOS? OTHER FIELDS?

IDEA: RULE OUT ALL COMMUNICATIONS THAT ARE NOT SUPERLUMINAL!

CHOOSE SETTINGS JUST BEFORE RESULT LIGHTS UP, SUCH THAT NOT EVEN LIGHT CAN REACH THE OTHER SIDE

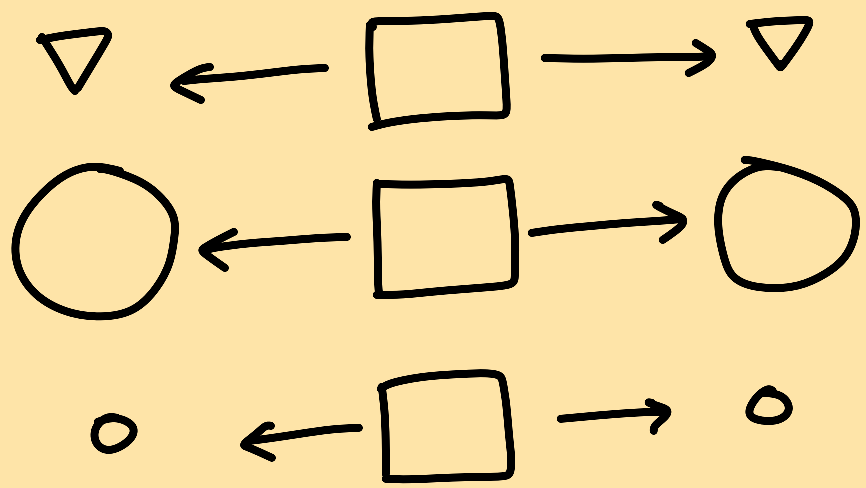


$c\Delta t < d$

$\Delta t =$ TIME SETTING $\leftrightarrow$ RESULT

$\Rightarrow$ EVEN THEN: STATISTICS HOLD!

CORRELATIONS <u>WITHOUT</u> COMMUNICATION?



SHAPE / SIZE / COLOR
1 2 3

MORE GENERAL HYPOTHESIS:

MEASURE <u>PRE-EXISTING PROPERTIES</u>
WHICH ARE <u>CORRELATED</u>
BETWEEN BOTH SIDES!

FOR EACH INDIVIDUAL RUN, ASK:
"WHAT WOULD BE THE RESULT, GIVEN ANY SETTING?"

AT "A":

1	2	3
R	G	R

'TABLE'

- NEED THIS: CANNOT KNOW IN ADVANCE THE SETTING THAT WILL BE CHOSEN
- SEPARATE TABLES FOR BOTH SIDES, NO DEPENDENCE ON SETTING COMBINATION
  (NO SIGNALS!)
- SAME TABLE FOR BOTH SIDES (SAME SETTINGS → SAME COLORS)
- NO RANDOMNESS IN OUTCOMES
  (PERFECT CORRELATIONS!)
- RANDOMLY CHOSEN TABLES

STATISTICS BASED ON <u>ANY</u> SUCH MODEL

IF $\boxed{\begin{matrix} 1 & 2 & 3 \\ R & R & R \end{matrix}}$ <u>OR</u> $\boxed{\begin{matrix} 1 & 2 & 3 \\ G & G & G \end{matrix}}$: NEVER RG OR GR ! $P(RG)=0$

IF $\boxed{\begin{matrix} 1 & 2 & 3 \\ R & R & G \end{matrix}}$ OR $\boxed{\begin{matrix} 1 & 2 & 3 \\ G & R & G \end{matrix}}$ OR ....

$\Rightarrow$ IF SETTINGS RANDOM :

$$P(\underset{(A)}{R}\underset{(B)}{G}) = \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9} \leq \frac{2}{8} = \frac{1}{4}$$

WE WILL <u>NEVER</u> BE ABLE TO ACHIEVE

$$P(RG) = \frac{1}{4}$$

OBSERV. #2

$\Rightarrow$ OBS. #1 HAS ENFORCED TOO MUCH OF A TENDENCY TOWARDS SAME COLORS!

ANY WAY OUT OF THIS?

- "MAYBE NO SUCH EXPT. EXISTS?"
  (NO, THERE IS!)
- SUPERLUMINAL SIGNALLING?

→ JOINT INSTRUCTION SETS

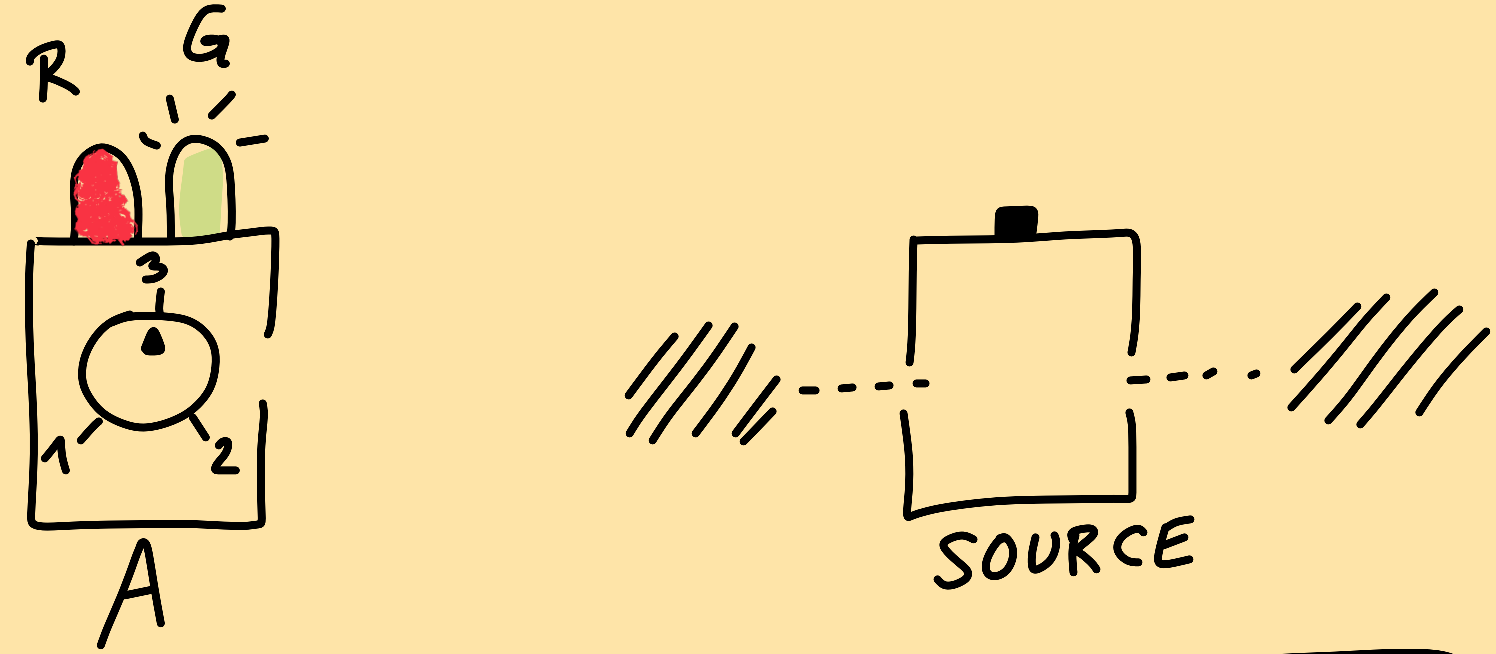
1 1	1 2	
R R	G R	...

Lecture 4

Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

Start at 6pm CET



"NATURE DISPLAYS MYSTERIOUS CORRELATIONS"
(NO NEED TO KNOW QUANTUM THEORY!)

HOW DID WE FIND OUT?

EPR 1935 → Schrödinger → Bohm → Bell

2.2 EINSTEIN, PODOLSKY, ROSEN (EPR)

"IS QM THE RESULT OF SOME UNDERLYING DEEPER THEORY?"

PROBLEM: NO TRAJECTORIES
x, p CANNOT BE MEASURED SIMULTANEOUSLY

EPR 1935: CONSTRUCT SITUATION WHERE BOTH x & p CAN BE KNOWN SIMULTANEOUSLY

(IF TRUE → x & p "ELEMENTS OF REALITY"
→ "QM INCOMPLETE", SINCE IT DOES NOT TALK ABOUT x & p AT THE SAME TIME)

TRICK: USE 2 PARTICLES IN

$$\psi(x_1, x_2) \sim \underline{\delta(x_1 - x_2)} \Rightarrow \underline{\text{"}x_1 = x_2\text{"}}$$

$$= \left.\int_{-\infty}^{+\infty} \frac{dp}{2\pi\hbar}\, \underbrace{e^{i\frac{p}{\hbar}(x_1 - x_2)}}_{\underbrace{e^{i\frac{p}{\hbar}x_1}}_{\underline{p_1 = +p}} \cdot \underbrace{e^{-i\frac{p}{\hbar}x_2}}_{\underline{p_2 = -p}}}\right\} \Rightarrow \underline{\text{"}p_1 = -p_2\text{"}}$$

(1) $x_1 = x_2$ in any msmt of x_1, x_2

(2) $p_1 = -p_2$ —— " —— of p_1, p_2

$\Rightarrow$ IDEA:

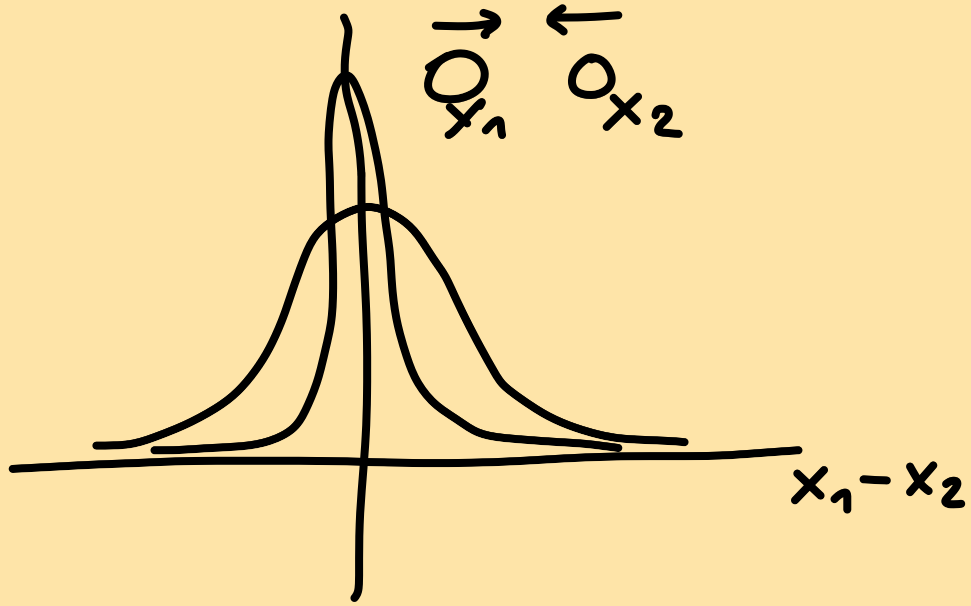
MEASURE x_1 $\Rightarrow$ DEDUCE VALUE $x_2 \overset{!}{=} x_1$

($\downarrow$ p_1 IS PERTURBED)

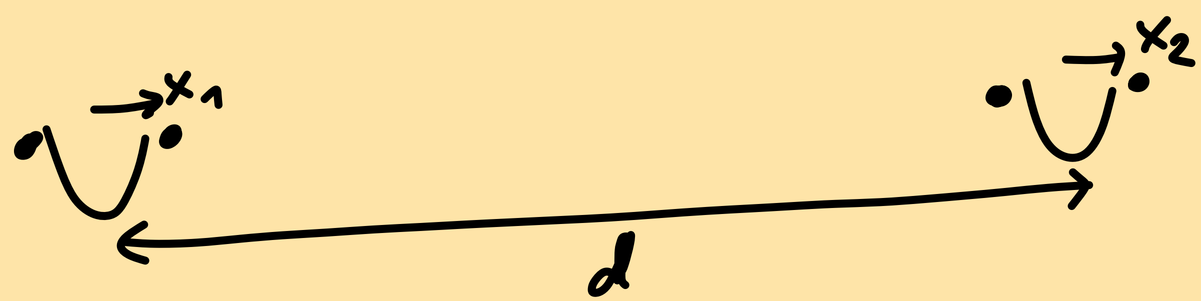
BUT MEASURE p_2 !

($\downarrow$ x_2 IS PERTURBED)

(ANSWER TO QUESTION)



$$\delta(x_1 - x_2 - d)$$



$$\delta(\underbrace{x_1 + x_2})$$

$$x_1 = -x_2$$
$$p_1 = +p_2$$

NOTE: MSMT OF x_1 CANNOT PERTURB $x_2 p_2$ (IF FAR APART)

$\Longrightarrow$ WE KNOW BOTH x_2 & p_2

x_2 → BY DEDUCTION

p_2 → MSMT

SUMMARY:

QM DESCRIBES CORRECTLY RESULTS OF SUCH A 2-PARTICLE EXPT.

NO "SPOOKY ACTION AT A DISTANCE"

$\xRightarrow{\text{EPR}}$

QM "INCOMPLETE"
$\Longrightarrow$ CONSTRUCT MORE COMPLETE THEORY
= "HIDDEN VARIABLE THEORY"

BOHR'S REPLY:

- DO NOT TREAT PARTICLES SEPARATELY!
- EACH MSMT COMBINATION (LIKE "x_1/p_2" OR "x_1/x_2") IS A SEPARATE EXPT.
- DEDUCTION "WHAT WOULD* HAVE BEEN" NOT ALLOWED!

* 'COUNTERFACTUAL REASONING'

SCHRÖDINGER (1935)

EPR WORKS BECAUSE:

$\psi(x_1, x_2) \neq$ PRODUCT $\phi_1(x_1) \cdot \phi_2(x_2)$

$\Rightarrow$ "ψ ENTANGLED"

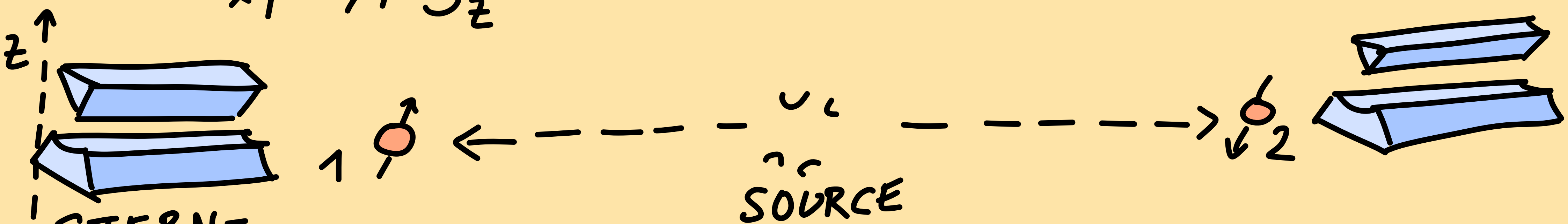
BOHM'S VERSION OF EPR (1951)

CONSIDER SPIN-$\frac{1}{2}$ PARTICLE(S)

$|\uparrow\rangle \quad |\downarrow\rangle$

z-BASIS

$\hat{S}_x, \hat{S}_y, \hat{S}_z$



STERN-GERLACH APPARATUS

ASSUME: "SINGLET STATE" (S=0)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\underbrace{|\uparrow\rangle_1}|\downarrow\rangle_2 - \underbrace{|\downarrow\rangle_1}|\uparrow\rangle_2\right)$$

ENTANGLED!

MEASURE SPIN 1 ALONG z-AXIS $\Rightarrow$

50% "UP" $|\uparrow\rangle_1 \underline{|\downarrow\rangle_2}$

50% "DOWN" $|\downarrow\rangle_1 \underline{|\uparrow\rangle_2}$

$\Rightarrow$ MSMT OF 2: EXACTLY THE <u>OPPOSITE</u>

(ANALOGOUS TO "$p_1 = -p_2$")

EXCURSION: SPIN

$\hat{\vec{L}} = \vec{r} \times \hat{\vec{p}}$

$$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$$

$\hookrightarrow$ CAN ALSO HAVE THIS 'ALGEBRA' IN EVEN-DIM. HILBERT SPACES, E.G. 2-DIM: "SPIN $\frac{1}{2}$"

$$\hat{S}_{x,y,z} = \frac{\hbar}{2} \cdot \hat{\sigma}_{x,y,z}$$

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

PAULI MATRICES

$$\hat{\sigma}_x \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

EIGENSTATES:

$$\hat{\sigma}_z \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (+1) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\hat{\sigma}_z \begin{pmatrix} 0 \\ 1 \end{pmatrix} = (-1) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

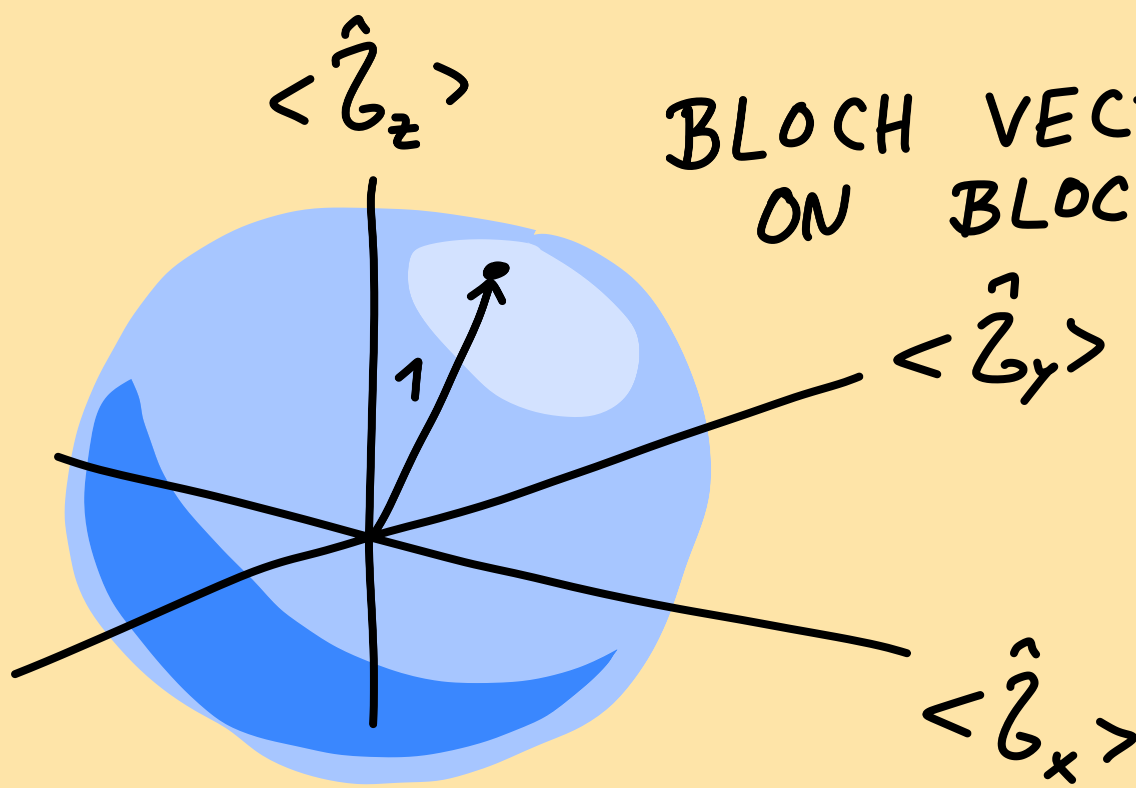
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \equiv |\uparrow\rangle \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \equiv |\downarrow\rangle$$

$$\hat{\sigma}_x \begin{pmatrix}1\\1\end{pmatrix} = (+1)\begin{pmatrix}1\\1\end{pmatrix} \qquad |\rightarrow\rangle_x \equiv \begin{pmatrix}1\\1\end{pmatrix}\frac{1}{\sqrt{2}} \equiv \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$\hat{\sigma}_x \begin{pmatrix}1\\-1\end{pmatrix} = (-1)\begin{pmatrix}1\\-1\end{pmatrix} \qquad |\leftarrow\rangle_x \equiv \begin{pmatrix}1\\-1\end{pmatrix}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$$

$$\langle \vec{\hat{\sigma}} \rangle = \langle\psi| \begin{pmatrix}\hat{\sigma}_x\\ \hat{\sigma}_y\\ \hat{\sigma}_z\end{pmatrix} |\psi\rangle = \begin{pmatrix}\langle\psi|\hat{\sigma}_x|\psi\rangle\\ \dots\\ \dots\end{pmatrix}$$

BLOCH VECTOR
ON BLOCH SPHERE



PROJECTION OF SPIN ON ARBITRARY DIRECTION

$$\vec{n}\cdot\hat{\vec{S}} = \frac{\hbar}{2}\,\underbrace{\vec{n}\cdot\hat{\vec{\sigma}}}$$

unit vector

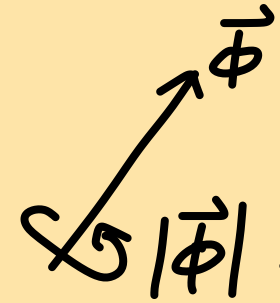
$$\underbrace{(\underbrace{\vec{n}\cdot\hat{\vec{\sigma}}})^2} = (n_x\hat{\sigma}_x + n_y\hat{\sigma}_y + n_z\hat{\sigma}_z)^2$$

$$= n_x^2\,\underbrace{\hat{\sigma}_x^2}_{\substack{\equiv\\ \mathbb{1}}} + n_x n_y\,\underbrace{(\hat{\sigma}_x\hat{\sigma}_y + \hat{\sigma}_y\hat{\sigma}_x)}_{\substack{\equiv\\ 0}} + \ldots$$

$$= \mathbb{1}(n_x^2 + n_y^2 + n_z^2) = \mathbb{1}\,\vec{n}^2 = \underline{\underline{\mathbb{1}}}$$

$\Longrightarrow$ $\vec{n}\cdot\hat{\vec{\sigma}}$ HAS EIGENVALUES ± 1

ROTATE: $\underbrace{e^{-\frac{i}{\hbar}\vec{\phi}\cdot\hat{\vec{S}}}}\,|\psi\rangle$



$|\vec{\phi}|$ = rotation angle

SPIN $\frac{1}{2}$: $\hat{R}_z(\phi) = e^{-\frac{i}{2}\phi\,\hat{\sigma}_z} = \mathbb{1}\cdot\cos\left(\frac{\phi}{2}\right) - i\sin\left(\frac{\phi}{2}\right)\hat{\sigma}_z$

<u>TWO</u> SPIN $\frac{1}{2}$

PRODUCT HILBERT SPACE

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$$

PRODUCT BASIS e.g. $|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 = |\uparrow\downarrow\rangle$

$$\hat{S}_{z2} \, |\uparrow \underline{\underline{\downarrow}}\rangle = -\frac{\hbar}{2} |\uparrow\downarrow\rangle$$

$$\hat{\vec{S}} = \hat{\vec{S}}_1 + \hat{\vec{S}}_2$$ TOTAL SPIN

$$\underline{\underline{\hat{\vec{S}}^2}} = \hat{\vec{S}}_1^{\,2} + \hat{\vec{S}}_2^{\,2} + 2\,\hat{\vec{S}}_1 \hat{\vec{S}}_2$$

EIGENSTATE FOR EIGENVALUE 0:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$ 'SINGLET'

$$\hat{\vec{S}}\,|\psi\rangle = 0 \cdot |\psi\rangle$$

$$\underbrace{e^{-\frac{i}{\hbar}\vec{\Phi}\cdot\hat{\vec{S}}}}_{}\,\underbrace{|\psi\rangle}_{} = \underline{|\psi\rangle}$$ ROTATIONALLY INVARIANT

MEASURE $\hat{S}_x \Rightarrow$?

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$$

$$|\leftarrow\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle - |\downarrow\rangle\right)$$

$$|\psi\rangle \underset{\substack{\uparrow \\ \text{EXERCISE}}}{=} \frac{1}{\sqrt{2}}\left(|\rightarrow\rangle_1|\leftarrow\rangle_2 - |\leftarrow\rangle_1|\rightarrow\rangle_2\right)$$

MSMT S_x

$$\xrightarrow[50\%]{} |\rightarrow\rangle_1 |\leftarrow\rangle_2$$

Lecture 5

Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

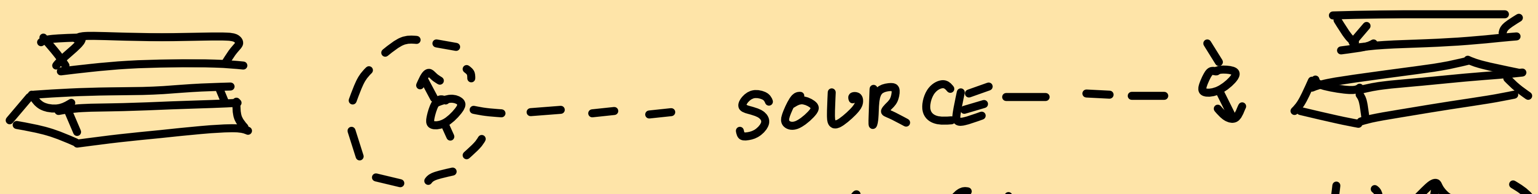
Start at 6pm CET

NEXT UP:

JOHN BELL:

2.3 Bell's Inequalities

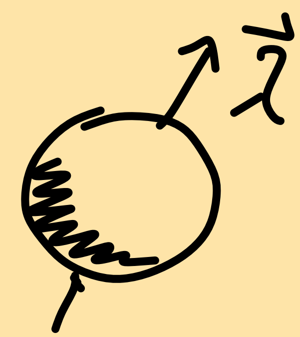
Q: UNDERLYING THEORY TO EXPLAIN EPR-EXPT. ?



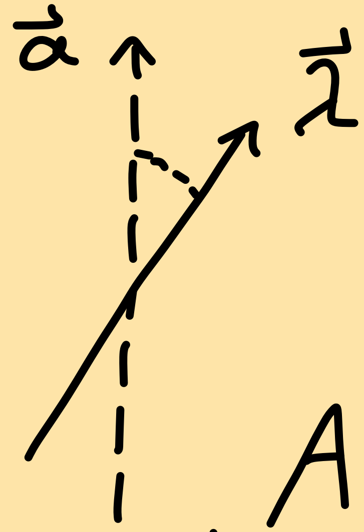
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

MODEL: SPIN & MSMT

SPIN $\equiv$ UNIT VECTOR $\vec{\lambda}$



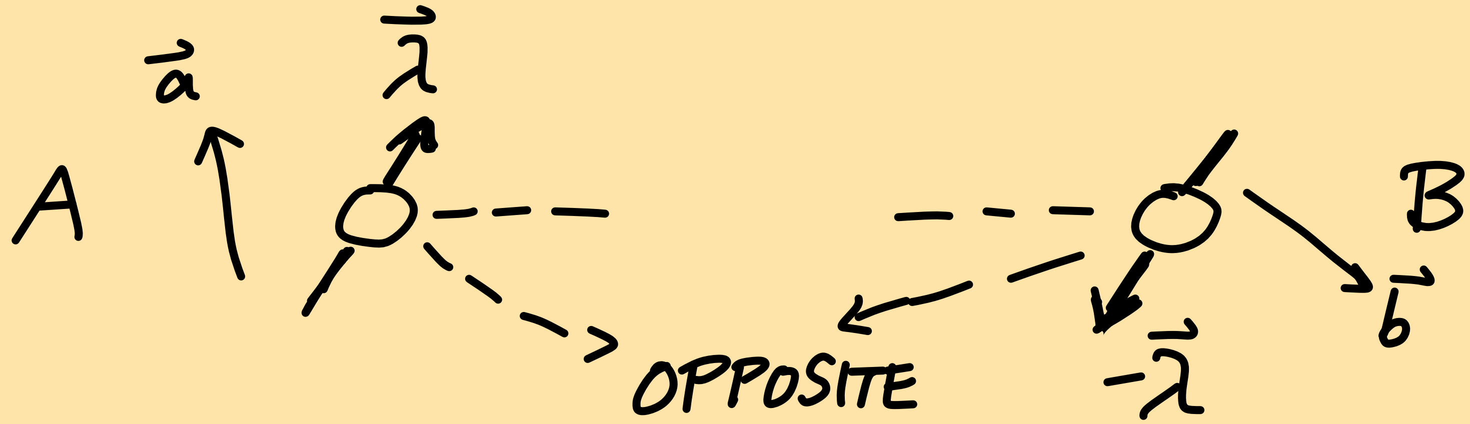
MSMT OF SPIN ALONG AXIS $\vec{a}$



"UP" $\Leftrightarrow$ $\vec{\lambda}\cdot\vec{a} > 0$

"DOWN" $\Leftrightarrow$ $\vec{\lambda}\cdot\vec{a} < 0$

$$\underbrace{A}_{\text{MSMT RESULT}}(\vec{a},\vec{\lambda}) = \underline{\mathrm{sign}(\vec{\lambda}\cdot\vec{a})} = \pm 1$$



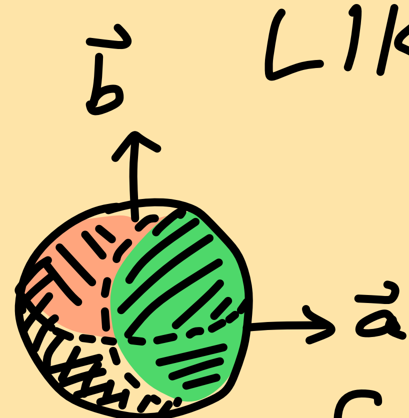
$\vec{\lambda}$ RANDOM

$$B(\vec{b}, \vec{\lambda}) = \text{sign}(\vec{\lambda}_B \cdot \vec{b}) = -\text{sign}(\vec{\lambda} \cdot \vec{b})$$
$$= -A(\vec{b}, \vec{\lambda})$$

IF $\vec{a} = \vec{b} \Rightarrow A = -B$ ✓

LIKE IN QM ✓

IF $\vec{a} \perp \vec{b}$:



$A \cdot B > 0$ AS OFTEN AS $A \cdot B < 0$

$$\langle A \cdot B \rangle = \int d\vec{\lambda}\, \rho(\vec{\lambda})\, A(\vec{a}, \vec{\lambda})\, B(\vec{b}, \vec{\lambda}) = 0$$

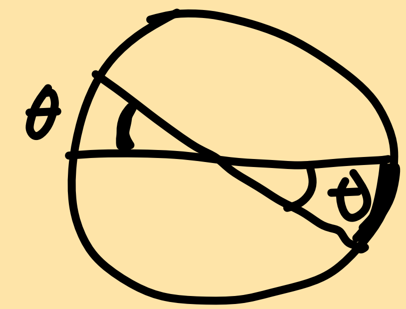
$\Rightarrow$ UNCORRELATED, LIKE IN QM ✓

BELL'S IDEA:

CHECK OTHER ANGLES $\sphericalangle(\vec{a},\vec{b})$

IN THIS MODEL:

$$\theta = \sphericalangle(\vec{a},\vec{b})$$

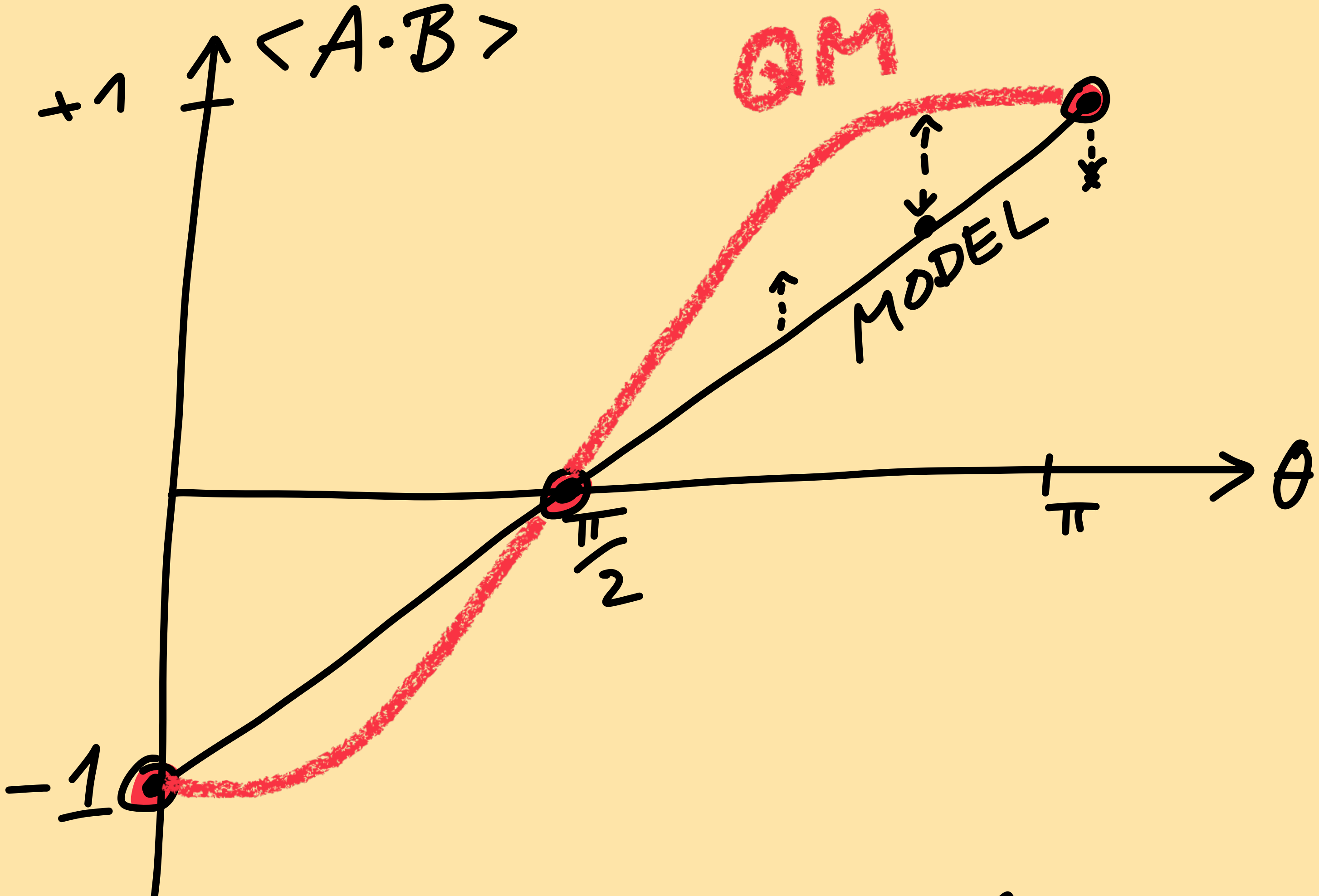


$\underbrace{A \cdot B > 0}$ IN A FRACTION

$$\frac{2\theta}{2\pi} = \frac{\theta}{\pi}$$

$$\langle A \cdot B \rangle = \underbrace{(-1)\cdot\left(1 - \frac{\theta}{\pi}\right)} + (+1)\cdot\frac{\theta}{\pi}$$

$$= \left(2\frac{\theta}{\pi} - 1\right)$$



$$\hat{\vec{\sigma}} = \begin{pmatrix} \hat{\sigma}_x \\ \hat{\sigma}_y \\ \hat{\sigma}_z \end{pmatrix}$$

COMPARE QM:

$$\langle AB \rangle \Big|_{\substack{QM \\ SINGLET}} = \langle \psi | (\vec{a} \cdot \hat{\vec{\sigma}}_1)(\vec{b} \cdot \hat{\vec{\sigma}}_2) | \psi \rangle = -\, \vec{a} \cdot \vec{b} = -\cos\theta$$

☹

CANNOT REPRODUCE QM CORRELATIONS!

IS THIS A <u>GENERAL</u> PROBLEM?

DEFINE GENERAL

LOCAL HIDDEN VARIABLE MODEL (LHV)

FOR SPIN-VERSION OF EPR

MSMT RESULTS:

$$A(\vec{a}, \lambda) \in \{-1, +1\}$$

$$B(\vec{b}, \lambda) \in \{-1, +1\}$$

HIDDEN VARIABLE λ (VECTOR, FIELD, ...)

PROBABILITY DENSITY $\rho(\lambda)$

$\vec{a}, \vec{b}$ MSMT DIR.

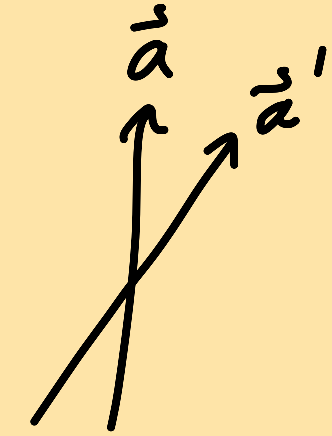
LOCAL BECAUSE WE DO NOT ALLOW:

$$A(\vec{a}, \cancel{\vec{b}}, \lambda)$$

$\Longrightarrow$ CONSTRAINTS FOR STATISTICS?

$$\langle AB \rangle = \int d\lambda \, \rho(\lambda) \, A(\vec{a},\lambda) \, B(\vec{b},\lambda) \equiv E(\vec{a},\vec{b})$$

LOOK AT DIFFERENT ANGLES $\sphericalangle(\vec{a},\vec{b})$



BELL'S ORIGINAL VERSION

$\vec{a}, \vec{a}'$ $\qquad$ $\vec{b}, \vec{b}'$

$$\underbrace{A(\underline{\vec{a}}', \lambda)} \equiv A'$$

IF $\vec{a}' = \vec{b}'$ $\Rightarrow$ IF RESULTS COMPATIBLE WITH QM:

$$A' = -B'$$
$$\forall \lambda$$

$|AB' - AB|$ (FOR A PARTICULAR λ)

$$= |A(B'-B)| \underset{A=\pm 1}{=} |B'-B| \underset{B'=-A'}{=} |-A'-B| = |A'+B| \checkmark$$

$$\underset{\substack{A'=\pm 1\\ B=\pm 1}}{=} 1 + A'B \checkmark$$

$$\langle |X| \rangle \geq |\langle X \rangle| \Rightarrow$$

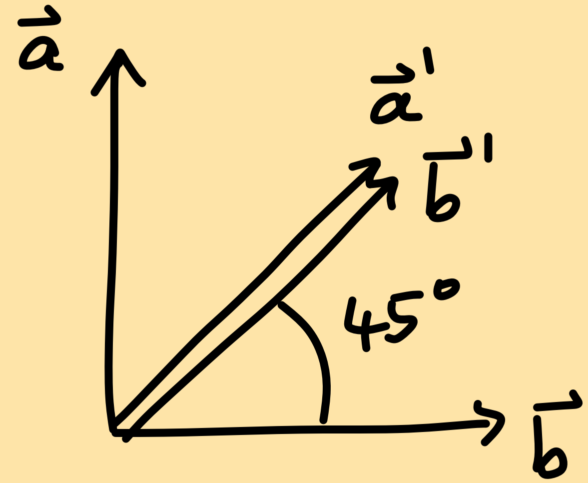
$$\boxed{|\langle AB' \rangle - \langle AB \rangle| \leq 1 + \langle A'B \rangle}$$

BELL 1964

OBEYED BY EVERY LHV MODEL

(THAT SHOWS PERFECT ANTICORR. FOR $\vec{a}' = \vec{b}'$)

COMPARE QM: $\langle AB\rangle = -\vec{a}\cdot\vec{b}$



$\langle AB'\rangle_{QM} = -\frac{1}{\sqrt{2}}$ $\qquad$ $\langle AB\rangle_{QM} = 0$

$\langle A'B\rangle_{QM} = -\frac{1}{\sqrt{2}}$

$$|\langle AB'\rangle_{QM} - \langle AB\rangle_{QM}| = \frac{1}{\sqrt{2}}$$

$$1 + \langle A'B\rangle_{QM} = 1 - \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} \not\leq 1 - \frac{1}{\sqrt{2}}$$

$$1 \not\leq \sqrt{2} - 1 \approx 0.4\ldots$$

QM VIOLATES B.I.!

$\Rightarrow \nexists$ LHV THAT MATCHES QM!

Lecture 6

Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

Start at 6pm CET

GENERALIZATION (1969, CLAUSER, HOLT, HORNE, SHIMONY)

$A, B, A', B' \in \{-1, 0, +1\}$ ↳ "NO DETECTION"

DO <u>NOT</u> ASSUME "$A' = -B'$".

$$|\underbrace{AB + AB'}_{A(B+B')}| + |A'B - A'B'|$$

$$\leq |B+B'| + |B-B'| \leq 2$$

$$\& \quad \langle |X| \rangle \geq |\langle X \rangle|$$

$$\boxed{|\langle AB\rangle + \langle AB'\rangle| + |\langle A'B\rangle - \langle A'B'\rangle| \leq \ldots \leq 2}$$

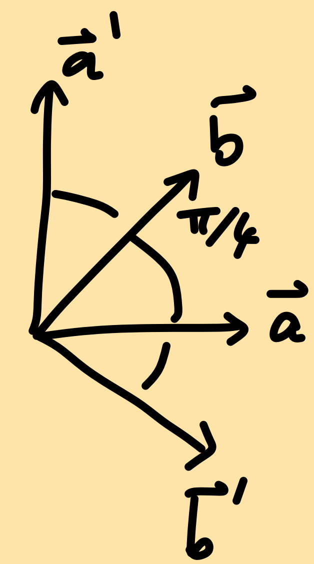
CHHS INEQUALITY

LOCALITY : BECAUSE THE VALUE $A = A(\vec{a}, \lambda)$ INDEPENDENT OF $\vec{b}$

$$AB = A(\vec{a}, \lambda)\, B(\vec{b}, \lambda)$$
$$AB' = A(\vec{a}, \lambda)\, B(\vec{b}', \lambda)$$

THE SAME

FOR QM SINGLET:

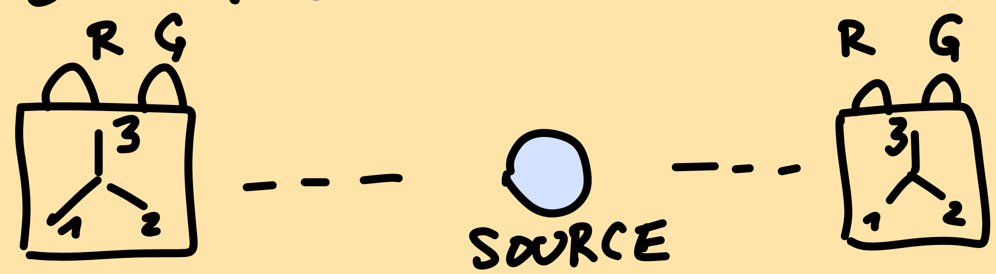


$$\langle AB \rangle_{QM} = -\cos \measuredangle(\vec{a}, \vec{b}) = -\frac{1}{\sqrt{2}}$$

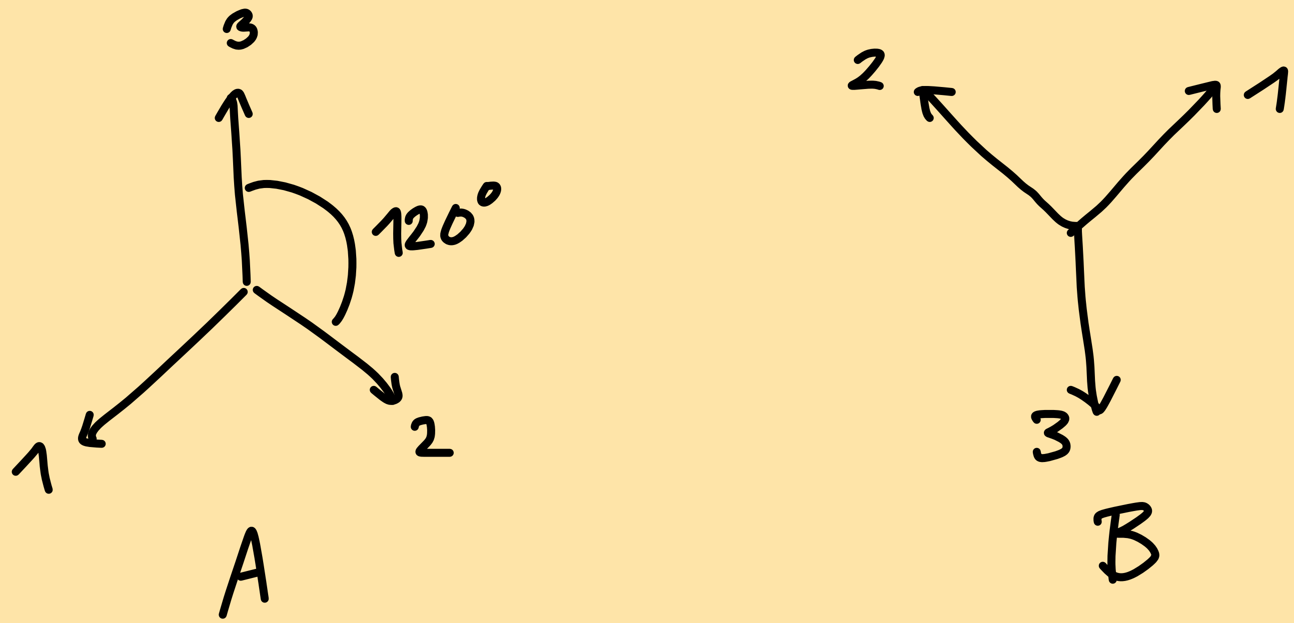
lhs of CHHS: $2\sqrt{2} \overset{!}{>} 2$

↯ CONFLICT BETWEEN QM & LHV!

MERMIN'S GEDANKENEXP.



