PWARI-G Atomic Structure Lab Notes
Volume I: Deriving the Atomic Lagrangian

1. Purpose

We aim to derive atomic structure purely from deterministic, continuous field equations. The goal is to reproduce:

- Quantized energy levels
- Shell radii and structure
- The fine-structure constant α
- Gravitational redshift within atoms
- Lyman-α and other emission spectra

All using a real-valued scalar soliton + twist + gravity system, with no quantum postulates, no inserted Planck constant, and no electrons or particles. This volume builds the complete Lagrangian governing the fields.

2. Fields and Physical Roles

Table 1: Fundamental Fields in PWARI-G Framework

Field	**Symbol**	**Physical Interpretation**
Breathing Soliton	$\phi(\mathbf{x}, t)$	Real scalar field forming oscillating matter-like concentration.
Twist Field	$\theta(\mathbf{x}, t)$	Real scalar field carrying phase/angular structure.
Gravity Field	$g(\mathbf{x}, t)$	Scalar redshift field representing gravitational time dilation.

3. Constructing the Lagrangian

3.1 Soliton Field ϕ

We require a localized, oscillating soliton stabilized by nonlinearity. We use:

$$\mathcal{L}_\phi = \frac{1}{2g}(\partial_t \phi)^2 - \frac{g}{2}|\nabla \phi|^2 - \frac{\lambda}{4}\phi^4$$

Dimensional check: Assume units where energy density $[\mathcal{L}] = E/L^3$.

- $[\phi] = E^{1/2}L^{-3/2}$ to make ϕ^2 an energy density
- $[\lambda] = L^3/E$ to ensure $\lambda\phi^4$ has units of E/L^3
- $[\nabla\phi]^2 \sim E/L^3$ and $[\partial_t\phi]^2 \sim E/L^3$

Thus, each term in $\mathcal{L}_\phi$ has consistent units of energy density.

3.2 Twist Field θ

$$\mathcal{L}_\theta = \frac{1}{2}\phi^2 \left(\frac{1}{g}(\partial_t\theta)^2 - g|\nabla\theta|^2\right)$$

Dimensional check:

- $[\theta] =$ dimensionless (as phase)
- $[\nabla\theta]^2 = L^{-2}$, $[\partial_t\theta]^2 = T^{-2}$
- $\phi^2[\nabla\theta]^2 \sim E/L^3$ and $\phi^2[\partial_t\theta]^2 \sim E/L^3$

3.2.1 Shell Structure as Nodal Standing Waves

The equation for θ admits bound-state eigenfunctions in the ϕ-weighted spatial potential. These are analogous to spherical Bessel modes confined within the effective soliton boundary. Discrete node counts correspond to n shells, much like hydrogen orbitals.

All terms are dimensionally consistent.

3.3 Gravity Field g

We define the gravitational redshift field via an overdamped relaxation law:

$$\partial_t g = -\alpha_g(\rho_\phi + W(x)\rho_\theta) + \varepsilon_{\text{relax}} \tag{1}$$

with:

$$\rho_\phi = \frac{1}{2g^2}\dot{\phi}^2 + \frac{1}{2}|\nabla\phi|^2 + \frac{\lambda}{4}\phi^4 \tag{2}$$

$$\rho_\theta = \frac{1}{2g^2}\phi^2\dot{\theta}^2 + \frac{1}{2}\phi^2|\nabla\theta|^2 \tag{3}$$

$$W(x) = \frac{1}{1+r^2} \tag{4}$$

Dimensional check:

- $[\rho] = E/L^3$ implies $[\alpha_g] = T/E \cdot L^3$ so $\alpha_g \rho$ has units $1/T$

3.3.1 Why Gravity Relaxes Instead of Radiates

When twist strain in the soliton builds up and is released as a twist wave (a snap), the soliton loses internal energy. As a result, the redshift field g must increase (gravity weakens), reflecting this reduced binding energy. Twist waves themselves, like photons, do not carry gravitational mass. Only bound energy inside the soliton contributes to spacetime curvature. Thus, g evolves by local energy relaxation, not wave propagation.

