PWARI-G Soliton Atom System: Complete Field-Theoretic Model

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Abstract

We present a fully wave-based model of atomic structure in which particles, charge, spin, and quantum statistics emerge naturally from nonlinear field interactions. The PWARI-G framework (Photon Wave Absorption and Reshaping Interpretation with Gravity) eliminates the need for point particles and collapse-based quantum mechanics. Instead, atoms are composed of breathing scalar solitons, topological twist fields, and gauge/spinor dynamics—all coupled to a gravitational redshift background. We derive the full set of field equations from a Lagrangian, unify U(1) and SU(2) gauge interactions, couple the system to curved spacetime through Einstein's equations, and reproduce fermionic behavior via deterministic multi-shell structure. The model resolves singularities, conserves energy dynamically, and provides testable predictions spanning spectroscopy, redshift, and gravitational wave emission.

1 Lagrangian and Field Content

The PWARI-G model describes matter using a fully nonlinear wave Lagrangian composed of scalar, twist, gauge, and gravitational components. The four key fields are:

- $\phi(x,t)$: Breathing scalar field (real), modeling energy density and localization
- $\theta(x,t)$: Phase/twist field (real), encoding angular structure and spin topology
- $A_0(x,t)$: Gauge potential (U(1)), coupling to the twist to simulate charge
- $A(x,t)$: Gravitational redshift scalar, controlling time dilation from energy concentration

1.1 Full PWARI-G Lagrangian Density

$$\mathcal{L} = \frac{1}{2A}(\partial_t\phi)^2 - \frac{A}{2}(\nabla\phi)^2 - \frac{\lambda}{4}\phi^4 - \frac{1}{2}\phi^2\left[(\partial_t\theta - A_0)^2 - (\nabla\theta)^2\right] + \frac{1}{2}(\nabla A_0)^2 \tag{1}$$

This Lagrangian includes:

- Time dilation from redshift A
- Nonlinear self-interaction of the scalar core ($\lambda\phi^4$)
- Coupling between twist (θ) and gauge potential (A_0)
- Electrostatic energy from ∇A_0

1.2 Master Field Equation (Unified Evolution)

From the Euler-Lagrange equations, a master wave equation emerges:

$$\frac{1}{A}\partial_t^2\phi - A\nabla^2\phi + \lambda\phi^3 + \phi(\partial_t\theta - A_0)^2 + \phi(\nabla\theta)^2 = 0 \tag{2}$$

This equation governs the breathing dynamics of the scalar field, including:

- Oscillation driven by the potential well
- Redshifted propagation from curved background
- Angular feedback from twist tension
- Local coupling to gauge field energy

2 Gravitational Coupling

PWARI-G introduces gravity through a scalar redshift field $A(x,t)$, derived from energy density and representing time dilation. While not a full dynamical metric in its original form, PWARI-G extends to curved spacetime via coupling to general relativity using a stress-energy tensor constructed from the breathing scalar, twist phase, gauge fields, and spinors.

2.1 Stress-Energy Tensor

From the Lagrangian density, the stress-energy tensor $T^{\mu\nu}$ is derived using:

$$T^{\mu\nu} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\partial^\nu\psi - g^{\mu\nu}\mathcal{L} \tag{3}$$

This includes contributions from:

- Scalar breathing field ϕ
- Twist phase θ
- Gauge field A_0
- (If included) Dirac spinor ψ

The full energy-momentum content acts as the source for curvature in Einstein's equations:

$$G^{\mu\nu} = 8\pi G\left(T^{\phi\,\mu\nu} + T^{\theta\,\mu\nu} + T^{A_0\,\mu\nu} + T^{\psi\,\mu\nu}\right) \tag{4}$$

2.2 Redshift as Scalar Metric Field

In the weak-field regime or as a reduced model, gravitational effects are encoded through a scalar redshift function $A(x, t)$ obeying:

$$\partial_t A = -\frac{1}{\tau}\left(A - e^{-\alpha\rho}\right) \tag{5}$$

where:

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\phi^2\dot{\theta}^2 + \frac{1}{2}\phi^2(\nabla\theta)^2 + V(\phi) \tag{6}$$

This field lags behind the matter distribution and creates a form of gravitational memory or drag.

2.3 Post-Newtonian and Wave Regime Behavior

Simulations and derivations confirm that:

- In the static limit, $A(x)$ produces a gravitational potential matching Newtonian $\Phi = -GM/r$
- In curved spacetime, solitons reproduce redshift and time dilation consistent with general relativity
- In time-dependent systems, lagged $A(x, t)$ results in energy dispersion and effective gravitational drag
- The full Einstein equation coupling (used in PWARI-G extensions) recovers Schwarzschild behavior at large radius, and avoids singularities at the core due to wave smoothing

Thus, PWARI-G reproduces gravitational effects both in the Newtonian and post-Newtonian regimes and admits full wave dynamics when coupled to a dynamic metric $g_{\mu\nu}$.

