periodogram

Periodogram power spectral density estimate

Syntax

```
pxx = periodogram(x)                                      example
pxx = periodogram(x,window)                               example
pxx = periodogram(x,window,nfft)                          example

[pxx,w] = periodogram( ___ )
[pxx,f] = periodogram( ___ ,fs)                           example

[pxx,w] = periodogram(x,window,w)                         example
[pxx,f] = periodogram(x,window,f,fs)                      example

[ ___ ] = periodogram(x,window, ___ ,freqrange)           example
[ ___ ] = periodogram(x,window, ___ ,spectrumtype)        example

[ ___ ,pxxc] = periodogram( ___ ,'ConfidenceLevel',probability)   example

periodogram( ___ )                                        example
```

Description

`pxx = periodogram(x)` returns the periodogram power spectral density (PSD) estimate, `pxx`, of the input signal, `x`, found using a rectangular window. When `x` is a vector, it is treated as a single channel. When `x` is a matrix, the PSD is computed independently for each column and stored in the corresponding column of `pxx`. If `x` is real-valued, `pxx` is a one-sided PSD estimate. If `x` is complex-valued, `pxx` is a two-sided PSD estimate. The number of points, `nfft`, in the discrete Fourier transform (DFT) is the maximum of 256 or the next power of two greater than the signal length. example

`pxx = periodogram(x,window)` returns the modified periodogram PSD estimate using the window, `window`. `window` is a vector the same length as `x`. example

`pxx = periodogram(x,window,nfft)` uses `nfft` points in the discrete Fourier transform (DFT). If `nfft` is greater than the signal length, `x` is zero-padded to length `nfft`. If `nfft` is less than the signal length, the signal is wrapped modulo `nfft` and summed using `datawrap`. For example, the input signal `[1 2 3 4 5 6 7 8]` with `nfft` equal to 4 results in the periodogram of `sum([1 5; 2 6; 3 7; 4 8],2)`. example

`[pxx,w] = periodogram( ___ )` returns the normalized frequency vector, `w`. If `pxx` is a one-sided periodogram, `w` spans the interval [0,π] if `nfft` is even and [0,π) if `nfft` is odd. If `pxx` is a two-sided periodogram, `w` spans the interval [0,2π).

`[pxx,f] = periodogram( ___ ,fs)` returns a frequency vector, `f`, in cycles per unit time. The sampling frequency, `fs`, is the number of samples per unit time. If the unit of time is seconds, then `f` is in cycles/second (Hz). For real-valued signals, `f` spans the interval [0,`fs`/2] when `nfft` is even and [0,`fs`/2) when `nfft` is odd. For complex-valued signals, `f` spans the interval [0,`fs`). example `[pxx,w] = periodogram(x,window,w)` returns the two-sided periodogram estimates at the normalized frequencies specified in the vector, `w`. `w` must contain at least two elements. example

`[pxx,f] = periodogram(x,window,f,fs)` returns the two-sided periodogram estimates at the frequencies specified in the vector, `f`. `f` must contain at least two elements. The frequencies in `f` are in cycles per unit time. The sampling frequency, `fs`, is the number of samples per unit time. If the unit of time is seconds, then `f` is in cycles/second (Hz). example

`[ ___ ] = periodogram(x,window, ___ ,freqrange)` returns the periodogram over the frequency range specified by `freqrange`. Valid options for `freqrange` are: `'onesided'`, `'twosided'`, or `'centered'`. example

`[ ___ ] = periodogram(x,window, ___ ,spectrumtype)` returns the PSD estimate if `spectrumtype` is specified as `'psd'` and returns the power spectrum if `spectrumtype` is specified as `'power'`. example

`[ ___ ,pxxc] = periodogram( ___ ,'ConfidenceLevel',probability)` returns the `probability` × 100% confidence intervals for the PSD estimate in `pxxc`. example

`periodogram( ___ )` with no output arguments plots the periodogram PSD estimate in dB per unit frequency in the current figure window. example

