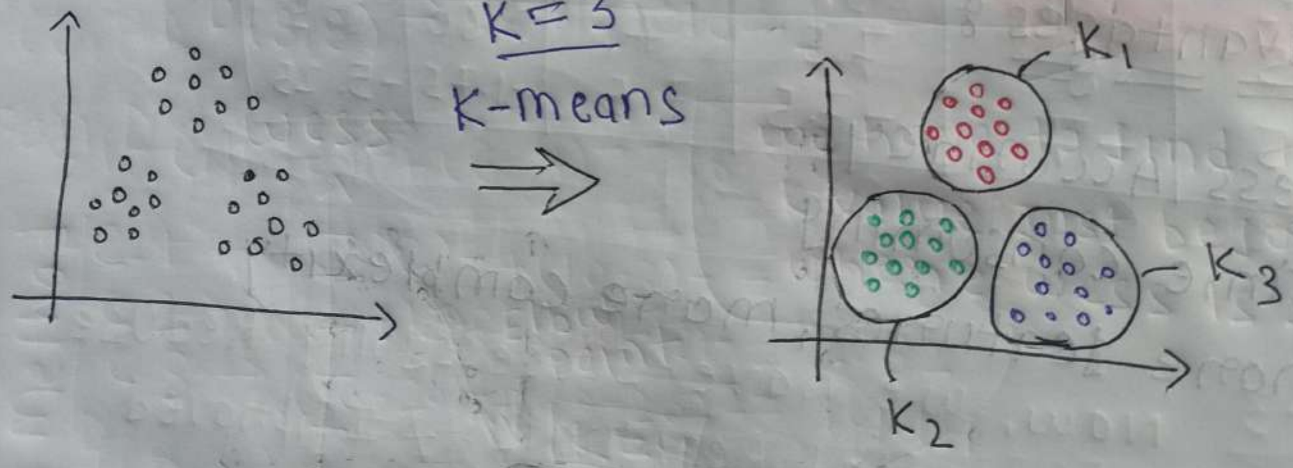
K-means clustering

K-means clustering is popular unsupervised ML. Algo. used to partition data into groups or clusters.

'K' in K-means represent No. of clusters you want to create. Goal is to partition data into 'K' clusters.

centroid: center of mass or AVG. of All data points in cluster.

$$\text{Euclidean Distance} = \sqrt{\left((x_2 - x_1)^2 + (y_2 - y_1)^2\right)}$$



↳ steps

working :

↳ select K.

↳ select centroids.

↳ Assign each datapoint to their closest centroid.

