Coding Exercise 3: Solving the HJB Equation of the Huggett Model

ECON 202A

November 5, 2024

In this problem set, you will solve the Huggett Model using the Hamilton-Jacobi-Bellman (HJB) equation in continuous time with an upwind scheme and implicit method. Your task is to produce plots that illustrate key results of the model and to provide the code used to generate these plots.

The Huggett Model equations and relevant parameters are defined below.[1]

$$\begin{aligned} \rho V_e(a) &= \max_c \left\{ U(c) + V_e'(a)(z_e + ra - c) + \lambda_e(V_u(a) - V_e(a)) \right\}, \\ \rho V_u(a) &= \max_c \left\{ U(c) + V_u'(a)(z_u + ra - c) + \lambda_u(V_e(a) - V_u(a)) \right\} \end{aligned} \tag{1}$$

where the optimal consumption $c_j = c_j(a)$ is derived from the first-order condition:

$$U'(c_j) = V_j'(a), \tag{2}$$

and $s_j(a) = z_j + ra - c_j$ represents optimal savings with $j \in \{e, u\}$.

The utility function is assumed to be of constant relative risk aversion (CRRA), and the borrowing constraint is $a \geq -0.02$.

The model parameters are as follows:

- Relative risk aversion coefficient: $\sigma = 2$
- Income: $z = [z_e, z_u] = [0.2, 0.1]$
- Transition rates: $\lambda = [\lambda_e, \lambda_u] = [0.03, 0.02]$
- Interest rate: $r = 0.03$
- Discount rate: $\rho = 0.05$

Please complete the following tasks and submit both your written answers and the code you used. Ensure that your submission includes plots and a brief explanation of your results. Refer to the course syllabus for detailed code submission guidelines.

[1] Refer to the section slides for any notational details.

1. Plot the optimal consumption for both employed and unemployed states across the asset grid. Discuss any notable features and their economic interpretations.

2. Plot the optimal savings for both employed and unemployed states across the asset grid. Discuss any notable features and their economic interpretations.

3. Plot the value function across the asset grid for both employed and unemployed states. Discuss any notable features and their economic interpretations.

4. Based on the generated plots, discuss the macroeconomic implications of the Huggett model.