Graph Theory - Part III

Foundation Course on Discrete Mathematics for GATE

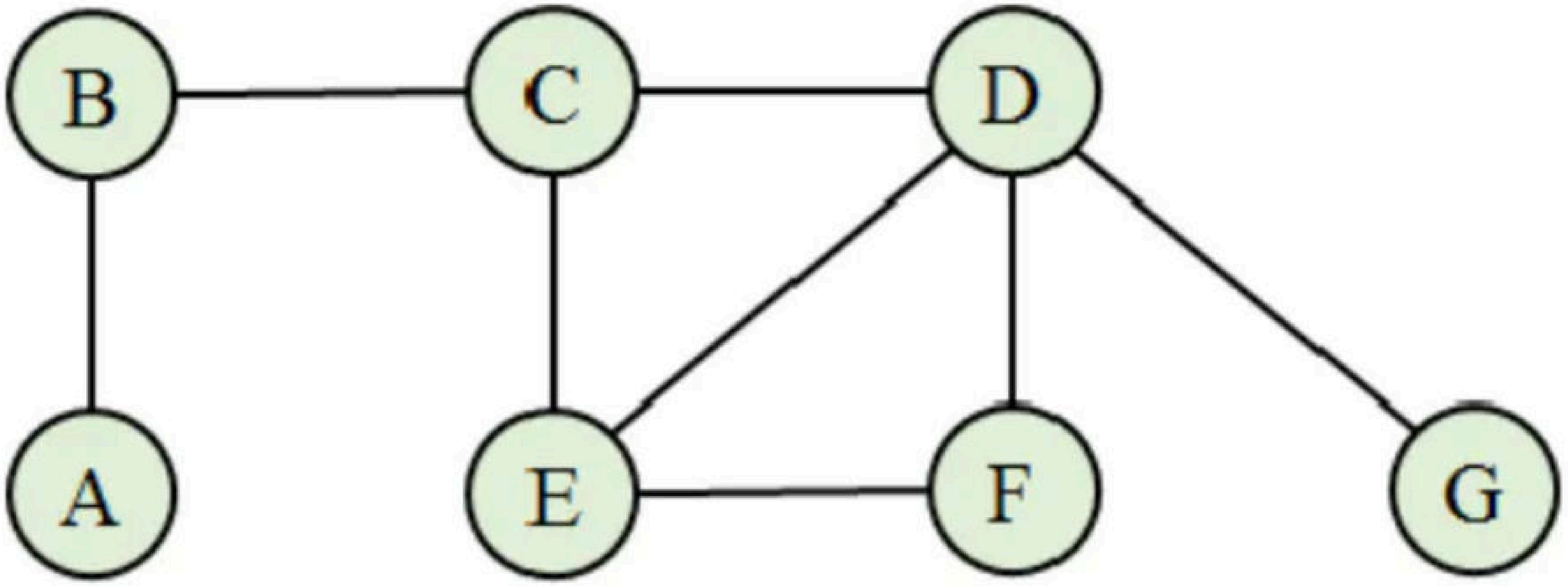
Sanchit Jain • Lesson 21 • Mar 1, 2022

Graph Theory

1. A graph G (V, E) consist of a set off objects $V = \{V_1, V_2, V_3....,V_N\}$ called vertices and another set $E = \{E_1, E_2, E_3,....,E_n\}$ whose elements are called edges.

2. Each edge e_k is identified with an unordered pair (v_i, v_j) of vertices.

3. The vertices $v_{i,}$ v_j associated with edge e_k are called the end vertices of e_k.

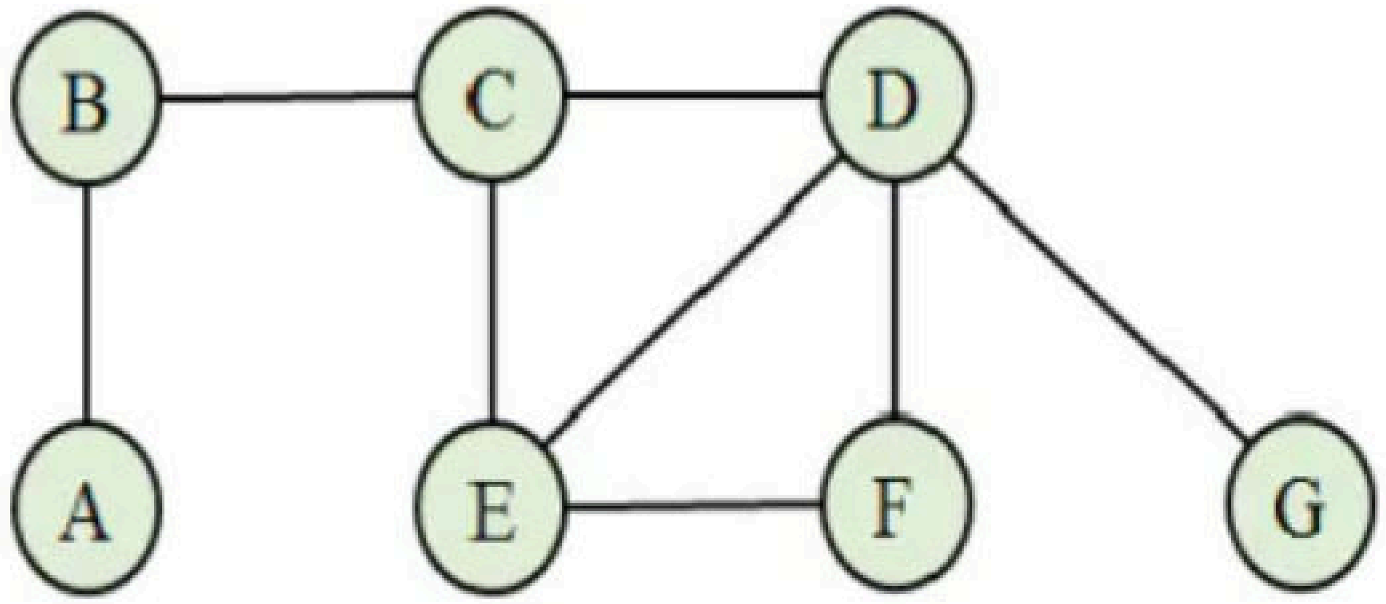


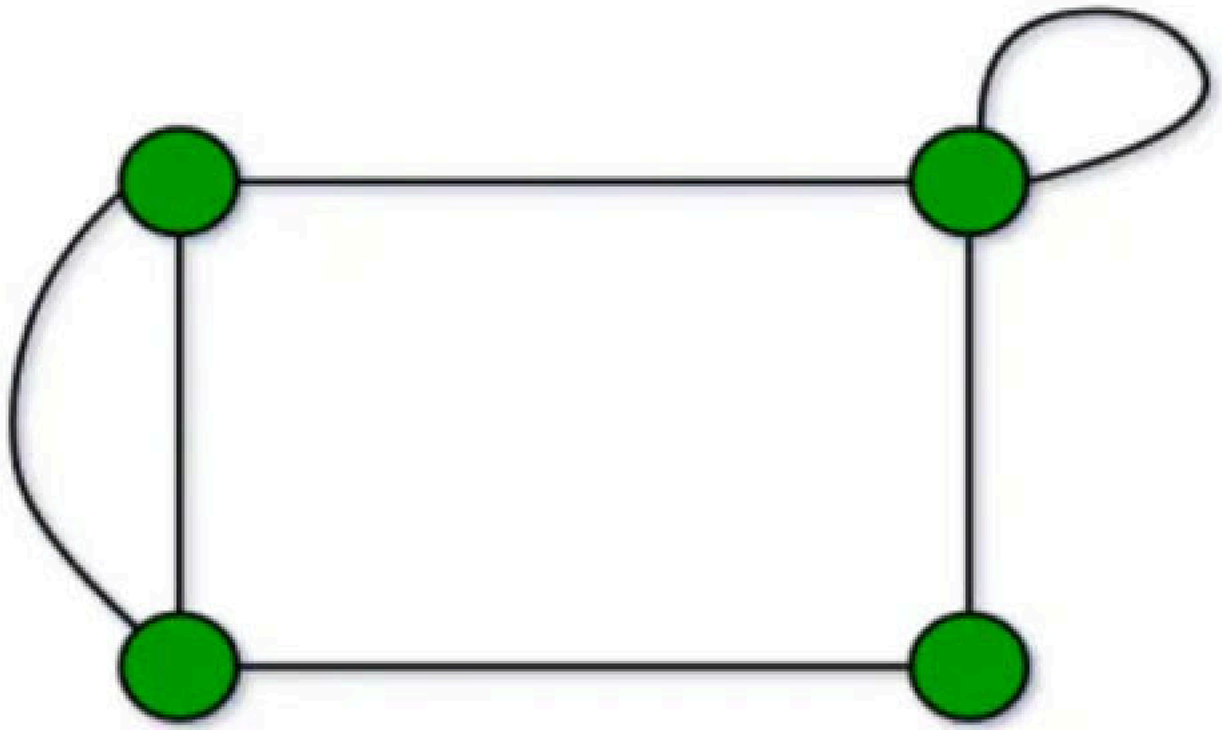
1. **Self-Loop:** Edge having the same vertex (v_i, v_i) as both its end vertices is called self-loop.

2. **Parallel Edge:** When more than one edge associated with a given pair of vertices such edges are referred as parallel edges.

3. **Adjacent Vertices:** If two vertices are joined by the same edges, they are called adjacent vertices.

4. **Adjacent Edges:** If two edges are incident on some vertex, they are called adjacent edges.

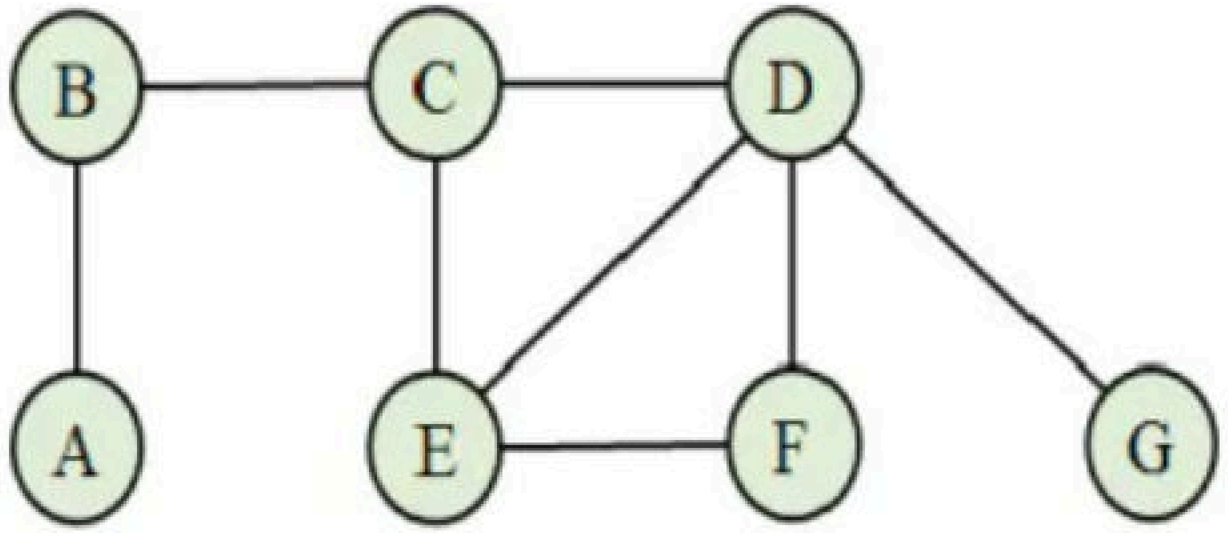


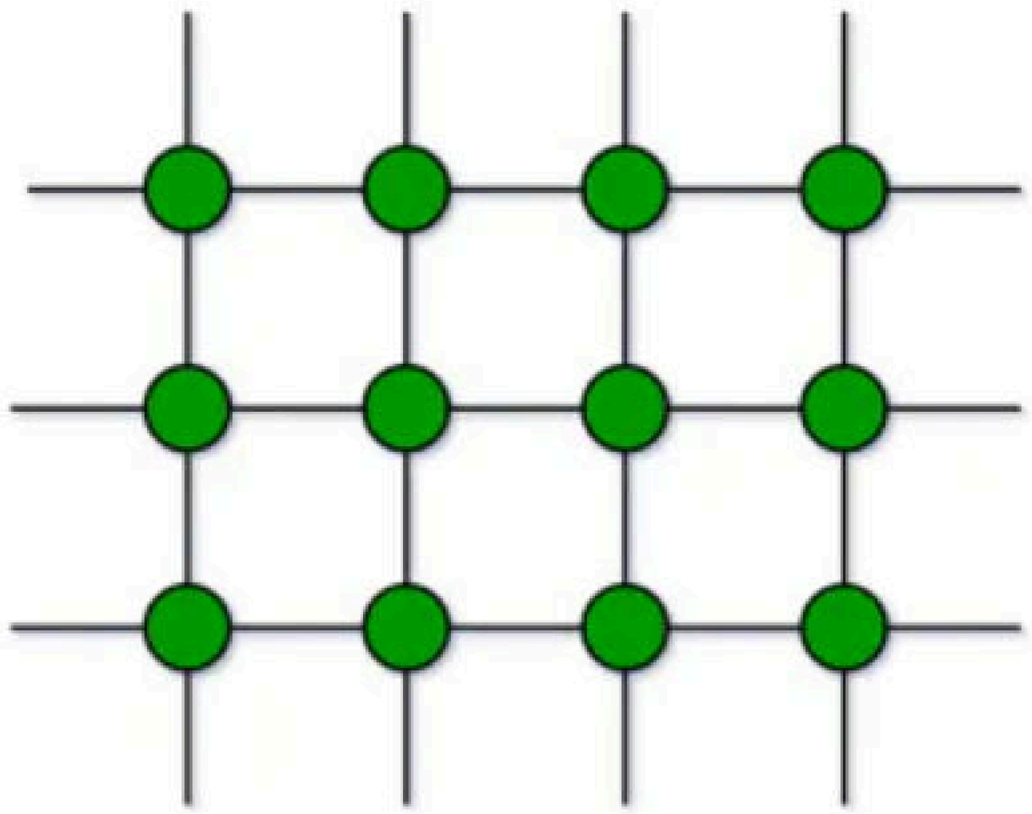


	Self-Loop	Parallel Edge
Simple Graph	No	No
Multi/Pseudo Graph	Yes	Yes

1. **Finite graph**: - A graph with finite number of vertices as well as the finite number of edges is called a finite graph.

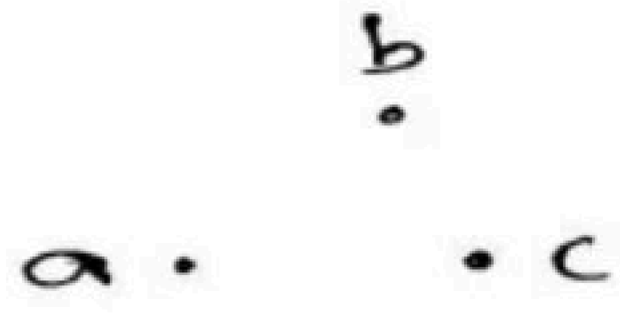
2. For simple graph we can say if the number of vertices are finite then number of edges will also be finite.



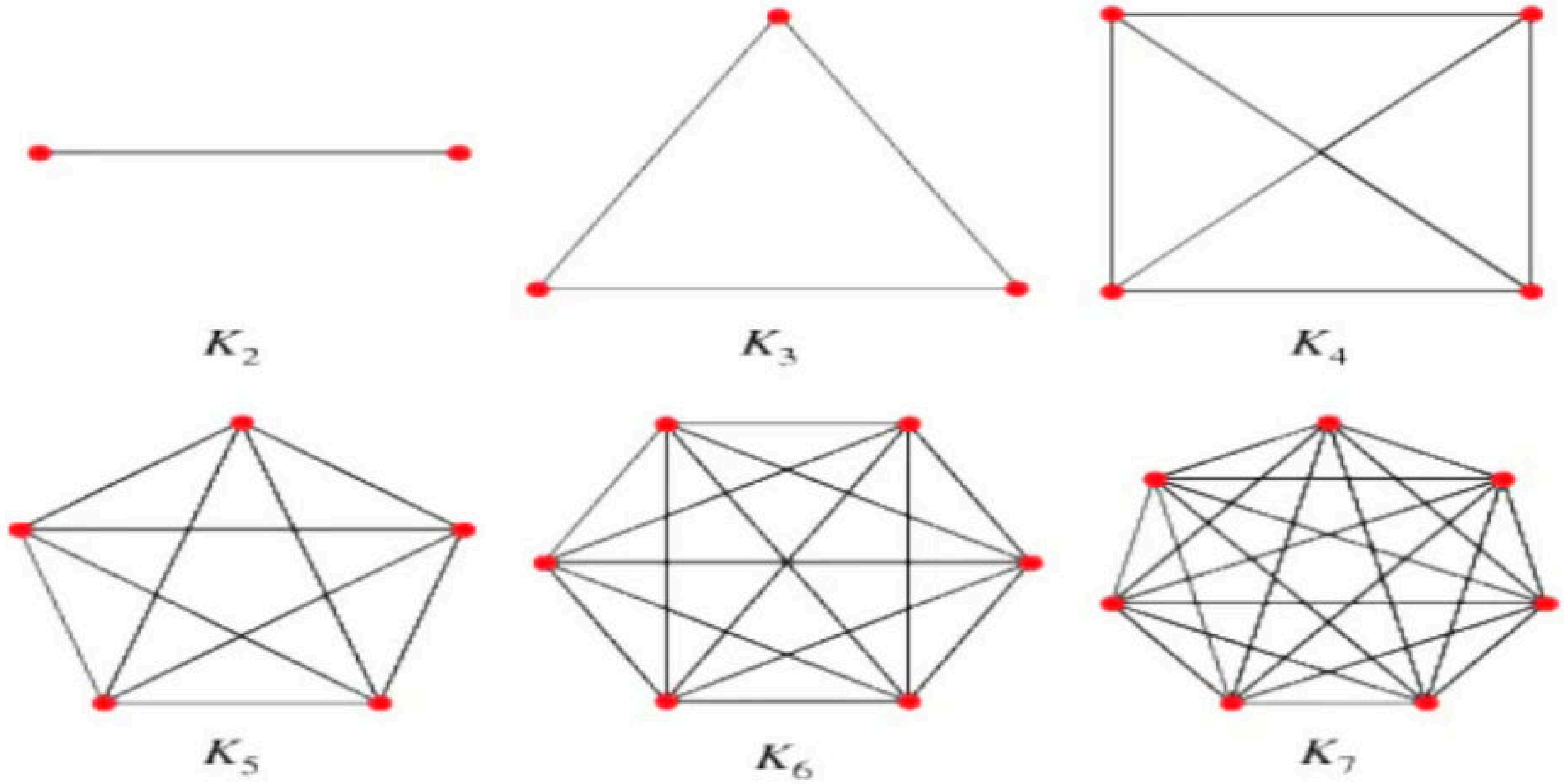


1. **Null Graph:** A graph is said to be null if edge set is empty E = {}, that is a graph with only vertices but no edges.

2. **Trivial Graph:** A graph with only one vertex without an edge is called trivial graph. It is the smallest possible.



- **<u>Complete or Full Graph:</u>** In a simple graph there exist an edge between each and every pair of vertices i.e. every vertex are adjacent to each other, then the graph is said to be a complete graph, denoted by K_n.



1. A simple graph with maximum number of edges are called Complete Graph.
2. Number of edges in a simple graph is $n(n-1)/2$

Q1 The number of edges in a complete graph with N vertices is equal to: **(NET-DEC-2006) (NET-DEC-2007)**

a) N (N - 1) **b)** N(N-1)]/2 **c)** N^2 **d)** 2N-1

Q2 The number of edges in a complete graph of n vertices is **(NET-DEC-2009)**

(A) n **(B)** $n(n - 1)/2$ **(C)** $n(n + 1)/2$ **(D)** $n^2/2$

Q3 The complete graph with four vertices has k edges where k is: **(NET-JUNE-2009)**

a) 3 **b)** 4 **c)** 5 **d)** 6

Q Maximum number of edges in a n node undirected graph without self-loops is **(GATE-2002) (1 Marks) (NET-DEC-2011)**

(A) n^2 **(B)** $n(n-1)/2$ **(C)** $n-1$ **(D)** $(n+1)(n)/2$

Q Number of simple graph possible with n vertices?

Q Number of simple graph possible with n vertices and e edges?

Q The number of distinct simple graphs with up to three nodes is **(GATE-1994) (1 Marks)**

a) 15 **b)** 11 **c)** 8 **d)** 9

Q How many undirected graphs (not necessarily connected) can be constructed out of a given set $V = \{v_1, v_2, \ldots v_n\}$ of n vertices? **(GATE-2001) (2 Marks)**

(A) $n(n-1)/2$ **(B)** $2n$ **(C)** $n!$ **(D)** $2^{n(n-1)/2}$

Q Let G be a complete undirected graph on 6 vertices. If vertices of G are labeled, then the number of distinct cycles of length 4 in G is equal to **(GATE-2012) (2 Marks)**

(A) 15 **(B)** 30 **(C)** 90 **(D)** 360

Q Consider an undirected graph G where self-loops are not allowed. The vertex set of G is $\{(i, j): 1 <= i <= 12, 1 <= j <= 12\}$. There is an edge between (a, b) and (c, d) if $|a - c| <= 1$ and $|b - d| <= 1$. **(GATE-2014) (2 Marks)** **(NET-AUG-2016)**
The number of edges in this graph is __________.

