Fast Fourier transform in the analysis of biomedical data*

Ajit P. Yoganathan Ramesh Gupta William H. Corcoran

Chemical Engineering Laboratory, California Institute of Technology, Pasadena, California 91125, USA

Abstract—*The fast Fourier transform (f.f.t.) is a powerful technique which facilitates analysis of signals in the frequency domain. This paper reviews some of the important features of the fast Fourier transform which are relevant to its increasing application to biomedical data. A distinction is made between the power spectrum of ergodic signals, computed from the autocorrelation function, and the frequency spectrum of nonstationary biomedical signals. The major practical pitfalls that are encountered in applying the f.f.t. technique to biomedical data are discussed, and practical hints for avoiding such pitfalls are suggested.*

Keywords—*Biomedical data, Frequency analysis, Fast Fourier transform*

Introduction

THIS article is written as a review paper on the application of the fast Fourier transform (f.f.t.) in the analysis of biomedical data. Many articles have been written about the fast Fourier transform in the electrical-engineering literature (Special issue on fast Fourier transform, 1969; BERGLAND, 1969; COCHRAN *et al.*, 1967). As expected, however, these articles are oriented towards applications in the electrical-engineering field. Since the f.f.t. algorithm for the computation of Fourier coefficients was reported by COOLEY and TUKEY (1965), this technique has found increasing applications in the biomedical field (RAUTERKUS *et al.*, 1966; GUPTA *et al.*, 1975; YOGANATHAN *et al.*, 1975).

Physically, the Fourier transform $S(f)$ represents the distribution of signal strength with frequency (i.e. a density function). The fast-Fourier-transform algorithm is a method for computing the finite discrete Fourier transform (d.f.t.) of a series of N complex data points in approximately $N \log_2 N$ operations. Prior to the formulation of the f.f.t. algorithm, computing the finite discrete Fourier transform involved approximately N^2 operations. The f.f.t. algorithm has not only reduced the computation time from several minutes to seconds, but has also substantially reduced the round-off errors and the computation costs from dollars to cents. In fact, both computation time and round-off error are reduced by a factor of about $1/N \log_2 N$. If $N = 2048$ (i.e. 2^{11}), then $N \log_2 N = 22\,528$ and $N^2 = 4\,193\,304$. Hence the conventional method would require an effort of more than 180 times that required by the f.f.t.

The discrete Fourier transform is a transform in its own right, like the Fourier integral transform or the Fourier series transform. It is a powerful reversible mapping operation for time series. Its mathematical properties are analogous to those of the Fourier integral transform.

The discrete Fourier transform is defined by

$$S(k) = \frac{1}{N} \sum_{r=0}^{N-1} s(r) \exp(-2\pi irk/N) \quad . \quad . \quad (1)$$

for $r = 0, 1, 2, \ldots, N-1$, and $k = 0, 1, \ldots M-1$. $i = \sqrt{-1}$ where $S(k)$ is the kth coefficient of the d.f.t. and $s(r)$ is the rth sample of the time series, consisting of N samples. The inverse transform is given by

$$s(r) = \sum_{k=0}^{N-1} S(k) \exp(2\pi irk/N) \quad . \quad . \quad . \quad . \quad (2)$$

The Fourier transform for a continuous signal may be written as:

$$S(f) = \int_{-\alpha}^{\alpha} s(t) \exp(-2\pi ift) dt \quad . \quad . \quad . \quad . \quad (3)$$

and the inverse transform as

$$s(t) = \int_{-\alpha}^{\alpha} S(f) \exp(2\pi ift)\, df \quad . \quad . \quad . \quad . \quad (4)$$

for $-\alpha < f < \alpha$, and $-\alpha < t < \alpha$

where $S(f)$ describes the frequency-domain function and $s(t)$ describes the time-domain function.

In reality, however, the signal is usually of finite duration. Therefore, one sets $s(t) = 0$ for $t < 0$, and $t > T$ when the signal is available over the time range $(0, T)$.