3 Energy Conservation and Shell Emission

PWARI-G conserves total energy through its Lagrangian formulation. Each component—scalar, twist, gauge, and gravitational fields—contributes to the overall stress-energy budget, and energy transfers between these components are governed by local dynamics.

3.1 Energy Terms

- Core Energy: Stored in the breathing field and self-interaction of ϕ
- Twist/Shell Energy: Stored in the angular gradients and oscillations of θ
- Gauge Energy: From gradients of A_0, associated with electrostatic field energy
- Gravitational Redshift Energy: Encoded in deviations of A from unity; quantifies curved background

Representative integrals:

$$E_{\text{core}} = \int \left[\frac{1}{2A}(\partial_t\phi)^2 + \frac{A}{2}(\nabla\phi)^2 + V(\phi)\right] d^3x \tag{7}$$

$$E_{\text{twist}} = \int \left[\frac{1}{2}\phi^2(\partial_t\theta)^2 + \frac{1}{2}\phi^2(\nabla\theta)^2\right] d^3x \tag{8}$$

$$E_{\text{gauge}} = \int \frac{1}{2}(\nabla A_0)^2 d^3x \tag{9}$$

$$E_{\text{gravity}} = \int \frac{1}{\alpha}(1-A)^2 d^3x \tag{10}$$

3.2 Shell Snapping and Twist Threshold

As twist energy accumulates in a localized region, the shell reaches a maximum tension it can contain:

$$\frac{1}{2}\phi^2(\nabla\theta)^2 \geq \sigma_{\max} \tag{11}$$

where:

$$\sigma_{\max} \sim \frac{2\pi^2\phi_0^2}{R^2} \tag{12}$$

At this point, energy rapidly disperses outward into a propagating shell—a solitonic pulse containing both ϕ and θ. This is a physical, nonlinear relaxation mechanism.

3.3 Nonlinear Stiffness Regulation

The twist field resists unbounded growth due to its nonlinear coupling:

$$\tau_{\text{twist}} = \frac{\nabla\theta}{1 + \alpha_{\text{twist}}(\nabla\theta)^2} \tag{13}$$

When $\alpha_{\text{twist}}(\nabla\theta)^2 \gg 1$, twist energy saturates and becomes prone to releasing stored energy as radiation (the shell).

3.4 Prevention of Infinite Shell Generation

PWARI-G enforces energy bounds through:

- Finite support of ϕ: energy is localized by construction
- Dissipation of twist into shells: each emission reduces available energy
- Redshift feedback: gravitational lag slows further instability

Once a shell has been emitted, the core relaxes and may retain insufficient energy to emit another. This creates a natural equilibrium state with one stable shell per soliton core under most configurations.

4 Soliton Tails and Redshift Memory

PWARI-G incorporates a form of gravitational memory via delayed redshift response. As solitons move, the redshift field $A(x,t)$—or the broader gravitational metric—is updated non-instantaneously, leaving behind dynamically inert curvature structures known as gravitational tails.

4.1 Formation of Gravitational Tails

The redshift field A is governed by either:

- a diffusion-relaxation equation in reduced models:

$$\partial_t A = -\frac{1}{\tau}(A - A_{\text{target}}) \quad \text{with} \quad A_{\text{target}} = e^{-\alpha\rho} \tag{14}$$

- or a delayed action formulation from your gravitational lag Lagrangian:

$$\mathcal{L}_{\text{lag}} = -\alpha\phi(x^\mu)\,\Box\phi(x^\mu - \Delta x^\mu) \tag{15}$$

This term introduces time-retarded curvature that cannot respond instantly to field motion. As a soliton moves through space, it leaves behind a trail of redshift curvature that does not overlap with ϕ, forming a gravitational tail.

4.2 Energy Dissipation and Inert Curvature

These tail regions:

- No longer influence ϕ directly (since energy density is zero there)
- Store redshift curvature as a "memory" of the soliton's prior motion
- Do not contribute to ongoing shell or twist dynamics
- Cannot be reabsorbed unless a new soliton traverses the same location

This leads to effective energy dissipation into space, without radiative loss.

4.3 Lagged Redshift and Drag Mechanism

The delayed redshift field resists soliton motion. The field tries to reshape itself to match the moving ϕ but lags behind due to the $\Box\phi(x^\mu - \Delta x^\mu)$ term.