Examples

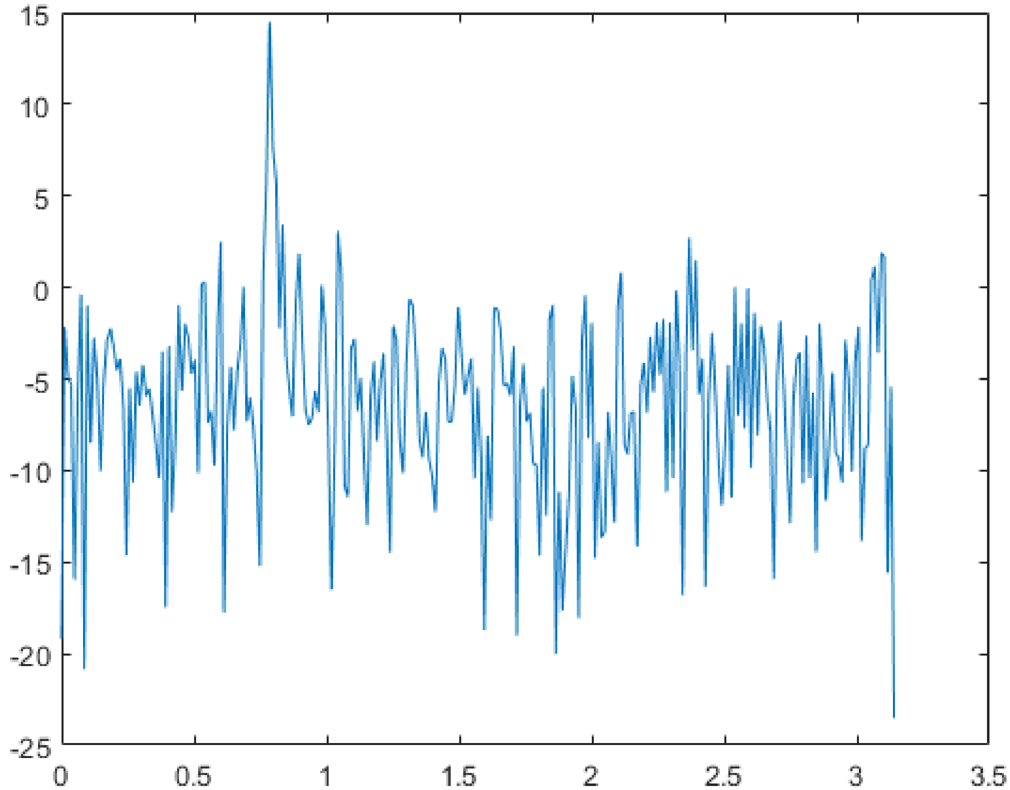
collapse all

Periodogram Using Default Inputs

Obtain the periodogram of an input signal consisting of a discrete-time sinusoid with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise.

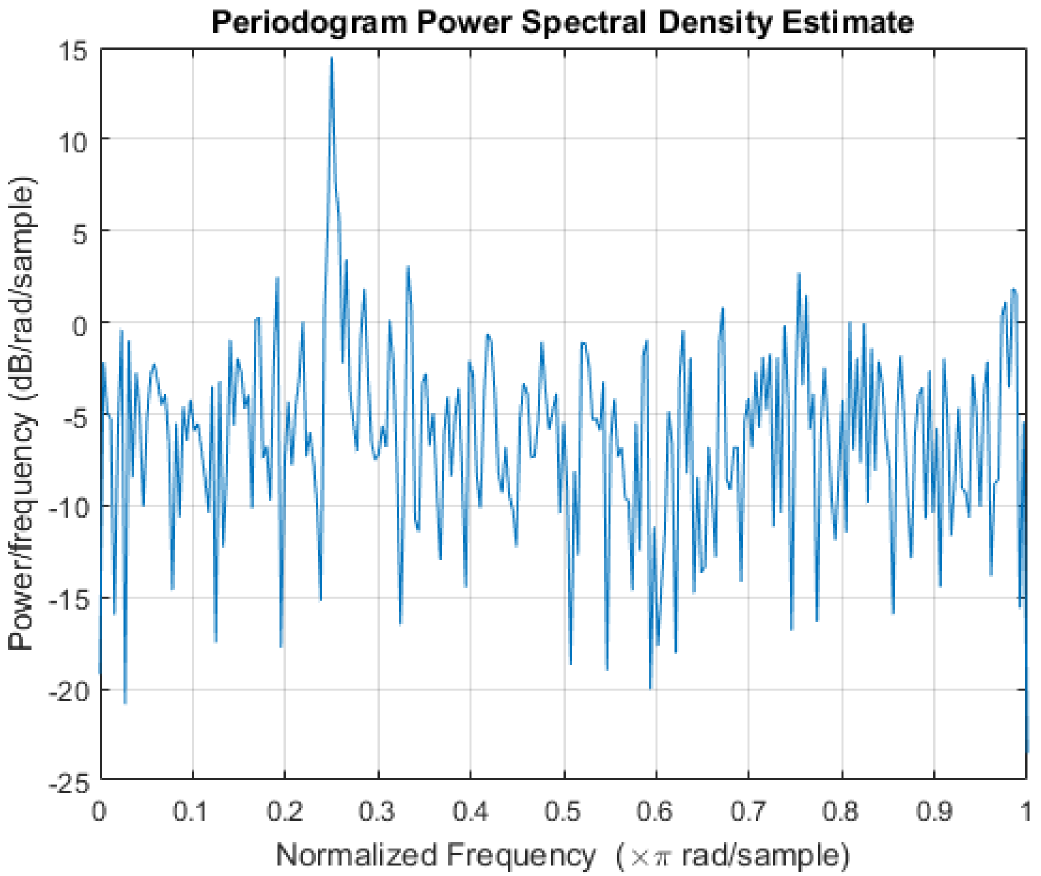
Create a sine wave with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise. The signal is 320 samples in length. Obtain the periodogram using the default rectangular window and DFT length. The DFT length is the next power of two greater than the signal length, or 512 points. Because the signal is real-valued and has even length, the periodogram is one-sided and there are 512/2+1 points.

```
n = 0:319;
x = cos(pi/4*n)+randn(size(n));
[pxx,w] = periodogram(x);
plot(w,10*log10(pxx))
```