Q Consider an undirected random graph of eight vertices. The probability that there is an edge between a pair of vertices is 1/2. What is the expected number of unordered cycles of length three? **(GATE-2013) (1 Marks)**

(A) 1/8 **(B)** 1 **(C)** 7 **(D)** 8

Q How many graphs on n labeled vertices exist which have at least $(n^2 - 3n)/2$ edges? **(GATE-2004) (2 Marks)**

a) $^{((n2 - n)/2)}C_{((n2) - 3n)/2}$

b) $\sum_{K=0}^{((n\text{^}2) - 3n)/2} \; ((n^2 - n)/2)C_K$

c) $^{((n2) - n)/2)}C_n$

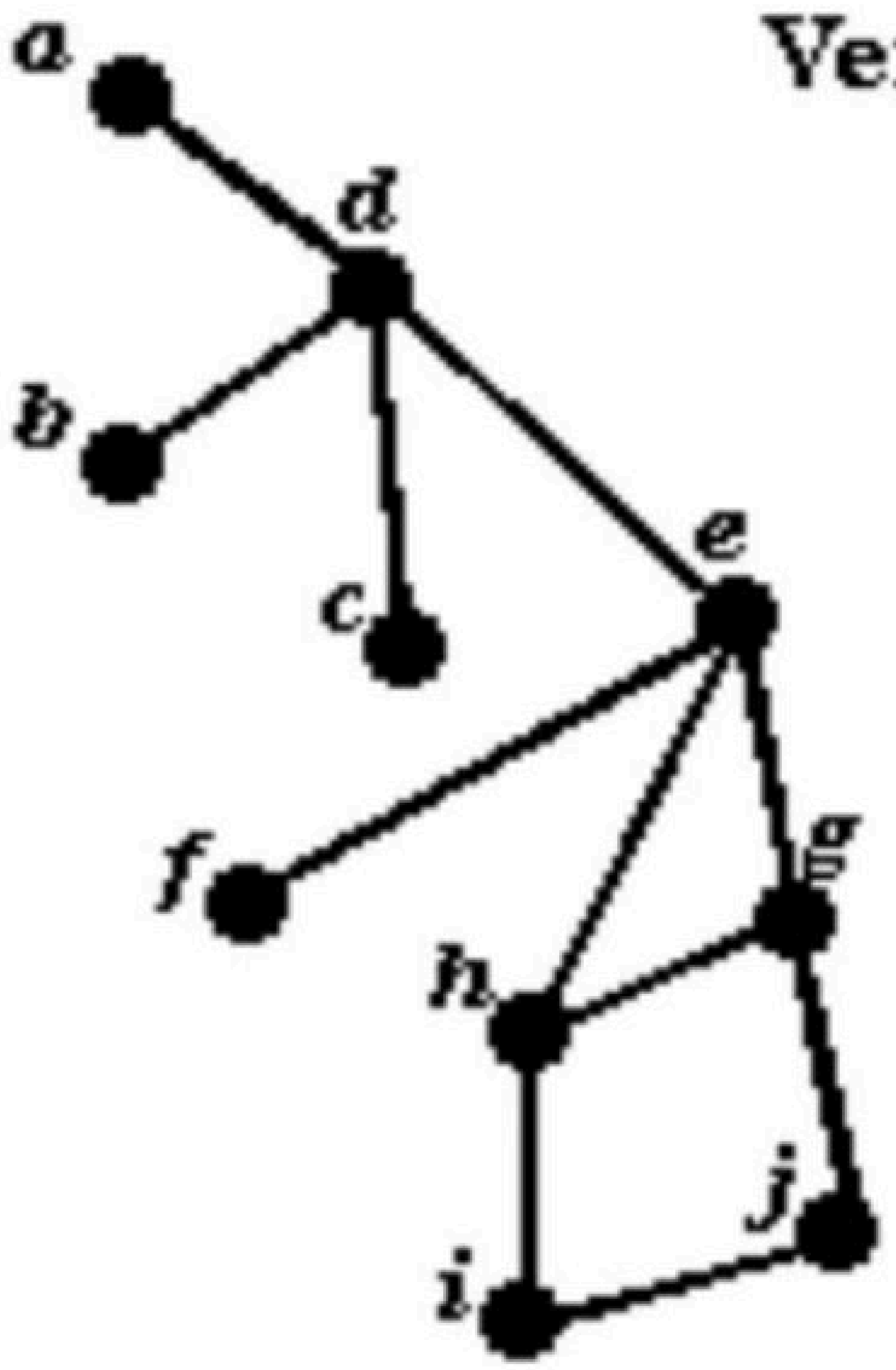
d) $\sum_{K=0}^{n} \; ((n^2 - n)/2)C_K$

Q The 2^n vertices of a graph G corresponds to all subsets of a set of size n, for n >= 6. Two vertices of G are adjacent if and only if the corresponding sets intersect in exactly two elements. The number of vertices of degree zero in G is **(GATE-2006) (2 Marks)**

(A) 1 **(B)** n **(C)** n+1 **(D)** 2n

Degree

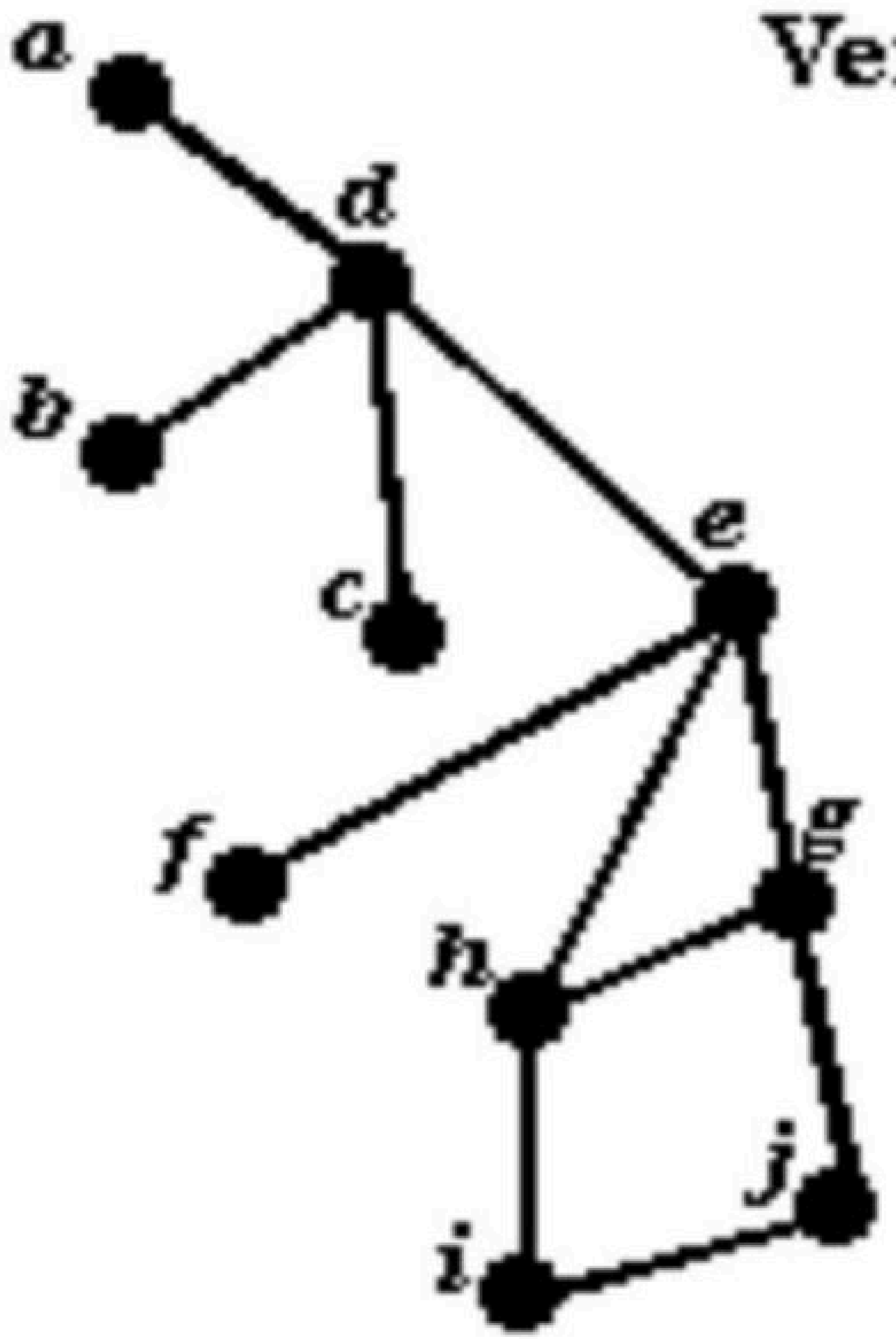
- **Degree of a Vertex:** The degree of a vertex in an undirected graph is the number of edges associated with it, denoted by deg(v_i).



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

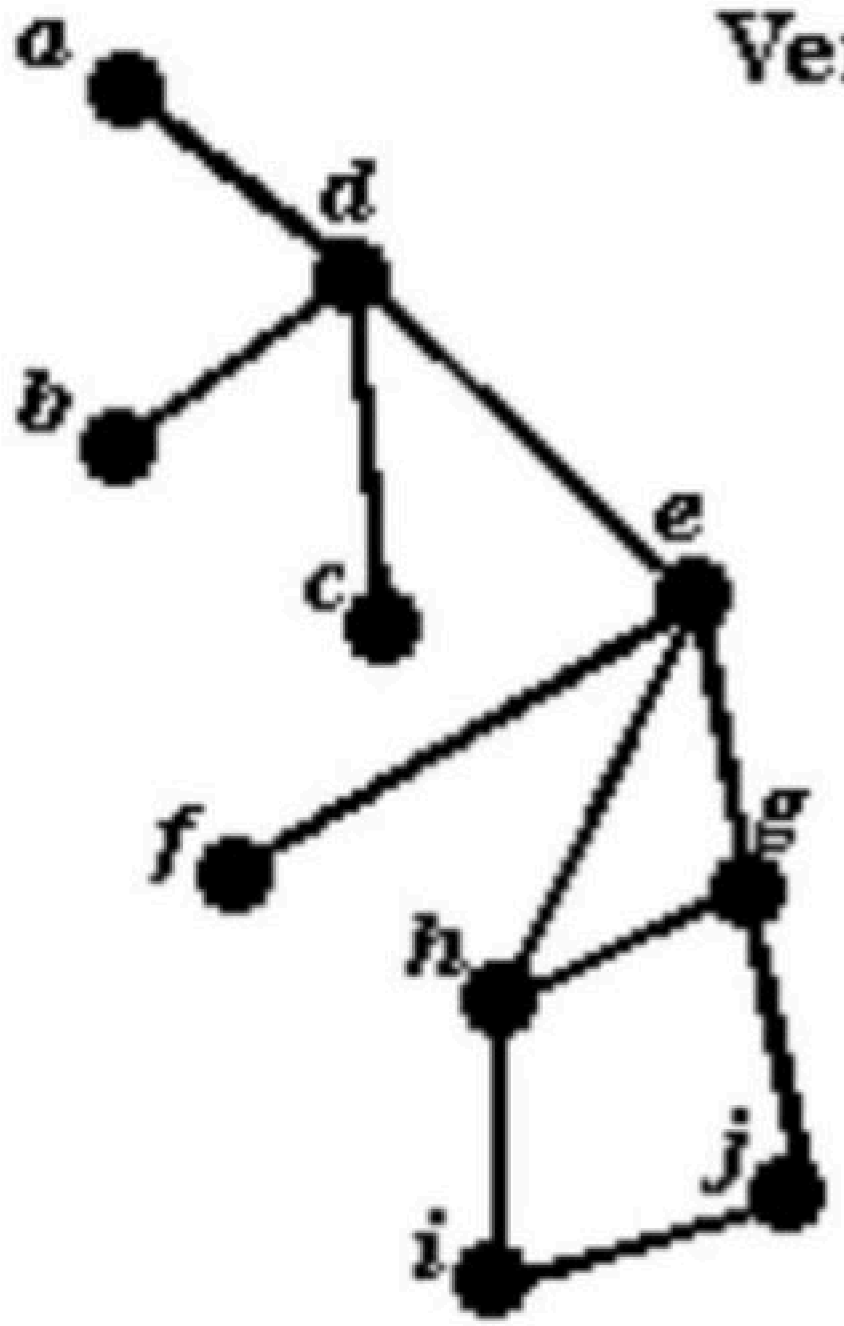
- **Isolated vertex:** A vertex with degree zero is called isolated vertex.

- **Pendant vertex:** A vertex with degree one is called pendant vertex.



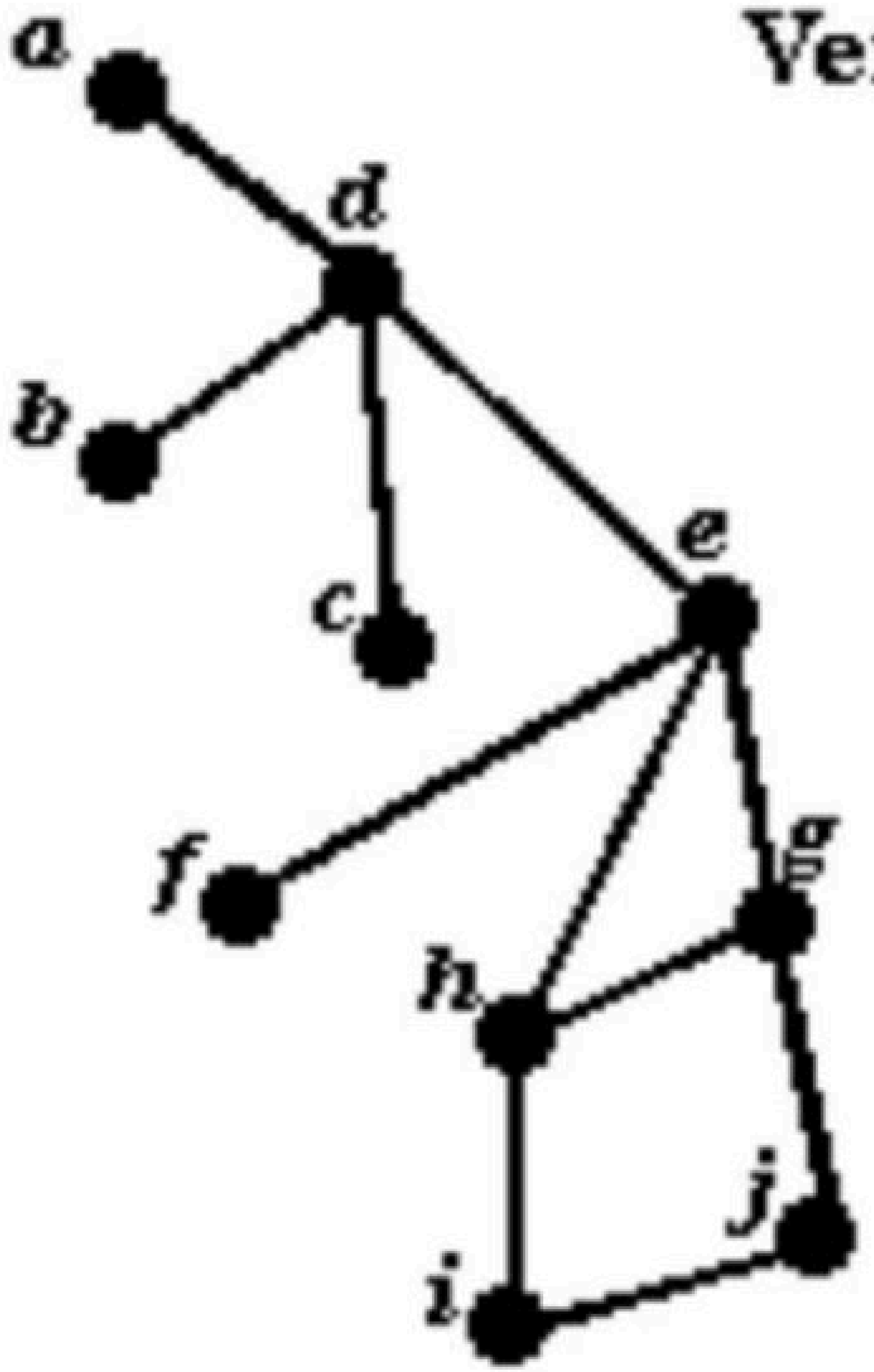
Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

- **<u>Hand-shaking theorem:</u>** - Since each edge contribute two degree in the graph, the sum of the degree of all vertices in G is twice the number of edges in g.
  - $\sum_{i=1}^{n} d(vi)$ = 2|E|



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

- The number of vertices of odd degree in a graph is always even.
  - $\sum_{i=1}^{n} d(vi) = \sum_{even} d(vi) + \sum_{odd} d(vi)$



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

Q Which of the following statements is/are TRUE for undirected graphs? **(GATE-2013) (1 Marks)**

P: Number of odd degree vertices is even.

Q: Sum of degrees of all vertices is even.

a) P only

b) Q only

c) Both P and Q

d) Neither P nor Q

Q Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices?**(GATE-2009) (1 Marks)**

(A) No two vertices have the same degree.

(B) At least two vertices have the same degree.

(C) At least three vertices have the same degree.

(D) All vertices have the same degree.

Q A simple graph G contains 21 edges, 3 vertices of degree 4 and all the remaining vertices are of degree 2. Then number of vertices |v| is?

Q A simple non-directed graph G has 24 edges and degree of each vertex is 4, then find the $|v|$?

Q Consider a simple graph with 35 edges such that 4 vertex of degree 5, 5 vertex of degree 4, 4 vertex of degree 3, find the number of vertex with degree 2?

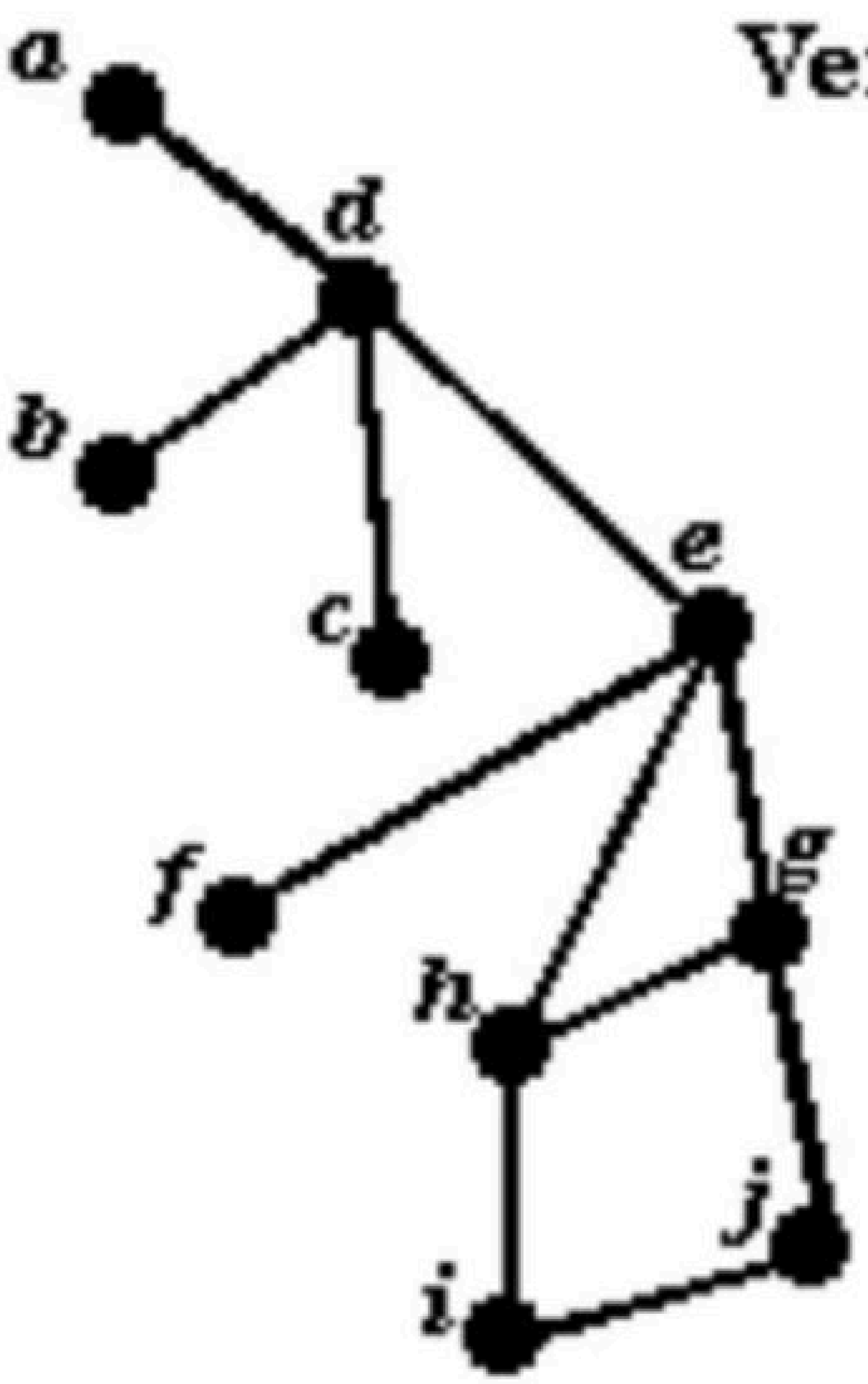
Q What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3? **(GATE-2004) (2 Marks)**

(A) 10 **(B)** 11 **(C)** 18 **(D)** 19

Q simple non-directed graph G has 24 edges and degree of each vertex is K, the which of the following is possible no of vertices?

a) 20 b) 15 c) 10 d) 8

- δ(G) is the minimum possible degree of any vertex in a graph
- Δ(G) is the maximum possible degree of any vertex in a graph.



Vertex	Degree
a	1
b	1
c	1
d	4
e	4
f	1
g	3
h	3
i	2
j	2

$$\boxed{\delta(G) * |V(G)| <= 2|E|} <= \Delta(G) * |V(G)|$$

Q G is undirected graph with n vertices and 25 edges such that each vertex has degree at least 3. Then the maximum possible value of n is __________ **(GATE-2017) (2 Marks)**

Q Maximum no of vertex in a simple graph with 35 edges and degree of each vertex is at least 3 is _________?

Q Minimum number of vertices possible in a simple graph if 41 edges and degree of each vertex is at most 5?

Q Which of the following degree sequence represent a simple non-directed graph?

1) {2, 3, 3, 4, 4, 5}

2) {2, 3, 4, 4, 5}

3) {3, 3, 3, 1}

4) {1, 3, 3, 4, 5, 6, 6}

5) {2, 3, 3, 3, 3}

6) {6, 6, 6, 6, 4, 3, 3, 0}

7) {6, 5, 5, 4, 3, 3, 2, 2, 2}

Q An ordered n-tuple (d1, d2, …, dn) with d1 >= d2 >= ⋯ >= dn is called graphic if there exists a simple undirected graph with n vertices having degrees d1, d2, …, dn respectively. Which of the following 6-tuples is NOT graphic? **(GATE-2014) (2 Marks)**

(A) (1, 1, 1, 1, 1, 1)

(B) (2, 2, 2, 2, 2, 2)

(C) (3, 3, 3, 1, 0, 0)

(D) (3, 2, 1, 1, 1, 0)

Q The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences cannot be the degree sequence of any graph? **(GATE-2010) (2 Marks)**

I. 7, 6, 5, 4, 4, 3, 2, 1

II. 6, 6, 6, 6, 3, 3, 2, 2

III. 7, 6, 6, 4, 4, 3, 2, 2

IV. 8, 7, 7, 6, 4, 2, 1, 1

(A) I and II **(B)** III and IV **(C)** IV only **(D)** II and IV

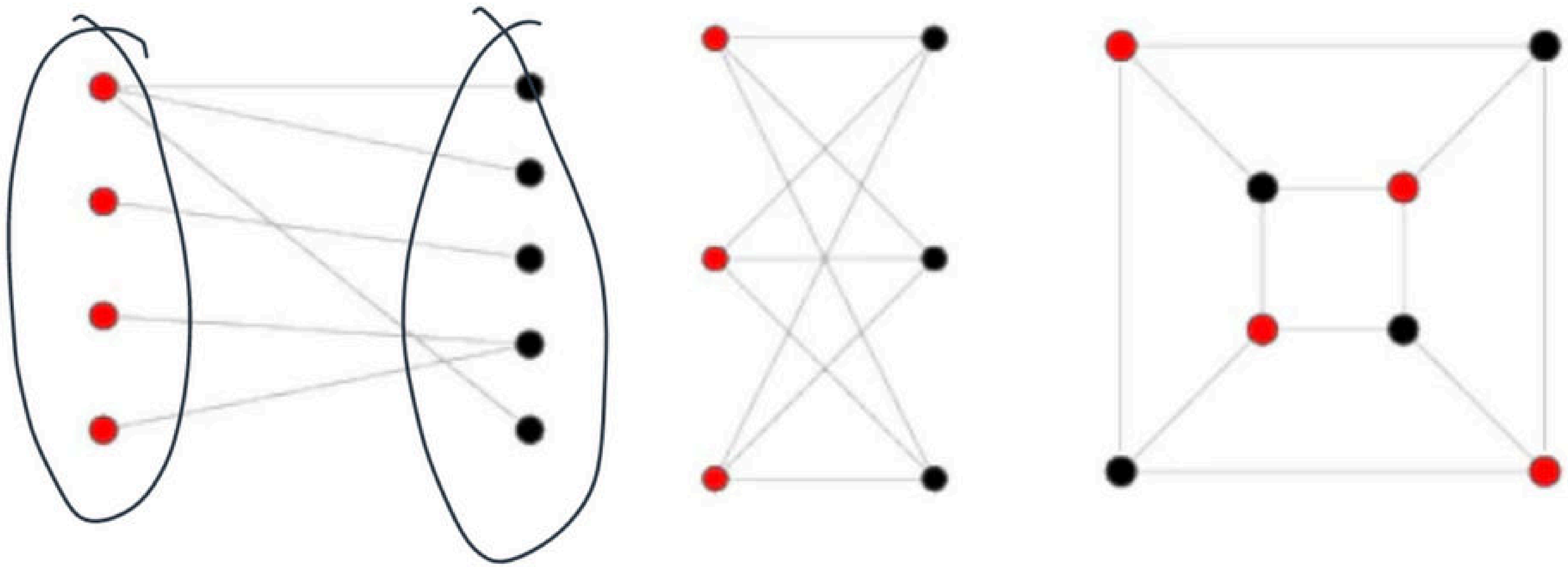
1. The **Havel–Hakimi algorithm** is an algorithm in graph theory solving the graph realization problem. That is, it answers the following question: Given a finite list of nonnegative integers, is there a simple graph such that its degree sequence is exactly this list.

