AI2101: Convex Optimization

Assignment - 4

Handed out: 18 - Feb - 2024   **Due : 28 - Feb - 2024 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.

2. Submissions received after the deadline will attract negative marking.

3. It is suggested that you attempt all the problems. However, it is sufficient to submit solutions for problems that total 10 points.

4. The submission must be named in the following format: RollNo-Assignment-X.pdf .

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1. (4 Points) Compute the gradient and hessian of the function $f(\mathbf{x})$ which is defined as

$$f(\mathbf{x}) \;=\; \frac{c}{\|\mathbf{x}\|} \;\forall\; \mathbf{x} \neq \mathbf{0} \tag{1}$$

   at a point $\mathbf{x}^0 \neq \mathbf{0}$. (Hint: The function $f(\mathbf{x})$ represents a typical potential function in Physics).

2. (2 Points) Compute the derivative matrix of the function $\mathbf{g}(\mathbf{x})$ at a point $\mathbf{x}^0 \neq \mathbf{0}$

$$\mathbf{g}(\mathbf{x}) \;=\; \frac{c}{\|\mathbf{x}\|^2}\mathbf{e}_x \;\forall\; \mathbf{x} \neq \mathbf{0} \tag{2}$$

   where $\mathbf{e}_x$ is the unit vector in the direction of $\mathbf{x}$. (Hint: The function $g(\mathbf{x})$ represents a typical expression for Force).

3. (4 Points) We use the least squares method very often, especially in regression problems. Determine the gradient of $\|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2$. Using this, find the least squares solution for the equation $\mathbf{A}\mathbf{x} = \mathbf{b}$

4. (6 Points) Compute the hessian of the following functions at $\mathbf{x}^0$ (where $\mathbf{x} \in \mathcal{R}^2$). Also, plot/comment on the nature (or shape) of level sets for each function.

   - $f_1(\mathbf{x}) \;=\; 2x_1^2 - 2x_2^2$
   - $f_2(\mathbf{x}) \;=\; x_1^4 + x_2^4 + 2x_1x_2$
   - $f_3(\mathbf{x}) \;=\; \cos^2(x_1) + \sin^2(x_2)$

   For plotting the level sets, you are free to choose a programming language of your choice and are also allowed to use the built-in functions/modules.