WORKS FOR SINGLET
IF MAPPING



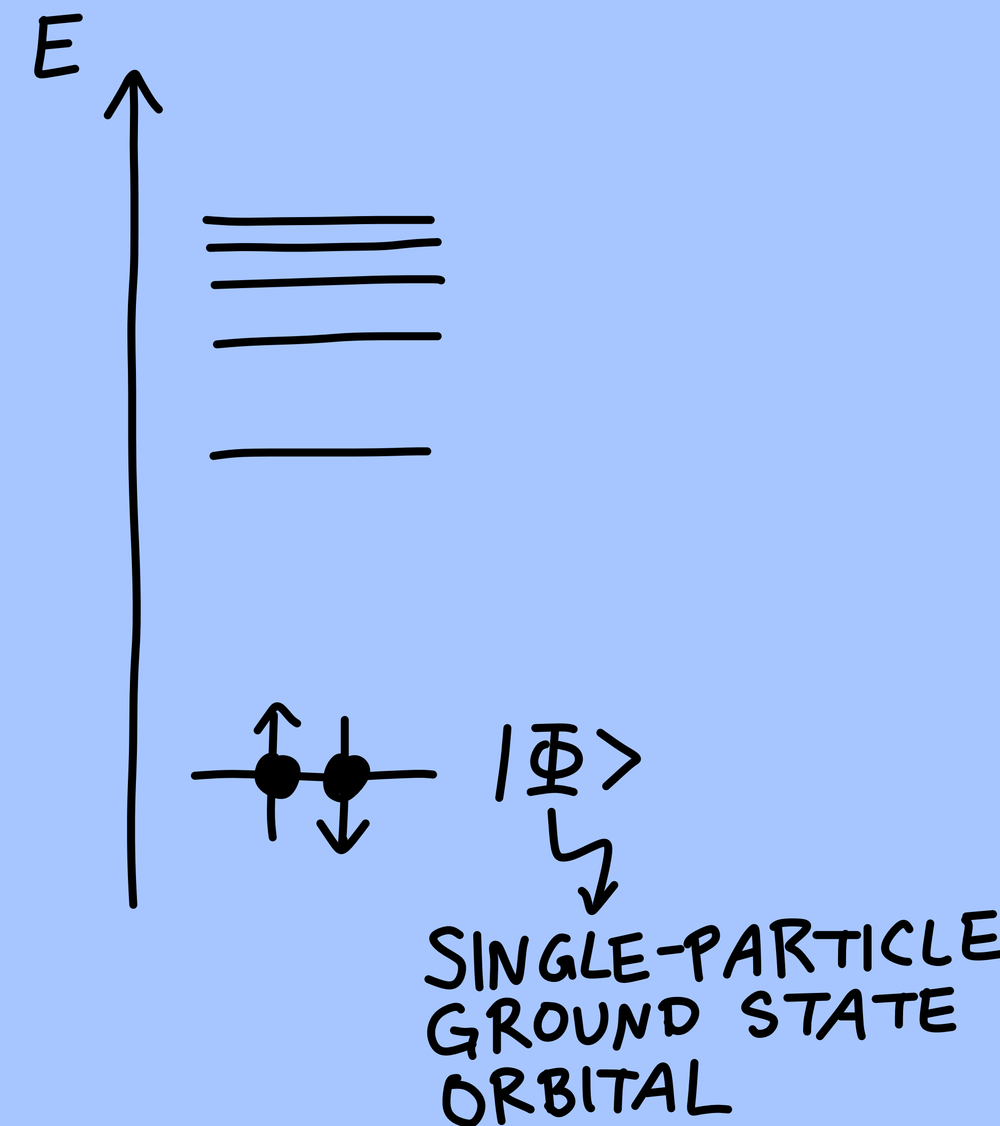
2.4 BELL TEST EXPERIMENTS

SPIN

PHOTONS

QUBITS

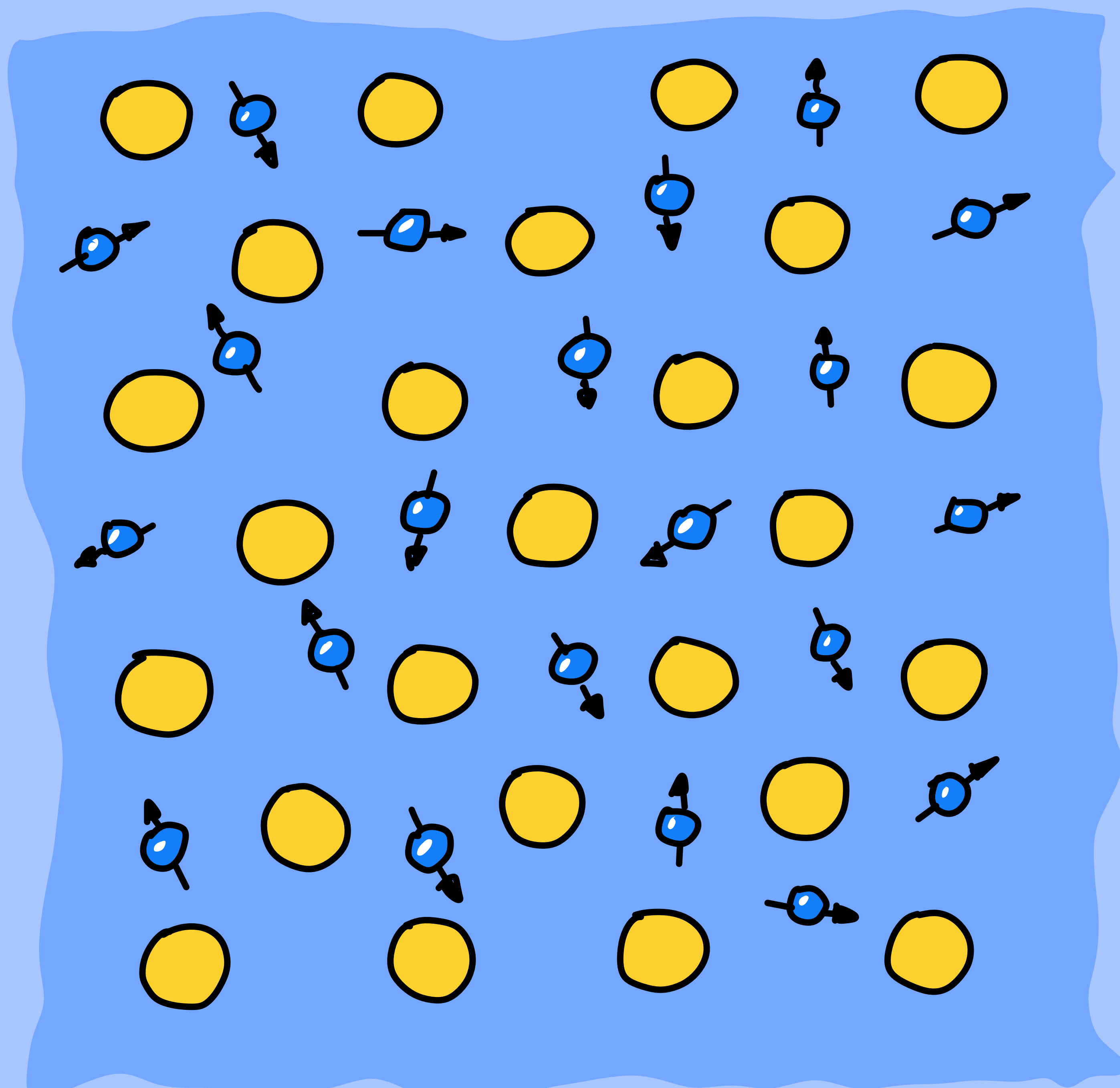
HELIUM ATOM

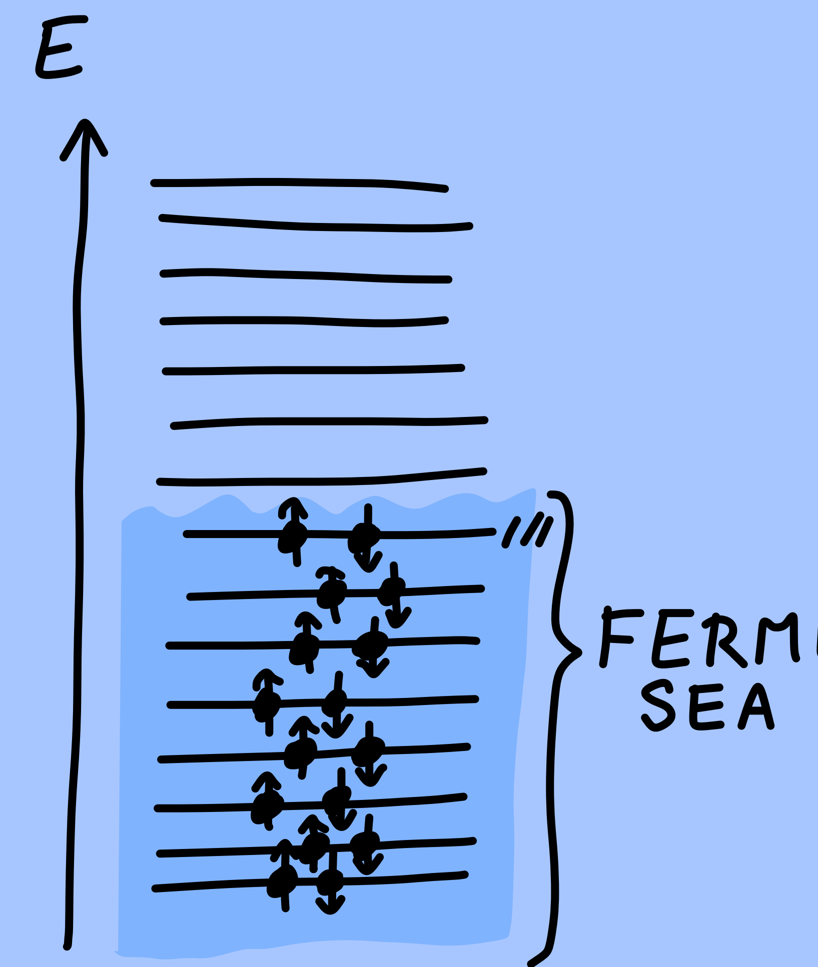


SINGLET STATE

$$|\Psi\rangle = \underbrace{|\Phi\rangle_1 \otimes |\Phi\rangle_2}_{\text{SYMMETRIC}} \otimes \underbrace{\left(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2\right)}_{\text{ANTI-SYMMETRIC}}$$

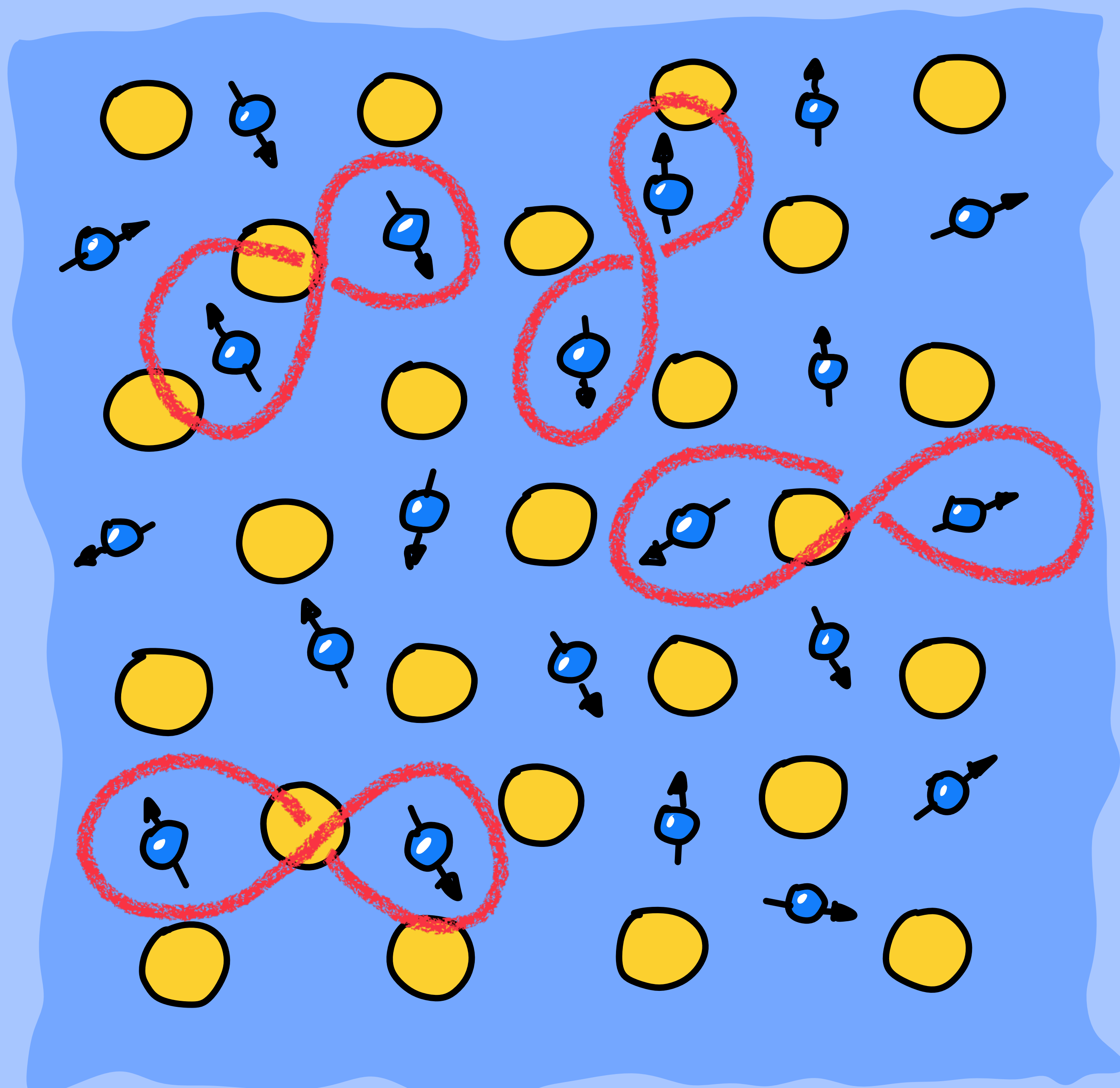
FERMI SEA





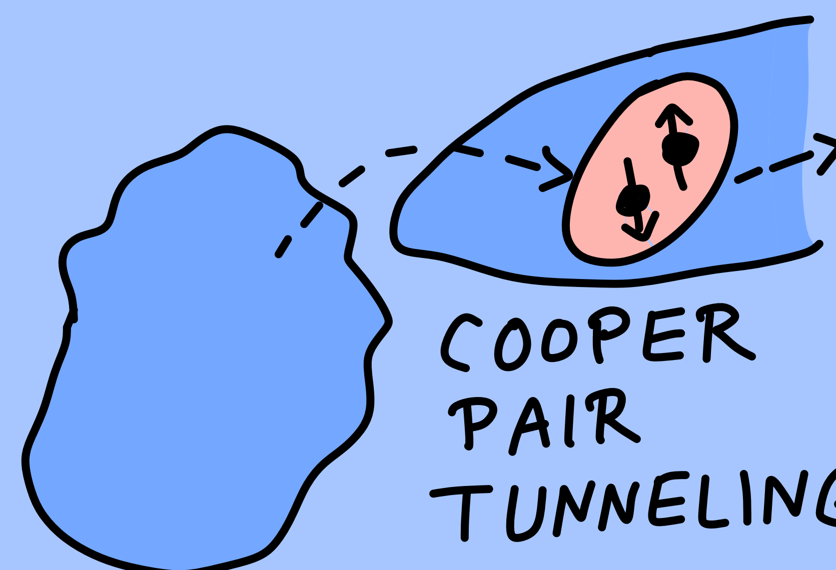
CAN EXTRACT SINGLET (IF DONE PROPERLY)

COOPER PAIRS

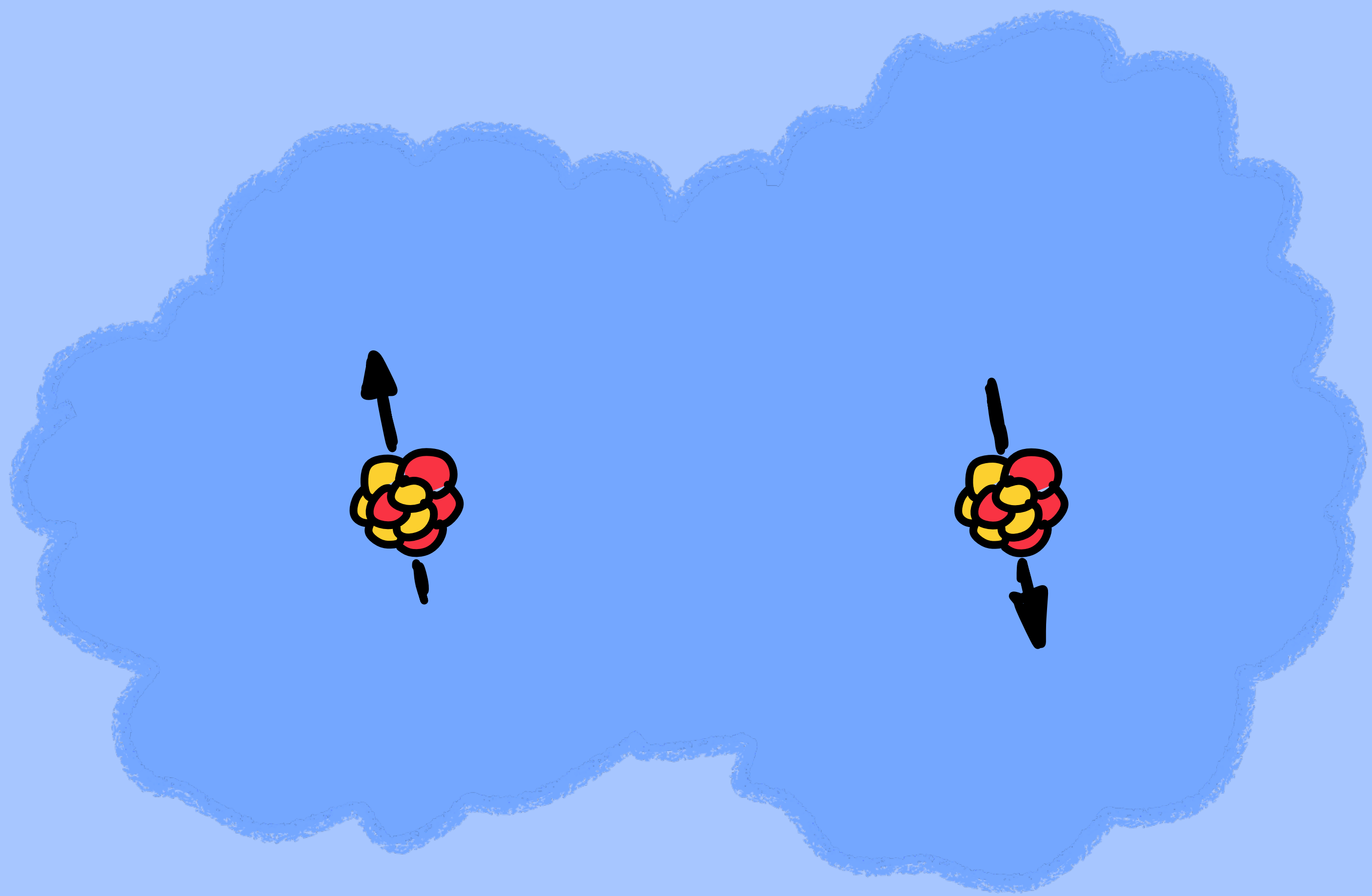


SUPERCONDUCTOR:

ELECTRONS PAIRED UP → SINGLETS!



NUCLEAR SPINS



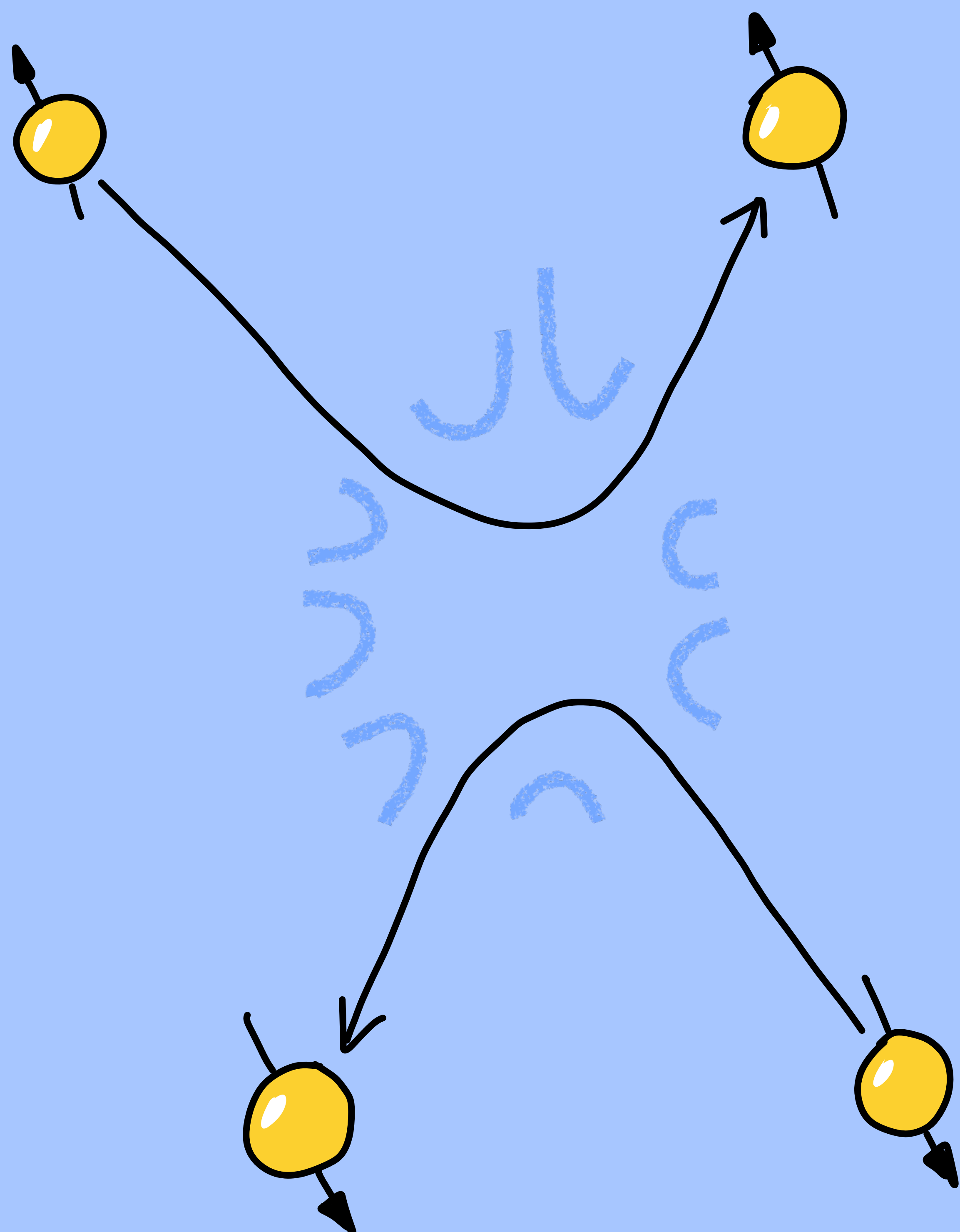
EXAMPLE:

H_2

"PARA-HYDROGEN":

NUCLEI IN SINGLET-STATE

SPIN-DEPENDENT SCATTERING



SPIN-DEPENDENT ORBITAL STATE

$\Rightarrow$

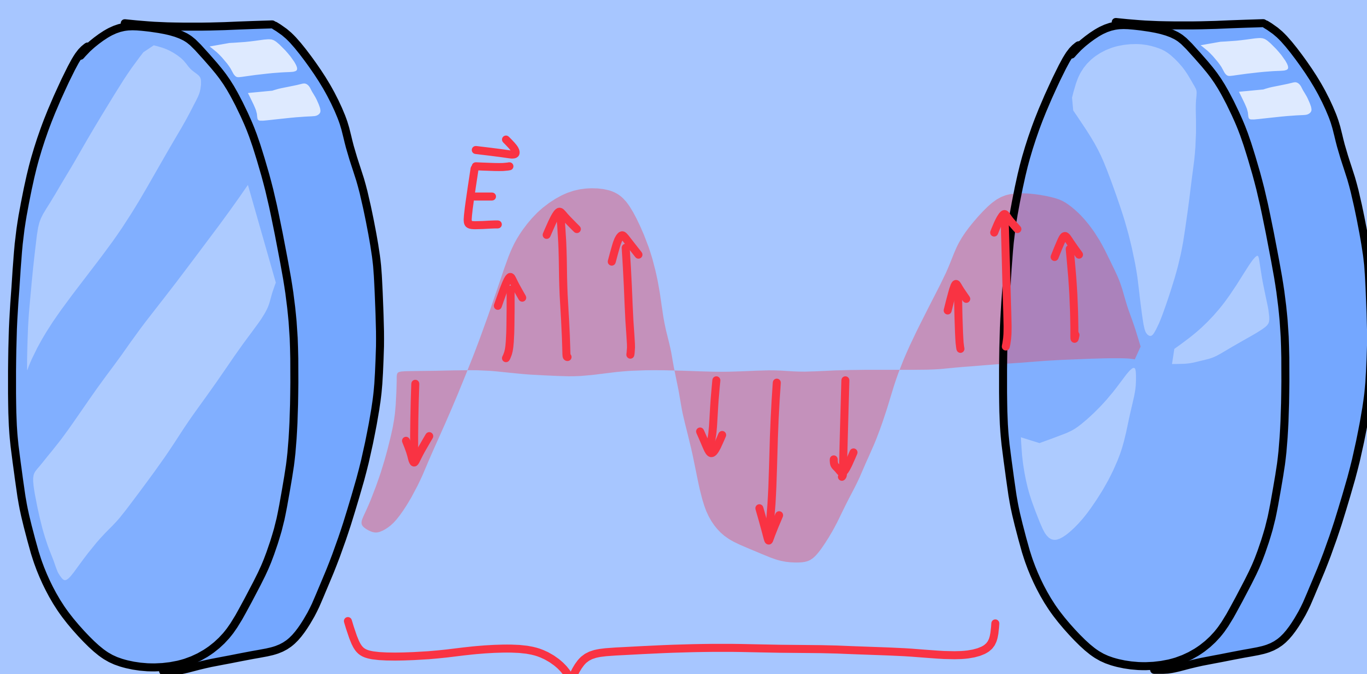
SCATTERING DEPENDS ON SPIN

EXAMPLE:

FERMIONS, SHORT-RANGE INTERACTION

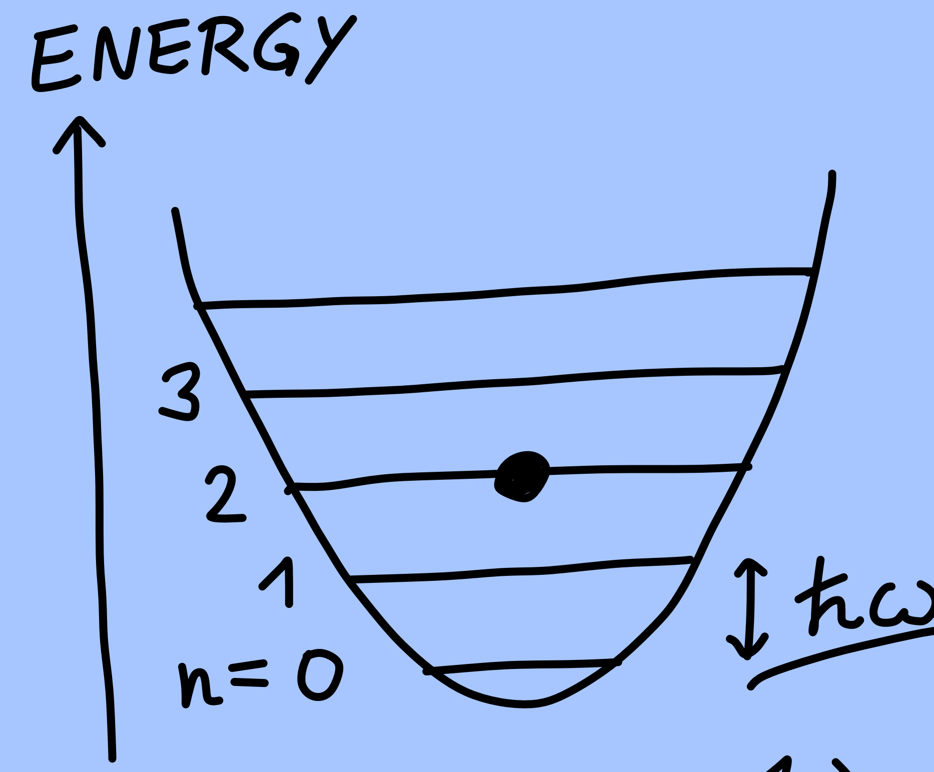
$\Rightarrow$ SINGLET SCATTERING DOMINATES!

EXCURSION : PHOTONS



CAVITY MODE

≡ HARMONIC OSCILLATOR

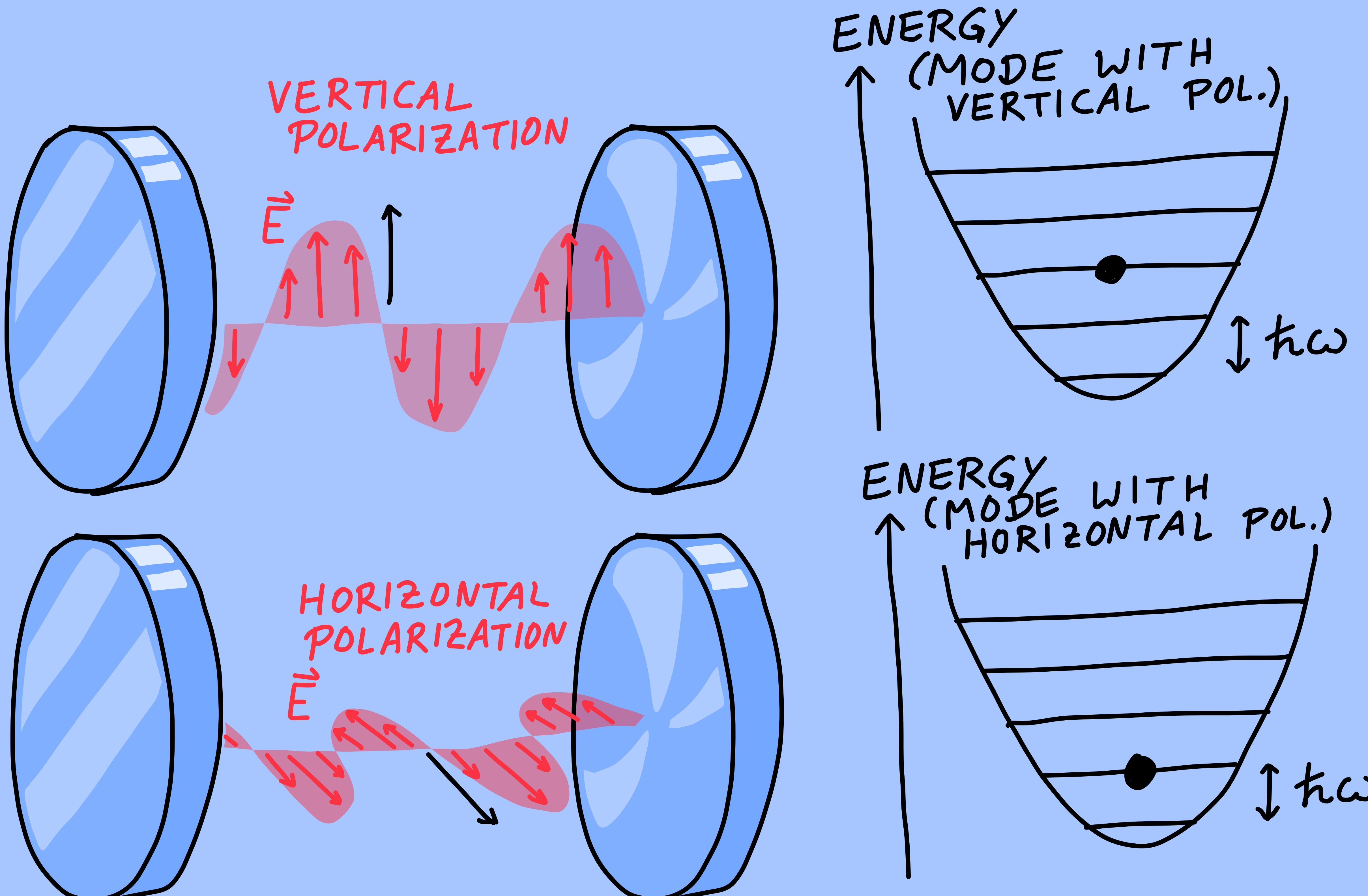


$$E_n = \hbar\omega\left(n + \frac{1}{2}\right)$$

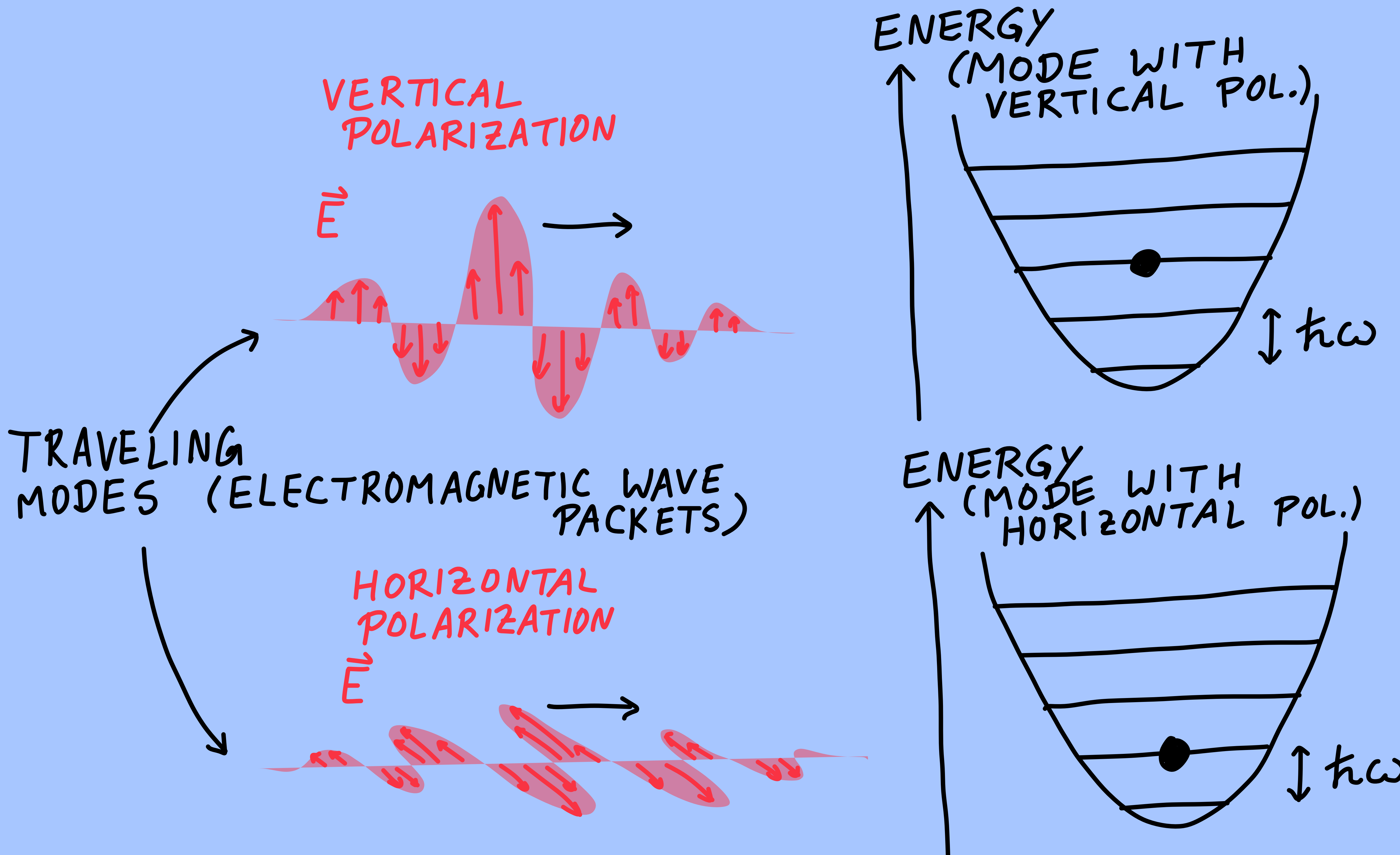
"NUMBER OF PHOTONS"

= NUMBER OF EXCITATIONS IN THE MODE

EXCURSION : PHOTONS



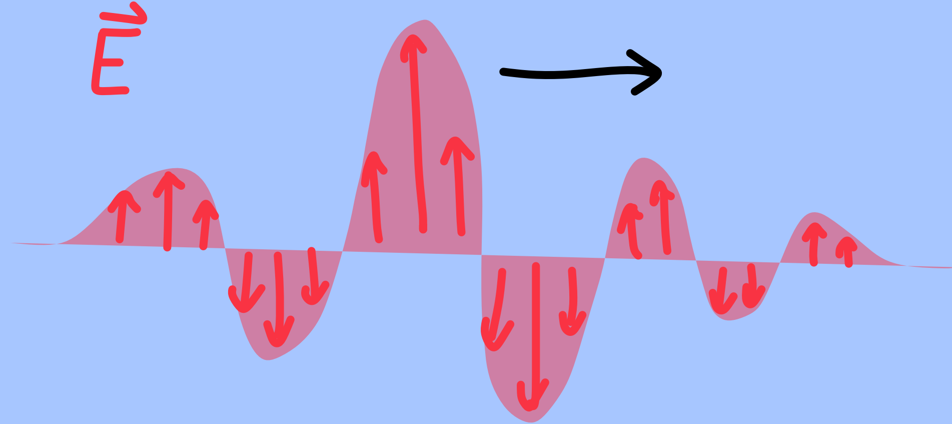
EXCURSION : PHOTONS



EXCURSION : PHOTONS

VERTICAL POLARIZATION

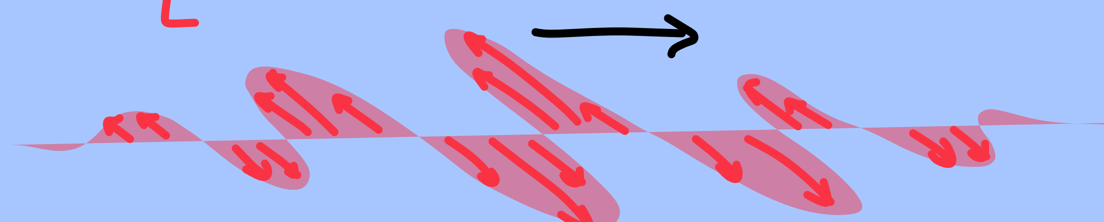
$\vec{E}$



TRAVELING MODES (ELECTROMAGNETIC WAVE PACKETS)

HORIZONTAL POLARIZATION

$\vec{E}$



PHOTON MODES:

$|V\rangle$

→ CAN BE OCCUPIED WITH SEVERAL PHOTONS

→ OMITTED HERE: SPATIAL MODE (SHAPE & LOCATION)

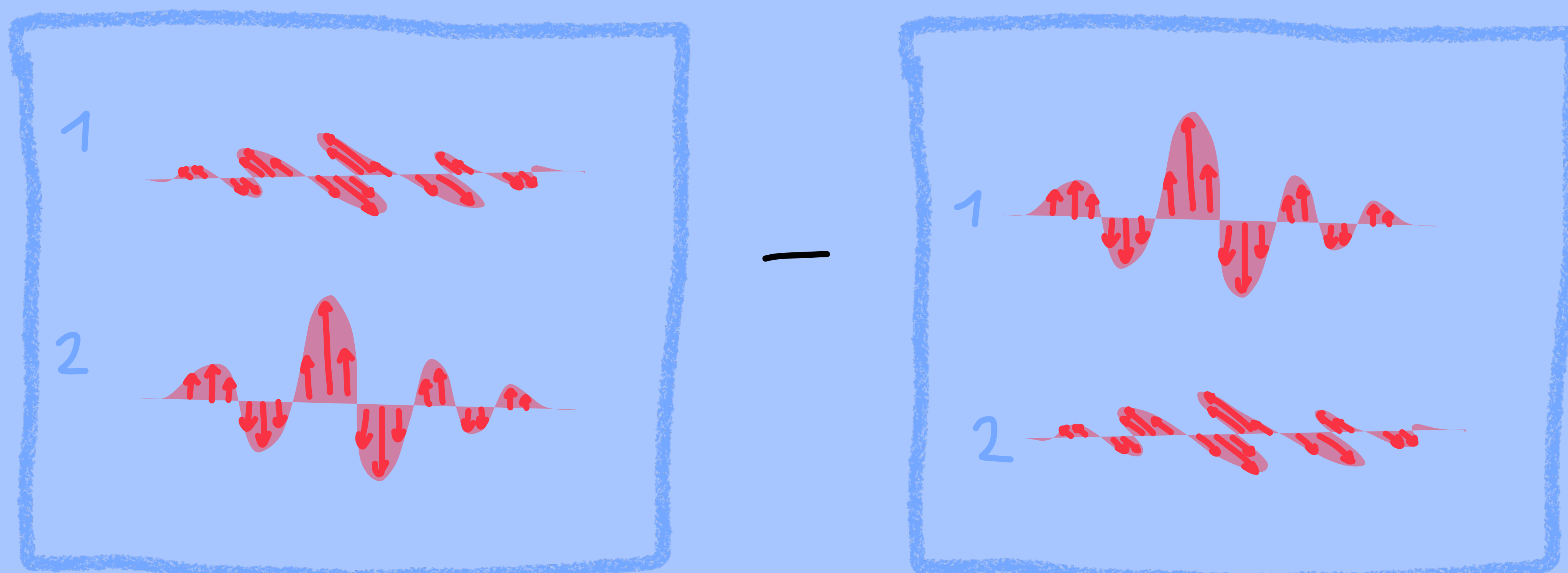
$|H\rangle$

EXCURSION : PHOTONS

PHOTONIC SINGLET STATE:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(\overbrace{|H\rangle_1}^{\text{ONE PHOTON IN THIS MODE}} \otimes |V\rangle_2 - |V\rangle_1 \otimes |H\rangle_2\Big)$$

SUBSCRIPT INDICATES SPATIAL MODE

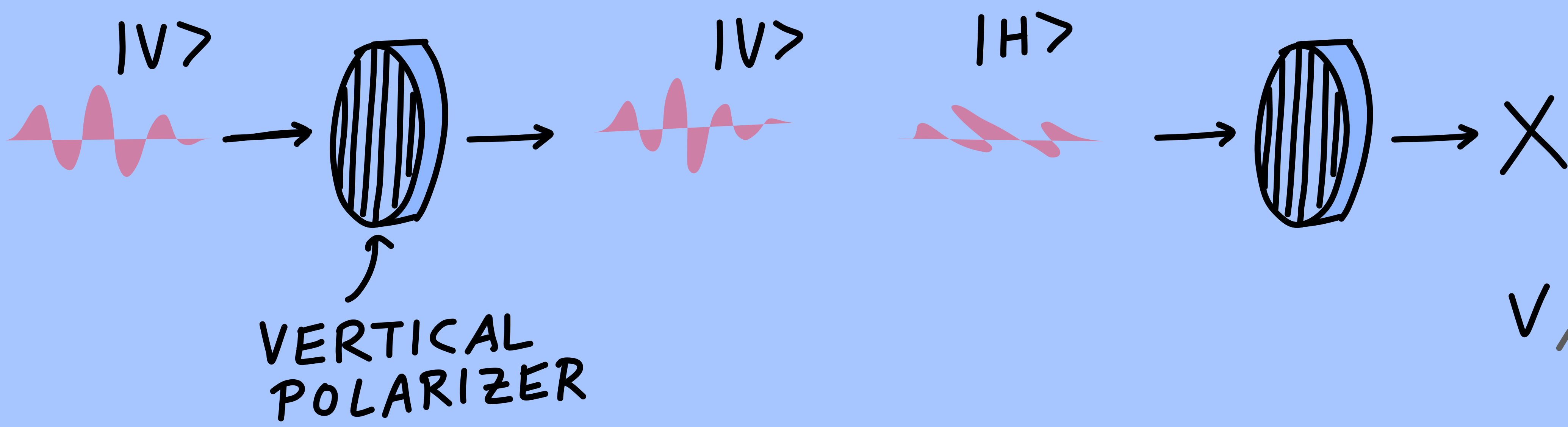


FOUR MODES INVOLVED:

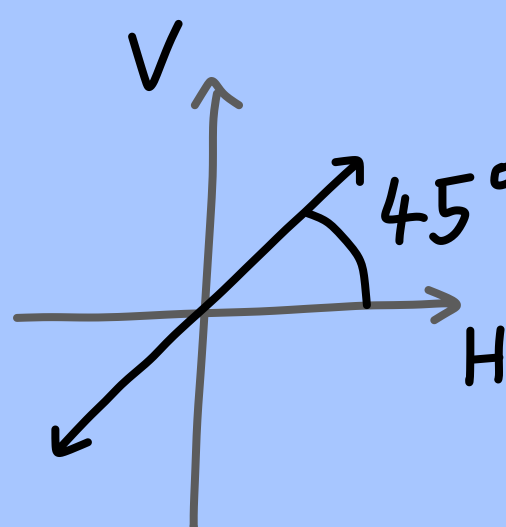
THIS CONFIGURATION:
$|H\rangle_1 \otimes |V\rangle_2$

EXCURSION: PHOTONS

MEASUREMENT VIA POLARIZER



OTHER ANGLES: $\frac{1}{\sqrt{2}}\left(|H\rangle + |V\rangle\right)$

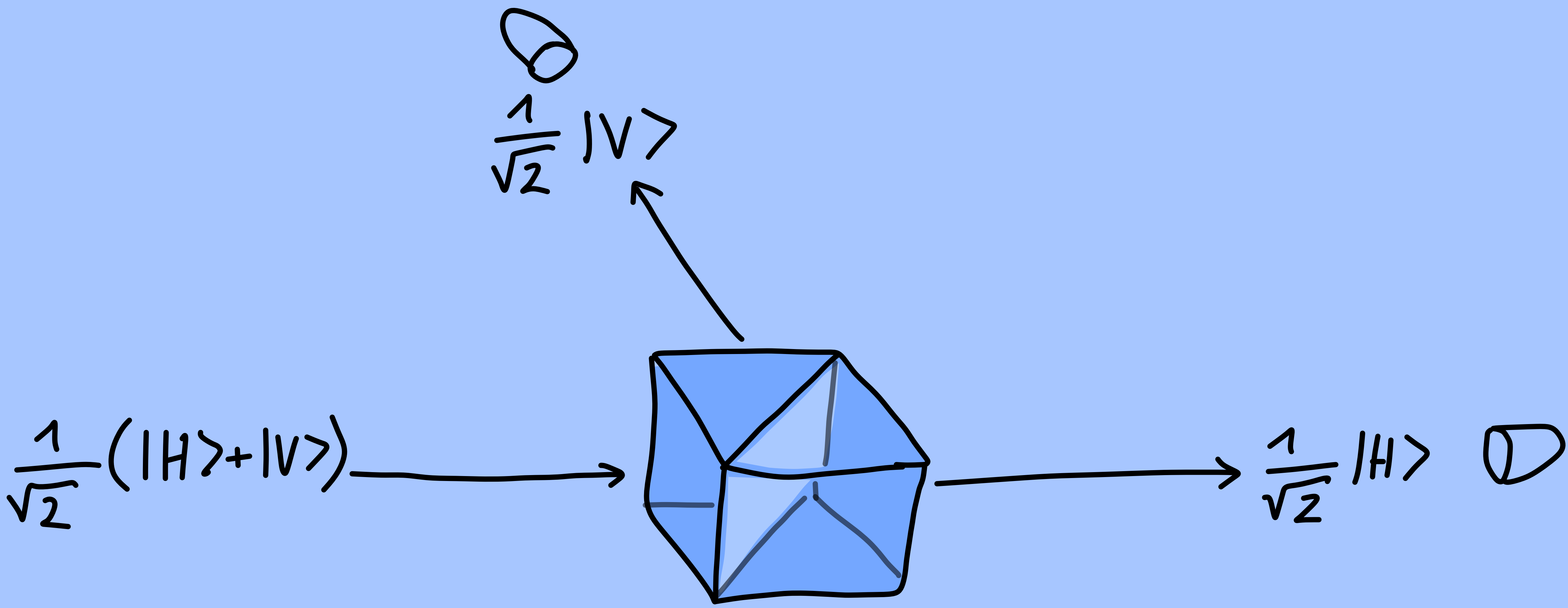


$\Rightarrow$ TRANSMIT ONLY $\frac{1}{\sqrt{2}}|V\rangle$

$\Rightarrow$ 50% TRANSMISSION

EXCURSION : PHOTONS

POLARIZING BEAMSPLITTER:



EXCURSION : PHOTONS

SPIN 1/2

$|\uparrow\rangle_z \qquad |\downarrow\rangle_z$

$$|\uparrow\rangle_x = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_z + |\downarrow\rangle_z\right)$$

$$|\downarrow\rangle_x = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_z - |\downarrow\rangle_z\right)$$

$$|\uparrow\rangle_y = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_z + \underline{i}\,|\downarrow\rangle_z\right)$$

$$|\downarrow\rangle_y = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_z - i\,|\downarrow\rangle_z\right)$$

PHOTON

$\underline{|H\rangle} \qquad \underline{|V\rangle}$

$$|\swarrow\!\!\!\nearrow\rangle = \frac{1}{\sqrt{2}}\left(\underline{|H\rangle + |V\rangle}\right)$$

$$|\nwarrow\!\!\!\searrow\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - |V\rangle\right)$$

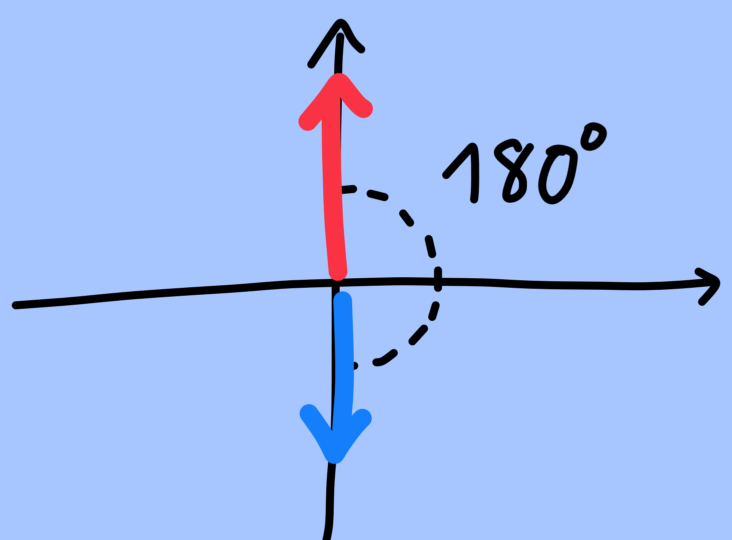
$$|\underline{\circlearrowleft}\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + i\,|V\rangle\right)$$

$$|\circlearrowright\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - i\,|V\rangle\right)$$

EXCURSION : PHOTONS

SPIN 1/2

$|\uparrow\rangle_z \qquad |\downarrow\rangle_z$



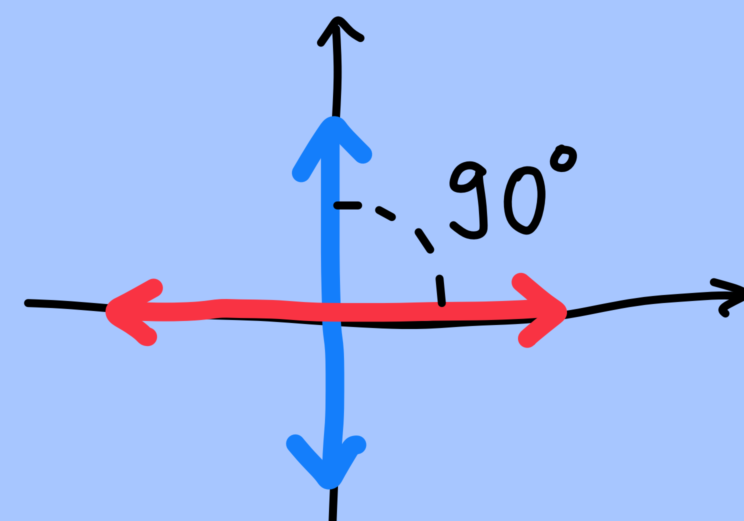
$$\langle AB\rangle_{QM} = -\cos\measuredangle(\vec{a},\vec{b})$$