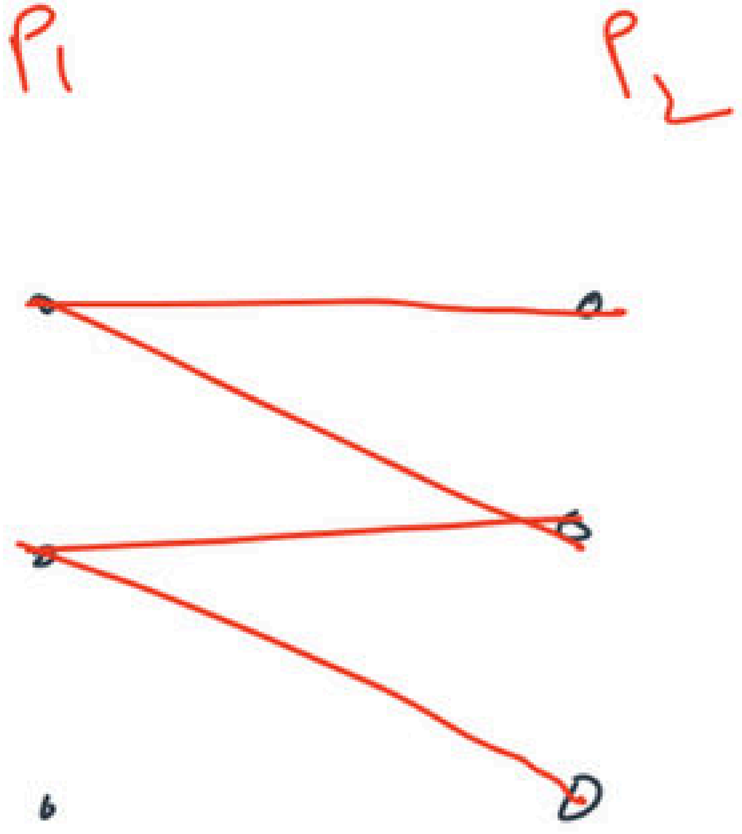
2. Here, the "degree sequence" is a list of numbers that for each vertex of the graph states how many neighbors it has. For a positive answer the list of integers is called *graphic*.

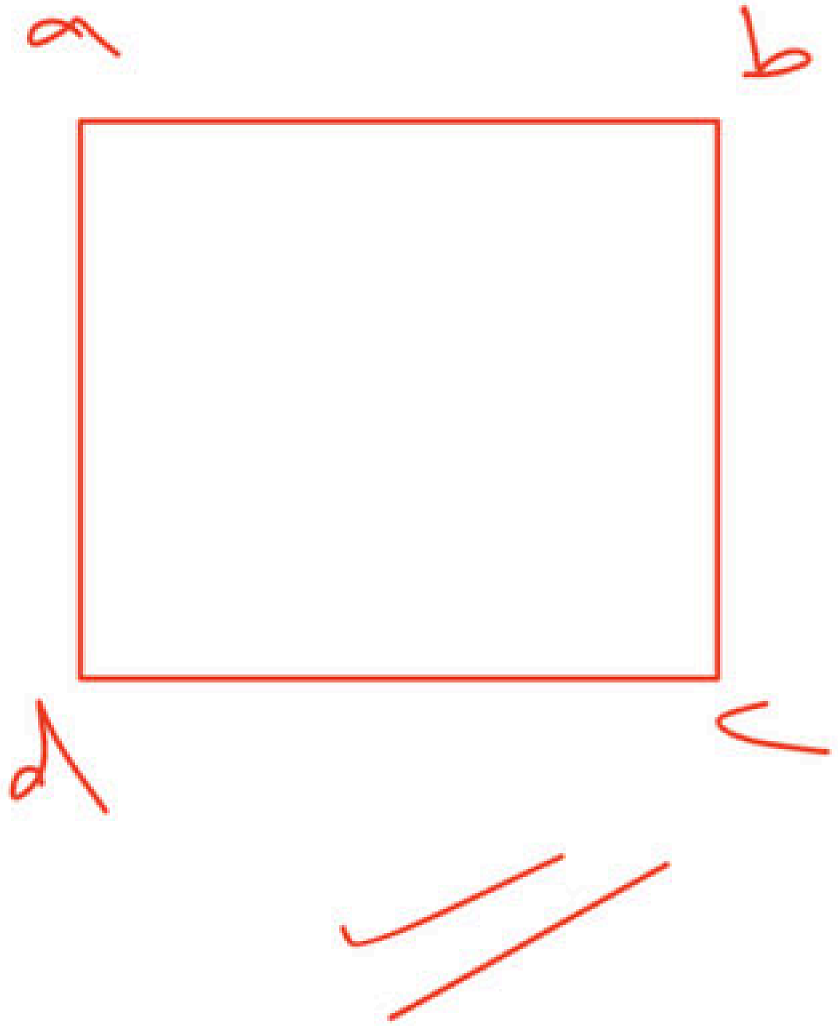
3. The algorithm constructs a special solution if one exists or proves that one cannot find a positive answer. This construction is based on a recursive algorithm. The algorithm was published by Havel (1955), and later by Hakimi (1962).

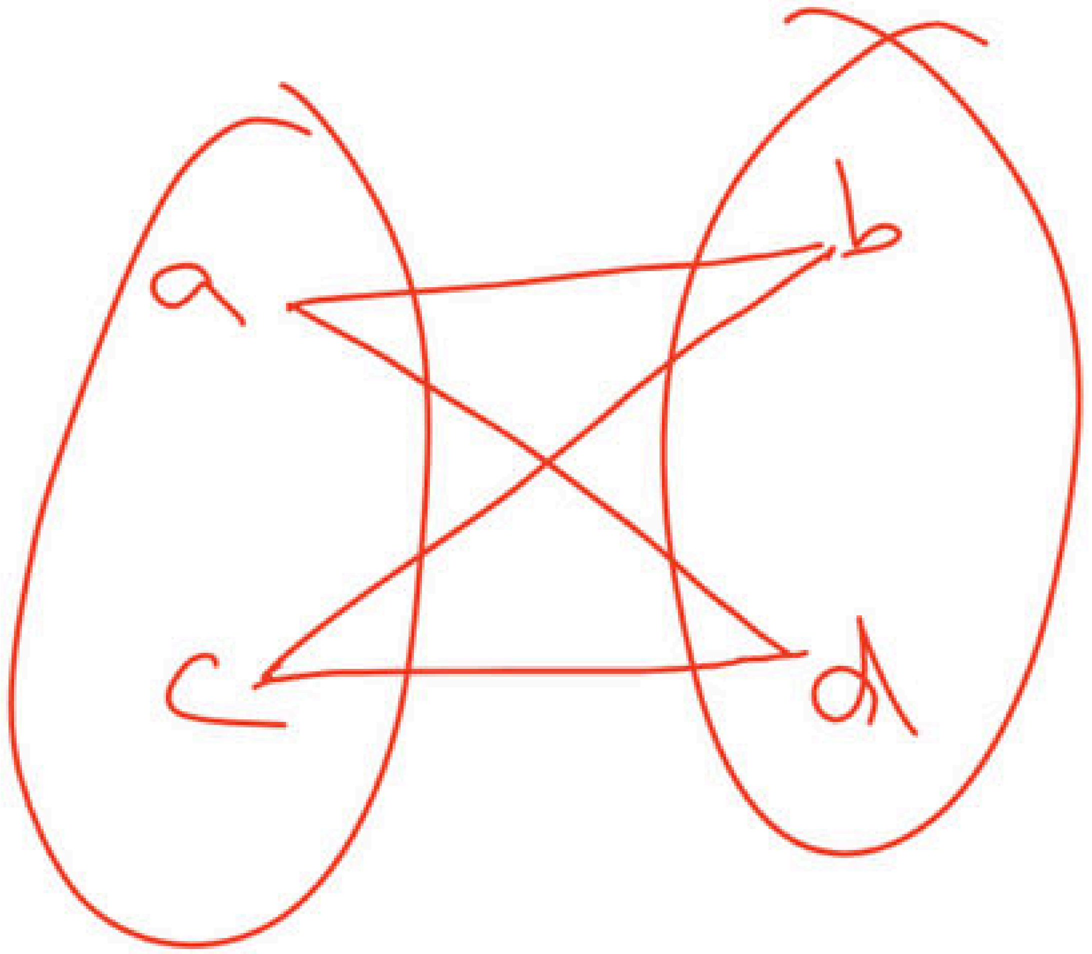
Some Popular Graph

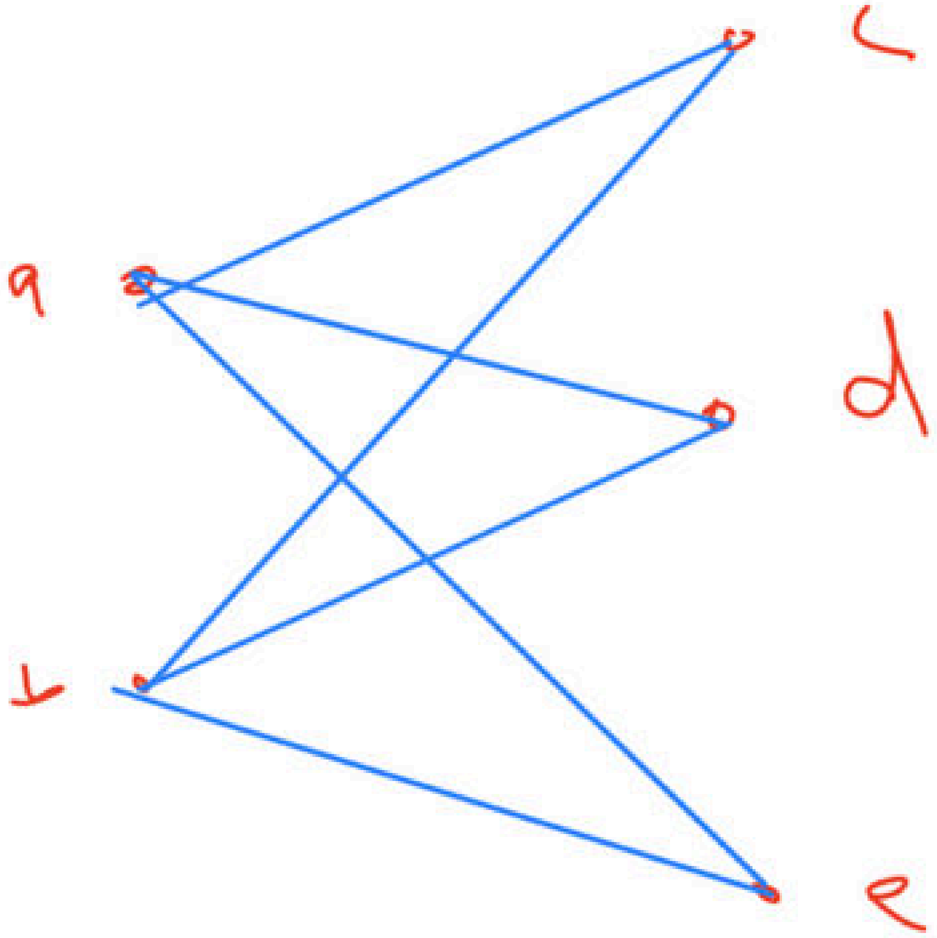
1. **Bi-partite graph:** - A graph G(V, E) is called bi-partite graph if it's vertex set V(G) can be partitioned into two non-empty disjoint subset $V_1(G)$ and $V_2(G)$ in such a way that each edge e ϵ E(G) has it's one end point in $V_1(g)$ and other end point in $V_2(g)$. The partition $V = V_1 \cup V_2$ is called bipartition of G.
2. **Complete Bi-partite graph:** - A Bi-partite graph G(V, E) is called Complete bi-partite graph if every vertex in the first partition is connected to every vertex in the second partition, denoted by $K_{m,n}$.



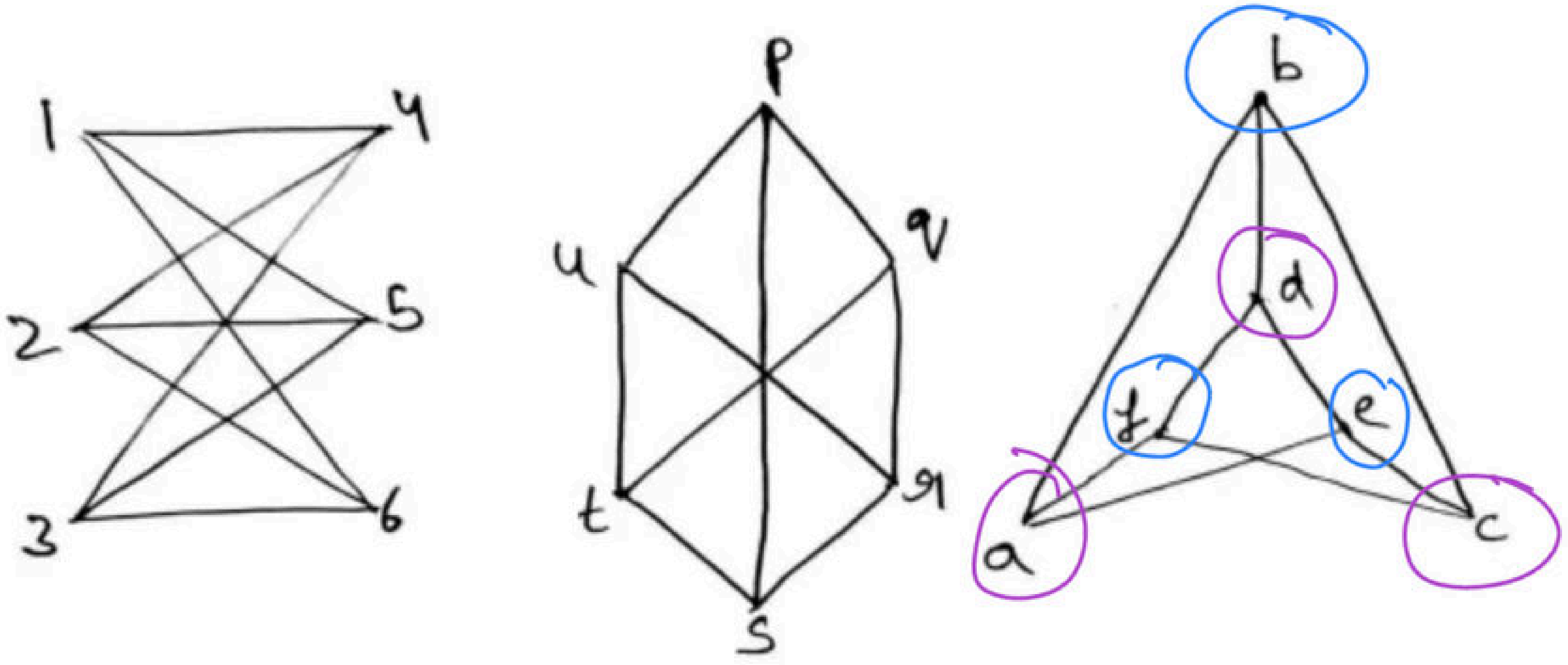








$K_{m,n} = K_{2,3}$



1 2 3

$K_{m,n}$

$m \times n = \underline{\underline{m \cdot n}}$

Q The maximum number of edges in a bipartite graph on 12 vertices is ______________________________. **(GATE-2014) (1 Marks)**

$|V| = 12$

$1 \times 11 = 11$

$2 \times 10 = 20$

$3 \times 9 = 27$

$4 \times 8 = 32$

$5 \times 7 = 35$

$6 \times 6 = 36$

$7 \times 5 = 35$

11 → a) 24

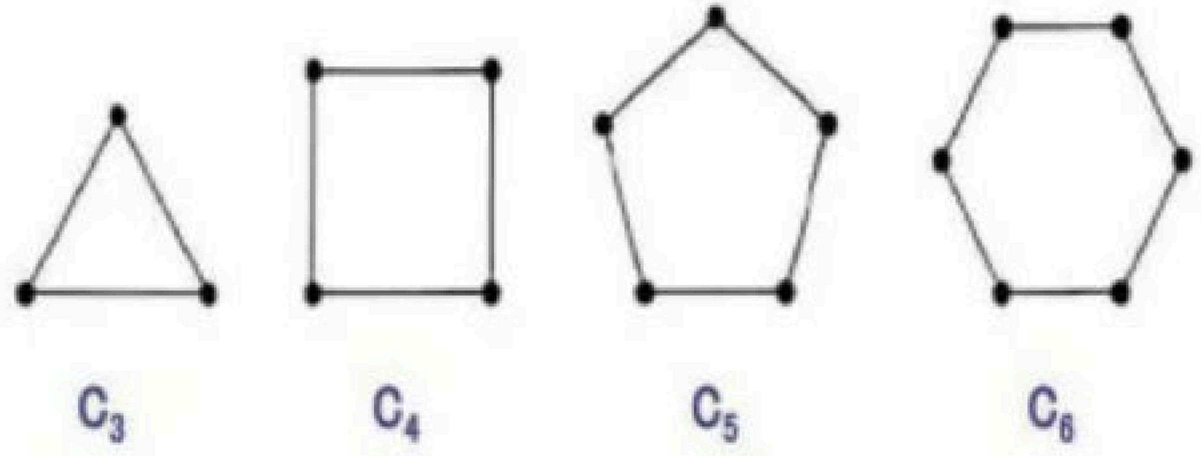
77 → b) ~~36~~

2 → c) 9

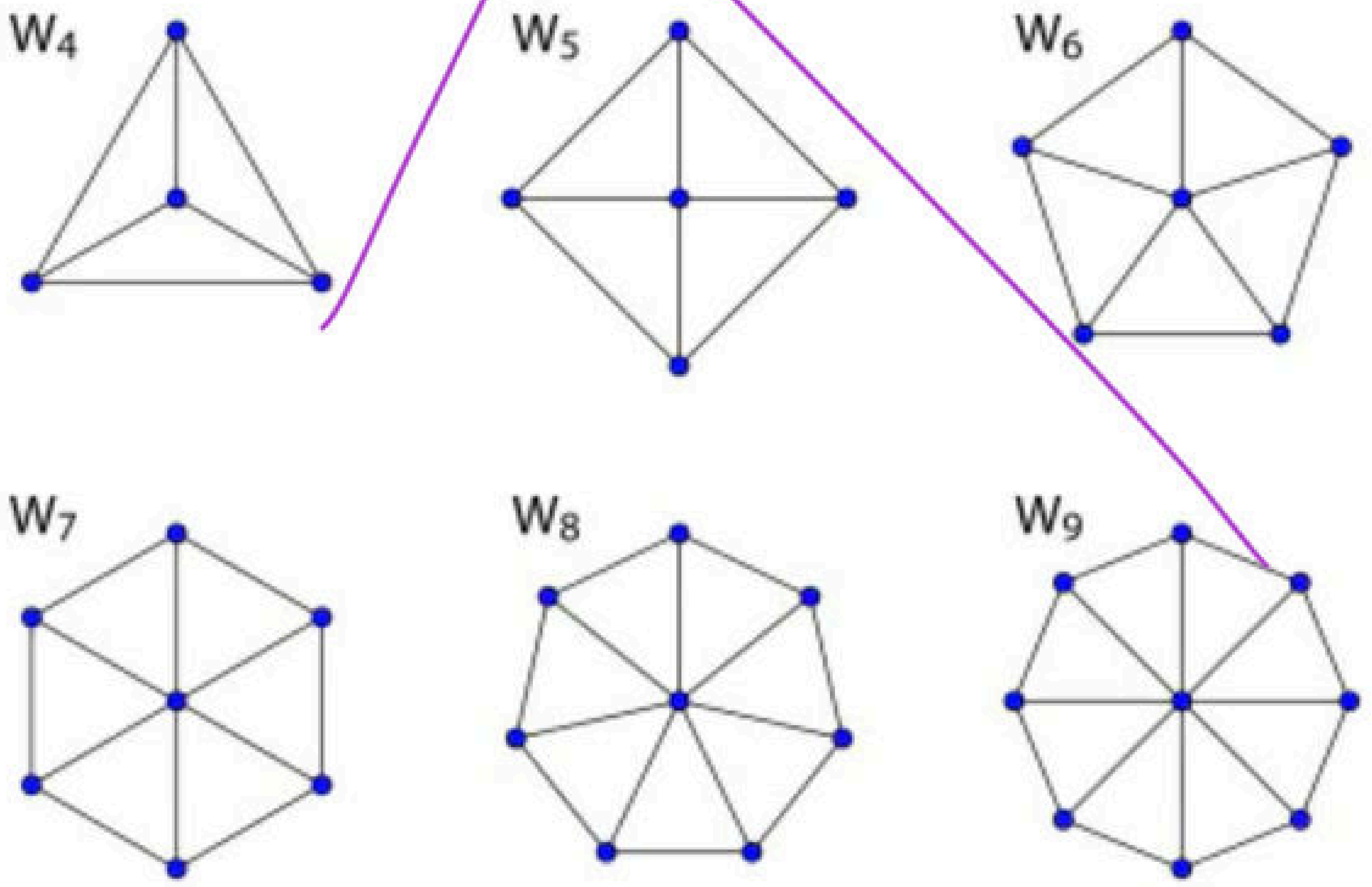
2 — d) 16

This video
Typical performance
6.0K
4.0K
2.0K
0
0
1
2
3
4
5
6
7 days

1. **Cycle Graph:** - A cycle graph or circular graph is a graph that consists of a single cycle, or in other words, some number of vertices (at least 3) connected in a closed chain. The cycle graph with *n* vertices is called C_n. The number of vertices in C_n equals the number of edges, and every vertex has degree 2; that is, every vertex has exactly two edges incident with it.
2. **Wheel graph:** - A wheel graph is a graph formed by connecting a single universal vertex to all vertices of a cycle. Some authors write W_n to denote a wheel graph with *n* vertices (n ≥ 4); other authors instead use W_n to denote a wheel graph with *n*+1 vertices (n ≥ 3), which is formed by connecting a single vertex to all vertices of a cycle of length *n*.



