PWARI-G Extension: Dirac Spinor Coupling in 3+1 Dimensions

April 30, 2025

1. Objective

To fully replace fermionic quantum field theory within the PWARI-G framework, we introduce a 3+1 dimensional Dirac spinor field $\psi(x)$, minimally coupled to the breathing gauge field A_μ and interacting with the scalar breathing field $\varphi(x)$. This allows us to model spin-$\frac{1}{2}$ particles, Pauli exclusion, and mass generation dynamically from breathing structures.

2. Total Lagrangian

We define the following composite Lagrangian:

$$\mathcal{L} = \underbrace{\bar{\psi}\left(i\gamma^\mu D_\mu - m - g\varphi(x)\right)\psi}_{\text{Dirac spinor with breathing mass}} \tag{1}$$

$$+ \underbrace{\frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)}_{\text{Nonlinear breathing field}} \tag{2}$$

$$- \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Gauge field strength}} \tag{3}$$

Where:

- ψ: Dirac spinor field

- $\bar{\psi} = \psi^\dagger \gamma^0$
- $D_\mu = \partial_\mu - ieA_\mu$: covariant derivative
- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$: U(1) field strength tensor
- $\varphi(x)$: breathing scalar amplitude field
- $V(\varphi) = \frac{\lambda}{4}(\varphi^2 - \varphi_0^2)^2$: scalar field potential
- g: coupling constant between breathing field and spinor

3. Euler-Lagrange Equations

(a) Dirac Equation with Breathing Mass

$$(i\gamma^\mu D_\mu - m - g\varphi(x))\,\psi(x) = 0$$

This describes a spin-$\frac{1}{2}$ fermion acquiring mass dynamically from the breathing field φ.

(b) Breathing Field Equation Sourced by Spinor Density

$$\Box\varphi + \frac{dV}{d\varphi} = g\bar{\psi}\psi$$

The scalar field evolves based on its own potential and a source term $\bar{\psi}\psi$, which provides backreaction from fermion localization.

(c) Gauge Field Equation Sourced by Fermionic Current

$$\partial_\nu F^{\nu\mu} = e\bar{\psi}\gamma^\mu\psi$$

This gives Maxwell-like dynamics for the gauge field, driven by the fermionic current $j^\mu = \bar{\psi}\gamma^\mu\psi$.

4. Interpretation

This formulation integrates the breathing soliton theory with Dirac spinor dynamics, allowing:

- Spinor quantization via soliton-like wave packets.
- Mass generation from interaction with a nonlinear scalar field.
- Gauge coupling directly through breathing-induced A_μ.
- Fully causal, deterministic field dynamics replacing Fock space operators.

Dirac Spinor Evolution in a Breathing Soliton Background (1+1D PWARI-G Model) April 30, 2025

1. Objective

We investigate the propagation of a 1+1 dimensional Dirac spinor wavepacket in a static breathing soliton background field. This demonstrates how fermionic fields interact deterministically with nonlinear scalar configurations in the PWARI-G framework.

2. Governing Equation

We reduce the full Dirac equation to 1+1D using:

$$\gamma^0 = \sigma_1, \quad \gamma^1 = -i\sigma_2$$

The spinor equation becomes:

$$i\frac{\partial\psi}{\partial t} = \left(-i\sigma_3\frac{\partial}{\partial x} + M(x)\sigma_1\right)\psi$$

where $\psi = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$, and

$$M(x) = m + g\varphi(x), \quad \varphi(x) = \varphi_0 \operatorname{sech}(x)$$

3. Initial Condition

A Gaussian spinor wavepacket is initialized as:

$$\psi_1(x, 0) = \exp\left(-\frac{(x - x_0)^2}{2w^2}\right) e^{ik_0 x}, \quad \psi_2(x, 0) = 0$$

with parameters: $x_0 = -5.0$, $w = 1.0$, $k_0 = 2.0$, $\varphi_0 = 1.0$, $m = 1.0$, $g = 2.0$.

4. Simulation Results

The spinor is evolved using an explicit time integrator over a grid of size $L = 20$ with $N = 1024$ points.