FOR

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

PHOTON

$|H\rangle \qquad |V\rangle$

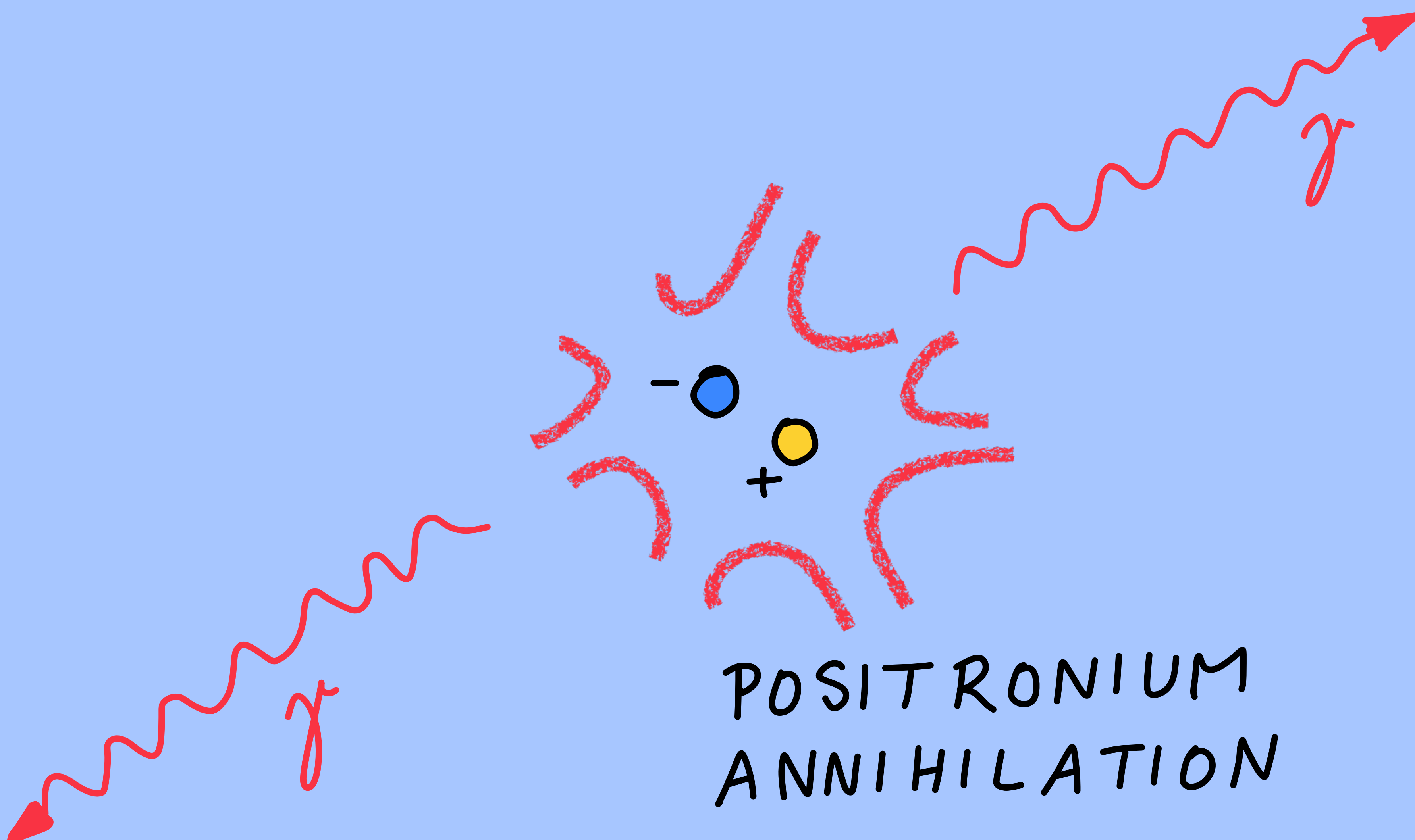


$$\langle AB\rangle_{QM} = -\cos 2\measuredangle(\vec{a},\vec{b})$$

FOR

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$

PHOTONS FROM ANNIHILATION

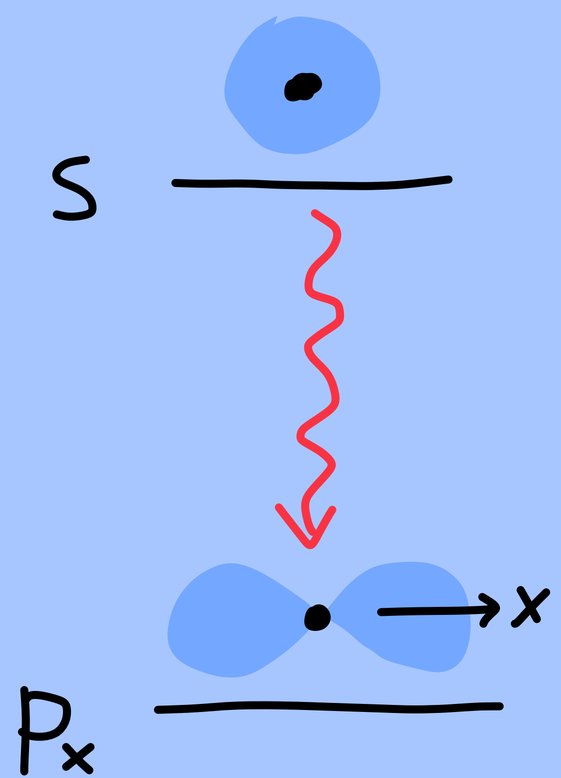


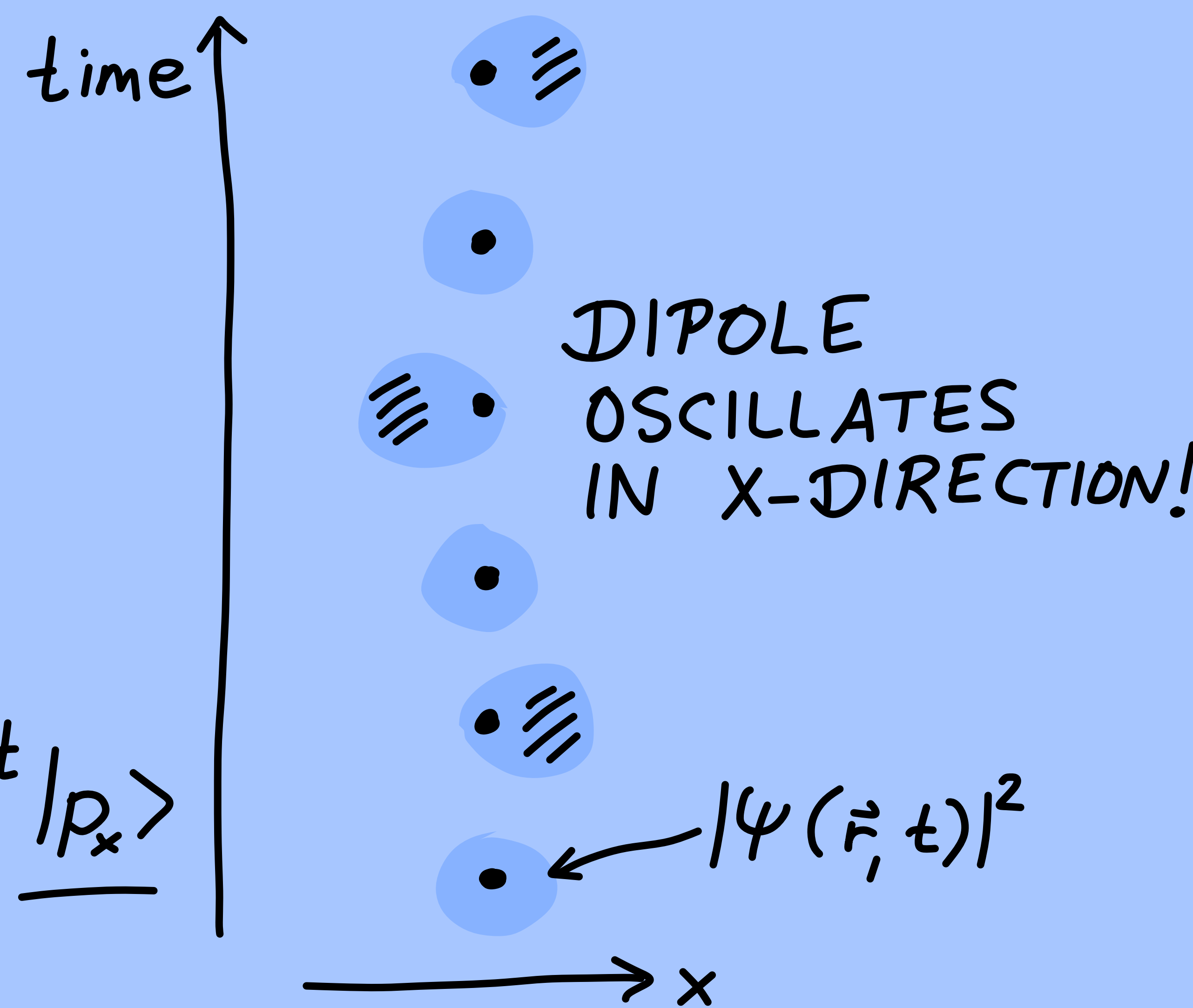
POSITRONIUM
ANNIHILATION

(WU & SHAKNOV 1950)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$

EXCURSION: TRANSITION DIPOLE MOMENT



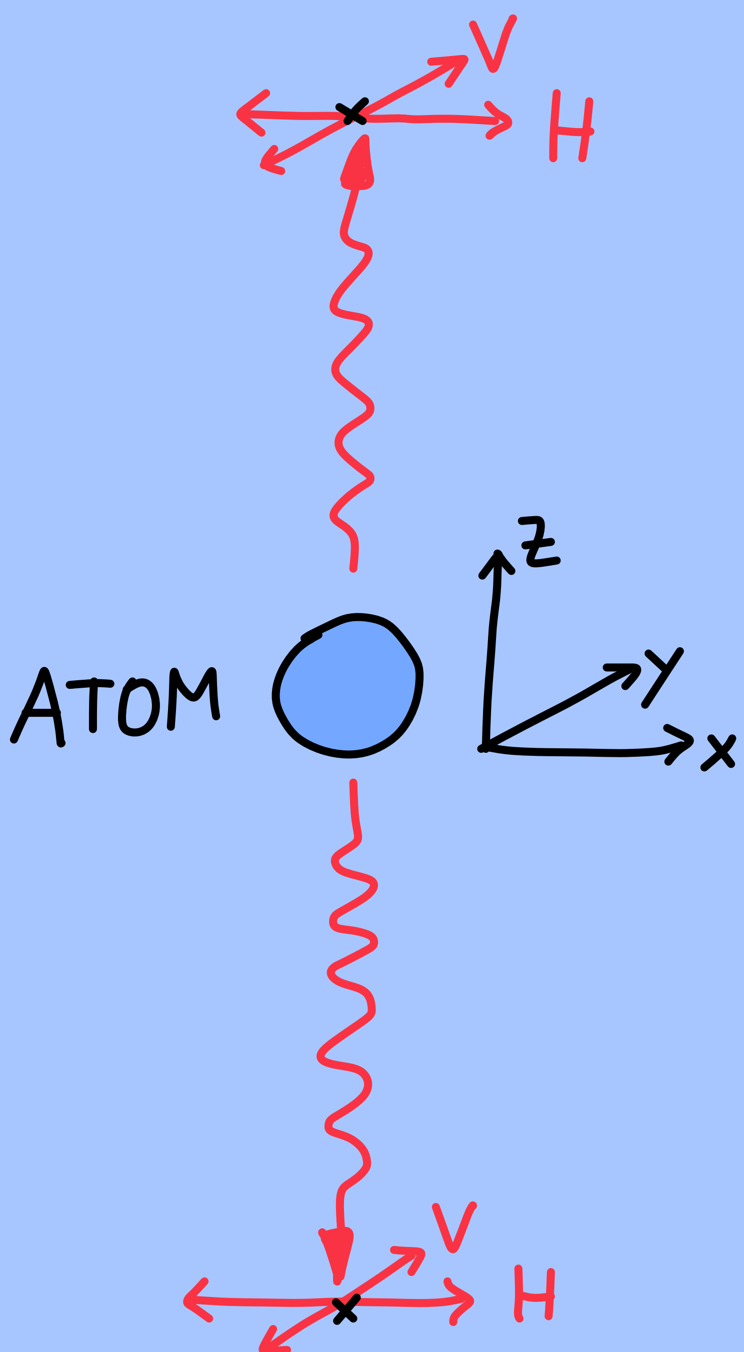


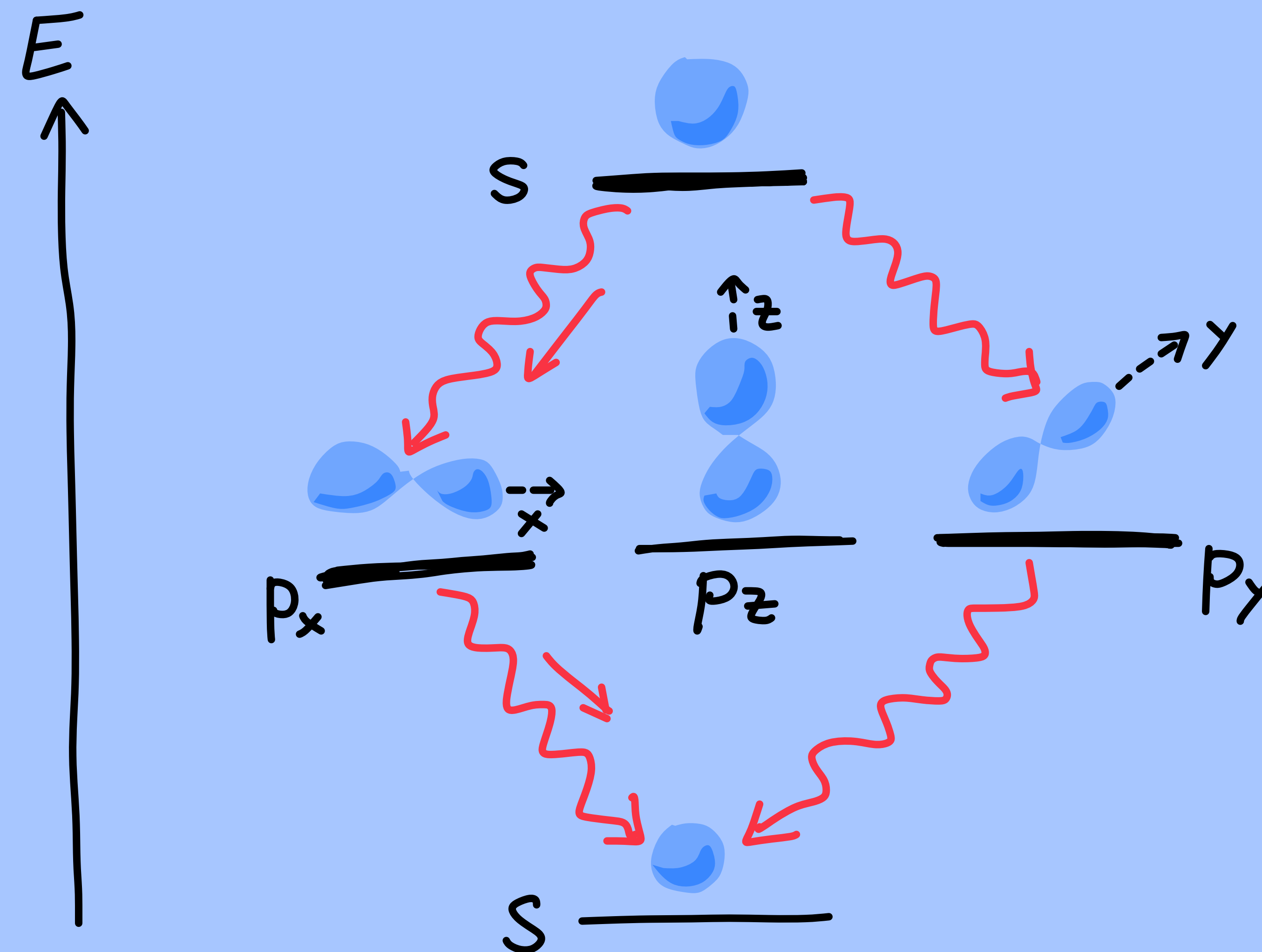
SUPERPOSITION:

$$|\psi\rangle = \underline{\alpha}\, e^{-\frac{i}{\hbar}E_s t}\,\underline{|s\rangle} + \underline{\beta}\, e^{-\frac{i}{\hbar}E_p t}\,\underline{|p_x\rangle}$$

FORMALLY: $\langle \underline{\underline{p_x}} | \underbrace{q\hat{\vec{r}}}_{\text{DIPOLE OPERATOR}} | \underline{\underline{s}} \rangle \sim \vec{e}_x$

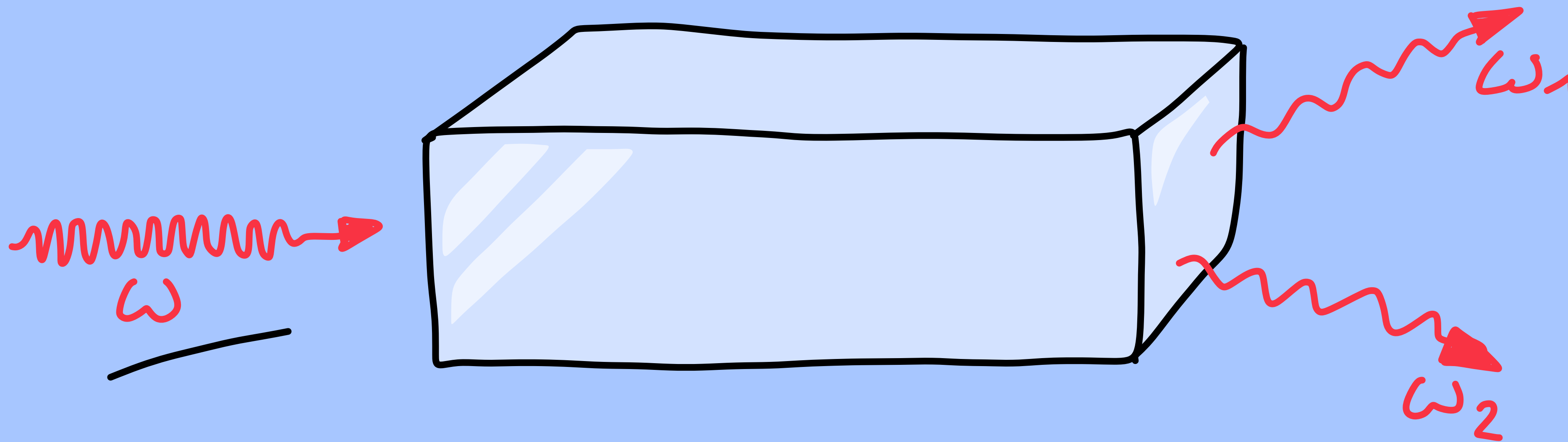
ATOMIC CASCADE





$$\underbrace{|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\underset{x}{H}\,\underset{x}{H}\rangle + |\underset{y}{V}\,\underset{y}{V}\rangle\right)}$$

PARAMETRIC DOWN-CONVERSION

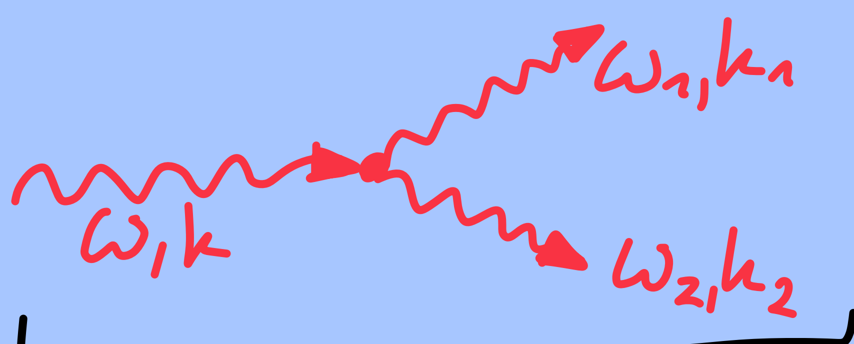


NONLINEAR OPTICAL MEDIUM:

$$\underbrace{P}_{\text{POLARIZATION}} = \ldots \cdot \underline{E} + \ldots \cdot \underline{\underline{E^2}} + \ldots \cdot \underline{\underline{E^3}} + \ldots$$

HAMILTONIAN:

$$\hat{H} = \sum_k \hbar \omega_k \hat{a}_k^+ \hat{a}_k + \sum_{k, k_1} \underbrace{\hbar g_{k,k_1}}_{\sim \chi^{(2)}} \hat{a}_{k_1}^+ \hat{a}_{k_2 = k - k_1}^+ \hat{a}_k + \ldots$$



ENERGY CONSERVATION

$$\omega = \omega_1 + \omega_2$$

MOMENTUM CONSERVATION

$$k = k_1 + k_2$$

PARAMETRIC DOWN-CONVERSION

POLARIZATION STRUCTURE:

$$|HH\rangle + |VV\rangle$$

OR

$$|HV\rangle + |VH\rangle$$

("TYPE II PDC")

ENTANGLEMENT ALSO IN:

FREQUENCY

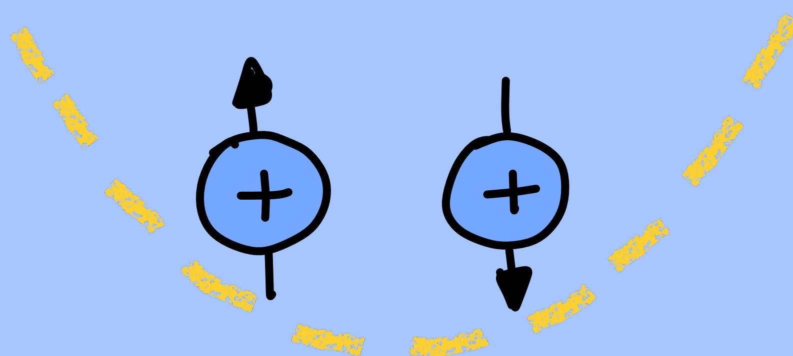
DIRECTION

TOTAL STATE:

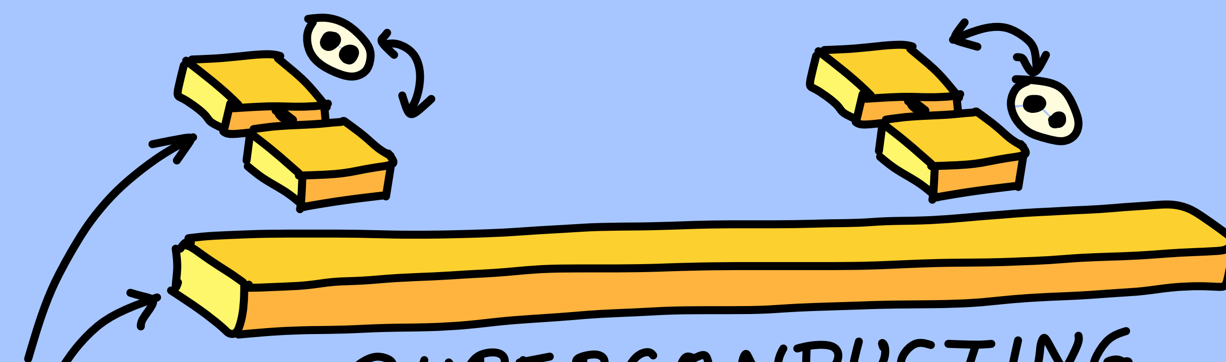
$$|\psi\rangle = \sum_{\vec{k}_1, \vec{k}_2, \vec{\zeta}_1, \vec{\zeta}_2} \psi_{\vec{k}_1, \vec{k}_2, \vec{\zeta}_1, \vec{\zeta}_2} \, |\underbrace{\vec{k}_1}_{\text{FREQ. + DIR.}}, \underbrace{\vec{\zeta}_1}_{\text{POLARIZATION}}\rangle \otimes |\vec{k}_2, \vec{\zeta}_2\rangle$$

QUBIT PLATFORMS

QUBITS = TWO-LEVEL SYSTEMS WITH ARBITRARY CONTROL (INCLUDING INTERACTIONS)



TRAPPED IONS



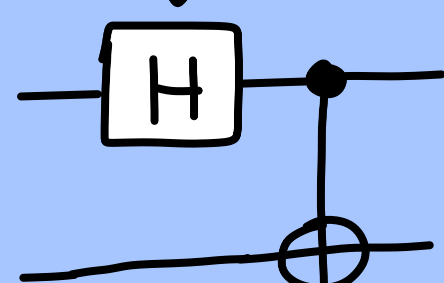
METAL

SUPERCONDUCTING QUBITS

HADAMARD GATE : $|0\rangle \mapsto \frac{1}{\sqrt{2}}\underline{(|0\rangle + |1\rangle)}$

QUBIT 1: $\underline{|0\rangle}$

QUBIT 2: $\underline{|0\rangle}$



FINAL STATE:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\underline{(|0\underline{0}\rangle + |1\underline{1}\rangle)}$$

"QUANTUM CIRCUIT"

$\underline{\text{CNOT GATE}}$:

$|00\rangle \mapsto |00\rangle$

$|\underline{1}0\rangle \mapsto |1\underline{1}\rangle$

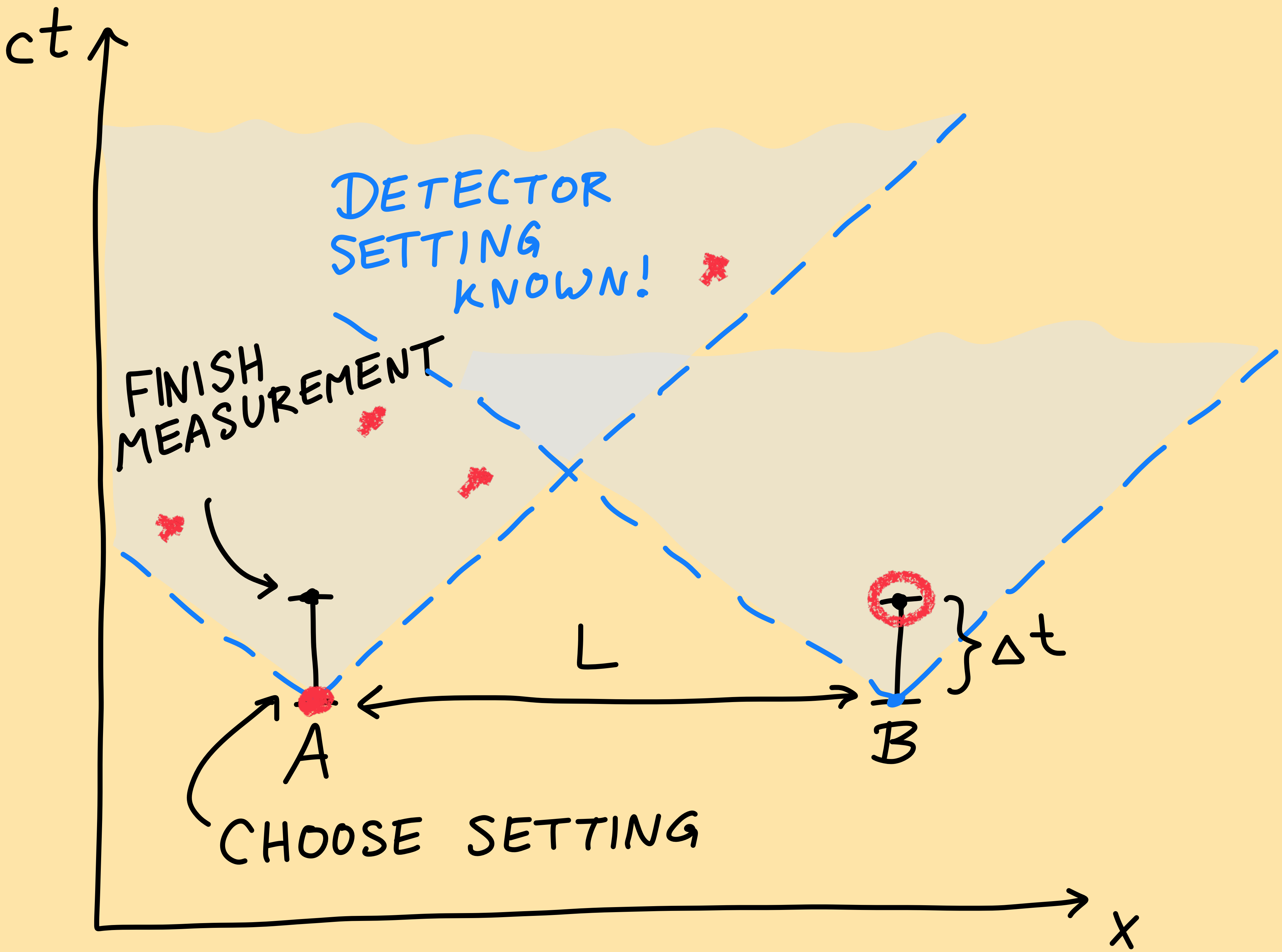
Lecture 7

Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

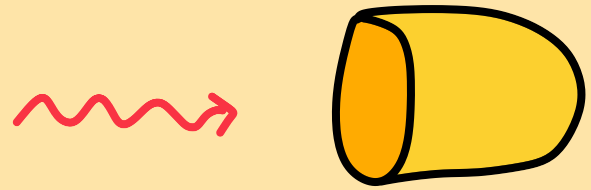
Start at 6pm CET

LOCALITY



$$L > c\Delta t$$

DETECTION



DETECTOR EFFICIENCY η

$$\langle AB\rangle_{\text{OBSERVED}} = \eta_A \eta_B \langle AB\rangle_{\text{IDEAL}}$$

0 IF NO DET.

FOR $\eta_A \eta_B < \frac{1}{\sqrt{2}}$

$$\Longrightarrow \; lhs_{\text{CHHS}}\Big|_{\text{OBSERVED}} \leq 2$$

RATE

$$\underbrace{R(A=+1,\, B=-1)}_{\text{OBSERVED}} = \underbrace{\eta_A}\underbrace{\eta_B} \cdot P(A=+1, B=-1) \cdot \underbrace{P_{\text{COLLECT}}} \cdot \underbrace{R_{\text{EMIT}}}$$

E.G. FROM LHV (OR QM)

$\Rightarrow$ EXTRACT $P(A=1,\, B=-1)$ ETC.

<u>IF</u> MODEL FOR RATE IS TRUE

"FAIR SAMPLING HYPOTHESIS"
(CHHS 69)

DETECTED PAIRS
ARE FAIR SAMPLE
OF ALL PAIRS
(NO BIAS)

"LOOPHOLES"

NONIDEAL BELL TEST

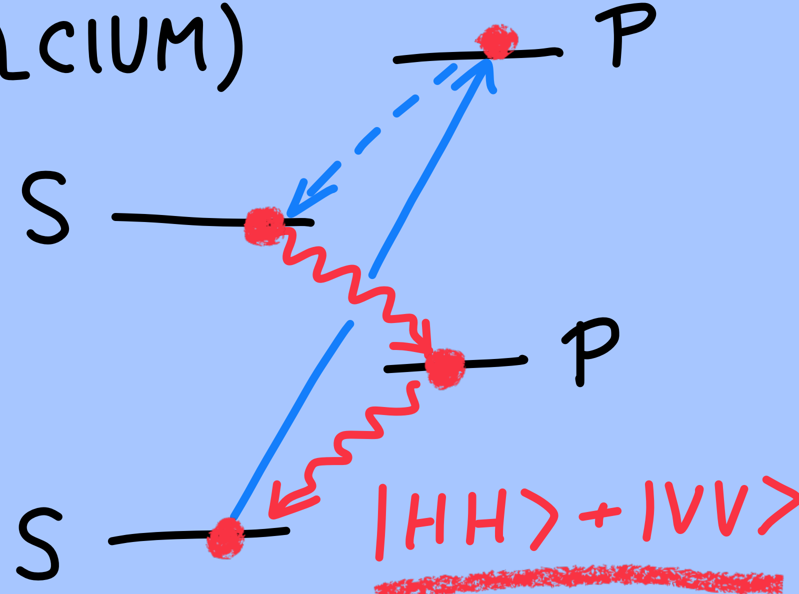
NEED EXTRA ASSUMPTIONS TO INTERPRET RESULTS

"LOOPHOLE":
LOCAL HIDDEN VARIABLE THEORIES COULD STILL BE TRUE!

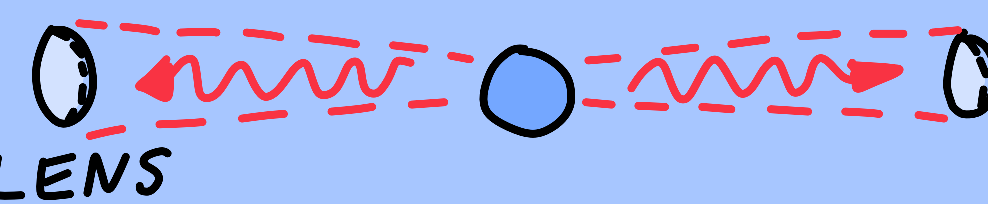
BELL TESTS: HISTORY

- POSITRONIUM ANNIHILATION (1950's & 70's)
- PROTON-PROTON SCATTERING
  (LAMEHI-RACHTIG, MITTIG 1976)
- FREEDMAN & CLAUSER 1972:
  FIRST ATOMIC CASCADE BELLTEST

(CALCIUM)



COLLECT ONLY
BACK-TO-BACK PAIRS



✓ AGREE WITH QM

✓ VIOLATE SUITABLY MODIFIED BELL INEQUALITY

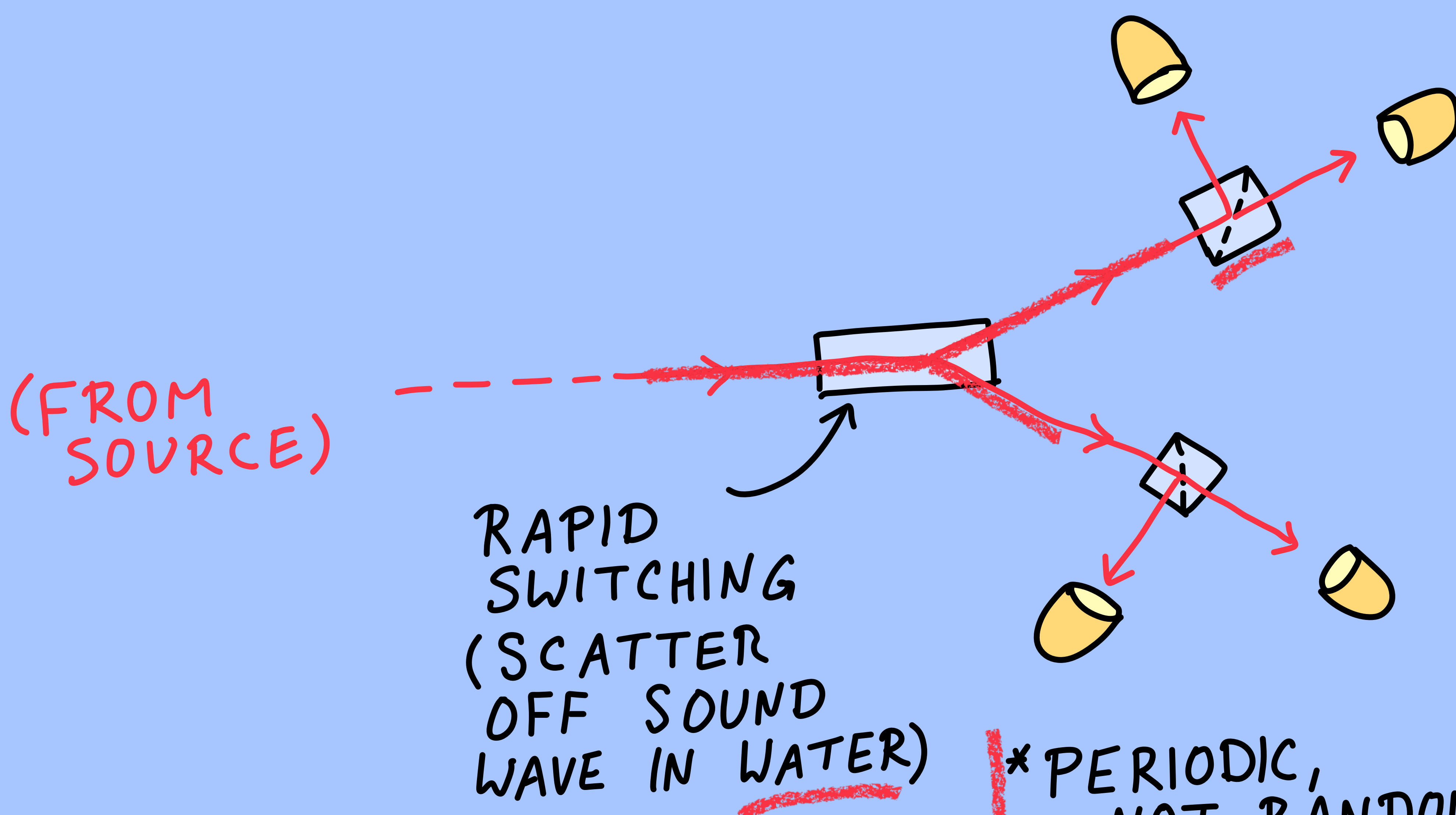
DETECTOR EFFICIENCY $\sim 10^{-3}$
(INCLUDING SOLID ANGLE)

COINCIDENCE RATE 0.1 Hz

DATA FOR 200 hrs

BELL TESTS: HISTORY

- ASPECT ET AL. 1980's
  ALSO ATOMIC CASCADE
  - FIRST TO DETECT <u>BOTH</u> POLARIZATION DIRECTIONS
  - FIRST TO SWITCH RAPIDLY MEASUREMENT DIRECTION → LOCALITY

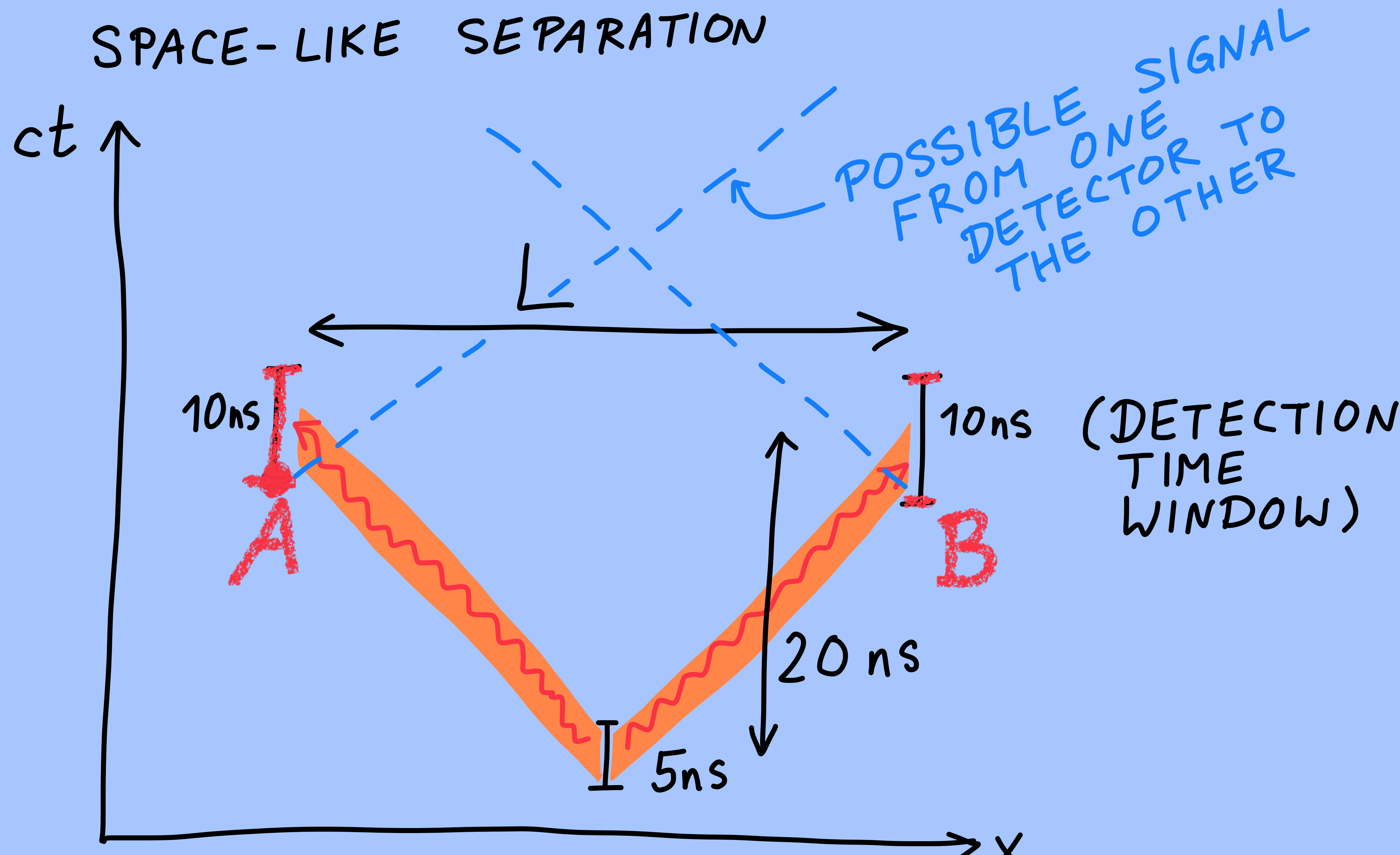


*PERIODIC, NOT RANDOM!

BELL TESTS: HISTORY

ASPECT (CONT'D)

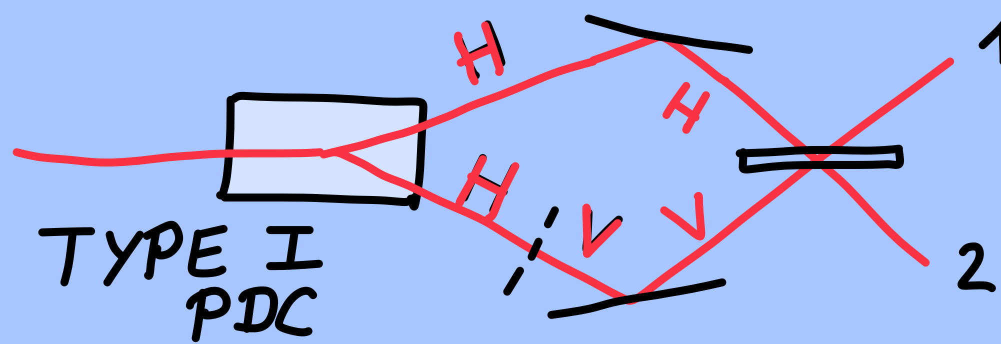
SPACE-LIKE SEPARATION



$L/c = 40\text{ns} > 10\text{ns}$

BELL TESTS: HISTORY

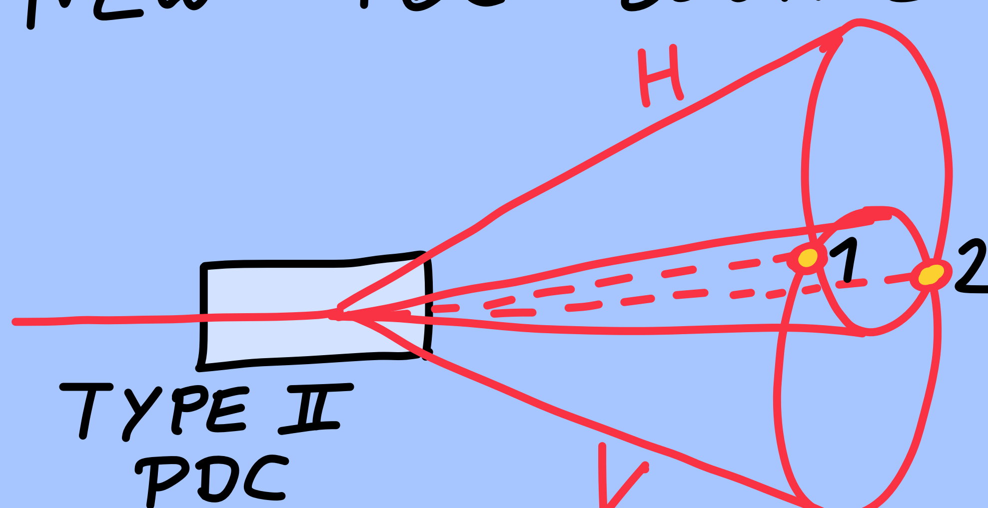
- FIRST PARAMETRIC DOWN-CONVERSION EXPERIMENTS
  1980s ALLEY & SHIH, HONG, OU, MANDEL



$$|\psi\rangle = \left(r\,|H\rangle_1 + t\,|H\rangle_2\right) \cdot \left(r\,|V\rangle_2 + t\,|V\rangle_1\right)$$

$$= \ldots + \underbrace{r^2\,|H_1V_2\rangle + t^2\,|V_1H_2\rangle}_{\text{POSTSELECT ONLY THESE}}$$

- 1995 KWIAT ET AL. (ZEILINGER GROUP)
  NEW PDC SOURCE



ALONG DIRECTIONS 1 & 2:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|H_1V_2\rangle + e^{i\alpha}|V_1H_2\rangle\right)$$

WITHOUT POSTSELECTION

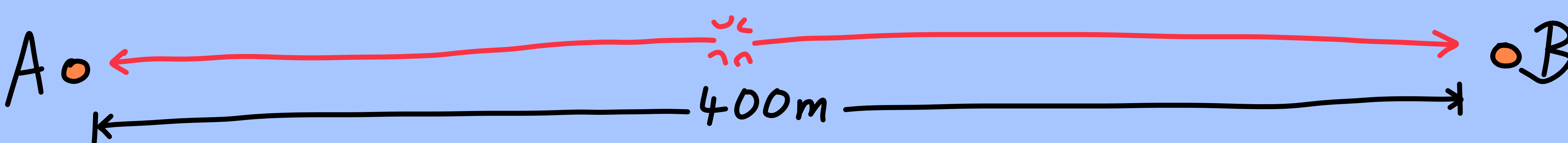
BELL TESTS: HISTORY

- 1998 IDEAL NONLOCALITY CONDITIONS
  WEIHS ET AL. (ZEILINGER GROUP)

SPACE-LIKE SEPARATION

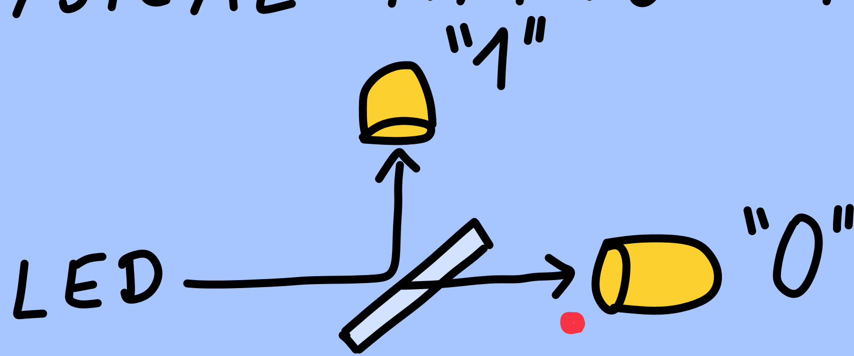
AND

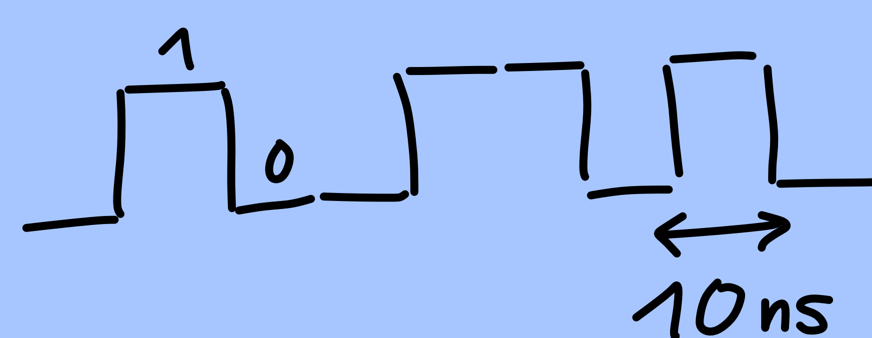
INDEPENDENT RANDOM SETTINGS



$L/c \approx 1.3\,\mu s$

PHYSICAL RANDOM NUMBER GENERATOR:





BELL TESTS: HISTORY

- 1998 IDEAL NONLOCALITY CONDITIONS
  WEIHS ET AL. (ZEILINGER GROUP)

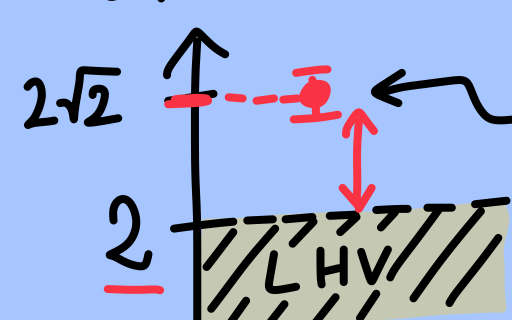
  TOTAL MEASUREMENT TIME ≤ 100ns

  DETECTION & COLLECTION EFFICIENCY: 5% !

  DETECTION RATE: 10 kHz
  DARK COUNT RATE: FEW 100 Hz

  TIME-TAG EVENTS USING INDEPENDENT ATOMIC CLOCKS
  ↳ COMPARE LATER
  ↳ VIOLATE "MODIFIED BELL INEQUALITY BY 30 STANDARD DEVIATIONS"

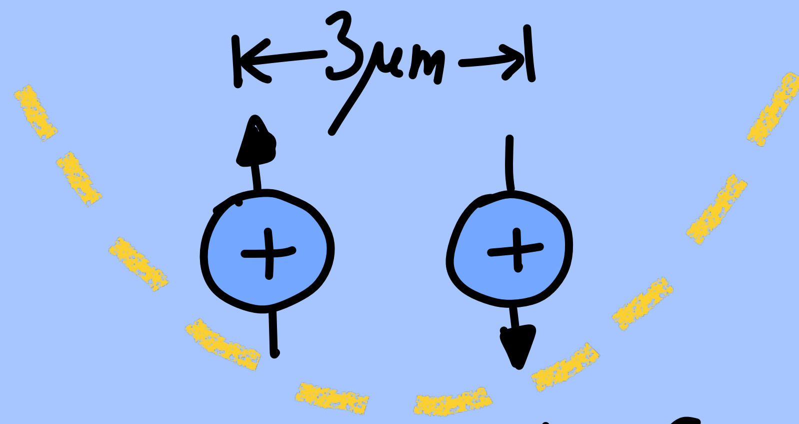
✓ COMPLETELY CLOSE LOCALITY LOOPHOLE
(BUT: DETECTION!)



VALUE ON LEFT-HAND-SIDE OF BELL INEQUALITY

BELL TESTS: HISTORY

- 2001 ION TRAP BELL EXPERIMENT (WINELAND GROUP)

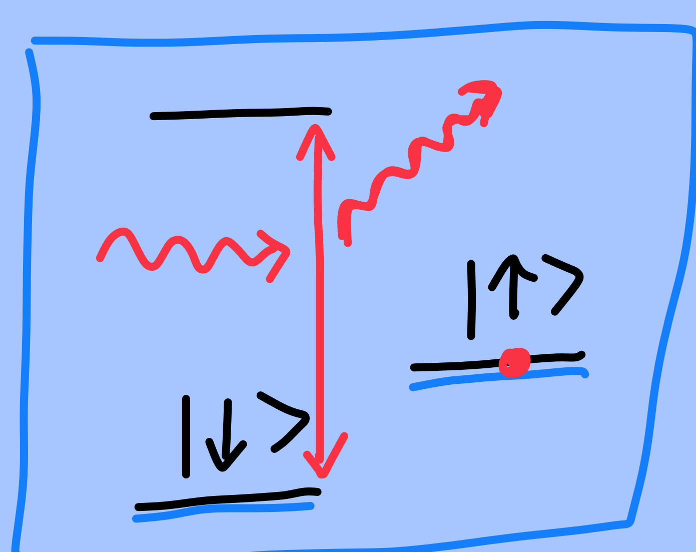


TRAPPED IONS

ENTANGLEMENT VIA

COULOMB INTERACTION
+
SPIN/MOTION CONTROL BY LASER FIELDS

MEASUREMENT VIA "RESONANCE FLUORESCENCE"



$|\downarrow\rangle \Rightarrow$ 60 PHOTONS DETECTED

$|\uparrow\rangle \Rightarrow$ DARK

✓ **DETECTION LOOPHOLE CLOSED**

(BUT: LOCALITY!)

BELL TESTS: HISTORY

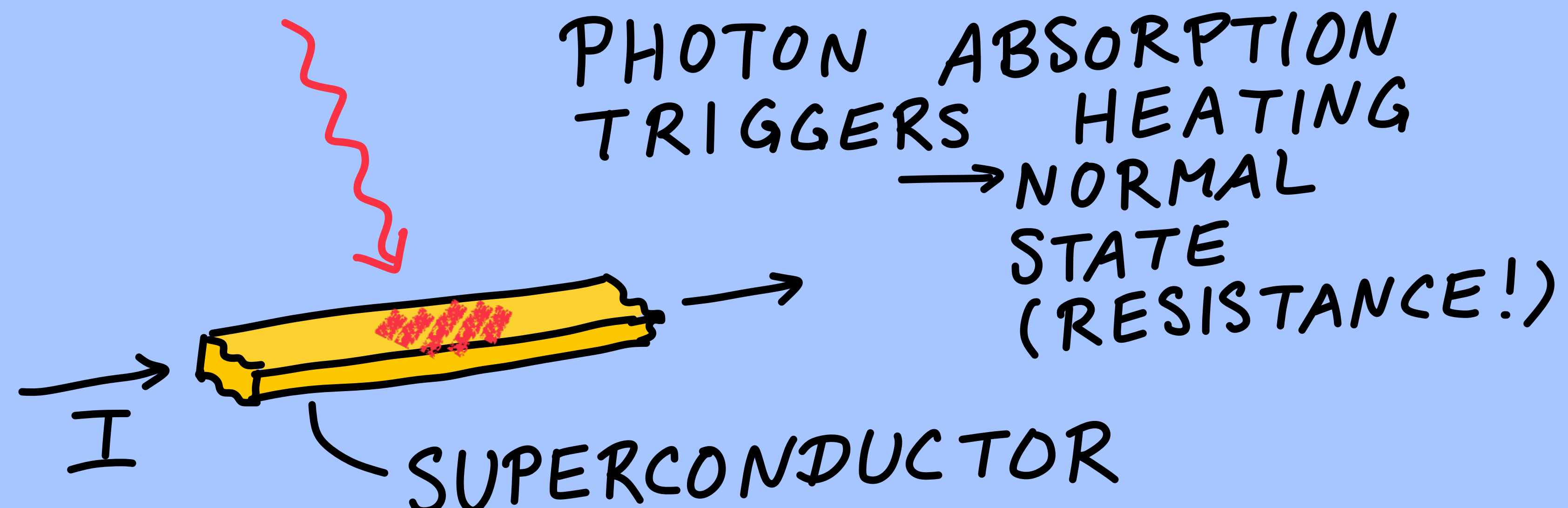
GOAL: CLOSE DETECTION <u>AND</u> LOCALITY LOOPHOLE SIMULTANEOUSLY

BELL TESTS: HISTORY

GOAL: CLOSE DETECTION AND LOCALITY LOOPHOLE SIMULTANEOUSLY

IMPORTANT TECHNICAL DEVELOPMENT:

HIGHLY EFFICIENT PHOTON DETECTORS



DETECTION EFFICIENCY >90% !

BELL TESTS: HISTORY

ENTANGLED PDC PHOTONS
+ SUPERCONDUCTING DETECTORS

DEC' 2015

NIST/BOULDER TEAM (KWIAT, NAM, ...)