C5



Q the graph $K_{3,4}$ has: **(NET-DEC-2008)**

a) 3 edges **b)** 4 edges **c)** 7 edges **d)** 12 edges

1 3 2 ans

Q Consider the graph given below: **(NET-DEC-2015)**

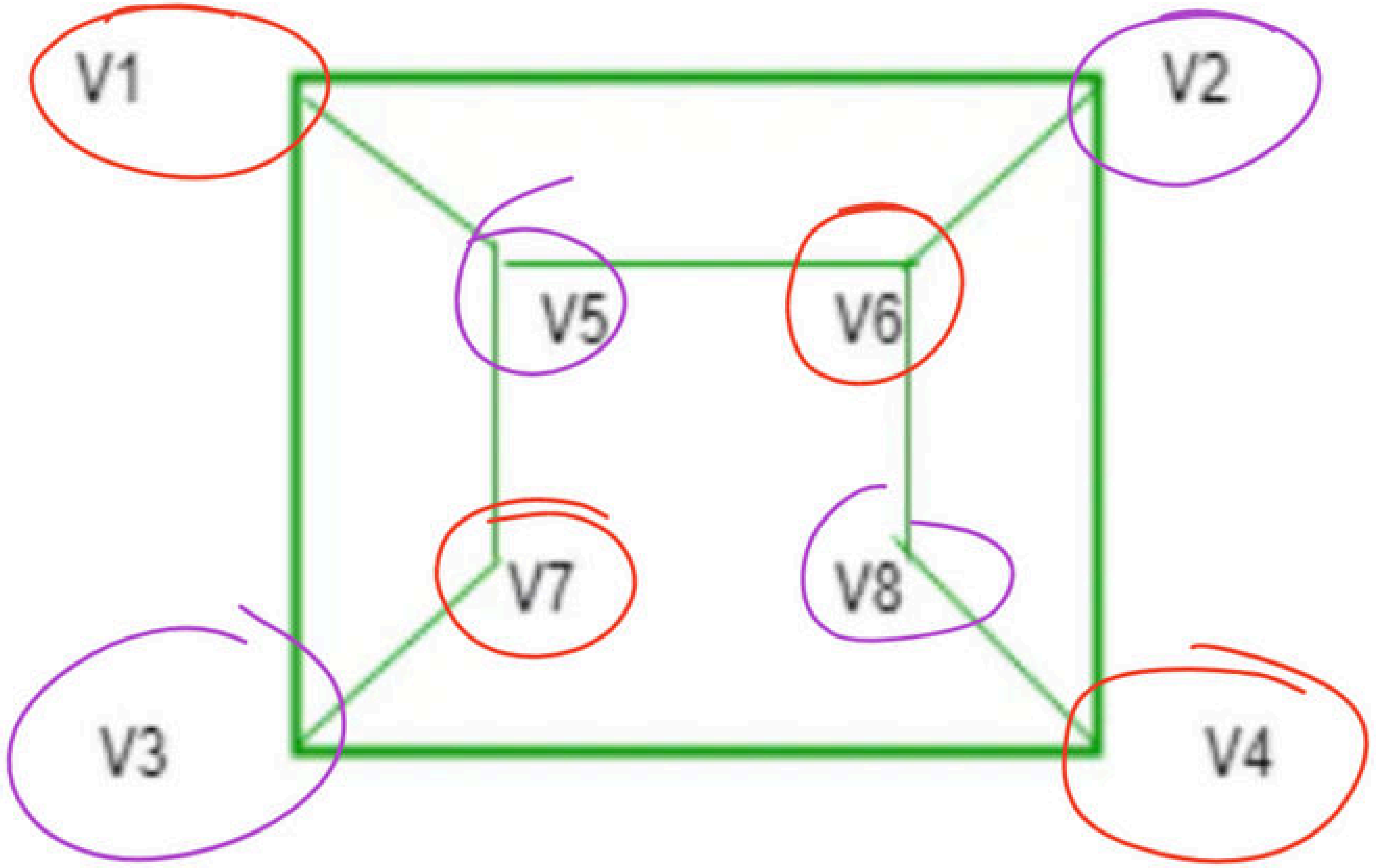
The two distinct sets of vertices, which make the graph bipartite are:

a) (v1, v4, v6); (v2, v3, v5, v7, v8) — 9

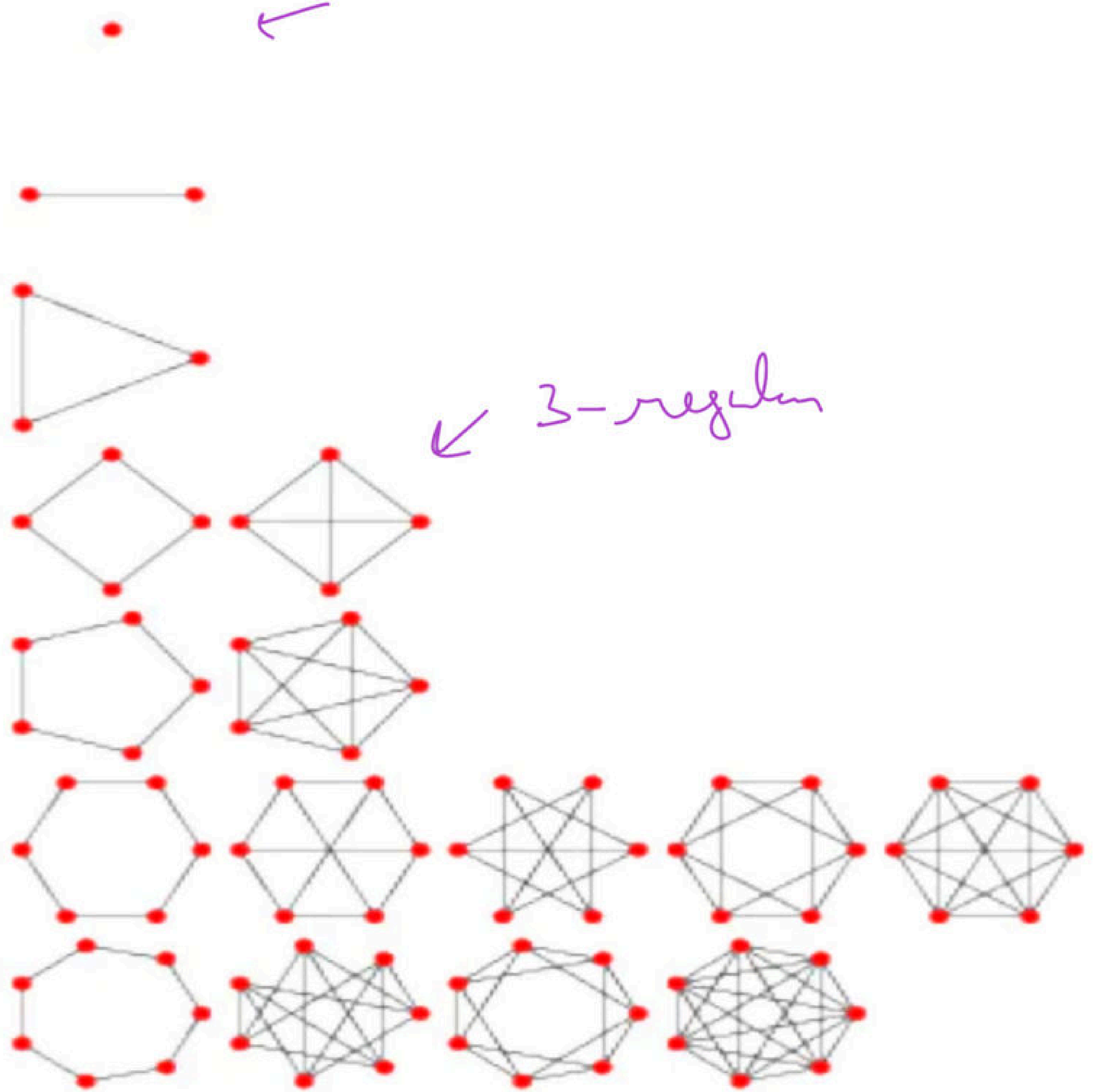
b) (v1, v7, v8); (v2, v3, v5, v6) — 8

c) (v1, v4, v6, v7); (v2, v3, v5, v8) — 65

d) (v1, v4, v6, v7, v8); (v2, v3, v5) — 13



- **Regular graph:** - A graph in which all the vertices are of equal degree is called a regular graph. E.g. 2-regular graph, 3-regular graph.



3-regular

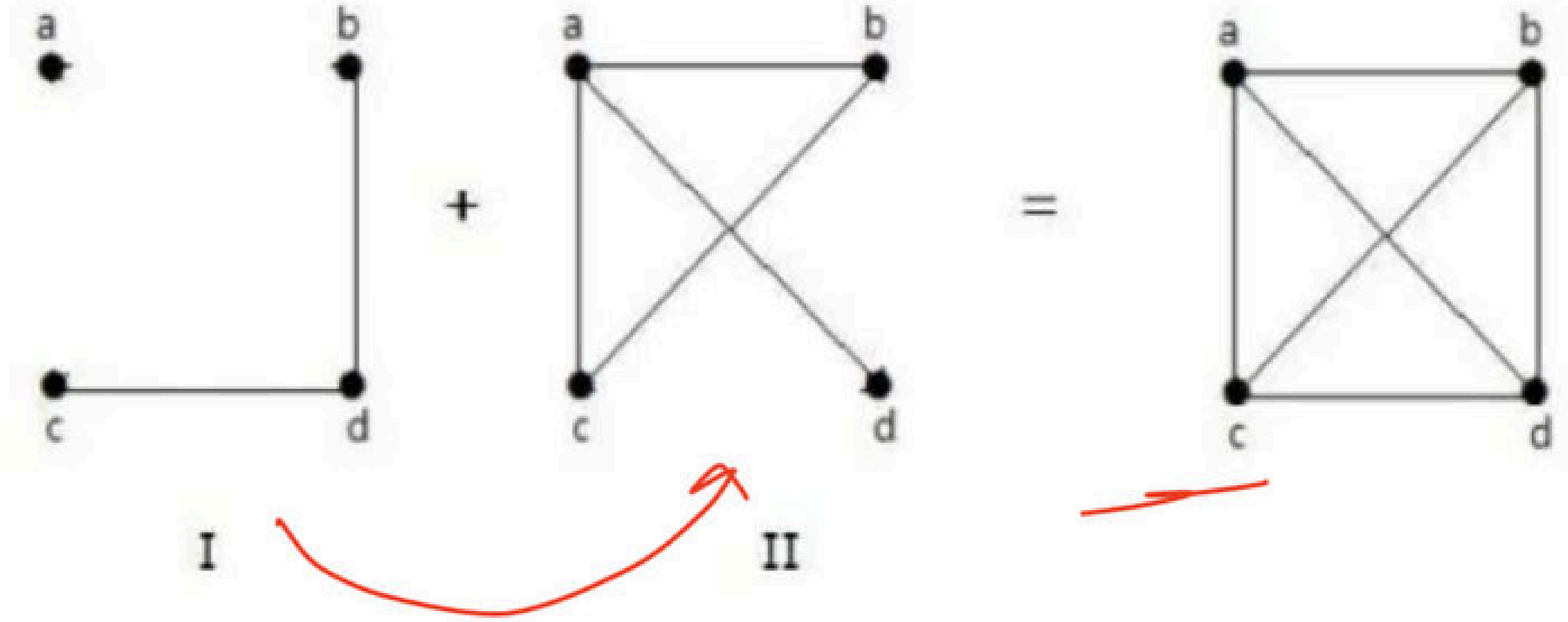
Complement of a Graph

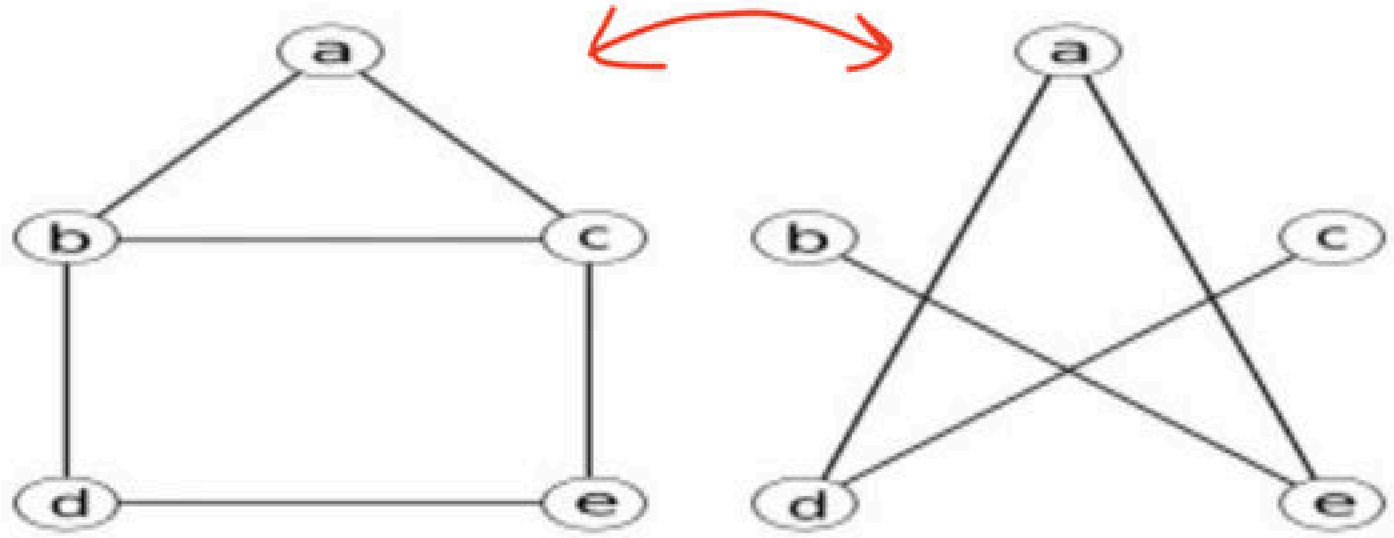
1. The complement of a simple graph G (V, E) is a graph G^c (V, E^c) on the same vertices set as of G, such that there will be an edge between two vertices u, v in G^c if ang only if there is no edge between u, v in G. i.e. two vertices of Gc are adjacent iff they are not adjacent in G.
2. $\boxed{V(G) = V(G^c)}$
3. $\boxed{E(G^c) = \{(u, v) \mid (u, v) \notin E(G)\}}$
4. $E(G^c) = E(K_n) - E(G)$

$A \longrightarrow A^c$

$\boxed{A^c = U - A}$

$G^c = K_n - G$





K_5

Properties

1. $G \cup G^c = K_n$

2. $G \cap G^c$ = null graph

3. $|E(G)| + |E(G^c)| = E(K_n) = n(n-1)/2$

Q A simple graph G has 30 edges and G^c has 36 edges, the number of vertices in G will be?

$$30 + 36 = |E(K_n)|$$

$$66 = \frac{n(n-1)}{2}$$

$$n(n-1) = 132$$

$$n^2 - n - 132 = 0$$

$$n^2 - 12n + 11n - 131 = 0$$

$$n(n-12) + 11(n-12) = 0$$

$$(n+11)(n-12) = 0$$

$$\cancel{n = -11} \quad | \quad n = 12$$

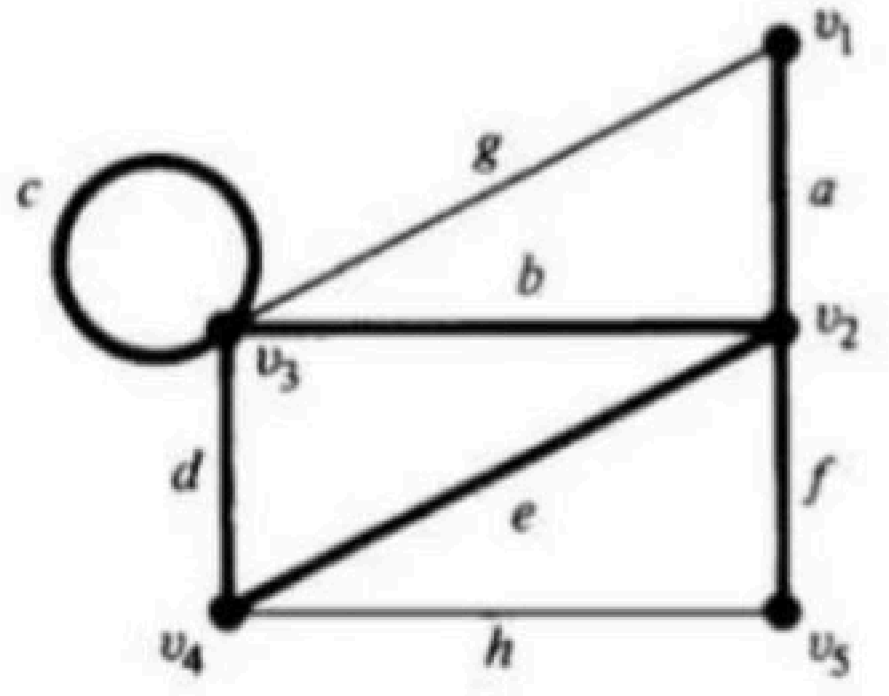
Q A simple graph G has 56 edges and G^c has 80 edges, the number of vertices in G will be?

Q a simple graph G has $|v|=8$ and $|E|=12$, find number of edges in $|E(G^c)|$?

Traversal

1. **Walk / Edge Train / Chain**: -A Walk is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it. No edge is allowed to appear more than once in a walk. A vertex, however, may appear more than once.

- Vertices with which a walk begins and ends are called its terminal vertices. It is possible for a walk to begin and end at the same vertex. Such a walk is called a closed walk. A walk that is not closed is called an open walk.
- An open walk in which no vertex appears more than once is called a path (a path does not interact itself). Number of edges in a path is called length of a path.



Traversal	Walk	Open Walk	Closed Walk	Path
V_1 g V_3 b V_2 e V_4 d V_3 b V_2				
V_1 a V_2 e V_4 d V_3 b V_2 f V_5				
V_1 g V_3 c V_3 b V_2 a V_1				
V_1 a V_2 b V_3 d V_4 h V_5				

True

$\downarrow$

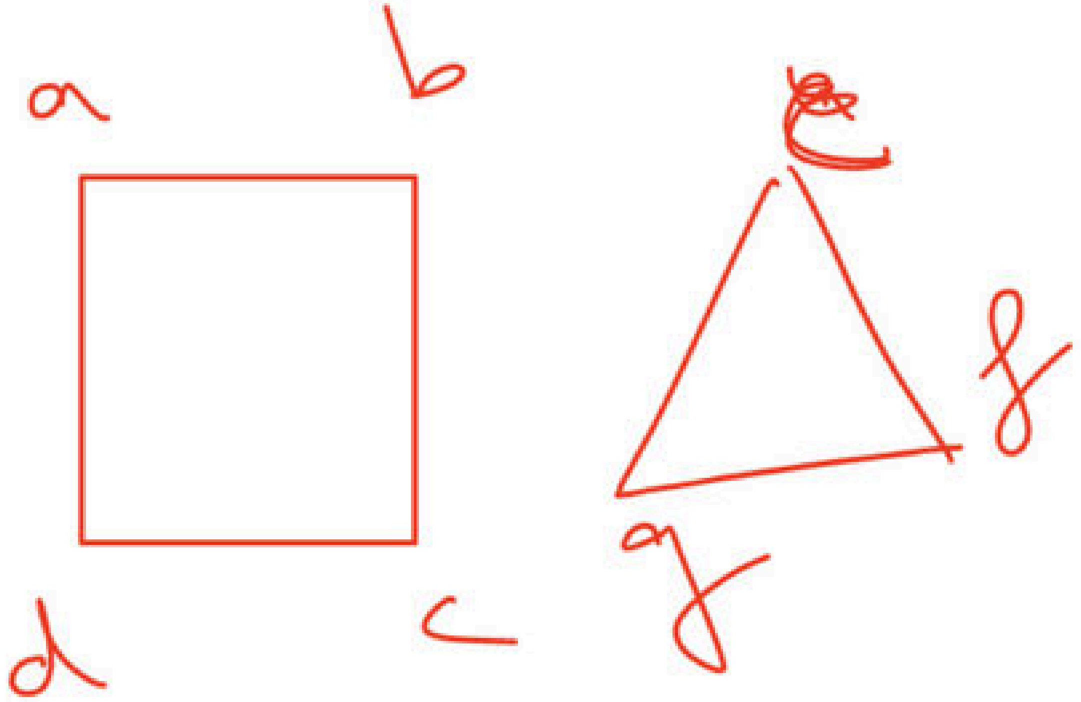
70

false

$\downarrow$

30

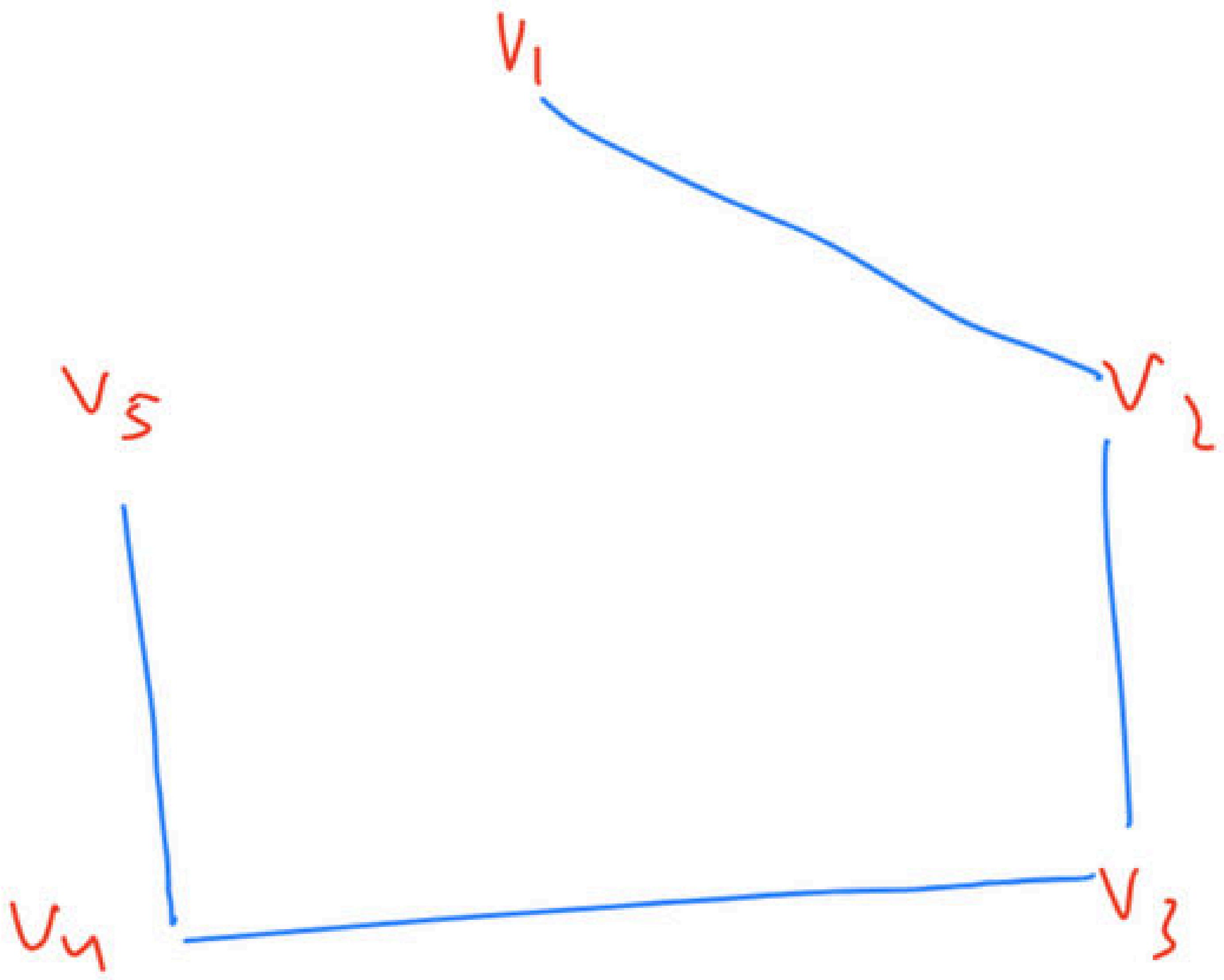
$\forall\ i, j$

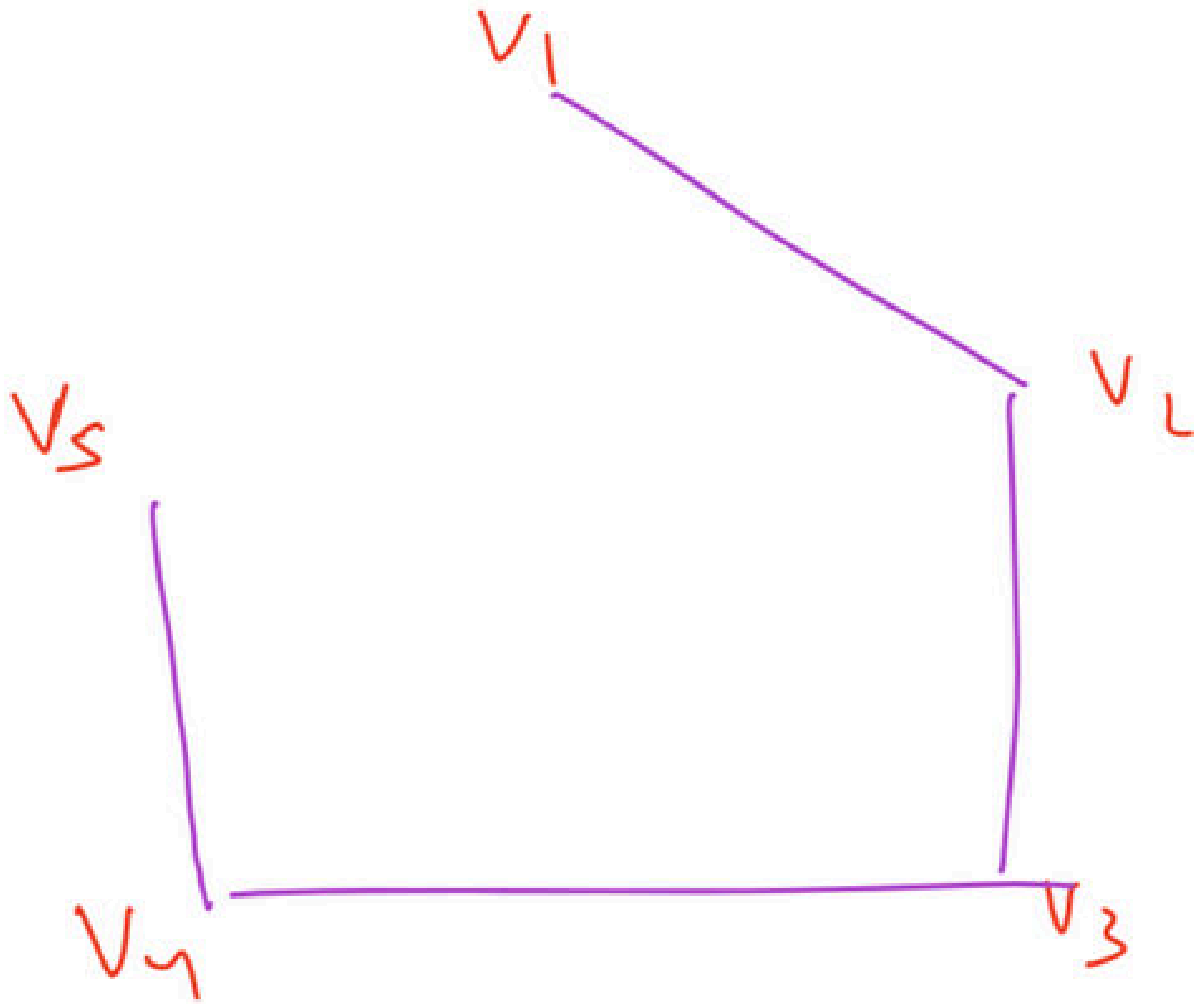


$|V| = n$

$\downarrow$

$E = (n-1)$





a) 4 / 68 b) 5 / 20

c) 6 / 7 d) 10 / 6

a) 4 – 65

b) 5 – 1

c) 6 – 20

d) 10 – 8

1. **<u>Connected Graph:</u>** A graph is said to be connected if there is at one path between every pair of vertices in G.
2. A graph with n vertices can be connected with minimum n -1 edges.
3. A graph with n vertices will necessary be connected if it has more than (n - 1) (n - 2)/2 edges.
4. if a graph (connected or disconnected) has exactly two vertices of odd degree, there must be a path joining these two vertices

Q A simple Graph with 'n' vertices and 'k' components can have at most (n - k) (n – k + 1)/2 edges?