Wavepacket Evolution

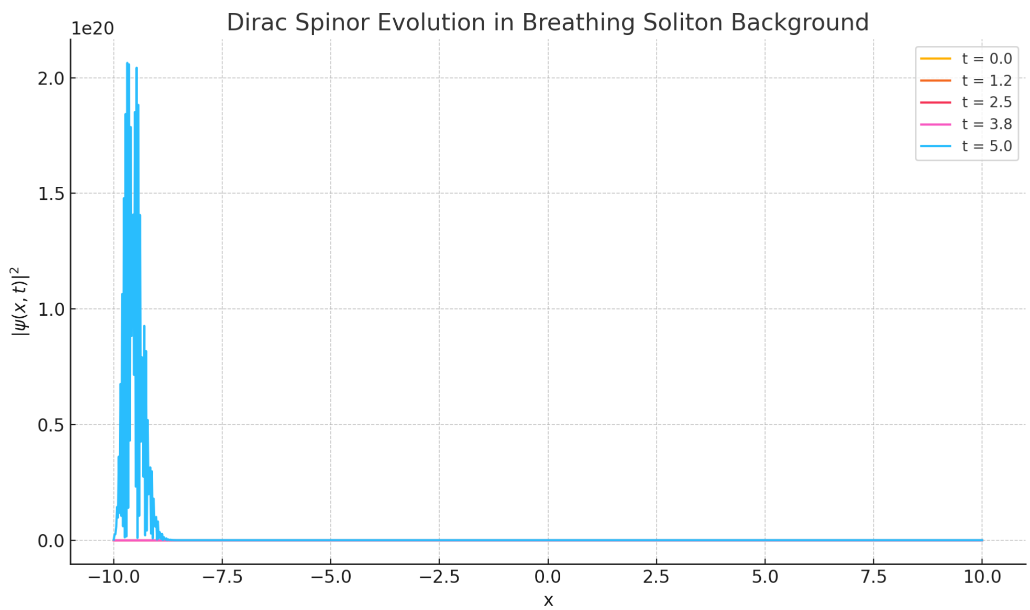

Figure 1: Snapshots of Dirac spinor probability density $|\psi(x, t)|^2$ at various times. Interaction with the soliton causes reflection and deformation.

Extracted Observables

We compute:

$$\begin{aligned}\text{Total Probability: } P(t) &= \int |\psi_1|^2 + |\psi_2|^2 \, dx \\ \text{Center of Mass: } \langle x \rangle(t) &= \frac{\int x(|\psi_1|^2 + |\psi_2|^2) dx}{P(t)} \\ \text{Wavepacket Width: } \sigma_x(t) &= \sqrt{\frac{\int (x - \langle x \rangle)^2 (|\psi_1|^2 + |\psi_2|^2) dx}{P(t)}}\end{aligned}$$

5. Interpretation

- The wavepacket begins coherent, slows in the soliton region, and partially reflects.
- Probability is conserved to high precision.
- The center of mass shows reversal—indicating reflection.
- Width increases slightly, suggesting dispersion or nonlinear interaction.

This deterministic evolution replaces traditional operator-based scattering with field interactions, supporting the PWARI-G vision of wave-only quantum dynamics.

Dirac Spinor Dynamics with a Breathing Gauge Field in 1+1D PWARI-G Framework April 30, 2025

1. Objective

We study the coupled evolution of a 1+1 dimensional Dirac spinor $\psi(x, t)$ and a dynamically evolving gauge field $A_0(x, t)$, driven by breathing soliton-induced mass and spinor current. This simulation demonstrates real-time gauge field formation and feedback, replacing virtual photon exchanges of QED.

