EE1203: Vector Calculus

Assignment - 1

Handed out on : 07 - Jan - 2024 **Due on : 17 - Jan - 2024 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the Google Classroom platform). Solutions submitted to the course page will only be evaluated.
2. Submissions received after the deadline will attract negative marking.
3. It is suggested that you attempt all the questions. However, it is sufficient to submit solutions for problems that total to at least 10 points.
4. Note: Vectors are indicated using bold interface.

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1. (5 Points) Representation of a Vector Fields: Represent the following (3D) vector fields
   (a) $\mathbf{F}(\mathbf{r}) = -2\frac{\mathbf{r}}{\|\mathbf{r}\|}$.
   (b) $\mathbf{F}(\mathbf{r}) = 2\frac{\mathbf{e}_2 \times \mathbf{r}}{\|\mathbf{r}\|^2}$.

   As mentioned in the tutorial, there are open source tools that readily helps you in visualizing the vector fields. However, for the purpose of this assignment, it is recommended that indicate the vector fields (without using these tools) in an appropriate manner.

2. (5 Points) An elementary example of computing flux of a Vector field: Let $\boldsymbol{F}(x, y, z)$ (or) $\boldsymbol{F}(\mathbf{r})$ denote a vector field whose value at point $(1, 1, 1)$ on a particular plane is $2\mathbf{e}_1 + 3\mathbf{e}_2 - \mathbf{e}_3$. The chosen plane for analysis has two other points $(3, 2, 1)$ and $(4, 1, 2)$.
   (a) (2 Points) Compute the equation of the plane chosen for analysis.
   (b) (1 Points) Compute a unit vector (say $\mathbf{e}_n$) that is normal to the plane chosen for analysis.
   (c) (2 Points) Compute $\boldsymbol{F}(\mathbf{r}) \cdot \mathbf{e}_n$ at $(1, 1, 1)$ (Later in the course, you will see that the scalar product of the form plays a crucial role in computing flux of a vector field through a given surface).

3. (5 Points) Consider two points $(1, 1, 1)$ (say p_0) and $(3, 4, 5)$ (say p_1).
   (a) (2 points) Compute the parametric equation of the line (in vector form) passing through p_0 and p_1.
   (b) (3 points) Compute the point on the line passing through p_0 and p_1 that is closest to $(2, 2, 2)$.

4. (5 Points) Consider $\mathbf{u} \in \mathcal{R}^3$ and $\mathbf{v} \in \mathcal{R}^3$ such that $\mathbf{e}_u \neq \mathbf{e}_v$. Simplify
   (a) (2 Points) $\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v})$
   (b) (3 Points) $(\mathbf{u} \times 3\mathbf{v}) \times \mathbf{v}$

5. (5 Points) Consider 2 vectors $\mathbf{a} \in \mathcal{R}^n$ and $\mathbf{b} \in \mathcal{R}^n$. Find the area of the parallelogram formed by the aforementioned two vectors: your answer should only involve norms of vectors and/or scalar products.

6. (5 Points) Higher Dimensional Analysis: Using the concepts you have learnt regarding planes in three dimensions, answer the following questions based on the information provided for a 4 dimensional space.

   Consider a plane $P_1 : \mathbf{n_1} \cdot (\mathbf{r} - \mathbf{r_0}) = \mathbf{m_1} \cdot (\mathbf{r} - \mathbf{r_0}) = 0$

   Consider another plane $P_2 : \mathbf{n_2} \cdot (\mathbf{r} - \mathbf{r_0}) = \mathbf{m_2} \cdot (\mathbf{r} - \mathbf{r_0}) = 0$

   (a) Will they intersect if they are not parallel?

   (b) What will be the condition for the two planes to be parallel?

   (c) What will be the condition for them to coincide?

   (d) When will they intersect at a single point?

   (e) When will they intersect in a line?

   (f) Are there any other forms of intersection possible, that haven't been enumerated above?

7. (5 Points) Find the volume of the largest cuboid whose three faces lie flat in the coordinate planes, but one vertex lies on the plane $ax + by + cz + d = 0$.