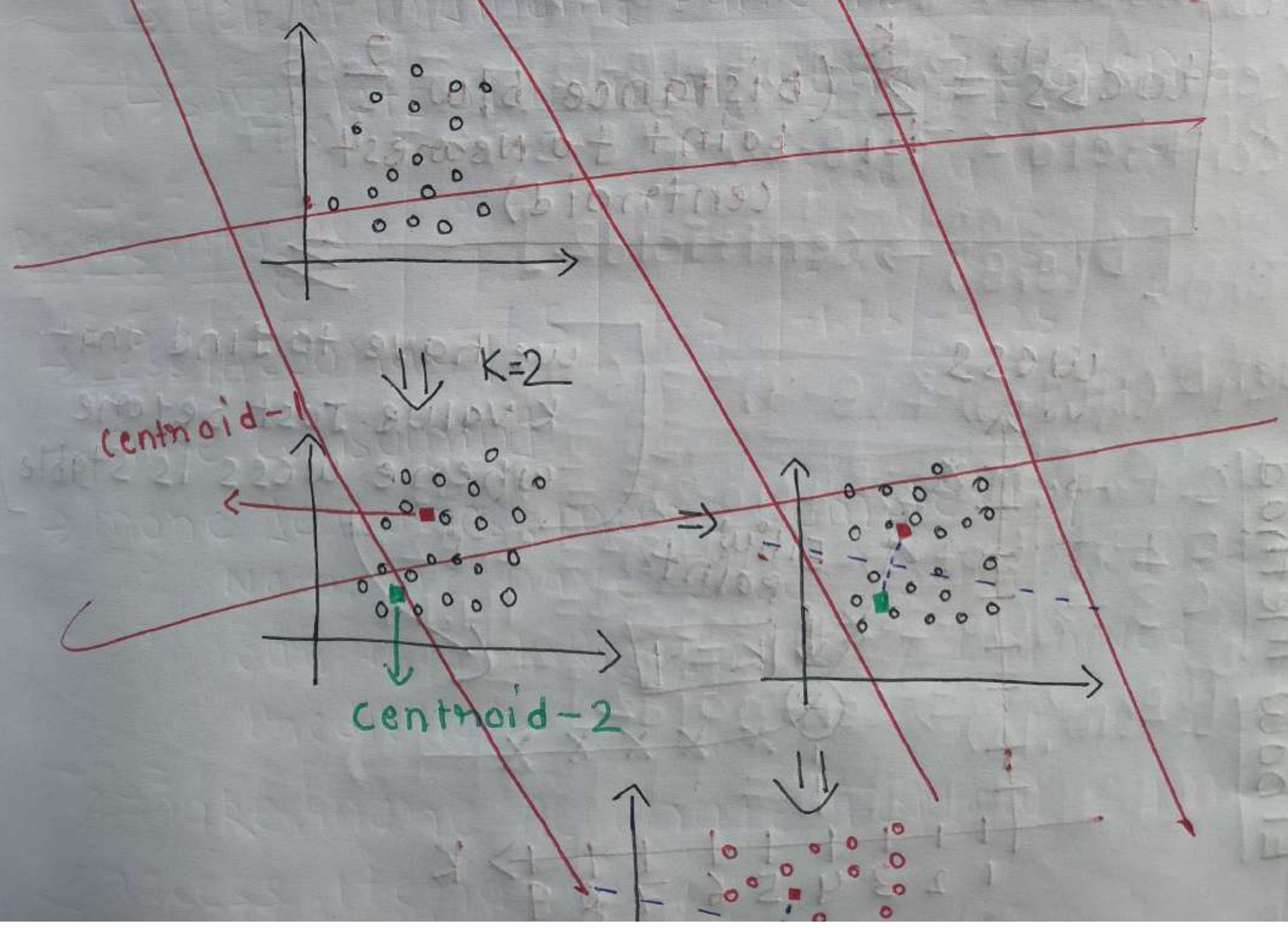
↳ calculate variance & place new centriod of each cluster.

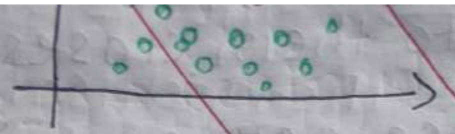
↳ Repeat 3rd step, reassign each datapoint to new closest centroid of each cluster.

↳ If any reassigment occurs, then go to step-4 else go to Finish.

↳ model is Ready.

Graphical intuition :