TOTAL DETECTION EFFICIENCY ~75%

180 m DISTANCE

RANDOMNESS FROM LASER FLUCTUATIONS, PHOTONS, AND MOVIES/TV SHOWS/ DIGITS OF π

VIENNA TEAM (ZEILINGER ET AL)

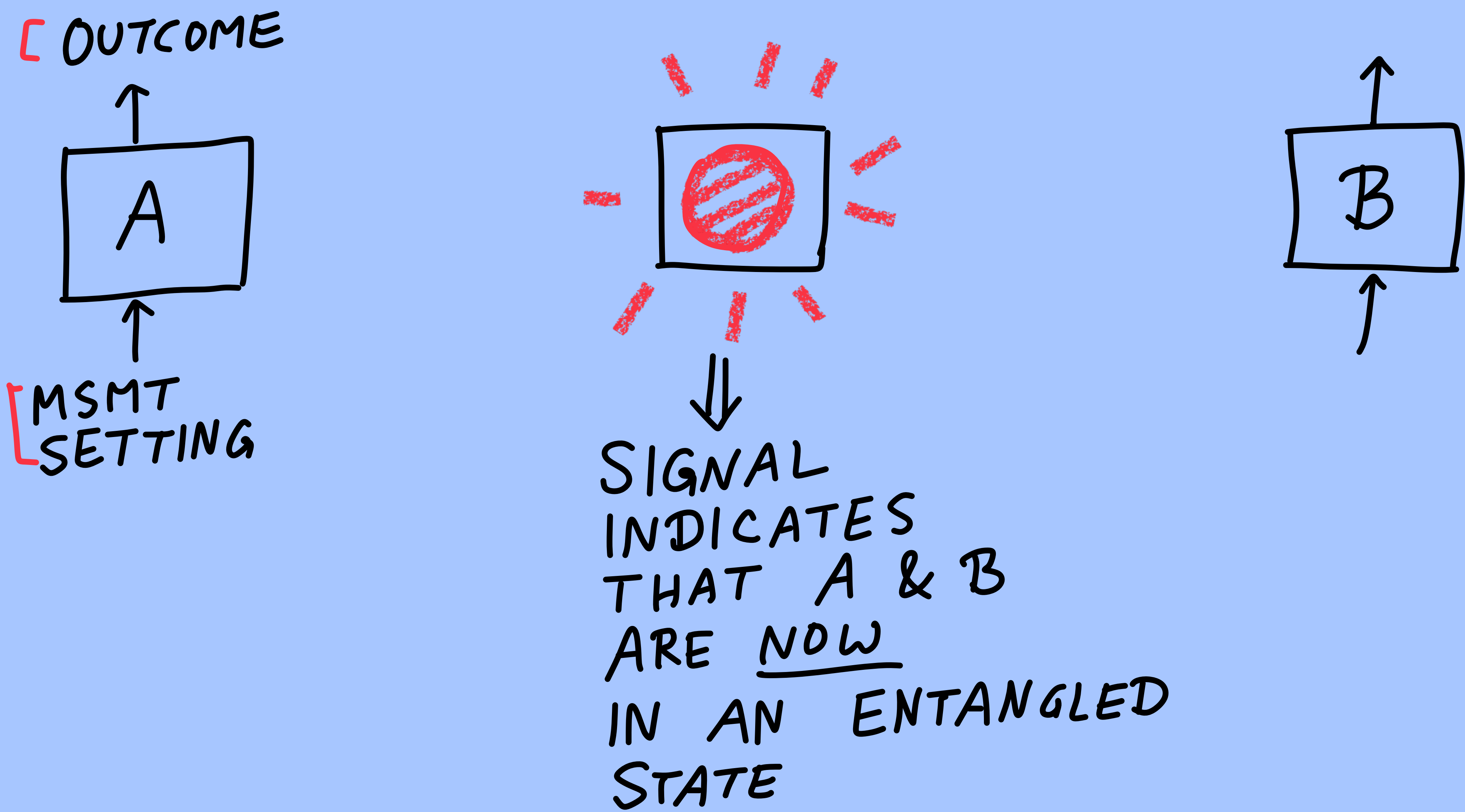
60 m DISTANCE

PROB. (OBS. | LHV) $\lesssim 4 \cdot 10^{-31}$

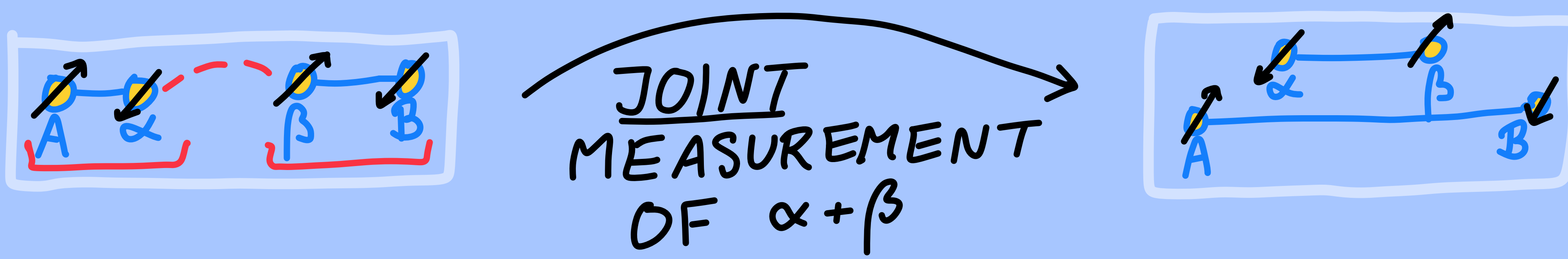
✓ DETECTION LOOPHOLE CLOSED

✓ LOCALITY LOOPHOLE CLOSED

PRINCIPLE: "EVENT-READY DETECTION"



EXCURSION: ENTANGLEMENT SWAPPING



$$|\Psi\rangle = |\Psi_{A\alpha}\rangle \otimes |\Psi_{B\beta}\rangle$$

$$|\Psi_{A\alpha}\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_A \downarrow_\alpha\rangle - |\downarrow_A \uparrow_\alpha\rangle \right) \quad \text{etc.}$$

USE <u>JOINT</u> MEASUREMENT OF α & β

$\Rightarrow$ E.G. DETECTING THEM IN SINGLET STATE IMPLIES:

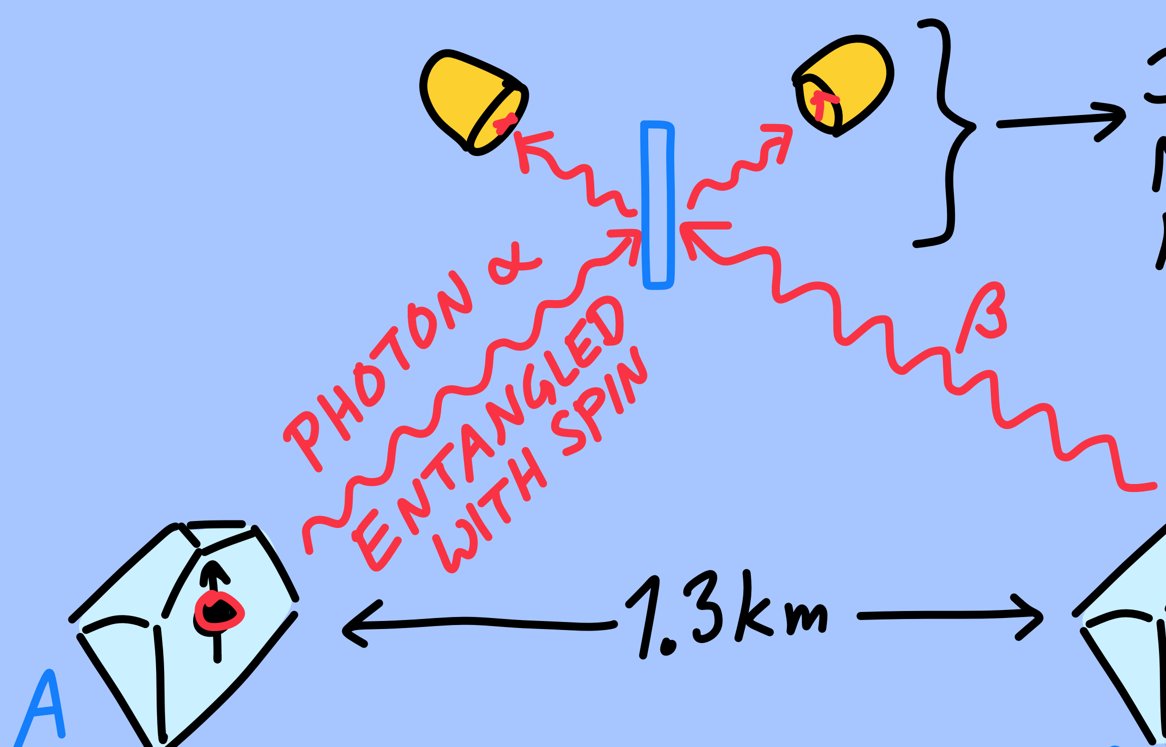
$$|\Psi^{AFTER}\rangle \sim |\Psi^{SINGLET}_{\alpha\beta}\rangle\langle\Psi^{SINGLET}_{\alpha\beta}|\Psi\rangle$$

$$= \dots = |\Psi^{SINGLET}_{\alpha\beta}\rangle \otimes |\Psi^{SINGLET}_{AB}\rangle$$

BELL TESTS: HISTORY

SPIN-PHOTON ENTANGLEMENT & "EVENT-READY DETECTION"

HANSON GROUP, DELFT (OCT' 2015)



DETECTION HERE MEANS: SPINS ARE IN SINGLET-STATE
⇒ MEASURE THEM!
(USING MICROWAVES AND RESONANCE FLUORESCENCE)

NV CENTRE IN DIAMOND

1 EVENT PER HOUR (!)

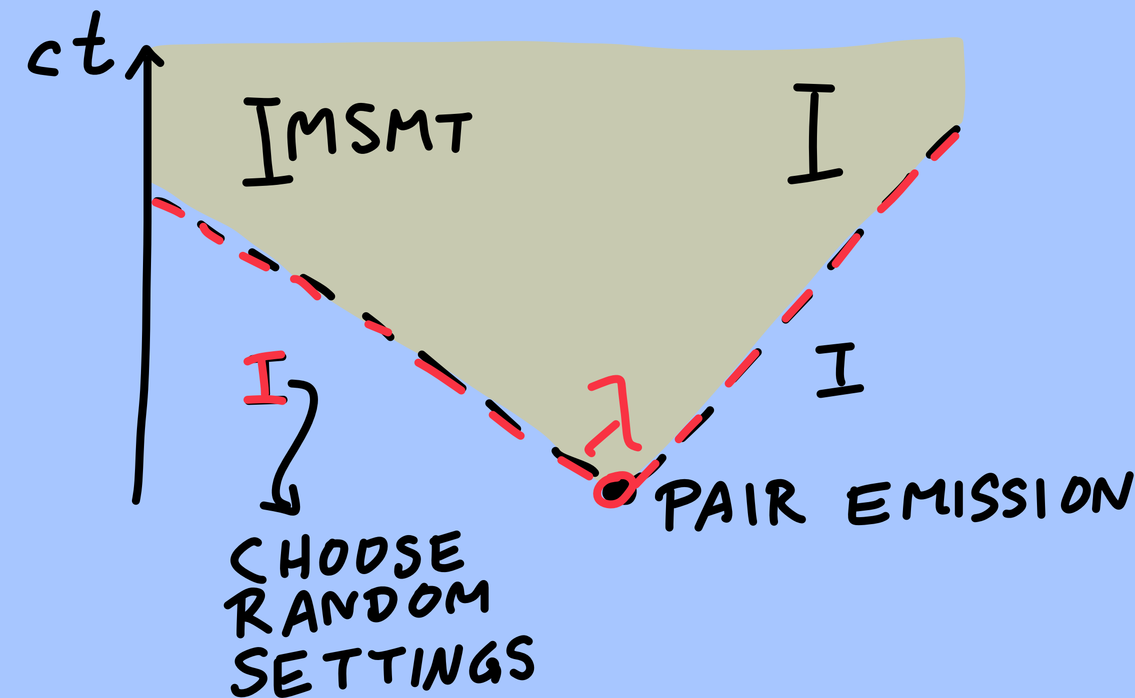
✓ **DETECTION LOOPHOLE CLOSED**

✓ **LOCALITY LOOPHOLE CLOSED**

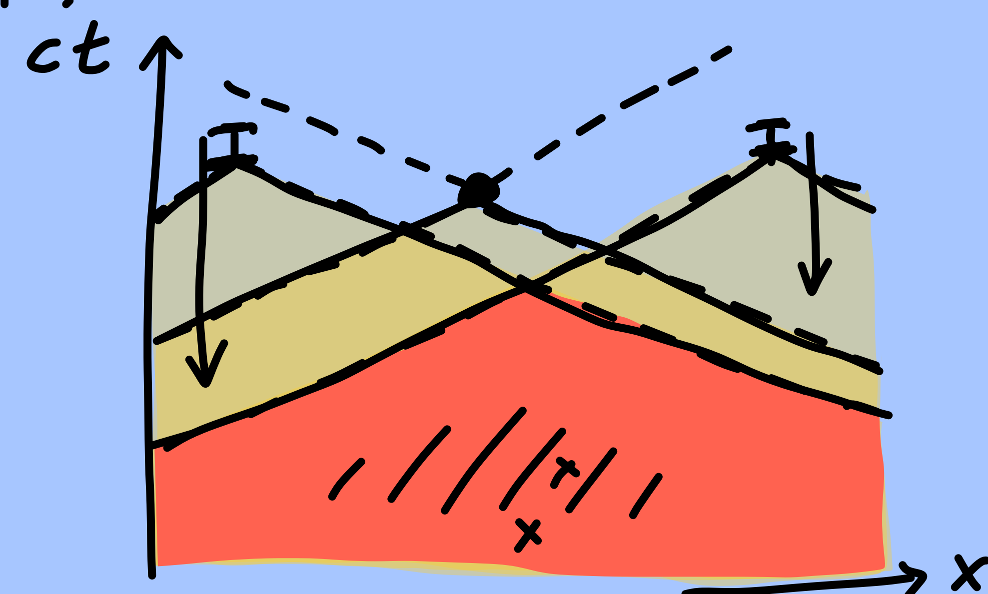
"FREEDOM-OF-CHOICE LOOPHOLE"

MAYBE SETTINGS ARE NOT TRULY RANDOM & INDEPENDENT OF λ?
MAYBE PRE-DETERMINED?

① MAKE SURE λ DOES NOT INFLUENCE SETTING:

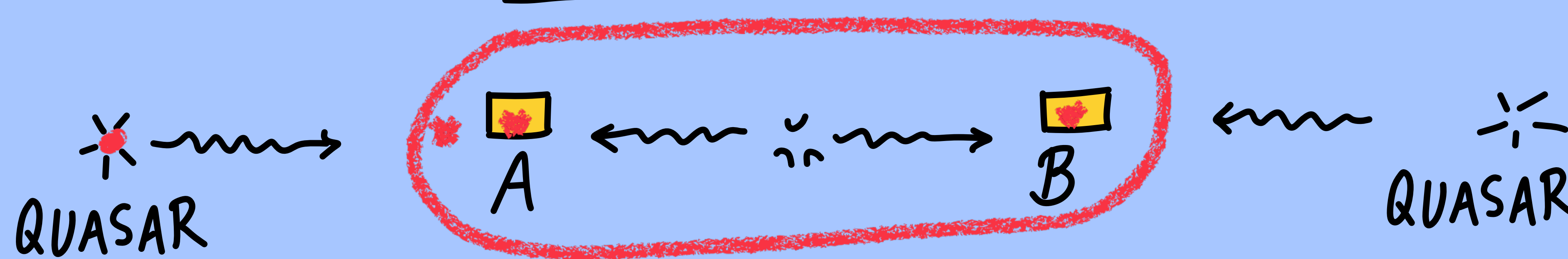


BUT: MAYBE EVERYTHING PREDETERMINED IN THE PAST?



② OBTAIN RANDOMNESS FROM UNCONVENTIONAL SOURCES: MOVIES, GAMES,

LIGHT FROM DISTANT QUASARS → SETTINGS



(ZEILINGER ET AL 2018)

LIGHT EMITTED ~3/7 BILLION YEARS AGO

2.5 FURTHER REMARKS

GREENBERGER-HORNE-ZEILINGER STATES (GHZ)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\underline{\uparrow}\,\underline{\uparrow}\,\underline{\uparrow}\rangle + |\downarrow\downarrow\downarrow\rangle\right)$$

$$\hat{X}_2 = \hat{\sigma}_{x2}$$

$$\hat{X}|\uparrow\rangle = |\downarrow\rangle$$

$$\hat{Y}|\downarrow\rangle = -i|\uparrow\rangle$$

$$\hat{Y}|\uparrow\rangle = i|\downarrow\rangle$$

$$\hat{X}_1\hat{X}_2\hat{X}_3|\psi\rangle = |\psi\rangle$$

$$\underline{\hat{Y}_1\hat{Y}_2\hat{X}_3}\,|\psi\rangle = \underline{-}|\psi\rangle$$

X_1 = MSMT RESULT FOR $\hat{X}_1$

MSMT COMBINATION	QM PREDICTION FOR $|\psi\rangle$
$\underline{X_1} \cdot X_2 \cdot X_3$	$+1$
$\underline{Y_1} \cdot Y_2 \cdot X_3$	-1
$\underline{Y_1} \cdot X_2 \cdot Y_3$	-1
$\underline{X_1} \cdot Y_2 \cdot Y_3$	-1

LHV MODEL ($X_1, Y_1, X_2, \ldots$ HAVE VALUES EVEN BEFORE MSMT)

$$\prod_{\text{RESULTS}} = (X_1 X_2 X_3)(Y_1 Y_2 X_3)(\quad)(\quad)$$

$$= X_1^2 Y_1^2 X_2^2 \cdot \ldots$$

$$= \underline{\underline{+1}}$$

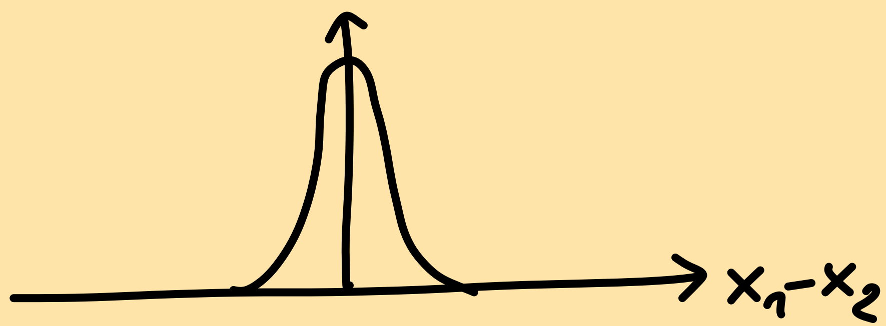
QM

$$\prod_{\text{RESULTS}} = \underline{\underline{-1}}$$

↯ QM $\leftrightarrow$ LHV

ORIGINAL EPR

$$\Psi(x_1, x_2) \sim \delta(x_1 - x_2)$$

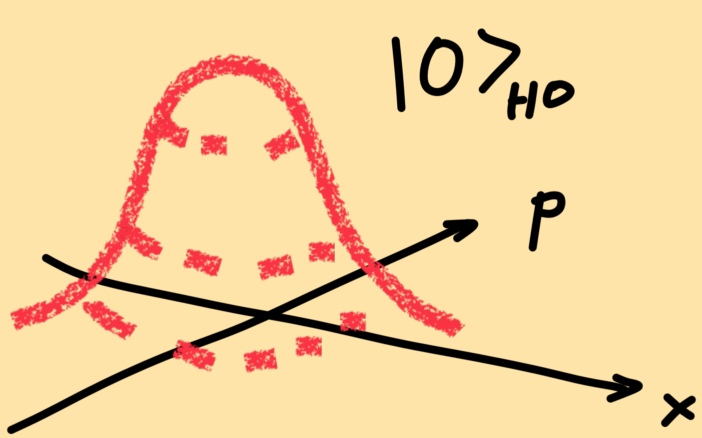


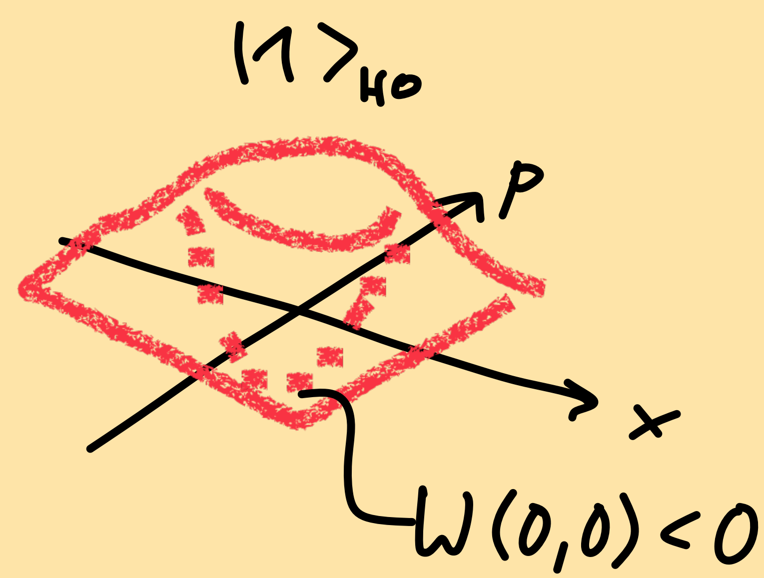
PROBLEM: $\exists$ LHV MODELS THAT DESCRIBE x, p MSMTS FOR SUCH STATES

EXCURSION : WIGNER DENSITY

$$W(x,p) = \frac{1}{\pi\hbar}\int_{-\infty}^{+\infty} e^{2i\frac{p}{\hbar}y}\,\psi^*(x+y)\,\psi(x-y)\,dy$$

- $W \in \mathbb{R}$
- $\int dx\,dp\, W(x,p) = 1$
- $|\psi(x)|^2 = \int W(x,p)\,dp$
- $|\tilde{\psi}(p)|^2 = \int W(x,p)\,dx$
- $\int dx\,dp\, \underbrace{f(x)}\, W(x,p) = \langle\psi|f(\hat{x})|\psi\rangle$
- IT CAN BECOME NEGATIVE





WIGNER DENSITIES OF GAUSSIAN STATES ARE ≥ 0

$$W(x_1, p_1, x_2, p_2)_{EPR} \geq 0$$

$\hookrightarrow$ EPR : LHV "HIDDEN VARIABLES" $= x_1, x_2, p_1, p_2$

WAY OUT: STATES WITH $W<0$

MORE MODEST GOAL:
CONFIRM ENTANGLEMENT

IDEAL EPR: $\langle (\hat{x}_1 - \hat{x}_2)^2 \rangle = 0 \quad \langle (\hat{p}_1 + \hat{p}_2)^2 \rangle = 0$

UNCORR. STATE: $\langle \quad \hat{} \quad \rangle = \langle \hat{x}_1^2 \rangle + \langle \hat{x}_2^2 \rangle > 0$

$$\langle (\hat{p}_1 + \hat{p}_2)^2 \rangle = \langle \hat{p}_1^2 \rangle + \langle \hat{p}_2^2 \rangle > 0$$

DUAN ET AL 2000

$$\hat{u} = \hat{x}_1 - \hat{x}_2$$
$$\hat{v} = \hat{p}_1 + \hat{p}_2$$
$$[\hat{x}, \hat{p}] = i$$

IF $\underbrace{\langle(\hat{u} - \langle\hat{u}\rangle)^2\rangle}_{\text{Var }\hat{u}} + \langle(\hat{v} - \langle\hat{v}\rangle)^2\rangle \geq 2$

IS VIOLATED $\Rightarrow$

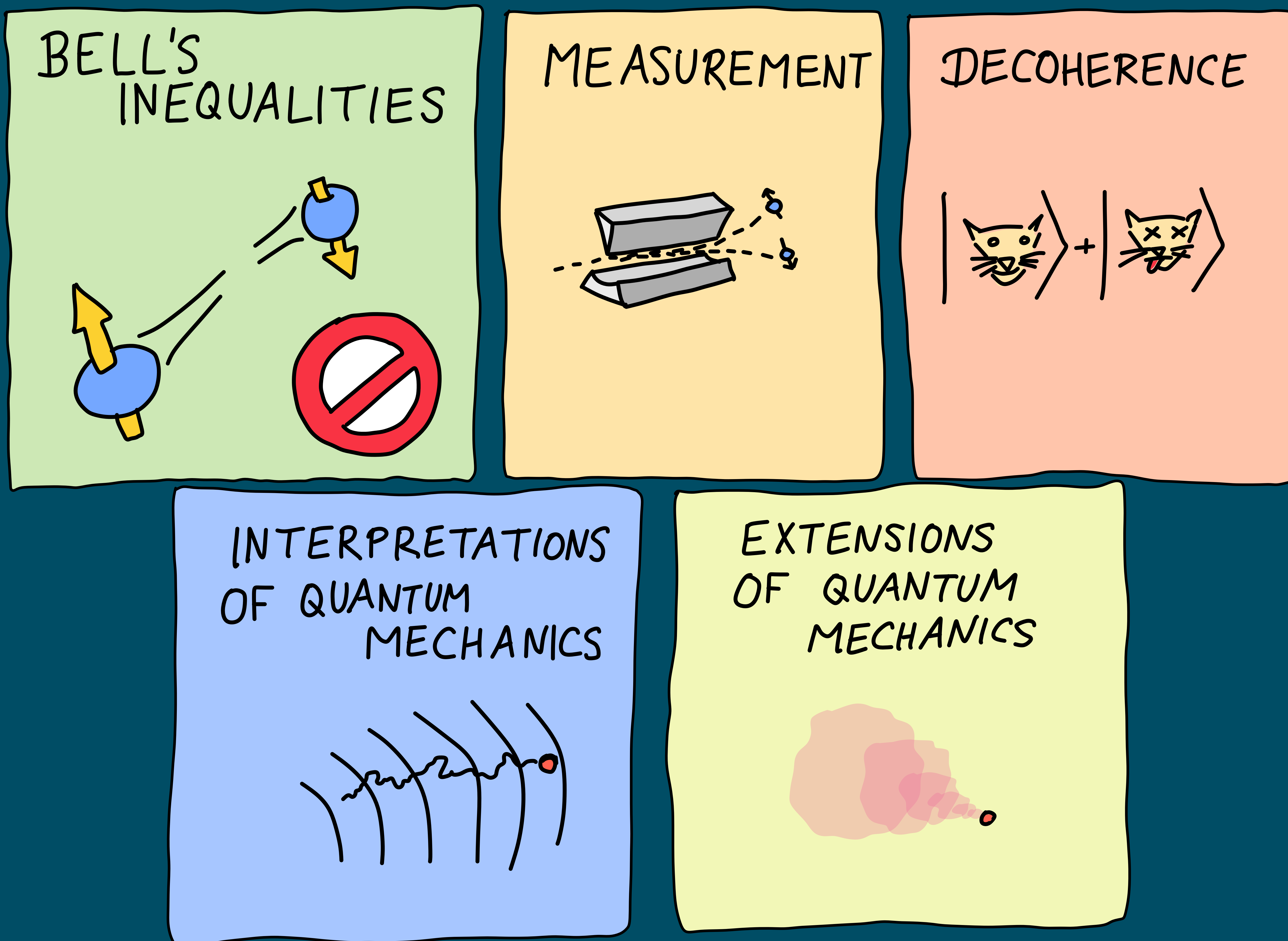
STATE IS ENTANGLED!

Lecture 8

Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

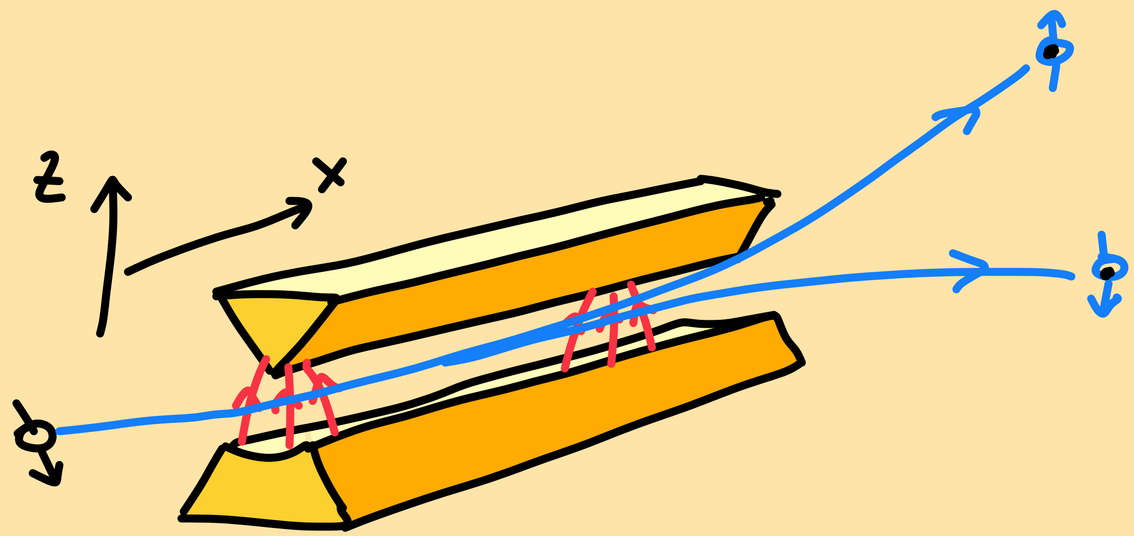
Start at 6pm CET


BELL'S INEQUALITIES
MEASUREMENT
DECOHERENCE
INTERPRETATIONS OF QUANTUM MECHANICS
EXTENSIONS OF QUANTUM MECHANICS

3. MEASUREMENT

3.1 BASIC FEATURES

EXAMPLE: STERN-GERLACH EXPERIMENT (1922)



$$-\hat{\vec{\mu}}\cdot\vec{B}(\vec{r})$$

$$= -\mu\,\hat{\sigma}_z\cdot\underbrace{B_z(z)}_{B_z(z)\,=\,\underline{B}\,+\,\underline{B'z}}$$

$\vec{B}\parallel\vec{z}$

$$\hat{H} = \frac{\hat{p}^2}{2m} - (\mu B)\hat{\sigma}_z - (\mu B')\,z\,\hat{\sigma}_z$$

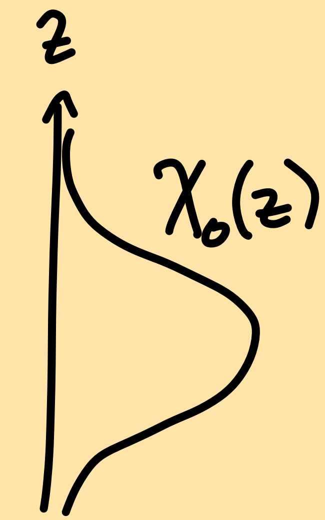
$$\hat{p} = \hat{p}_z = -i\hbar\frac{\partial}{\partial z}$$

$$-\underbrace{\overbrace{(\mu B')}^{F_0}\hat{\sigma}_z}_{F}\cdot z \quad = -\hat{F} z$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{\hbar\Omega}{2}\hat{\sigma}_z - \underbrace{F_0\,\hat{\sigma}_z\, z}_{\Rightarrow \text{MSMT}}$$

INITIAL STATE:

$$|\psi(t=0)\rangle = \underbrace{(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)}\otimes \underbrace{|\chi_0\rangle}_{\text{MOTIONAL STATE}}$$



$$[\hat{\sigma}_z, \hat{H}] = 0$$ "QUANTUM NON-DEMOLITION MEASUREMENT"

$$|\psi(t)\rangle = e^{-\frac{i}{\hbar}\hat{H}t}\,|\psi(t=0)\rangle$$

$$= \alpha \quad e^{-\frac{i}{\hbar}\hat{H}t}\,|\uparrow\rangle\otimes|\chi_0\rangle$$

$$+\beta \quad e^{-\frac{i}{\hbar}\hat{H}t}\,|\downarrow\rangle\otimes|\chi_0\rangle$$

$$= \alpha \;|\uparrow\rangle\,\underline{e^{-\frac{i}{\hbar}\hat{H}_\uparrow t}\,|\chi_0\rangle}$$

$$+\beta \;|\downarrow\rangle\,\underline{e^{-\frac{i}{\hbar}\hat{H}_\downarrow t}\,|\chi_0\rangle}$$

$$\hat{H}|\uparrow\rangle = \hat{H}_\uparrow|\uparrow\rangle$$

$$\hat{H}_\uparrow = \hat{H}\big|_{\hat{\sigma}_z \mapsto +1}$$

$$= \alpha |\uparrow\rangle \otimes |\chi_\uparrow\rangle + \beta |\downarrow\rangle \otimes |\chi_\downarrow\rangle$$

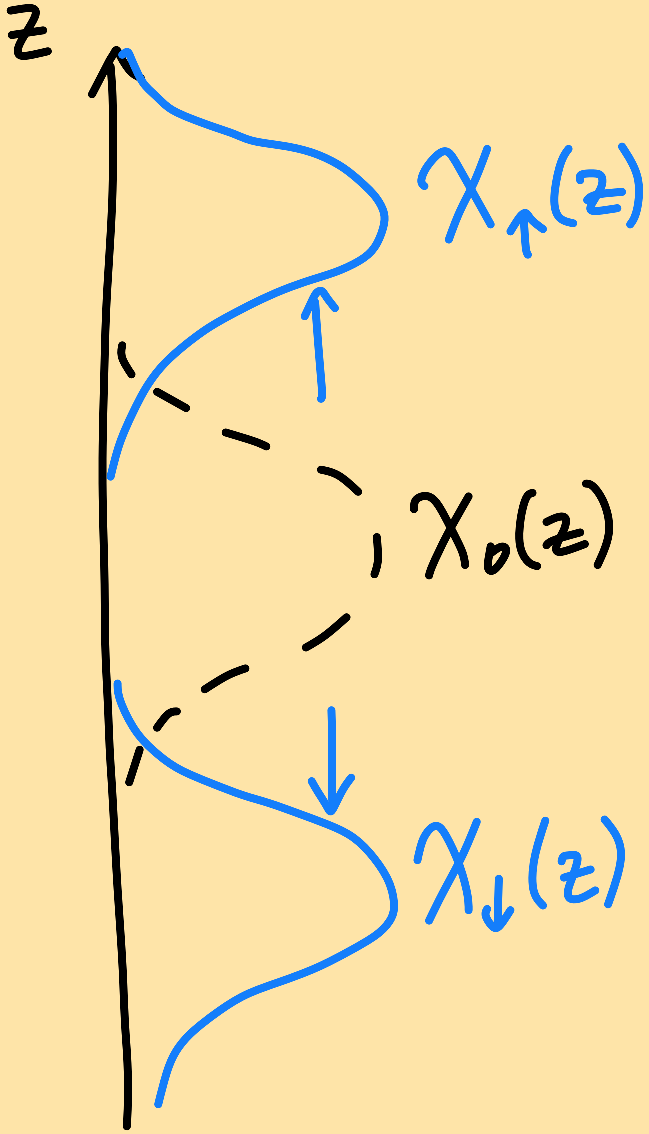
$$|\chi_\uparrow\rangle = e^{-\frac{i}{\hbar}\hat{H}_\uparrow t} |\chi_0\rangle$$

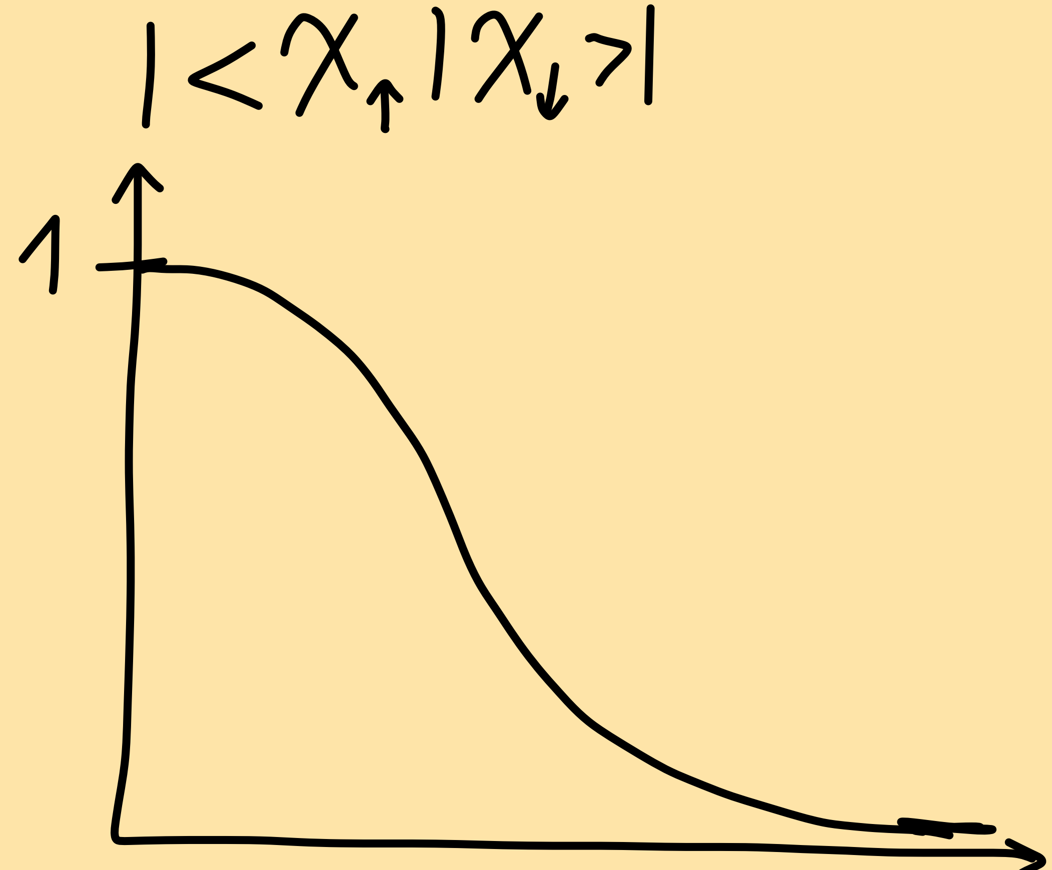
$$\hat{H}_{\uparrow/\downarrow} = \frac{\hat{p}^2}{2m} \pm \left(\frac{\hbar\Omega}{2} - F_0 z\right)$$

$|\chi_{\uparrow/\downarrow}\rangle$ = motion under force $\pm F_0$

$|\psi(t)\rangle$ = ENTANGLED STATE

$|\chi_{\uparrow/\downarrow}\rangle$ = "POINTER STATES"





$$\hat H_{\uparrow/\downarrow} = \frac{\hat p^2}{2m} \mp F_0 \underbrace{z}_{i\hbar \frac{\partial}{\partial p}} \pm \frac{\hbar\Omega}{2}$$

$$\tilde\chi(p) \equiv \frac{1}{\sqrt{2\pi\hbar}}\int dz\, e^{-i\frac{p}{\hbar}z}\,\chi(z)$$

$$i\hbar\, \partial_t \tilde\chi(p) = \mp i\hbar F_0 \frac{\partial}{\partial p}\tilde\chi(p) + \dots$$

$$\underline{\partial_t} \tilde\chi(p) = \underbrace{\mp F_0}\, \underline{\frac{\partial}{\partial p}}\, \tilde\chi(p) + \dots$$

→ SHIFT IN p SPACE

$\dot p = \pm F_0$

$$\tilde{\chi}_{\uparrow/\downarrow}(p,t) = e^{-i\varphi_{\uparrow/\downarrow}(t)}\,\tilde{\chi}(p \mp F_0 t, 0)$$

$$\varphi_{\uparrow/\downarrow}(t) = \pm\frac{\Omega}{2}t + \frac{1}{2m\hbar}\int_0^t \left(p \mp F_0(t-t')\right)^2 dt'$$

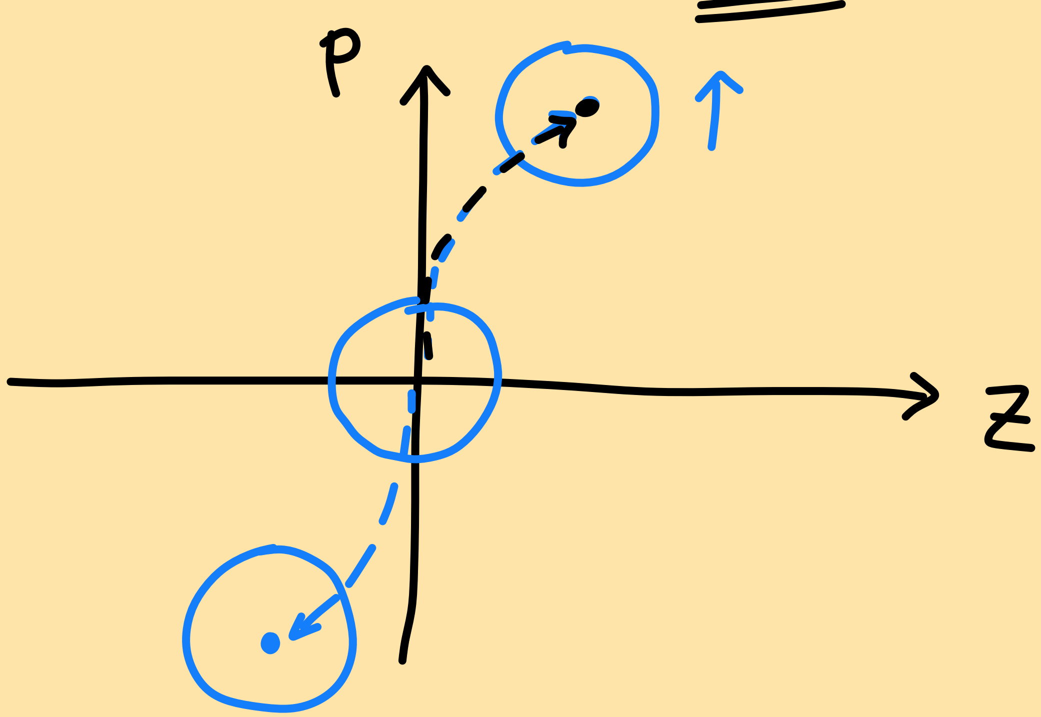
$$|\langle \chi_\uparrow | \chi_\downarrow \rangle| = \dots = \exp\left(\underline{-\frac{\bar{p}(t)^2}{2\Delta p^2} - \frac{\bar{z}(t)^2}{2\Delta z^2}}\right)$$

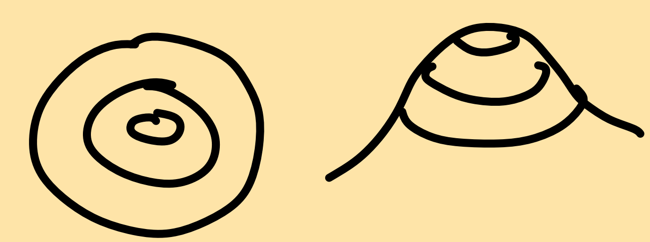
GAUSSIAN

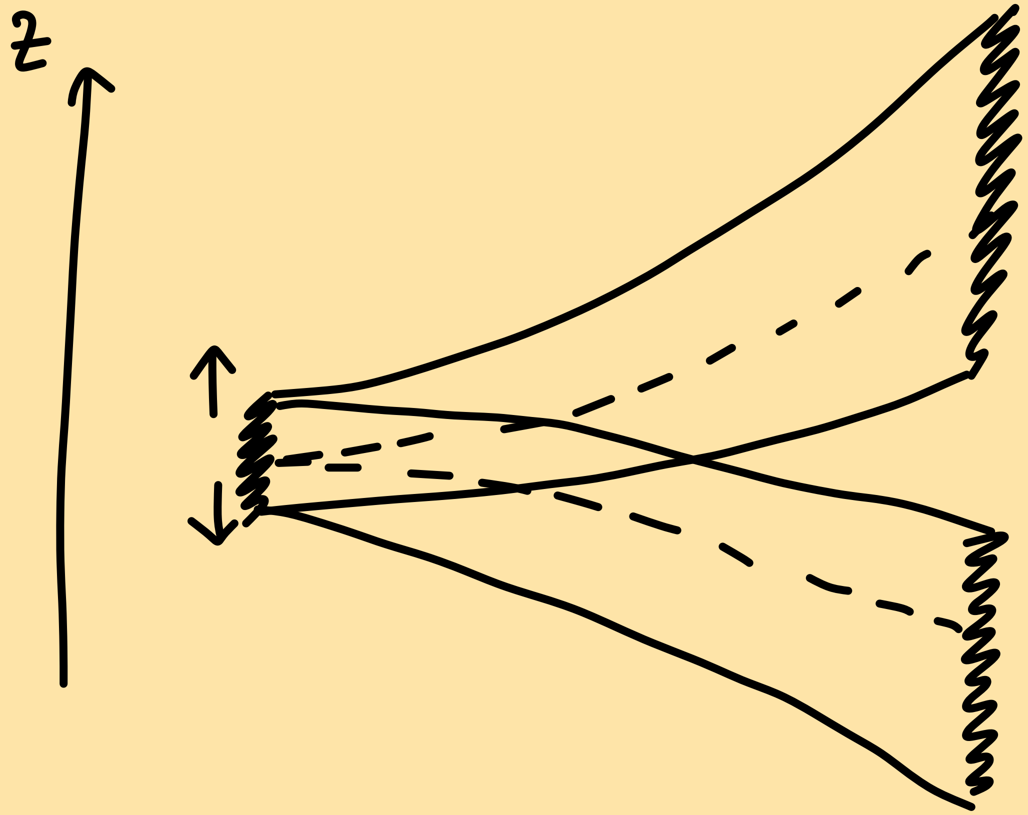
$$\Delta p^2 = \langle \chi_0 | (\hat{p} - \langle \hat{p} \rangle)^2 | \chi_0 \rangle$$

$$\Delta z = \frac{\hbar}{2\Delta p}$$

$$\bar{p}(t) = \underline{\underline{F_0 t}} \qquad \bar{z}(t) = \underline{\underline{\frac{F_0}{2m}t^2}}$$







NO FORCE $\Rightarrow$ OVERLAP REMAINS

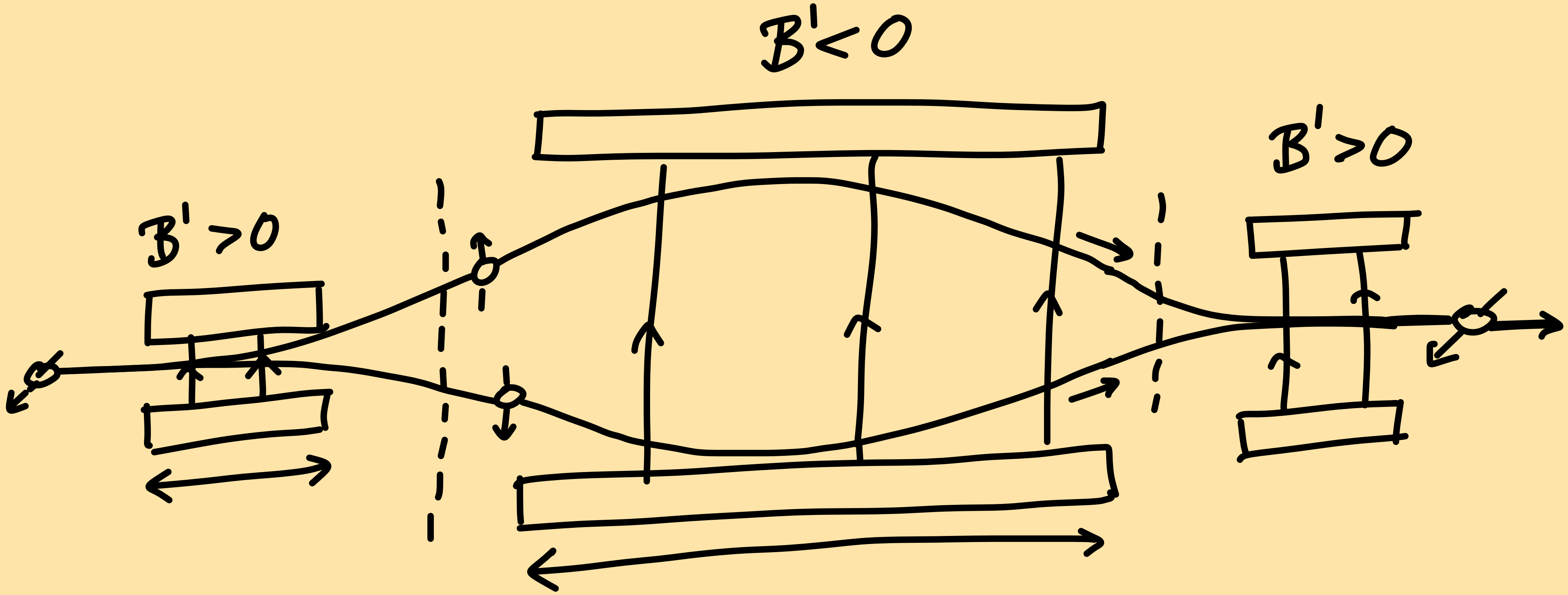
$$\langle \chi_\uparrow(t)|\chi_\downarrow(t)\rangle = \langle \hat{U}(t-t_0)\chi_\uparrow(t_0)|\hat{U}(t-t_0)\chi_\downarrow(t_0)\rangle$$

THE SAME!

$$= \langle \chi_\uparrow(t_0)|\chi_\downarrow(t_0)\rangle$$

$$\langle \hat{U}\psi|\hat{U}\phi\rangle = \langle \psi|\underbrace{\hat{U}^\dagger\hat{U}}_{\mathbb{1}}\phi\rangle = \langle\psi|\phi\rangle$$

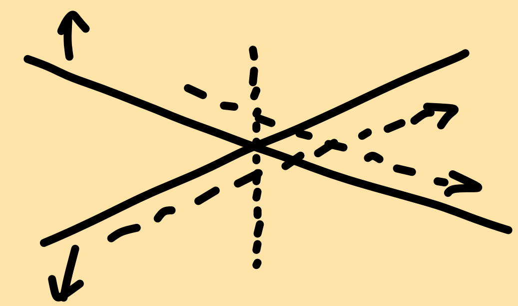
"QUANTUM ERASER"



UNDO ENTANGLEMENT!

$$\alpha|\uparrow\rangle \underbrace{|\chi_\uparrow\rangle} + \beta|\downarrow\rangle\underbrace{|\chi_\downarrow\rangle}$$

$$\mapsto (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |\chi_0\rangle$$



→ CRUCIAL ASPECT OF MSMT:

IRREVERSIBILITY

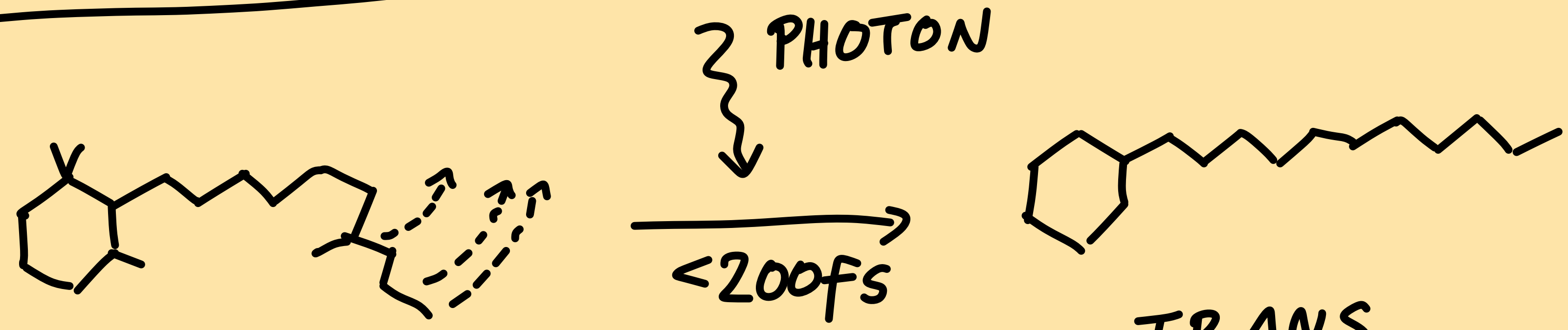
IRREVERSIBILITY

PHOTOMULTIPLIER

$\gamma \longmapsto$ MANY e^-

AMPL. $\sim 10^6$

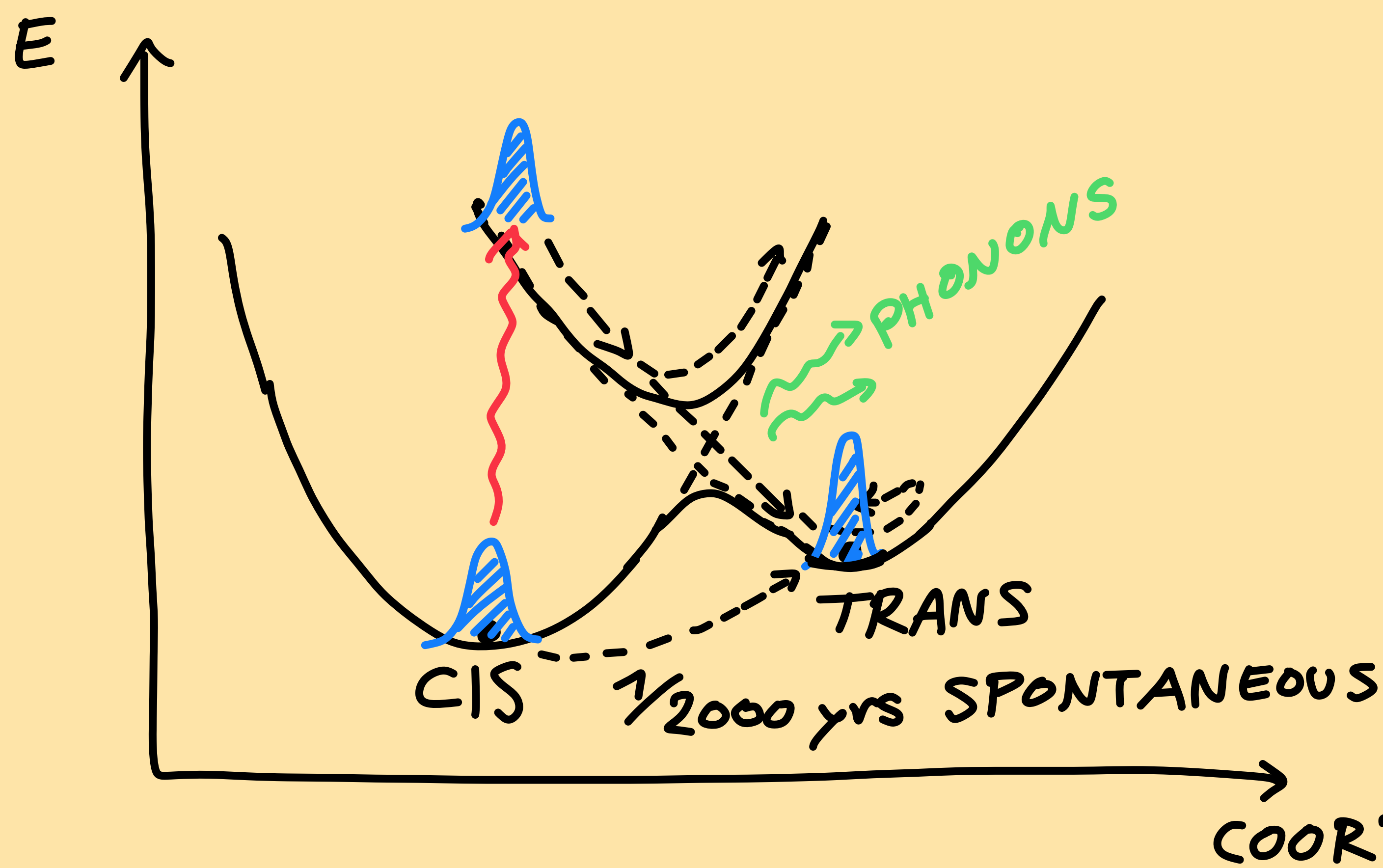
HUMAN EYE



CIS RHODOPSIN

TRANS

ION CHANNELS ETC. } ~0.1s

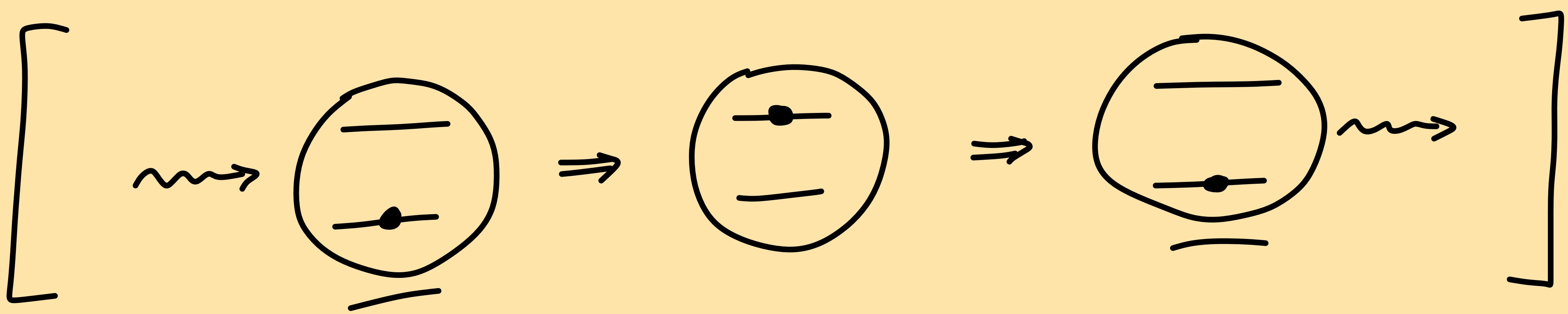


SIGNAL ONLY IF $\gtrsim 5$ PHOTONS WITHIN 0.1 sec

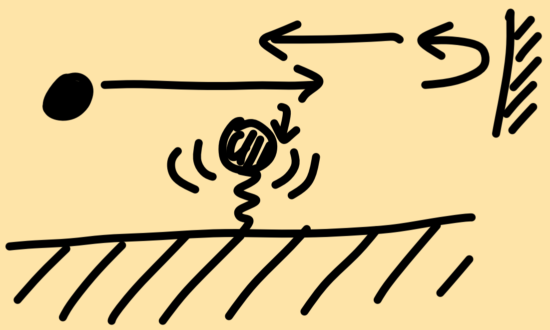
→ SUPPRESS "DARK COUNTS"