Repeat the plot using `periodogram` with no outputs.

```
periodogram(x)
```

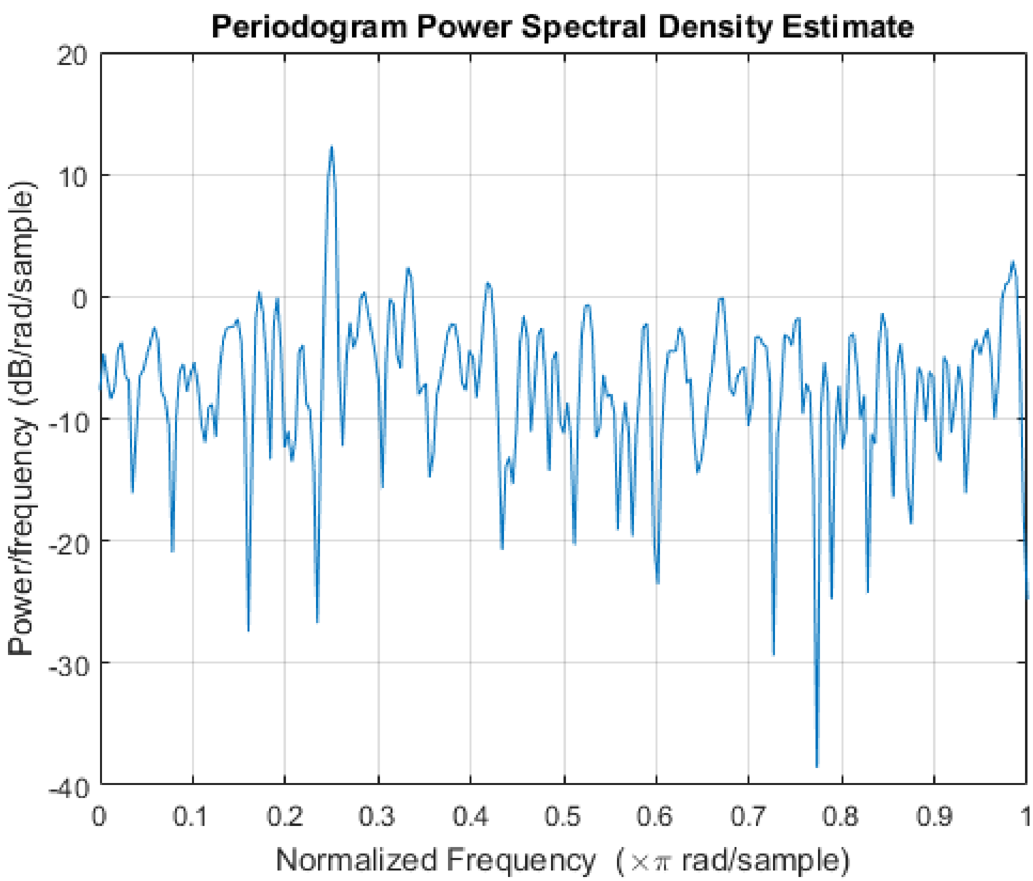


Modified Periodogram with Hamming Window

Obtain the modified periodogram of an input signal consisting of a discrete-time sinusoid with an angular frequency of $\pi/4$ radians/sample with additive $N(0,1)$ white noise.

Create a sine wave with an angular frequency of $\pi/4$ radians/sample with additive $N(0,1)$ white noise. The signal is 320 samples in length. Obtain the modified periodogram using a Hamming window and default DFT length. The DFT length is the next power of two greater than the signal length, or 512 points. Because the signal is real-valued and has even length, the periodogram is one-sided and there are 512/2+1 points.

```
n = 0:319;
x = cos(pi/4*n)+randn(size(n));
periodogram(x,hamming(length(x)))
```

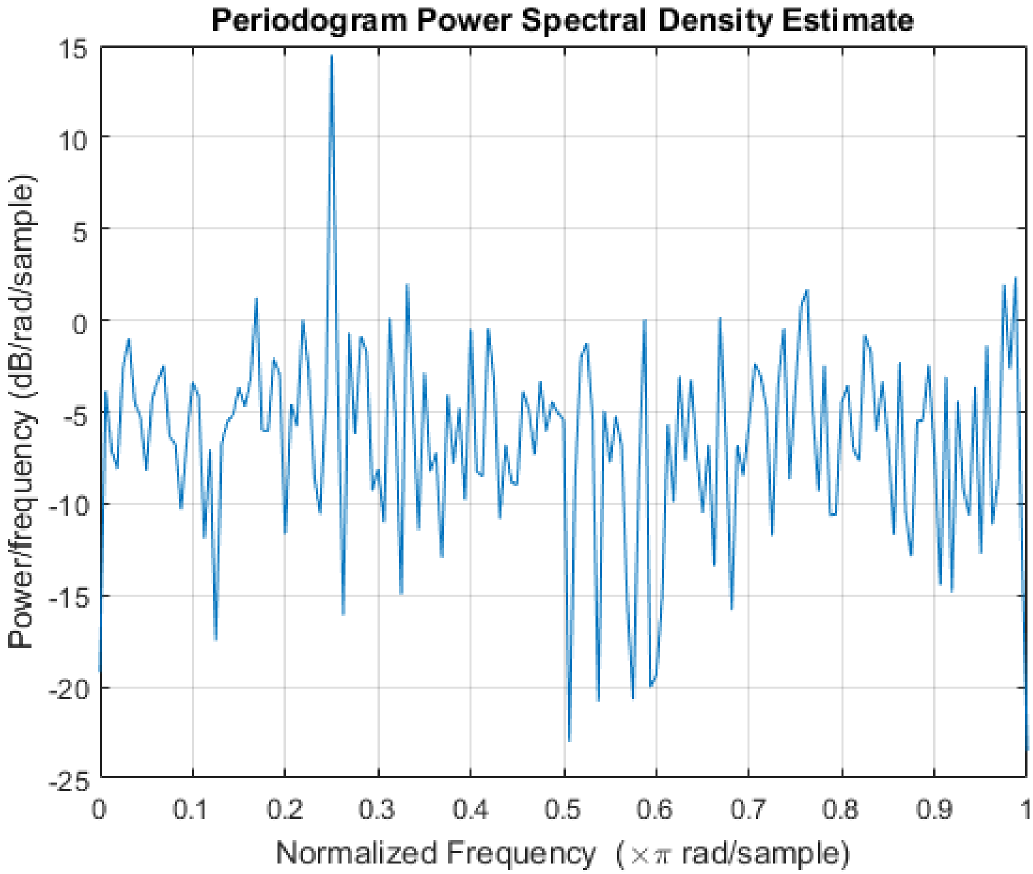


DFT Length Equal to Signal Length

Obtain the periodogram of an input signal consisting of a discrete-time sinusoid with an angular frequency of $\pi/4$ radians/sample with additive $N(0,1)$ white noise. Use a DFT length equal to the signal length.