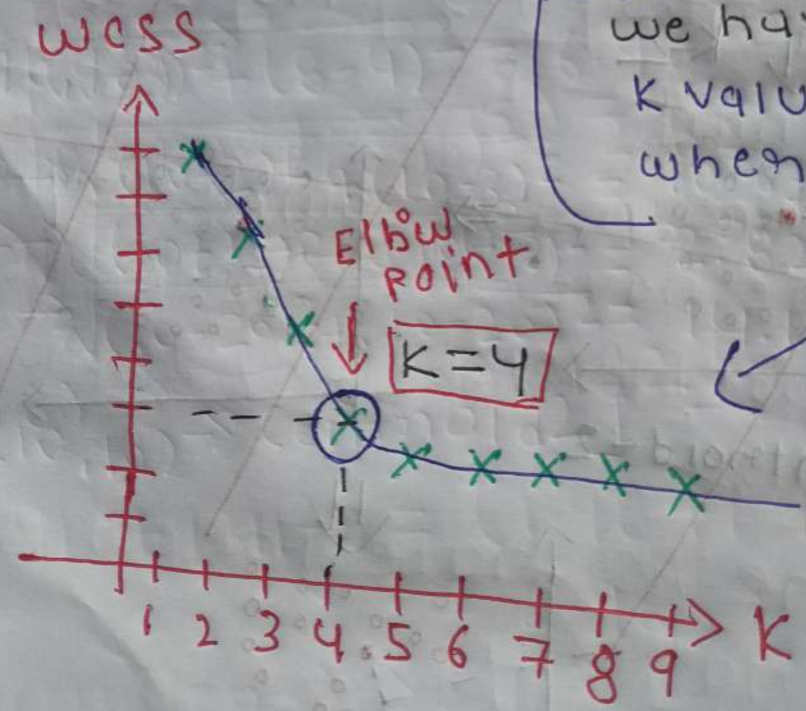
* How to choose "K No. of clusters":

↳ Elbow Method:

Elbow method is most optimal way to find No. of clusters. This method use concept of wcss value. (with in cluster sum of sq.)

$$\boxed{WCSS = \sum_{i=1}^{K} (\text{Distance b/w point to Nearest centroid})^2}$$

Elbow Method



we have to find that K value just before where wcss is stable

* K-Means Clustering Implimentation :

x_2 X	y_2 Y
2	3
3	4
5	6
8	8
10	10
12	12
~~14~~	~~14~~
~~16~~	~~16~~
18	18
20	20

x_1, y_1

Random centroid :

centroid 1 → (2 , 3)
centroid 2 → (12, 12)
centroid 3 → (20, 20)

If K = 3,

↳ clusters = 3
↳ No. of centroid = 3

For D(2,3) →

Data X   centroid X   Data Y   centroid Y

Centroid to Datapoint

$$\boxed{\text{Euclidean Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Distance to centroid 1 = $\sqrt{(2-2)^2 + (3-3)^2} = 0$

" " centroid 2 = $\sqrt{(2-12)^2 + (3-12)^2} = 13.45$

" " centroid 3 = $\sqrt{(2-20)^2 + (3-20)^2} = 24.75$

[D(2,3) → centroid 1]

For $D(3,4) \rightarrow$

centroid 1 $\rightarrow \sqrt{(3-2)^2+(4-3)^2} = 1.41$

centroid 2 $\rightarrow \sqrt{(3-12)^2+(4-12)^2} = 12.04$

centroid 3 $\rightarrow \sqrt{(3-20)^2+(4-20)^2} = 23.34$

[$D(3,4) \rightarrow$ centroid 1]

For $D(5,6) \rightarrow$

centroid 1 $\rightarrow \sqrt{(5-2)^2+(6-3)^2} = 4.24$

centroid 2 $\rightarrow \sqrt{(5-12)^2+(6-12)^2} = 9.21$

centroid 3 $\rightarrow \sqrt{(5-20)^2+(6-20)^2} = 20.51$

[$D(5,6) \rightarrow$ centroid 1]

For $D(8,8) \rightarrow$

centroid 1 $\rightarrow \sqrt{(8-2)^2+(8-3)^2} = 7.81$

centroid 2 $\rightarrow \sqrt{(8-12)^2+(8-12)^2} = 5.65$

centroid 3 $\rightarrow \sqrt{(8-20)^2+(8-20)^2} = 16.97$

[$D(8,8) \rightarrow$ centroid 2]

For $D(10,10) \rightarrow$

$C1 \rightarrow \sqrt{(10-2)^2+(10-3)^2} = 10.63$

$C2 \rightarrow \sqrt{(10-12)^2+(10-12)^2} = 2.82$

$C3 \rightarrow \sqrt{(10-20)^2+(10-20)^2} = 14.14$

[$D(10,10) \rightarrow$ centroid 2]

