pwelch

Welch's power spectral density estimate

Syntax

```
pxx = pwelch(x)                                          example
pxx = pwelch(x,window)                                   example
pxx = pwelch(x,window,noverlap)                          example
pxx = pwelch(x,window,noverlap,nfft)                     example
```

```
[pxx,w] = pwelch( __ )
[pxx,f] = pwelch( __ ,fs)                                example
```

```
[pxx,w] = pwelch(x,window,noverlap,w)
[pxx,f] = pwelch(x,window,noverlap,f,fs)
```

```
[ __ ] = pwelch(x,window, __ ,freqrange)                 example
[ __ ] = pwelch(x,window, __ ,spectrumtype)              example
[ __ ] = pwelch(x,window, __ ,trace)                     example
```

```
[ __ ,pxxc] = pwelch( __ ,'ConfidenceLevel',probability)  example
```

```
pwelch( __ )                                             example
```

Description

`pxx = pwelch(x)` returns the power spectral density (PSD) estimate, `pxx`, of the input signal, `x`, found using Welch's overlapped segment averaging estimator. When `x` is a vector, it is treated as a single channel. When `x` is a matrix, the PSD is computed independently for each column and stored in the corresponding column of `pxx`. If `x` is real-valued, `pxx` is a one-sided PSD estimate. If `x` is complex-valued, `pxx` is a two-sided PSD estimate. By default, `x` is divided into the longest possible sections to obtain as close to but not exceed 8 segments with 50% overlap. Each section is windowed with a Hamming window. The modified periodograms are averaged to obtain the PSD estimate. If you cannot divide the length of `x` exactly into an integer number of sections with 50% overlap, `x` is truncated accordingly. example

`pxx = pwelch(x,window)` uses the input vector or integer, `window`, to divide the signal into sections. If `window` is a vector, `pwelch` divides the signal into sections equal in length to the length of `window`. The modified periodograms are computed using the signal sections multiplied by the vector, `window`. If `window` is an integer, the signal is divided into sections of length `window`. The modified periodograms are computed using a Hamming window of length `window`. example

`pxx = pwelch(x,window,noverlap)` uses `noverlap` samples of overlap from section to section. `noverlap` must be a positive integer smaller than `window` if `window` is an integer. `noverlap` must be a positive integer less than the length of `window` if `window` is a vector. If you do not specify `noverlap`, or specify `noverlap` as empty, the default number of overlapped samples is 50% of the window length. example

`pxx = pwelch(x,window,noverlap,nfft)` specifies the number of discrete Fourier transform (DFT) points to use in the PSD estimate. The default `nfft` is the greater of 256 or the next power of 2 greater than the length of the segments. example

`[pxx,w] = pwelch( __ )` returns the normalized frequency vector, `w`. If `pxx` is a one-sided PSD estimate, `w` spans the interval [0,π] if `nfft` is even and [0,π) if `nfft` is odd. If `pxx` is a two-sided PSD estimate, `w` spans the interval [0,2π).

`[pxx,f] = pwelch( __ ,fs)` returns a frequency vector, `f`, in cycles per unit time. The sampling frequency, `fs`, is the number of samples per unit time. If the unit of time is seconds, then `f` is in cycles/sec (Hz). For real–valued signals, `f` spans the interval [0,`fs`/2] when `nfft` is even and [0,`fs`/2) when `nfft` is odd. For complex-valued signals, `f` spans the interval [0,`fs`). example

`[pxx,w] = pwelch(x,window,noverlap,w)` returns the two-sided Welch PSD estimates at the normalized frequencies specified in the vector, `w`. The vector, `w`, must contain at least 2 elements.

`[pxx,f] = pwelch(x,window,noverlap,f,fs)` returns the two-sided Welch PSD estimates at the frequencies specified in the vector, `f`. The vector, `f`, must contain at least 2 elements. The frequencies in `f` are in cycles per unit time. The sampling frequency, `fs`, is the number of samples per unit time. If the unit of time is seconds, then `f` is in cycles/sec (Hz).

`[ __ ] = pwelch(x,window, __ ,freqrange)` returns the Welch PSD estimate over the frequency range specified by `freqrange`. Valid options for `freqrange` are: `'onesided'`, `'twosided'`, or `'centered'`. example

`[ __ ] = pwelch(x,window, __ ,spectrumtype)` returns the PSD estimate if `spectrumtype` is specified as `'psd'` and returns the power spectrum if `spectrumtype` is specified as `'power'`. example

`[ __ ] = pwelch(x,window, __ ,trace)` returns the maximum-hold spectrum estimate if `trace` is specified as `'maxhold'` and returns the minimum-hold spectrum estimate if `trace` is specified as `'minhold'`. example

`[ __ ,pxxc] = pwelch( __ ,'ConfidenceLevel',probability)` returns the `probability` × 100% confidence intervals for the PSD estimate in `pxxc`. example

`pwelch( __ )` with no output arguments plots the Welch PSD estimate in the current figure window. example

Examples

collapse all

Welch Estimate Using Default Inputs

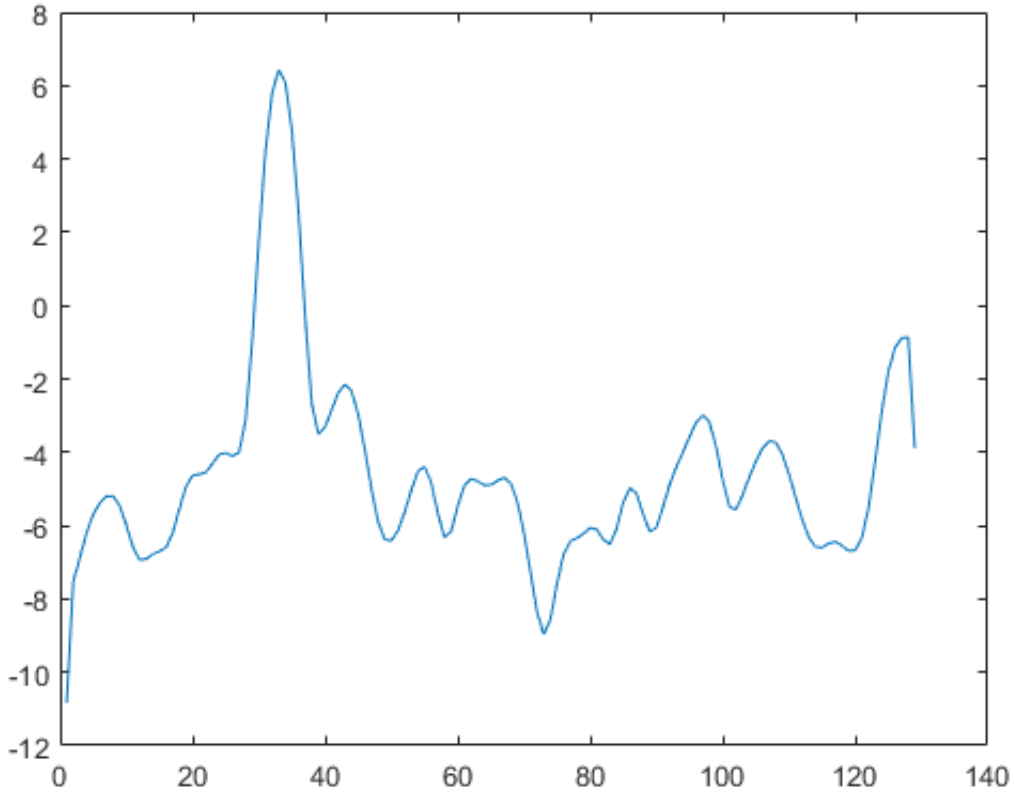
Obtain the Welch PSD estimate of an input signal consisting of a discrete-time sinusoid with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise.

