AI2101: Convex Optimization

Assignment - 1

Handed out: 20 - Jan - 2024 **Due : 30 - Jan - 2024 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the canvas platform). Solutions submitted to the course page will only be evaluated. Refer to the assignment guidelines mentioned on the course page.
2. Submissions received after the deadline will attract negative marking.
3. It is suggested that you attempt all the problems. However, it is sufficient to submit solutions for problems that total 10 points.
4. Please note that you should use the template provided along with the assignment for submitting the programming part. The submission must be named in the following format: RollNo-Assignment-1.ipynb (for programming) or RollNo-Assignment-X.pdf (for non-programming submissions).

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1. (10 Points) Using a programming language of choice, implement the Golden Section Search, the Bisection Search, and Newton's Method for the single variable optimization problem.

   (a) For Golden Section and Bisection Search Algorithms, plot the width of the search space and the approximate minima obtained at the end of each iteration.

   (b) For Newton's method, plot the value of the derivative evaluated at the trail points obtained after each iteration.

   Also, please include the following in your solution:

   - Take 2 new differentiable convex objective functions (double differentiable for Newton's Method)
   - For both a) and b), plot the function you have chosen and its first and second derivatives. Show the actual optimum on all these plots.
   - For both a) and b), plot a "zoomed-in" view of the obtained optimum v/s the actual optimum.
   - Make a bar plot of the error in the value of the function at the optimum obtained by your algorithm and another bar plot indicating the error in the optimum obtained by your algorithm for all three search methods.
   - Label all plots clearly- each one must include a title, x-axis labels, y-axis labels, legend, etc.

   Please note: There is an edge case in Newton's Method- try to devise your algorithm such that it is robust to such a case. The edge case is as follows: When a window is provided in Newton's method, what happens when your algorithm results in the current optimum exceeding the bounds of the window? How can this case be handled? Is this problem even feasible?

Practice Questions (Need Not be Submitted)

1. Considering the property that for a unimodal function, say $f(x)$(with minima) always satisfies $(f'(x))$ [We'll formally discuss this property a little later in this course), show that the Bisection Search Method converges to the minima (Mathematically, show that $x^k \to x^*$ as $k \to \infty$).

2. By what factor does the initial interval's size decrease for each step of the Golden Section Search?

3. Compute the number of iterations needed for the Bisection Search and the Golden Section Search approaches to have the final result to be $\epsilon-$close to the minima of the convex objective function. In terms of the number of iterations alone, which seems to be the better method?

4. In which of the following cases the sets $\mathbb{A}$ and $\mathbb{B}$ span the same subspace:

   (a) $\mathbb{A} = \left\{ \begin{bmatrix} 5 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \right\}$ and $\mathbb{B} = \left\{ \begin{bmatrix} 7 \\ 1 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 8 \\ -1 \end{bmatrix} \right\}$

   (b) $\mathbb{A} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} \right\}$ and $\mathbb{B} = \left\{ \begin{bmatrix} 8 \\ 9 \\ 10 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix} \right\}$