[D(12,12) → centroid 2]

For D(18,18) →

$C1 \rightarrow \sqrt{(18-2)^2+(18-3)^2} = 21.93$

$C2 \rightarrow \sqrt{(18-12)^2+(18-12)^2} = 8.48$

$C3 \rightarrow \sqrt{(18-20)^2+(18-20)^2} = 2.82$

[D(18,18) → centroid 3]

[For D(20,20) → centroid 3]

Update centroid :

X	Y	centroid
2	3	1
3	4	1
5	6	1
8	8	2
10	10	2
12	12	2
18	18	3
20	20	3

New centroid 1 = (3,4)
New centroid 2 = (10,10)
New centroid 3 = (19,19)

Now calculate New centroids →

New centroid 1 = X : (2+3+5)/3 = 3.33 → 3
Y : (3+4+6)/3 = 4.33 → 4

New centroid 2 = X : (8+10+12)/3 = 10
Y : (8+10+12)/3 = 10

New centroid 3 = X : (18+20)/2 = 19
Y : (18+20)/2 = 19

make Clusters with New centroid:

D(2,3) →

$C1 \rightarrow \sqrt{(2-3)^2+(3-4)^2} = 1.41$
$C2 \rightarrow \sqrt{(2-10)^2+(3-10)^2} = 10.63$
$C3 \rightarrow \sqrt{(2-19)^2+(3-19)^2} = 23.34$

[D(2,3) → centroid 1]

[D(3,4) → centroid 1]

D(5,6) →

$C_1 \rightarrow \sqrt{(5-3)^2+(6-4)^2} = 2.82$
$C_2 \rightarrow \sqrt{(5-10)^2+(6-10)^2} = 6.40$
$C_3 \rightarrow \sqrt{(5-19)^2+(6-19)^2} = 19.10$

[D(5,6) → centroid 1]

D(8,8) →

$C_1 \rightarrow \sqrt{(8-3)^2+(8-4)^2} = 6.40$
$C_2 \rightarrow \sqrt{(8-10)^2+(8-10)^2} = 2.82$
$C_3 \rightarrow \sqrt{(8-19)^2+(8-19)^2} = 15.55$

[D(8,8) → ~~2~~ centroid 2]

$D(10,10) \rightarrow$

$C1 \rightarrow \sqrt{(10-3)^2+(10-4)^2}$

$C2 \rightarrow \sqrt{(10-10)^2+(10-10)^2} = 0$ ✓

$C3 \rightarrow \sqrt{(10-19)^2+(10-19)^2}$

[$D(10,10) \rightarrow$ centroid 2]

$D(12,12) \rightarrow$

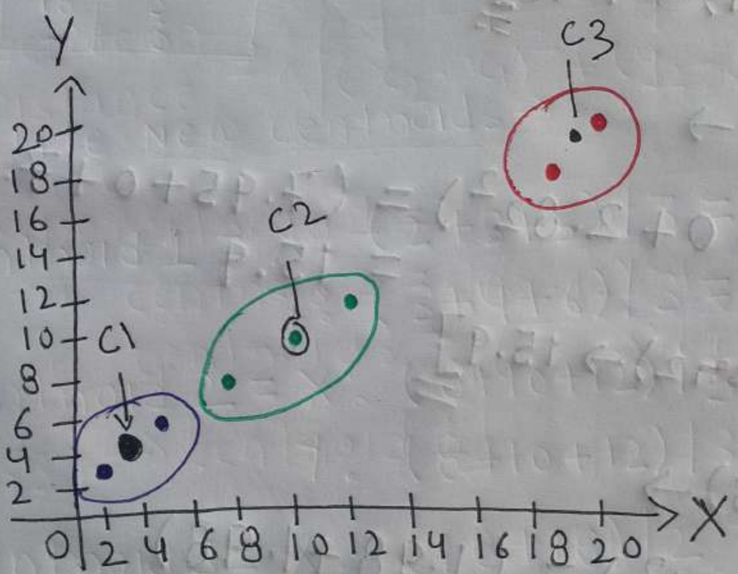
$C1 \rightarrow \sqrt{(12-3)^2+(12-4)^2} = 12.04$