This produces a gravitational analog of drag or inertial resistance, manifesting as:

- A rise in $\dot{\phi}^2$ during soliton motion
- Local increase in apparent kinetic energy
- Non-radiative but irreversible energy transfer to lagging curvature

4.4 Quantified Energy Loss from Tail Regions

While not directly radiative, energy stored in the tail regions can be estimated as:

$$E_{\text{tail}} = \int_{\text{tail}} \frac{1}{\alpha}(1 - A)^2 d^3x \tag{16}$$

Where $A < 1$ due to past energy density, even though $\phi \approx 0$ locally. This value represents curvature not participating in active dynamics—a form of geometric entropy or gravitational irreversibility.

5 Quantum Interpretation

PWARI-G is a classical field theory in formulation, yet it gives rise to structures that resemble quantum particles through the nonlinear interplay of breathing, twist, and gauge dynamics. Several routes to quantum correspondence are built into its architecture.

5.1 Topological Twist → Spin-$\frac{1}{2}$ Analogue

The twist field $\theta(x)$ defines a topological phase wrapped around the soliton structure. When the twist completes a 2π cycle around a spatial loop, it resembles a winding number of one—analogous to spin-$\frac{1}{2}$ behavior.

Systems where θ must rotate 4π to return to their original state emulate the two-valuedness of spinors.

The field $\phi e^{i\theta}$ becomes a complex scalar with nontrivial monodromy, which under rotation transforms similarly to a spin-$\frac{1}{2}$ object.

This emergent spin arises without invoking a spinor field, enabling what you've termed "spin without spinors."

5.2 Path Integral Over Soliton Sectors

The action functional for the full PWARI-G system is:

$$S[\phi, \theta, A_0, A] = \int d^4x \, \mathcal{L} \tag{17}$$

A quantum version of PWARI-G can be constructed using a semiclassical path integral over topologically distinct field configurations:

$$Z = \int \mathcal{D}\phi \, \mathcal{D}\theta \, \mathcal{D}A_0 \, e^{iS[\phi,\theta,A_0]} \tag{18}$$

Each sector corresponds to a different soliton number, twist winding, or shell state.

Transitions between sectors correspond to particle-like events.

Shell emission resembles a tunneling event in this landscape.

5.3 Bound-State Quantization

From the Dirac-bound state analysis, soliton backgrounds support discrete spinor modes due to the breathing scalar mass:

$$(i\gamma^\mu \nabla_\mu - m - g\phi)\psi = 0 \tag{19}$$

These modes form a discrete spectrum analogous to atomic orbitals.

Only a finite number of fermion shells can fit due to energy constraints and exclusion.

The breathing field ϕ dynamically shapes the allowed bound states—quantization arises geometrically, not probabilistically.

5.4 Dirac Emergence from Soliton Structure

Even without explicit fermionic fields, the scalar-twist combination behaves in a spinor-like way:

- The twist encodes intrinsic angular momentum
- Gauge coupling via A_0 gives rise to charge-like phase locking
- Multishell configurations mimic exclusion and shell-filling (fermionic statistics)

When spinors are explicitly included, the breathing scalar acts as a mass term, and Pauli exclusion naturally emerges from deterministic wavefield orthogonality rather than antisymmetrized states.

6 Spin Representation Options

6.1 Scalar Winding and Topological Spin

The PWARI-G framework permits a purely scalar mechanism for modeling spin through the topological behavior of the twist field $\theta(x)$. Rather than introducing explicit spinor degrees of freedom, spin is encoded as a geometric property of scalar winding—specifically, how the phase θ wraps over the soliton's support.

6.1.1 Two-Valued Phase and Spin-$\frac{1}{2}$ Analogue

In conventional quantum mechanics, spin-$\frac{1}{2}$ particles require a 4π rotation to return to their original state. PWARI-G captures this behavior naturally:

- The combined scalar-twist field $\phi e^{i\theta}$ undergoes a sign reversal under a 2π rotation if the winding number is half-integer.
- Systems with total twist phase accumulation of π or 3π, not 2π, must rotate twice to return to initial configuration.
- This two-valuedness mimics the transformation behavior of spinors, suggesting an emergent spin-$\frac{1}{2}$ property.

6.1.2 Integer and Half-Integer Winding

The total twist around a closed loop defines a topological charge:

$$n = \frac{1}{2\pi} \oint \nabla\theta \cdot d\ell$$

- Integer windings ($n = 1, 2, \dots$) represent bosonic-like structures.
- Half-integer windings ($n = \frac{1}{2}, \frac{3}{2}, \dots$) yield soliton states with spinor-like behavior.
- The twist's direction also sets the chirality (left vs right-handedness) of the soliton.

This continuous-but-topologically-constrained winding mirrors the structure of angular momentum quantization without requiring operator formalism.