*First received 10th February and in final form 7th March 1975

It is more customary to use the power spectrum $P(f)$ instead of the amplitude or voltage spectrum. The power spectrum is given by

$$P(f) \quad \lim_{T\to\alpha} \frac{1}{T}\,|S(f)|^2 \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In order to make $P(f)$ relatively independent of the time duration T of the data, the factor $1/T$ is inserted in the above equation.

Concept of the power spectrum

Using the property of Fourier transforms that

$$S^*(f) = \tilde{F}\,[s(-t)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where the asterisk indicates a complex conjugate, and $\tilde{F}$ indicates Fourier transform of [ ], eqn. 5 may be rewritten as:

$$P(f) = \lim_{T\to\alpha} \frac{1}{T}\,S(f)\,S^*(f) \quad . \quad . \quad . \quad . \quad (7)$$

From the convolution theorem and eqn. 6

$$P(f) = \lim_{T\to\alpha} \frac{1}{T}\,\tilde{F}\left[\int_{-T}^{+T} s(-t')\,s(t-t')\,dt'\right] \quad (8)$$

Now we can define

$$R(\tau) = \lim_{T\to\alpha} \frac{1}{T} \int_{-T}^{+T} s(t'')\,s(\tau+t'')\,dt'' \quad . \quad . \quad (9)$$

where $t'' = -t'$.

From eqn. 9, it is seen that $P(f)$ is just the Fourier transform of $R(\tau)$, i.e.

$$P(f) = \int_{-\alpha}^{\alpha} R(\tau)\exp(-2\pi i f\tau)\,d\tau \quad . \quad . \quad . \quad (10)$$

The function $R(\tau)$ is commonly known as the autocorrelation function of the continuous time signal $s(t)$. Eqn. 10 is widely used in the field of electrical engineering. The equation states that the power spectrum of a random continuous signal is the Fourier transform of its autocorrelation function. This is known as the Wiener–Kinchine theorem (Lee, 1960). Another form of this theorem is:

$$P(f) = \tilde{F}[R(\tau)]$$

$$= \int_{-\alpha}^{\alpha} \overline{s(t)\,s(t+\tau)}\,\exp(-2\pi i f t)\,d\tau \quad . \quad (10a)$$

The above power-spectrum approach is profitably applied when the signal consists of a recurring phenomena, plus any random noise. The autocorrelation step (eqn. 9) in the power-spectrum-analysis technique helps to nullify the random components (i.e. noise) in the signal. Therefore, the Fourier transform of the resulting autocorrelation function (eqn. 10) then yields the relative power spectrum of the recurring components of the original time signal. This technique is extremely attractive, especially since it more or less eliminates the contribution of random noise to the original signal.

If we step back, however, and look at the underlying conditions in the derivation of eqn. 10, it becomes apparent that this technique cannot be used in most biomedical applications (e.g. heart sounds, arterial sounds etc.). Such signals cannot be subject to Fourier analysis by this technique because they are nonstationary in nature, transient, and changeable in form. Implicit in eqn. 10a is the condition that $\overline{s(t)\,s(t+\tau)}$ depends only on τ, i.e. the source must be ergodic. For nonstationary sources, the power spectrum obtained from eqn. 10 would be a function of both frequency and time. This condition is in total contradiction to the concept of frequency-domain descriptions. Therefore the autocorrelation step to yield the power spectrum can only be applied to ergodic, and hence stationary, signals or sounds.

Fig. 1 Aliasing effect: a high-frequency signal impersonating a low-frequency signal

In the case of biomedical data, for example the heart sounds, the Fourier analysis is applied directly, using the Cooley and Tukey algorithm (1965), to each individual sound, and then an average frequency spectrum may be obtained for 10 such sounds. This method of analysis is described in detail in the papers by Gupta *et al.* (1975) and Yoganathan *et al.* (1975). The approach yields the frequency distribution of the relative sound intensity for each sound, without assuming that the sounds are stationary or periodic. The frequency spectrum obtained in the analysis of such biomedical data should not be confused with the power spectrum obtained via the Fourier transform of the autocorrelation function.