FEATURE:

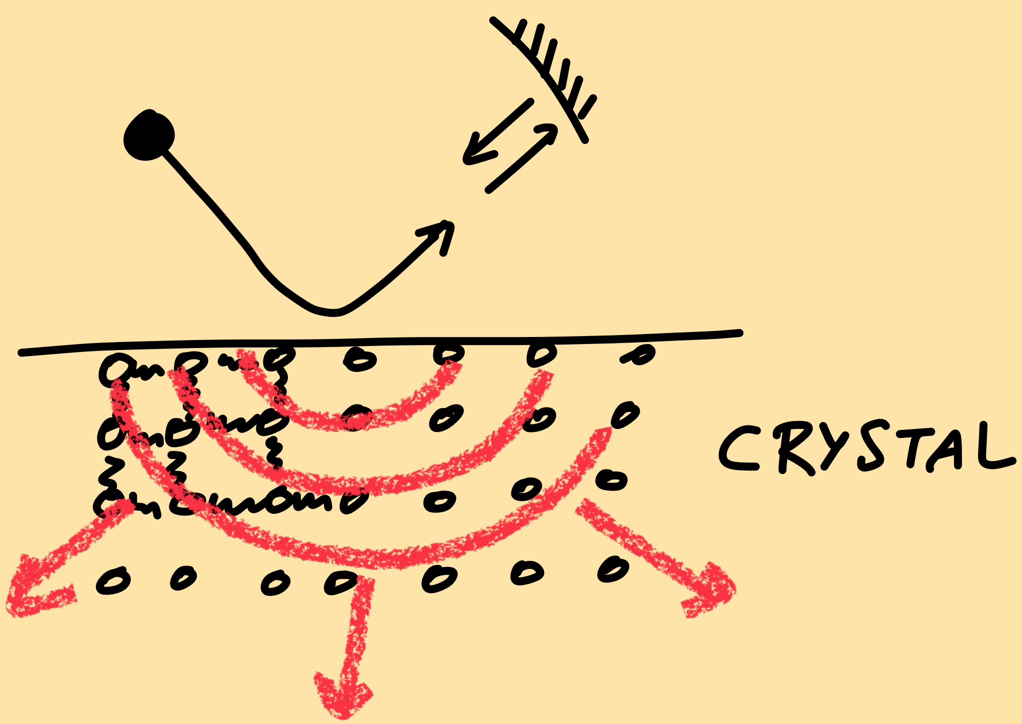
- ONE QUANTUM
  ⟶ MANY QUANTA!
- IRREVERSIBLE

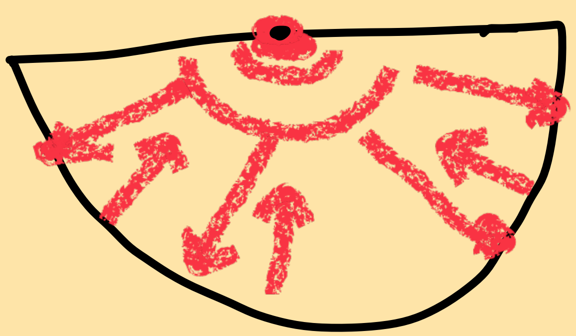


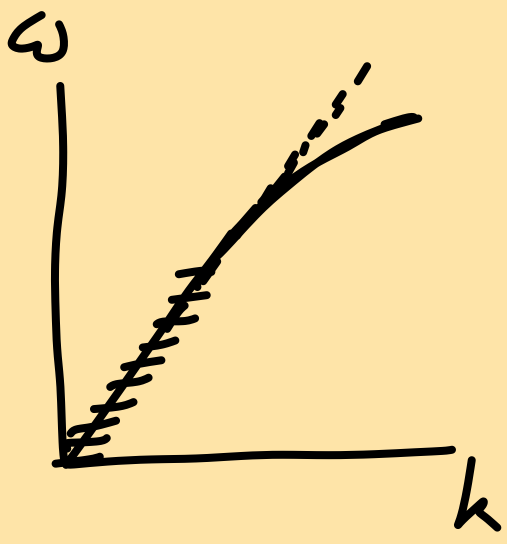
BAD DETECTOR:

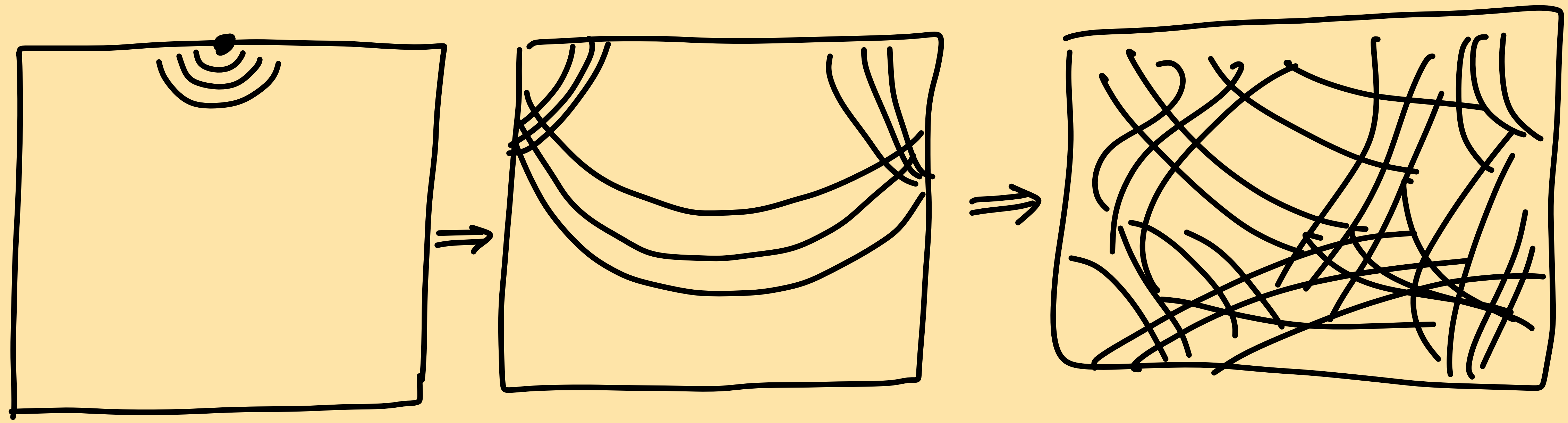


SIMPLE MODEL FOR IRREVERSIBILITY:









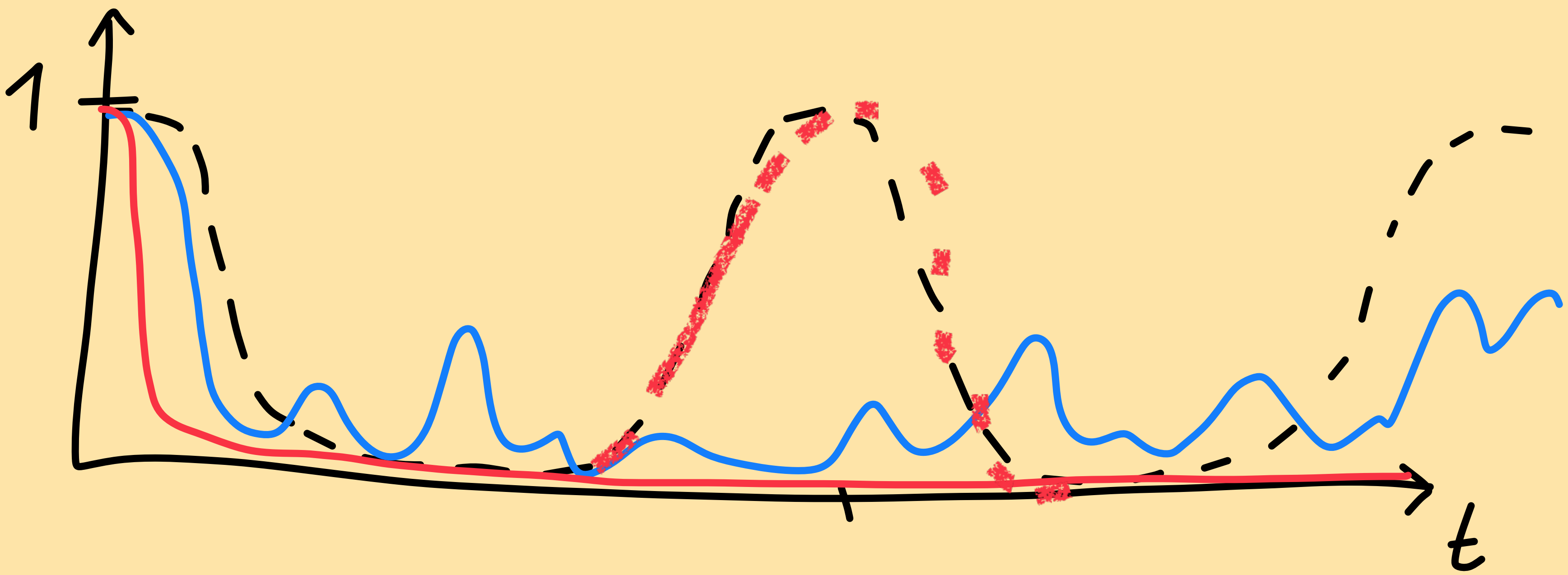
TIME-REVERSAL

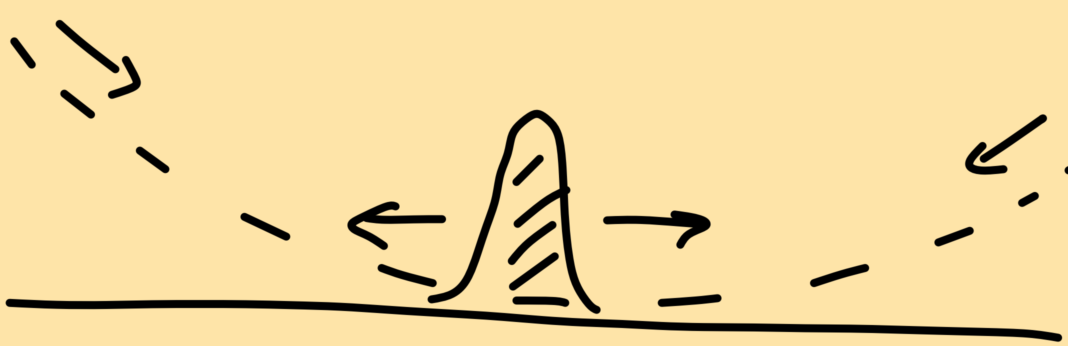
$$\vec{v}_j \longmapsto -\vec{v}_j$$

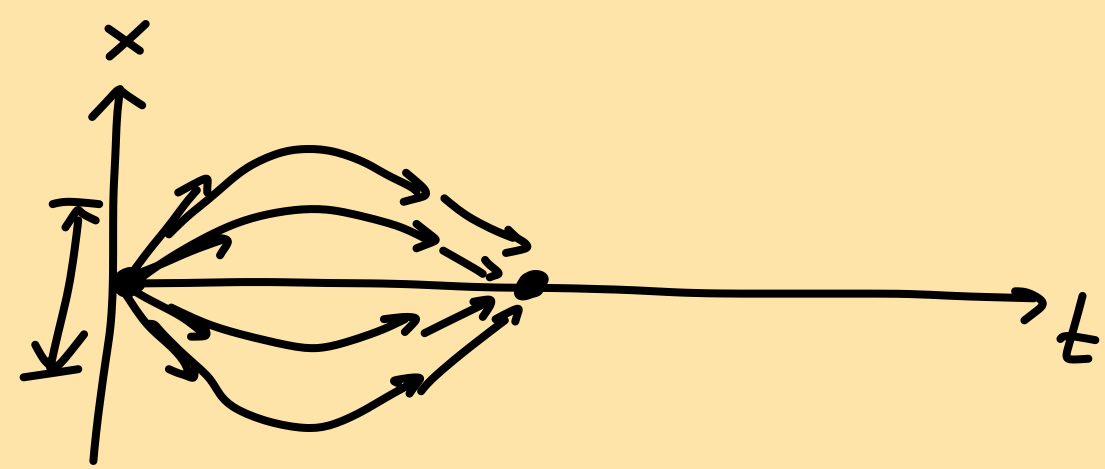
$$\Psi(\vec{r}_1, \vec{r}_2, \dots) \longmapsto \Psi^*(\vec{r}_1, \vec{r}_2, \dots)$$

$$\left(e^{i\vec{k}\vec{r}} \longmapsto e^{-i\vec{k}\vec{r}} \right)$$

$$|\langle \Psi(t=0) | \Psi(t) \rangle|$$







Lecture 9

Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

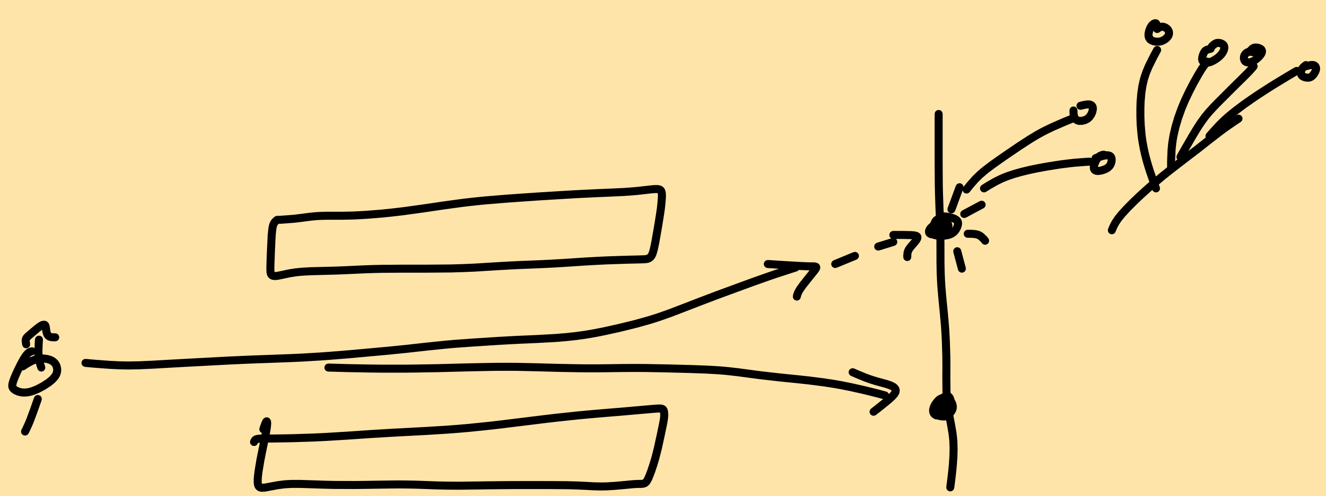
Start at 6pm CET

RECAP

QUANTUM MEASUREMENT
OF $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$ LEADS TO:

ORTHOGONAL &
MACROSCOPICALLY
DISTINCT

$$|\psi\rangle = \alpha \underbrace{|\uparrow\rangle \otimes |\chi_\uparrow\rangle}_{|\psi_\uparrow\rangle} + \beta \underbrace{|\downarrow\rangle \otimes |\chi_\downarrow\rangle}_{|\psi_\downarrow\rangle}$$



3.2 QUANTUM-CLASSICAL "CUT"

AT WHAT POINT ARE WE ALLOWED TO REPLACE THE SUPERPOSITION

ORTHOGONAL & MACROSCOPICALLY DISTINCT

$$|\psi\rangle = \alpha \underbrace{|\uparrow\rangle \otimes |\chi_\uparrow\rangle}_{|\psi_\uparrow\rangle} + \beta \underbrace{|\downarrow\rangle \otimes |\chi_\downarrow\rangle}_{|\psi_\downarrow\rangle}$$

BY AN "INCOHERENT MIXTURE" (CLASSICAL-LIKE)

$|\psi_\uparrow\rangle$ WITH PROBABILITY $|\alpha|^2$

$|\psi_\downarrow\rangle$ WITH PROBABILITY $|\beta|^2$

AND STILL GET CORRECT PREDICTIONS FOR ALL FUTURE MEASUREMENTS OF RELEVANT OBSERVABLES?

QUANTUM-CLASSICAL "CUT"

FOR ALL RELEVANT OBSERVABLES $\hat{A}$:

$$\langle \psi(t) | \hat{A} | \psi(t) \rangle \stackrel{!}{=} |\alpha|^2 \langle \psi_\uparrow(t) | \hat{A} | \psi_\uparrow(t) \rangle + |\beta|^2 \langle \psi_\downarrow(t) | \hat{A} | \psi_\downarrow(t) \rangle$$

FOR NOW AND ALL FUTURE TIMES t.

REQUIREMENTS FOR THIS TO WORK:

1. EXCLUDE HIGHLY NONLOCAL MANY-PARTICLE OBSERVABLES THAT ACT ON SYSTEM + APPARATUS

EXAMPLE:

$$|\psi\rangle = \alpha|\overset{1}{\uparrow}\rangle \otimes |\overset{2}{\uparrow}\overset{3}{\uparrow}\overset{4}{\uparrow}\overset{5}{\uparrow}\overset{6}{\uparrow}\rangle + \beta|\downarrow\rangle \otimes |\downarrow\downarrow\downarrow\downarrow\downarrow\rangle$$

$$\hat{A} = \hat{\sigma}_{x_1}\hat{\sigma}_{x_2}\hat{\sigma}_{x_3}\hat{\sigma}_{x_4}\hat{\sigma}_{x_5}\hat{\sigma}_{x_6}$$

$$\langle\psi|\hat{A}|\psi\rangle = \alpha^*\beta + \beta^*\alpha \neq |\alpha|^2\langle\psi_\uparrow|\hat{A}|\psi_\uparrow\rangle + |\beta|^2\langle\psi_\downarrow|\hat{A}|\psi_\downarrow\rangle$$

$$\alpha|\uparrow\rangle \otimes \overbrace{\underline{|\underline{\uparrow}\uparrow\uparrow\uparrow\uparrow\rangle}} + \beta|\downarrow\rangle \otimes \overbrace{\underline{|\underline{\nearrow}\nearrow\nearrow\nearrow\nearrow\rangle}}$$

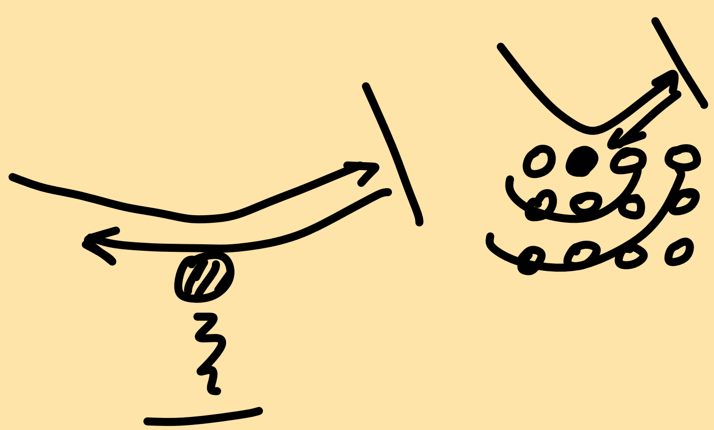
$$|\nearrow\rangle = \cos\theta|\uparrow\rangle + \sin\theta|\downarrow\rangle$$

2. MAKE SURE $|\chi_\uparrow\rangle$ AND $|\chi_\downarrow\rangle$ REMAIN ORTHOGONAL

OR

NO FUTURE SYSTEM-APPARATUS INTERACTION

$$\Rightarrow \langle \chi_\uparrow(t) | \chi_\downarrow(t) \rangle$$
$$= \langle \hat{U}(t)\chi_\uparrow(0) | \hat{U}(t)\chi_\downarrow(0) \rangle$$
$$= \langle \chi_\uparrow(0) | \chi_\downarrow(0) \rangle$$



FUTURE SYS.-APP. INTERACTION CANNOT UNDO ENTANGLEMENT

AND

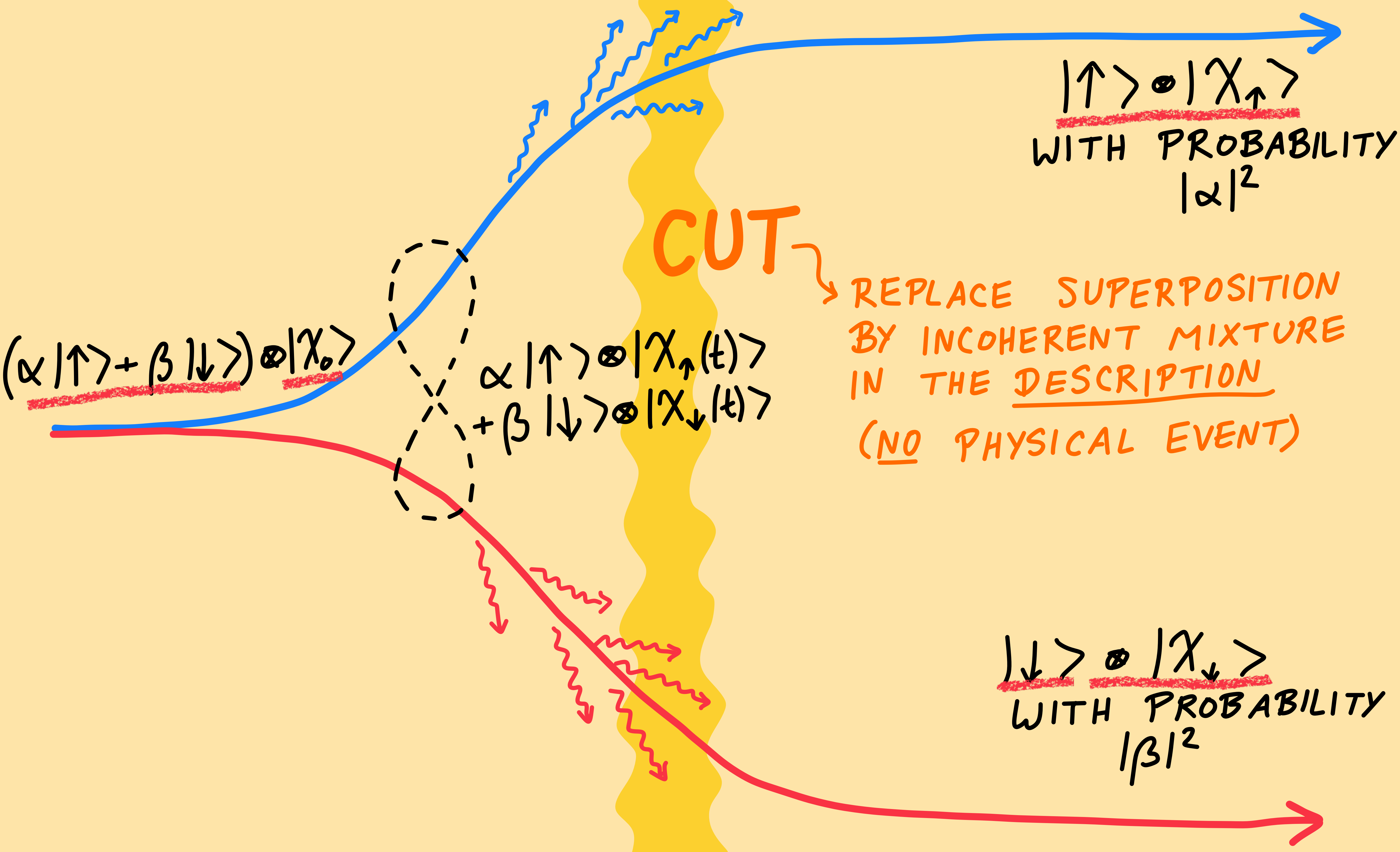
MUST EXCLUDE HIGHLY NONLOCAL INTERACTION

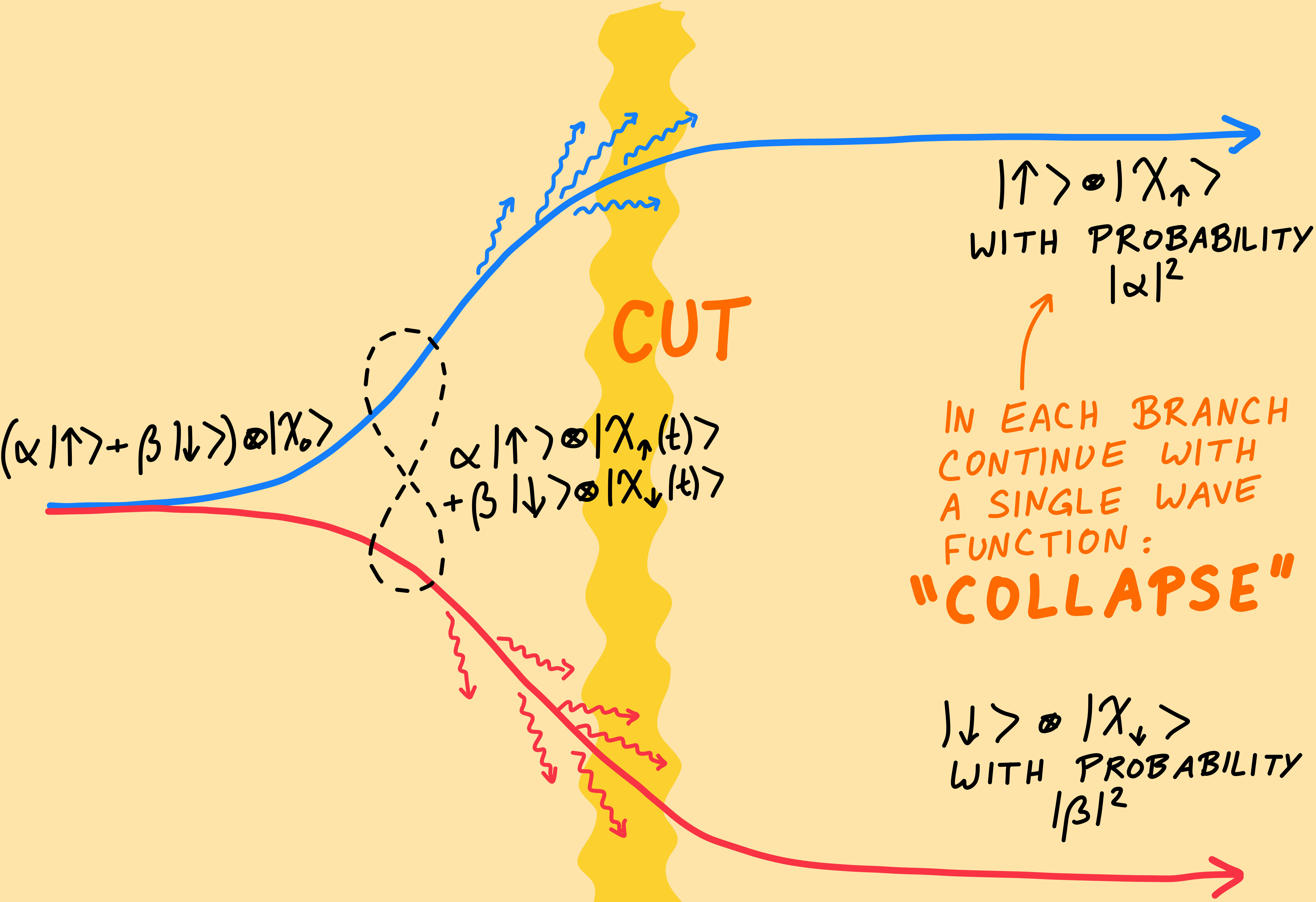
NO MERELY LOCAL DISTINCTION BETWEEN $|\chi_\uparrow\rangle$ & $|\chi_\downarrow\rangle$

PARTICULARLY: NO RECURRENCE $|\chi_{\uparrow/\downarrow}(t)\rangle \approx |\chi_{\uparrow/\downarrow}(0)\rangle$ BY DESIGN OR ACCIDENT

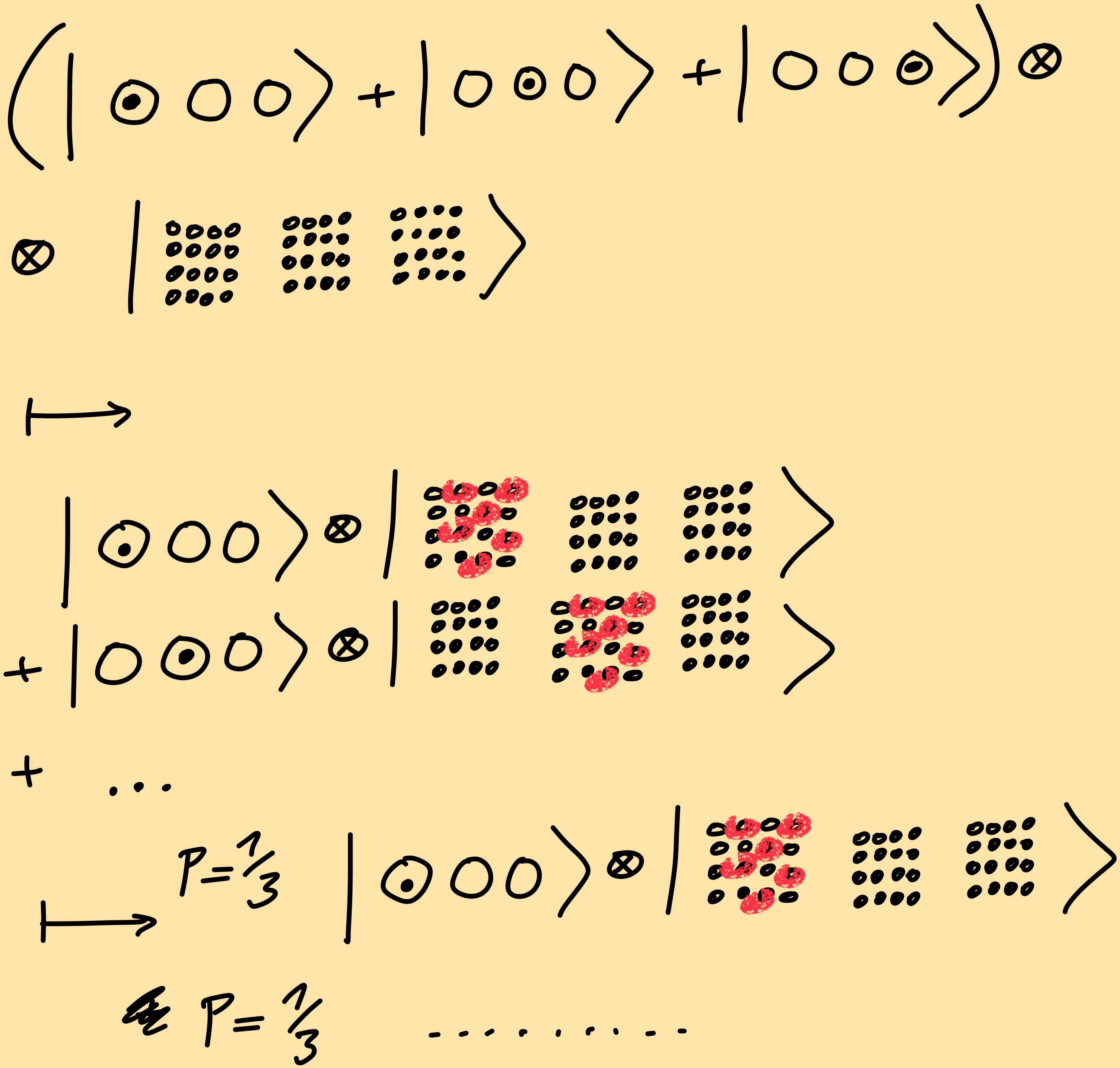
UNLIKELY FOR MACROSCOPIC SYSTEMS

MEASUREMENT LEADS TO DIFFERENT BRANCHES THAT WILL FOREVER REMAIN INCOHERENT FOR ALL PRACTICAL PURPOSES (NO FUTURE INTERFERENCE)

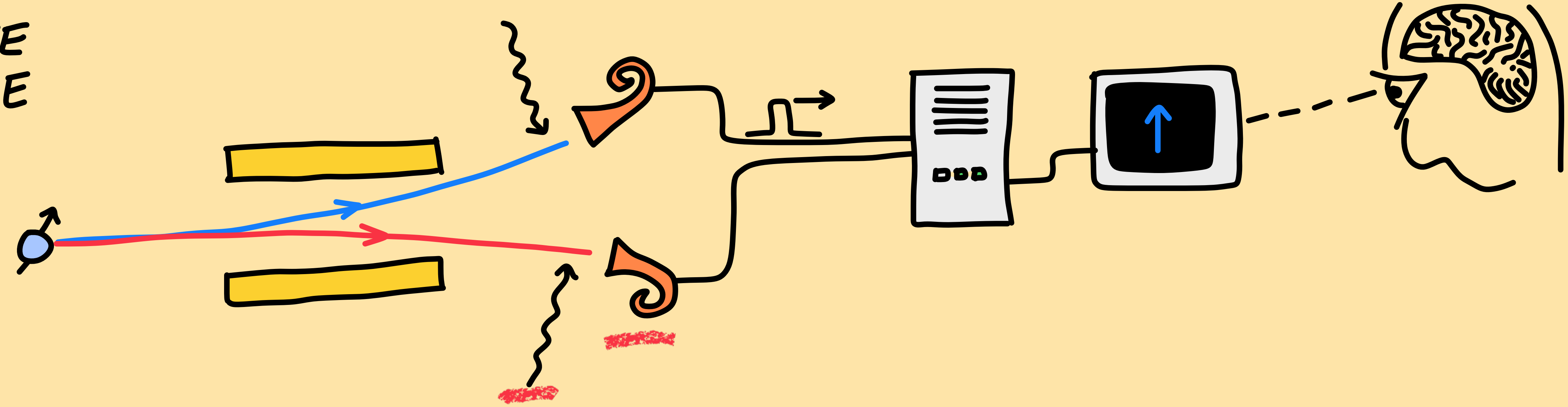




$(\alpha|\uparrow\rangle+\beta|\downarrow\rangle)\otimes|\chi_0\rangle$
$\alpha|\uparrow\rangle\otimes|\chi_\uparrow(t)\rangle$
$+\beta|\downarrow\rangle\otimes|\chi_\downarrow(t)\rangle$
CUT
$|\uparrow\rangle\otimes|\chi_\uparrow\rangle$
WITH PROBABILITY $|\alpha|^2$
IN EACH BRANCH CONTINUE WITH A SINGLE WAVE FUNCTION: "COLLAPSE"
$|\downarrow\rangle\otimes|\chi_\downarrow\rangle$
WITH PROBABILITY $|\beta|^2$

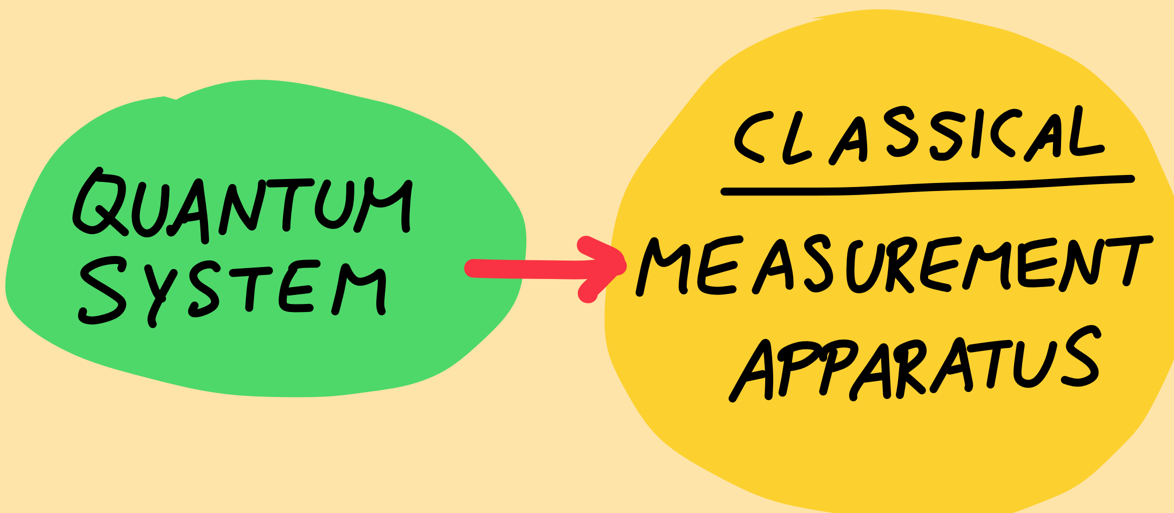

P=1/3
P=1/3

COMPLETE EXAMPLE



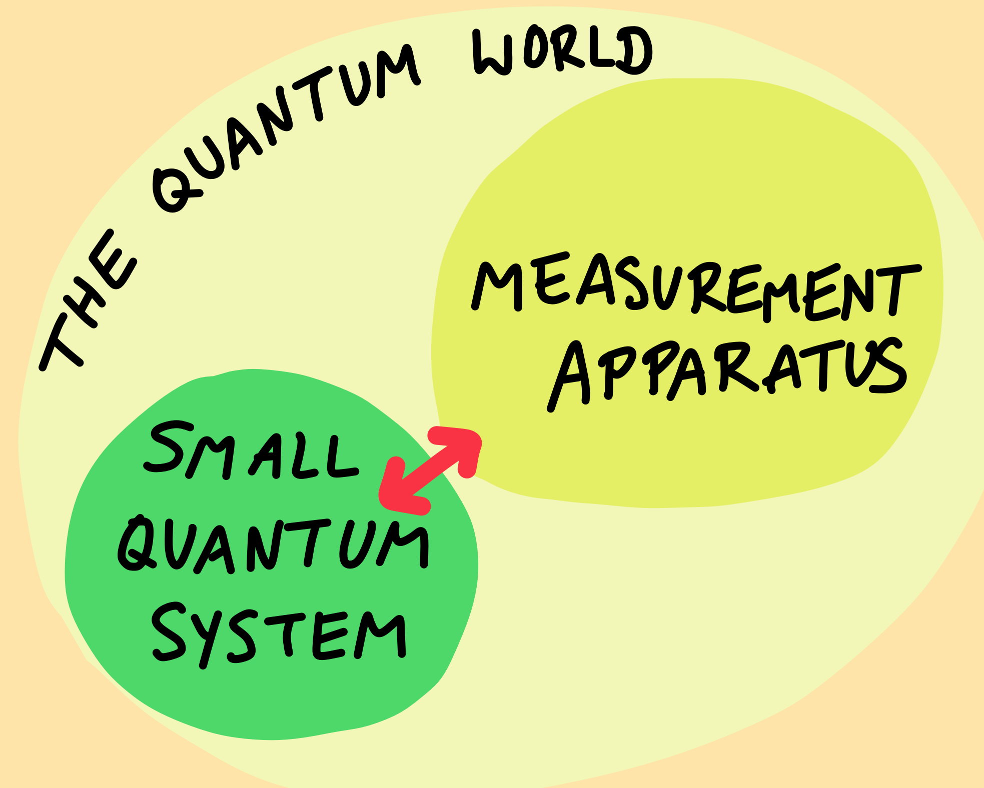
	TIME t	QUANTUM STATE $\vert\chi_{\uparrow/\downarrow}(t)\rangle$
	~ 1 ms	DEFLECTED WAVE PACKET
CUT ?	+ 1 μs	ATOM $\xrightarrow{\text{LASER}}$ ION + e^-
CUT	+ 1 ns	AVALANCHE OF e^- (IN e^- MULTIPLIER)
	+ 1 μs	CURRENT PULSE AMPLIFIED & REGISTERED
	+ 10 ms	RESULT DISPLAYED ON SCREEN
	+ 1 ns	PHOTONS HIT RETINA
	+ 100 fs	RHODOPSIN CONFORMATION CHANGE
	+ 0.1 s	CHEM. CASCADE → NERVE PULSE
	+ 0.2 s	EXPERIMENTALIST NOTICES "↑"

ORIGINAL COPENHAGEN INTERPRETATION



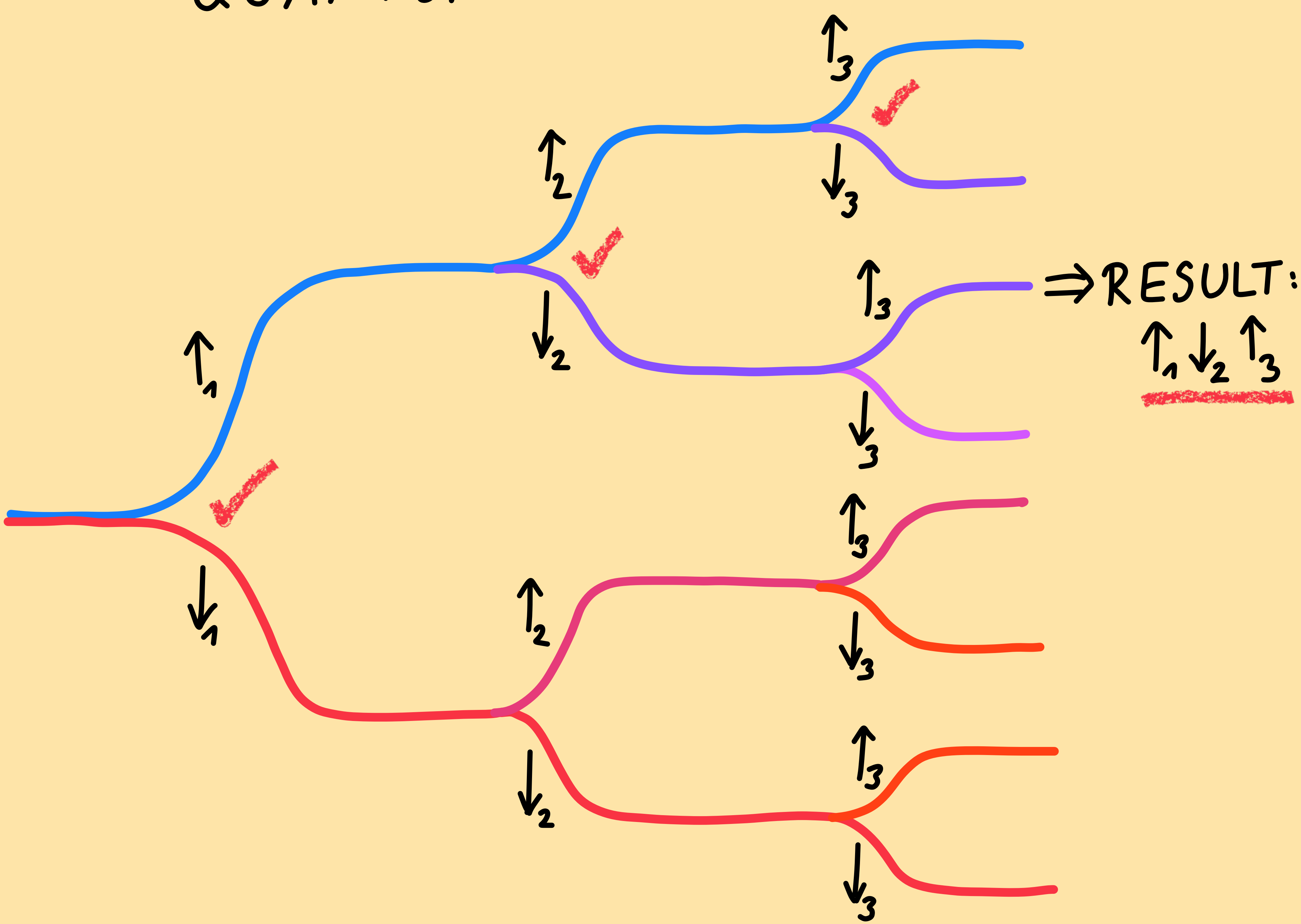
- MEASUREMENT PROCESS OUTSIDE CONTINUOUS SCHRÖDINGER EVOLUTION
- POSTULATE: "COLLAPSE OF THE WAVE FUNCTION"
- EMPHASIS ON CLASSICAL, MACROSCOPIC MSMT APPARATUS

MODERN THEORY OF QUANTUM MEASUREMENTS (v. NEUMANN, EVERETT, ...)



- MEASUREMENT PROCESS DESCRIBED USING SCHRÖDINGER EQUATION
- "QUANTUM-CLASSICAL CUT" IN OUR DESCRIPTION
- IMPORTANCE OF IRREVERSIBILITY VIA MANY DEGREES OF FREEDOM

3.3 MULTIPLE MEASUREMENTS & EVERETT'S THEORY OF QUANTUM MEASUREMENTS



$\Rightarrow$ USE

$$|\psi\rangle = \alpha_1\alpha_2\alpha_3 \; |\uparrow_1\rangle \otimes |\uparrow_2\rangle \otimes |\uparrow_3\rangle \otimes |\chi^{(1)}_{\uparrow}\rangle \otimes |\chi^{(2)}_{\uparrow}\rangle \otimes |\chi^{(3)}_{\uparrow}\rangle$$
$$+ \underline{\alpha_1}\,\underline{\beta_2}\,\underline{\alpha_3} \; \underline{|\uparrow_1\rangle} \otimes \underline{|\downarrow_2\rangle} \otimes \underline{|\uparrow_3\rangle} \otimes \underline{|\chi^{(1)}_{\uparrow}\rangle} \otimes \underline{|\chi^{(2)}_{\downarrow}\rangle} \otimes \underline{|\chi^{(3)}_{\uparrow}\rangle}$$
$$+ \ldots \qquad (2^3 \text{ TERMS})$$

OR SAY:

" $|\uparrow_1\rangle \otimes |\downarrow_2\rangle \otimes |\uparrow_3\rangle \otimes |\chi^{(1)}_{\uparrow}\rangle \otimes |\chi^{(2)}_{\downarrow}\rangle \otimes |\chi^{(3)}_{\uparrow}\rangle$

WITH PROBABILITY $|\alpha_1|^2 |\beta_2|^2 |\alpha_3|^2$ "

ETC. (INCOHERENT BRANCHES)

PhD THESIS

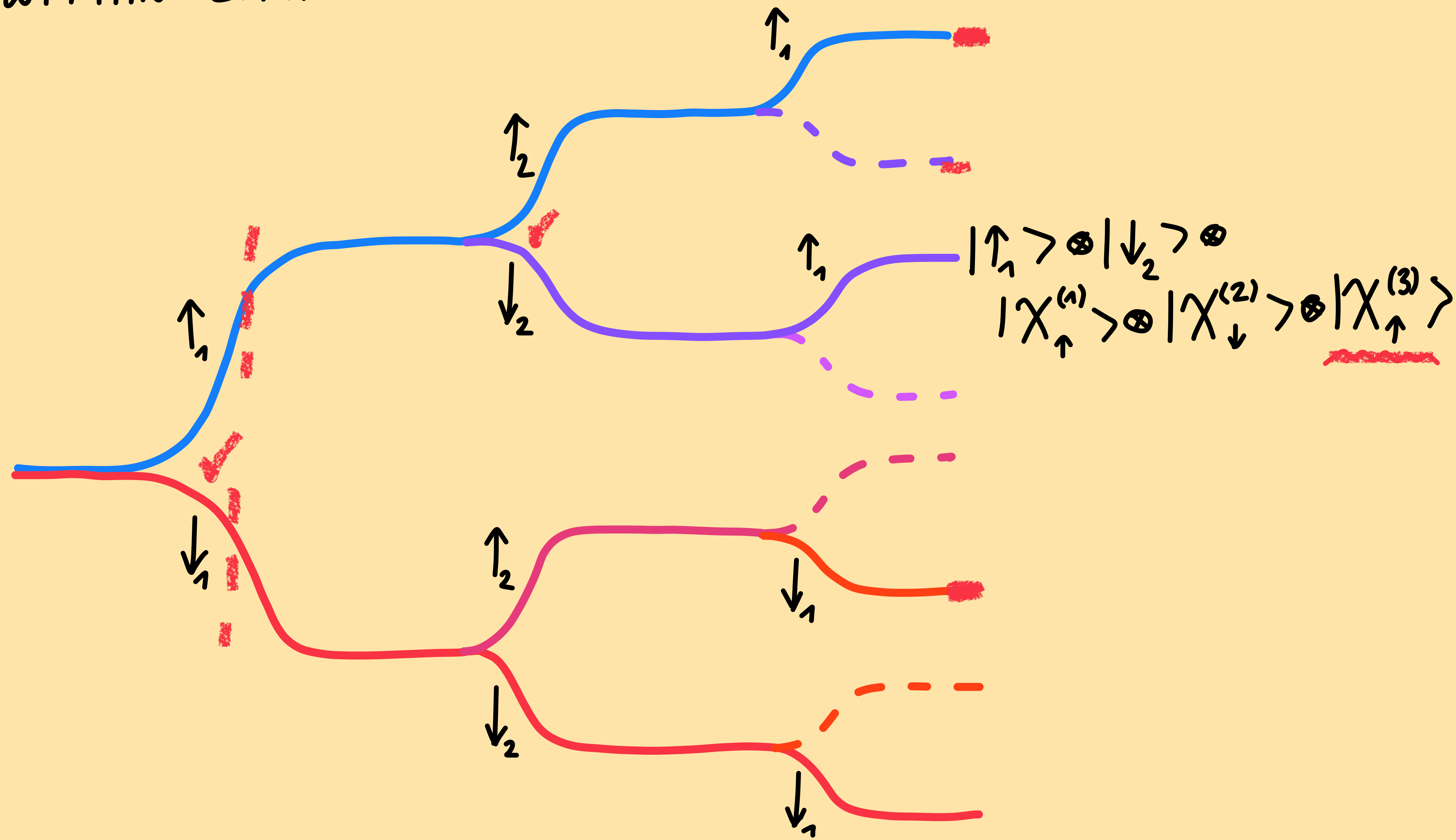
EVERETT 1957 :

- USE <u>SCHRÖDINGER EQUATION</u> FOR <u>ALL</u> OF MEASUREMENT BUT ALSO RECOGNIZE THAT ONE MAY USE SEPARATE WAVE FUNCTIONS FOR PREDICTIONS WITHIN EACH BRANCH
- DEMONSTRATE CONSISTENCY FOR REPEATED MEASUREMENTS
- 'DERIVE' BORN RULE ($|\psi|^2$) FROM WEAK ASSUMPTIONS

CONSISTENCY

MEASURE SAME SYSTEM AGAIN ⇒ SAME RESULT ✓

⇒ WITHIN EACH BRANCH, IT LOOKS LIKE COLLAPSE HAPPENED



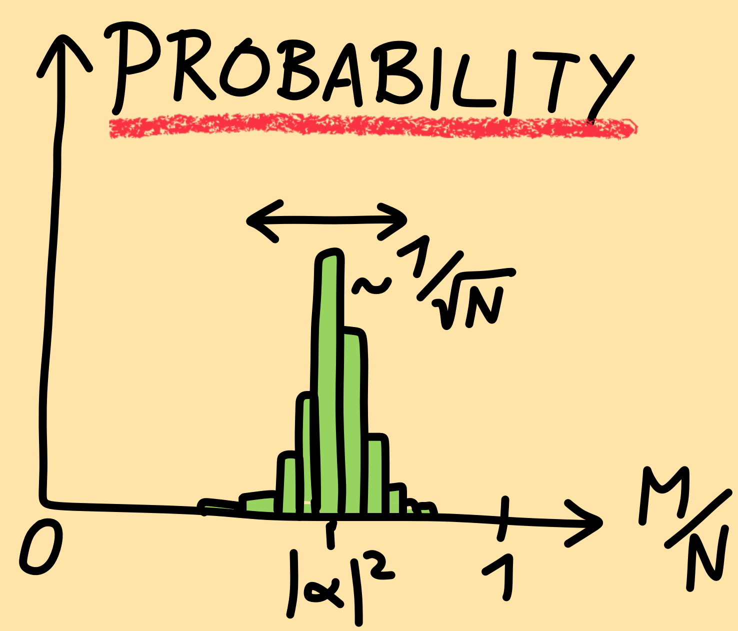
"LAW OF AVERAGES"

OBSERVE N SYSTEMS, ALL PREPARED IDENTICALLY IN $|\phi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$

$$|\Psi\rangle = (\alpha|\uparrow_1\rangle + \beta|\downarrow_1\rangle) \otimes (\alpha|\uparrow_2\rangle + \beta|\downarrow_2\rangle) \otimes \dots$$

P ("RECORD SHOWS M TIMES ↑")

$$= \binom{N}{M} (|\alpha|^2)^M (|\beta|^2)^{N-M}$$



OBSERVED FREQUENCY $\frac{M}{N} \xrightarrow{N\to\infty} |\alpha|^2$

IN ANY* OBSERVATION RECORD

$\Rightarrow$ PREDICTIONS OF COPENHAGEN WILL BE TRUE "ALMOST ALWAYS"*

ASSIGNING PROBABILITIES

BORN RULE SAYS: $P = |\Psi|^2$

EVERETT: SUPPOSE WE DON'T KNOW $P = |\Psi|^2 \Rightarrow ?$

EACH BRANCH HAS A QUANTUM STATE

e.g. $|\phi\rangle = \alpha_1 \beta_2 |\uparrow_1\rangle |\downarrow_2\rangle |\chi^{(1)}_{\uparrow}\rangle |\chi^{(2)}_{\downarrow}\rangle$

(NOT NORMALIZED)

$\rightarrow$ HOW TO ASSIGN $P(|\phi\rangle)$ CONSISTENTLY?