Q Which condition is necessarily for a graph to be connected?

a) ~~A graph with 6 vertices and 10 edges~~

b) ~~A graph with 7 vertices and 14 edges~~

c) A graph with 8 vertices and 22 edges

d) ~~A graph with 9 vertices and 28 edges~~

$|V| = 6$

K_5

$\frac{7(7-1)}{2} = \frac{7\times 6}{2} = 21$

$\frac{6(6-1)}{2} = \frac{6\times 5}{2} = 15$

$\frac{9(9-1)}{2} = \frac{9\times 8}{2} = 36$

Q A simple graph G with n-vertices is connected if the graph has **(NET-SEP-2013)**

(A) $(n - 1)(n - 2)/2$ edges

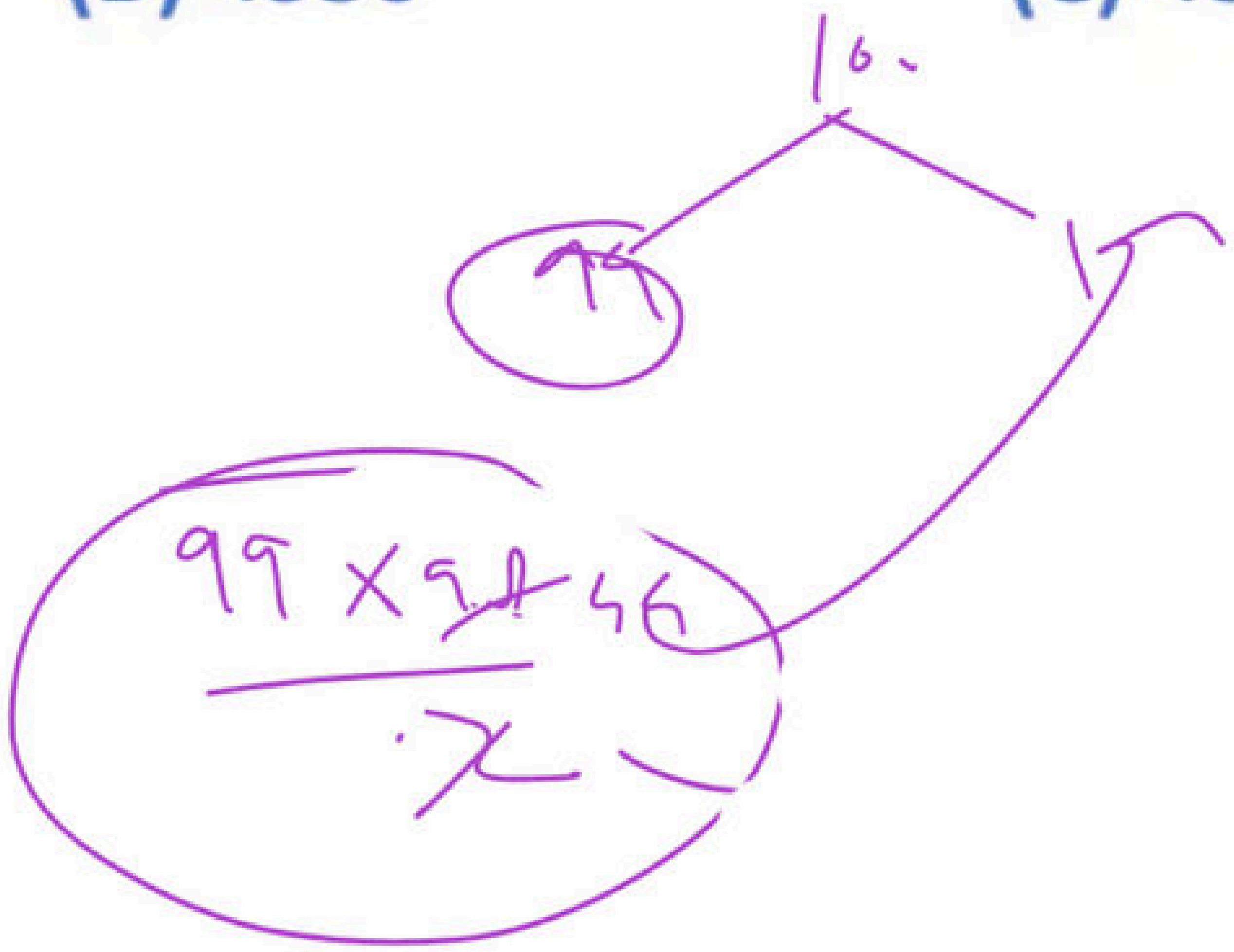
(B) More than $(n - 1)(n - 2)/2$ edges

(C) Less than $(n - 1)(n - 2)/2$ edges

(D) $\sum^{k}_{i=1} C(n_i, 2)$ edges

Q Consider an undirected graph G with ~~100 nodes~~. The maximum number of edges to be included in G so that the graph is not connected is **(NET-SEP-2013)**

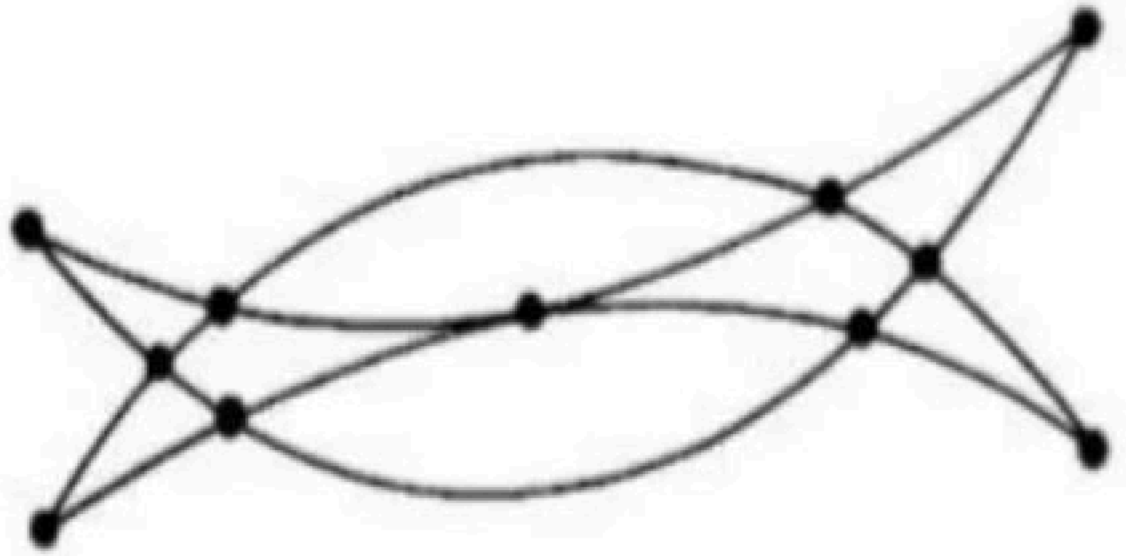
(A) 2451 **(B)** 4950 **(C)** 4851 **(D)** 9900

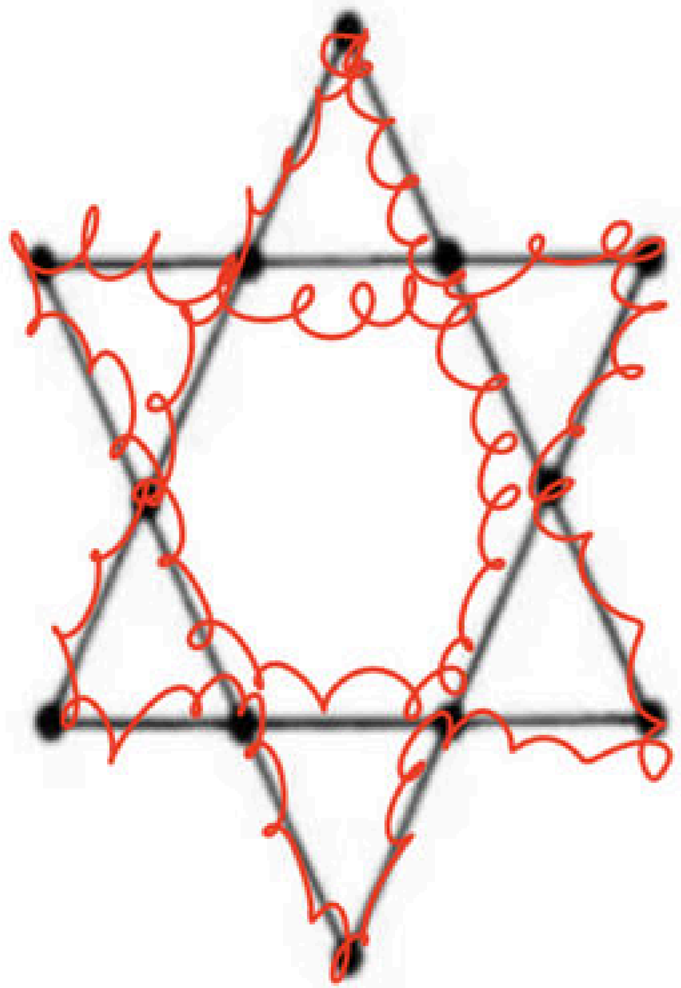


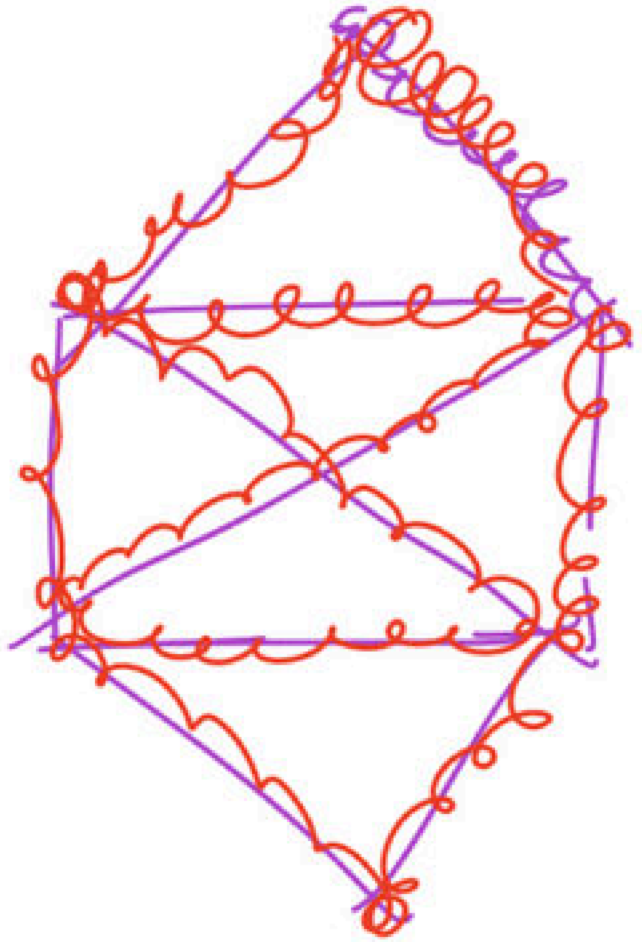
4852

Euler Graph

1. **Euler Graph:** - If some closed walk in a graph contains all the edges of the graph(connected), then the walk is called a Euler line and the graph a Euler Graph.
2. A given connected graph G is a Euler graph if and only if all vertices of G are of even degree.

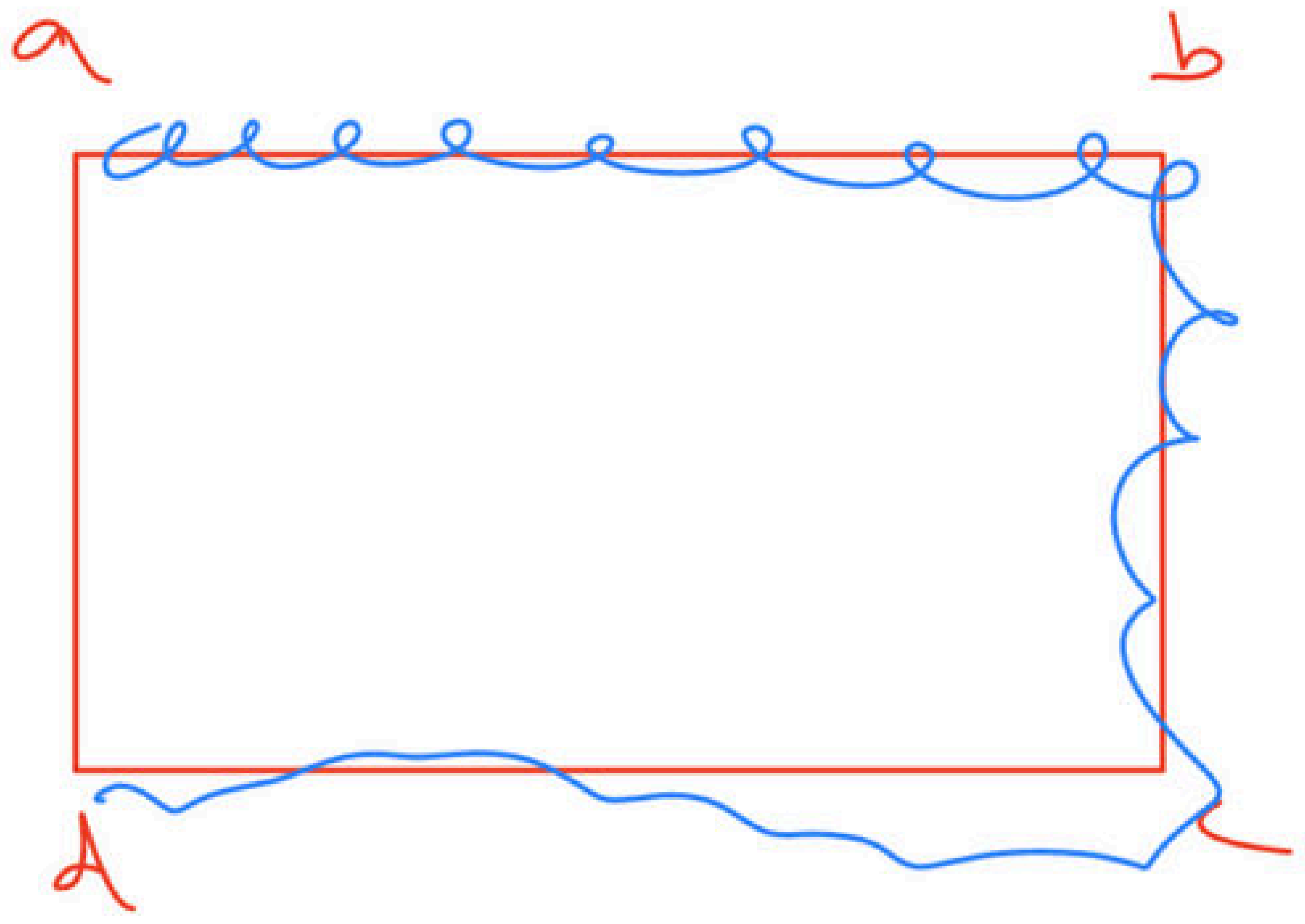






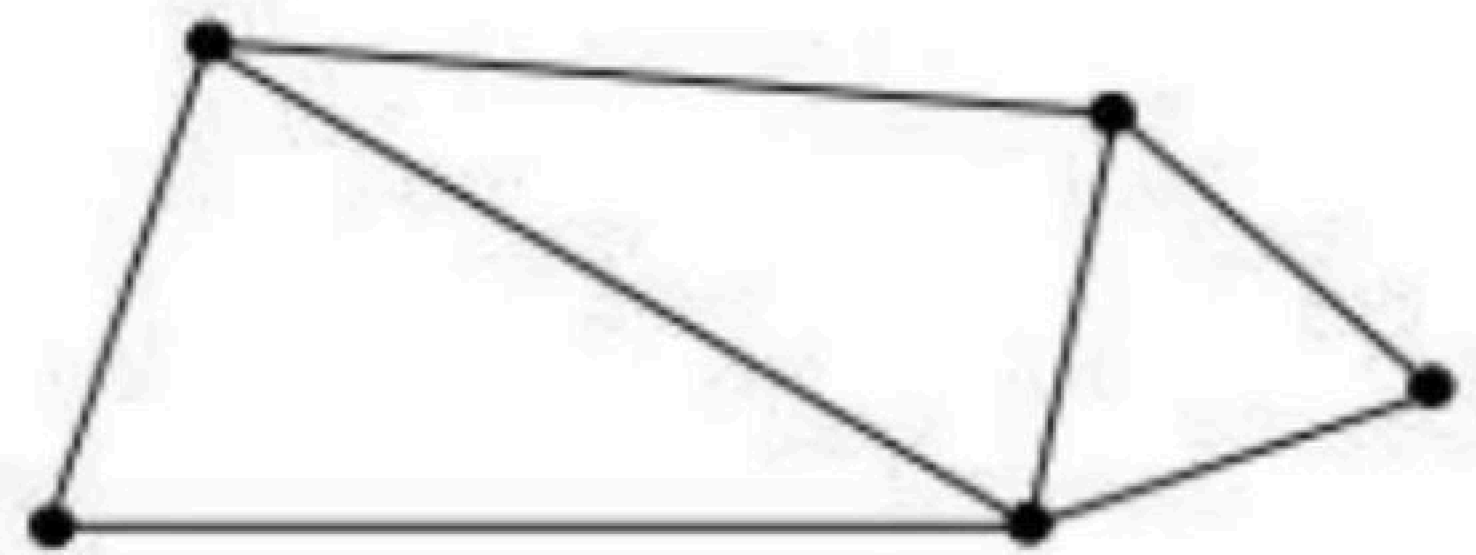
c → E

H → v



a
b
c
d

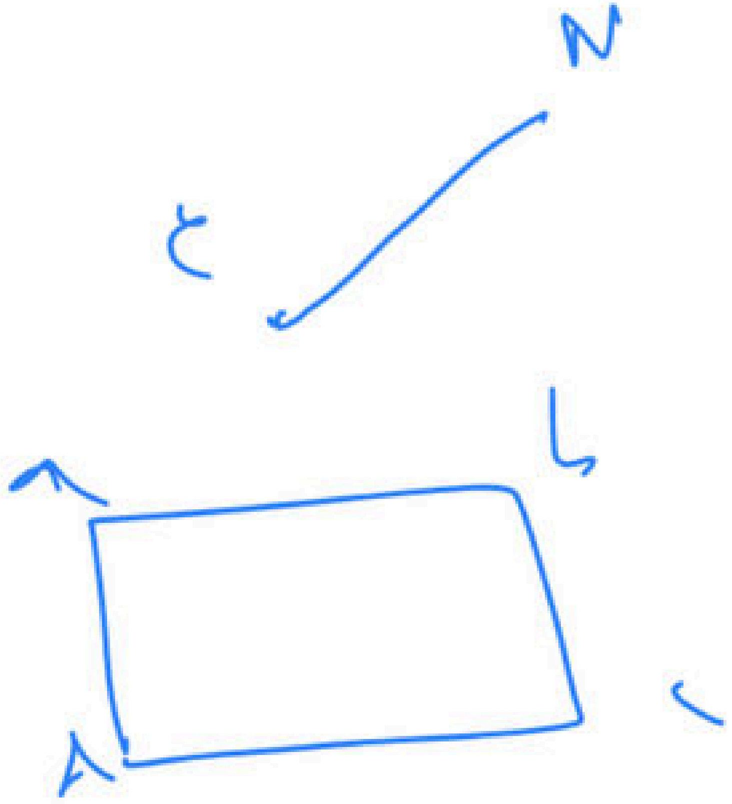
1. A connected graph G is Eulerian if and only if its edge set can be decomposed into cycles.
2. The number of edge−disjoint paths between any two vertices of an Euler graph is even.
3. An open walk that includes (or traces) all edges of a graph without retracing any edge is called a unicursal line or open Euler line. A connected graph that has a unicursal line is called a unicursal graph.
4. Clearly by adding an edge between the initial and final vertices of a unicursal line, we get an Euler line.