Create a sine wave with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise. Reset the random number generator for reproducible results. The signal is 320 samples in length.

```
rng default

n = 0:319;
x = cos(pi/4*n)+randn(size(n));
```

Obtain the Welch PSD estimate using the default Hamming window and DFT length. The default segment length is 71 samples and the DFT length is the 256 points yielding a frequency resolution of $2\pi/256$ rad/sample. Because the signal is real-valued, the periodogram is one-sided and there are 256/2+1 points.

```
pxx = pwelch(x);
plot(10*log10(pxx))
```



Welch Estimate Using Specified Segment Length

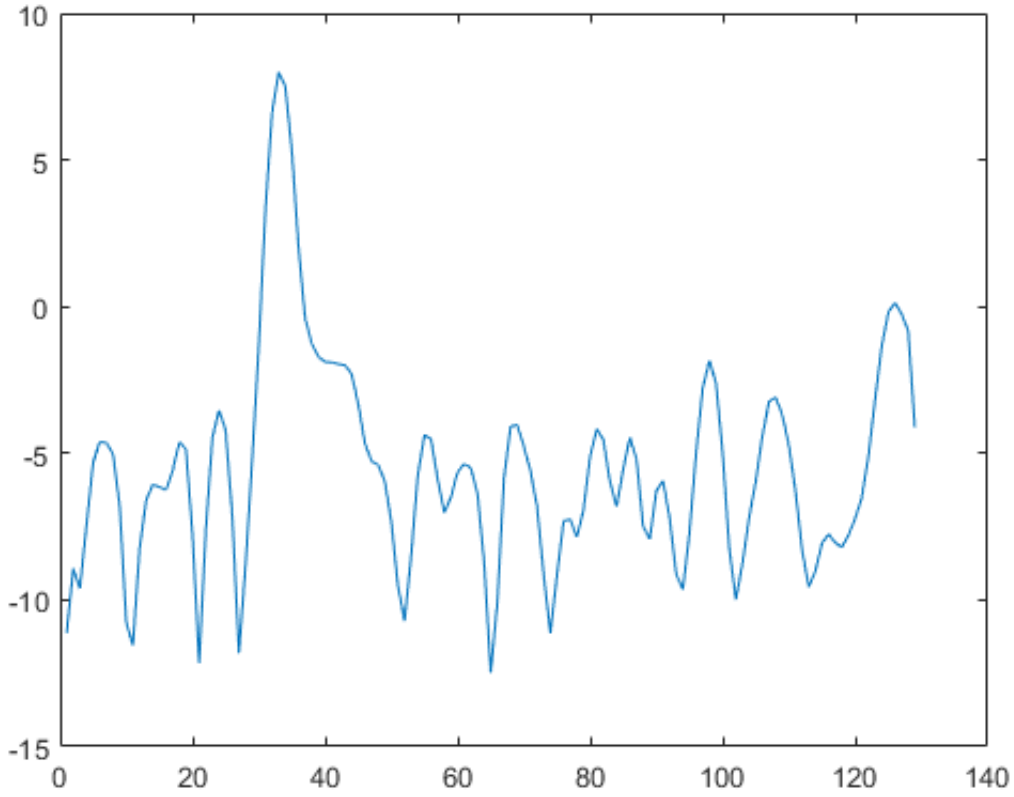
Obtain the Welch PSD estimate of an input signal consisting of a discrete-time sinusoid with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise.

Create a sine wave with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise. Reset the random number generator for reproducible results. The signal is 320 samples in length.

```
rng default

n = 0:319;
x = cos(pi/4*n)+randn(size(n));
```

Obtain the Welch PSD estimate dividing the signal into segments 100 samples in length. The signal segments are multiplied by a Hamming window 100 samples in length. The number of overlapped samples is 25. The DFT length is 256 points, yielding a frequency resolution of $2\pi/256$ rad/sample. Because the signal is real-valued, the PSD estimate is one-sided and there are 256/2+1 points.

```
segmentLength = 100;
noverlap = 25;
pxx = pwelch(x,segmentLength,noverlap);

plot(10*log10(pxx))
```