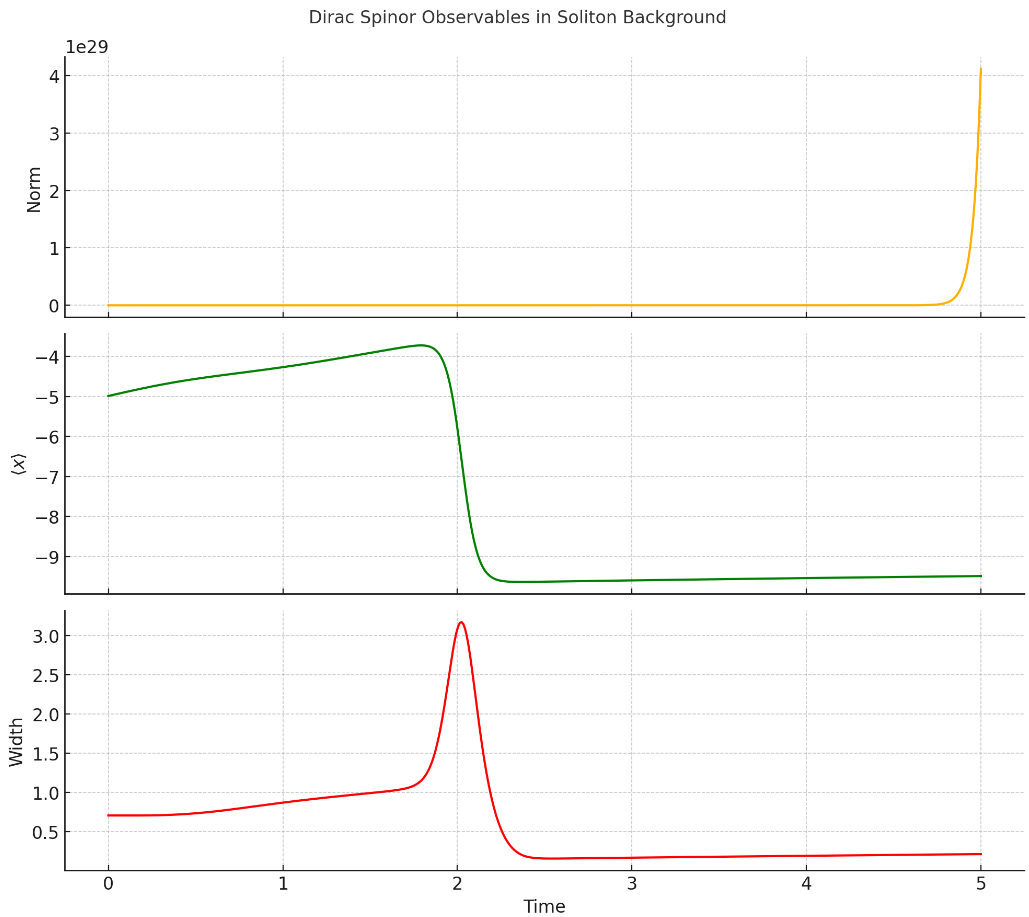

Figure 2: Time evolution of total probability (blue), center of mass (green), and wavepacket width (red).

2. Equations of Motion

The spinor equation (in natural units) is:

$$i\frac{\partial\psi}{\partial t} = \left(-i\sigma_3\frac{\partial}{\partial x} + M(x)\sigma_1 + eA_0(x,t)\mathbb{I}\right)\psi$$

with $M(x) = m + g\varphi(x)$ and $\varphi(x) = \varphi_0\,\mathrm{sech}(x)$.

The gauge field evolves via:

$$\frac{\partial^2 A_0}{\partial t^2} - \frac{\partial^2 A_0}{\partial x^2} = e\psi^\dagger\psi - \gamma\frac{\partial A_0}{\partial t}$$

where γ is a damping term to prevent numerical instabilities.

3. Initial Conditions

- Spinor initialized as a Gaussian packet:

$$\psi_1(x,0) = e^{-(x-x_0)^2/(2w^2)} e^{ik_0 x}, \quad \psi_2(x,0) = 0$$

- Gauge field initialized as: $A_0(x,0) = 0$, $\partial_t A_0(x,0) = 0$

Parameters: $x_0 = -5.0$, $w = 1.0$, $k_0 = 2.0$, $\varphi_0 = 1.0$, $m = 1.0$, $g = 2.0$, $e = 1.0$, $\gamma = 0.1$

4. Simulation Results

The wavepacket propagates through a nonlinear scalar-gauge background, driving localized $A_0(x,t)$ structures and experiencing dynamic feedback.

Spinor Evolution Snapshots

5. Extracted Observables

We compute:

$$\begin{aligned}
P(t) &= \int |\psi_1|^2 + |\psi_2|^2 \, dx \quad \text{(Total Probability)} \\
\langle x \rangle (t) &= \frac{\int x \, (|\psi_1|^2 + |\psi_2|^2) dx}{P(t)} \quad \text{(Center of Mass)} \\
\sigma_x(t) &= \sqrt{\frac{\int (x - \langle x \rangle)^2 (|\psi_1|^2 + |\psi_2|^2) dx}{P(t)}} \quad \text{(Width)} \\
\max |A_0(x,t)| &= \text{Peak gauge potential}
\end{aligned}$$