$C2 \rightarrow \sqrt{(12-10)^2+(12-10)^2} = 2.82$

$C3 \rightarrow \sqrt{(12-19)^2+(12-19)^2} = 9.89$

same For $D(18,18)$ & $D(20,20)$

we get same clusters :-



$$\boxed{WCSS = \sum_{i=1}^{n} \sum_{j=1}^{k} d(c_j, x_i)^2}$$

WCSS → within clusters sum of square

n → Total No. of data points.

k → Total No. of clusters.

c_j → centroid of j^{th} clusters.

x_i → i^{th} data point

$d(c_j, x_i)$ → Euclidean Distance

WCSS(K=3) = WCSSS (cluster1) + WCSS(cluster2) + WCSS(cluster3)

WCSS(cluster1) →

$$(1.41^2 + 0^2 + 2.82^2) = (1.98 + 0 + 7.95)$$
$$= 9.93$$

[WCSS(cluster1) → 9.93]

WCSS(cluster2) →

$$(2.82^2 + 0 + 2.82^2) = (7.95 + 0 + 7.95)$$
$$= 15.9$$

[WCSS(cluster2) → 15.9]

WCSS(cluster3) →

$$(1.41^2 + 1.41^2) = (1.98 + 1.98)$$
$$= 3.96$$

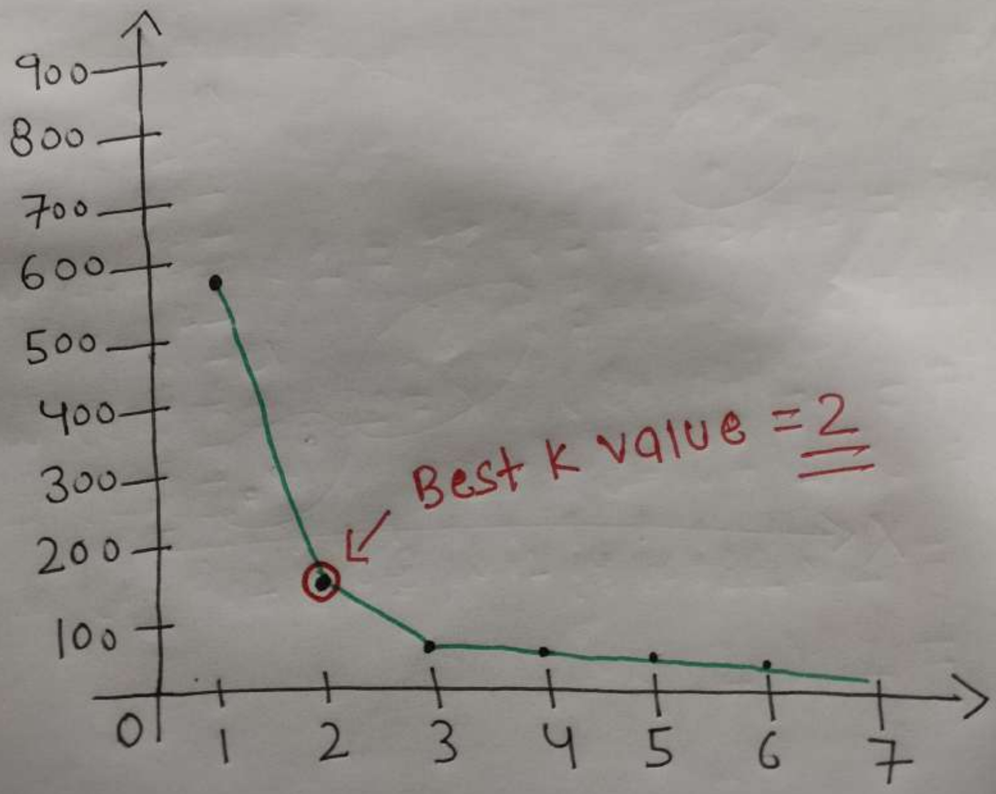
[WCSS(cluster3) → 3.96]