Welch Estimate Specifying Segment Overlap

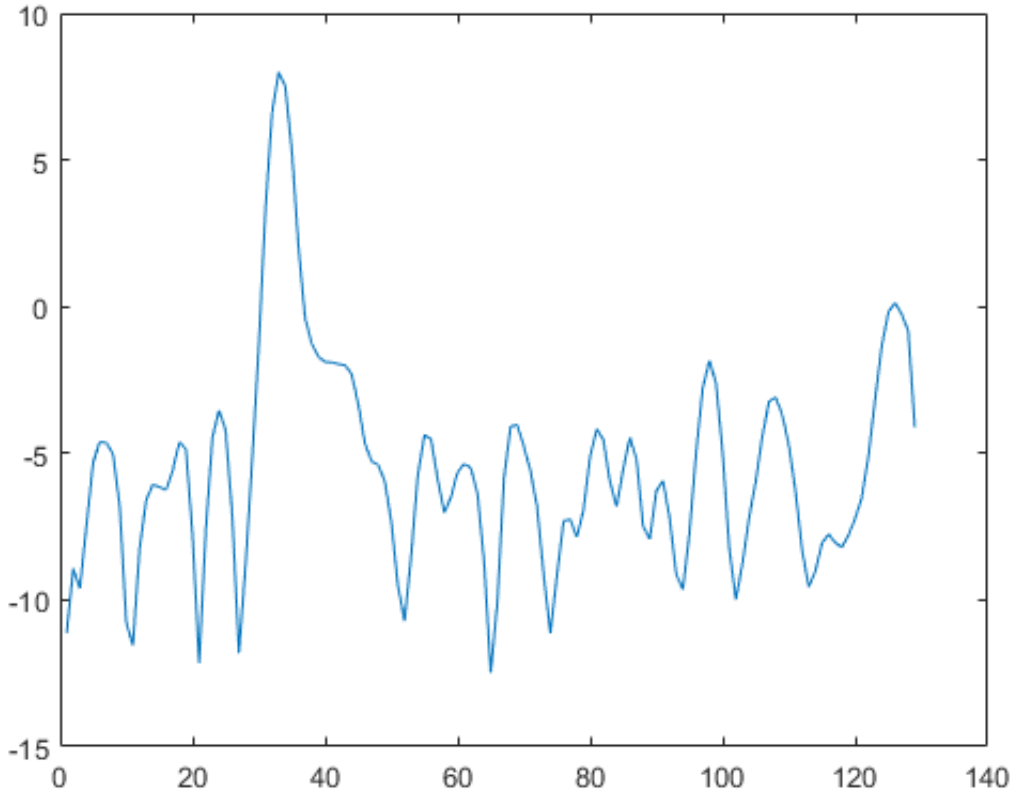
Obtain the Welch PSD estimate of an input signal consisting of a discrete-time sinusoid with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise.

Create a sine wave with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise. Reset the random number generator for reproducible results. The signal is 320 samples in length.

```
rng default

n = 0:319;
x = cos(pi/4*n)+randn(size(n));
```

Obtain the Welch PSD estimate dividing the signal into segments 100 samples in length. The signal segments are multiplied by a Hamming window 100 samples in length. The number of overlapped samples is 25. The DFT length is 256 points yielding a frequency resolution of $2\pi/256$ rad/sample. Because the signal is real-valued, the PSD estimate is one-sided and there are 256/2+1 points.

```
segmentLength = 100;
noverlap = 25;
pxx = pwelch(x,segmentLength,noverlap);

plot(10*log10(pxx))
```



Welch Estimate Using Specified DFT Length

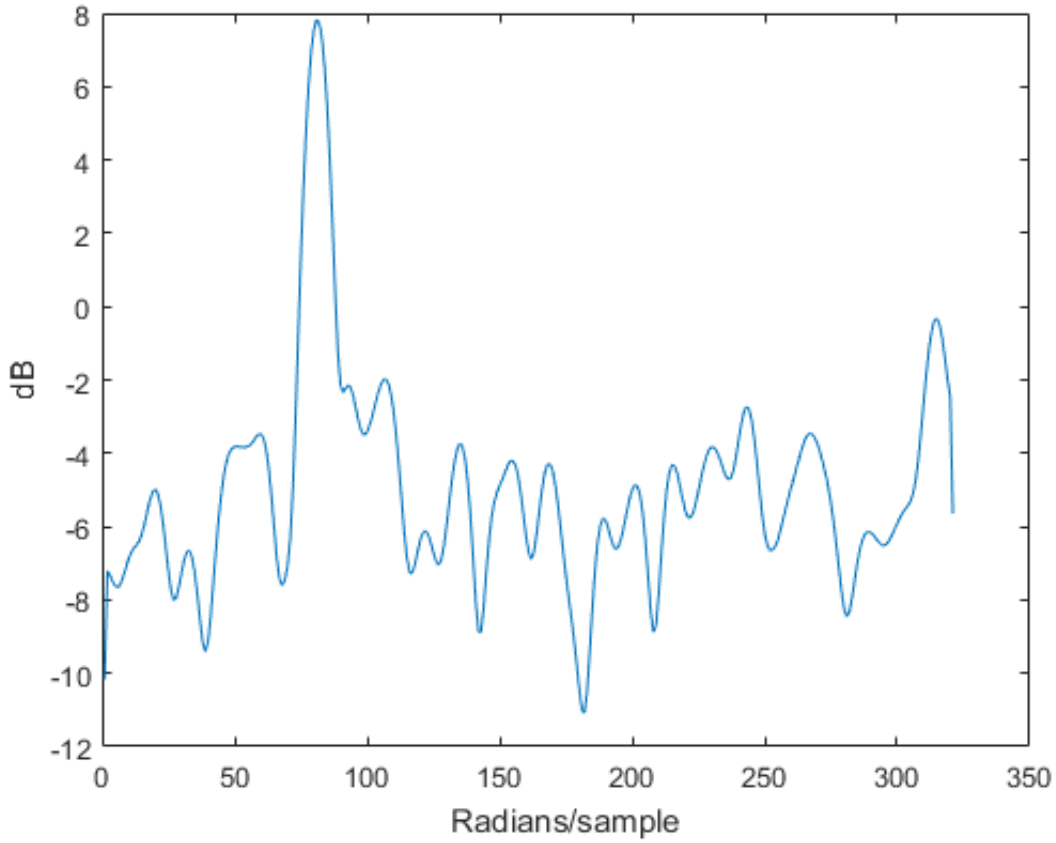
Obtain the Welch PSD estimate of an input signal consisting of a discrete-time sinusoid with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise.

Create a sine wave with an angular frequency of $\pi/4$ rad/sample with additive $N(0,1)$ white noise. Reset the random number generator for reproducible results. The signal is 320 samples in length.

```
rng default

n = 0:319;
x = cos(pi/4*n)+randn(size(n));
```

Obtain the Welch PSD estimate dividing the signal into segments 100 samples in length. Use the default overlap of 50%. Specify the DFT length to be 640 points so that the frequency of $\pi/4$ rad/sample corresponds to a DFT bin (bin 81). Because the signal is real-valued, the PSD estimate is one-sided and there are 640/2+1 points.

```
segmentLength = 100;
nfft = 640;
pxx = pwelch(x,segmentLength,[],nfft);

plot(10*log10(pxx));
xlabel('Radians/sample')
ylabel('dB')
```



