PWARI-G Breathing Gauge Field Theory

1 Field Definitions

- Breathing amplitude field: $\phi(x)$
- Breathing phase field: $\theta(x)$
- Gauge connection field: $A_\mu(x)$

2 Lagrangian Density

In natural units ($c = 1$):

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2}_{\text{Kinetic term}} + \underbrace{\frac{1}{2}\phi^2(\mathcal{D}_\mu \theta)^2}_{\text{Twist coupling}} - \underbrace{\frac{\lambda}{4}(\phi^2 - \phi_0^2)^2}_{V(\phi)} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Gauge field}}$$

2.1 Component Definitions

$$\begin{aligned} \text{Covariant derivative:} \quad & \mathcal{D}_\mu \theta = \partial_\mu \theta - eA_\mu \\ \text{Field strength tensor:} \quad & F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\ \text{Quartic potential:} \quad & V(\phi) = \frac{\lambda}{4}(\phi^2 - \phi_0^2)^2 \end{aligned}$$

3 Euler-Lagrange Equations

3.1 For Breathing Amplitude Field ϕ

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$
$$\Rightarrow \Box \phi - \phi(\mathcal{D}_\mu \theta)^2 + \frac{dV}{d\phi} = 0$$

3.2 For Breathing Phase Field θ

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \theta)} \right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$
$$\Rightarrow \partial_\mu(\phi^2 \mathcal{D}^\mu \theta) = 0$$

3.3 For Gauge Field A_μ

$$\partial_\nu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)} \right) - \frac{\partial \mathcal{L}}{\partial A_\mu} = 0$$

$$\Rightarrow \partial_\nu F^{\nu\mu} = j^\mu \quad \text{where} \quad j^\mu = e\phi^2 \mathcal{D}^\mu \theta$$

4 Physical Interpretation

- ϕ: Nonlinear breathing amplitude with self-interaction potential
- θ: Phase field generating breathing current j^μ
- A_μ: Mediates breathing phase distortions through $F_{\mu\nu}$
- Current conservation: $\partial_\mu j^\mu = 0$ (from $\partial_\mu \partial_\nu F^{\nu\mu} = 0$)

5 Extensions

- Higher-order potential: $V(\phi) = \sum_{n=2}^{\infty} \frac{\lambda_n}{n!} (\phi^2 - \phi_0^2)^n$
- Spontaneous symmetry breaking: $\phi_0 \neq 0$ vacuum expectation value
- Solitonic solutions: Localized $\phi(x)$ configurations coupled to A_μ

6 Breathing Amplitude Field $\phi(x)$

The Lagrangian terms involving ϕ are:

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\phi^2 (D_\mu \theta)^2 - V(\phi)$$

Applying the Euler-Lagrange equation:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

Compute each piece:

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi, \quad \partial_\mu(\partial^\mu \phi) = \Box \phi, \quad \frac{\partial \mathcal{L}}{\partial \phi} = \phi (D_\mu \theta)^2 + \frac{dV}{d\phi}$$

Resulting field equation:

$$\Box \phi - \phi (D_\mu \theta)^2 + \frac{dV}{d\phi} = 0$$

green **Modified by gauge interactions.**

7 Breathing Phase Field $\theta(x)$

Relevant Lagrangian terms:

$$\mathcal{L}_\theta = \frac{1}{2}\phi^2(D_\mu\theta)^2$$

Euler-Lagrange equation for θ:

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\theta)}\right) - \frac{\partial\mathcal{L}}{\partial\theta} = 0$$

Compute:

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\theta)} = \phi^2 D^\mu\theta, \quad \partial_\mu(\phi^2 D^\mu\theta) = 0$$

Resulting continuity equation:

$$\partial_\mu(\phi^2 D^\mu\theta) = 0$$

green **Breathing charge conservation.**

8 Gauge Field $A_\mu(x)$

Relevant terms:

$$\mathcal{L}_A = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\phi^2(D_\mu\theta)^2$$

Varying with respect to A_μ:

$$\partial_\nu F^{\nu\mu} = e\phi^2 D^\mu\theta \quad \text{or equivalently} \quad \partial_\nu F^{\nu\mu} = j^\mu \quad \text{with} \quad j^\mu = e\phi^2 D^\mu\theta$$

green **Breathing Maxwell equation.**

Summary of Field Equations

Field	Evolution Equation
$\phi(x)$	$\Box\phi - \phi(D_\mu\theta)^2 + \frac{dV}{d\phi} = 0$
$\theta(x)$	$\partial_\mu(\phi^2 D^\mu\theta) = 0$
$A_\mu(x)$	$\partial_\nu F^{\nu\mu} = e\phi^2 D^\mu\theta$

Physical Interpretation

- ϕ: Breathes and reshapes via energy exchange between twist and potential
- θ: Governs "breathing charge" flow dynamics
- A_μ: Mediates breathing phase distortions (analogous to EM but geometric)