6.1.3 Hopf Index and Twist Preservation

Topological spin is robust against continuous deformations of the soliton:

- The winding number is a homotopy invariant—a conserved quantity under smooth evolution.
- Soliton breathing and twisting may alter local field configuration but cannot unwind topological spin without crossing a discontinuity or forming a node.
- This ensures that the spin value of a soliton is preserved unless a shell is emitted or the soliton is destroyed.

6.2 Spinor Field Coupling

PWARI-G supports explicit fermionic behavior through the inclusion of Dirac spinors coupled to the breathing scalar field $\phi(x)$. These spinors experience both gauge and gravitational background effects, enabling bound-state formation, exclusion, and shell-dependent structure.

6.2.1 Dirac Equation in Soliton Background

Spinor fields $\psi(x)$ are governed by the Dirac equation in curved spacetime:

$$i\gamma^{\mu}(x)\nabla_{\mu}\psi - m\psi - g\phi\psi = 0$$

- The breathing scalar $\phi(x)$ acts as a mass-generating background, modulating the effective mass $m_{\text{eff}}(x) = m + g\phi(x)$.
- The spin connection ∇_{μ} ensures covariant derivatives in curved redshift geometry.
- Bound states form where $\phi(x)$ is large and localized—these match soliton "orbitals".

6.2.2 Yukawa-like Breathing Mass Coupling

The interaction term:

$$\mathcal{L}_{\text{int}} = -g\phi\bar{\psi}\psi$$

resembles a Yukawa coupling, but here the scalar is not a quantum field—it is a classical breathing soliton. The breathing of ϕ results in oscillating mass effects for the spinor, leading to dynamic bound energy levels.

- Spinor binding occurs in regions where ϕ oscillates around nonzero values.
- These bound states show discrete energy modes, as shown in numerical solutions to the static background Dirac equation.

6.2.3 Curved-Space Spinor Solutions

When spinors evolve within a nontrivial redshift geometry $A(x)$, the equation includes gravitational time dilation. The local energy of each spinor mode becomes:

$$E_n = \int \bar{\psi}_n \left[-i\gamma^0 \partial_t + \text{curvature terms}\right] \psi_n \, d^3x$$

- Redshift modifies the mode energies via $E \to E/\sqrt{A(x)}$
- Multi-mode filling forms structured "shells" similar to electronic orbitals.
- These solutions naturally exhibit shell exclusion and stability without requiring antisymmetrization—fermionic behavior emerges deterministically from soliton-bound mode orthogonality.

6.3 Emergent Fermionic Statistics

PWARI-G solitons support discrete fermionic-like behavior through multi-shell spinor bound states and deterministic field dynamics. This provides an alternative to antisymmetrization in quantum field theory, allowing exclusion to emerge naturally from mode structure.

6.3.1 Multi-Shell Mode Filling

Spinors bound within a breathing soliton background form discrete energy levels—analogous to atomic orbitals. These arise from the coupling:

$$(i\gamma^\mu \nabla_\mu - m - g\phi)\,\psi = 0$$

The breathing scalar $\phi(x)$ defines a potential well. Only a finite number of orthogonal spinor modes can fit due to:

- Finite spatial support of ϕ
- Energetic separation between levels
- Coupling strength g

These fill from lowest to highest energy, creating a layered shell structure.

6.3.2 Deterministic Exclusion Behavior

Unlike standard QFT, PWARI-G does not require symmetrization of wavefunctions. Exclusion arises from:

- Orthogonality of spinor modes
- Mutual backreaction: each filled mode modifies ϕ and A
- Nonlinear deformation of the background from accumulated spinor density

As more modes are added:

- The scalar field adjusts shape and redshift
- Higher-energy states eventually delocalize and are not bound

This imposes a hard cutoff on the number of allowed fermionic shells.

6.3.3 Pauli-Like Orbital Structure

Each bound spinor mode corresponds to a distinct spatial profile—analogous to s, p, d, etc. orbitals. These modes:

- Avoid spatial overlap due to orthogonality
- Occupy well-defined radial shells
- Evolve deterministically under the field equations

This leads to an effective "Pauli principle" where no two modes occupy the same shell configuration. It arises from classical nonlinear dynamics rather than wavefunction antisymmetry.

7 Gauge Field Structure (U(1) and SU(2))

7.1 U(1) Electrodynamics

PWARI-G incorporates U(1) gauge symmetry through the coupling of the twist field $\theta(x,t)$ to a scalar gauge potential $A_0(x,t)$. This enables the emergence of electrostatic-like behavior, where charge and field arise from soliton structure rather than point particles.