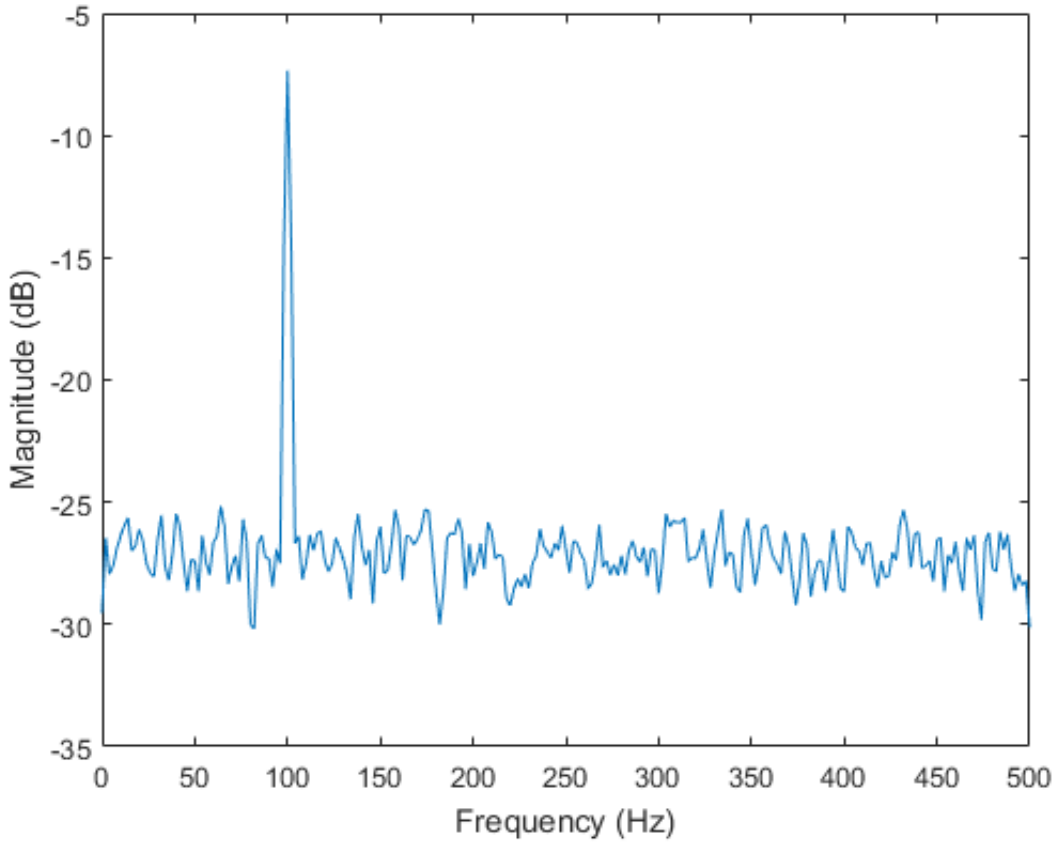
Welch PSD Estimate of Signal with Frequency in Hertz

Create a signal consisting of a 100 Hz sinusoid in additive $N(0,1)$ white noise. Reset the random number generator for reproducible results. The sample rate is 1 kHz and the signal is 5 seconds in duration.

```
rng default

fs = 1000;
t = 0:1/fs:5-1/fs;
x = cos(2*pi*100*t)+randn(size(t));
```

Obtain Welch's overlapped segment averaging PSD estimate of the preceding signal. Use a segment length of 500 samples with 300 overlapped samples. Use 500 DFT points so that 100 Hz falls directly on a DFT bin. Input the sample rate to output a vector of frequencies in Hz. Plot the result.

```
[pxx,f] = pwelch(x,500,300,500,fs);

plot(f,10*log10(pxx))

xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)')
```



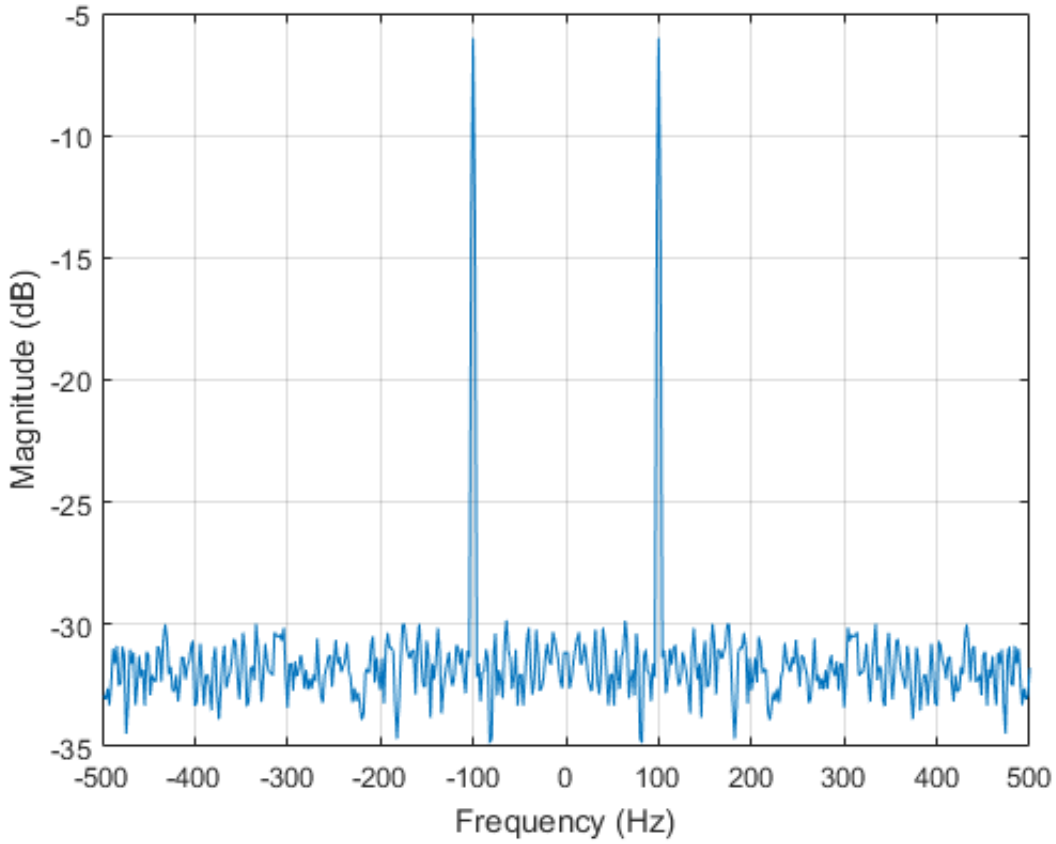
DC-Centered Power Spectrum

Create a signal consisting of a 100 Hz sinusoid in additive $N(0, 1/4)$ white noise. Reset the random number generator for reproducible results. The sample rate is 1 kHz and the signal is 5 seconds in duration.

```
rng default

fs = 1000;
t = 0:1/fs:5-1/fs;

noisevar = 1/4;
x = cos(2*pi*100*t)+sqrt(noisevar)*randn(size(t));
```

Obtain the DC-centered power spectrum using Welch's method. Use a segment length of 500 samples with 300 overlapped samples and a DFT length of 500 points. Plot the result.

```
[pxx,f] = pwelch(x,500,300,500,fs,'centered','power');

plot(f,10*log10(pxx))
xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)')
grid
```



You see that the power at -100 and 100 Hz is close to the expected power of 1/4 for a real-valued sine wave with an amplitude of 1. The deviation from 1/4 is due to the effect of the additive noise.