Create a sine wave with an angular frequency of $\pi/4$ radians/sample with additive $N(0,1)$ white noise. The signal is 320 samples in length. Obtain the periodogram using the default rectangular window and DFT length equal to the signal length. Because the signal is real-valued, the one-sided periodogram is returned by default with a length equal to 320/2+1.

```
n = 0:319;
x = cos(pi/4*n)+randn(size(n));
nfft = length(x);
periodogram(x,[],nfft)
```



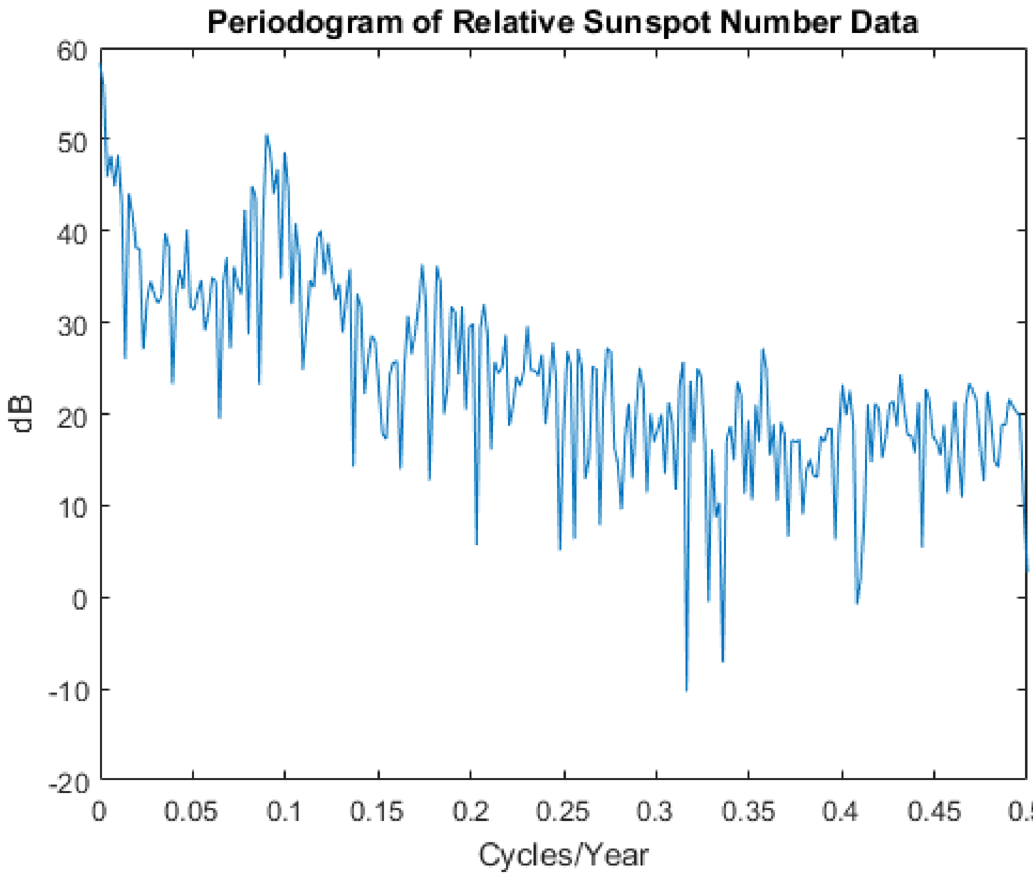
Periodogram of Relative Sunspot Numbers

Obtain the periodogram of the Wolf (relative sunspot) number data sampled yearly between 1700 and 1987.

Load the relative sunspot number data. Obtain the periodogram using the default rectangular window and number of DFT points (512 in this example). The sample rate for these data is 1 sample/year. Plot the periodogram.

```
load sunspot.dat
relNums=sunspot(:,2);

[pxx,f] = periodogram(relNums,[],[],1);
plot(f,10*log10(pxx))
xlabel('Cycles/Year')
ylabel('dB')
title('Periodogram of Relative Sunspot Number Data')
```



You see in the preceding figure that there is a peak in the periodogram at approximately 0.1 cycles/year, which indicates a period of approximately 10 years.

Periodogram at a Given Set of Normalized Frequencies

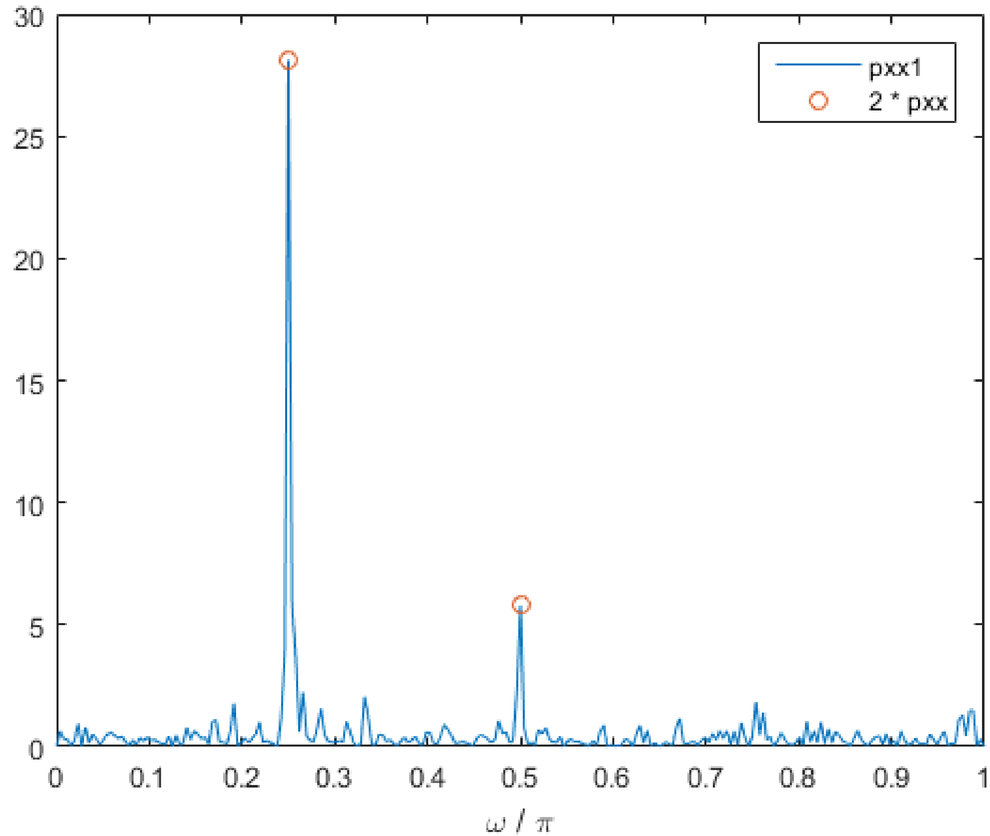
Obtain the periodogram of an input signal consisting of two discrete-time sinusoids with an angular frequencies of $\pi/4$ and $\pi/2$ rad/sample in additive $N(0,1)$ white noise. Obtain the two-sided periodogram estimates at $\pi/4$ and $\pi/2$ rad/sample. Compare the result to the one-sided periodogram.

```
n = 0:319;
x = cos(pi/4*n)+0.5*sin(pi/2*n)+randn(size(n));

[pxx,w] = periodogram(x,[],[pi/4 pi/2]);
pxx
[pxx1,w1] = periodogram(x);
plot(w1/pi,pxx1,w/pi,2*pxx,'o')
legend('pxx1','2 * pxx')
xlabel('\omega / \pi')
```

```
pxx =

   14.0589    2.8872
```