$$WCSS(K=3) = 9.93 + 15.9 + 3.96$$
$$= 29.79$$

Like this,

$WCSS(K=1) = 582.37$
$WCSS(K=2) = 144.16$
$WCSS(K=3) = 29.79$
$WCSS(K=4) = 15.50$
$WCSS(K=5) = 9.0$
$WCSS(K=6) = 5.0$

Elbow method:



In [1]:
```
import pyforest
from sklearn.cluster import KMeans
import warnings
warnings.filterwarnings('ignore')
%matplotlib inline
```

In [2]:
```
# Create a dictionary with the data
data = {
    'X': [2, 3, 5, 8, 10, 12,  18, 20],
    'Y': [3, 4, 6, 8, 10, 12,  18, 20]
}

# Create a DataFrame from the dictionary
df = pd.DataFrame(data)

# Display the DataFrame
df
```

Out[2]:

	X	Y
0	2	3
1	3	4
2	5	6
3	8	8
4	10	10
5	12	12
6	18	18
7	20	20

Find K Value (No. Of Clusters):

$$WCSS = \sum_{C_k}^{C_n} \left(\sum_{d_i\, in\, C_i}^{d_m} distance(d_i, C_k)^2 \right)$$

Where,

C is the cluster centroids and d is the data point in each Cluster.

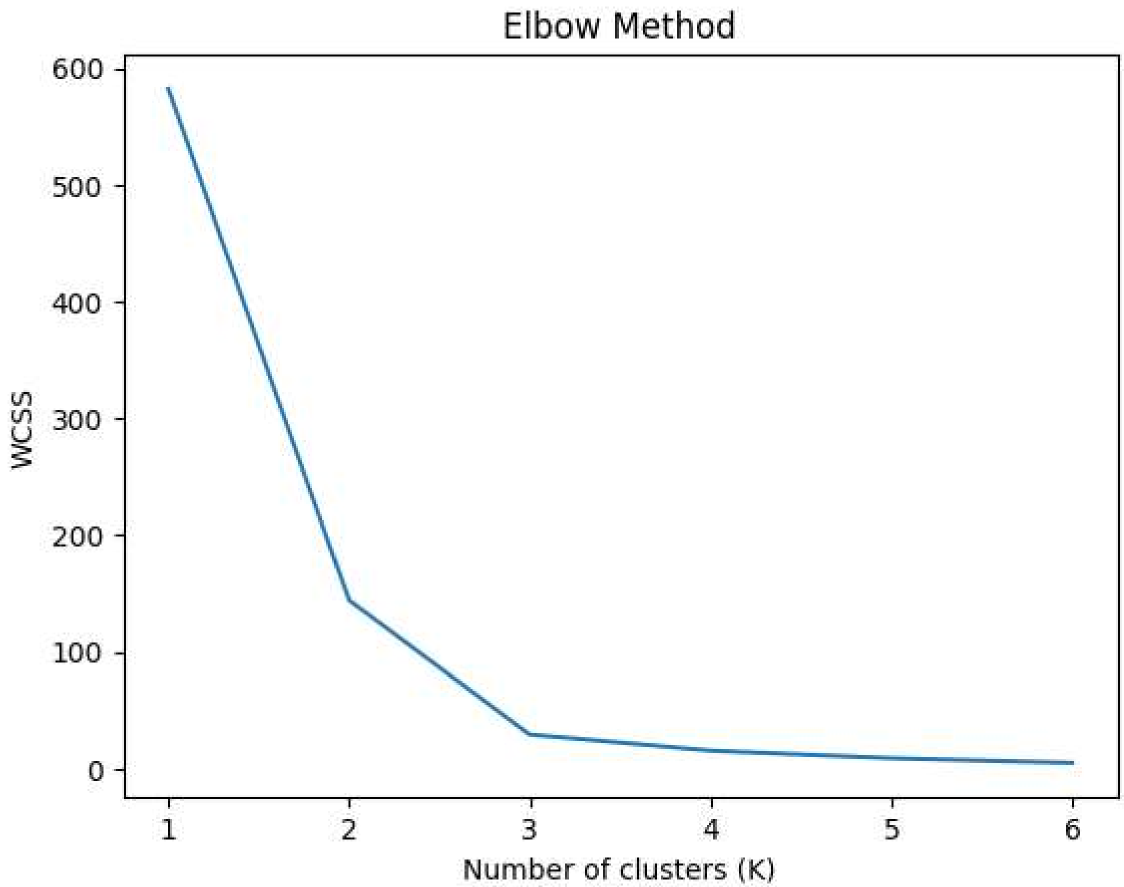
In [ ]:
```python
wcss= []
for k in range(1,7):
    kmeans= KMeans(n_clusters=k,init='k-means++')
    kmeans.fit(df)
    wcss.append(kmeans.inertia_)
```