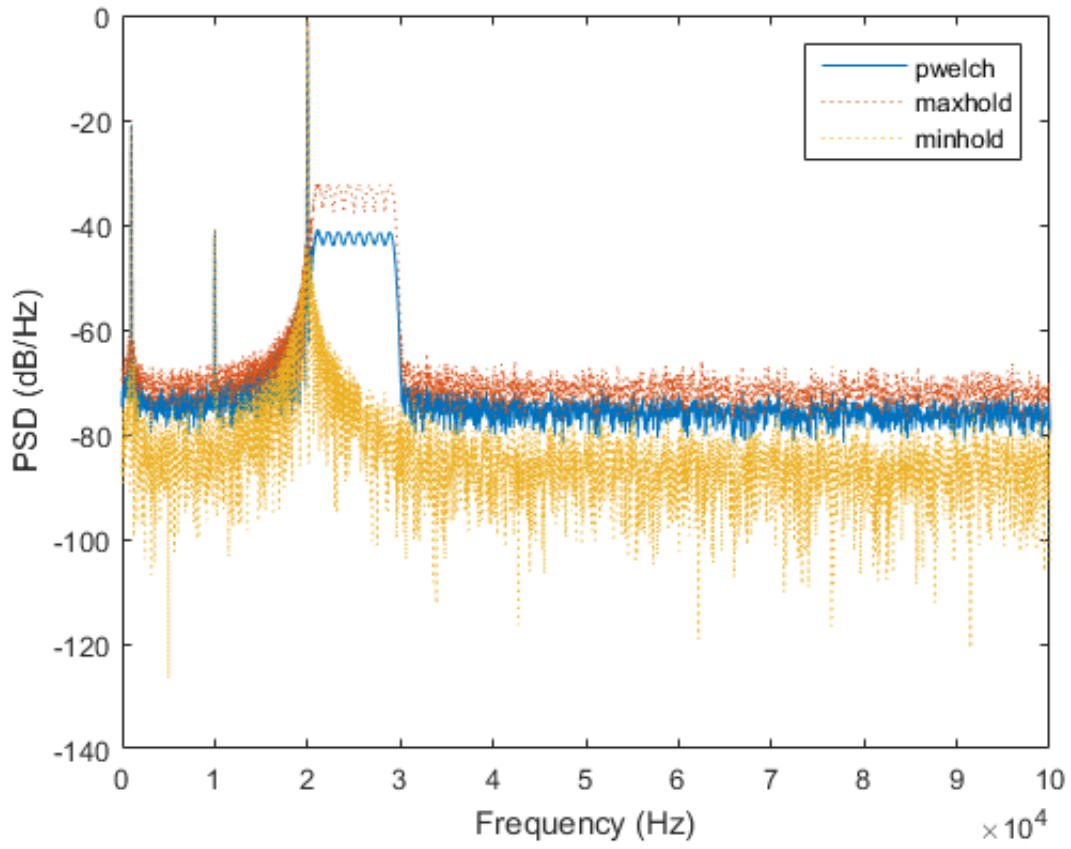
Maximum-Hold and Minimum-Hold Spectra

Create a signal consisting of three noisy sinusoids and a chirp, sampled at 200 kHz for 0.1 second. The frequencies of the sinusoids are 1 kHz, 10 kHz, and 20 kHz. The sinusoids have different amplitudes and noise levels. The noiseless chirp has a frequency that starts at 20 kHz and increases linearly to 30 kHz during the sampling.

```
Fs = 200e3;
Fc = [1 10 20]'*1e3;
Ns = 0.1*Fs;

t = (0:Ns-1)/Fs;
x = [1 1/10 10]*sin(2*pi*Fc*t)+[1/200 1/2000 1/20]*randn(3,Ns);
x = x+chirp(t,20e3,t(end),30e3);
```