Computing the frequency spectrum by the direct-Fourier-transformation method does have one drawback. In this analysis technique, it is not possible to distinguish between the signal and background noise. This problem is not major, however, in most experimental situations because it is possible to produce signal levels which are large compared to the baseline noise level (i.e. a high signal-to-noise ratio).

Convolution theorem

The convolution theorem states that the product of two Fourier transforms $S(f)\,R(f)$ is itself the Fourier transform of a third time function, which is is known as the convolution of $s(t)$ and $r(t)$. It is defined as:

$$s(t) \circledast r(t) = \int_{-\infty}^{\infty} s(t')\, r(t-t')\, dt' \quad . \quad . \quad . \quad . \quad (11)$$

Thus the product of two Fourier transforms $S(f)\,R(f)$ may be written as

$$S(f)\,R(f)$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} s(t')\, r(t-t')\, dt' \right] \exp(-2\pi i f t)\, dt$$

$$= \iint_{-\infty}^{\infty} s(t')\, r(t'') \exp[-2\pi i f(t'+t'')]\, dt''\, dt' \quad (12)$$

where t' and t'' are dummy variables, and $t = t'+t''$. Similarly, we have

$$s(t)\, r(t) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} S(g)\, R(f-g)\, dg \right] \times \exp(2\pi i f t)\, df \quad . \quad (13)$$

The above mathematical expression states that the product of $s(t)\, r(t)$ in the time domain is the inverse Fourier transform of the frequency convolution

$$S(f) \circledast R(f) = \int_{-\infty}^{\infty} S(g)\, R(f-g)\, dg \quad . \quad . \quad (14)$$

Pitfalls

In the following Sections we will discuss the major pitfalls that are encountered in analysing biomedical data using the f.f.t. technique. If care is taken in avoiding these pitfalls, the f.f.t. analysis will give a high degree of accuracy.

(*a*) *Time sampling and aliasing:* For purposes of analysis, most continuous signals $s(t)$ will be digitised at a fixed rate and converted to digital signals via an analogue-to-digital convertor. These digital data may then be used for calculations on the computer. The digital signal may be considered as the result of multiplying the original continuous signal by a signal $a(t)$ which consists of a train of delta functions. The quantity $a(t)$ is defined as:

$$a(t) = \sum_{n=-\infty}^{+\infty} \delta(t - n\Delta t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where $\Delta t =$ (digitisation rate)$^{-1}$. This procedure produces an impulse-modulated signal $s_i(t)$, where

$$s_i(t) = s(t)\, a(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Using the convolution theorem

$$S_i(f) = \int_{-\infty}^{\infty} S(f-f')\, A(f')\, df' \quad . \quad . \quad . \quad (17)$$

where $A(f)$ is the Fourier transform of $a(t)$. Using the result that the Fourier transform of a train of delta functions

$$a(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta t),$$

is

$$A(f) = \frac{1}{\Delta t} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{\Delta t}\right) \quad . \quad . \quad . \quad . \quad (18)$$

and substituting this into eqn. 17 yields

$$S_i(f) = \int_{-\infty}^{\infty} S(f-f') \frac{1}{\Delta t} \sum_{n=-\infty}^{\infty} \delta\left(f' - \frac{n}{\Delta t}\right) df'$$

$$= \frac{1}{\Delta t} \sum_{n=-\infty}^{\infty} S\left(f - \frac{n}{\Delta t}\right) \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Eqn. 19 shows that the impulse-modulated signal $s_i(t)$ has a transform period of $1/\Delta t$, and if $S(f)$ is zero when $f \geq 1/2\Delta t$, then $S_i(f)$ is simply a periodic version of $S(f)$. Therefore it is possible to recover $S(f)$ from $A_i(f)$ by multiplying $S_i(f)$ by $R(f)$ where

$$R(f) = \begin{cases} \Delta t, \text{ when } |f| \leq \dfrac{1}{2\Delta t} \\ 0, \text{ when } |f| > \dfrac{1}{2\Delta t} \end{cases} \quad . \quad . \quad . \quad (20)$$