The periodogram values obtained are 1/2 the values in the one-sided periodogram. When you evaluate the periodogram at a specific set of frequencies, the output is a two-sided estimate.

Periodogram at a Given Set of Cyclical Frequencies

Create a signal consisting of two sine waves with frequencies of 100 and 200 Hz in *N*(0,1) white additive noise. The sampling frequency is 1 kHz. Obtain the two-sided periodogram at 100 and 200 Hz.

```
fs = 1000;
t = 0:0.001:1-0.001;
x = cos(2*pi*100*t)+sin(2*pi*200*t)+randn(size(t));

freq = [100 200];
pxx = periodogram(x,[],freq,fs)
```

```
pxx =

    0.2647    0.2313
```

Upper and Lower 95%-Confidence Bounds

The following example illustrates the use of confidence bounds with the periodogram. While not a necessary condition for statistical significance, frequencies in the periodogram where the lower confidence bound exceeds the upper confidence bound for surrounding PSD estimates clearly indicate significant oscillations in the time series.

Create a signal consisting of the superposition of 100 Hz and 150 Hz sine waves in additive white *N*(0,1) noise. The amplitude of the two sine waves is 1. The sampling frequency is 1 kHz.

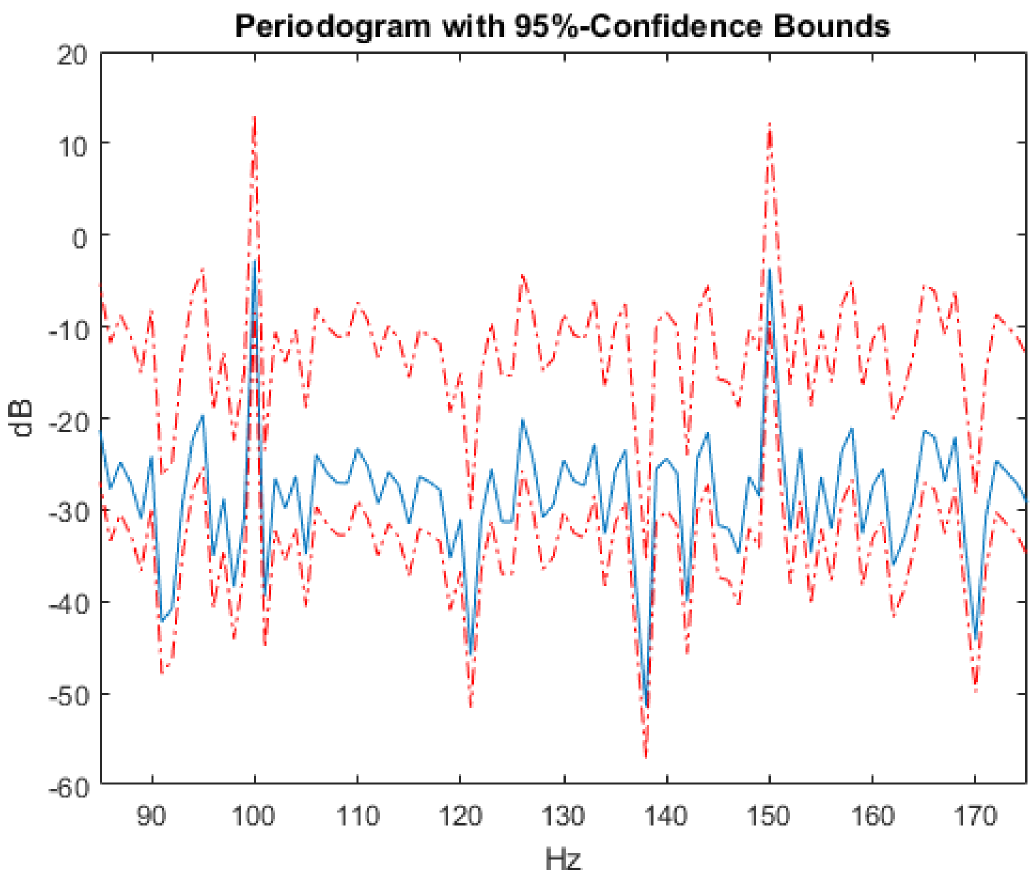
```
fs = 1000;
t = 0:0.001:1-0.001;
x = cos(2*pi*100*t)+sin(2*pi*150*t)+randn(size(t));
```

Obtain the periodogram with 95%-confidence bounds. Plot the periodogram along with the confidence interval and zoom in on the frequency region of interest near 100 and 150 Hz.

```
[pxx,f,pxxc] = periodogram(x,rectwin(length(x)),length(x),fs,...
    'ConfidenceLevel', 0.95);

plot(f,10*log10(pxx))
hold on
plot(f,10*log10(pxxc),'r-.')

xlim([85 175])
xlabel('Hz')
ylabel('dB')
title('Periodogram with 95%-Confidence Bounds')
```



The lower confidence bound in the immediate vicinity of 100 and 150 Hz is significantly above the upper confidence bound outside the vicinity of 100 and 150 Hz.