7.1.1 Phase-Locking of A_0 and θ

The Lagrangian includes a gauge-invariant term for twist dynamics:

$$\mathcal{L}_\theta = -\frac{1}{2}\phi^2\left[(\partial_t\theta - A_0)^2 - (\nabla\theta)^2\right]$$

This term:

- Enforces U(1) gauge invariance under:

$$\theta \to \theta + \alpha(x), \quad A_0 \to A_0 + \partial_t\alpha$$

- Acts as a "charge binding" mechanism: the soliton must breathe and twist in phase with A_0

The Euler-Lagrange equation for A_0 gives:

$$\phi^2(A_0 - \partial_t\theta) + \nabla^2 A_0 = 0$$

This relaxes to:

$$A_0 \approx \partial_t\theta \quad \text{in static or bound systems}$$

Here, A_0 tracks the twist frequency—a form of phase-locking.

7.1.2 Electrostatic Field Sourcing by Soliton Twist

The effective charge density arises from the twist dynamics:

$$\rho_q = \phi^2(\partial_t\theta - A_0)$$

In equilibrium, A_0 adjusts so that $\rho_q \to 0$, but any change in twist rate induces localized charge-like excitations.

The electrostatic energy is given by:

$$E_{\text{gauge}} = \int \frac{1}{2}(\nabla A_0)^2 \, d^3x$$

This term quantifies:

- Stored field energy in the gauge sector
- The electrostatic self-energy of the twist configuration
- Shell-like structures can support isolated radial charge-like fields

7.1.3 Interpretation

- The gauge field A_0 is not sourced by point charges but by spatial and temporal variation of the twist phase
- Local fluctuations in twist speed generate charge-like behavior
- Static soliton cores with stable twist exhibit neutral configurations
- Moving or snapping twist shells can eject charged pulses—analogous to radiation or ionization

7.2 SU(2) Gauge Extension

Beyond the Abelian U(1) field A_0, PWARI-G supports non-Abelian SU(2) gauge structures which couple to solitonic fields and spinor doublets. These SU(2) gauge fields introduce internal symmetry dynamics, topologically enriched breathing solitons, and field-sourced shells analogous to nuclear-level structure.

7.2.1 SU(2) Gauge Breathing Solitons

PWARI-G can generalize the scalar field to a doublet $\Phi = (\phi_1, \phi_2)^T$, minimally coupled to an SU(2) gauge field A_μ^a. The scalar sector Lagrangian becomes:

$$\mathcal{L}_{\text{scalar}} = \frac{1}{2}(D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi)$$

where:

$$D_\mu = \partial_\mu - igA_\mu^a T^a, \quad T^a = \frac{1}{2}\sigma^a$$

$$V(\Phi) = \frac{\lambda}{4}(\Phi^\dagger\Phi - v^2)^2$$

These scalar doublets support radially breathing SU(2) solitons whose energy is stabilized by gauge curvature.

7.2.2 Scalar and Spinor Doublets

SU(2) naturally extends to spinor fields:

$$\Psi = (\psi_1, \psi_2)^T$$

which couple via:

$$\mathcal{L}_{\text{Dirac}} = i\bar{\Psi}\gamma^\mu D_\mu \Psi - g\bar{\Psi}\Phi\Psi$$

- Φ acts as a chiral mass generator for spinors
- Dirac bound states evolve in a curved gauge and gravitational background
- Spinor wavefunctions carry isospin and experience symmetry-breaking breathing dynamics from the scalar field

7.2.3 Covariant Coupling via D_μ

All matter fields are promoted to gauge-covariant derivatives:

$$\begin{aligned} \text{Scalar:} \quad & D_\mu \Phi = \partial_\mu \Phi - igA_\mu^a T^a \Phi \\ \text{Spinor:} \quad & D_\mu \Psi = \partial_\mu \Psi - igA_\mu^a T^a \Psi \end{aligned}$$

These terms ensure full SU(2) invariance of the system. The breathing and shell dynamics then follow from the gauge-aligned field oscillations and backreaction.

7.2.4 Field Strength Tensor Dynamics

The SU(2) gauge field evolves via the non-Abelian field strength tensor:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$$

The gauge sector Lagrangian is:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

This introduces:

- Self-interacting gauge curvature
- Shell-pulsing field emissions from fermionic transitions
- SU(2) symmetry preservation during shell evolution

7.3 Gauge-Sourced Structure from Spinors

The PWARI-G framework unifies gauge field sourcing with matter by coupling spinor fields directly to U(1) and SU(2) gauge potentials. Unlike in standard QED/QCD where point charges act as external sources, here the distributed density and twist of bound Dirac spinors actively shape the gauge field structure.

7.3.1 Dirac Shell Sourcing of U(1) and SU(2)

Spinor fields ψ act as source terms for both U(1) and SU(2) gauge sectors.

- **U(1) gauge current:**

$$j^{\mu}_{\mathrm{U}(1)} = \bar{\psi}\gamma^{\mu}\psi \quad \Rightarrow \quad \nabla^2 A_0 = \rho_q = \psi^{\dagger}\psi$$

  This creates a radial electrostatic field sourced by the spinor shell density.