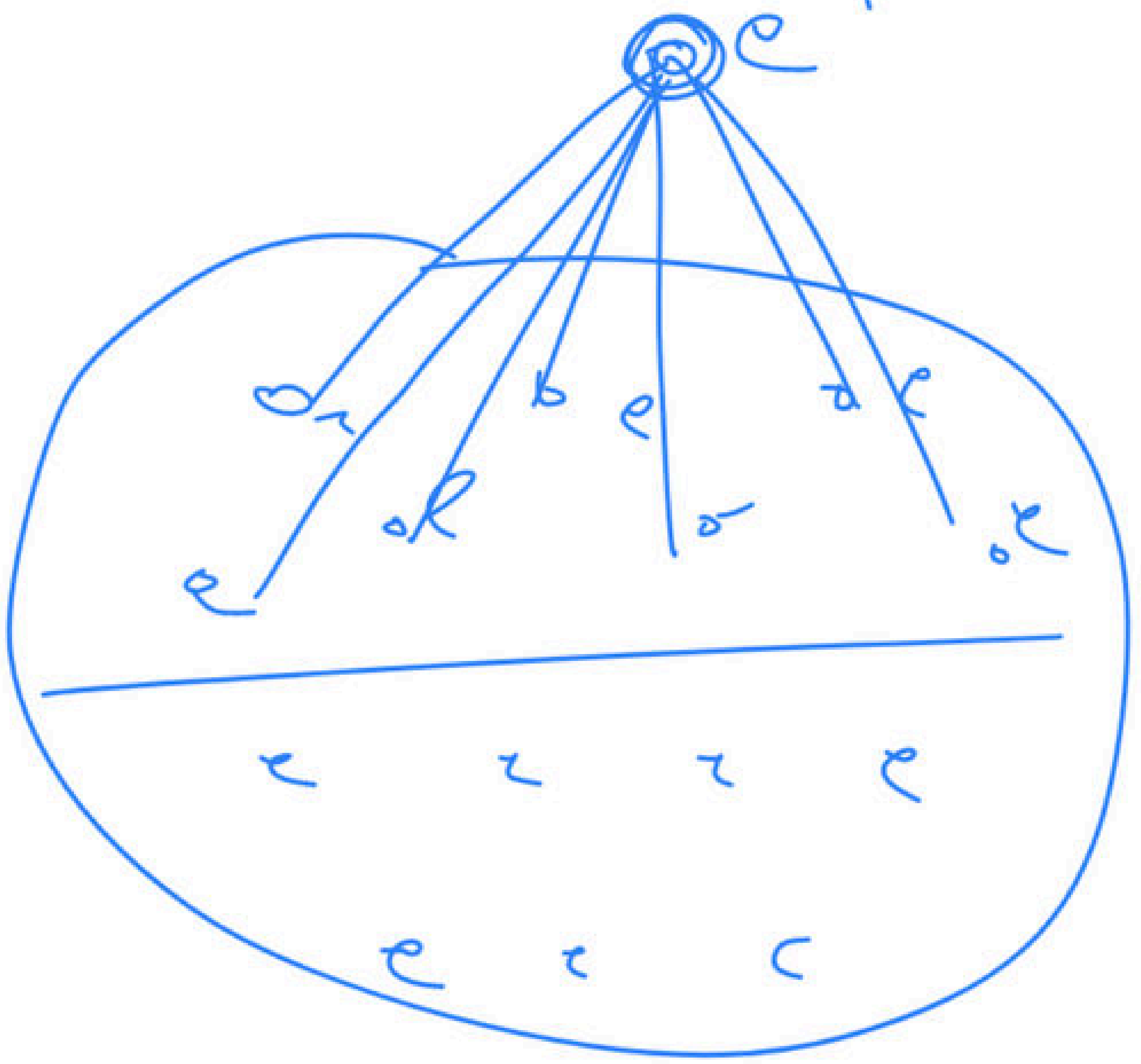


Unicursal graph

Q G is a simple undirected graph. Some vertices of G are of odd degree. Add a node v to G and make it adjacent to each odd degree vertex of G. The resultant graph is sure to be **(GATE-2008) (2 Marks)**

(A) regular **(B)** Complete **(C)** Hamiltonian **(D)** ~~Euler~~





Q An undirected graph possesses an eulerian circuit if and only if it is connected and its vertices are **(NET-DEC-2010)**

(A) all of even degree

(B) all of odd degree

(C) of any degree

(D) even in number

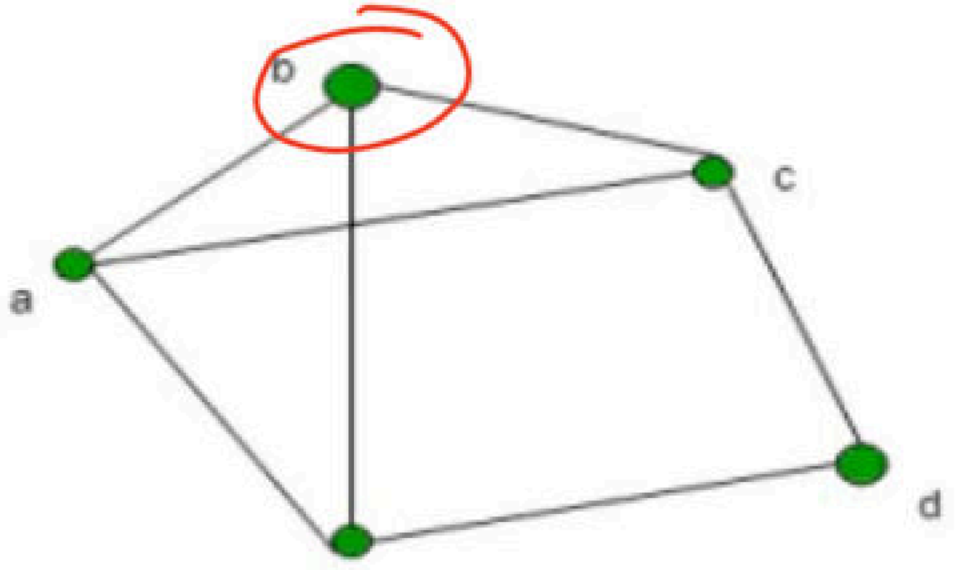
a) (a) only

b) (b) and (c)

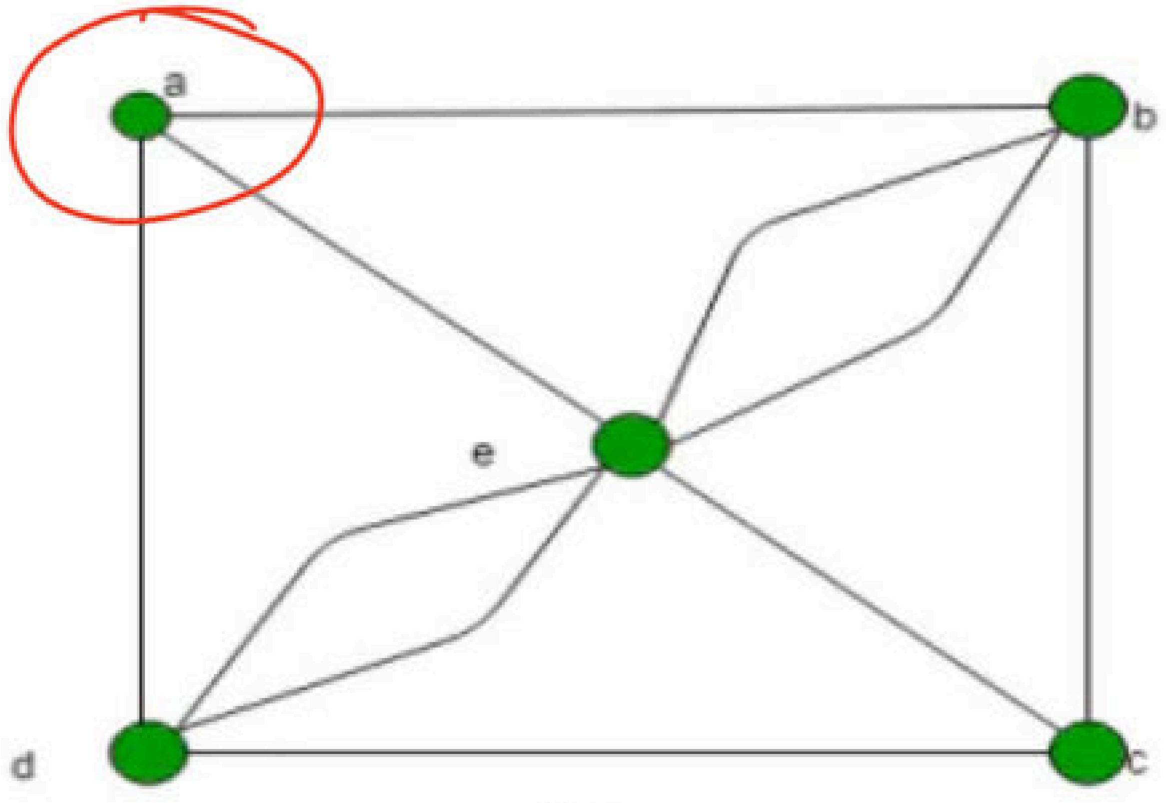
c) (c) only

d) (d) only

Q Given the following graphs: **(NET-AUG-2016)**



G1



G2

Which of the following is correct?

a) G1 contains Euler circuit and G2 does not contain Euler circuit. — 2

b) G1 does not contain Euler circuit and G2 contains Euler circuit. — 6

c) Both G1 and G2 do not contain Euler circuit. — 82

d) Both G1 and G2 contain Euler circuit. — 7

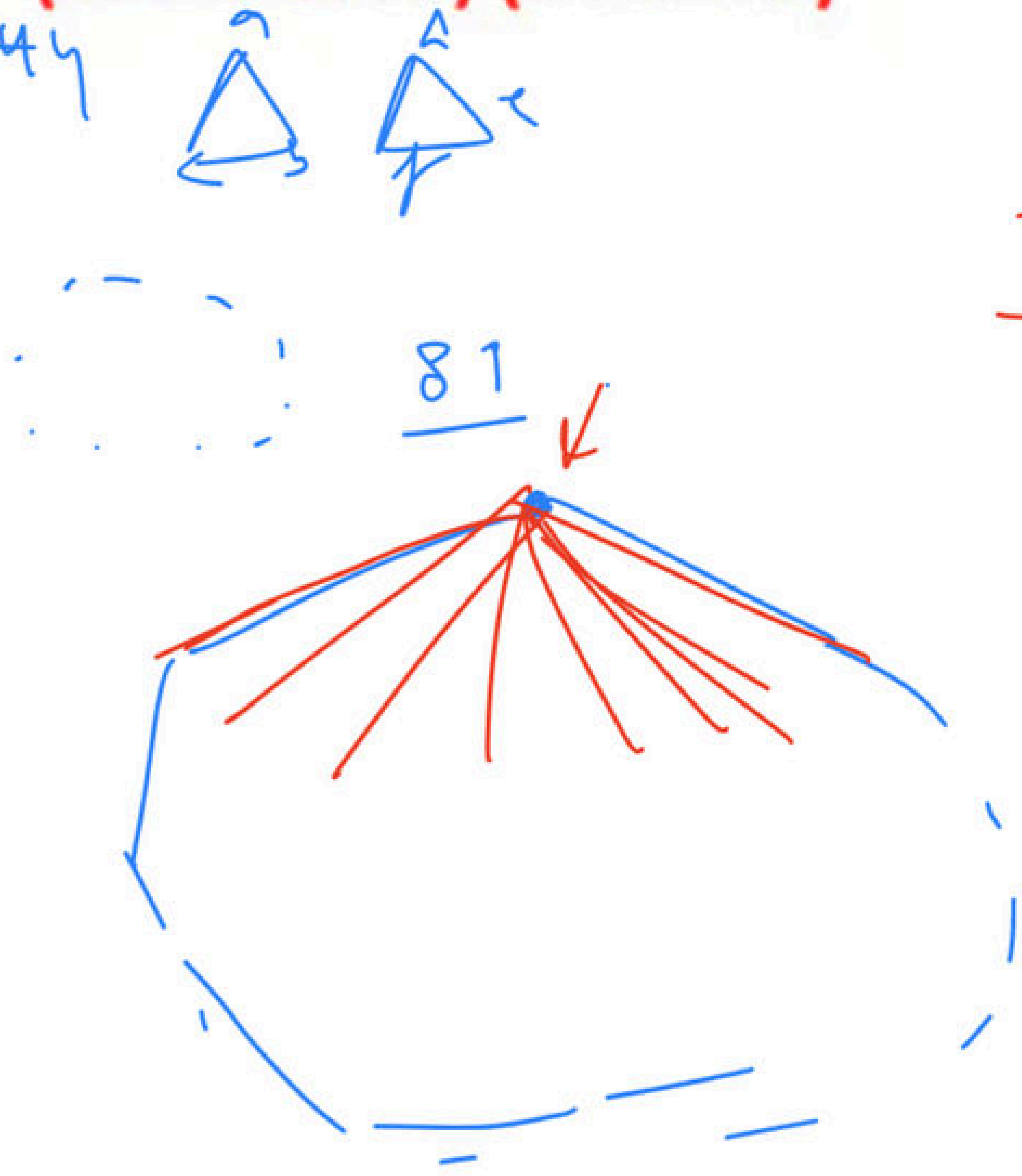
Q Which of the following graphs has a Eulerian circuit? **(GATE-2007) (2 Marks)**

(A) ~~Any k-regular graph where k is an even number.~~ — 44

(B) ~~A complete graph on 90 vertices~~ — 11

(C) The complement of a cycle on 25 vertices — 29

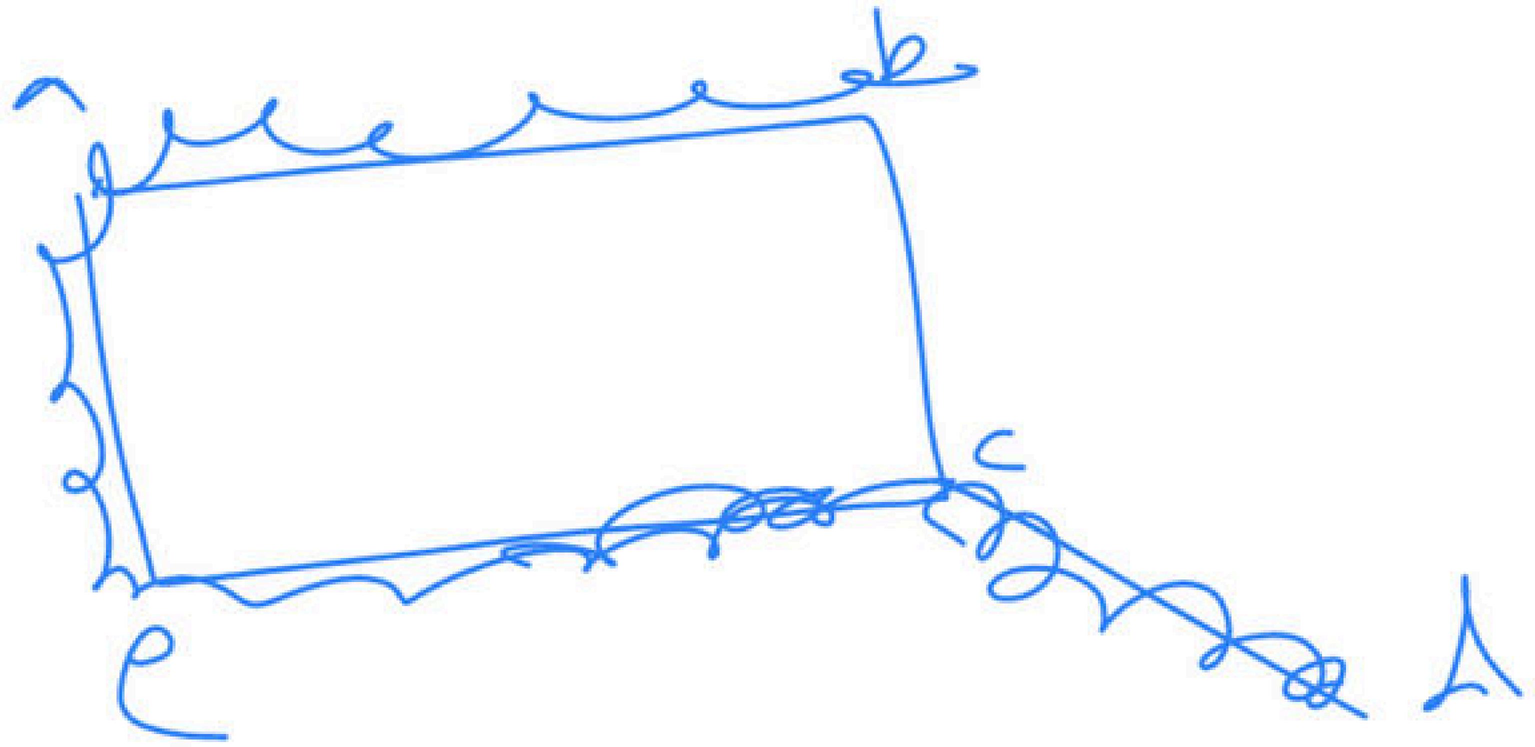
(D) None of the above — 15



25
−1
24
−2
22

Hamiltonian

1. **Hamiltonian Graph:** - A Hamiltonian circuit in a connected graph is defined as a closed walk that traverses every vertex of G exactly once, except of course the starting vertex, at which the walk also terminates. A graph containing Hamiltonian circuit is called Hamiltonian graph.
2. Finding weather a graph is Hamiltonian or not is a NCP problem.



1. If we remove any one edge from a Hamiltonian circuit, we are left with a path. This path is called a Hamiltonian path.
2. If a graph has Hamiltonian circuit then it also has Hamiltonian path, but vice versa is not true.
3. In a complete graph with n vertices there are (n - 1)/2 edge-disjoint Hamiltonian circuits, if n is odd number >= 3

- A sufficient (but by no means necessary) condition for a simple graph G to have a Hamiltonian circuit is that the degree of every vertex in G be at least n/2, where n is the number of vertices in G. (if this condition satisfy graph will be Hamiltonian but to be a Hamiltonian graph this condition is not required to be true)

Q Let G be an undirected complete graph on n vertices, where $n > 2$. Then, the number of different Hamiltonian cycles in G is equal to **(GATE-2019) (2 Marks)**

(A) $n!$ **(B)** $n - 1!$ **(C)** 1 **(D)** $(n-1)! / 2$

Q Let G be an undirected complete graph on n vertices, where n > 2. Then, the number of different Hamiltonian cycles in G is equal to **(GATE-2019) (2 Marks)**

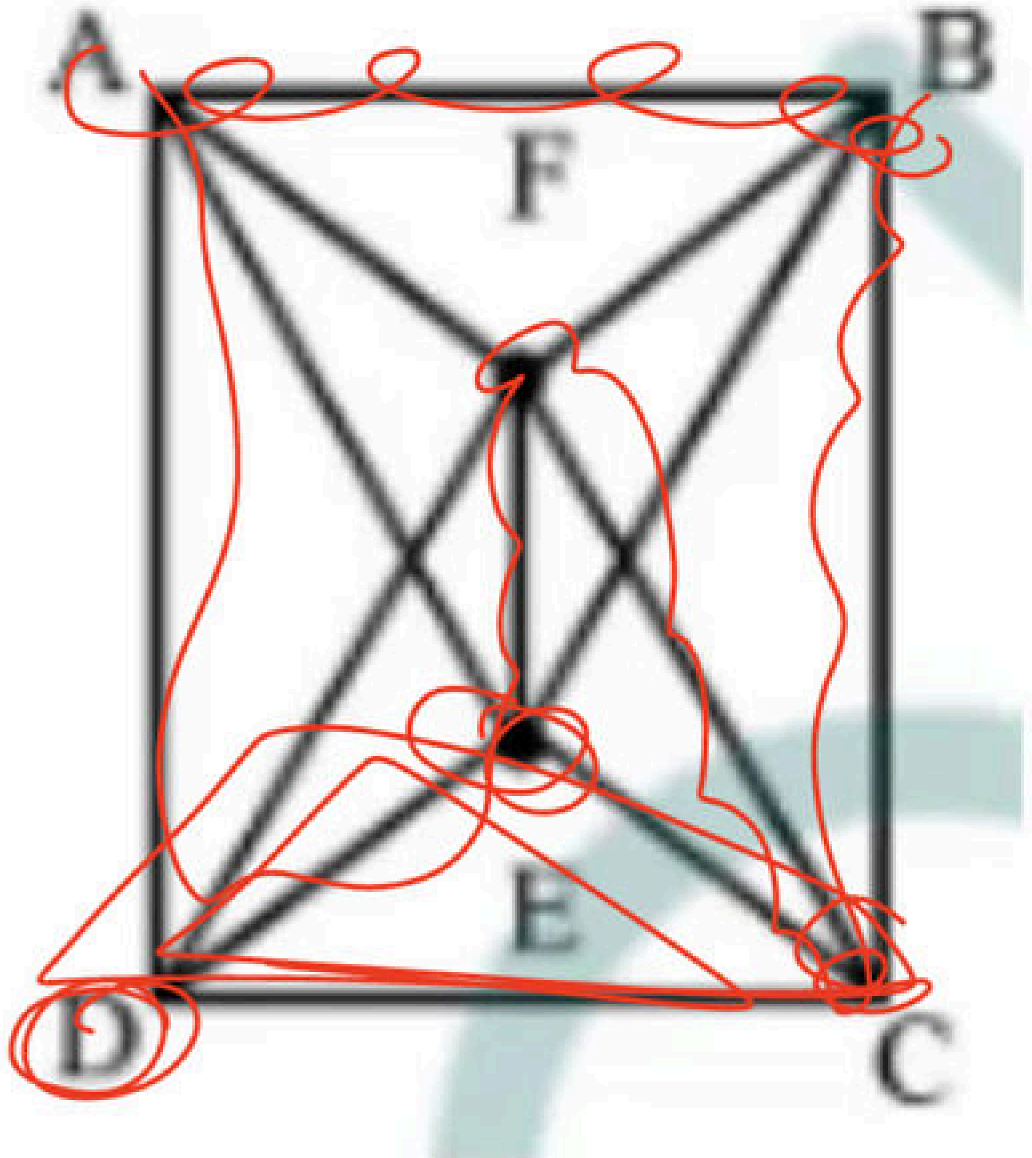
(A) n! **(B)** n – 1! **(C)** 1 **(D)** (n-1)! / 2

A simple circuit in a graph G that passes through every vertex exactly once is called a Hamiltonian circuit.
In an undirected complete graph on n vertices, there are n permutations are possible to visit every node. But from these permutations, there are: n different places (i.e., nodes) you can start; 2 (clockwise or anticlockwise) different directions you can travel.
So any one of these n! cycles is in a set of 2n cycles which all contain the same set of edges. So there are,
= (n)! / (2n)
= (n−1)! / 2 distinct Hamilton cycles.

Q Consider a complete bipartite graph $k_{m,n.}$ For which values of m and n does this, complete graph have a Hamilton circuit **(NET-JUNE-2014)**

(A) $m = 3, n = 2$ **(B)** $m = 2, n = 3$ **(C)** $m = n > 2$ **(D)** $m = n > 3$

Q Consider the graph shown below:
This graph is a ________ **(NET-DEC-2014)**



~~a) Complete graph~~ - 5
~~**b)** Bipartite graph~~ - 4
c) Hamiltonian graph - 12
~~**d)** All of the above~~ - 22

Q Which of the following statement(s) is/are false? **(NET-DEC-2015)**

(a) A connected multigraph has an Euler Circuit if and only if each of its vertices has even degree.

(b) A connected multigraph has an Euler Path but not an Euler Circuit if and only if it has exactly two vertices of odd degree.

(c) A complete graph (K_n) has a Hamilton Circuit whenever $n \geq 3$.

(d) A cycle over six vertices (C6) is not a bipartite graph but a complete graph over 3 vertices is bipartite.

Q Consider a Hamiltonian Graph (G) with no loops and parallel edges. Which of the following is true with respect to this Graph (G)? **(NET-JUNE-2015)** (NET-JAN-2017)

(a) deg (v) ≥ n / 2 for each vertex of G

(b) |E(G)| ≥ 1 / 2 (n - 1) (n - 2) + 2 edges

(c) deg (v) + deg (w) ≥ n for every v and w not connected by an edge.

a) (a) and (b) **b)** (b) and (c) **c)** (a) and (c) **d)** (a), (b) and (c)

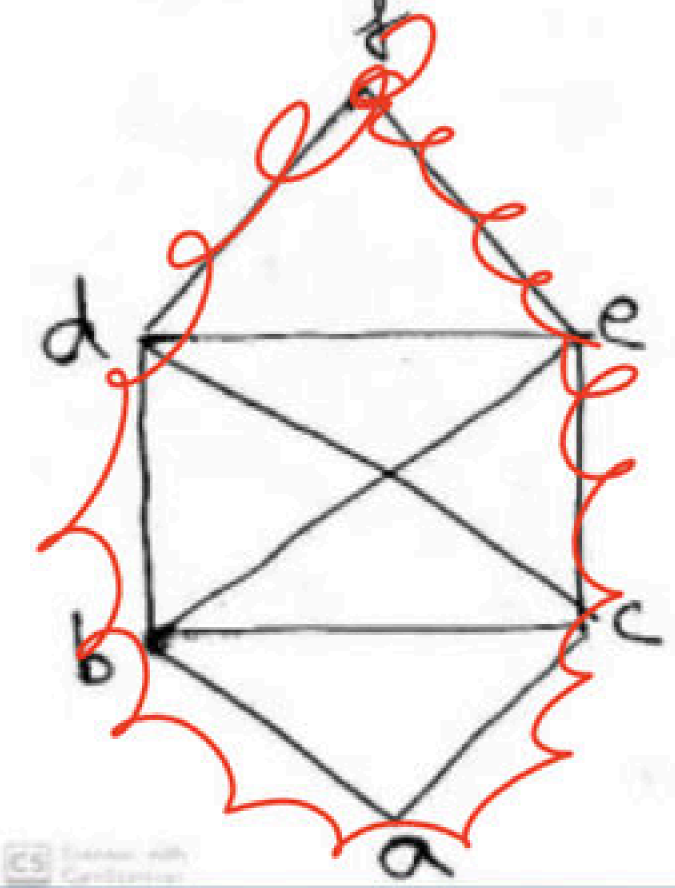
Q if a graph(g) has no loops or parallel edges, and if the number of vertices(n) in the graph is n>=3 , then graph G is Hamiltonian if **(NET-DEC-2018)**

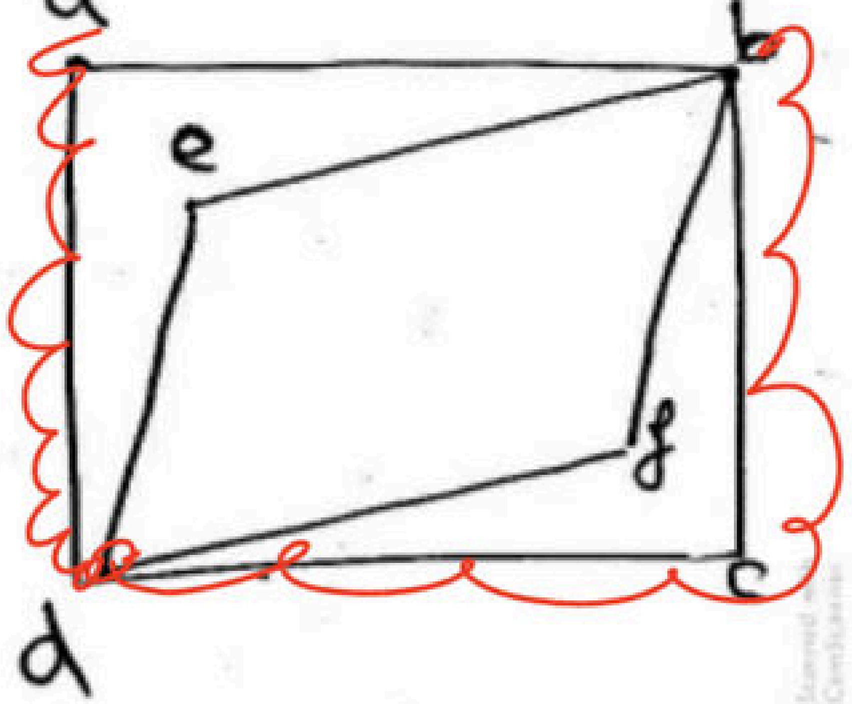
(i) $deg(v) \geq n/3$ for each vertex v

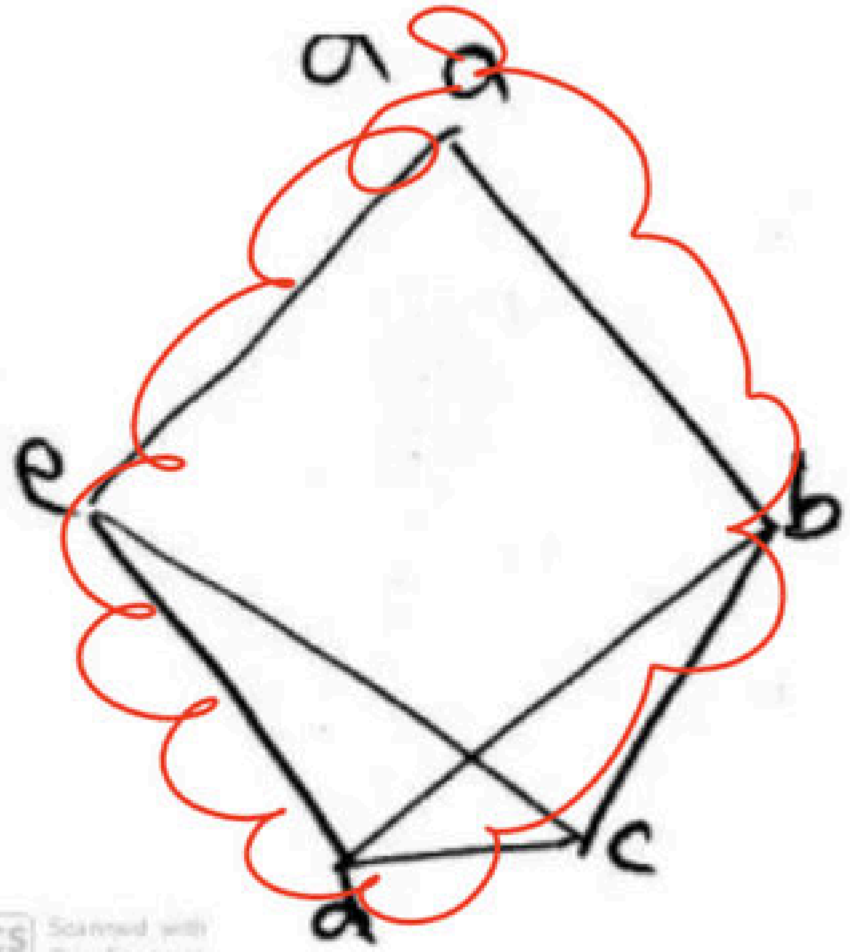
(ii) $deg(v) + deg(w) \geq n$ whenever v and w are not connected by an edge.