Power Estimate of Sinusoid

Estimate the power of sinusoid at a specific frequency using the `'power'` option.

Create a 100 Hz sinusoid one second in duration sampled at 1 kHz. The amplitude of the sine wave is 1.8, which equates to a power of $1.8^2/2 = 1.62$. Estimate the power using the `'power'` option.

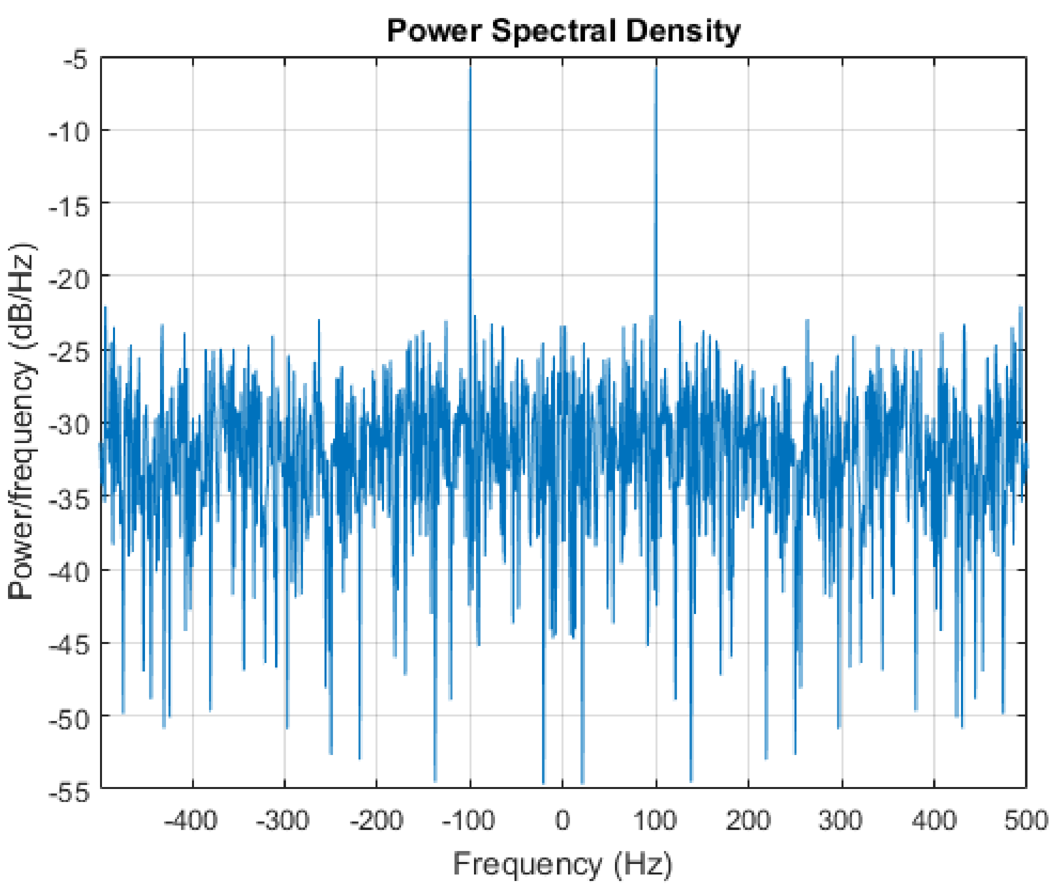
```
fs = 1000;
t = 0:1/fs:1-1/fs;
x = 1.8*cos(2*pi*100*t);
[pxx,f] = periodogram(x,hamming(length(x)),length(x),fs,'power');
[pwrest,idx] = max(pxx);
fprintf('The maximum power occurs at %3.1f Hz\n',f(idx));
fprintf('The power estimate is %2.2f\n',pwrest);
```

```
The maximum power occurs at 100.0 Hz
The power estimate is 1.62
```

DC-Centered Periodogram

Obtain the periodogram of a 100 Hz sine wave in additive $N(0,1)$ noise. The data are sampled at 1 kHz. Use the `'centered'` option to obtain the DC-centered periodogram and plot the result.

```
fs = 1000;
t = 0:0.001:1-0.001;
x = cos(2*pi*100*t)+randn(size(t));
periodogram(x,[],length(x),fs,'centered')
```