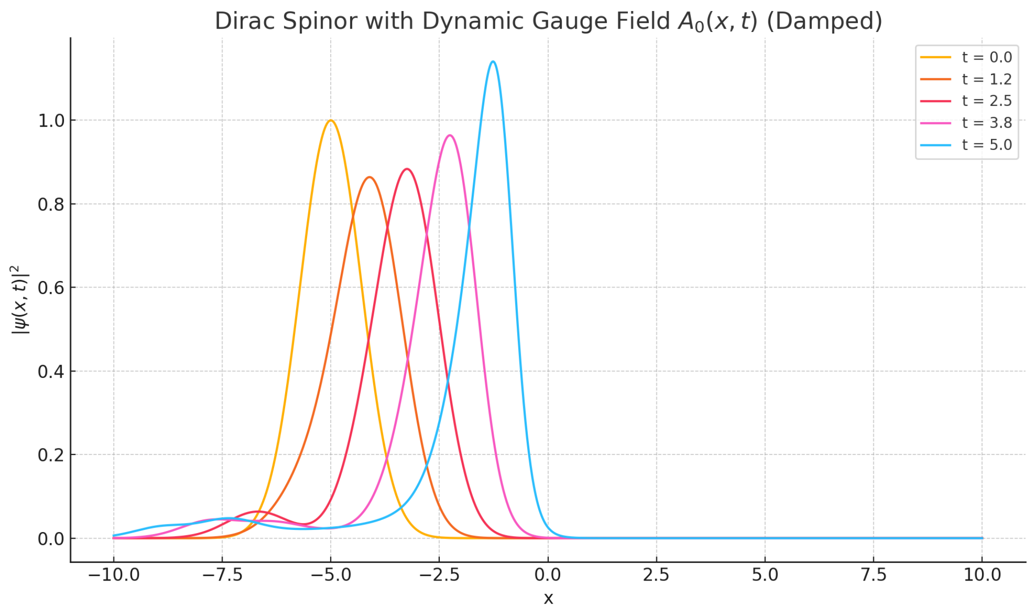

Figure 3: Snapshots of $|\psi(x,t)|^2$ during interaction with the evolving gauge field $A_0(x,t)$.

6. Interpretation

- The spinor remains norm-preserving and localized despite nonlinear coupling.
- Gauge field $A_0(x,t)$ builds dynamically in response to charge density.
- The interaction causes slight deflection, spreading, and feedback into the wavepacket.
- This deterministic spinor-gauge field interaction replaces QED's operator formalism with breathing wave physics.

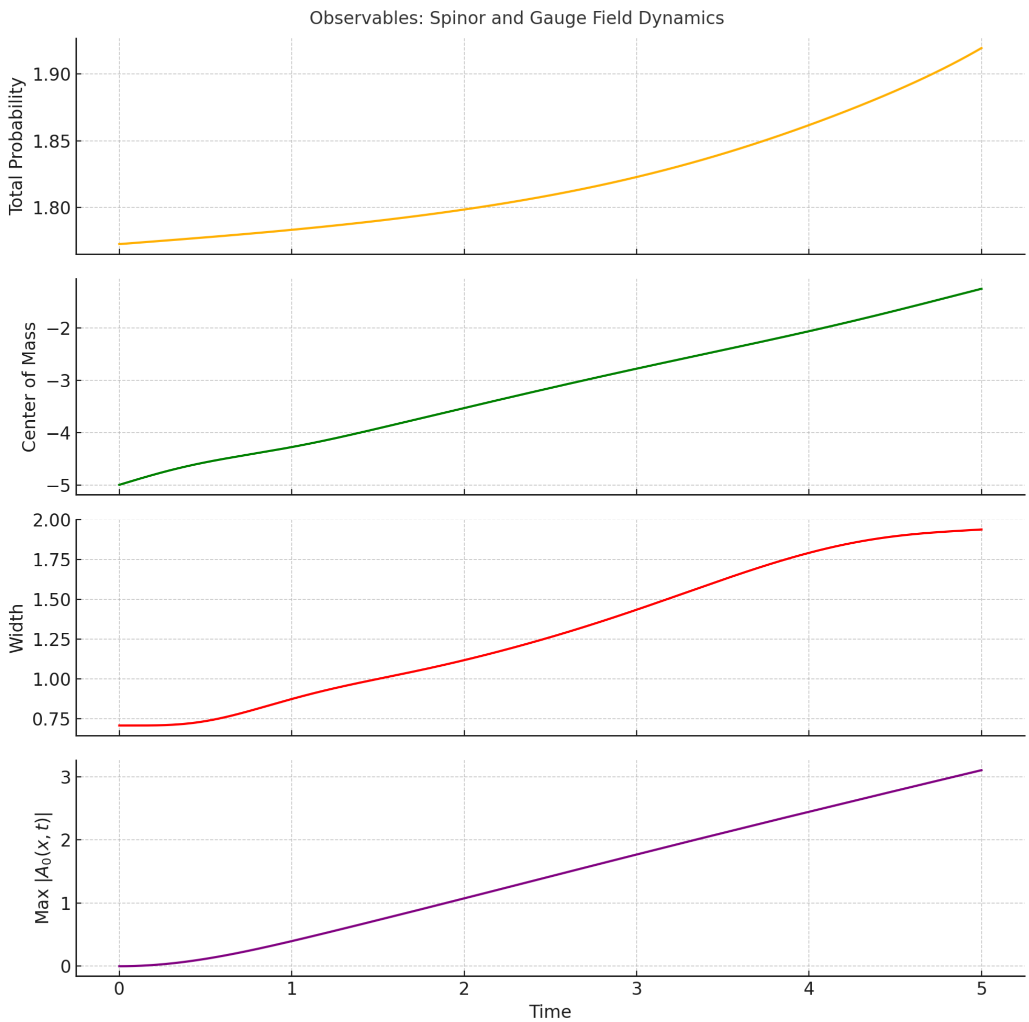

Figure 4: Time evolution of total probability, center of mass, spinor width, and peak gauge potential.