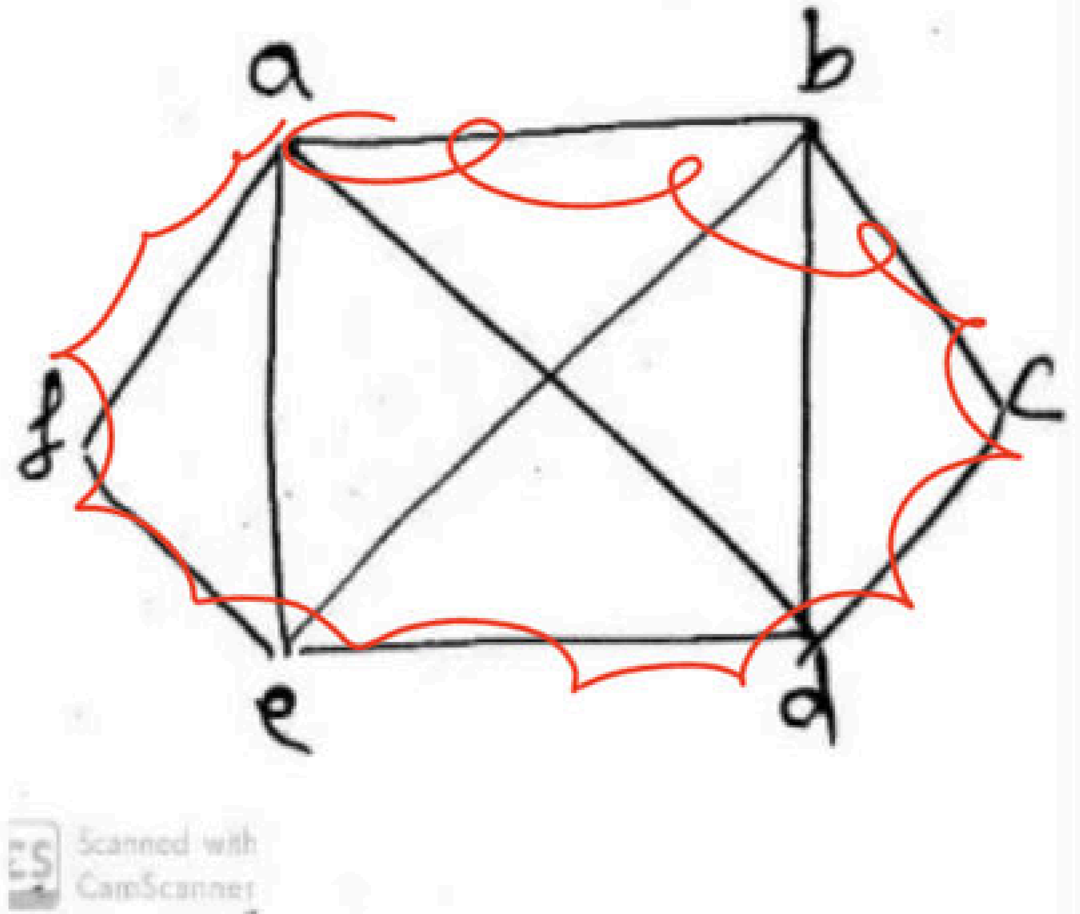
(iii) $|E(G)| \geq 1/3(n-1)(n-2) + 2$

a) (i) and (iii) only **b)** (ii) only **c)** (ii) and (iii) only **d)** (ii) only

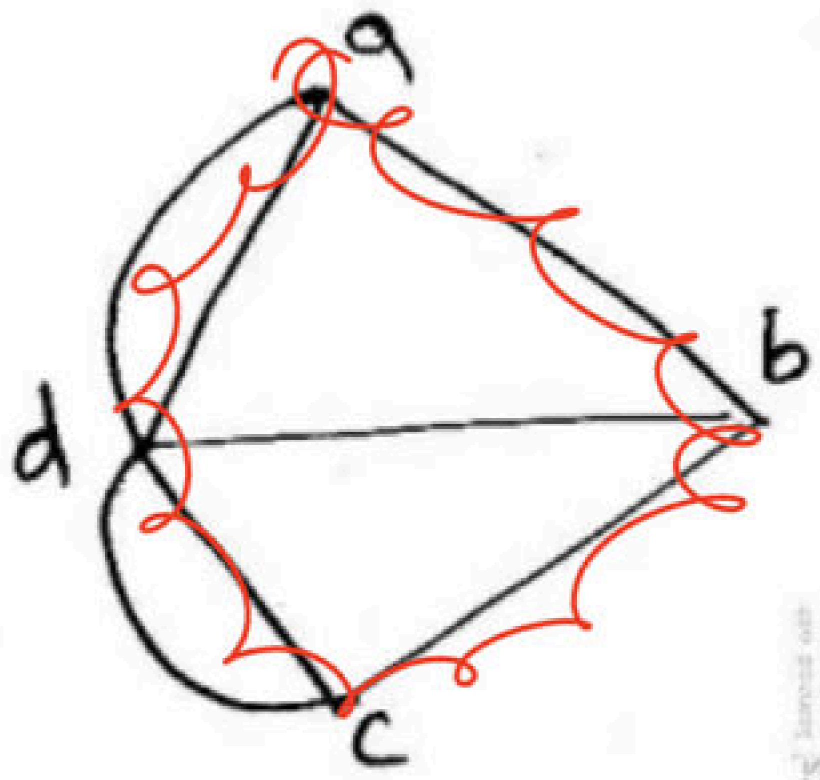


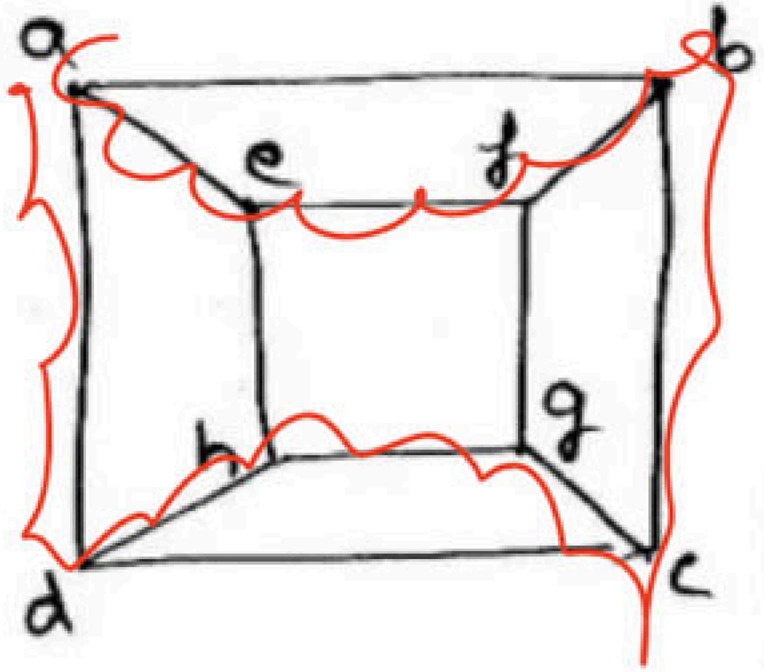


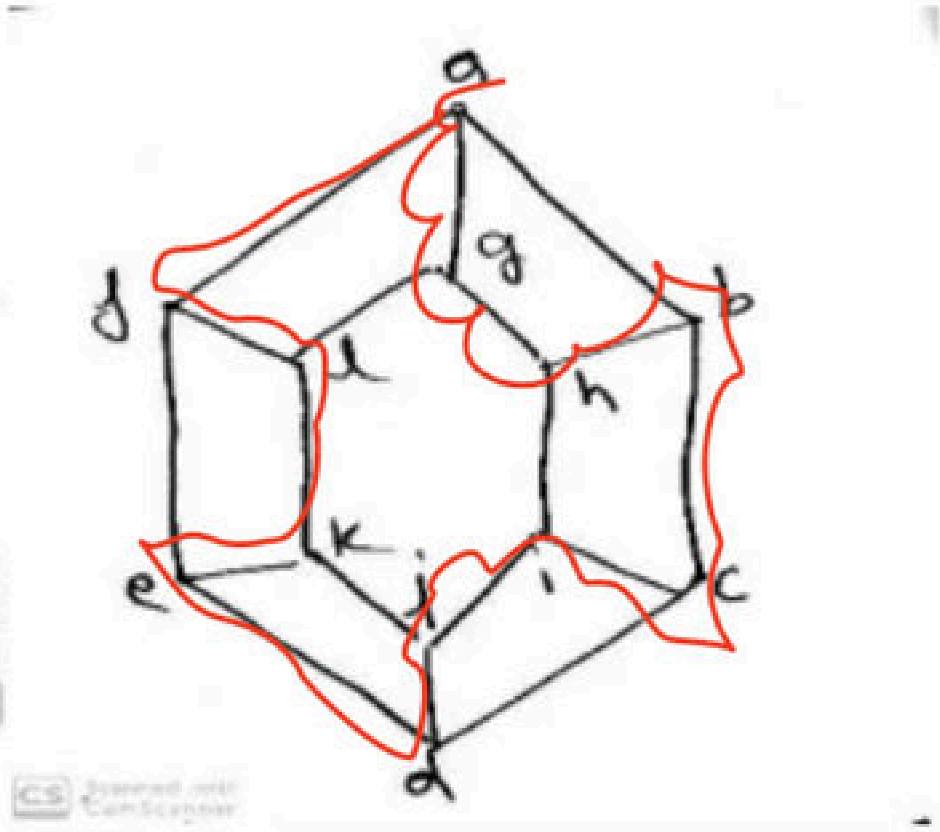


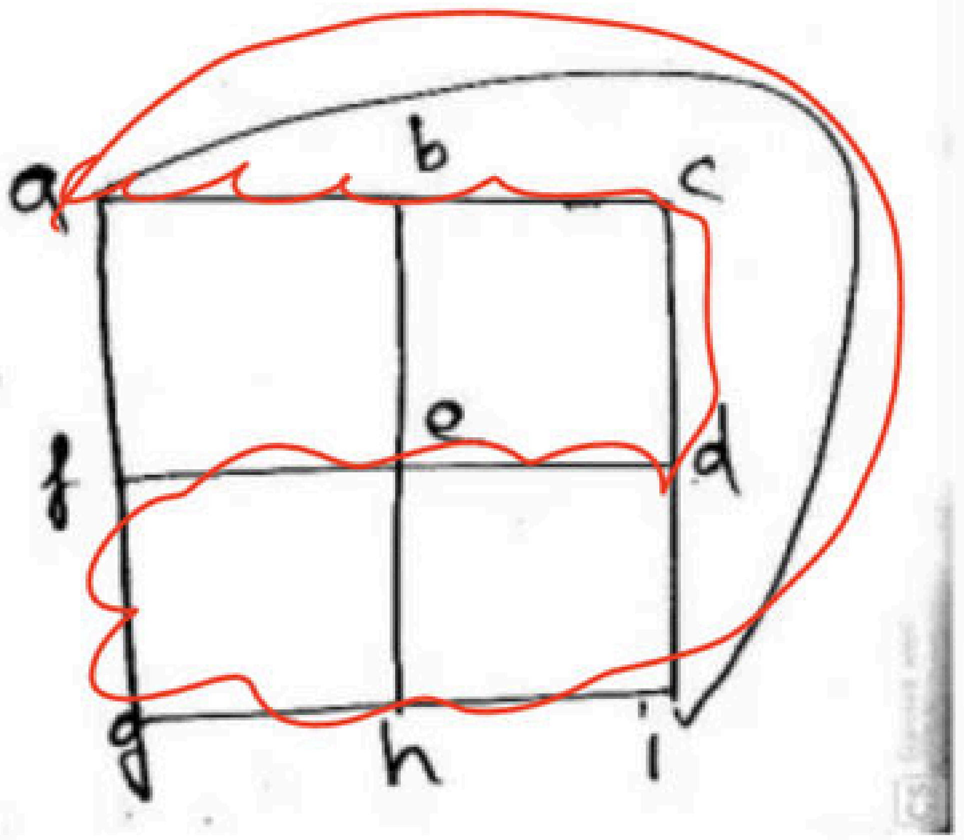


Euler Graph	✓	✓	✗	✓
Hamiltonian Graph	✓	✗	✓	✓
	1	2	3	4

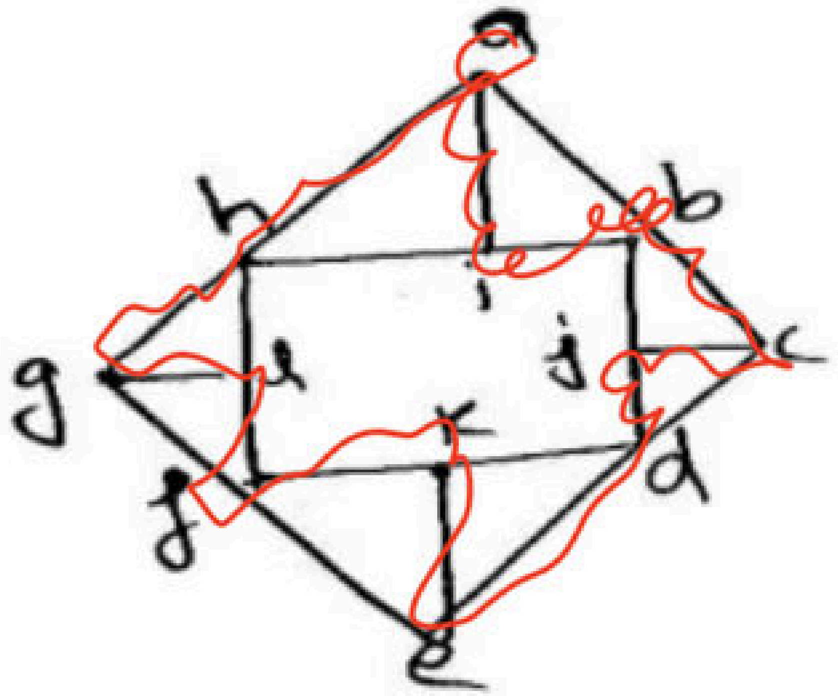
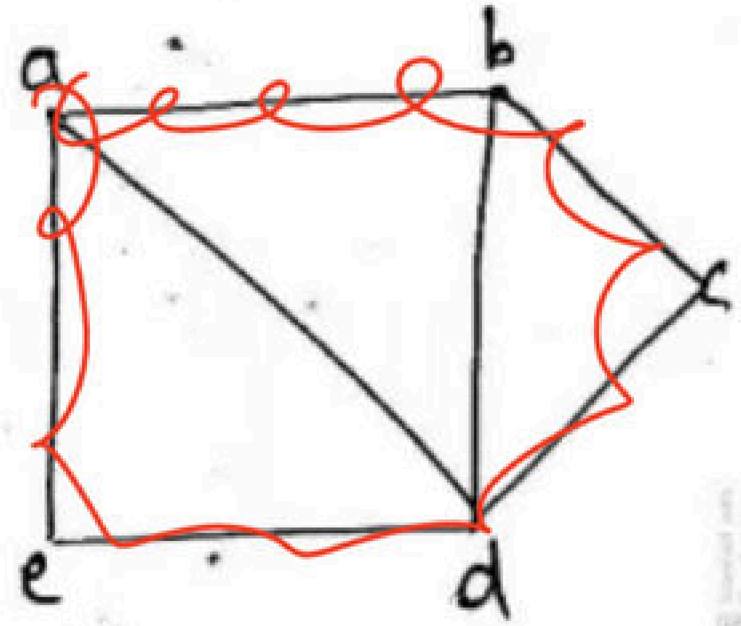
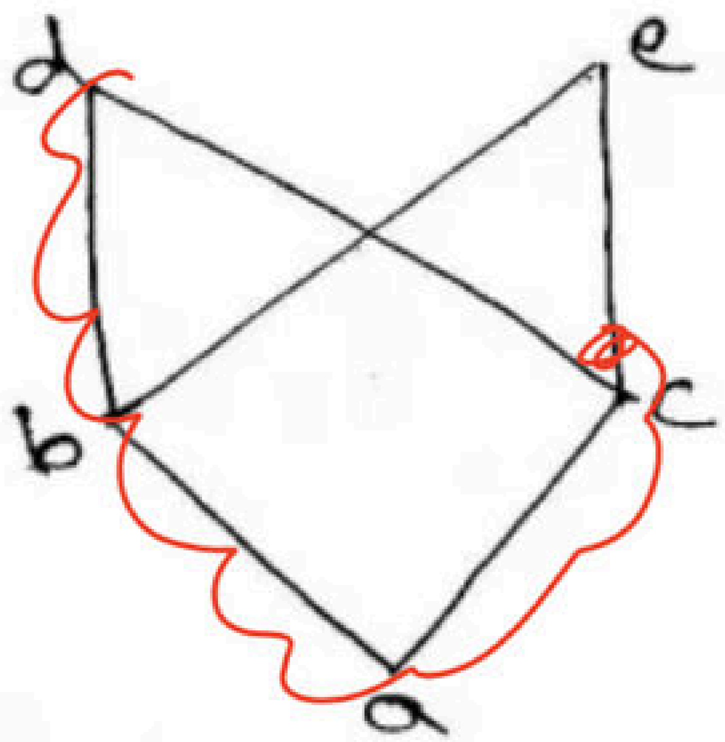
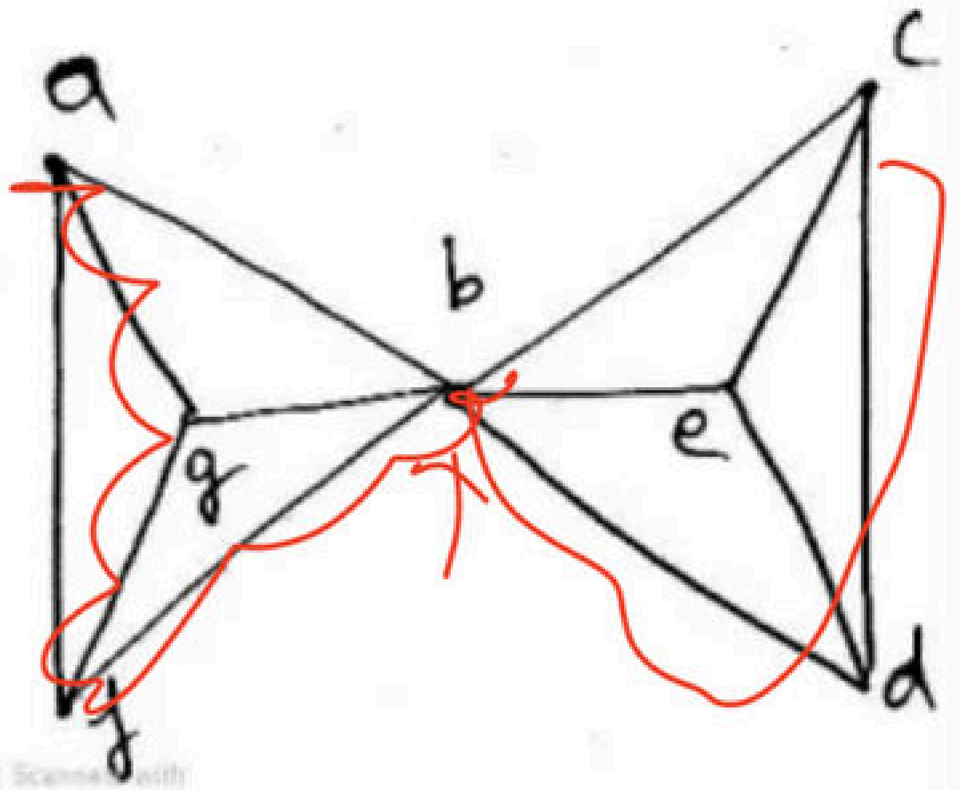