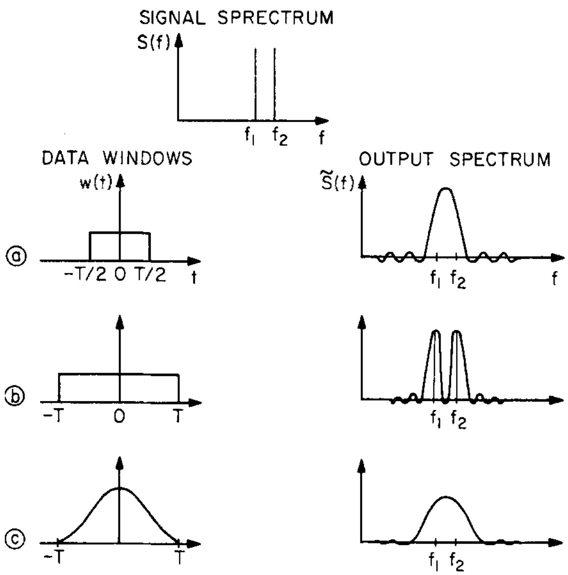

Fig. 2 Effect of data-window duration and shape on the frequency spectrum

The term 'aliasing' refers to the fact that high-frequency components of a time function are capable of impersonating low frequencies if the digitisation rate is too low. This fact is illustrated in Fig. 1. Because the digitisation rate is too low, a high-frequency and a low-frequency sine wave share identical points. To ensure that such an error does not creep in, the Nyquist criterion is used. This criterion demands that the digitisation rate should be at least large enough to be able to sample the highest frequency present in the signal at least twice during each cycle. The Nyquist criterion may be mathematically obtained by referring to eqns. 19 and 20. If the sampling interval is such that $S(f)$ falls to zero before $|f| = 1/2\Delta t$, then it is possible to recover $S(f)$ from $S_i(f)$ [i.e. $s(t)$ from $s_i(t)$]. If $S(f)$ is, however, not zero above $f_{Ny} = 1/2\Delta t$, then frequency components above $1/2\Delta t$ in $S(f)$ appear in $S_i(f)$ in the frequency range $-(1/2\Delta t) \leqslant f \leqslant (1/2\Delta t)$. The frequency f_{Ny} is commonly known as the Nyquist frequency.

Thus the operation of analysing a finite length of recorded data is equivalent to multiplying the actual signal $s(t)$ by the window $w(t)$. Use of eqn. 14 shows that the finite-time interval transformation $\tilde{S}(f)$ is the convolution of the transform of $s(t)$ and $w(t)$, i.e.

$$\tilde{S}(f) = \int_{-\alpha}^{\alpha} S(f')\, W(f-f')\, df' \quad . \quad . \quad . \quad (23)$$

where $W(f)$ is the transform $w(t)$.

The data window given in eqn. 21 is only an illustrative example. One may use any reasonable data window. Such a window will produce a spectral window $W(f)$ which will be centred around $f \equiv 0$ (see Fig. 3), but with sidelobes which will dampen out as f moves away from zero. Further mention will be made about these sidelobes in the section called 'Leakage'. For a small time interval T, $\tilde{S}(f)$ may give a very distorted representation of $S(f)$.