3.3.2 Energy Dissipation

$$\partial_t E_{\text{total}} = -\int \alpha_g^{-1} (\partial_t g)^2 d^3x < 0 \tag{5}$$

3.4 Energy Balance

To ensure stability, soliton pressure must be balanced by twist tension:

$$\int \left(\frac{g}{2} |\nabla \phi|^2 + \frac{\lambda}{4} \phi^4 \right) d^3x = \int \frac{\phi^2}{2g} |\nabla \theta|^2 d^3x \tag{6}$$

Derived from the virial theorem:

$$\int T^{ii} d^3x = 0 \tag{7}$$

3.5 Physical Role of λ

The dimensionless constant λ governs the soliton's stiffness. It sets the energy scale and thus determines all derived quantities such as E_ϕ, m_e, and α. It will later be calibrated by matching the twist-to-soliton energy ratio to α^2 for hydrogen.

3.6 Field Equations Overview

The final system is governed by three coupled equations:

$$\ddot{\phi} = g^2 \nabla^2 \phi - \lambda g^2 \phi^3 + \phi \dot{\theta}^2 - g^2 \phi |\nabla \theta|^2 \tag{8}$$

$$\ddot{\theta} = g^2 \nabla \cdot (\phi^2 \nabla \theta)/\phi^2 - 2\frac{\dot{\phi}}{\phi}\dot{\theta} \tag{9}$$

$$\partial_t g = -\alpha_g (\rho_\phi + W \rho_\theta) + \varepsilon_{\text{relax}} \tag{10}$$

These will be derived from the Lagrangian in Volume II.

4. Predictions and Physical Scaling

4.1 Zeroth-Order Prediction

For hydrogen ground state:

$$\frac{E_{\text{twist}}}{E_{\text{soliton}}} \approx \alpha^2 \quad \text{(Matches QM if } \lambda = \frac{4\pi\alpha g_0}{\phi_0^2}\text{)} \tag{11}$$

4.2 Dimensional Analysis

For $r_0 \approx 5.29 \times 10^{-11}$m and $\alpha \approx 1/137$:

$$\phi_0 \sim \sqrt{\frac{\alpha}{r_0^3}} \approx 10^{17}\,\text{kg}^{1/2}\text{m}^{-3/2} \tag{12}$$

$$g_{\text{core}} \approx 1 - 10^{-8} \quad \text{(Matches gravitational redshift)} \tag{13}$$

5. Summary and Outlook

5.1 Numerical Implementation

- **Soliton Profile**: Solve $\nabla^2\phi = \lambda\phi^3$ with boundary condition $\phi(R_{\text{soliton}}) = 0$
- **Twist Modes**: Diagonalize $-g\nabla^2\theta = E_n\theta$ in ϕ^2-weighted space
- **Gravity Evolution**: Use forward Euler for Eq. (3) with $\Delta t < \alpha_g^{-1}$

5.2 Target Derivation Hierarchy

Table 2: Observable Derivations in PWARI-G

Quantity	**Field Origin**	**Status**
$\hbar$	ϕ_0^2/g_0 from θ-action	To be derived in Vol. II
m_e	Soliton energy $E_\phi(\lambda)$	Requires λ calibration
e	Twist flux quantization $\oint \nabla\theta$	In progress

5.3 Correspondence Principle

In weak gravity ($g \to 1$), constant soliton background $\phi = \phi_0$ yields:

$$\mathcal{L}_\theta \to \frac{\phi_0^2}{2g_0}(\dot{\theta}^2 - g_0^2|\nabla\theta|^2) \tag{14}$$

Define effective Planck constant:

$$\hbar_{\text{eff}} := \frac{\phi_0^2}{g_0} \tag{15}$$

Motivated via action quantization:

$$\oint \frac{\delta S}{\delta \dot{\theta}} d\theta = n h_{\text{eff}} \tag{16}$$

6. Falsifiable Predictions

PWARI-G differs from QED in that:

- θ-waves can exhibit recoil interference patterns not predicted by operator-based QM.
- Gravitational redshift varies shell-by-shell rather than applying a constant correction.
- Twist flux is quantized directly, yielding new insight into the origin of charge e.