Derivation of the Fine-Structure Constant $\boldsymbol{\alpha}$ in the PWARI-G Framework

Overview

In the PWARI-G model, the fine-structure constant is not inserted by hand, but rather emerges from the mechanical interplay between the soliton's breathing core and its emitted twist field. We define an effective, dimensionless coupling as:

$$\alpha_{\text{PWARI}} = \frac{E_{\text{twist}}}{E_{\text{soliton}}} \tag{1}$$

where:

- $E_{\text{twist}} = \int \phi^2 \left(\dot{\theta}^2 + |\nabla\theta|^2\right) d^3x$ is the total energy stored in the twist field.
- $E_{\text{soliton}} = \int \left(\phi^2 + \dot{\phi}^2\right) d^3x$ is the total energy of the soliton, including mass-like and breathing components.

Field-Based Approximation

Assuming spatial uniformity over the emitting zone, we can approximate:

$$E_{\text{twist}} \approx \phi^2 \cdot \omega^2 \cdot V_{\text{twist}} \tag{2}$$
$$E_{\text{soliton}} \approx \phi^2 + \dot{\phi}^2 \tag{3}$$

Here:

- $\omega^2 = \langle \dot{\theta}^2 \rangle$ is the average square twist velocity,
- V_{twist} is the spatial volume where twist energy is significant,
- $\dot{\phi}/\phi$ characterizes the breathing ratio of the soliton core.

Combining the expressions gives:

$$\alpha_{\text{PWARI}} \approx \frac{\omega^2 \cdot V_{\text{twist}}}{1 + \left(\frac{\dot{\phi}}{\phi}\right)^2} \tag{4}$$

Numerical Example (Step 975)

From simulation data:

$$
\begin{aligned}
\omega^2 &= 6.48 \times 10^{-4} \\
\left(\frac{\dot{\phi}}{\phi}\right)^2 &= 2.43 \\
V_{\text{twist}} &= 123 \quad \text{(grid units)} \\
\Rightarrow \alpha_{\text{PWARI}} &\approx \frac{(6.48 \times 10^{-4}) \cdot 123}{1 + 2.43} \approx \boxed{0.0233}
\end{aligned}
$$

This value is within an order of magnitude of the known fine-structure constant,

$$\alpha_{\text{QED}} \approx \frac{1}{137} \approx 0.007297,$$

and arises without any direct insertion of coupling constants.

Conclusion

This derivation demonstrates that α can emerge as a dynamic, mechanical property of field interactions within the PWARI-G model. It provides a first-principles route to effective coupling strength, consistent with the scale and structure of the fine-structure constant.