- **SU(2) gauge current:**

$$j^{a\mu}_{\mathrm{SU}(2)} = \bar{\psi}\gamma^{\mu}T^a\psi \quad \Rightarrow \quad D_{\mu}F^{a\mu\nu} = j^{a\nu}$$

  where $T^a = \sigma^a/2$ are the SU(2) generators. This yields rotationally structured gauge fields.

The strength and configuration of these fields depend on:

- The number of filled spinor modes
- Their spatial overlap and radial separation
- Backreaction from the gauge field onto the soliton core

7.3.2 Electromagnetic-like Fields from Fermionic Twist

Even without explicit charge insertion, rotating or twisting spinor modes generate effective "electromagnetic" field configurations:

- Time-varying phase of ψ sources longitudinal components in A_0
- Spinor shell rotation induces transverse curvature in SU(2) fields
- Breathing shells with twist imbalance radiate small gauge pulses

This reproduces many electromagnetic field effects:

- Field lines emerge from soliton structures, not point charges
- Gauss-like laws appear as surface integrals over spinor current density
- Gauge fields self-consistently reflect soliton-bound fermionic motion

7.3.3 Gauge Shell Symmetry Preservation and Propagation

Each spinor shell maintains its symmetry through the structure of the SU(2) gauge field:

- The field strength tensor $F^a_{\mu\nu}$ evolves radially outward
- Symmetry constraints enforce conservation of internal angular momentum
- Breathing soliton core + radiating gauge shell creates a layered, stable configuration

Soliton atoms therefore possess:

- Discrete shell-filling rules
- Structured internal gauge fields
- Quantized-like charge distribution without fundamental particles

8 Fully Unified Matter-Gravity Coupling

8.1 Einstein Field Equations

PWARI-G extends classical general relativity by coupling nonlinear soliton fields—scalar, spinor, and gauge—to curvature via the Einstein field equations. Each field contributes to the total energy-momentum tensor, sourcing spacetime curvature in a self-consistent, wave-based manner.

8.1.1 Total Stress-Energy Source

The Einstein field equations relate curvature to total matter content:

$$G_{\mu\nu} = 8\pi G \left(T^{\phi}_{\mu\nu} + T^{\text{spinor}}_{\mu\nu} + T^{\text{gauge}}_{\mu\nu} \right)$$

Each component of $T^{\mu\nu}$ is derived from its respective Lagrangian:

- $T^{\phi}_{\mu\nu}$: breathing scalar contributions (gradient, potential, kinetic)
- $T^{\text{spinor}}_{\mu\nu}$: Dirac field stress-energy from bound fermionic modes
- $T^{\text{gauge}}_{\mu\nu}$: field strength tensor terms for U(1) and SU(2)

These combine to generate curvature dynamically, in contrast to fixed background models.

8.1.2 Redshift, Spatial Curvature, and Mass Function

The spherically symmetric spacetime metric is taken to be:

$$ds^2 = -A(x)dt^2 + B(x)dx^2 + x^2 d\Omega^2$$

From this, the Einstein equations reduce to radial functions $A(x)$, $B(x)$, and a mass function $m(x)$:

- $A(x)$ describes time dilation (redshift function)
- $B(x)$ controls radial distance scaling (spatial curvature)
- The enclosed mass function is defined as:

$$m(x) = 4\pi \int_0^x \rho_{\text{total}}(r)\, r^2\, dr$$

These quantities satisfy Oppenheimer–Volkoff–like structure equations under distributed wave-based energy.

8.1.3 Asymptotic Behavior and Horizon Avoidance

Far from the soliton, spacetime approaches the Schwarzschild limit:

$$A(x) \to 1 - \frac{2GM}{x}, \quad B(x) \to \left(1 - \frac{2GM}{x}\right)^{-1}$$

However, unlike black holes:

- No singularity forms at the origin—the wave field ϕ smooths energy density at $r = 0$
- The redshift $A(x)$ never vanishes completely, avoiding a true event horizon
- The mass function $m(x)$ saturates at finite radius due to soliton localization

This ensures a horizonless, nonsingular geometry—a soliton atom whose mass-energy is self-contained, not point-sourced.

8.2 Spherically Symmetric Soliton Atom

The PWARI-G soliton atom forms a stable, horizonless field structure in curved spacetime, organized around a spherically symmetric metric and layered internal composition. The solution emerges from the full set of coupled scalar, spinor, and gauge field equations sourced by a self-consistent stress-energy tensor.

