EE1203: Vector Calculus

Assignment - 3

Handed out on : 23 - Jan - 2024 **Due on : 31 - Jan - 2024 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the Google Classroom platform). Only solutions submitted to the course page will be evaluated.
2. Submissions received after the deadline will attract negative marking.
3. It is suggested that you attempt all the questions. However, submitting solutions for problems totalling at least 10 points is sufficient.
4. Note: Vectors are indicated using the bold interface.

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1. (2 points) Consider a function $f(x, y, z) = e^x cos(yz)$. Find the directional derivative of this function along the vector $2\hat{x} + \hat{y} - 2\hat{z}$
2. (10 points) Based on the idea of simple (a curve that doesn't cross itself), smooth(differentiable), closed (the curve has no endpoints and encloses an area) curves, consider the following:
   Say a curve as described above has been parameterized as follows:
   $$\underline{g}(\tau), \tau \in \mathbb{R}$$
   $$\|\underline{g}(\tau)\|_2 = 2$$
   then either prove or disprove the following:
   (a) $\underline{g}(\tau).\frac{d\underline{g}(\tau)}{d\tau} = 0 \ \forall\tau$
   (b) $\underline{g}(\tau).\frac{d\underline{g}(\tau)}{d\tau} \leq 0 \ \forall\tau$
   (c) $\underline{g}(\tau) \times \frac{d^2\underline{g}(\tau)}{d\tau^2} = 0 \ \forall\tau$
3. (8 points) In class, the basic definition of gradient has been discussed- the direction of greatest change of a scalar function. You might have noticed the inherent use of basis vectors in this definition. Based on the fact that in spherical coordinates, the basis vectors themselves are a function of the point in space that is under consideration, derive the gradient vector in spherical coordinates, giving reasons at each step as to how you are circumventing the aforementioned issue.
4. (5 points) Consider a sphere centred at the origin. Let it be defined as follows
   $$S: \ x^2 + y^2 + z^2 = 9$$

Consider another curve defined as follows:

$$C: \ x^2 + y^2 - z - 3 = 0$$

Find the angle between these surfaces. Can there be multiple answers to this problem? If yes, find all the answers. If no, argue why such an event is not possible.

5. (4 points) Evaluate the following expressions:

   (a) $\nabla \times (\nabla \times g)$, where $g = x^2 y \ \hat{x} + 2yz \ \hat{z} - 2zx \ \hat{y}$

   (b) $\nabla \cdot (\nabla \times g)$, where $g = x^{27} y^9 \ \hat{x} + 2yz \ \hat{z} - 2zxy^{81} \ \hat{y}$

6. (8 points) Find the minimum value of the function:

$$g(x, y) = x^3 + y^3 - 3xy$$

Now, find the minimum value of the function

$$h(x, y) = \frac{1}{3}x^3 + y^2 + 2xy - 6x - 3y + 4$$

Let their respective minimum values be g_0 and h_0. Do any of the following hold true:

   (a) minimum value$(g(x, y) + h(x, y)) = g_0 + h_0$

   (b) minimum value$(g(x, y) \times h(x, y)) = g_0 h_0$

Can you conclude that a) and b) will either always be true or always be false irrespective of the function? Provide a justification for your answer.