- WANT $P(\underline{\hat{U}}|\phi\rangle) \overset{!}{\equiv} P(|\phi\rangle)$ FOR ALL UNITARIES → P IN BRANCH DOES NOT CHANGE

$\Rightarrow P(|\phi\rangle) =$ SOME FUNCTION OF $\langle\phi|\phi\rangle$

- SPLITTING OF BRANCHES CONSERVES P

$$\Rightarrow P(\underline{(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)} \otimes |\chi_0\rangle) = P(\alpha|\uparrow\rangle|\chi_\uparrow\rangle) + P(\beta|\downarrow\rangle|\chi_\downarrow\rangle)$$

$$P(|\phi_\uparrow\rangle + |\phi_\downarrow\rangle) = P(|\phi_\uparrow\rangle) + P(|\phi_\downarrow\rangle)$$

$$\Rightarrow \boxed{P(|\phi\rangle) = \langle\underline{\phi|}\phi\rangle}$$

$\Rightarrow$ BORN, AGAIN!

Lecture 10

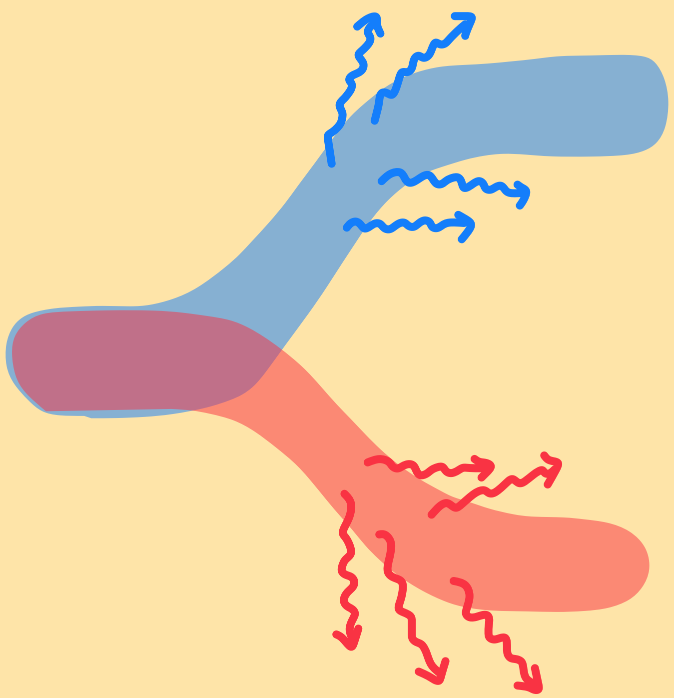
Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

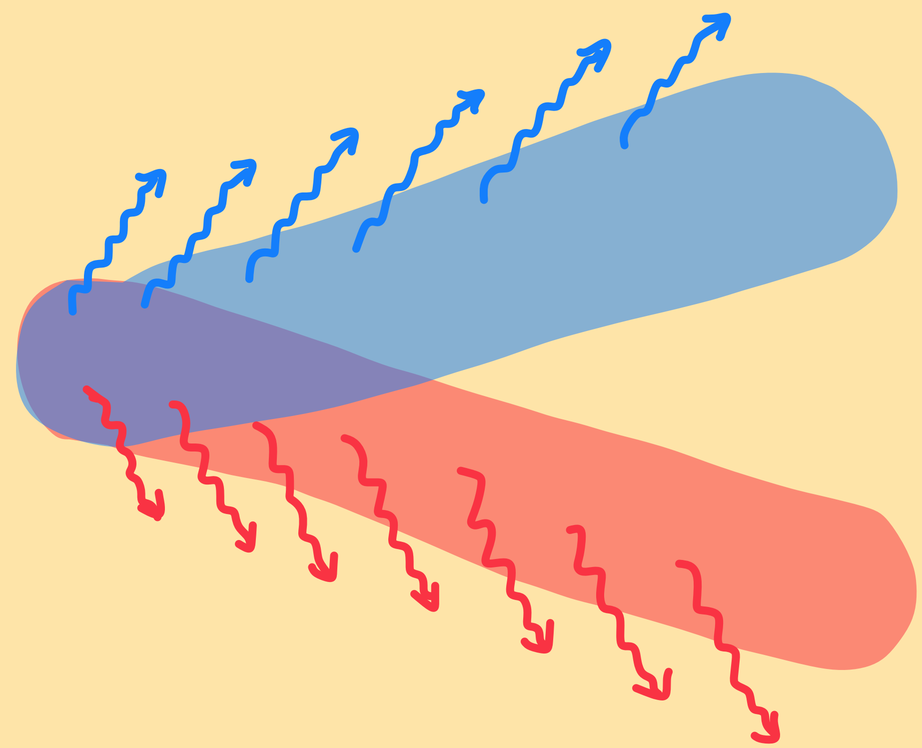
Start at 6pm CET

3.4 WEAK MEASUREMENTS

(TIME-RESOLVING REALISTIC MEASUREMENTS)

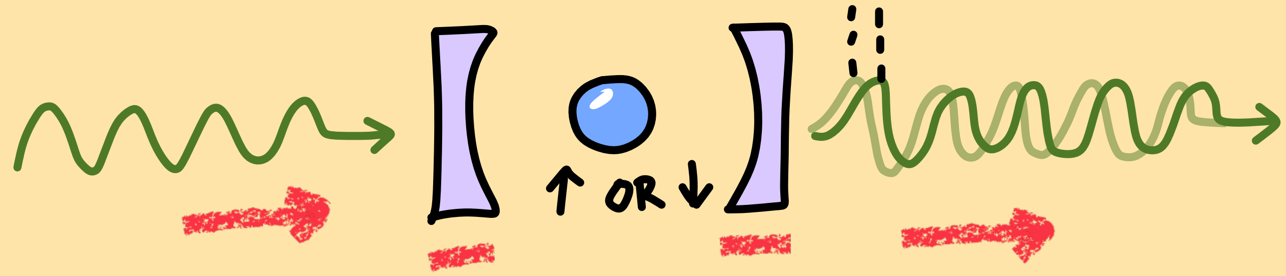


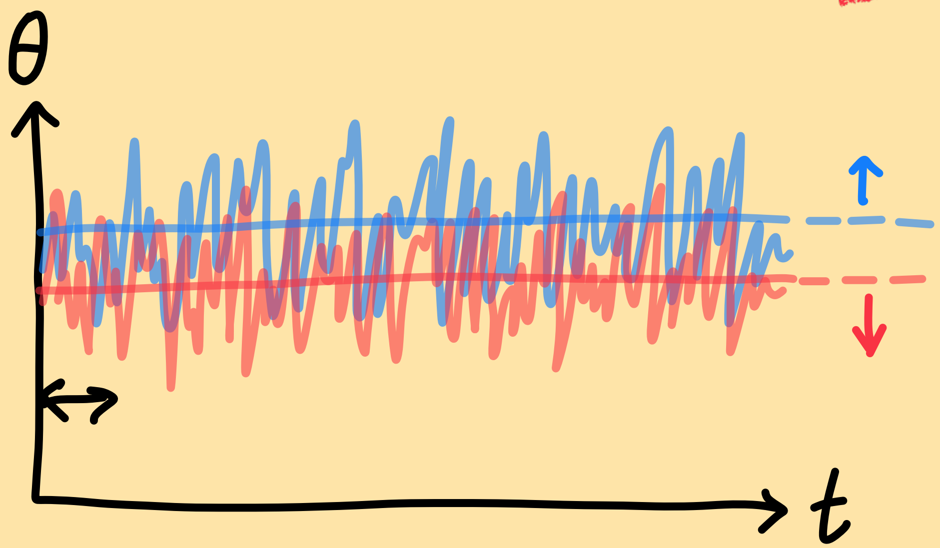
STERN-GERLACH:
FIRST $\langle \chi_\uparrow | \chi_\downarrow \rangle \to 0$
THEN IRREVERSIBLE



WEAK MEASUREMENT:
$\langle \chi_\uparrow | \chi_\downarrow \rangle \to 0$ AND
IRREVERSIBLE ACCUMULATION
OF INFORMATION SIMULTANEOUSLY

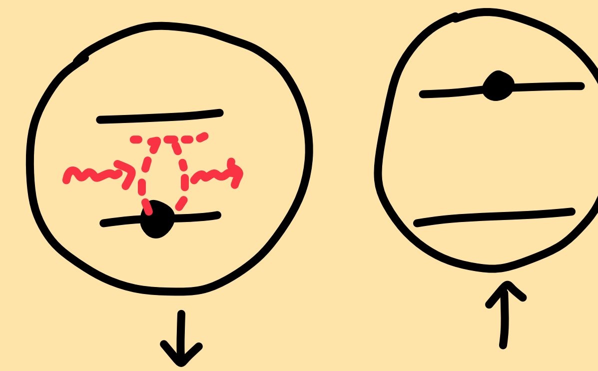
PHASE SHIFT $\theta = \underbrace{\text{SIGNAL}}_{\text{DEPENDS ON STATE}} + \text{NOISE}$

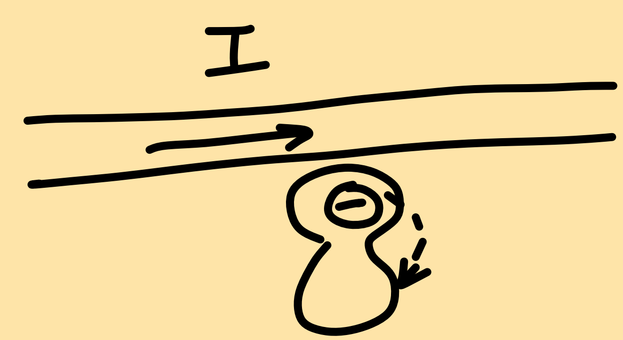




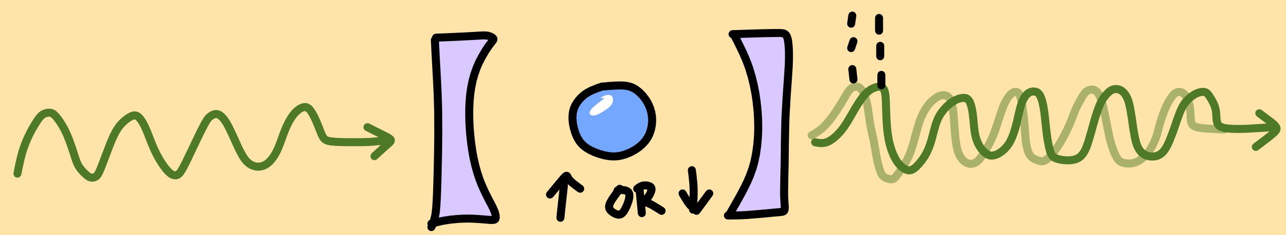
NOISE PREVENTS IMMEDIATE IDENTIFICATION OF STATE ↑ vs ↓

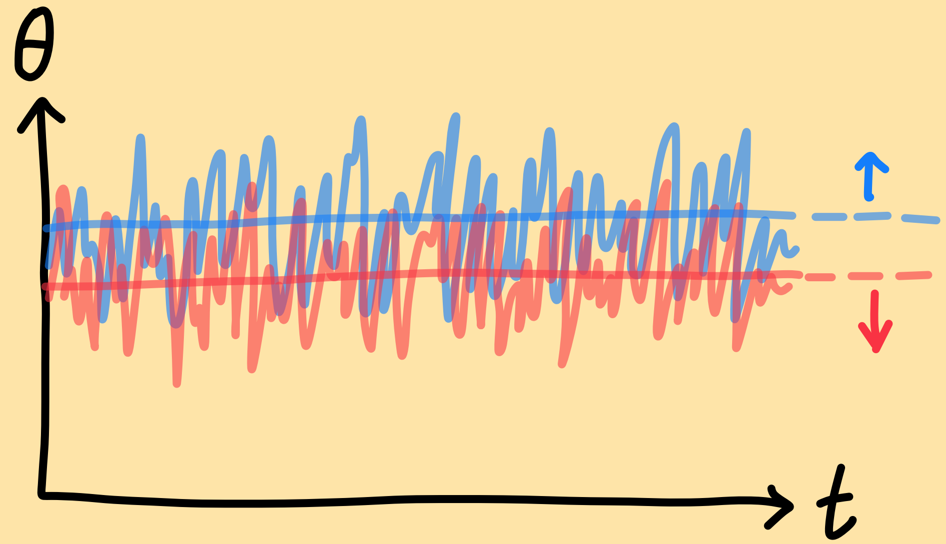
⇒ NEED TIME-INTEGRAL (OR AVERAGE)





PHASE SHIFT $\theta = \underbrace{\text{SIGNAL}}_{\text{DEPENDS ON STATE}} + \text{NOISE}$

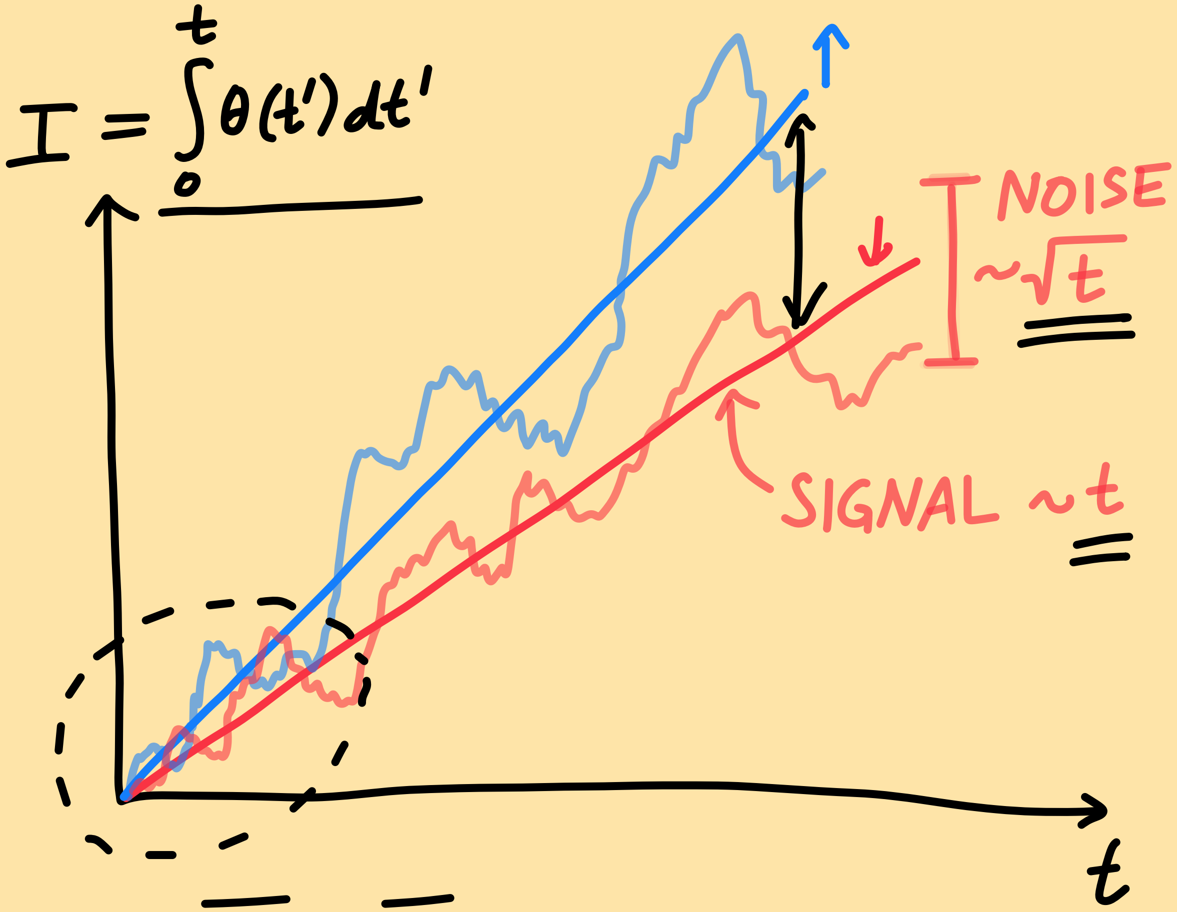




NOISE PREVENTS IMMEDIATE IDENTIFICATION OF STATE ↑ vs ↓

⇒ NEED TIME-INTEGRAL (OR AVERAGE)

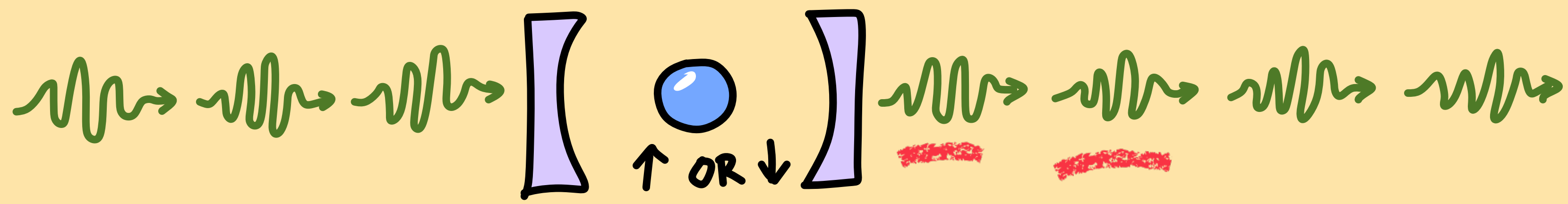
$$I = \int_0^t \theta(t')\,dt'$$



$$\langle \chi_\uparrow(t) | \chi_\downarrow(t) \rangle \longrightarrow 0 \quad \text{AS} \quad I_\uparrow - I_\downarrow > \text{NOISE}$$

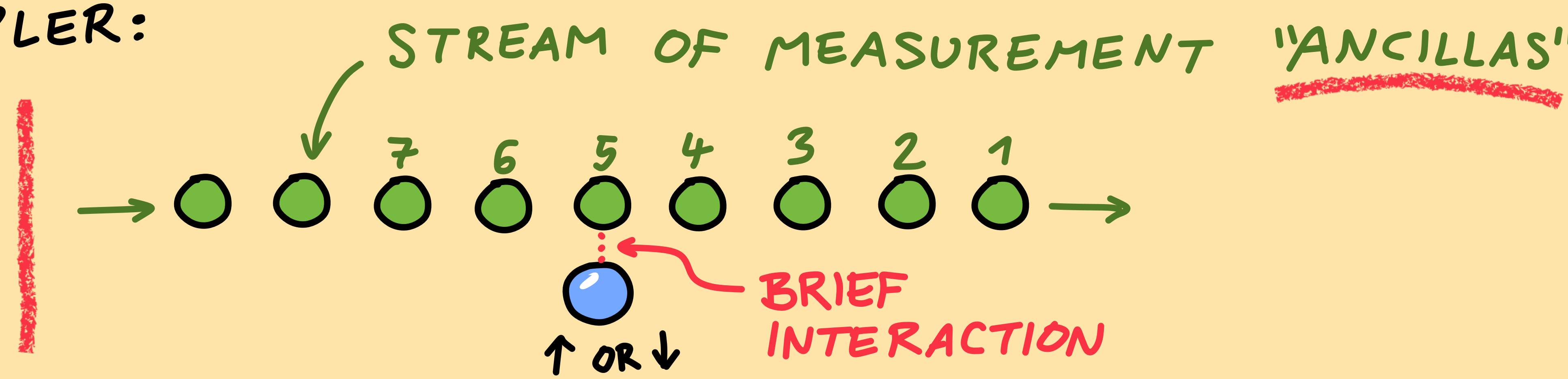
(APPROACH: "STOCHASTIC MASTER EQUATIONS")

SIMPLIFIED PICTURE:



"QUANTUM NON-DEMOLITION MSMT"

EVEN SIMPLER:



$$(\alpha|\uparrow\rangle+\beta|\downarrow\rangle)\otimes|\chi_1^{(0)}\rangle\otimes|\chi_2^{(0)}\rangle\otimes|\chi_3^{(0)}\rangle\otimes\dots$$

$$(\alpha|\uparrow\rangle\otimes|\chi_1^{\uparrow}\rangle+\beta|\downarrow\rangle\otimes|\chi_1^{\downarrow}\rangle)\otimes|\chi_2^{(0)}\rangle\otimes|\chi_3^{(0)}\rangle\otimes\dots$$

$$(\alpha|\uparrow\rangle\otimes|\chi_1^{\uparrow}\rangle\otimes|\chi_2^{\uparrow}\rangle+\beta|\downarrow\rangle\otimes|\chi_1^{\downarrow}\rangle\otimes|\chi_2^{\downarrow}\rangle)\otimes|\chi_3^{(0)}\rangle\otimes\dots$$

AND SO ON...

$$|\langle\chi_j^{\uparrow}|\chi_j^{\downarrow}\rangle|\approx 1$$

$\Rightarrow$ "WEAK" MSMT

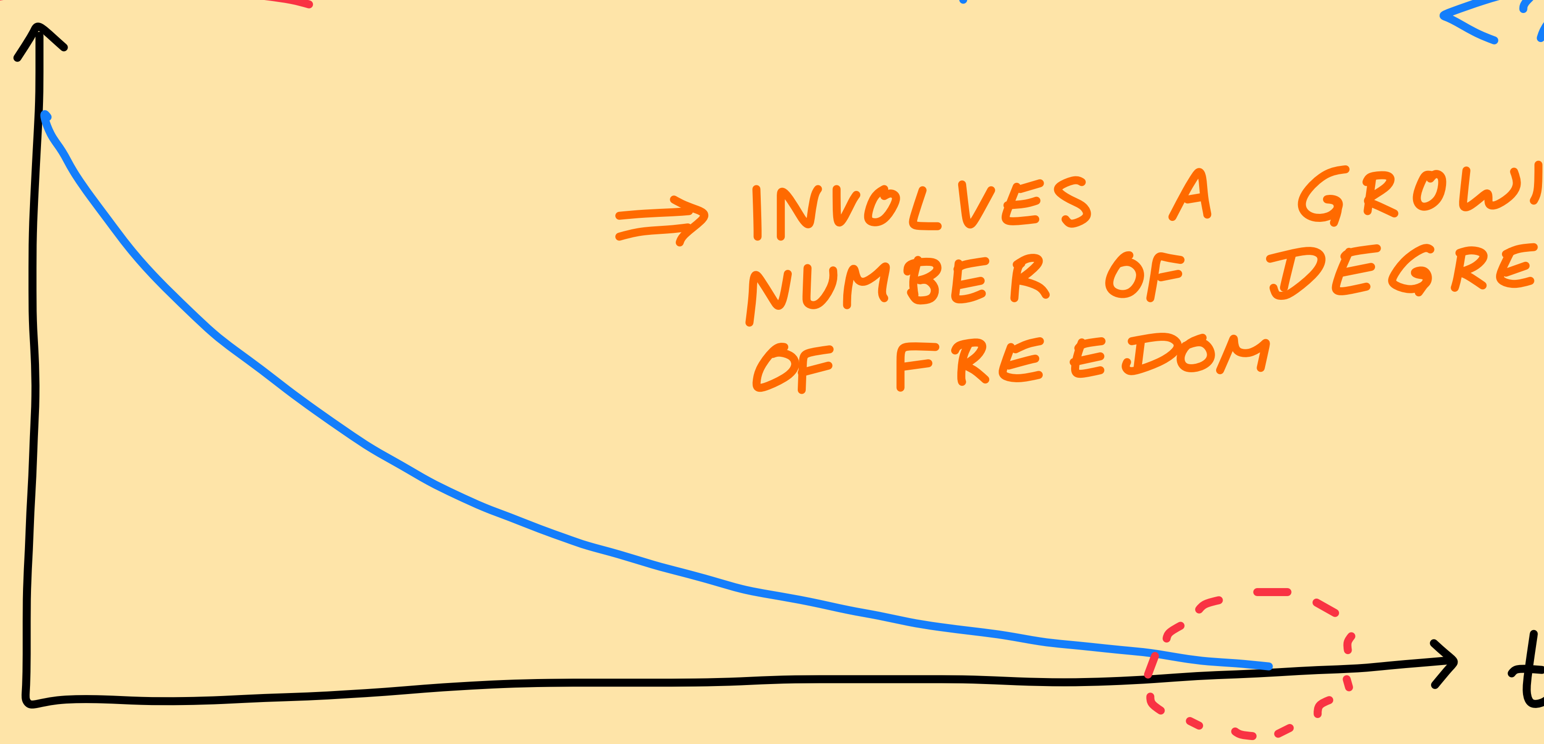
$$\underbrace{|\chi^{\uparrow}(t)\rangle}_{\substack{\text{FULL STATE} \\ \text{OF MSMT} \\ \text{APPARATUS} \\ \text{(= STRING OF} \\ \text{ANCILLAS)}}} = \underbrace{\prod_{t' \leq t} |\chi^{\uparrow}_{t'}\rangle}_{\substack{\text{ALREADY} \\ \text{INTERACTED} \\ \text{WITH SYSTEM}}} \otimes \underbrace{\prod_{t' > t} |\chi^{(0)}_{t'}\rangle}_{\substack{\text{STILL} \\ \text{UNAFFECTED}}}$$

$$\Rightarrow \quad \langle \chi^{\uparrow}(t) | \chi^{\downarrow}(t) \rangle = \prod_{t' \leq t} \langle \chi^{\uparrow}_{t'} | \chi^{\downarrow}_{t'} \rangle$$

$$= e^{-\Gamma t}$$

(IF ALL OVERLAPS IDENTICAL, $\langle \chi^{\uparrow}_{t} | \chi^{\downarrow}_{t} \rangle \equiv e^{-\Gamma}$)

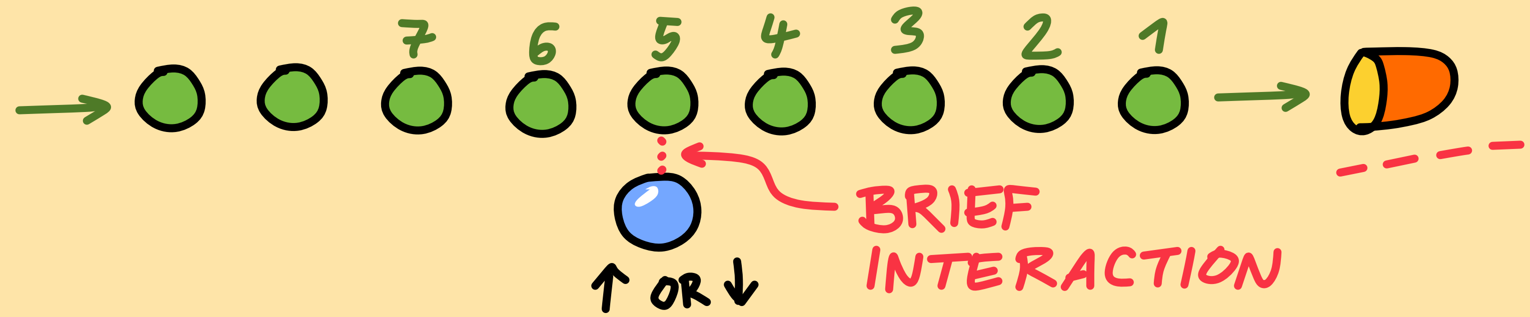
$\langle \chi^{\uparrow}(t) | \chi^{\downarrow}(t) \rangle$



$\Rightarrow$ INVOLVES A GROWING NUMBER OF DEGREES OF FREEDOM

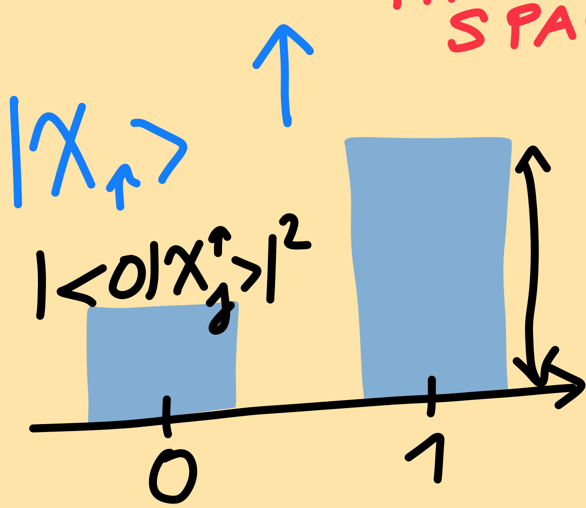
"MEASUREMENT TRAJECTORY" (OUTCOME)

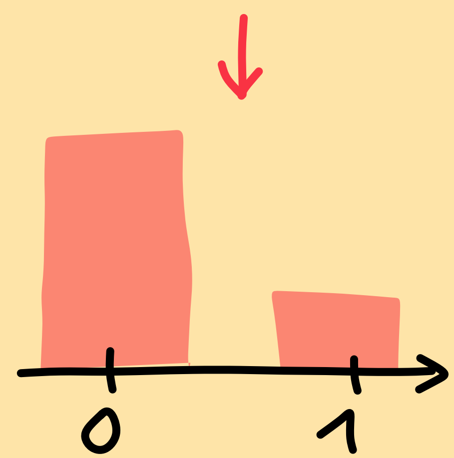
IMAGINE STRONG (PROJECTIVE) MSMT OF ANCILLAS:

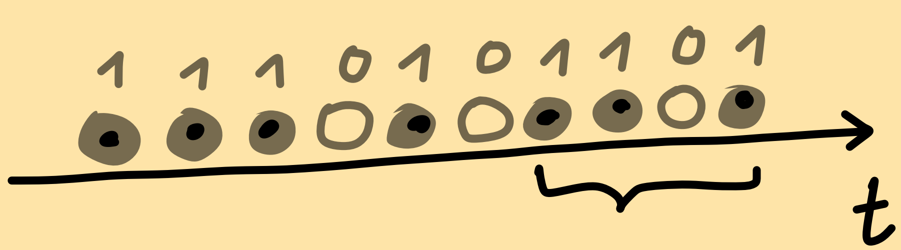


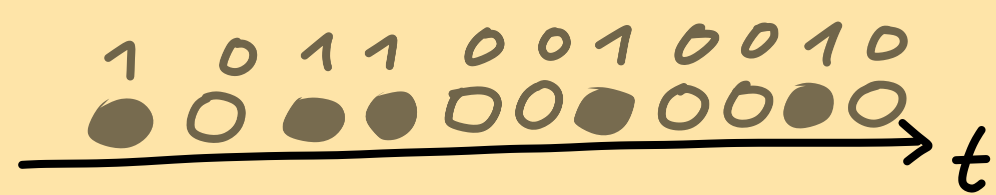
FOR SUITABLE MSMT BASIS: $\underbrace{\{|0\rangle, |1\rangle\}}_{\text{ANCILLA HILBERT SPACE}}$

<u>SLIGHT</u> DIFFERENCE IN PROBABILITIES FOR ↑ vs ↓









3.5 NON-COMMUTING MEASUREMENTS AND TIME ORDER: LEGGETT-GARG INEQUALITY ("TEMPORAL BELL INEQUALITY")

"CAN WE RULE OUT A CERTAIN CLASS OF HIDDEN-VARIABLE THEORIES VIA MEASUREMENTS ON A <u>SINGLE</u> QUANTUM SYSTEM?"

QM: OUTCOMES DEPEND ON <u>ORDER</u> OF MSMTS

$$\hat{x} \quad \hat{p} \quad \neq \quad \hat{p} \quad \hat{x}$$

↳ "SCRAMBLES $\hat{x}$"

$$\hat{S}_x(t_1) \quad \hat{S}_x(t_3) \quad \neq \quad \hat{S}_x(t_1) \quad \hat{S}_z(t_2) \quad \hat{S}_x(t_3)$$

CONTRAST CLASSICAL PICTURE:

(A1) AN OBSERVABLE $Q(t)$ HAS A DEFINITE VALUE AT ANY TIME

(A2) $Q(t)$ CAN BE MEASURED (IN PRINCIPLE) WITHOUT AFFECTING SUBSEQUENT MEASUREMENTS

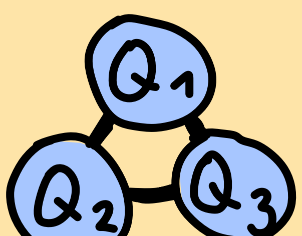
CONSIDER POSSIBLE MEASUREMENTS AT 3 TIMES:

$Q_1 \quad Q_2 \quad Q_3$ ASSUME $Q \in \{-1, +1\}$

$$(A1),(A2) \Rightarrow \boxed{1 + \langle Q_1 Q_2 \rangle + \langle Q_1 Q_3 \rangle + \langle Q_2 Q_3 \rangle \geq 0}$$

LEGGETT-GARG INEQUALITY (1985)

PROOF: L.H.S. $= 0$ OR 4 BEFORE AVERAGING



(NOT ALL $Q_i Q_j$ CAN BE -1 SIMULTANEOUSLY)

THIS INEQUALITY HOLDS REGARDLESS OF WHETHER ALL 3 ARE MEASURED OR ONLY THE 2 NEEDED FOR EACH $\langle Q_i Q_j \rangle$

THIS IS EASILY VIOLATED BY QUANTUM MECHANICS!

EXAMPLE: $\underline{\hat{Q} = \hat{\sigma}_x}$ $\qquad \hat{H} = \frac{\hbar\omega}{2}\hat{\sigma}_z$

$$\Rightarrow \hat{Q}(t) = \underline{\hat{\sigma}_x(t)} = \hat{\sigma}_x \cos(\omega t) - \hat{\sigma}_y \sin(\omega t)$$

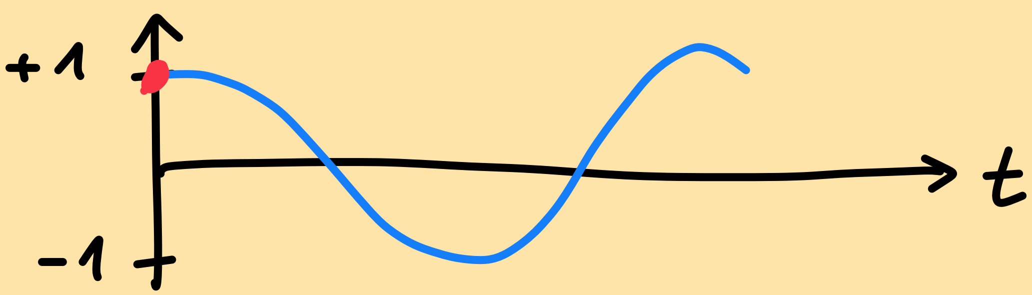
SOLVES $\underline{i\hbar \frac{d}{dt}\hat{\sigma}_x(t) = [\hat{\sigma}_x(t), \hat{H}(t)]}$

INITIAL STATE $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right)$

$\Rightarrow$

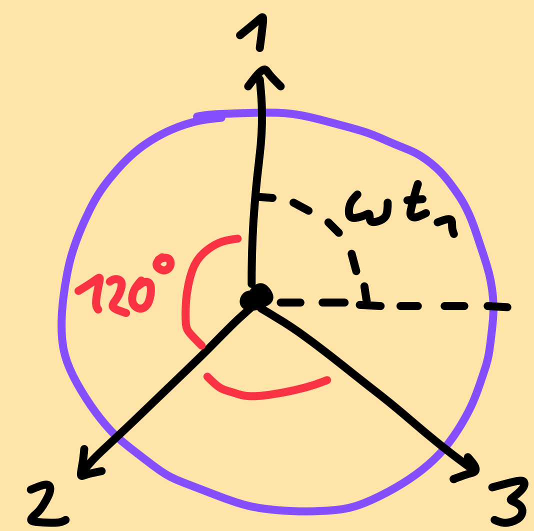
EXPECTATION VALUE:

$\langle \hat{Q}(t) \rangle = \cos(\omega t)$



CORRELATOR:

$\langle \hat{Q}(t)\, \hat{Q}(t') \rangle = \cos\left(\omega (t - t')\right)$



$\Rightarrow$ FOR SMART CHOICE OF t_1, t_2, t_3 WE GET

$$1 + \underbrace{\langle \hat{Q}_1 \hat{Q}_2 \rangle}_{-\frac{1}{2}} + \underbrace{\langle \hat{Q}_1 \hat{Q}_3 \rangle}_{-\frac{1}{2}} + \underbrace{\langle \hat{Q}_2 \hat{Q}_3 \rangle}_{-\frac{1}{2}} < 0$$

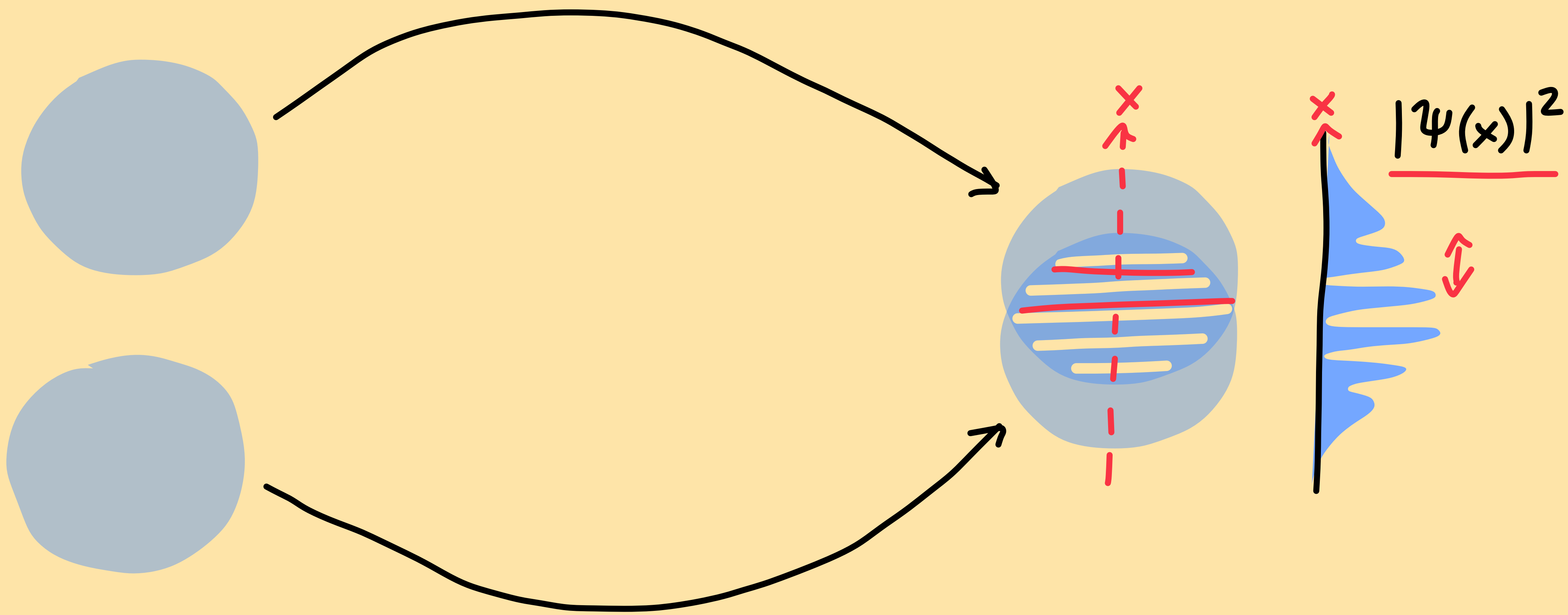
QM ↯ A1, A2

L.G. NOT AS POWERFUL AS BELL:

(A2) "NO PERTURBATION BY MSMT" IS A VERY STRONG ASSUMPTION!

4. DECOHERENCE

"LOSS OF INTERFERENCE"

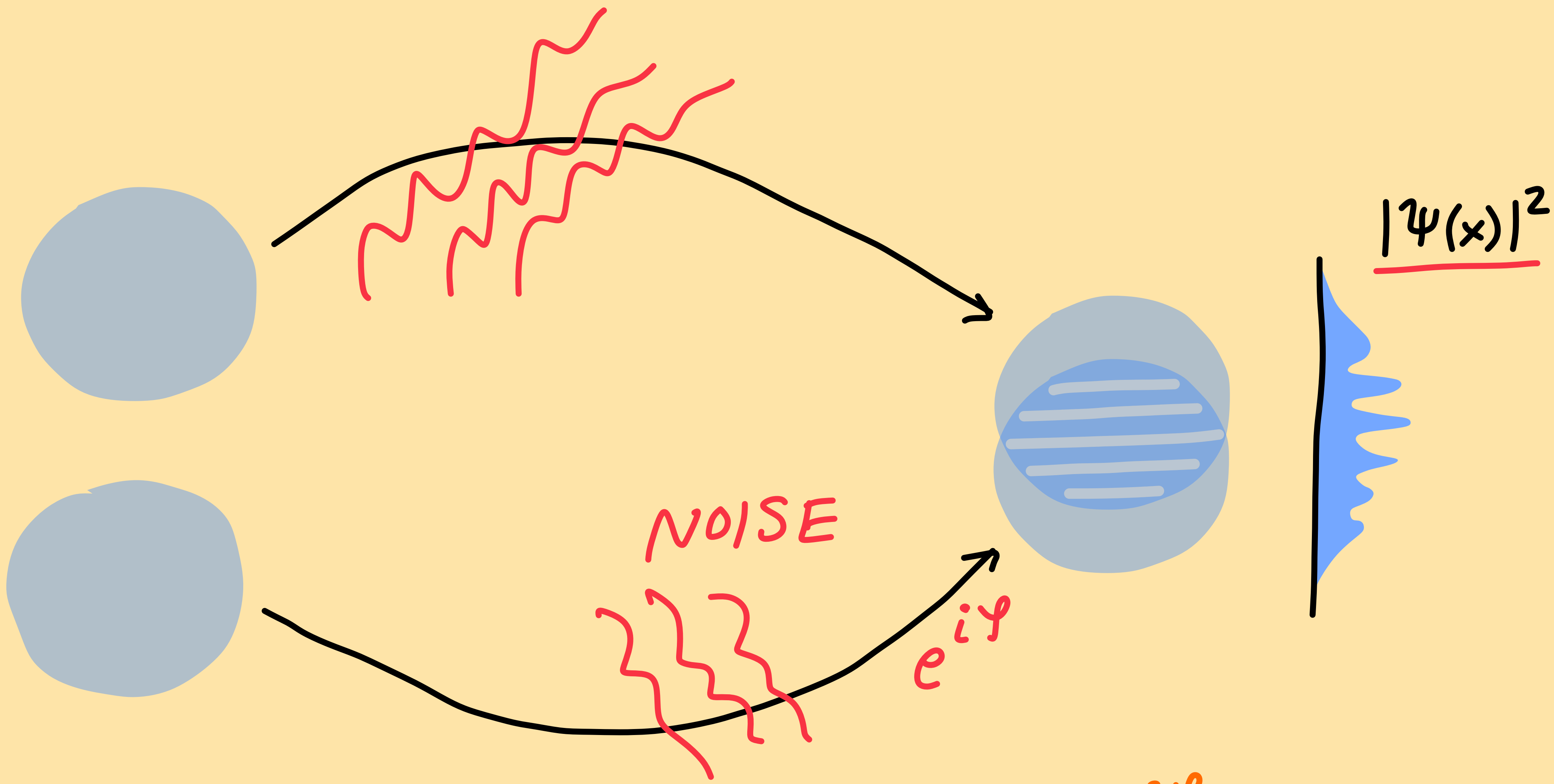


$$|\psi(x)|^2 = \frac{1}{2}|\psi_\uparrow(x)|^2 + \frac{1}{2}|\psi_\downarrow(x)|^2 + \underbrace{\frac{1}{2}\psi_\uparrow^*(x)\psi_\downarrow(x) + \frac{1}{2}\psi_\uparrow(x)\psi_\downarrow^*(x)}_{\text{INTERFERENCE TERMS, SENSITIVE TO RELATIVE PHASE}}$$

$$\psi(x) = \frac{1}{\sqrt{2}}\left(\psi_\uparrow(x) + \psi_\downarrow(x)\right)$$

4. DECOHERENCE

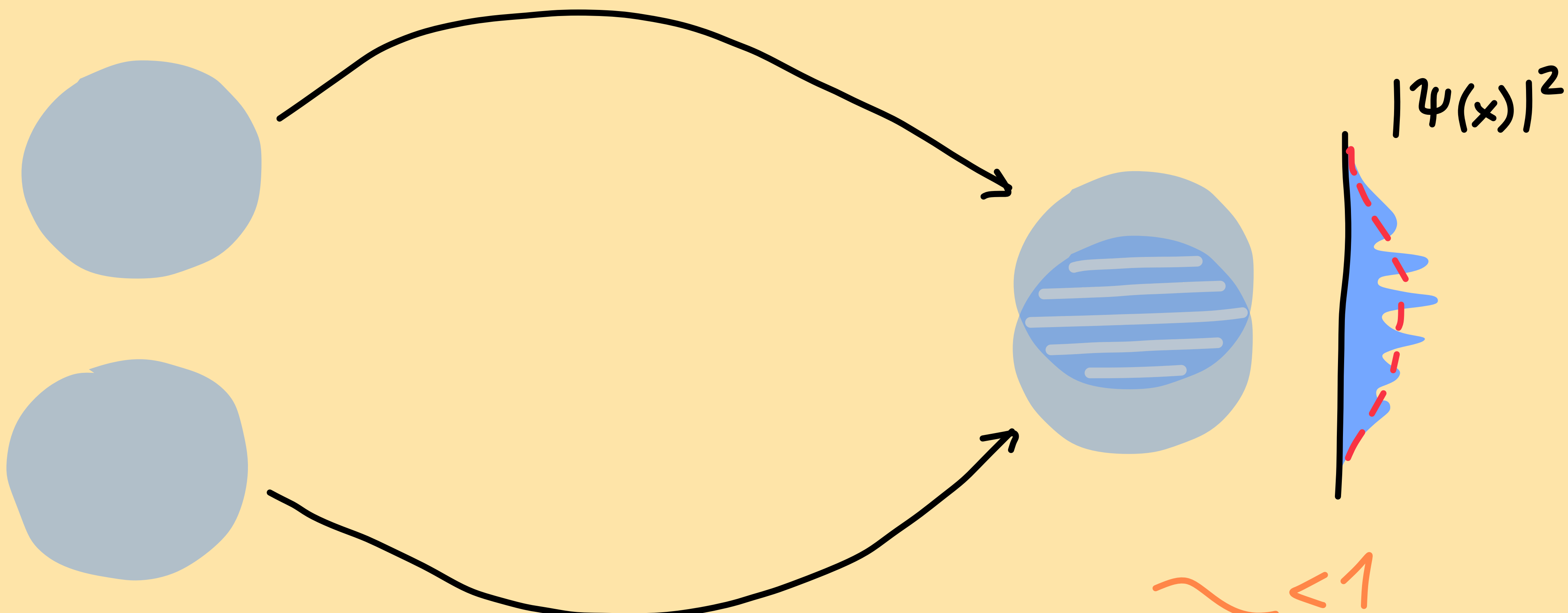
"LOSS OF INTERFERENCE"



$$|\psi(x)|^2 = \frac{1}{2}|\psi_\uparrow(x)|^2 + \frac{1}{2}|\psi_\downarrow(x)|^2 + \underbrace{\frac{1}{2}\psi_\uparrow^*(x)\psi_\downarrow(x)e^{i\varphi} + \frac{1}{2}\psi_\uparrow(x)\psi_\downarrow^*(x)e^{-i\varphi}}_{\text{INTERFERENCE TERMS, SENSITIVE TO RELATIVE PHASE}}$$

4. DECOHERENCE

"LOSS OF INTERFERENCE"



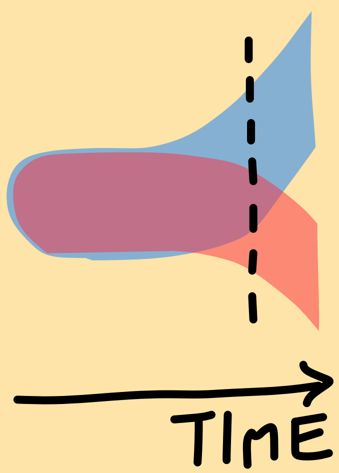
$$|\psi(x)|^2 = \frac{1}{2}|\psi_\uparrow(x)|^2 + \frac{1}{2}|\psi_\downarrow(x)|^2 + \underbrace{\frac{1}{2}\psi_\uparrow^*(x)\psi_\downarrow(x)\,\overbrace{\overline{e^{i\varphi}}}^{<1} + \frac{1}{2}\psi_\uparrow(x)\psi_\downarrow^*(x)\,\overline{e^{-i\varphi}}}$$

SUPPRESSED $\Longleftarrow$ INTERFERENCE TERMS, SENSITIVE TO RELATIVE PHASE

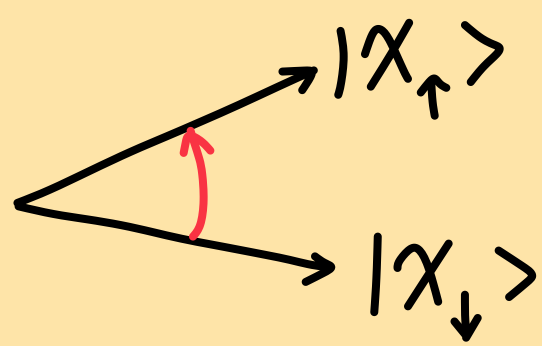
4.1 DECOHERENCE vs INFORMATION GAIN

INTERMEDIATE STATE DURING MEASUREMENT:

$$|\psi\rangle = \alpha\,\underline{|\uparrow\rangle} \otimes \underline{|\chi_\uparrow\rangle} + \beta\,\underline{|\downarrow\rangle} \otimes \underline{|\chi_\downarrow\rangle}$$



$\langle \chi_\uparrow | \chi_\downarrow \rangle \neq 0 \Rightarrow$ POINTER STATES NOT ORTHOGONAL



QUESTIONS:

- HOW MUCH ARE INTERFERENCE EFFECTS SUPPRESSED?
- HOW MUCH DID WE LEARN ABOUT THE SYSTEM STATE?