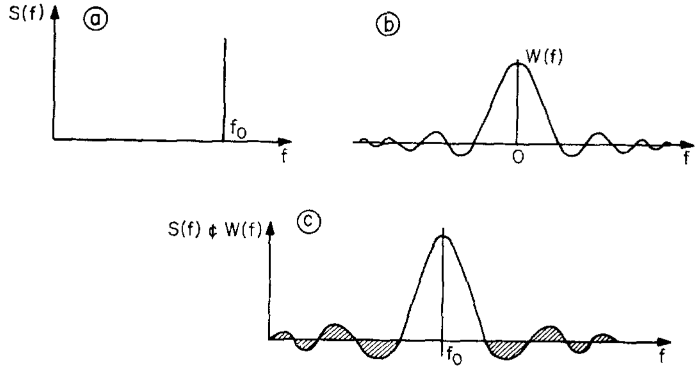

Fig. 3 Leakage of energy into the side lobes due to the analysis of a finite record of data

(b) *Finite-length records*: In real life, it is only possible to obtain a signal of finite length. Therefore a certain amount of truncation error arises when $s(t)$ is known only for a finite time interval

$$-T/2 \leqslant t \leqslant T/2.$$

Consider a rectangular data window (Fig. 2*a*) defined by

$$w(t) = \begin{cases} 1 & |t| \leqslant T/2 \\ 0 & |t| > T/2 \end{cases} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

where $-\alpha < t < \alpha$. Then the signal actually measured in the time interval T is given by

$$\tilde{s}(t) = s(t)\, w(t) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

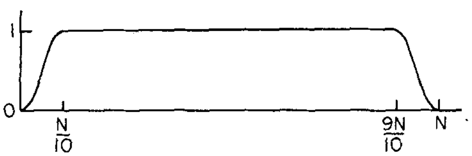

Fig. 4 Cosine data window

That, results because the window $W(f-f')$ will be wide, which leads to values of $S(f')$ that are far removed from $f'-f$ contributing to $\tilde{S}(f)$. As T becomes large, however, the distortion diminishes, and in the limit as $T \rightarrow \alpha$, the data window becomes a generalised function $w(t) = 1$, and the transform component at frequency f can be determined. Thus, as $T \rightarrow \alpha$, $W(f-f')$ becomes a delta function about $f' = f$, and $\tilde{S}(f) \rightarrow S(f)$.

Fig. 2 shows the effect of window shape and duration on the final transform. Only one main peak appears in the output transform of windows *a* and *c* because the two input peaks at frequencies f_1 and f_2 are fused into one peak. This result appears because a data window which was too narrow was used. In windows a and b, the sharp corners also contribute to the spurious peaks which appear in the frequency spectum.

It is possible to divide the finite record length into subsegments and take the Fourier transform over each subsegment. The power spectrum is then estimated by averaging these spectra. This approach is useful when computations have to be performed on a machine with very limited core storage. The problem of statistical stability of the estimated spectrum has been discussed by WELCH (1967).

(*c*) *Leakage:* The problem of leakage is inherent in the Fourier analysis of any finite record analysis. Consider a rectangular data window and a pure sine wave; if the sine wave is infinite in the time domain, its Fourier transform will result in a single value in the frequency domain (i.e. its fundamental frequency). In Fig. *3a*, this is shown as a single-impulse function of frequency f_0. As we have seen earlier, multiplication of this impulse by the data window in the time domain is equivalent to convolution in the frequency domain. The resulting function is shown in Fig. 3*c*.

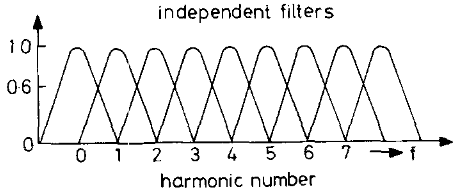

Fig. 5 Picket-fence effect

This function is not localised about a single frequency, but contains a series of spurious peaks known as side lobes as also seen in Figs. 2*a* and *b*. When such a phenomenon occurs, the object is to localise the contribution of a given frequency by reducing the amount of leakage through these spurious peaks. One way to achieve that goal is to apply a window to the time series which has lower side lobes in the frequency domain than a rectangular window. One such data window is the cosine window shown in Fig. 4 (BINGHAM *et al.*, 1967; BERGLAND, 1969). Other windows that can be used are the Hanning window (BLACKMAN and TUKEY, 1958), the Dolph–Chebyshev window (HELMS, 1968), and the Parzan window (WELCH, 1967). As a word of caution, whatever the type of window used, it must be applied to only actual physical data and not to any of the zeros which are added to increase the duration of the time signal.