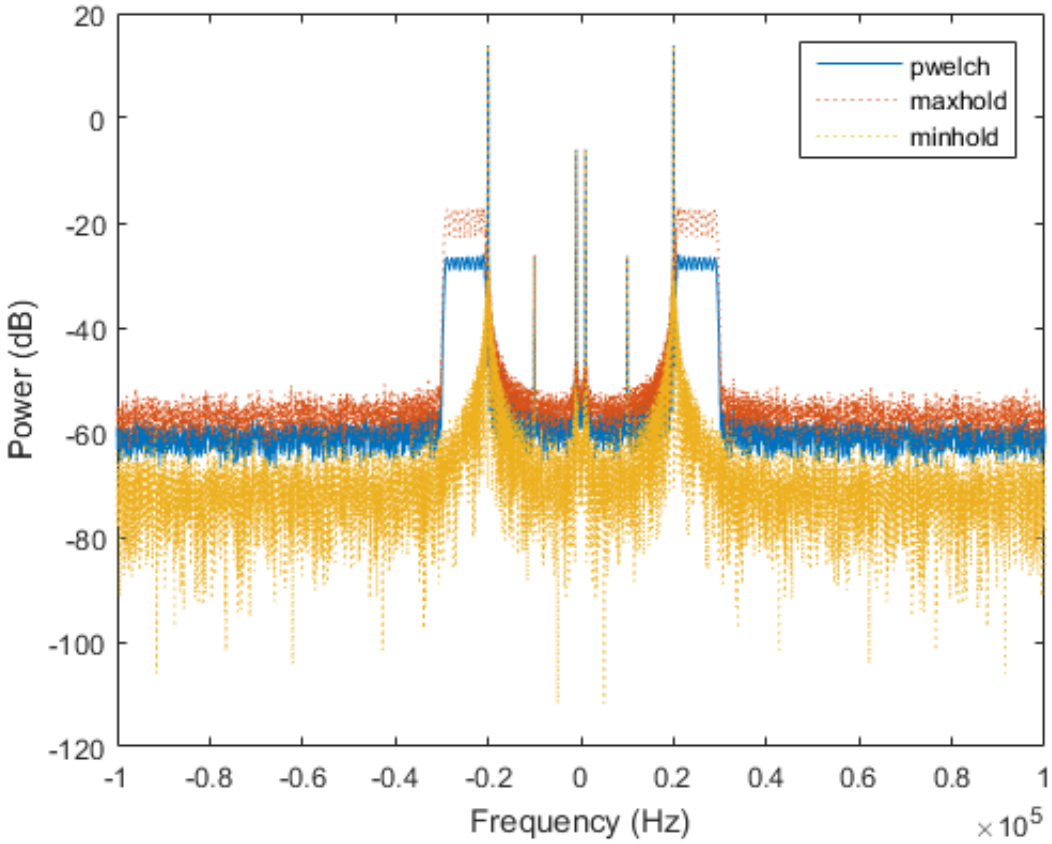
Compute the Welch PSD estimate and the maximum-hold and minimum-hold spectra of the signal. Plot the results.

```
[pxx,f] = pwelch(x,[],[],[],Fs);
pmax = pwelch(x,[],[],[],Fs,'maxhold');
pmin = pwelch(x,[],[],[],Fs,'minhold');

plot(f,pow2db(pxx))
hold on
plot(f,pow2db([pmax pmin]),':')
hold off
xlabel('Frequency (Hz)')
ylabel('PSD (dB/Hz)')
legend('pwelch','maxhold','minhold')
```



Repeat the procedure, this time computing centered power spectrum estimates.

```
[pxx,f] = pwelch(x,[],[],[],Fs,'centered','power');
pmax = pwelch(x,[],[],[],Fs,'maxhold','centered','power');
pmin = pwelch(x,[],[],[],Fs,'minhold','centered','power');

plot(f,pow2db(pxx))
hold on
plot(f,pow2db([pmax pmin]),':')
hold off
xlabel('Frequency (Hz)')
ylabel('Power (dB)')
legend('pwelch','maxhold','minhold')
```



Upper and Lower 95%-Confidence Bounds

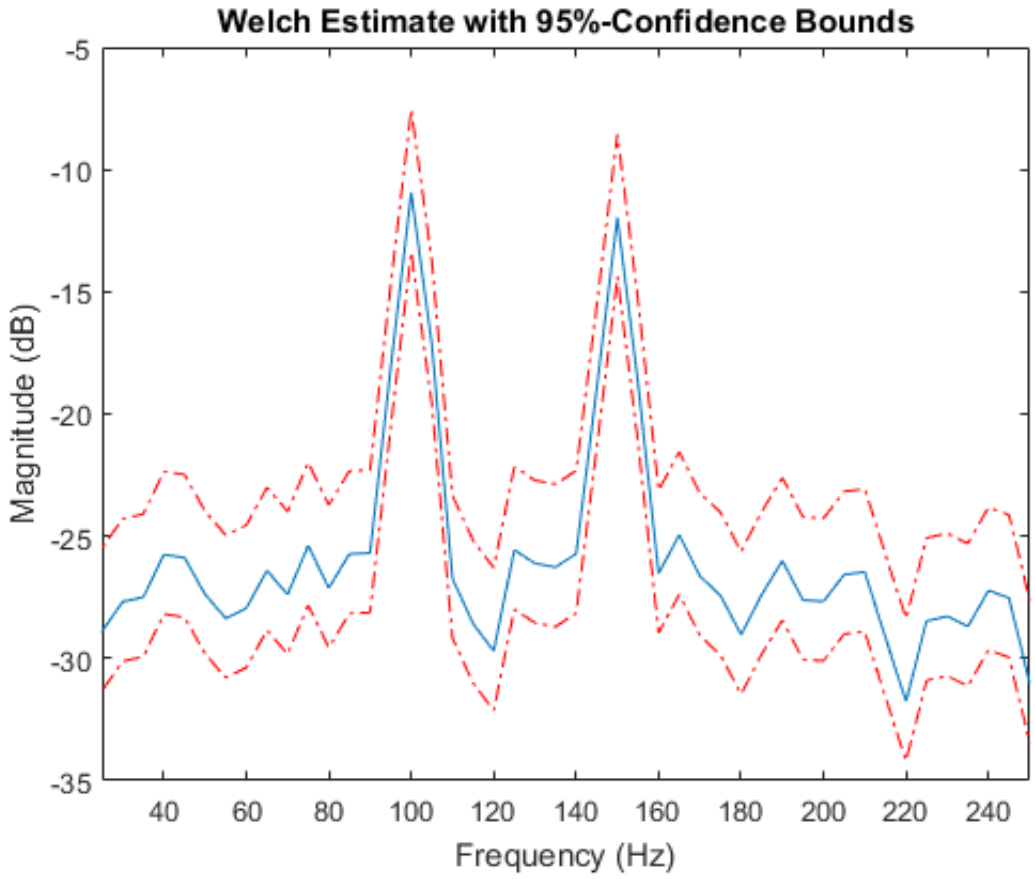
This example illustrates the use of confidence bounds with Welch's overlapped segment averaging (WOSA) PSD estimate. While not a necessary condition for statistical significance, frequencies in Welch's estimate where the lower confidence bound exceeds the upper confidence bound for surrounding PSD estimates clearly indicate significant oscillations in the time series.