8.2.1 Spacetime Metric Structure

The geometry is described by a static, spherically symmetric line element:

$$ds^2 = -A(x)dt^2 + B(x)dx^2 + x^2 d\Omega^2$$

Where:

- $A(x)$ is the redshift function
- $B(x)$ is the spatial curvature
- $d\Omega^2$ is the angular part of the 3D volume element

This metric is solved numerically using the total energy-momentum contributions from all fields and is asymptotically flat.

8.2.2 Breathing Scalar, Bound Spinors, and Gauge Emission

The structure is composed of:

- Central scalar core $\phi(x)$: localized, breathing, and responsible for sourcing both mass and redshift curvature
- Bound spinor shells $\psi_n(x)$: form discrete energy levels and contribute pressure and mass-energy
- Gauge fields $A_0(x)$, $A^a_\mu(x)$: sourced by spinor density and twist, radiating curvature and electromagnetic-like fields

The scalar field profile determines:

- The depth of the potential well
- The location of redshift peaks
- The number of spinor modes that can stably bind

8.2.3 Multi-Shell Formation and Redshift Spacing

The soliton atom organizes into nested shells:

- Each shell corresponds to a bound spinor state
- Shells are spaced due to energy quantization and redshift separation:

$$\Delta E_n \sim \frac{1}{\sqrt{A(x_n)}}$$

- Breathing dynamics compress inner shells and stretch outer shells
- Redshift $A(x)$ decreases toward the core, spacing shells nonlinearly

The combination of twist energy, gauge feedback, and gravitational redshift leads to self-regulating multi-shell stability. Each shell adjusts the background fields, influencing the configuration of the next—creating a deterministic wave-based analogue to atomic orbitals.

8.3 Stability and Regularity Conditions

The PWARI-G soliton atom avoids the traditional singularities of general relativity through its nonlinear, self-regularizing wave structure. All fields contribute to the balance of forces, ensuring spatial localization, finite total energy, and dynamical stability without divergence.

8.3.1 No Singularities or Divergences

Unlike black hole or point-charge models:

- The scalar field $\phi(x)$ is smooth and differentiable everywhere
- The redshift $A(x)$ never vanishes entirely
- The curvature function $B(x)$ is sourced by finite energy densities

Since the soliton is composed of extended wave profiles with soft boundaries, all physical quantities such as $R_{\mu\nu}R^{\mu\nu}$, ρ, and $T_{\mu\nu}$ remain bounded across space.

8.3.2 Finite Energy and Self-Regulating Core

The total energy is given by:

$$E_{\text{total}} = E_\phi + E_\theta + E_{A_0} + E_A$$

Each term is spatially localized due to:

- Finite support of $\phi(x)$
- Shell emission as a release mechanism for excess twist
- Energy loss into inert gravitational tails, preventing runaway accumulation

Backreaction from filled spinor modes and gauge fields redistributes energy, maintaining internal pressure against collapse.

8.3.3 Numerical Evolution and Backreaction Balance

Simulations show that:

- The breathing scalar stabilizes via nonlinear potential and redshift feedback
- Each filled spinor mode modifies the scalar and redshift profile
- Gauge field energy is absorbed into the total stress-energy budget
- Shell formation only occurs when twist tension exceeds a critical threshold

After shell emission, energy is depleted and the system returns to a stable, bound configuration. This creates a robust balance between curvature, matter, and field dynamics.

These mechanisms collectively ensure that the PWARI-G soliton atom evolves stably, resists divergence, and retains finite energy density without collapse or singular behavior.

9 Discussion and Future Work

9.1 Emergence of Atomic Structure

PWARI-G offers a fully wave-based reconstruction of atomic structure without invoking point particles, quantum collapse, or dualistic interpretations. Instead, structure emerges from deterministic field interactions and energy-constrained shell dynamics.

9.1.1 One Core One Shell Stability

The breathing scalar field ϕ naturally forms a localized soliton core, stabilized by nonlinear self-interaction and gravitational redshift. The core accumulates twist energy via the phase field θ until a threshold is reached, leading to the emission of a shell.

- Shell formation is a regulated process: one core typically supports only one stable shell.
- Additional energy causes transient shells that dissipate, restoring equilibrium.
- This creates a quantized, self-limiting core shell configuration.

9.1.2 Emergent Charge, Spin, and Exclusion

PWARI-G reproduces atomic properties as emergent field phenomena:

- Charge arises from twist phase dynamics sourcing the gauge field A_0.
- Spin appears through topological winding of the twist field and spinor alignment in SU(2) configurations.
- Pauli Exclusion emerges from deterministic shell-filling and backreaction—no two spinor modes can occupy the same bound state.
- Each shell corresponds to a specific energy level shaped by curvature, gauge field, and nonlinear coupling—no probability or operator quantization is required.