HOW MUCH ARE INTERFERENCE EFFECTS SUPPRESSED?

LOOK AT OBSERVABLES SENSITIVE TO RELATIVE PHASE BETWEEN $|\uparrow\rangle$ & $|\downarrow\rangle$

$\Rightarrow$ $\hat{\sigma}_x$ OR $\hat{\sigma}_y$ (BUT NOT $\hat{\sigma}_z$)

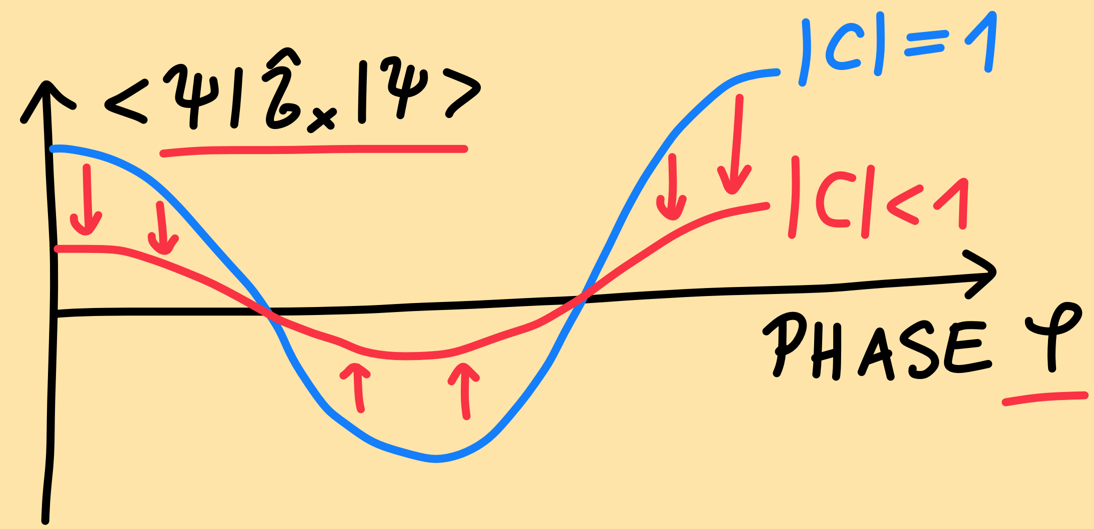
$\hat{\sigma}_x |\uparrow\rangle = |\downarrow\rangle$

$$\langle \psi | \hat{\sigma}_x | \psi \rangle = \alpha^* \beta \underbrace{\langle \chi_\uparrow | \chi_\downarrow \rangle}_{C} + \alpha \beta^* \underbrace{\langle \chi_\downarrow | \chi_\uparrow \rangle}_{C^*}$$

"COHERENCE FACTOR" $\equiv$ OVERLAP OF POINTER STATES

$|C| < 1 \Rightarrow$ INTERFERENCE SUPPRESSED!

LET $\alpha^* \beta = |\alpha\beta| e^{i\varphi}$



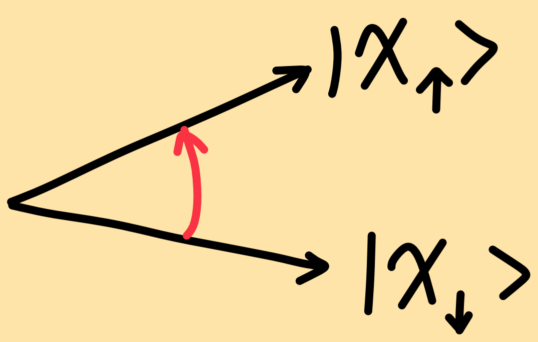
HOW MUCH DID WE LEARN ABOUT THE SYSTEM STATE ?

→ INDIRECTLY, VIA THE MSMT APPARATUS STATE!

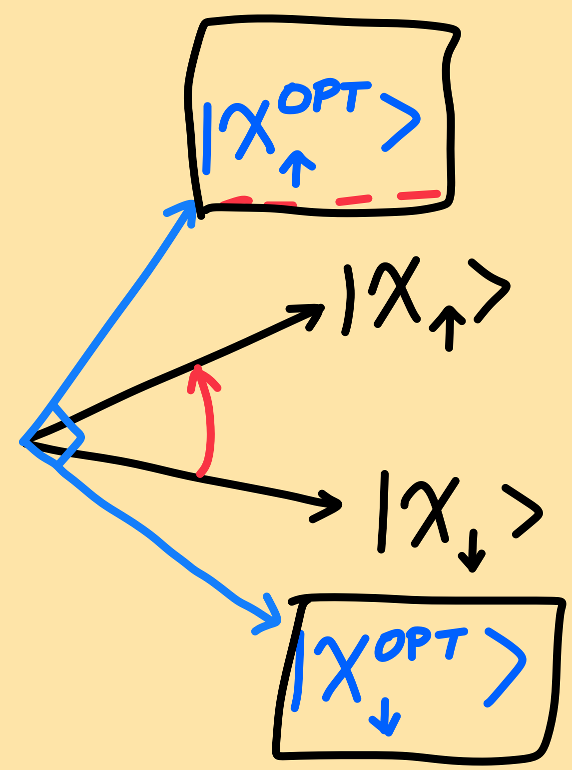
IDEA: IMAGINE PROJECTIVE MSMT OF APPARATUS
⇒ "HOW MUCH DOES THIS REVEAL ABOUT SYSTEM?"

CHALLENGE: POINTER STATES NOT ORTHOGONAL
→ <u>NOT</u> PERFECTLY CORRELATED WITH SPIN!

CHALLENGE: CHOOSE <u>OPTIMAL</u> ORTHOGONAL BASIS FOR MSMT OF APPARATUS!



CHALLENGE: CHOOSE OPTIMAL ORTHOGONAL BASIS FOR MSMT OF APPARATUS!



(ASSUMING $C = \underline{\langle \chi_\uparrow | \chi_\downarrow \rangle} \in \mathbb{R}$)

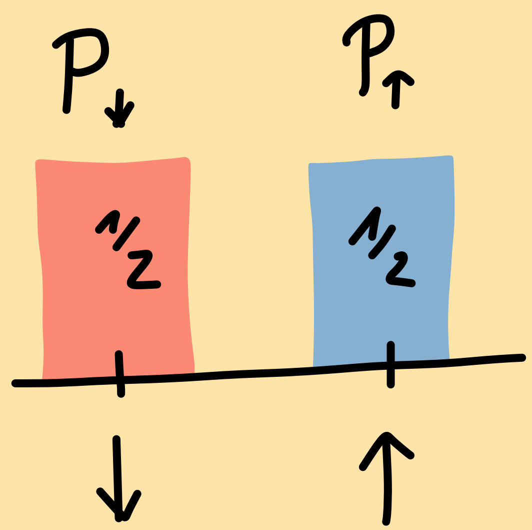
$$|\chi_\uparrow^{OPT}\rangle = \frac{1}{\sqrt{2}} \left\{ \frac{|\chi_+\rangle}{\| \, |\chi_+\rangle \|} + \frac{|\chi_-\rangle}{\| \, |\chi_-\rangle \|} \right\} \qquad |\chi_\pm\rangle = |\chi_\uparrow\rangle \pm |\chi_\downarrow\rangle$$

EXAMPLE: WE FIND "$|\chi^{OPT}_{\uparrow}\rangle$"

$$|\psi\rangle \mapsto |\psi'_{SYS}\rangle = \frac{\langle\chi^{OPT}_{\uparrow}|\psi\rangle}{\text{NORMALIZATION}}$$

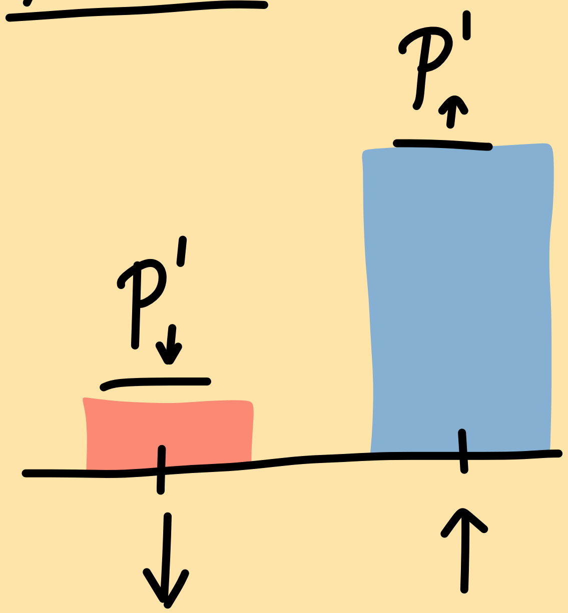
$\Rightarrow$ WHAT HAPPENS TO THE SYSTEM STATE?

BEFORE



(EXAMPLE: $|\alpha|^2 = |\beta|^2 = \frac{1}{2}$)

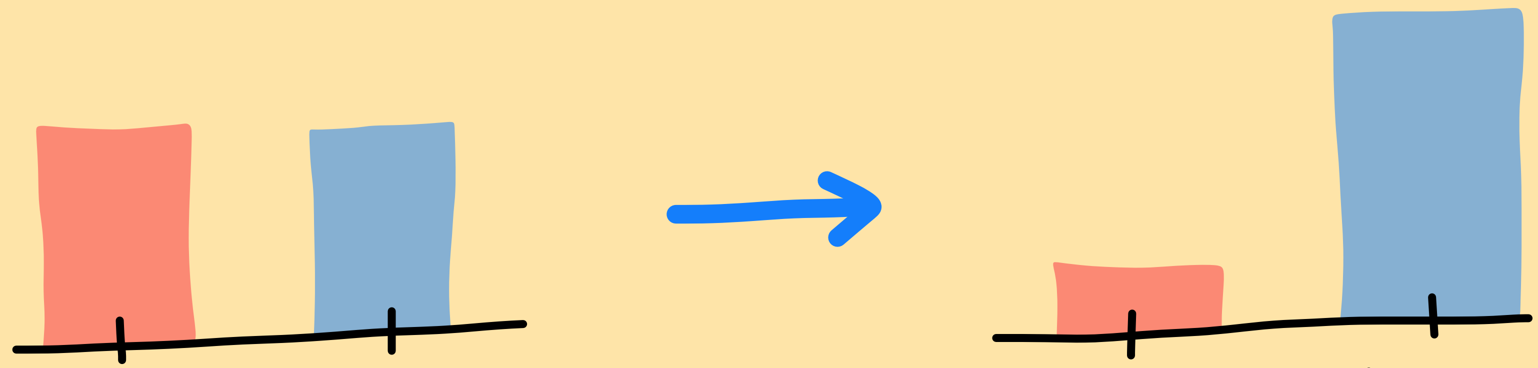
AFTER



$$P'_{\uparrow} = \frac{1}{2}\left(1 + \sqrt{1 - C^2}\right)$$

NOW WE KNOW MORE !

(BUT HOW MUCH MORE?)



$$P'_\uparrow = \frac{1}{2}\left(1 + \sqrt{1-C^2}\right)$$

QUANTIFY INFORMATION GAIN !

GENERAL IDEA: MEASURE IGNORANCE BY ENTROPY

$$S = -\sum_j P_j \ln P_j$$

(IS MAXIMAL FOR UNIFORM DISTRIBUTION)

"SHANNON ENTROPY"

INFORMATION GAIN = DECREASE OF ENTROPY

HERE: $\Delta S = \Delta S(C) = \Delta S(\langle \chi_\uparrow | \chi_\downarrow \rangle)$

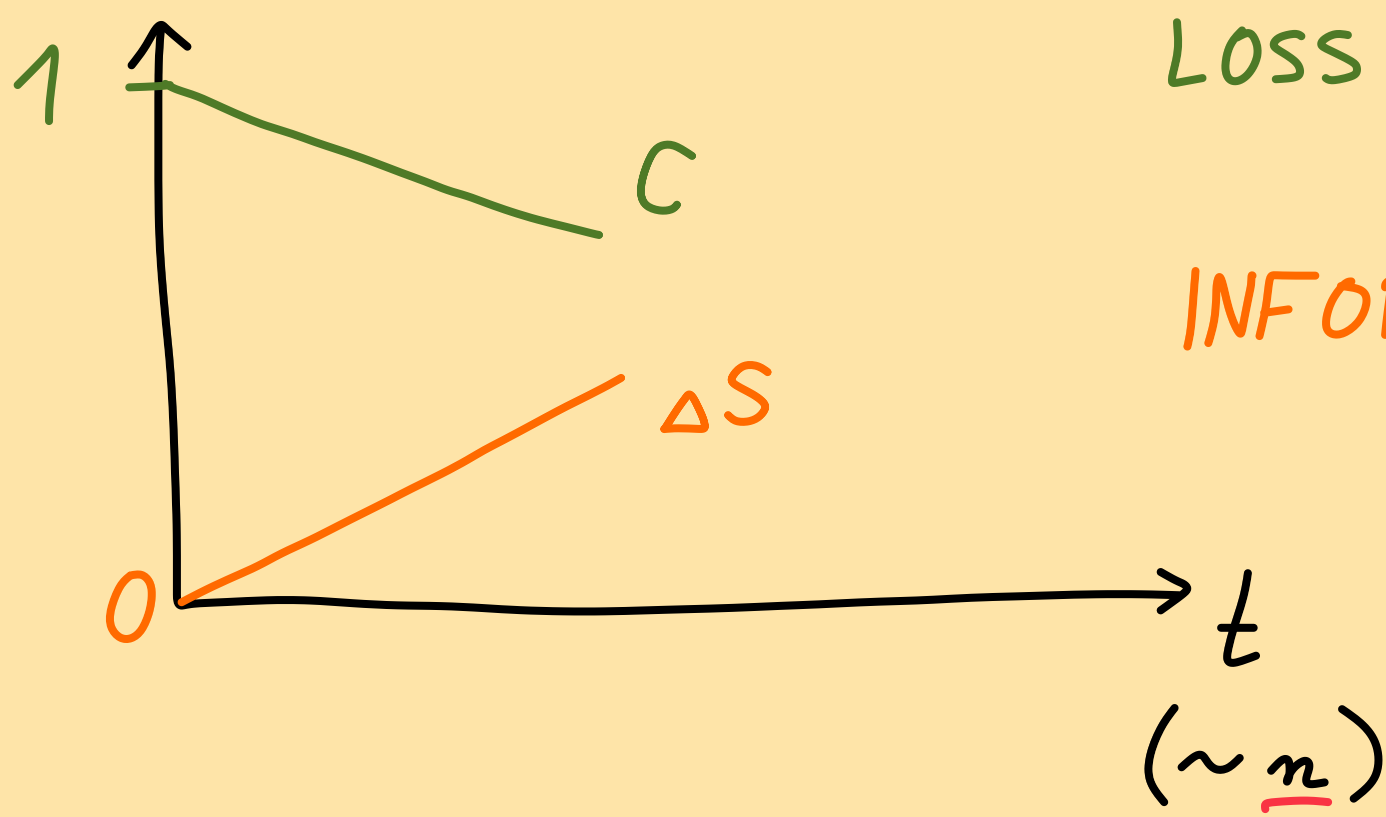
$\Rightarrow$ DIRECT RELATION BETWEEN INFORMATION GAIN AND DECOHERENCE!

FOR WEAK MSMT : $C = \langle \chi_\uparrow | \chi_\downarrow \rangle = \cos\theta \approx 1 - \frac{\theta^2}{2}$

$$\Rightarrow \quad \Delta S \approx \frac{\theta^2}{2}$$

MANY LITTLE STEPS $\Rightarrow$ $C \approx \left(1 - \frac{\theta^2}{2}\right)^n \approx 1 - n\frac{\theta^2}{2}$

$$\Delta S \approx n\frac{\theta^2}{2}$$



LOSS OF COHERENCE

$\Updownarrow$

INFORMATION GAIN

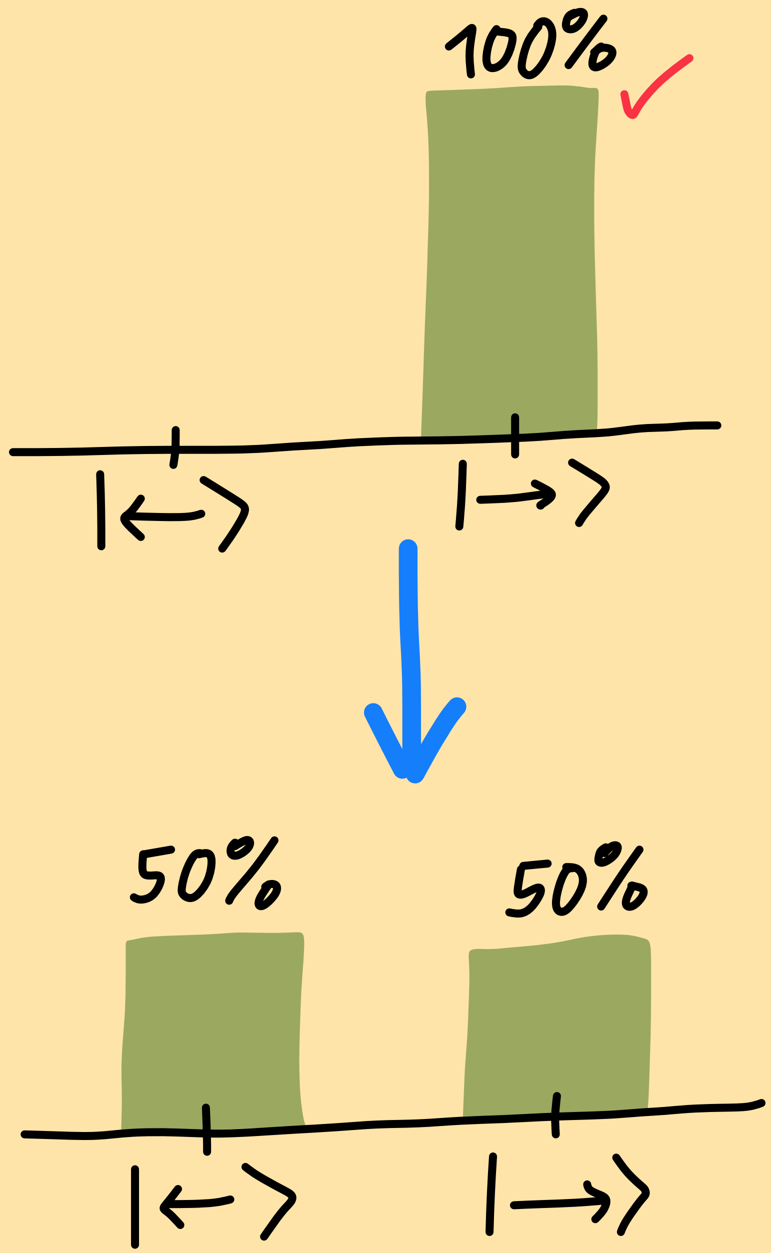
DECOHERENCE RATE $\geq$ MEASUREMENT RATE

(DECAY OF C) (DECREASE OF S)

SLIGHTLY DIFFERENT PERSPECTIVE

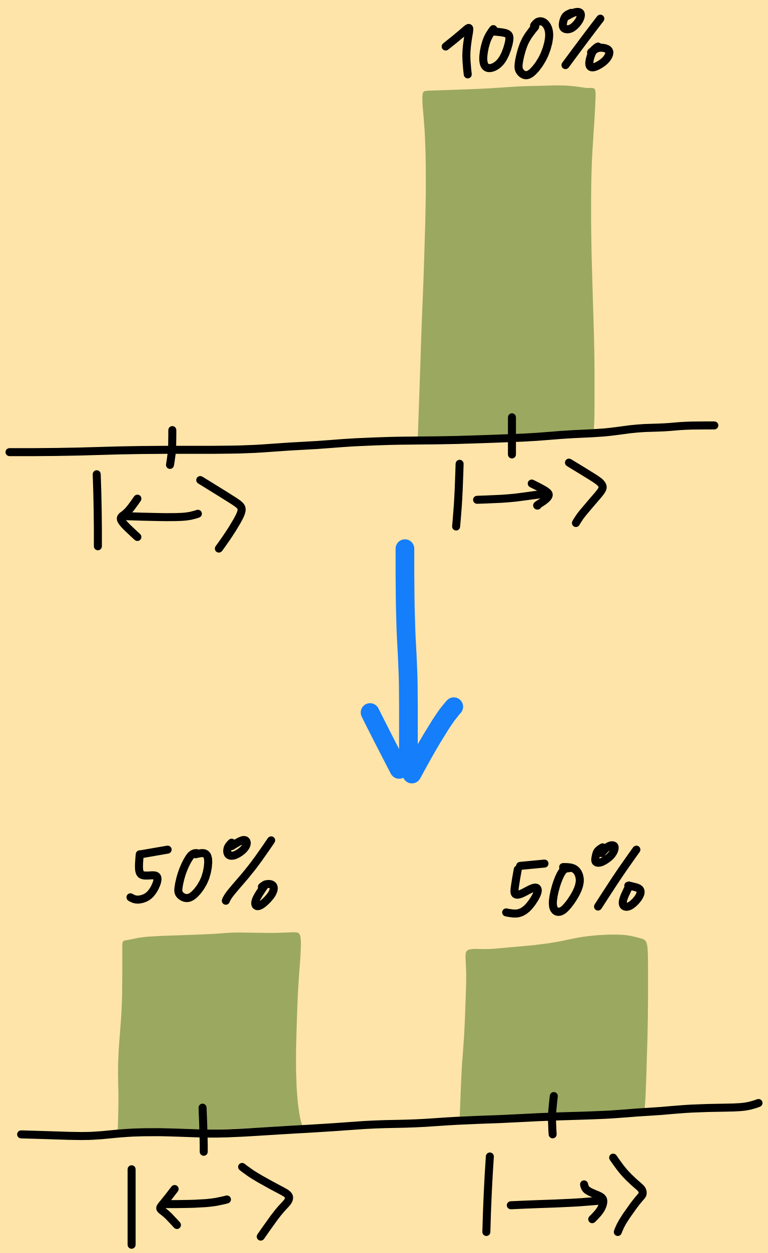
(RELATION TO SYSTEM-APPARATUS ENTANGLEMENT)

LET $\alpha = \beta = \frac{1}{\sqrt{2}} \Rightarrow$ SYSTEM IN STATE $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \equiv |\rightarrow\rangle$ ×



SLIGHTLY DIFFERENT PERSPECTIVE
(RELATION TO SYSTEM-APPARATUS ENTANGLEMENT)

LET $\alpha = \beta = \frac{1}{\sqrt{2}} \Rightarrow$ SYSTEM IN STATE $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \equiv |\rightarrow\rangle$

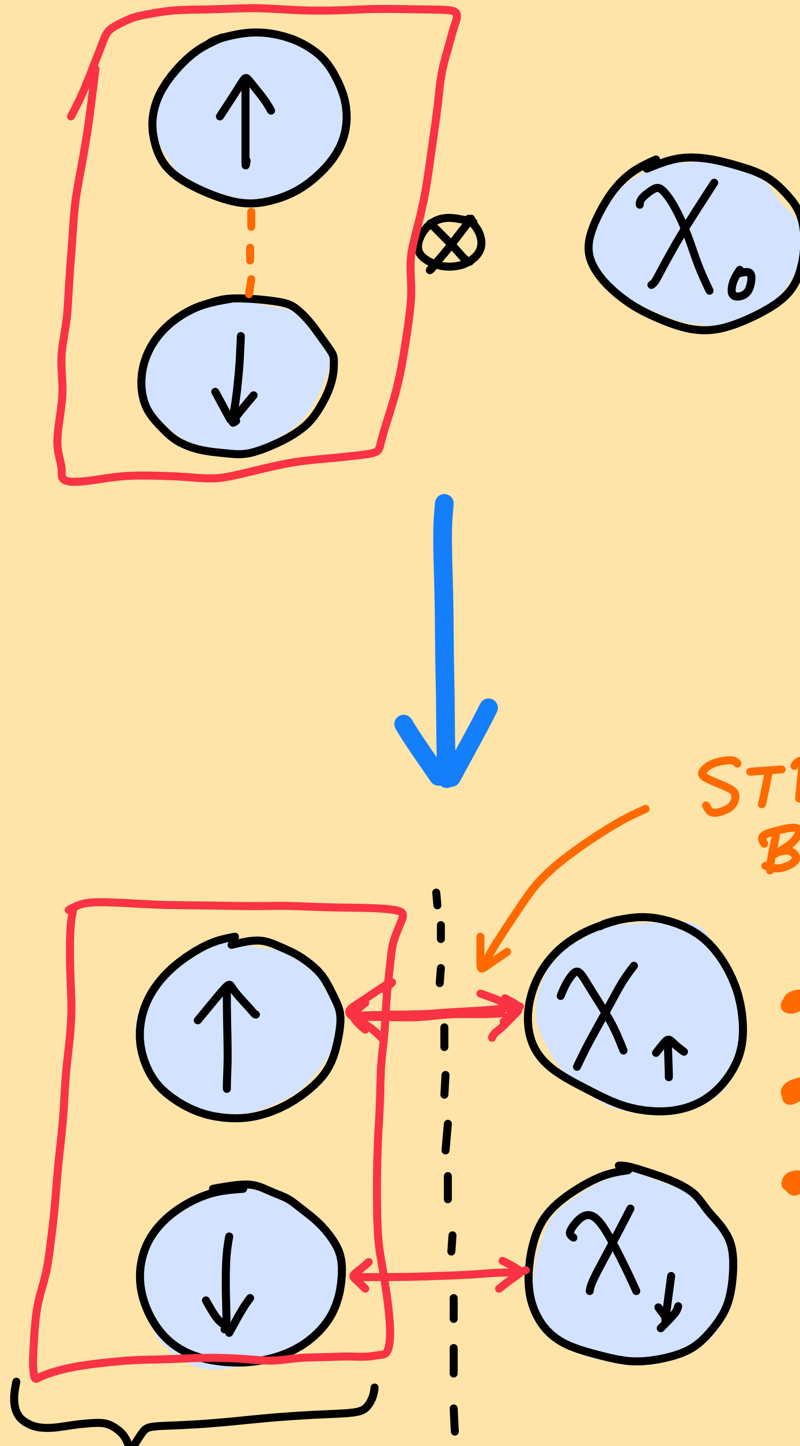


SYSTEM STATE (NOT CONDITIONED ON APPARATUS STATE) BECOMES INCOHERENT MIXTURE

$\Rightarrow$ ITS ENTROPY INCREASES

SYSTEM STATE CONDITIONED ON APPARATUS STATE BECOMES MORE CERTAIN

$\Rightarrow$ THAT ENTROPY DECREASES
(SEE DISCUSSION BEFORE)



STRONG CORRELATIONS BETWEEN APPARATUS & SYSTEM $\Rightarrow$

- INFORMATION GAIN LARGE
- LOW CONDITIONAL ENTROPY
- SYS+APPARATUS ENTANGLED

SUB-SYSTEM INCOHERENT
$\Rightarrow$ ENTROPY HIGH

$\Rightarrow$ A WAY TO QUANTIFY ENTANGLEMENT (FOR PURE JOINT STATES)

(CALC. DETAILS)

$$\langle \chi^{OPT} | \psi \rangle = \frac{1}{2} \left\{ \frac{\langle \chi_+ | \psi \rangle}{\sqrt{1+C}} + \frac{\langle \chi_- | \psi \rangle}{\sqrt{1-C}} \right\}$$

$$\langle \chi_\pm | \chi_\pm \rangle = 2\,(1 \pm \underbrace{\langle \chi_\uparrow | \chi_\downarrow \rangle}_{\in \mathbb{R} \text{ (ASSUMED W.L.O.G.)}})$$

$$\langle \chi_+ | \psi \rangle = \alpha\,(1+C)\,|\uparrow\rangle + \beta\,(1+C)\,|\downarrow\rangle$$

$$\langle \chi_- | \psi \rangle = \alpha\,(1-C)\,|\uparrow\rangle + \beta\,(C-1)\,|\downarrow\rangle$$

$$\Rightarrow \langle \chi^{OPT} | \psi \rangle = \frac{1}{2} \left\{ \alpha \left(\sqrt{1+C} + \sqrt{1-C} \right) |\uparrow\rangle + \beta \left(\sqrt{1+C} - \sqrt{1-C} \right) |\downarrow\rangle \right\}$$

$$\Rightarrow \quad P'_\uparrow \sim \frac{|\alpha|^2}{4} \left(1 + \cancel{C} + 1 - \cancel{C} + 2\sqrt{1-C^2} \right) = \frac{|\alpha|^2}{2} \left(1 + \sqrt{1-C^2} \right)$$

$$P'_\downarrow \sim \frac{|\beta|^2}{4} \left(2 - 2\sqrt{1-C^2} \right) = \frac{|\beta|^2}{2} \left(1 - \sqrt{1-C^2} \right)$$

FOR $|\alpha|^2 = |\beta|^2 = \frac{1}{2}$:

$$P'_\uparrow = \frac{1 + \sqrt{1-C^2}}{2}$$

$$P'_\downarrow = \frac{1 - \sqrt{1-C^2}}{2}$$

$$\ln\left(1 \pm \sqrt{1-C^2}\right) \approx \pm \sqrt{\ldots} - \frac{1}{2}(\ldots)$$

$$S^{AFTER} = -P'_{\uparrow} \ln P'_{\uparrow} - P'_{\downarrow} \ln P'_{\downarrow}$$

$$\approx - P'_{\uparrow} \left(-\ln 2 + \sqrt{} - \frac{1}{2}(..)\right) - P'_{\downarrow} \left(-\ln 2 - \sqrt{..} - \frac{1}{2}(..)\right)$$

$$= \ln 2 - \sqrt{} \underbrace{\left(P'_{\uparrow} - P'_{\downarrow}\right)}_{\sqrt{}} + \frac{1}{2}(..)$$

$$= \underbrace{\ln 2}_{S^{BEFORE}} + \frac{1}{2}(1 - C^2)$$

Lecture 11

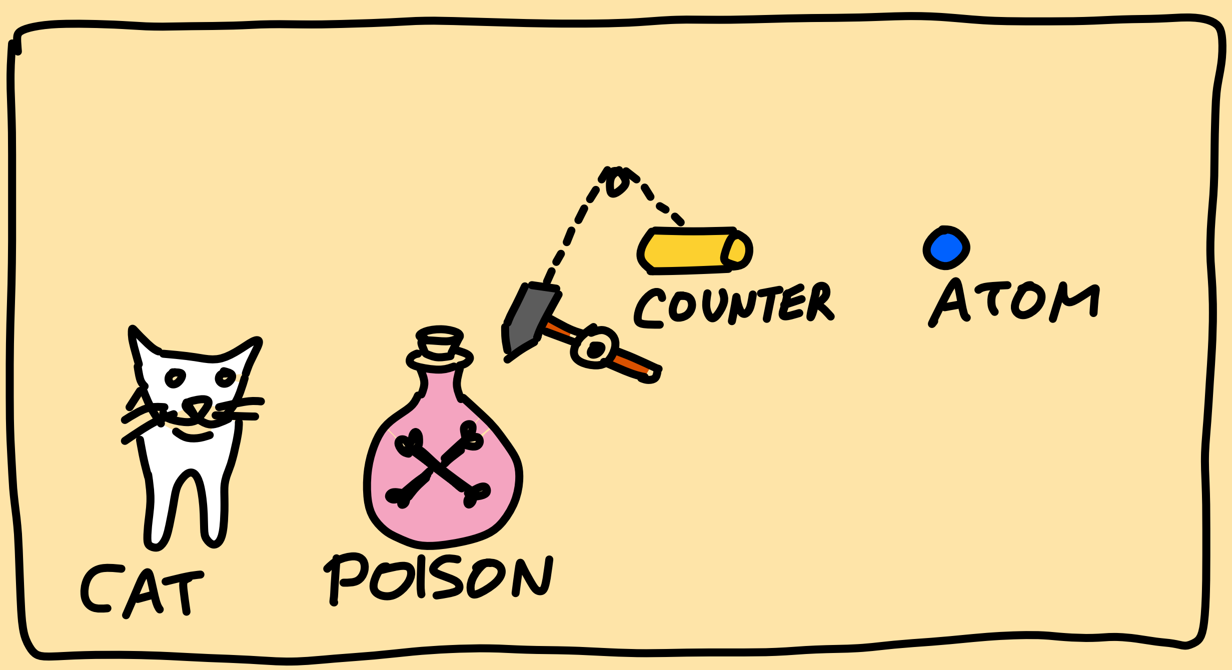
Foundations of Quantum Mechanics

Winter term 2020/21 Florian Marquardt

Start at 6pm CET

4.2 DECOHERENCE OF MACROSCOPIC OBJECTS

MACROSCOPIC SUPERPOSITION STATES:
"SCHRÖDINGER'S CAT" (1935)

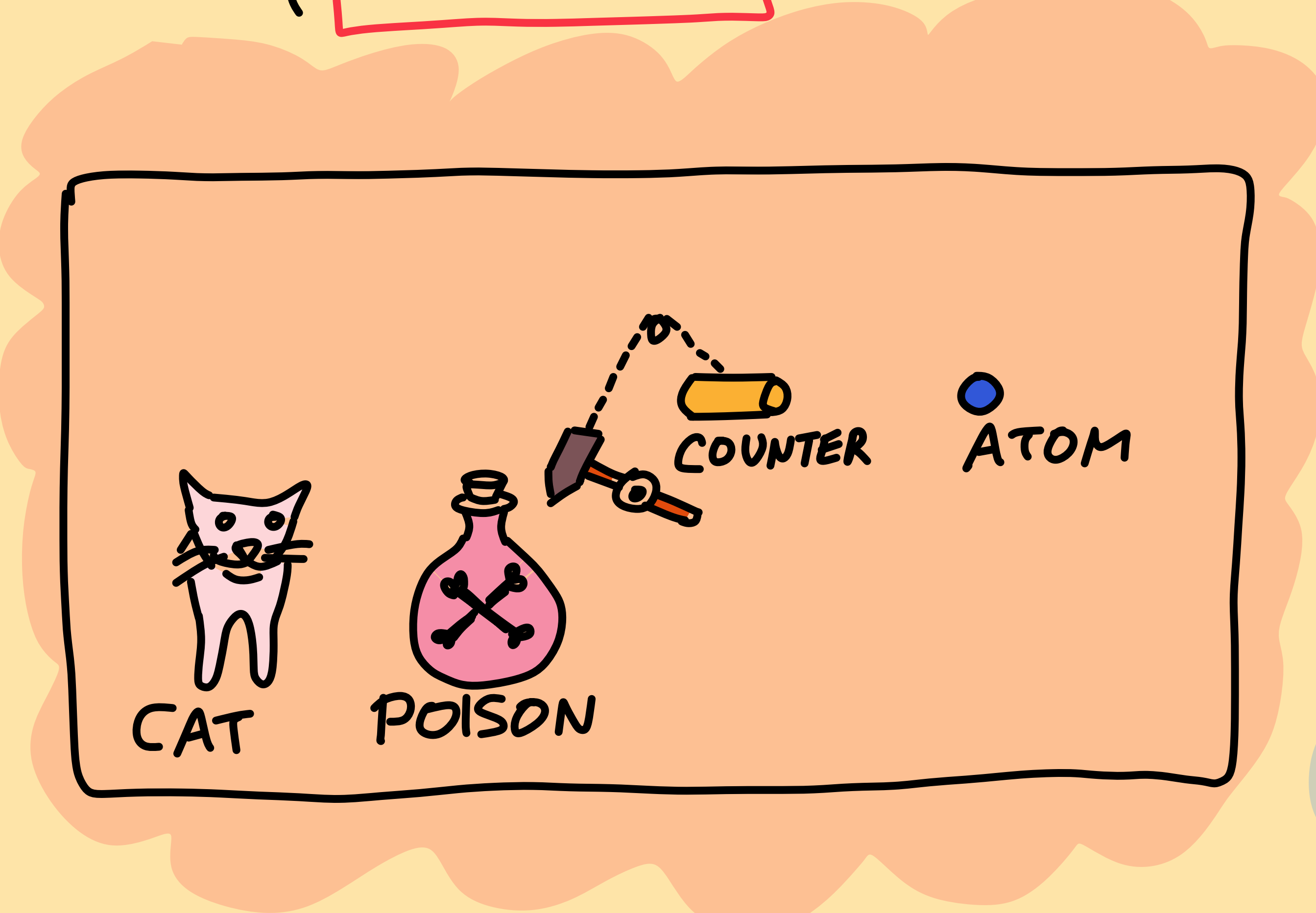


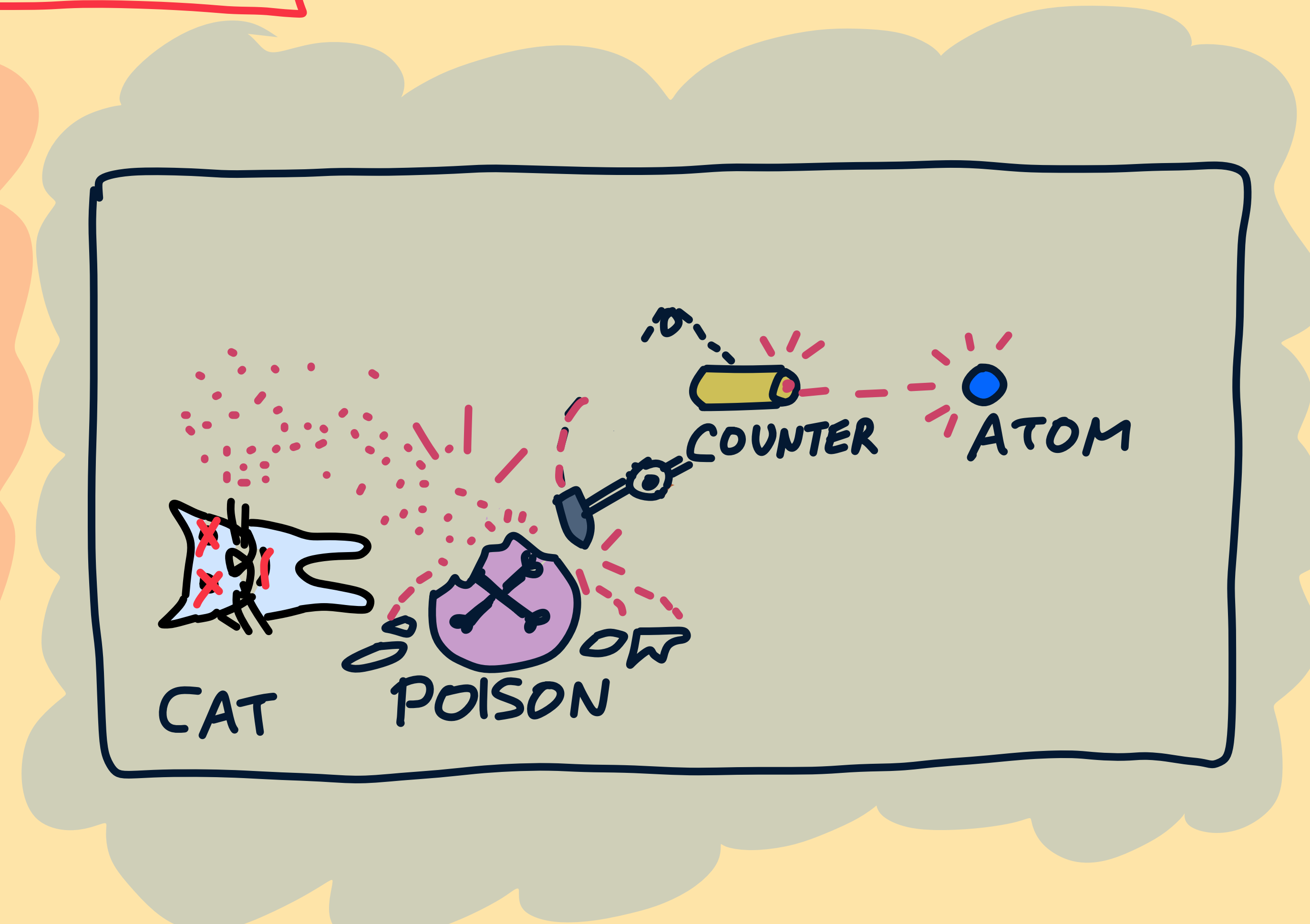
4.2 DECOHERENCE OF MACROSCOPIC OBJECTS

MACROSCOPIC SUPERPOSITION STATES:
"SCHRÖDINGER'S CAT"

$$\underset{\text{(DECAYED)}}{|\downarrow\rangle} \otimes |\text{DEAD}\rangle + |\uparrow\rangle \otimes |\text{ALIVE}\rangle$$

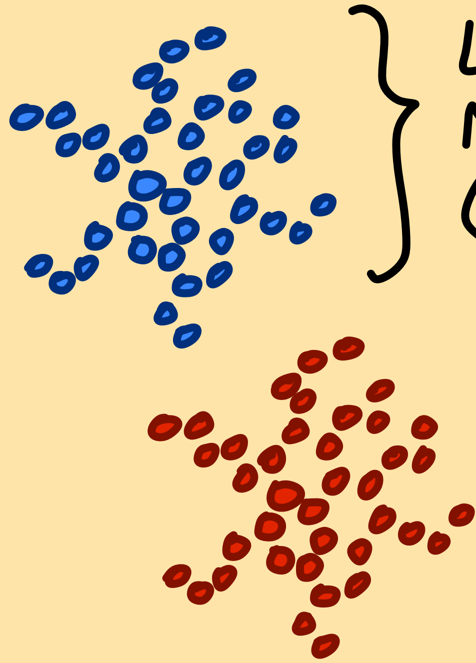
$$= \underbrace{(|\uparrow\rangle + |\downarrow\rangle)} \otimes \underbrace{(|\text{ALIVE}\rangle + |\text{DEAD}\rangle)} + (\quad - \quad) \otimes (\quad - \quad)$$



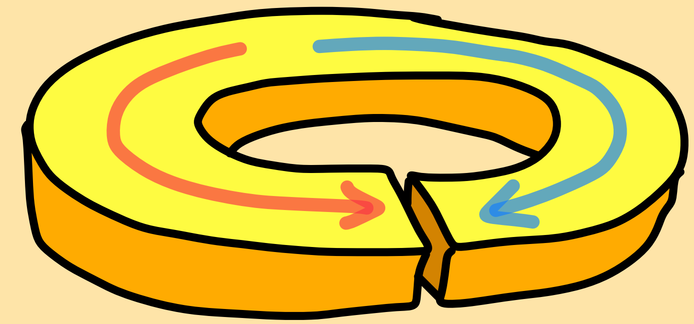


$$|ALIVE\rangle + |DEAD\rangle$$

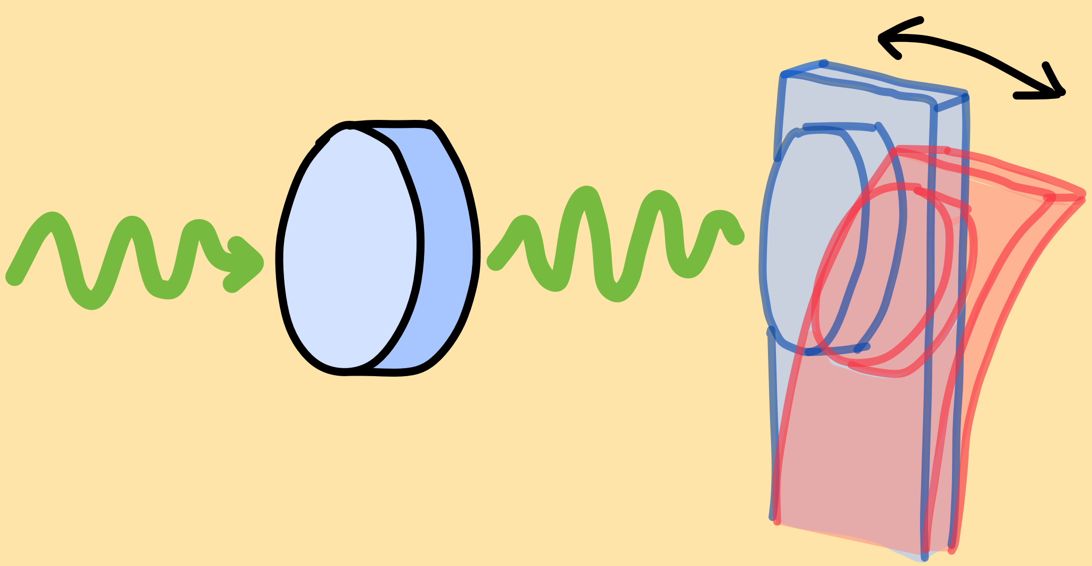
MACROSOPIC SUPERPOSITIONS



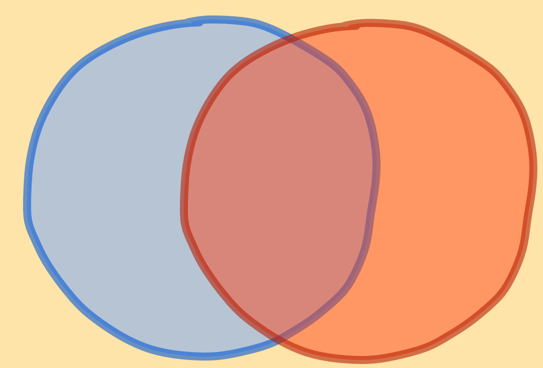
LARGE MOLECULES (>1000 ATOMS)



ELECTRIC CURRENT IN SUPERCONDUCTING QUANTUM INTERFERENCE DEVICE

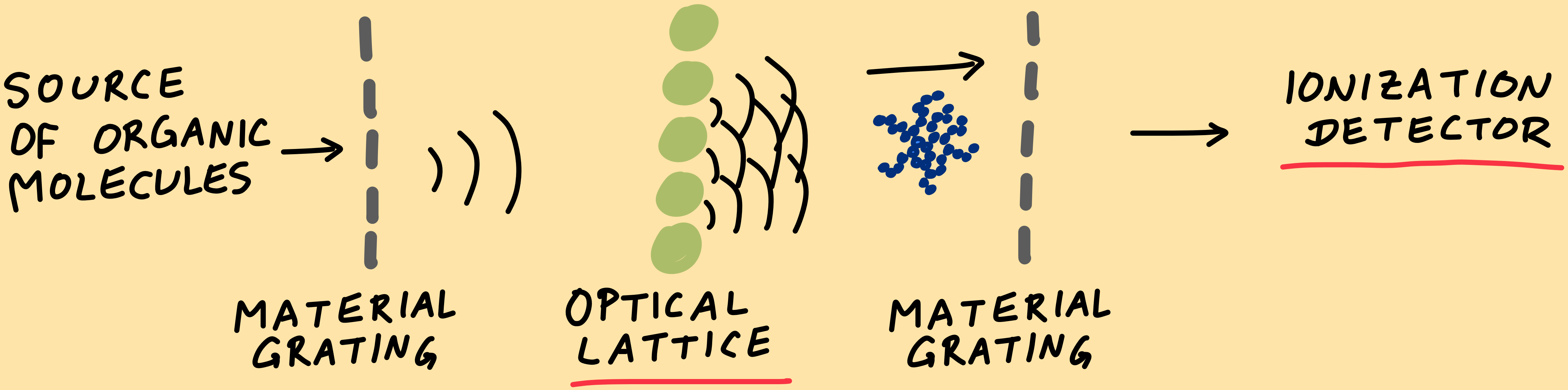


VIBRATIONS OF A MIRROR (CAVITY OPTOMECHANICS)



LEVITATED SPHERES

MATTER WAVE INTERFERENCE



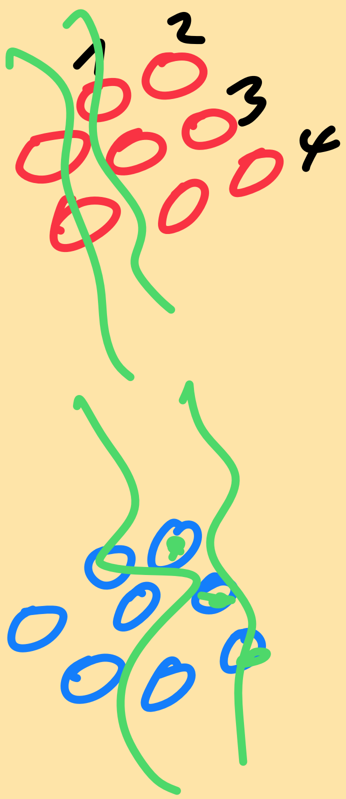
ARNDT GROUP, VIENNA

- UP TO 2000 ATOMS SO FAR!
- SUSPENDED SETUP & COMPENSATE CORIOLIS FORCE (OF EARTH ROTATION!)

DECOHERENCE OF LARGE OBJECTS

SCALING WITH PARTICLE NUMBER N ?