Create a signal consisting of the superposition of 100 Hz and 150 Hz sine waves in additive white $N(0,1)$ noise. The amplitude of the two sine waves is 1. The sample rate is 1 kHz. Reset the random number generator for reproducible results.

```
rng default
t = 0:0.001:1-0.001;
fs = 1000;
x = cos(2*pi*100*t)+sin(2*pi*150*t)+randn(size(t));
```

Obtain the WOSA estimate with 95%-confidence bounds. Set the segment length equal to 200 and overlap the segments by 50% (100 samples). Plot the WOSA PSD estimate along with the confidence interval and zoom in on the frequency region of interest near 100 and 150 Hz.

```
L = 200;
noverlap = 100;
[pxx,f,pxxc] = pwelch(x,hamming(L),noverlap,200,fs,...
    'ConfidenceLevel',0.95);

plot(f,10*log10(pxx))
hold on
plot(f,10*log10(pxxc),'r-.')

xlim([25 250])
xlabel('Frequency (Hz)')
ylabel('Magnitude (dB)');
title('Welch Estimate with 95%-Confidence Bounds');
```

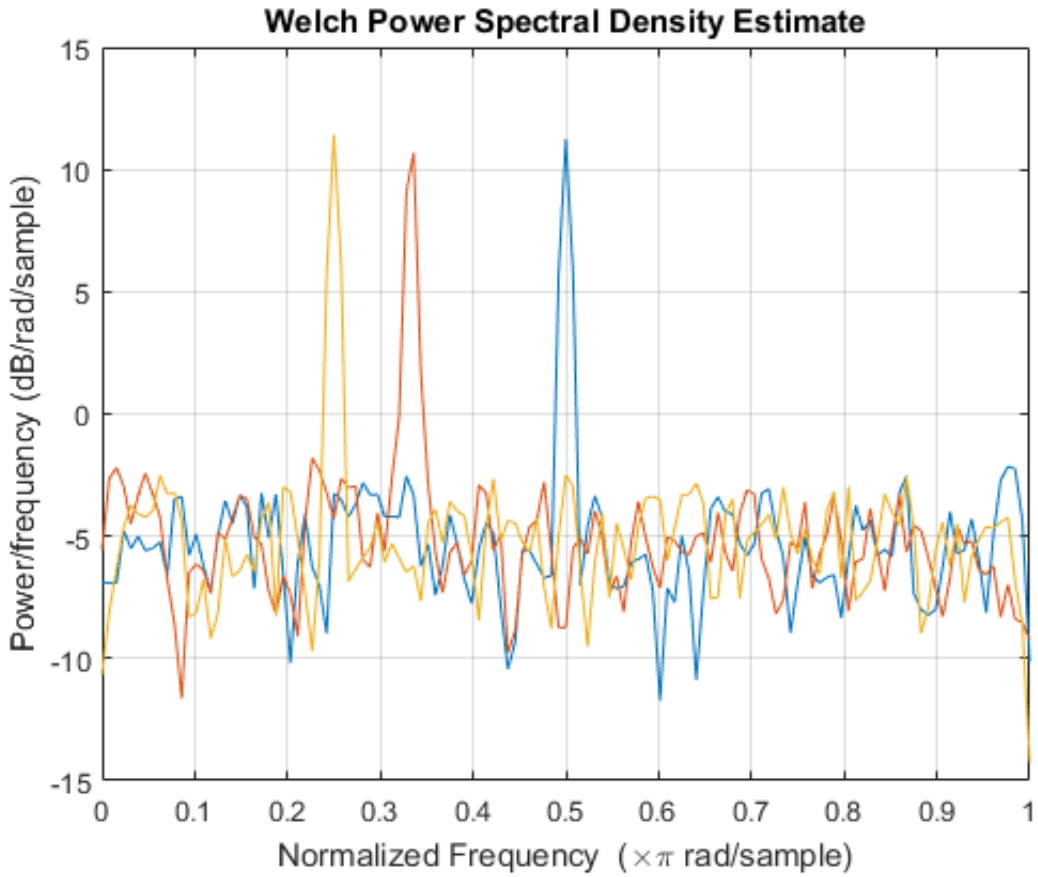


The lower confidence bound in the immediate vicinity of 100 and 150 Hz is significantly above the upper confidence bound outside the vicinity of 100 and 150 Hz.

Welch PSD Estimate of a Multichannel Signal

Generate 1024 samples of a multichannel signal consisting of three sinusoids in additive $N(0,1)$ white Gaussian noise. The sinusoids' frequencies are $\pi/2$, $\pi/3$, and $\pi/4$ rad/sample. Estimate the PSD of the signal using Welch's method and plot it.

```
N = 1024;
n = 0:N-1;

w = pi./[2;3;4];
x = cos(w*n)' + randn(length(n),3);

pwelch(x)
```



Related Examples

- Bias and Variability in the Periodogram

Input Arguments

collapse all

x — Input signal
vector | matrix

Input signal, specified as a row or column vector, or as a matrix. If x is a matrix, then its columns are treated as independent channels.

Example: `cos(pi/4*(0:159))+randn(1,160)` is a single-channel row-vector signal.

Example: `cos(pi./[4;2]*(0:159))'+randn(160,2)` is a two-channel signal.

Data Types: `single` | `double`
Complex Number Support: Yes

`window` — Window

integer | vector | []

Window, specified as a row or column vector or an integer. If `window` is a vector, `pwelch` divides `x` into overlapping sections of length equal to the length of `window`, and then multiplies each signal section with the vector specified in `window`. If `window` is an integer, `pwelch` is divided into sections of length equal to the integer value, and a Hamming window of equal length is used. If the length of `x` cannot be divided exactly into an integer number of sections with `noverlap` number of overlapping samples, `x` is truncated accordingly. If you specify `window` as empty, the default Hamming window is used to obtain eight sections of `x` with `noverlap` overlapping samples.

Data Types: `single` | `double`

`noverlap` — Number of overlapped samples

positive integer | []

Number of overlapped samples, specified as a positive integer smaller than the length of `window`. If you omit `noverlap` or specify `noverlap` as empty, a value is used to obtain 50% overlap between segments.

`nfft` — Number of DFT points

`max(256,2^nextpow2(length(window)))` (default) | integer | []

Number of DFT points, specified as a positive integer. For a real-valued input signal, `x`, the PSD estimate, `pxx` has length (`nfft`/2 + 1) if `nfft` is even, and (`nfft` + 1)/2 if `nfft` is odd. For a complex-valued input signal,`x`, the PSD estimate always has length `nfft`. If `nfft` is specified as empty, the default `nfft` is used.

If `nfft` is greater than the segment length, the data is zero-padded. If `nfft` is less than the segment length, the segment is wrapped using `datawrap` to make the length equal to `nfft`.

Data Types: `single` | `double`

`fs` — Sampling frequency

positive scalar

Sampling frequency, specified as a positive scalar. The sampling frequency is the number of samples per unit time. If the unit of time is seconds, the sampling frequency has units of hertz.

`w` — Normalized frequencies

vector

Normalized frequencies, specified as a row or column vector with at least 2 elements. Normalized frequencies are in rad/sample.

Example: `w = [pi/4 pi/2]`

Data Types: `double`

`f` — Cyclical frequencies

vector

Cyclical frequencies, specified as a row or column vector with at least 2 elements. The frequencies are in cycles per unit time. The unit time is specified by the sampling frequency, `fs`. If `fs` has units of samples/second, then `f` has units of Hz.