9.1.3 No Particle Duality or Collapse Assumptions

The model avoids all unresolved aspects of conventional quantum mechanics:

- There is no wavefunction collapse.
- No hidden variables or many-worlds interpretations.
- No dual particle-wave ontology.

The PWARI-G soliton atom is a self-consistent, physical wave object that obeys conservation laws and forms stable, quantized internal structure—just as atoms do, but with deeper transparency into the origin of their behavior.

9.2 Quantum-Classical Boundary

PWARI-G operates as a classical field theory, yet many of its emergent features parallel quantum behavior. The boundary between wave dynamics and quantized observation in this framework is defined not by measurement postulates, but by the structure and stability of nonlinear solitonic solutions.

9.2.1 Semi-Classical Interpretations

The breathing scalar field, twist modes, and gauge configurations form discrete, dynamically stable solutions. These correspond to what quantum mechanics would call "particle states," but in PWARI-G:

- No probabilistic collapse is invoked.
- State transitions (e.g., shell emission) arise from nonlinear dynamics.
- Quantization of energy levels, spin, and charge emerges geometrically.

Thus, PWARI-G can be interpreted as a semi-classical theory where classical fields give rise to quantized observables through topological and energetic constraints.

9.2.2 Overlap with QFT/QED

PWARI-G overlaps with quantum field theory in several critical ways:

- Bound state quantization of spinor fields matches QED orbital structure.
- Redshift-based energy level shifts mimic Lamb shifts and gravitational corrections.
- Charge and spin arise from gauge and twist field structure, analogous to conserved currents in QFT.

However, PWARI-G avoids:

- Canonical quantization of operators
- Fock space construction
- Renormalization of point-like divergences

This provides a deterministic foundation underneath QFT-like behavior, which may be especially relevant in curved spacetime or nonperturbative regimes.

9.2.3 Soliton Decoherence and Measurement Effects

In traditional quantum mechanics, measurement collapses the wavefunction. In PWARI-G:

- Observation corresponds to external perturbation of the soliton.
- Decoherence could arise via energy loss into gravitational tails or gauge field radiation.
- Shell emission may function analogously to quantum jumps, but follows from deterministic field instability.

This opens the possibility of modeling measurement and classical emergence as specific dynamic processes—especially in systems where energy thresholds and spatial separability are important (e.g., ionization, decay, or entanglement breakdown).

9.3 Predictions and Simulations

PWARI-G, while fundamentally theoretical, produces a range of clear, testable predictions across atomic, gravitational, and quantum domains. These predictions can be pursued through both numerical simulations and experimental analogues.

9.3.1 Spectra and Redshift of Bound States

- Bound spinor modes within the breathing scalar field yield discrete energy levels.
- These levels shift with the redshift function $A(x)$, leading to:
  - Gravitational redshift of shell energies
  - Predicted spectral lines that shift based on total soliton mass
  - Analogous to hydrogen-like spectral transitions, but influenced by curvature rather than Coulomb force
- Numerical simulations can extract:
  - Mode spacing as a function of redshift depth
  - Shell overlap and resonance patterns
  - Dependence on gauge charge and SU(2) symmetry configuration

9.3.2 Gravitational Lensing and Time Dilation

Soliton atoms produce measurable redshift and time dilation:

$$\Delta t = \frac{1}{\sqrt{A(x)}}\Delta\tau$$

- Light or other wave packets passing near a soliton will undergo lensing or delay.
- This can be probed in analog simulations or curved-space wave equations.
- Predictions include:
  - Lensing angles for field-generated mass distributions
  - Slow-light propagation through artificial gravitational shells

9.3.3 Soliton Shell Emission as Gravitational Wave Source

- Twist-shell snapping events eject radial pulses, dynamically deforming spacetime.
- These produce burst-like gravitational wave signatures with characteristic frequency:

$$f \sim \frac{1}{R}\sqrt{\frac{\sigma_{\max}}{\rho}}$$

- The wave is sourced not by quadrupole moment but by nonlinear breathing and redshift drag.
- Simulations can explore:
  - Burst profiles
  - Energy radiated per shell emission
  - Conditions for repeated emission (multi-shell instability)

9.3.4 Matching to Atomic and Subatomic Structure

- Each shell mimics an orbital level; Pauli-like exclusion emerges through deterministic mode saturation.
- Effective atomic number Z corresponds to the number of filled bound spinor modes.
- Gauge fields mimic electromagnetic charge; SU(2) symmetry leads to internal structure reminiscent of nuclear layers.
- These mappings allow PWARI-G to simulate:
  - Hydrogen and helium analogues
  - Charge shell radius scaling
  - Energy level convergence for high-Z configurations

Together, these predictions provide a blueprint for matching PWARI-G soliton structures to physical atomic phenomena—without invoking discrete particles or quantum collapse.