AI2101: Convex Optimization

Assignment - 3

Handed out: 09 - Feb - 2024 **Due : 19 - Feb - 2024 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.
2. Submissions received after the deadline will attract negative marking.
3. It is suggested that you attempt all the problems. However, it is sufficient to submit solutions for problems that total 10 points.
4. The submission must be named in the following format: RollNo-Assignment-X.pdf .

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1. (5 Points) In which of the following cases is the affine set generated by the given set of vectors in $\mathbb{R}^3$ a subspace? Justify your answer.
   (a) $\left\{ \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 7 \end{bmatrix} \right\}$
   (b) $\left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ 3 \end{bmatrix} \right\}$
2. (5 Points) Let $\mathbf{v_1} = \begin{pmatrix} 1 & 2 & 0 \end{pmatrix}^T$, $\mathbf{v_2} = \begin{pmatrix} -1 & 1 & 3 \end{pmatrix}^T$, $\mathbf{v_3} = \begin{pmatrix} 0 & -1 & 2 \end{pmatrix}^T$ be vectors in $\mathbb{R}^3$. Let T be a map from $\mathbb{R}^3$ to $\mathbb{R}^2$ such that $T(\mathbf{v_1}) = \begin{pmatrix} 1 & 2 \end{pmatrix}^T$, $T(\mathbf{v_2}) = \begin{pmatrix} 2 & 2 \end{pmatrix}^T$, and $T(\mathbf{v_3}) = \begin{pmatrix} 3 & 2 \end{pmatrix}^T$. Is it possible for the map T to be linear? If yes, find a matrix $\mathbf{A}$ of suitable dimension such that $T(\mathbf{x}) = \mathbf{Ax}$ for any $\mathbf{x} \in \mathbb{R}^3$.
3. (5 Points) Let $\mathbb{S} = \left\{ \begin{pmatrix} x_1, & x_2, & x_3 \end{pmatrix}^{\mathbf{T}} \in \mathbb{R}^{\mathbf{3}} : 0 \leq x_3, 0 \leq x_1 \leq 5, 0 \leq x_2 \leq 5 \right\}$.
   (a) Show that set S is a convex set.
   (b) Can set S be expressed as the convex hull generated by finitely many vectors in $\mathbb{R}^3$? Justify.
4. (5 Points) Define
   $$\mathcal{P}_m := \{f(x) = a_0 + a_1 x + \cdots + a_m x^m : a_i \in \mathbb{R}, 0 \leq i \leq m\}$$
   to be the set of all polynomials of degree at most m. Is this set convex, affine, subspace? Further consider the set of all positive polynomial i.e.

$$\mathcal{P}_{++}^{(m)} = \{f(x) \in \mathcal{P}_m : f(x) > 0 \forall x \in \mathbb{R}\}$$

Is this set convex? Is it affine?[1]

5. (5 Points) Consider the set of all the periodic functions with period 2. Is it affine? [2]

[1] EE5606 Convex Optimisation (Spring 2023) by Shashank Vatedka