(*d*) *Picket-fence effect:* Ideally, each Fourier coefficient in the output of the f.f.t. algorithm should represent a rectangular-shaped bandpass filter. In practice, however, because of the leakage effect as discussed in (*c*), each of these filters is deformed to a function of the form $\sin \pi f T/\pi f$, centred at the frequency of the corresponding filter. The main lobes of these filters have been plotted in Fig. 5 to show the nature of the output of the f.f.t. algorithm. As shown, these main lobes act as N independent filters. This means, for example, that an input of the form e^{iwt} with frequency $w = 2\pi/T\,n$, where n is an integer, would result in a response of unit amplitude at the nth harmonic and a response of zero at all other harmonics. Thus the picket-fence effect does not take place when the signal being analysed is one of these discrete frequencies. It will manifest itself when the signal frequency is not one of the above discrete frequencies. For example, a signal between the fourth and fifth harmonics will be seen by both the fourth and fifth harmonic filters but at a value lower than unity. In the worst case, when the signal frequency is exactly midway between two harmonics, the amplitude of the signal will be reduced by a factor of 0·637 in both the harmonic filters. This reduction produces a ripple in the resulting spectrum which is equivalent to viewing the real spectrum through a picket fence.

In practice, the picket-fence effect is not as severe as the above discussion may seem to imply. In the majority of cases, the signal being analysed is not a pure sinusoid but will have a broad frequency content.

It is possible to reduce the picket-fence effect by use of an interpolation function or by modification of the computation algorithm. The latter approach consists of computing the Fourier coefficients at frequencies between the original harmonics. Since the frequency spacing between harmonics is related to the reciprocal of the record length T, the above approach is carried out by increasing the length of the actual physical data by adding a set of samples which are identically zero. Also the use of a data window, other than the rectangular window discussed in (*b*), normally reduces the picket-fence effect by widening the main spectral lobes.

(*e*) *Frequency resolution.* The f.f.t. analysis is used as a tool to study the frequency content of a given time signal. In such an analysis, it may be required to resolve two frequency peaks, which are close to each other, in a continuous signal of finite time duration. The problem of resolving these peaks is analogous to the Rayleigh problem in optics. The Rayleigh criterion states that two sinusoids of frequencies f_1 and f_2 are just resolved if $T = 1/2(f_2 - f_1)$ and are well resolved if $T = 1/(f_2 - f_1)$. T is the record length of the actual physical data.

In practice, the biomedical signals are not pure sinewaves. Thus, to distinguish two peaks at frequencies g_1 and f_2, one may use the criterion

$$T > \frac{1}{f_2 - f_1} \quad \ldots \ldots \ldots \quad (24)$$

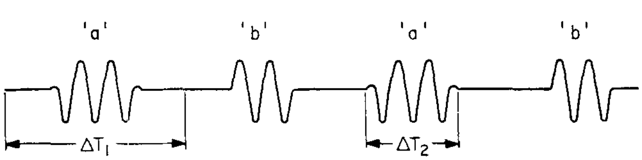

Fig. 6 (a) Proper selection of data window of sound 'a' ($\triangle T_1$)

(b) Incorrect selection of data window of sound 'a' ($\triangle T_2$)

If the lowest expected in the signal is f_{min}, then to observe it in the frequency spectrum, the value of T is computed from eqn. 24, and is

$$T > \frac{1}{f_{min}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

The ability to pick out peaks (resolution) depends not only on the duration of the data window but also on the shape of the window. This fact is illustrated in Fig. 2. In Figs. 2*a* and *b*, both windows have the same shape but different durations, while windows in Figs. 2*b* and *c* have the same duration but different shapes.