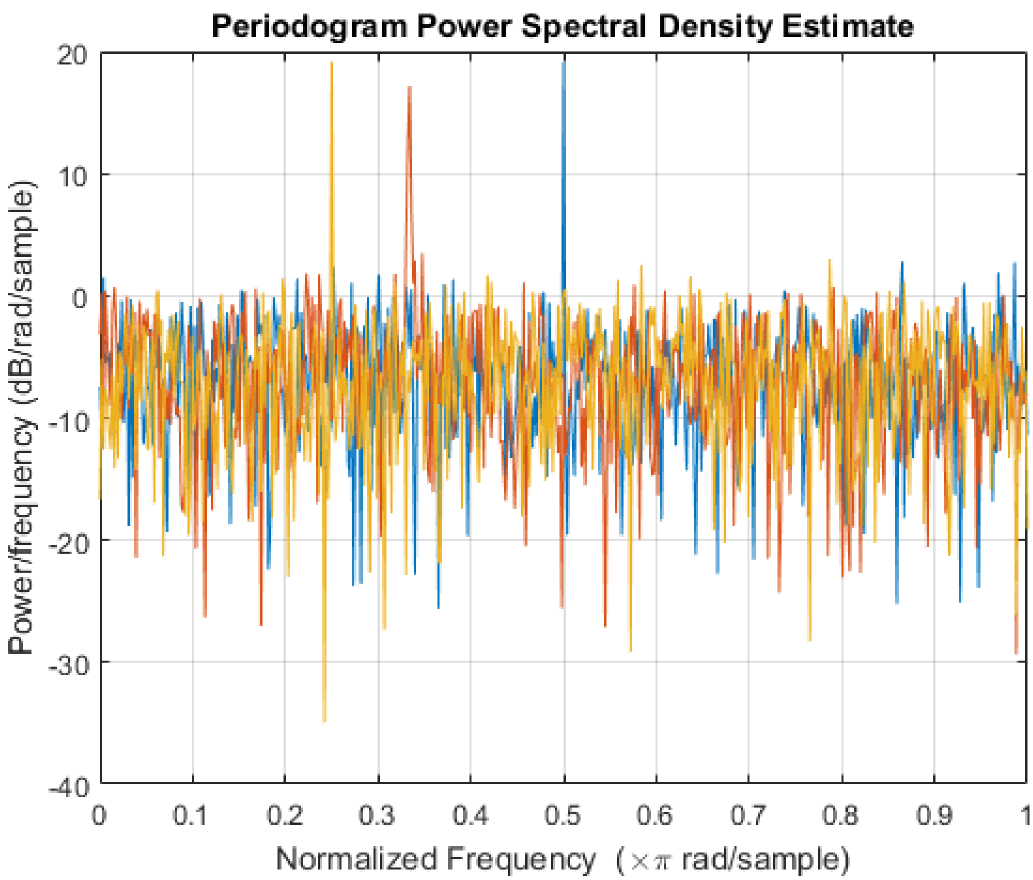
Periodogram PSD Estimate of a Multichannel Signal

Generate 1024 samples of a multichannel signal consisting of three sinusoids in additive $N(0,1)$ white Gaussian noise. The sinusoids' frequencies are $\pi/2$, $\pi/3$, and $\pi/4$ rad/sample. Estimate the PSD of the signal using the periodogram and plot it.

```
N = 1024;
n = 0:N-1;

w = pi./[2;3;4];
x = cos(w*n)' + randn(length(n),3);

periodogram(x)
```



Related Examples

- Bias and Variability in the Periodogram
- Power Spectral Density Estimates Using FFT

Input Arguments

collapse all

x — Input signal
vector | matrix

Input signal, specified as a row or column vector, or as a matrix. If x is a matrix, then its columns are treated as independent channels.

Example: `cos(pi/4*(0:159))+randn(1,160)` is a single-channel row-vector signal.

Example: `cos(pi./[4;2]*(0:159))'+randn(160,2)` is a two-channel signal.

Data Types: `single` | `double`
Complex Number Support: Yes

`window` — Window

`rectwin(length(x))` (default) | `[]` | vector

Window, specified as a row or column vector the same length as the input signal. If you specify `window` as empty, the default rectangular window is used.

Data Types: `single` | `double`

`nfft` — Number of DFT points

`max(256,2^nextpow2(length(x)))` (default) | integer | `[]`

Number of DFT points, specified as a positive integer. For a real-valued input signal, `x`, the PSD estimate, `pxx` has length (`nfft`/2 + 1) if `nfft` is even, and (`nfft` + 1)/2 if `nfft` is odd. For a complex-valued input signal,x, the PSD estimate always has length `nfft`. If `nfft` is specified as empty, the default `nfft` is used.

Data Types: `single` | `double`

`fs` — Sampling frequency

positive scalar

Sampling frequency, specified as a positive scalar. The sampling frequency is the number of samples per unit time. If the unit of time is seconds, the sampling frequency has units of hertz.

`w` — Normalized frequencies

vector

Normalized frequencies, specified as a row or column vector with at least 2 elements. Normalized frequencies are in rad/sample.

Example: `w = [pi/4 pi/2]`

Data Types: `double`

`f` — Cyclical frequencies

vector

Cyclical frequencies, specified as a row or column vector with at least 2 elements. The frequencies are in cycles per unit time. The unit time is specified by the sampling frequency, `fs`. If `fs` has units of samples/second, then `f` has units of Hz.

Example: `fs = 1000; f= [100 200]`

Data Types: `double`

`freqrange` — Frequency range for PSD estimate

`'onesided'` | `'twosided'` | `'centered'`

Frequency range for the PSD estimate, specified as a one of `'onesided'`, `'twosided'`, or `'centered'`. The default is `'onesided'` for real-valued signals and `'twosided'` for complex-valued signals. The frequency ranges corresponding to each option are

- `'onesided'` — returns the one-sided PSD estimate of a real-valued input signal, `x`. If `nfft` is even, `pxx` has length `nfft`/2 + 1 and is computed over the interval [0,π] rad/sample. If `nfft` is odd, the length of pxx is (`nfft` + 1)/2 and the interval is [0,π) rad/sample. When fs is optionally specified, the corresponding intervals are [0,fs/2] cycles/unit time and [0,fs/2) cycles/unit time for even and odd length nfft respectively.