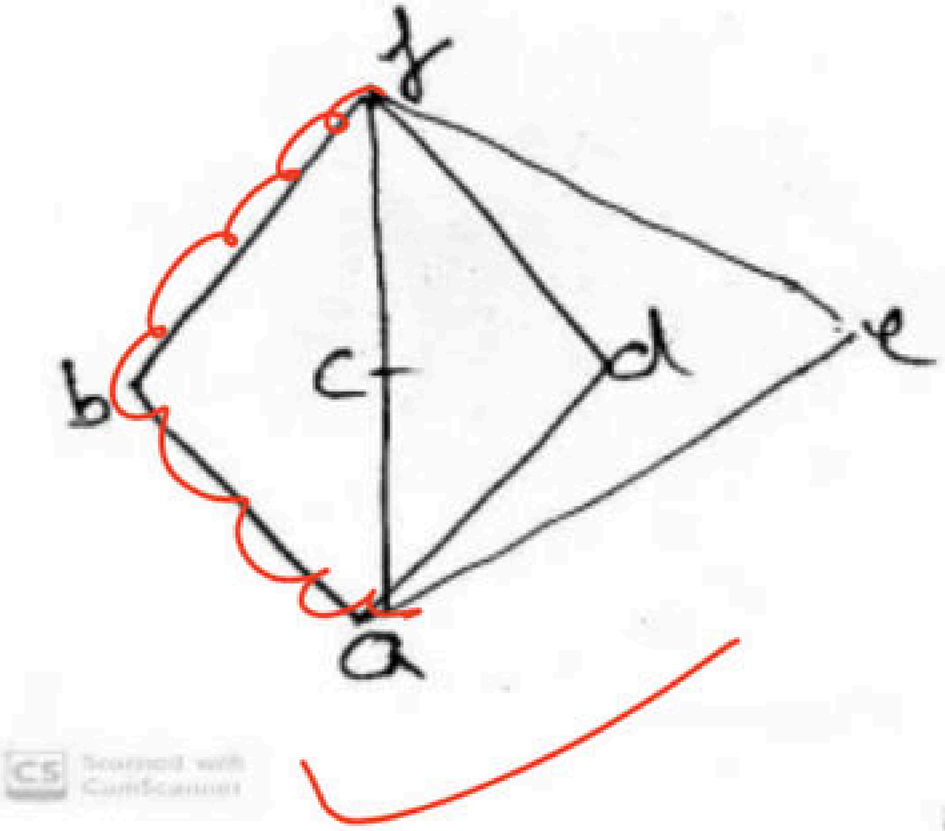


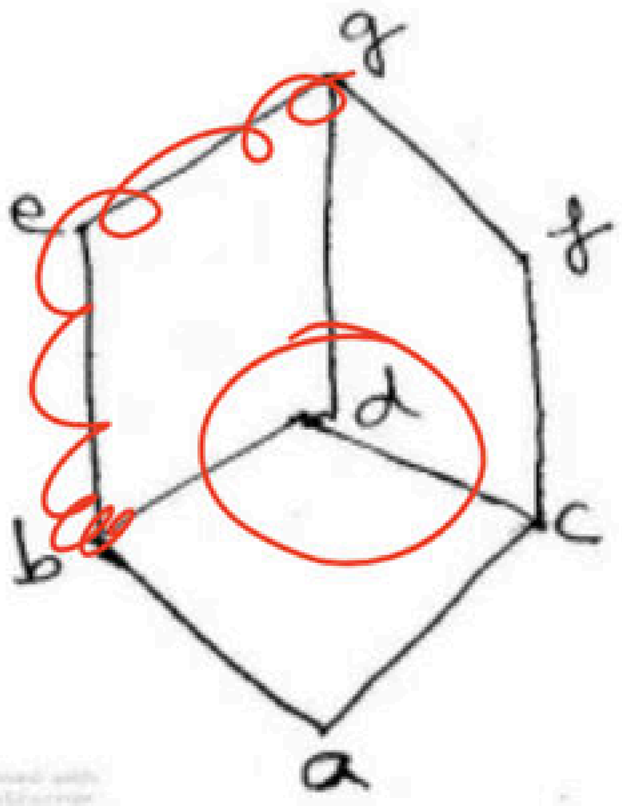


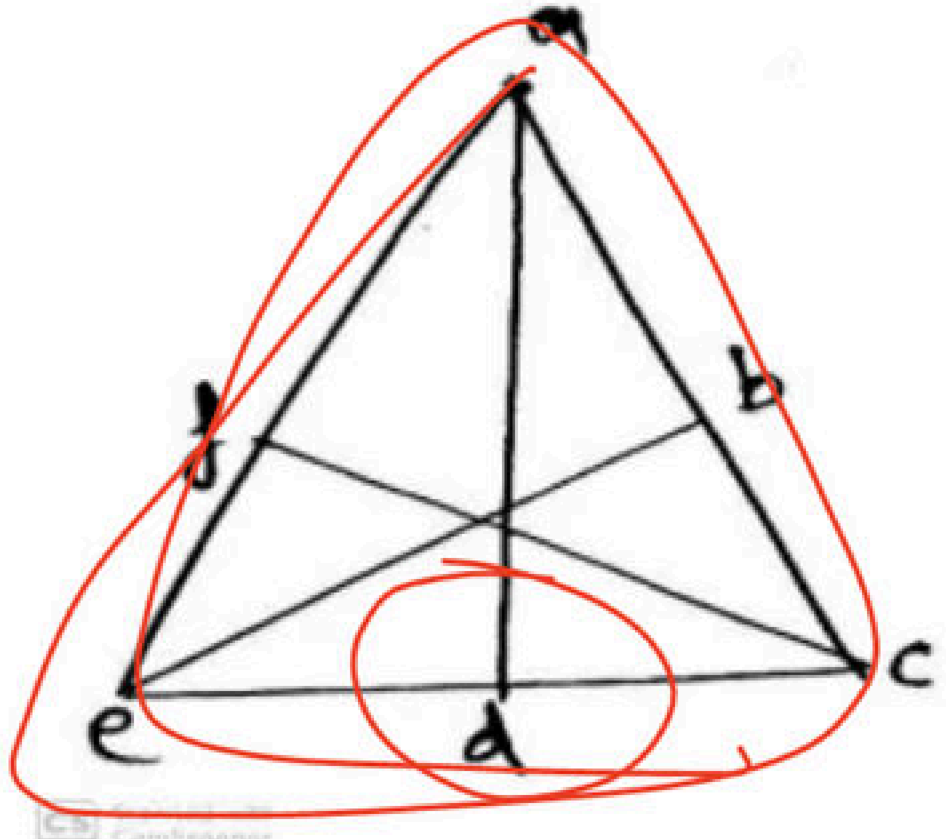
Euler Graph	✗	✗	✗	✓
Hamiltonian Graph				
	1	2	3	4

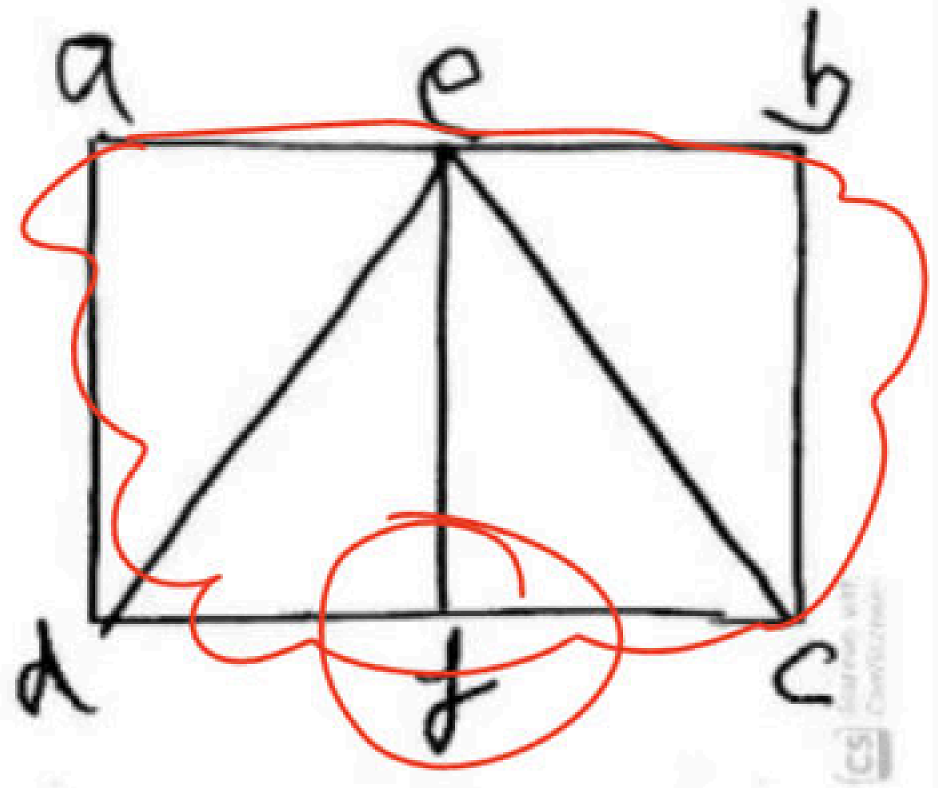





Euler Graph				
Hamiltonian Graph			X	X

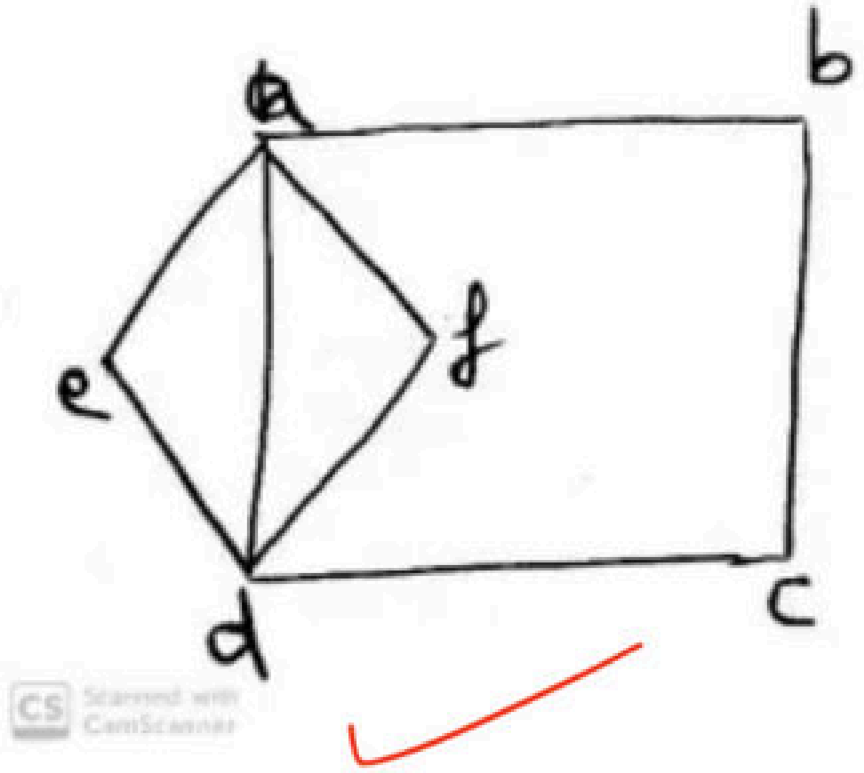


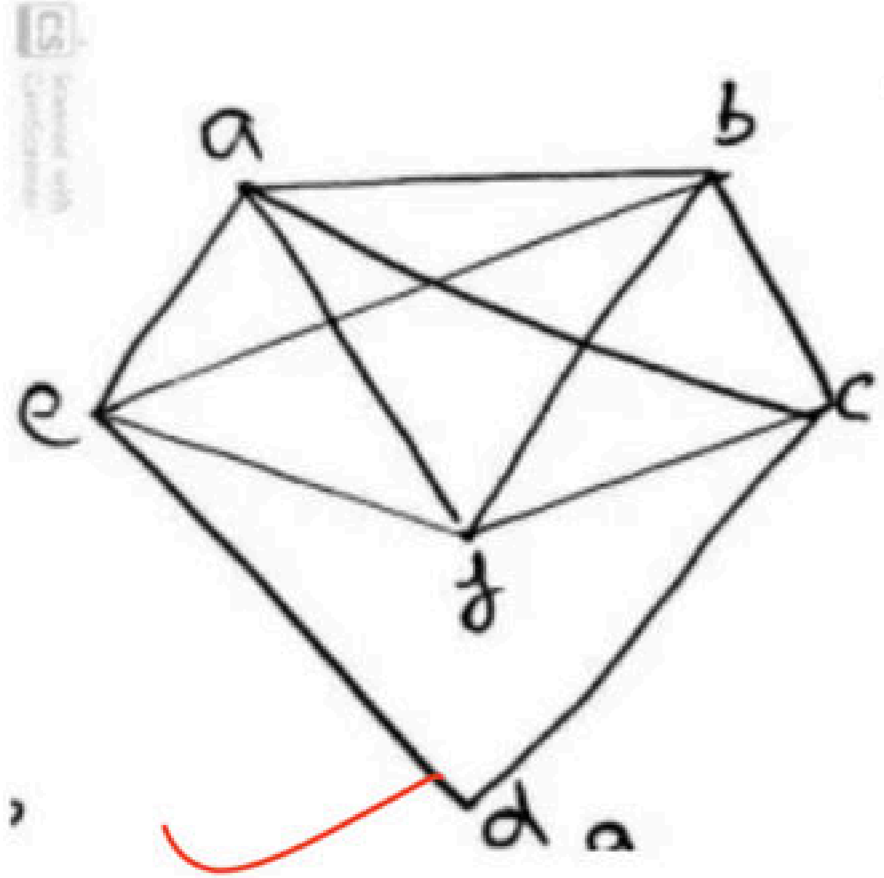


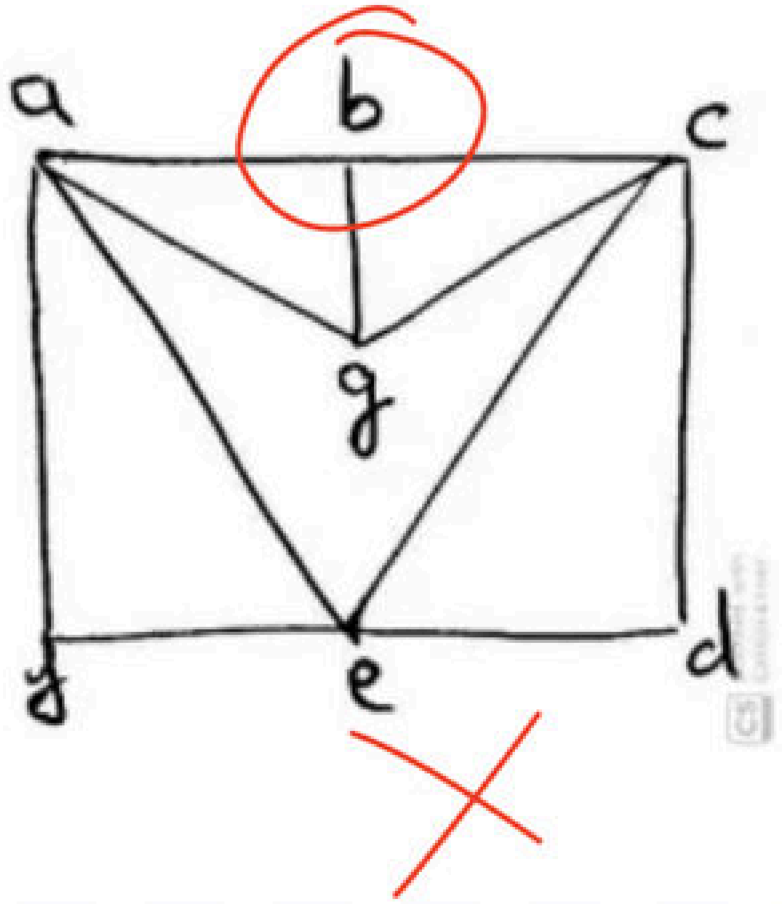


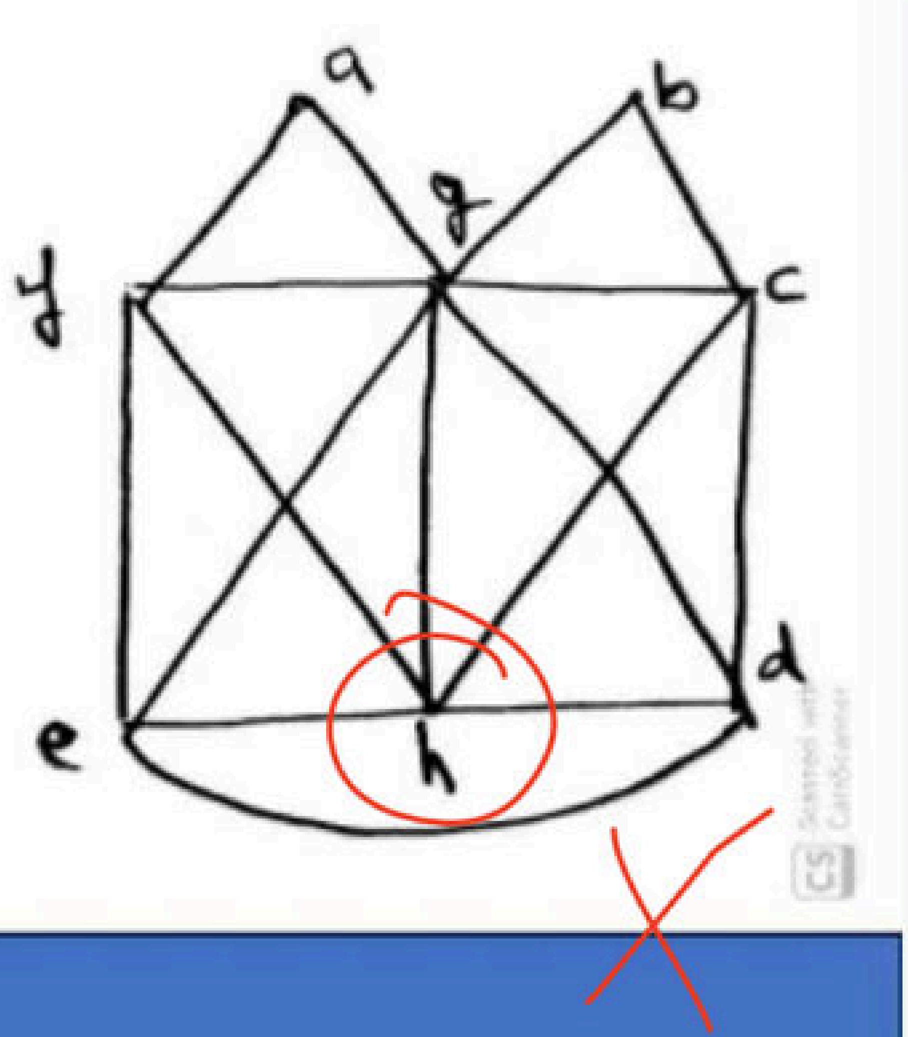


Euler Graph				
Hamiltonian Graph				

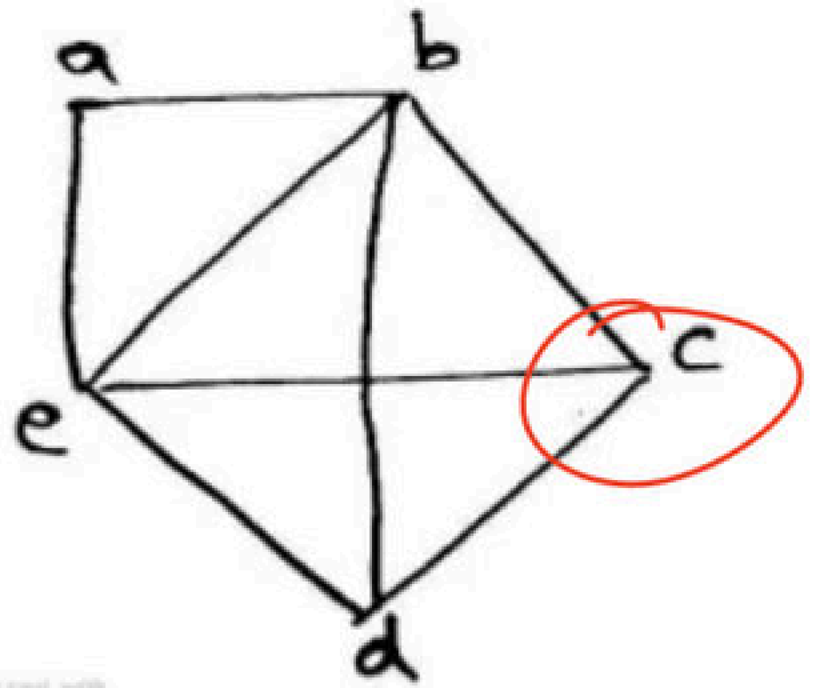


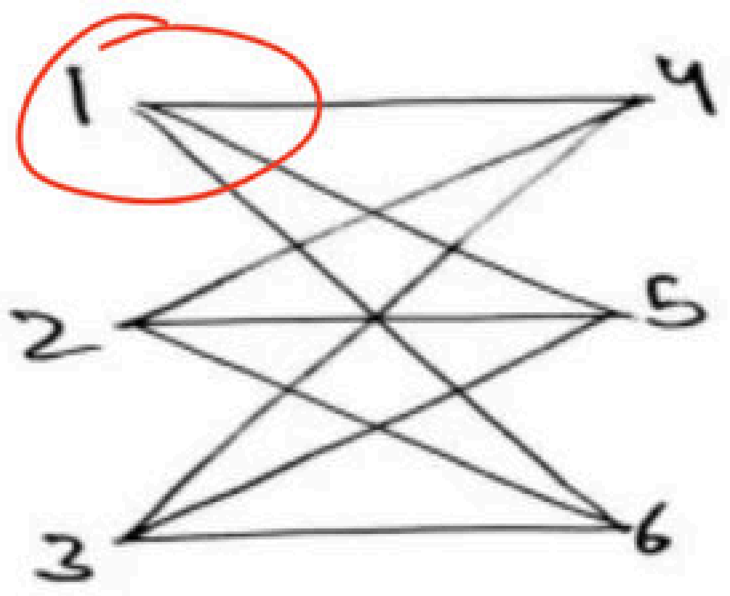


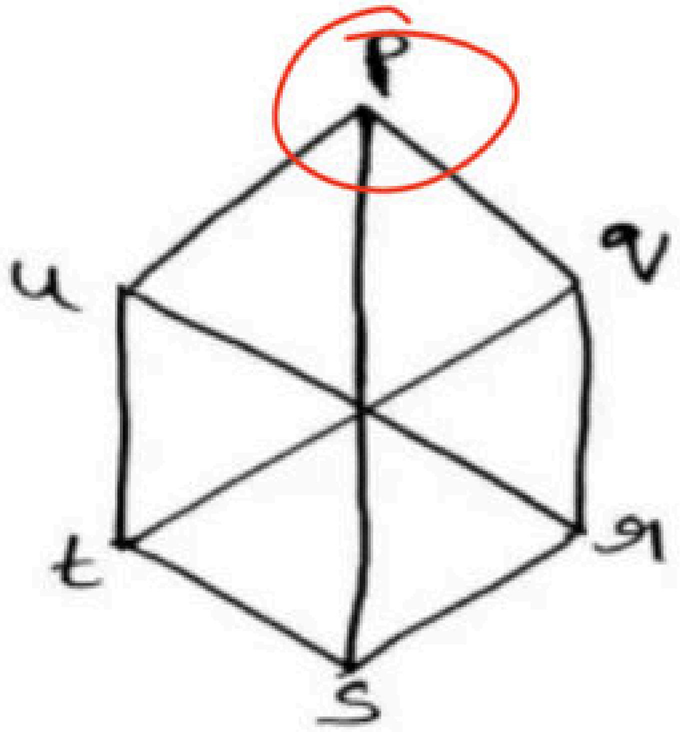


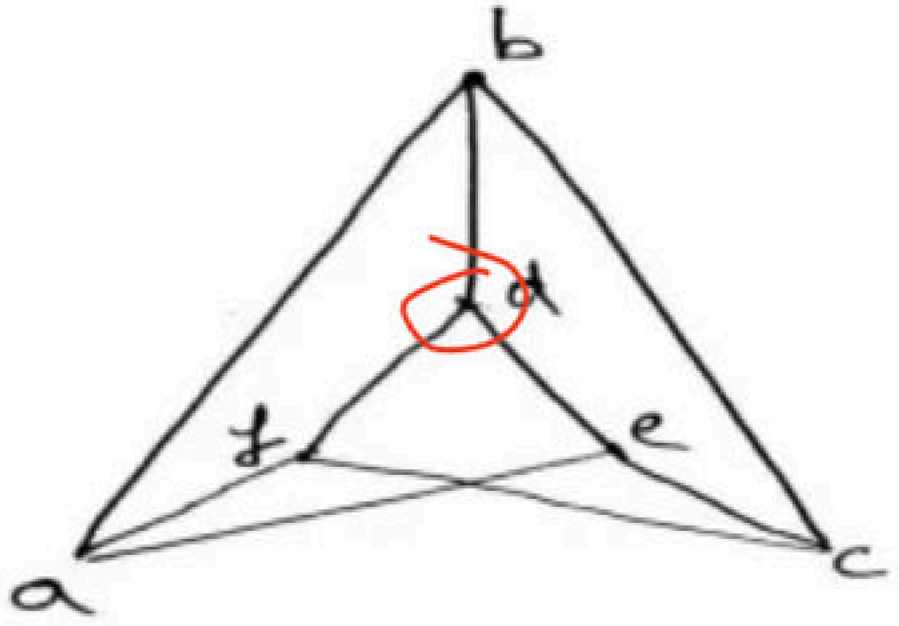


Euler Graph				
Hamiltonian Graph				









Euler Graph				
Hamiltonian Graph				