(*f*) *Selection of data for analysis:* Blind selection of data in the time domain usually leads to incorrect Fourier spectra. In most instances, the time signals obtained in biomedical data are nonstationary and transient in nature. The proper selection of data is of utmost importance.

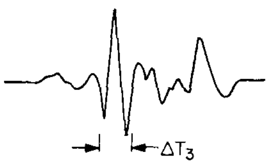

Fig. 7 Time signal of typical first heart sound

Referring to Fig. 6, for example, if the analysis of sounds 'a' are required, the data-window length must be chosen so that it covers the entire signal. Selection of a data window too close to the beginning and the end of the time signal can lead to artefacts in the f.f.t. analysis and produce spurious peaks in the spectrum. Also, the base line on either side of the signal being analysed should be relatively free of background noise.

In the analysis of transient biomedical signals, such as the first heart sound, one may want to use the f.f.t. algorithm for each component of the sound. This procedure may not be possible if the time duration of the component is shorter than the cycle time of the frequencies involved. In Fig. 7, for example the f.f.t. analysis cannot be used to analyse the first component contained in ΔT_3. In such situations, extreme care should be exercised in the analysis and interpretation of f.f.t. results.

Acknowledgment—Funds for support of the work described here were provided by the Donald E. Baxter Foundation. That support is gratefully acknowledged.

References

BERGLAND, G. D. (1969) A guided tour of the fast Fourier transform. *IEEE Spectrum* **6**, 41.

BINGHAM, C., TUKEY, J. W. and GODFREY, M. D. (1967) Modern techniques of power spectrum estimation. *IEEE Trans.* **AU-15**, 56.

BLACKMAN, R. and TUKEY, J. W. (1958) *The measurement of power spectra*. Dover, New York.

COCHRAN, W. T., COOLEY, J. W., FAVIN, D. L., HELMS, H. D., KAENEL, R. A., LANG, W. W., MALING, G. C. Jun., NELSON, D. E., RADER, C. M. and WELCH P. D. (1967) What is the fast Fourier transform? *Proc. IEEE* **55**, 1664.

COOLEY, J. W. and TUKEY, J. W. (1965) An algorithm for machine calculation of complex Fourier series. *Math, Comput.* **19**, 297.

GUPTA, R., MILLER, J. W., YOGANATHAN, A. P., KIM, B. M., UDWADIA, F. E. and CORCORAN, W. H. (1975) Spectral analysis of arterial sounds: a noninvasive method of studying arterial disease. Accepted for publication in the *Journal of Medical and Biological Engineering*

HELMS, H. D. (1968) Nonrecursive digital filters: design method for achieving specifications of frequency response. *IEEE Trans.* **AU-16**, 336.

LEE, Y. W. (1960) *Statistical theory of communications*. Wiley, New York.

RAUTERKUS, T., FELTZ, J. F. and FICKES, J. W. (1966) Frequency analysis of Korotkoff blood pressure sounds using Fourier transform. US Air Force School of Aerospace Medicine, Brooks AFB, Texas. Tech. Report 66–8.

WELCH, P. D. (1967) The use of the fast Fourier transform for the estimation of power spectra: a method based on time averaging over short, modified periodograms. *IEEE Trans.* **AU-15**, 70.

YOGANATHAN, A. P., GUPTA, R., UDEWADIA, F. E., MILLER, J. W., CORCORAN, W. H., SARMA, R., JOHNSON, J. L. and BING, R. J. (1975) Use of the fast Fourier transform for the frequency analysis of the first heart sound in normal man. Accepted for publication in the *Journal of Medical and Biological Engineering.*

Special issue on the fast Fourier transform (1969) *IEEE Trans.* **AU-17.**

La transformation rapide de Fourier dans l'analyse des donnèes biologiques

Sommaire—La transformation rapide de Fourier est une technique puissante qui facilite l'analyse des signaux dans le domaine des fréquences. Le présent article passe en revue quelques uns des traits importants de la transformation rapide de Fourier se rapportant à son application de plus en plus importante en données biomédicales. On fait la distinction entre le spectre de puissance des signaux ergodiques calculés à partir de la fonction d'autocorrélation, et le spectre de fréquence des signaux biomédicaux non stationnaires. Les pièges pratiques principaux rencontrés dans l'application de la technique de transformation rapide de Fourier sont débattus et des conseils pratiques pour éviter de tels pièges sont suggérés.