Example: `fs = 1000; f= [100 200]`

Data Types: `double`

`freqrange` — Frequency range for PSD estimate

`'onesided'` | `'twosided'` | `'centered'`

Frequency range for the PSD estimate, specified as a one of `'onesided'`, `'twosided'`, or `'centered'`. The default is `'onesided'` for real-valued signals and `'twosided'` for complex-valued signals. The frequency ranges corresponding to each option are

- `'onesided'` — returns the one-sided PSD estimate of a real-valued input signal, `x`. If `nfft` is even, `pxx` has length `nfft`/2 + 1 and is computed over the interval [0,π] rad/sample. If `nfft` is odd, the length of `pxx` is (`nfft` + 1)/2 and the interval is [0,π) rad/sample. When `fs` is optionally specified, the corresponding intervals are [0,`fs`/2] cycles/unit time and [0,`fs`/2) cycles/unit time for even and odd length `nfft` respectively.
- `'twosided'` — returns the two-sided PSD estimate for either the real-valued or complex-valued input, `x`. In this case, `pxx` has length `nfft` and is computed over the interval [0,2π) rad/sample. When `fs` is optionally specified, the interval is [0,`fs`) cycles/unit time.
- `'centered'` — returns the centered two-sided PSD estimate for either the real-valued or complex-valued input, `x`. In this case, `pxx` has length `nfft` and is computed over the interval (–π,π] rad/sample for even length `nfft` and (–π,π) rad/sample for odd length `nfft`. When `fs` is optionally specified, the corresponding intervals are (–`fs`/2, `fs`/2] cycles/unit time and (–`fs`/2, `fs`/2) cycles/unit time for even and odd length `nfft` respectively.

Data Types: `char`

`spectrumtype` — Power spectrum scaling

`'psd'` (default) | `'power'`

Power spectrum scaling, specified as one of `'psd'` or `'power'`. Omitting the `spectrumtype`, or specifying `'psd'`, returns the power spectral density. Specifying `'power'` scales each estimate of the PSD by the equivalent noise bandwidth of the window. Use the `'power'` option to obtain an estimate of the power at each frequency.

Data Types: `char`

`trace` — Trace mode

`'mean'` (default) | `'maxhold'` | `'minhold'`

Trace mode, specified as one of `'mean'`, `'maxhold'`, or `'minhold'`. The default is `'mean'`.

- `'mean'` — returns the Welch spectrum estimate of each input channel. `pwelch` computes the Welch spectrum estimate at each frequency bin by averaging the power spectrum estimates of all the segments.
- `'maxhold'` — returns the maximum-hold spectrum of each input channel. `pwelch` computes the maximum-hold spectrum at each frequency bin by keeping the maximum value among the power spectrum estimates of all the segments.
- `'minhold'` — returns the minimum-hold spectrum of each input channel. `pwelch` computes the minimum-hold spectrum at each frequency bin by keeping the minimum value among the power spectrum estimates of all the segments.

`probability` — Confidence interval for PSD estimate

0.95 (default) | scalar in the range (0,1)

Coverage probability for the true PSD, specified as a scalar in the range (0,1). The output, `pxxc`, contains the lower and upper bounds of the `probability` × 100% interval estimate for the true PSD.

Output Arguments

collapse all

`pxx` — PSD estimate
vector | matrix

PSD estimate, returned as a real-valued, nonnegative column vector or matrix. Each column of `pxx` is the PSD estimate of the corresponding column of `x`. The units of the PSD estimate are in squared magnitude units of the time series data per unit frequency. For example, if the input data is in volts, the PSD estimate is in units of squared volts per unit frequency. For a time series in volts, if you assume a resistance of 1 Ω and specify the sampling frequency in hertz, the PSD estimate is in watts per hertz.

Data Types: `single` | `double`

`w` — Normalized frequencies
vector

Normalized frequencies, returned as a real-valued column vector. If `pxx` is a one-sided PSD estimate, `w` spans the interval [0,π] if `nfft` is even and [0,π) if `nfft` is odd. If `pxx` is a two-sided PSD estimate, `w` spans the interval [0,2π). For a DC-centered PSD estimate, `f` spans the interval (–π,π] rad/sample for even length `nfft` and (–π,π) radians/sample for odd length `nfft`.

Data Types: `double`

`f` — Cyclical frequencies
vector

Cyclical frequencies, returned as a real-valued column vector. For a one-sided PSD estimate, `f` spans the interval [0,`fs`/2] when `nfft` is even and [0,`fs`/2) when `nfft` is odd. For a two-sided PSD estimate, `f` spans the interval [0,`fs`). For a DC-centered PSD estimate, `f` spans the interval (–`fs`/2, `fs`/2] cycles/unit time for even length `nfft` and (–`fs`/2, `fs`/2) cycles/unit time for odd length `nfft`.

Data Types: `double`

`pxxc` — Confidence bounds
matrix

Confidence bounds, returned as a matrix with real-valued elements. The row size of the matrix is equal to the length of the PSD estimate, `pxx`. `pxxc` has twice as many columns as `pxx`. Odd-numbered columns contain the lower bounds of the confidence intervals, and even-numbered columns contain the upper bounds. Thus, `pxxc(m,2*n-1)` is the lower confidence bound and `pxxc(m,2*n)` is the upper confidence bound corresponding to the estimate `pxx(m,n)`. The coverage probability of the confidence intervals is determined by the value of the `probability` input.

Data Types: `single` | `double`

More About

expand all

Welch's Overlapped Segment Averaging Spectral Estimation

The periodogram is not a consistent estimator of the true power spectral density of a wide-sense stationary process. Welch's technique to reduce the variance of the periodogram breaks the time series into segments, usually overlapping.

Welch's method computes a modified periodogram for each segment and then averages these estimates to produce the estimate of the power spectral density. Because the process is wide-sense stationary and Welch's method uses PSD estimates of different segments of the time series, the modified periodograms represent approximately uncorrelated estimates of the true PSD and averaging reduces the variability.

The segments are typically multiplied by a window function, such as a Hamming window, so that Welch's method amounts to averaging modified periodograms. Because the segments usually overlap, data values at the beginning and end of the segment tapered by the window in one segment, occur away from the ends of adjacent segments. This guards against the loss of information caused by windowing.

- Spectral Analysis

See Also

`periodogram` | `pmtm`

Introduced before R2006a