Derivation of Quantized Orbitals in PWARI-G Hydrogen Model

PWARI-G Collaboration

Overview

In the PWARI-G framework, quantized atomic orbitals emerge as standing wave solutions of the twist field θ evolving in the potential well formed by the soliton field ϕ. This derivation presents the mathematical formulation of these orbital modes and confirms their radial scaling behavior.

1 Twist Field Wave Equation

The twist field θ evolves in the background of the soliton (core) field ϕ via the nonlinear wave equation:

$$\Box\theta - (\nabla\theta)^2 - \phi^2\theta = 0 \tag{1}$$

For stationary eigenmodes, we assume $\theta(t, \vec{x}) = u(r)e^{-i\omega t}$ in spherical symmetry. The equation reduces to:

$$-\omega^2 u(r) = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{du}{dr}\right) - \phi(r)^2 u(r) \tag{2}$$

2 Numerical Solution as Boundary Value Problem

Given a numerically evolved $\phi(r)$ (from the stable soliton solution), we treat the twist field as a linear Schrödinger-type equation:

$$\frac{d^2u}{dr^2} + \frac{2}{r}\frac{du}{dr} + \left(\omega^2 - \phi(r)^2\right)u = 0 \tag{3}$$

This is solved as a boundary value problem:

$$u(0) = 0 \quad \text{(regularity)} \tag{4}$$

$$u(r_{\max}) \approx 0 \quad \text{(bound state decay)} \tag{5}$$

The numerical eigenvalue problem is solved using spectral or shooting methods to obtain the radial modes $u_n(r)$ and their eigenfrequencies ω_n.

3 Radial Peak and Shell Structure

Each eigenmode $u_n(r)$ exhibits exactly $n-1$ internal nodes and a well-defined radial peak, interpreted as the location of a quantized orbital shell:

$$r_n = \operatorname*{argmax}_r |u_n(r)| \tag{6}$$

4 Scaling Relationship

Plotting the radial peak positions r_n against $\sqrt{n}$ yields a nearly perfect linear relationship:

$$r_n \propto \sqrt{n} \tag{7}$$

This matches the orbital scaling found in the Bohr model but emerges here purely from field dynamics.

5 Interpretation

- The quantization of orbitals is not postulated but derived from the standing wave solutions of the twist field. - The ϕ field creates a potential well; θ forms eigenmodes in this background. - This mechanism reproduces shell quantization in hydrogen-like atoms within a deterministic wave framework.

6 Conclusion

We have shown that:

- The twist field supports discrete eigenmodes in the soliton core.
- These modes exhibit radial peaks consistent with quantized orbitals.
- The shell radii scale as $\sqrt{n}$, consistent with Bohr model predictions.

This derivation confirms that atomic structure in PWARI-G emerges from the underlying field equations, without invoking probabilistic quantum postulates.