Die Schnelle Fourier-Transformierte in der analyse von biologisch-medizinischen Daten

Zusammenfassung—Die schnelle Fourier-Transformierte (FFT) ist ein leistungsfähiges Verfahren, das die Analyse von Signalen im Frequenzbereich erleichtert. Diese Arbeit überprüft einige der wichtigen Eigenschaften der schnellen Fourier-Transformierten, die sich auf ihre wachsende Anwendung auf biologisch-medizinische Daten beziehen. Es wird zwischen dem Leistungsspektrum von ergodischen Signalen, die aus der Autokorrelationsfunktion errechnet werden, und dem Frequenzspektrum von nichtstationären biologisch-medizinischen Signalen unterschieden. Die wichtigsten praktischen Fallen, auf die man bei Anwendung des FFT-Verfahrens auf biologisch-medizinische Daten stieß, werden besprochen, und es werden praktische Hinweise zur Vermeidung solcher Fallen gegeben.

MBE letter

Vessel area detection with c.w. ultrasound

One of the major limitations to the use of c.w. Doppler ultrasound flowmeters in transcutaneous haemodynamic investigations is their inability to measure volume flowrate. Clearly an estimation of this parameter can be made if the cross-sectional area of the vessel being examined is known, and one approach to this problem is to estimate the vessel calibre by means of angiography. However, this has the disadvantages

(*a*) that it is an offline technique

(*b*) it assumes a uniform, circular vessel cross-section

(*c*) it does not reveal the variations in cross-sectional area that occur in arteries throughout the cardiac cycle.

In order accurately to estimate the time-varying volume flow patterns in the human arterial system two measurements are needed:

(*a*) the instantaneous mean flow velocity

(*b*) the instantaneous cross-sectional area of the vessel.

We have already reported the design of a new c.w. Doppler system for deriving the instantaneous mean and peak velocities (Sainz *et al.*, 1975). To complement this instrument we have now developed a relatively simple system for measuring vessel cross-section simultaneously from the velocity signal (Fig. 1).

The system functions by providing an output voltage that is proportional to the Doppler spectrum intensity. Spectrum intensity is here defined as the number of velocity vectors detectable by the area detector in unit time. This definition implies several restrictive conditions:

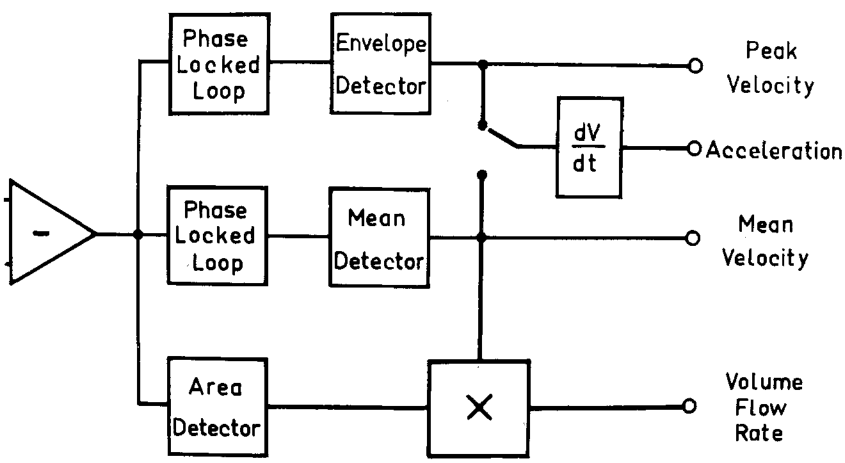

Fig. 1 Complete system