"UP" $= \uparrow$



$$|\uparrow_1 \uparrow_2 \uparrow_3 \uparrow_4 \dots\rangle$$

"DOWN" $= \downarrow$

$$+ \underbrace{|\downarrow_1 \downarrow_2 \downarrow_3 \dots\rangle}$$

$$= \underbrace{\left(|\uparrow_{c.o.m.}\rangle + |\downarrow_{c.o.m.}\rangle\right)} \otimes |\Psi_{relative}\rangle$$

DECOHERENCE OF LARGE OBJECTS

SCALING WITH PARTICLE NUMBER N ?

$$|\psi\rangle = \frac{1}{\sqrt{2}} |\overbrace{\uparrow \uparrow \uparrow \uparrow \uparrow \uparrow}^{N}\rangle$$

$$+ \frac{1}{\sqrt{2}} |\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow\rangle$$

$$\hat{A} = \underline{\hat{\sigma}_{x1} \cdot \hat{\sigma}_{x2} \cdot \ldots \cdot \hat{\sigma}_{xN}}$$

$$\underline{\langle \psi | \hat{A} | \psi \rangle = 1}$$

DECOHERENCE OF LARGE OBJECTS

SCALING WITH PARTICLE NUMBER N ?

$$|\psi\rangle = \frac{1}{\sqrt{2}}\,|\overbrace{\uparrow^{1}\uparrow^{2}\uparrow^{3}\uparrow\uparrow\uparrow}^{N}\rangle\, |\chi_{\uparrow}^{(1)}\rangle |\chi_{\uparrow}^{(2)}\rangle |\chi_{\uparrow}^{(3)}\rangle \otimes \dots \otimes |\chi_{\uparrow}^{(N)}\rangle$$

SPIN "BATH" FOR EACH OF THE "BIG" SPINS

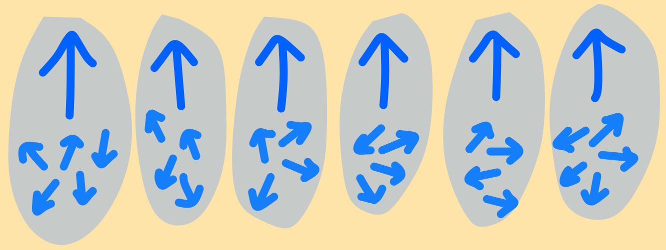
$$+\frac{1}{\sqrt{2}}\,|\downarrow\downarrow\downarrow\downarrow\downarrow\downarrow\rangle\, |\chi_{\downarrow}^{(1)}\rangle |\chi_{\downarrow}^{(2)}\rangle \otimes \dots \otimes |\chi_{\downarrow}^{(N)}\rangle$$

$$\hat{A} = \hat{\sigma}_{x1} \cdot \hat{\sigma}_{x2} \cdot \dots \cdot \hat{\sigma}_{xN}$$

$$\langle\psi|\hat{A}|\psi\rangle = \mathrm{Re}\Big(\langle\chi_{\uparrow}^{(1)}|\chi_{\downarrow}^{(1)}\rangle \cdot \dots \cdot \underbrace{\langle\chi_{\uparrow}^{(N)}|\chi_{\downarrow}^{(N)}\rangle}_{C}\Big)$$

$$\sim C^{N}$$

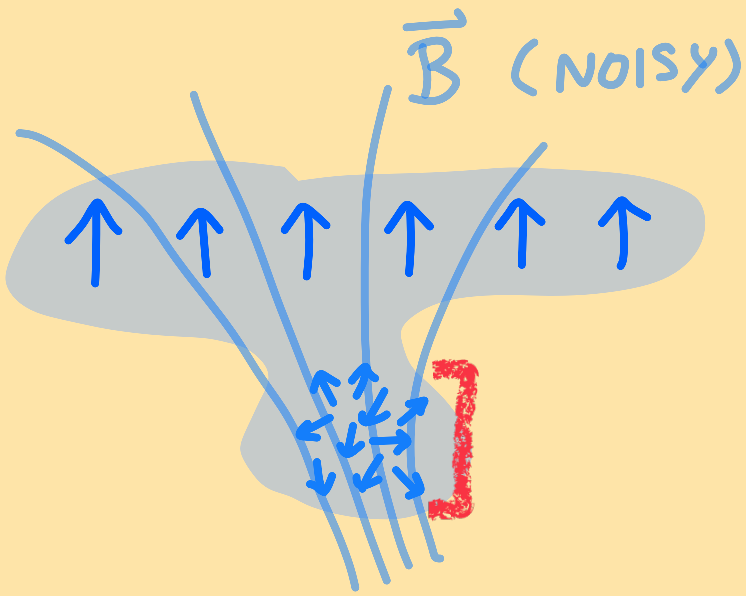
$$\langle \Psi | \hat{A} | \Psi \rangle \sim \underline{C^N}$$



SEPARATE "BATHS"

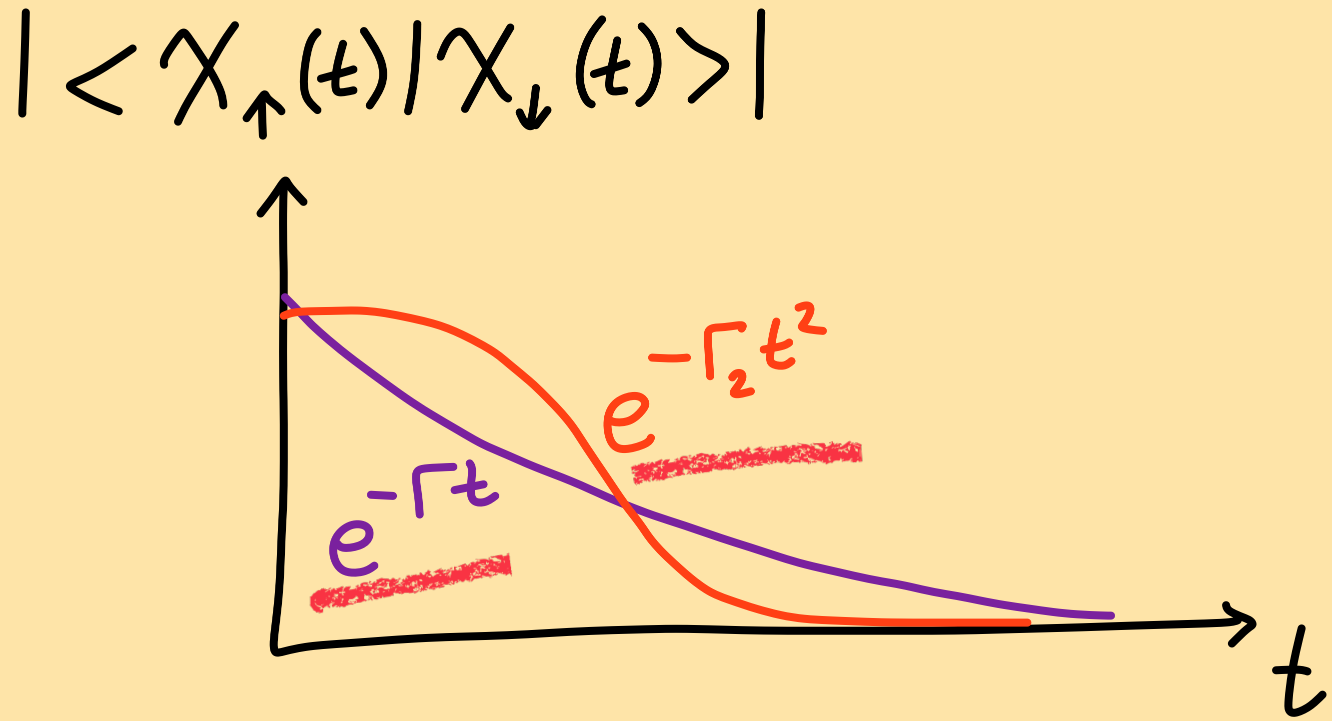
... SOMETIMES EVEN WORSE:

$$\langle \Psi | \hat{A} | \Psi \rangle \sim \mathrm{const}^{\underline{N^2}}$$



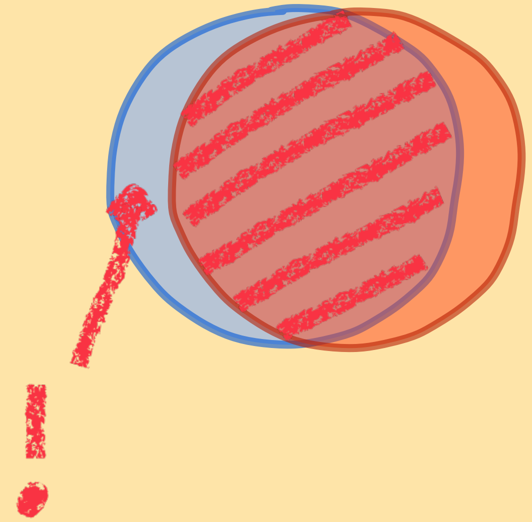
JOINT (GLOBAL) BATH

DECOHERENCE IN TIME:

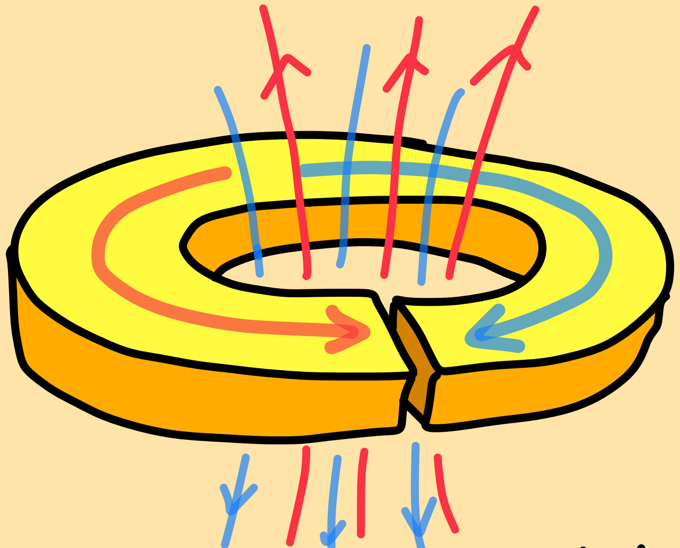


... AND MANY OTHER SCENARIOS

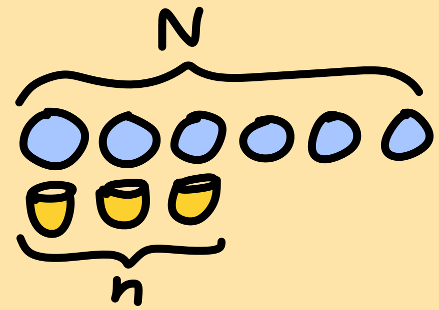
"MEASURING THE SIZE OF A SCHRÖDINGER CAT"



"MEASURING THE SIZE OF A SCHRÖDINGER CAT"



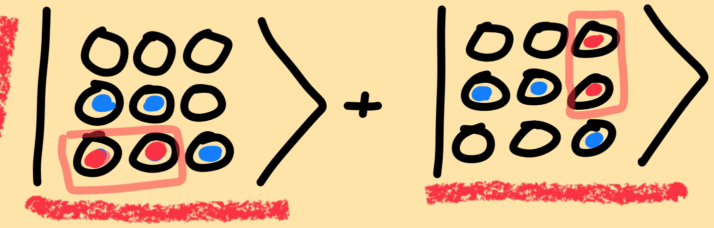
DIFFERENCE IN OBSERVABLES "$\Delta\mu \sim 10^{10}\mu_B$"
(LEGGETT)



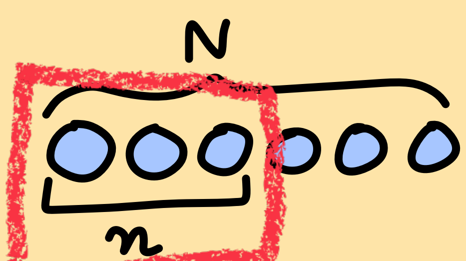
"HOW MANY PARTICLES NEED TO BE MEASURED TO DISTINGUISH BRANCHES?"

$\Rightarrow$ SIZE $= \frac{N}{n}$

(KORSBAKKEN ET AL)



"HOW MANY PARTICLES NEED TO BE MOVED TO ANOTHER ORBITAL?"
(ABEL ET AL)

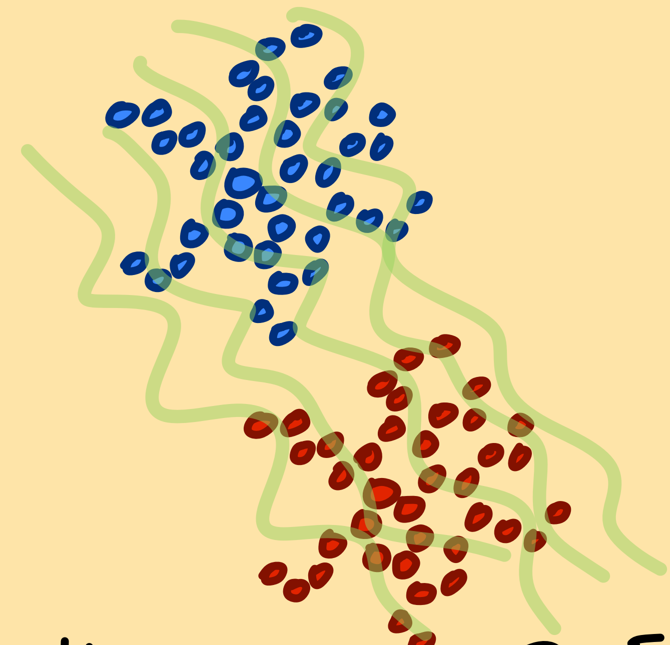


CONSIDER ENTROPY OF SUBSYSTEM AS FUNCTION OF n
"DISCONNECTIVITY"
(LEGGETT)

$|\uparrow\uparrow\uparrow\uparrow\rangle + |\uparrow\uparrow\uparrow\uparrow\rangle$

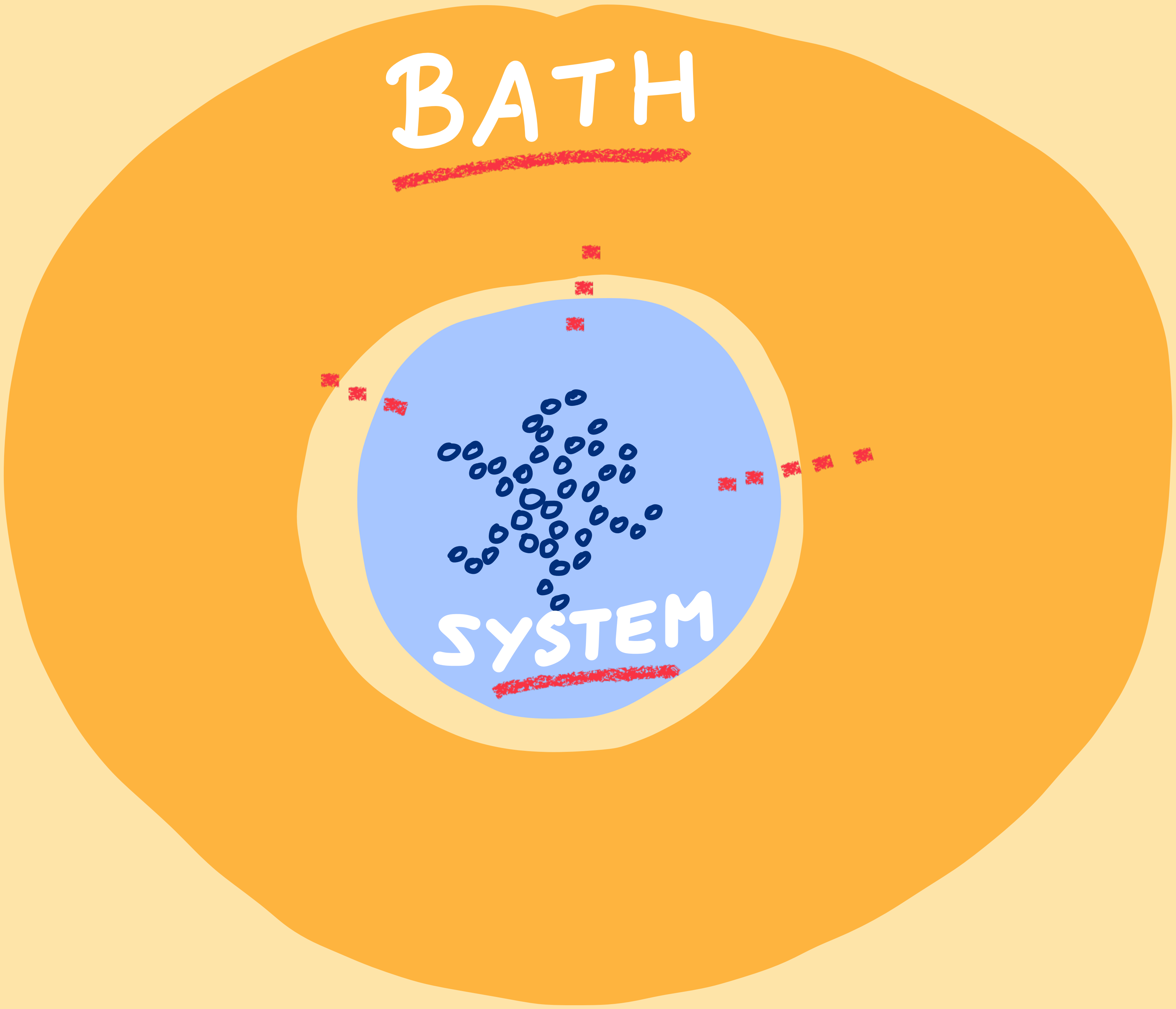
"GENERALIZED GHZ STATES"

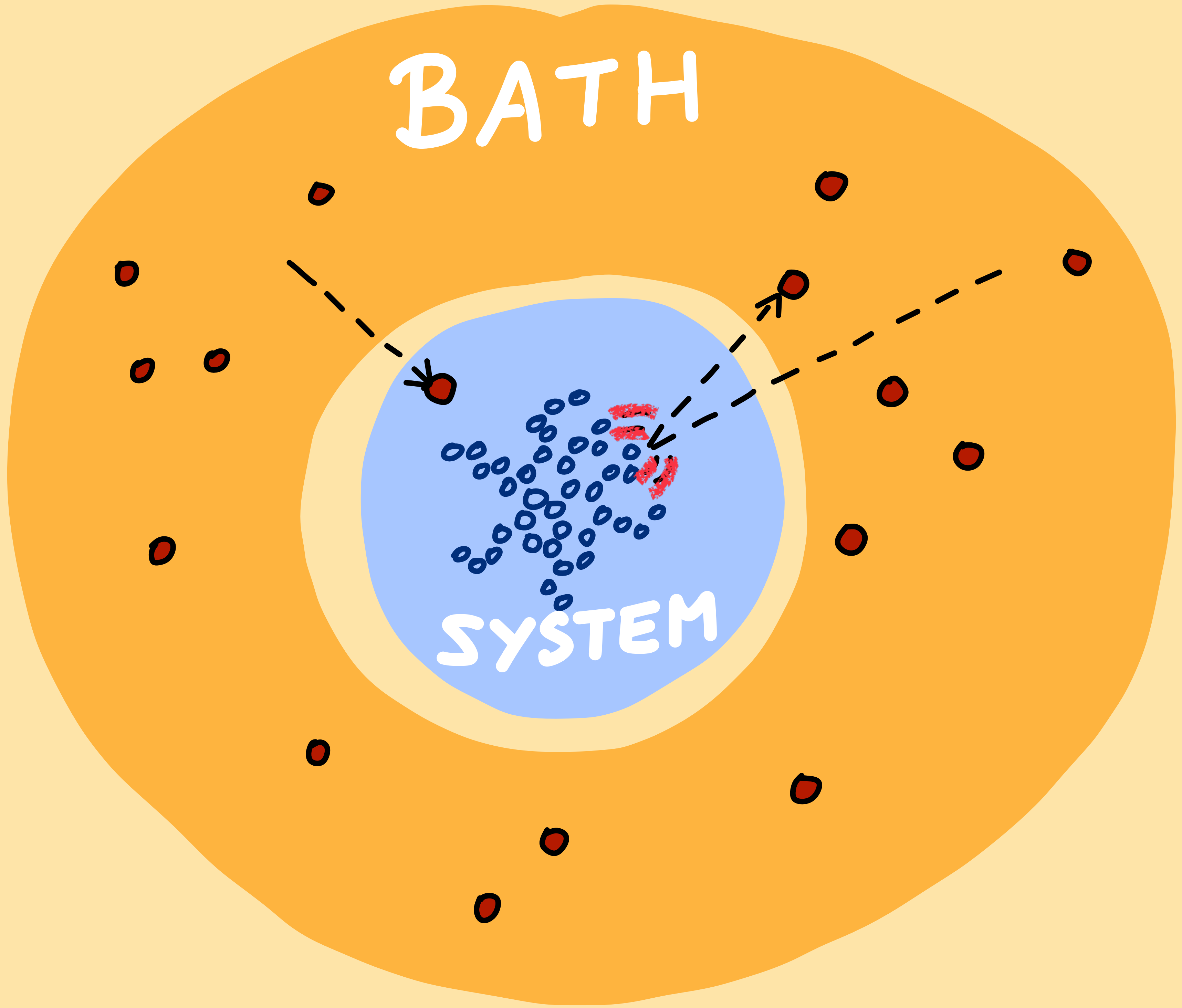
STABILITY VS DECOHERENCE, ENTANGLEMENT
(DÛR ET AL)



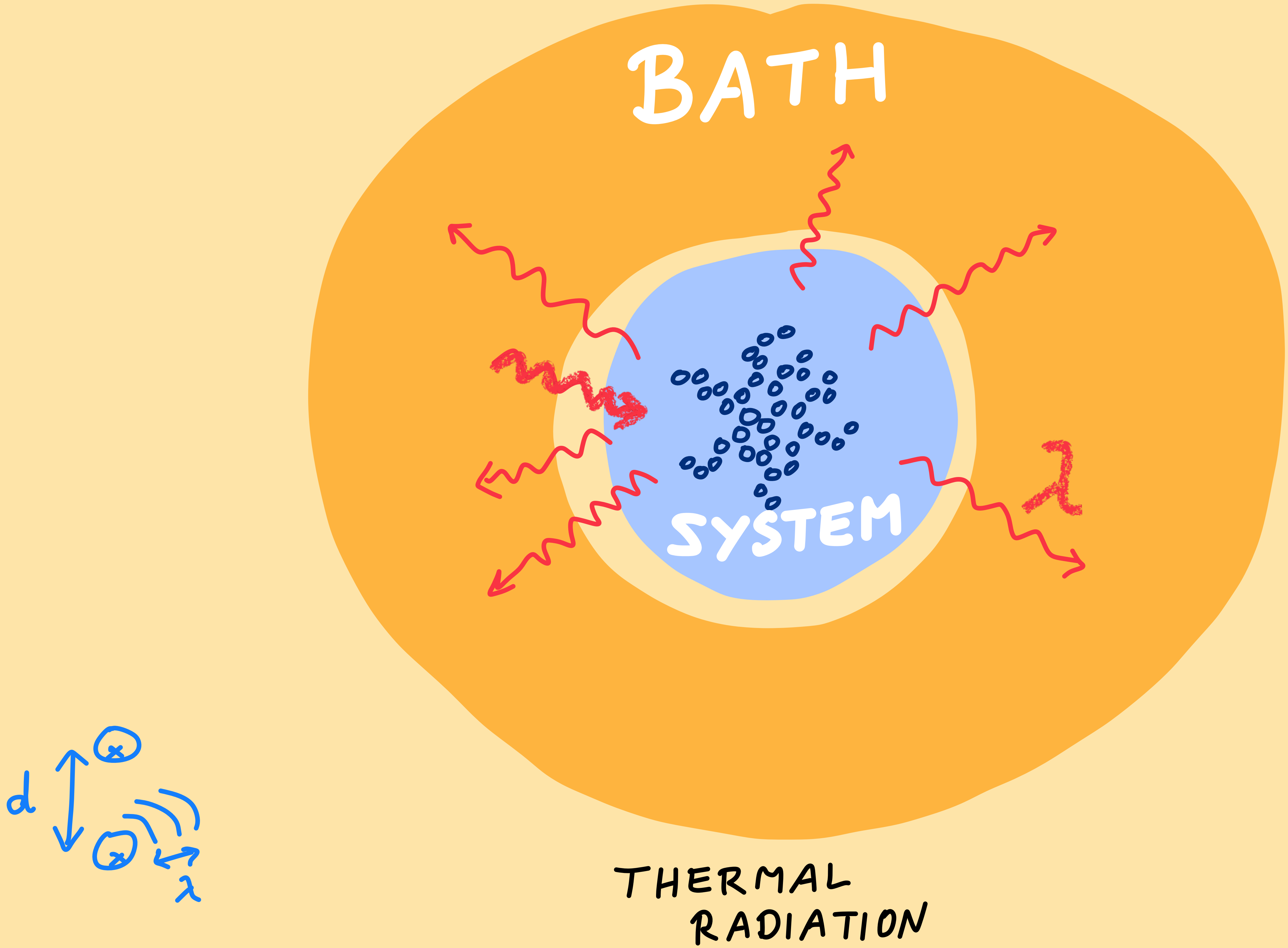
HOW WELL DOES THE CAT HELP TO RULE OUT EXTRA SOURCES OF DECOHERENCE?
(NIMMRICHTER, HORNBERGER)

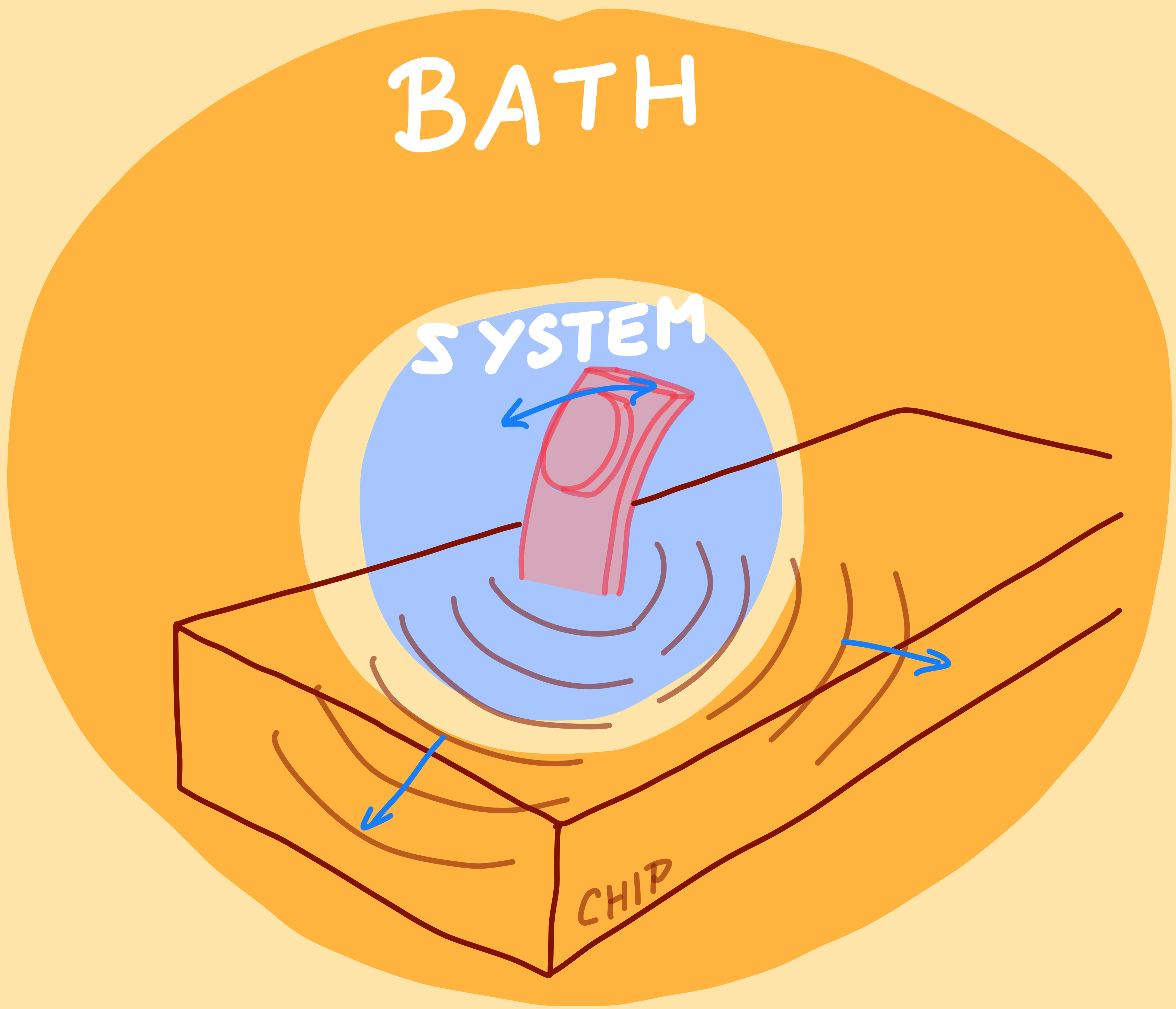
4.3 SYSTEM-BATH PICTURE AND TYPICAL SOURCES OF DECOHERENCE


BATH
SYSTEM

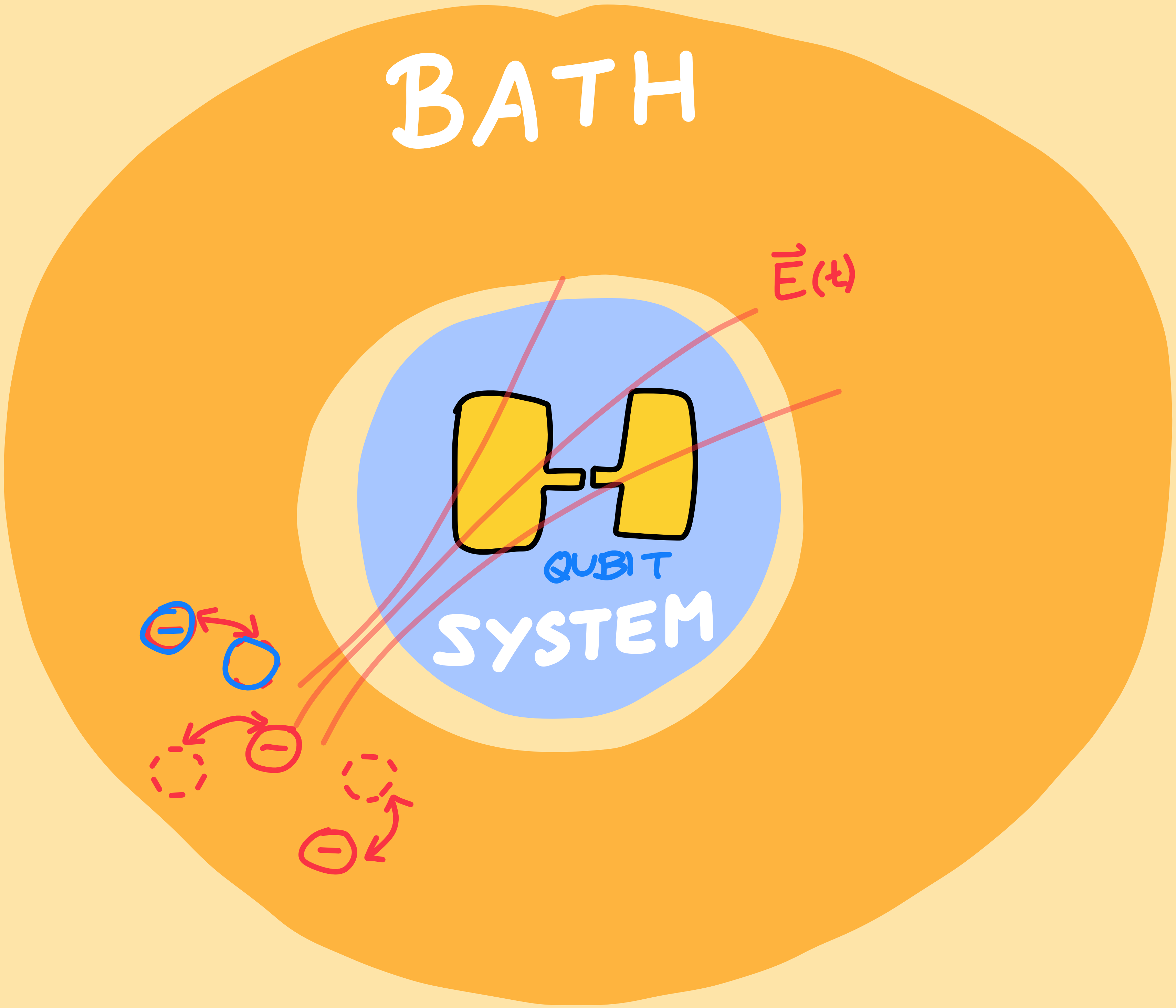


COLLISIONS WITH
GAS MOLECULES


BATH
SYSTEM
λ
d
λ
THERMAL
RADIATION



EMISSION OF
SOUND WAVES



FLUCTUATING ELECTRIC
OR MAGNETIC FIELDS

5. INTERPRETATIONS OF QUANTUM MECHANICS

→ UNDERSTANDING

→ POSSIBLE EXTENSION

5. INTERPRETATIONS OF QUANTUM MECHANICS

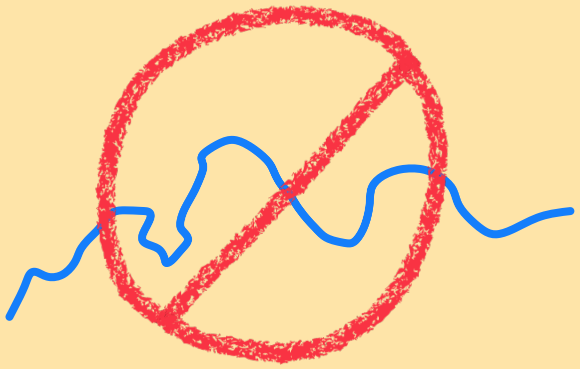
5.1 "COPENHAGEN INTERPRETATION"

- QUANTUM EVOLUTION $\quad i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi$
- MEASUREMENT
  - CLASSICAL, MACROSCOPIC APPARATUS
  - COLLAPSE
  - BORN RULE

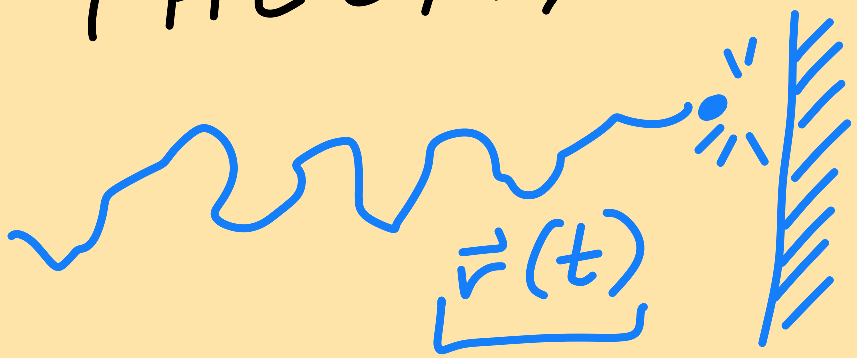
"OBSERVABLES BECOME REAL ONLY VIA MEASUREMENT"

→ "<u>NO</u> TRAJECTORIES!"

(DON'T EVEN THINK OF THEM!)



5.2 BOHM'S HIDDEN VARIABLE THEORY



$\vec{r}(t)$

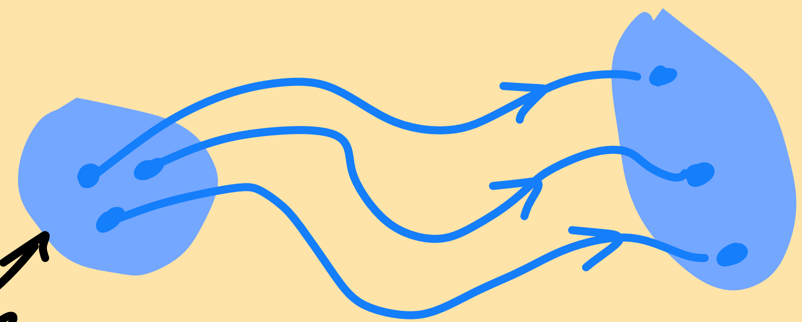
$\Downarrow$

$\dot{\vec{r}}(t)$

BOHM 1952 "PILOT WAVE THEORY"

de BROGLIE 1927
(EINSTEIN <1917)

- AT TIME t
  - $\psi(\vec{x},t)$
  - $\underbrace{\vec{r}(t)}$ ← POSITION OF PARTICLE
    HIDDEN VARIABLE

- 

$|\psi(\vec{r}(0),0)|^2$

AT $t=0$: $\vec{r}(0)$ DISTRIBUTED ACCORDING TO $|\psi(\vec{r}(0),0)|^2$

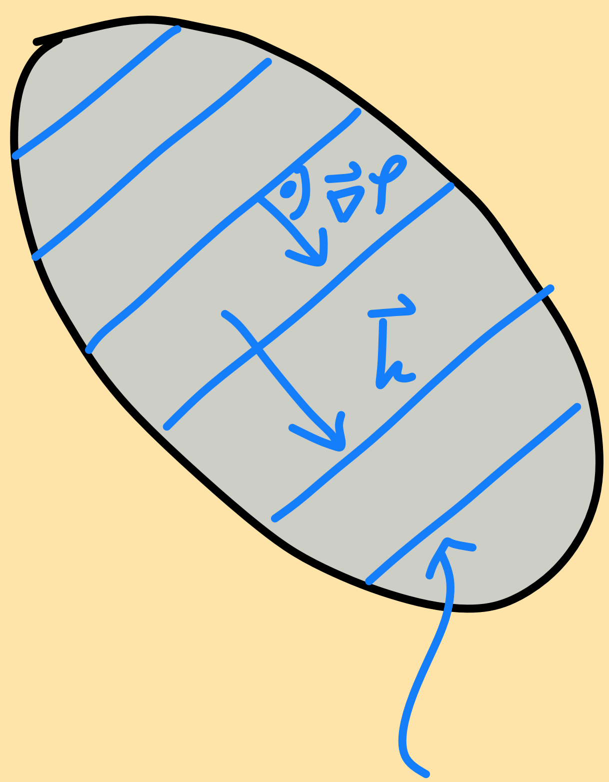
- AT $t>0$: ψ EVOLVES ACCORDING TO SCHRÖDINGER EQ.

  $\vec{r}(t)$ EVOLVES:

$$\dot{\vec{r}}(t) = \frac{\hbar}{m}(\vec{\nabla}\varphi)(\vec{r}(t),t) \equiv \vec{v}(\vec{r}(t),t)$$

$$\psi(\vec{x},t) = |\psi(\vec{x},t)| \cdot e^{i\underbrace{\varphi(\vec{x},t)}}$$

$$\underbrace{\phi(\vec{r})}\, e^{i\underbrace{\vec{k}\vec{r}}}$$

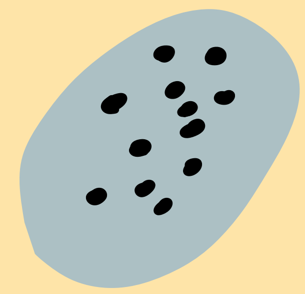


$\varphi = \mathrm{const}$

$$\vec{\nabla}\varphi \approx \vec{k}$$

$$\vec{v} = \frac{\hbar \vec{k}}{m} = \frac{\vec{p}}{m}$$

$\rho(\vec{r},t) =$ PROBABILITY DENSITY OF "BOHM'S ENSEMBLE"



CLAIM: IF $\rho(\vec{r},0) \equiv |\psi(\vec{r},0)|^2$

THEN $\rho(\vec{r},t) = |\psi(\vec{r},t)|^2 \quad \forall t>0$

PROOF: $\partial_t \rho = -\operatorname{div} \vec{j}_{BOHM} \overset{?}{=} \partial_t |\psi|^2 = -\operatorname{div} \vec{j}_{QM}$

$$\vec{j}_{BOHM} = \rho \cdot \vec{v}_{BOHM} = \rho \cdot \frac{\hbar \vec{\nabla} \varphi(\vec{r})}{m}$$

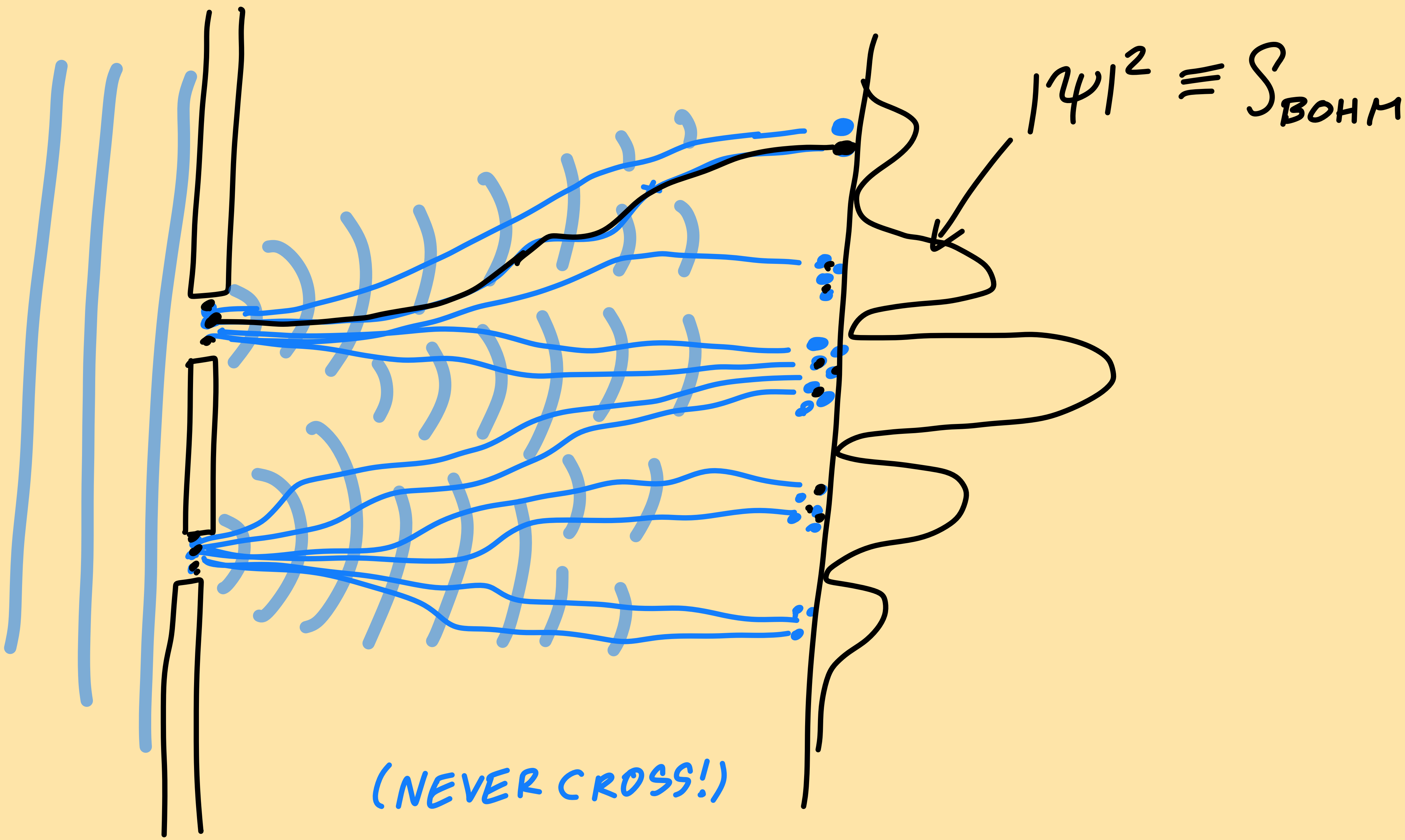
$$= |\psi|^2 \cdot \frac{\hbar}{m} \vec{\nabla} \varphi$$

$$\overset{!}{=} \mathrm{Re}\left[\psi^* \underbrace{\frac{-i\hbar\nabla}{m}}_{\hat{v}} \psi \right]$$

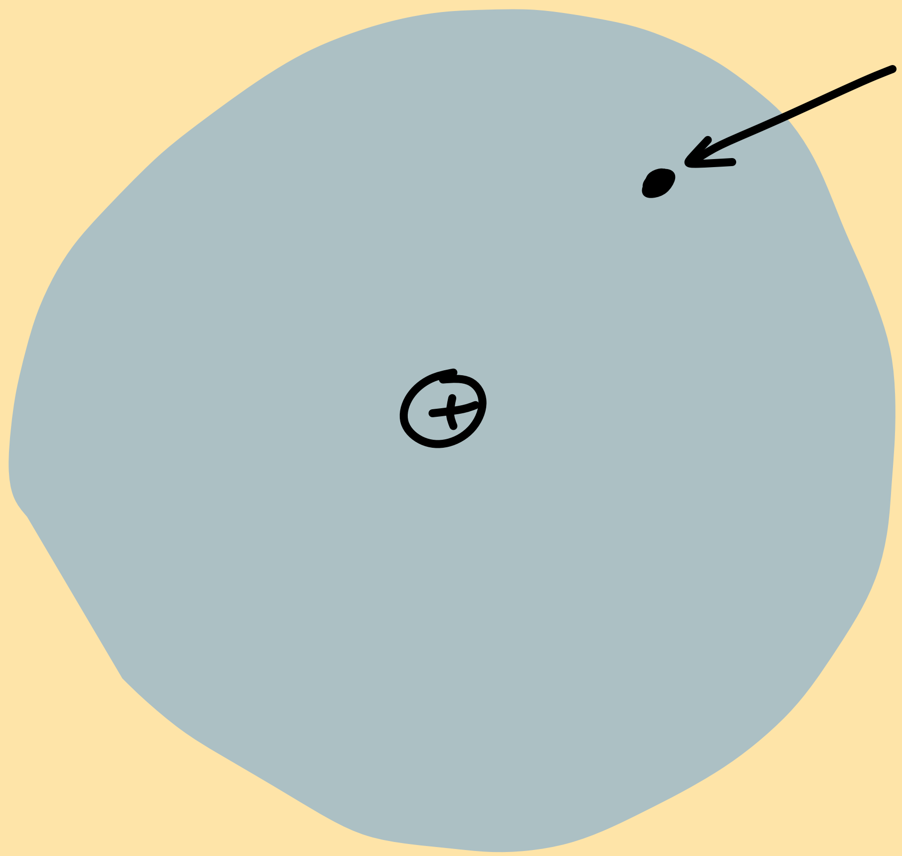
$$= \vec{j}_{QM}$$

$\Rightarrow$ IF ALL MSMTS ARE REDUCED TO $\vec{r}$- MSMTS THEN THIS REPRODUCES QM PREDICTIONS

EXAMPLE: DOUBLE SLIT



(HYDROGEN ATOM) GROUND STATE



e^- JUST SITS THERE!

$\vec{\nabla}\varphi \equiv 0$

IN ANY STATIONARY STATE (UNLESS $\vec{B}=0$)

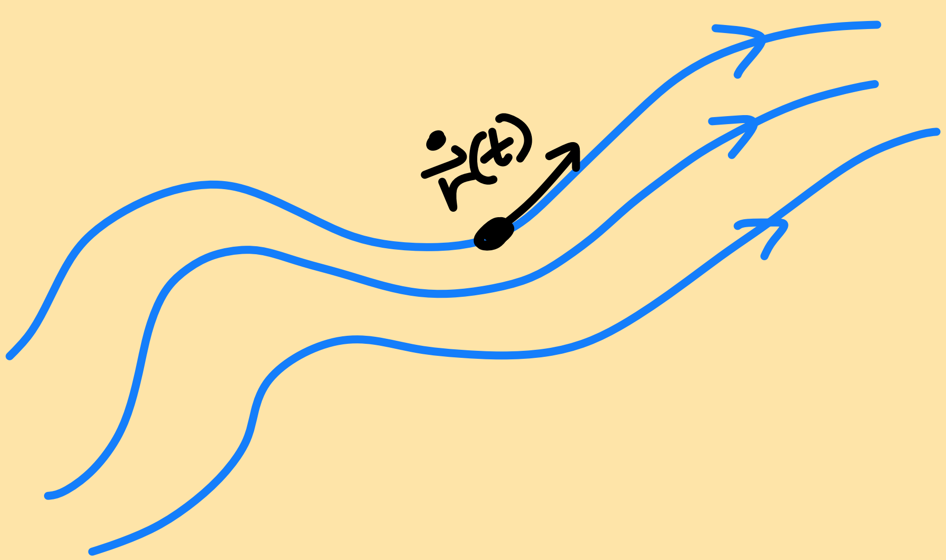
TIME EVOLUTION OF $\vec{v}(\vec{x},t)$?

$$\vec{v} = \frac{\hbar}{m}\vec{\nabla}\varphi$$

$$\partial_t \vec{v} = \; ? \; = \dots$$

$$i\hbar\,\partial_t \psi = -\frac{\hbar^2}{2m}\Delta\psi + V\psi$$

$$\partial_t \underbrace{\vec{v}}_{} = \underbrace{-\,(\vec{v}\vec{\nabla})\vec{v}}_{\text{"ADVECTIVE TERM"}} + \frac{1}{m}\underbrace{\vec{F}_{BOHM}}_{}$$



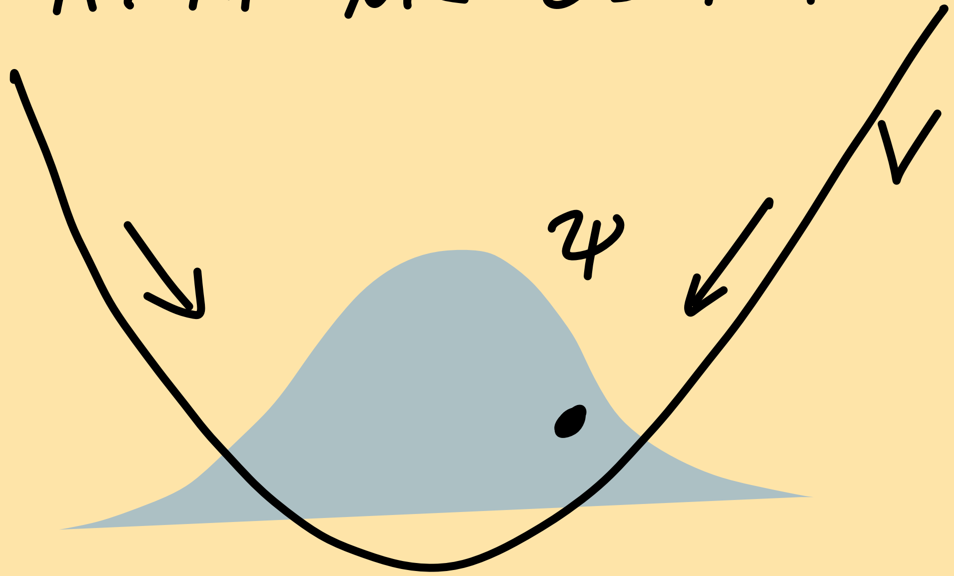
$$\vec{F}_{BOHM} = -\vec{\nabla}\Big[\underline{\underline{V}} \underbrace{-\frac{\hbar^2}{2m}\frac{\Delta|\psi|}{|\psi|}}_{V_{QUANTUM}}\Big]$$

"QUANTUM PRESSURE"

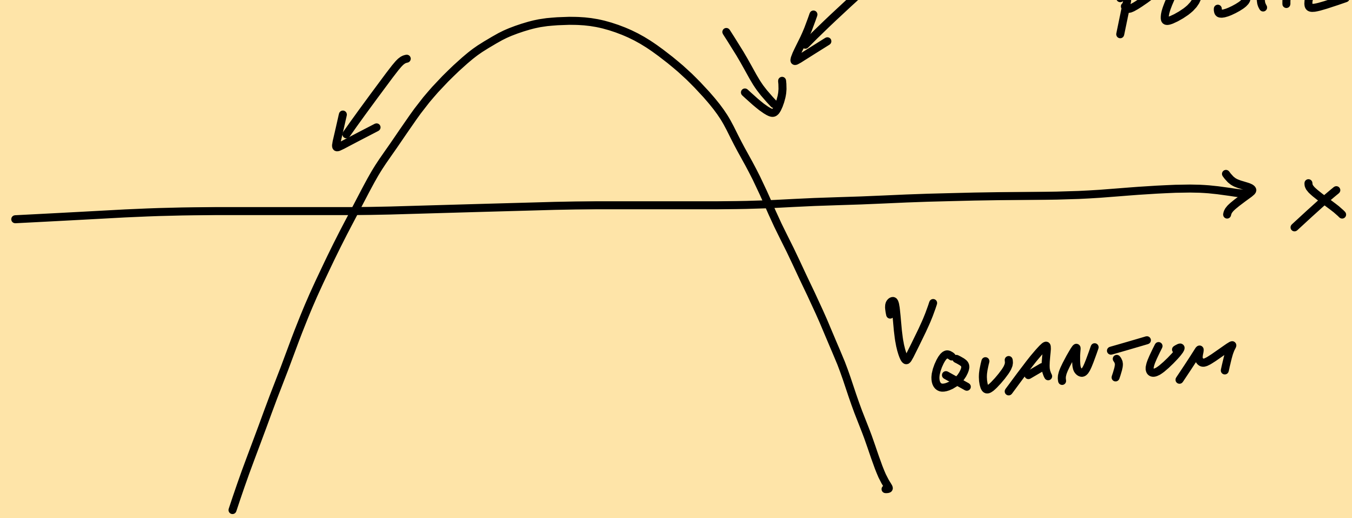
$$\psi(\vec{x}_1, \vec{x}_2, \dots.)$$

$$\dot{\vec{r}}_1(t) = \frac{\hbar}{m_1} \vec{\nabla}_{\vec{r}_1} \varphi(\vec{r}_1(t), \vec{r}_2(t), \dots)$$

EXAMPLE: HARMONIC OSCILLATOR



"QUANTUM PRESSURE" PUSHES AWAY FROM HIGH $|\psi|^2$



$$\underbrace{V + V_{\text{QUANTUM}}} \equiv \text{const} \implies \vec{F}_{\text{BOHM}} = 0$$

$$\left[\begin{aligned} &\partial_t \vec{v} + (\vec{v}\vec{\nabla})\vec{v} = \frac{1}{m}\vec{F}_{BOHM} \\ &\partial_t \rho + \operatorname{div}(\rho\vec{v}) = 0 \end{aligned}\right]$$

MADELUNG FORMULATION

$$\psi \in \mathbb{C} \leftrightarrow \rho, \vec{v}$$

NOTE: $\dot{\vec{r}} = \vec{v}$ IS <u>NOT</u> CONNECTED TO $\vec{P}_{(QM)}$!

$\vec{v} \equiv 0$ IN STATIONARY STATES
(QM: $\langle \hat{p}^2 \rangle \neq 0$)

$$\langle \dot{\vec{r}} \rangle_{BOHM} \equiv \left\langle \frac{\hat{\vec{p}}}{m} \right\rangle_{QM}$$

MEASURE $\vec{p}$ BY TIME-OF-FLIGHT

$\Rightarrow$ MEASURE $\vec{x}$-DISTR.
$\Rightarrow$ INFER $\vec{v}$-DISTR.