In [4]:
```python
WCSS_Values=pd.DataFrame({'K Value': [1,2,3,4,5,6], 'WCSS':wcss})
WCSS_Values
```

Out[4]:

	K Value	WCSS
0	1	582.375000
1	2	144.166667
2	3	29.333333
3	4	15.500000
4	5	9.000000
5	6	5.000000

In [5]:
```python
plt.plot(range(1,7), wcss)
plt.title('Elbow Method')
plt.xlabel('Number of clusters (K)')
plt.ylabel('WCSS')
plt.show()
```



Calculate Centroid and Eucliden Distance for Clustering:

$$d(x, y) = \sqrt{\sum_{i=1}^{n} (y_i - x_i)^2}$$

In [6]:
```
df
```

Out[6]:

	X	Y
0	2	3
1	3	4
2	5	6
3	8	8
4	10	10
5	12	12
6	18	18
7	20	20

In [7]:
```
# Random Centroids For Initialize: df index 0 and 7
c1= (3,4)
c2= (10,10)
```

In [8]:
```
#Euclidean Distance Each Data Point to Each(2) centoids:

# c1 and c2 to all data points euclidean distance:
df['Distance_to_c1']= np.sqrt((df['X']-c1[0])**2+(df['Y']-c1[1])**2)
df['Distance_to_c2'] = np.sqrt((df['X'] - c2[0])**2 + (df['Y'] - c2[1])**2)
```

In [9]:
```
df
```

Out[9]:

	X	Y	Distance_to_c1	Distance_to_c2
0	2	3	1.414214	10.630146
1	3	4	0.000000	9.219544
2	5	6	2.828427	6.403124
3	8	8	6.403124	2.828427
4	10	10	9.219544	0.000000
5	12	12	12.041595	2.828427
6	18	18	20.518285	11.313708
7	20	20	23.345235	14.142136

In [10]:
```
# Assign each data point to the cluster with the minimum distance
df['Cluster'] = np.where(df['Distance_to_c1'] < df['Distance_to_c2'], 'Cluster 1', 'Cluster 2')
```

In [11]:
```
df
```

Out[11]:

	X	Y	Distance_to_c1	Distance_to_c2	Cluster
0	2	3	1.414214	10.630146	Cluster 1
1	3	4	0.000000	9.219544	Cluster 1
2	5	6	2.828427	6.403124	Cluster 1
3	8	8	6.403124	2.828427	Cluster 2
4	10	10	9.219544	0.000000	Cluster 2
5	12	12	12.041595	2.828427	Cluster 2
6	18	18	20.518285	11.313708	Cluster 2
7	20	20	23.345235	14.142136	Cluster 2