- 'twosided' — returns the two-sided PSD estimate for either the real-valued or complex-valued input, x. In this case, pxx has length nfft and is computed over the interval [0,2π) rad/sample. When fs is optionally specified, the interval is [0,fs) cycles/unit time.
- 'centered' — returns the centered two-sided PSD estimate for either the real-valued or complex-valued input, x. In this case, pxx has length nfft and is computed over the interval (–π,π] rad/sample for even length nfft and (–π,π) rad/sample for odd length nfft. When fs is optionally specified, the corresponding intervals are (–fs/2, fs/2] cycles/unit time and (–fs/2, fs/2) cycles/unit time for even and odd length nfft respectively.

Data Types: char

spectrumtype — Power spectrum scaling

'psd' (default) | 'power'

Power spectrum scaling, specified as one of 'psd' or 'power'. Omitting the spectrumtype, or specifying 'psd', returns the power spectral density. Specifying 'power' scales each estimate of the PSD by the equivalent noise bandwidth of the window. Use the 'power' option to obtain an estimate of the power at each frequency.

Data Types: char

probability — Confidence interval for PSD estimate

0.95 (default) | scalar in the range (0,1)

Coverage probability for the true PSD, specified as a scalar in the range (0,1). The output, pxxc, contains the lower and upper bounds of the probability × 100% interval estimate for the true PSD.

Output Arguments

collapse all

pxx — PSD estimate

vector | matrix

PSD estimate, returned as a real-valued, nonnegative column vector or matrix. Each column of pxx is the PSD estimate of the corresponding column of x. The units of the PSD estimate are in squared magnitude units of the time series data per unit frequency. For example, if the input data is in volts, the PSD estimate is in units of squared volts per unit frequency. For a time series in volts, if you assume a resistance of 1 Ω and specify the sampling frequency in hertz, the PSD estimate is in watts per hertz.

Data Types: single | double

w — Normalized frequencies

vector

Normalized frequencies, returned as a real-valued column vector. If pxx is a one-sided PSD estimate, w spans the interval [0,π] if nfft is even and [0,π) if nfft is odd. If pxx is a two-sided PSD estimate, w spans the interval [0,2π). For a DC-centered PSD estimate, f spans the interval (–π,π] rad/sample for even length nfft and (–π,π) radians/sample for odd length nfft.

Data Types: double

`f` — Cyclical frequencies

vector

Cyclical frequencies, returned as a real-valued column vector. For a one-sided PSD estimate, `f` spans the interval [0,`fs`/2] when `nfft` is even and [0,`fs`/2) when `nfft` is odd. For a two-sided PSD estimate, `f` spans the interval [0,`fs`). For a DC-centered PSD estimate, `f` spans the interval (–`fs`/2, `fs`/2] cycles/unit time for even length `nfft` and (–`fs`/2, `fs`/2) cycles/unit time for odd length `nfft`.

Data Types: `double`

pxxc — Confidence bounds

matrix

Confidence bounds, returned as a matrix with real-valued elements. The row size of the matrix is equal to the length of the PSD estimate, `pxx`. `pxxc` has twice as many columns as `pxx`. Odd-numbered columns contain the lower bounds of the confidence intervals, and even-numbered columns contain the upper bounds. Thus, `pxxc(m,2*n-1)` is the lower confidence bound and `pxxc(m,2*n)` is the upper confidence bound corresponding to the estimate `pxx(m,n)`. The coverage probability of the confidence intervals is determined by the value of the `probability` input.

Data Types: `single` | `double`

More About

expand all

Periodogram

The periodogram is a nonparametric estimate of the power spectral density (PSD) of a wide-sense stationary random process. The periodogram is the Fourier transform of the biased estimate of the autocorrelation sequence. For a signal, x_n, sampled at `fs` samples per unit time, the periodogram is defined as

$$\hat{P}(f) = \frac{\Delta t}{N} \left| \sum_{n=0}^{N-1} x_n e^{-i\,2\pi\, fn} \right|^2 \quad -1/2\Delta t < f \le 1/2\Delta t$$

where Δt is the sampling interval. For a one-sided periodogram, the values at all frequencies except 0 and the Nyquist, 1/2Δt, are multiplied by 2 so that the total power is conserved.

If the frequencies are in radians/sample, the periodogram is defined as

$$\hat{P}(f) = \frac{1}{2\pi N} \left| \sum_{n=0}^{N-1} x_n e^{-i\,\omega n} \right|^2 \quad -\pi < \omega \le \pi$$

The frequency range in the preceding equations has variations depending on the value of the `freqrange` argument. See the description of `freqrange` in Input Arguments.

The integral of the true PSD, *P(f)*, over one period, *1/Δt* for cyclical frequency and 2π for normalized frequency, is equal to the variance of the wide-sense stationary random process.

$$\sigma^2 = \int_{-1/2\Delta t}^{1/2\Delta t} P(f)\, df$$

For normalized frequencies, replace the limits of integration appropriately.

Modified Periodogram

The modified periodogram multiplies the input time series by a window function. A suitable window function is nonnegative and decays to zero at the beginning and end points. Multiplying the time series by the window function *tapers* the data gradually on and off and helps to alleviate the leakage in the periodogram. See Bias and Variability in the Periodogram for an example.

If h_n is a window function, the modified periodogram is defined by

$$\hat{P}(f) = \frac{\Delta t}{N} \left| \sum_{n=0}^{N-1} h_n x_n e^{-i 2\pi fn} \right|^2 \quad -1/2\Delta t < f \le 1/2\Delta t$$

where Δt is the sampling interval.

If the frequencies are in radians/sample, the modified periodogram is defined as

$$\hat{P}(f) = \frac{1}{2\pi N} \left| \sum_{n=0}^{N-1} h_n x_n e^{-i \omega n} \right|^2 \quad -\pi < \omega \le \pi$$

The frequency range in the preceding equations has variations depending on the value of the `freqrange` argument. See the description of `freqrange` in Input Arguments.

- Nonparametric Methods

See Also

`bandpower` | `pburg` | `pcov` | `pmcov` | `pmtm` | `pwelch` | `sfdr`

Introduced before R2006a