EE1203: Vector Calculus

Assignment - 4

Handed out on : 30 - Jan - 2024 **Due on : 07 - Feb - 2024 (before 5 PM)**

Instructions :

1. Please submit the solutions to the assignment problems to the course page (on the Google Classroom platform). Solutions submitted to the course page will only be evaluated.
2. Submissions received after the deadline will attract negative marking.
3. It is suggested that you attempt all the questions. However, submitting solutions for problems totaling at least 10 points is sufficient.
4. Note: Vectors are indicated using the bold interface.

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1. (5 Points) Consider the vector field

$$\underline{f}(x,y) = 3x^2y^2\ \hat{x} + 2x^3y\ \hat{y}$$

Consider the curves:

$$\mathcal{C}:\ 2y^3 - x^3 - y^2 - x^2 + 2yx^2 - xy^2 - 2y + x + 1 = 0$$

Let C represent the paths on the XY plane starting at (-1,0) and terminating at (0.6, 0.8). Find $\int_{\mathcal{C}} \underline{f} \cdot d\underline{r}$

2. (6 Points) Consider the path formed by the following set of lines:

$$y = \frac{x}{2}, \quad x \in [0,4]$$
$$x = 4, \quad y \in [2,-8]$$

   (a) Find the equation of the line that will close the path.

   (b) Evaluate $\int_{\mathcal{C}} (x^2y^2\ dx + (yx^3 + y^2)\ dy)$ directly.

   (c) Evaluate the result using Green's Theorem.

   (d) How does the direction of traversal affect your result?

3. (2 Points) Along the path that traverses $y = x^2$, starting at (-1,1) and culminating at (2,4), evaluate the value of the integral:

$$\int_{\mathcal{C}} (xy\ dx + (x+y)\ dy)$$

4. (2 Points) Consider the expression:

$$\underline{g}(x,y,z) = (3x^2 + 6y)\hat{x} - 14yz\hat{y} + 20xz^2\hat{z}$$

Find the integral: $\int_{\mathcal{C}} \underline{g} \cdot d\underline{r}$, along the path defined by the straight lines from (0,0,0) to (1,0,0) and from (1,0,0) to (1,1,1).

5. (5 Points) Consider the following:

$$g(x,y) = (2sinx)(cos(x+y))(cosx)$$

Consider the clockwise and counterclockwise traversals along the curve

$$\mathcal{C} : x^2 + y^2 = 1$$

Find the line integral:

$$\int_{\mathcal{C}} (g) \nabla g \cdot d\underline{r}$$

6. (5 Points) Consider the vector field

$$\underline{g} = e^{xy}(2x + x^2 y)\ \hat{x} + e^{xy}(x^3 + 2ye^{-xy})\ \hat{y}$$

Find the value of: $\int_{\mathcal{C}} (g) \cdot d\underline{r}$
The curve $\mathcal{C}$ is parameterically defined as follows:

$$\mathcal{C} : \underline{r}(t) = tcos(\pi t)\ \hat{x} + tsin(\pi t)\ \hat{y} \quad t \in [0,1]$$

What is the maximum value of this integral in the given parametric range if the starting point corresponds to that at $t = 0$?