Update Centroids by Calculating Average:

In [12]:
```
new1_c1 = (df[df['Cluster'] == 'Cluster 1']['X'].mean(), df[df['Cluster'] == 'Cluster 1']['Y'].mean())
new1_c2 = (df[df['Cluster'] == 'Cluster 2']['X'].mean(), df[df['Cluster'] == 'Cluster 2']['Y'].mean())
```

In [13]:
```
new1_c1,new1_c2
```

Out[13]:
```
((3.3333333333333335, 4.333333333333333), (13.6, 13.6))
```

In [14]:
```
#Euclidean Distance Each Data Point to Each(2) new_centoids:

# new_c1 and new_c2 to all data points euclidean distance:
df['Distance_to_new1_c1']= np.sqrt((df['X']-new1_c1[0])**2+(df['Y']-new1_c1[1])**2)
df['Distance_to_new1_c2'] = np.sqrt((df['X'] - new1_c2[0])**2 + (df['Y'] - new1_c2[1])**2)
```

In [15]:
```
df
```

Out[15]:

	X	Y	Distance_to_c1	Distance_to_c2	Cluster	Distance_to_new1_c1	Distance_to_new1_c2
0	2	3	1.414214	10.630146	Cluster 1	1.885618	15.713688
1	3	4	0.000000	9.219544	Cluster 1	0.471405	14.301049
2	5	6	2.828427	6.403124	Cluster 1	2.357023	11.476933
3	8	8	6.403124	2.828427	Cluster 2	5.934831	7.919596
4	10	10	9.219544	0.000000	Cluster 2	8.749603	5.091169
5	12	12	12.041595	2.828427	Cluster 2	11.571037	2.262742
6	18	18	20.518285	11.313708	Cluster 2	20.047167	6.222540
7	20	20	23.345235	14.142136	Cluster 2	22.874051	9.050967

In [16]:

```
# Assign each data point to the cluster with the minimum distance
df['new_Cluster'] = np.where(df['Distance_to_new1_c1'] < df['Distance_to_new1_c2'], 'Cluster 1', 'Cluster 2')
```

In [17]:

```
df
```

Out[17]:

	X	Y	Distance_to_c1	Distance_to_c2	Cluster	Distance_to_new1_c1	Distance_to_new1_c2	new_Cluster
0	2	3	1.414214	10.630146	Cluster 1	1.885618	15.713688	Cluster 1
1	3	4	0.000000	9.219544	Cluster 1	0.471405	14.301049	Cluster 1
2	5	6	2.828427	6.403124	Cluster 1	2.357023	11.476933	Cluster 1
3	8	8	6.403124	2.828427	Cluster 2	5.934831	7.919596	Cluster 1
4	10	10	9.219544	0.000000	Cluster 2	8.749603	5.091169	Cluster 2
5	12	12	12.041595	2.828427	Cluster 2	11.571037	2.262742	Cluster 2
6	18	18	20.518285	11.313708	Cluster 2	20.047167	6.222540	Cluster 2
7	20	20	23.345235	14.142136	Cluster 2	22.874051	9.050967	Cluster 2

In [18]:

```
# Create a scatter plot for the data points in each cluster
plt.scatter(df[df['new_Cluster'] == 'Cluster 1']['X'], df[df['new_Cluster'] == 'Cluster 1']['Y'], label='Cluster 1',
plt.scatter(df[df['new_Cluster'] == 'Cluster 2']['X'], df[df['new_Cluster'] == 'Cluster 2']['Y'], label='Cluster 2',

# Plot the centroids as well
plt.scatter(new1_c1[0], new1_c1[1], label='Centroid 1', marker='x', c='black', s=100)
plt.scatter(new1_c2[0], new1_c2[1], label='Centroid 2', marker='x', c='green', s=100)

# Add labels and a legend
plt.xlabel('X')
plt.ylabel('Y')
plt.legend()

# Show the scatter plot
plt.show()
```