PWARI-G Derivation of Atomic Structure: Helium and Lithium from Merged Soliton Cores

PWARI-G Framework

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Abstract

This document presents a full derivation of atomic orbital structure in the PWARI-G framework, focusing on helium and lithium. We treat atoms as merged configurations of scalar solitons, and derive quantized twist modes (orbitals) from the resulting scalar field potential. Quantization, exclusion, and shell structure emerge naturally from field dynamics, without invoking postulated quantum numbers or spin.

1 Soliton Core Construction for Helium

We begin with the field ansatz for a single breathing scalar soliton:

$$\phi(x,t) = \phi_0(x)\cos(\omega t), \tag{1}$$

where $\phi_0(x)$ is localized and ω is the breathing frequency. The energy density of this soliton is

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi), \tag{2}$$

with potential $V(\phi) = \frac{1}{4}\lambda\phi^4$.

To construct helium, we model it as a tetrahedral merger of four such solitons. The spatial superposition:

$$\phi(x,0) = \sum_{i=1}^{4} \phi_0(x - x_i), \tag{3}$$

with x_i at tetrahedral positions, reorganizes into a deeper merged configuration:

$$\phi_4(x,t) = A' e^{-r^2/R'^2}\cos(\omega' t), \tag{4}$$

where $A' \sim 2A$ and $R' < R$.

Mass Defect and Energy Redistribution

Total initial energy:

$$E_{\text{init}} = 4E_{\phi} + E_{\text{int}}, \tag{5}$$

Final energy:

$$E_{\text{final}} = E_{\phi_{\text{merged}}} + E_{\theta} + E_{\text{radiated}}, \tag{6}$$

where E_{θ} is the bound twist energy and E_{radiated} includes emitted twist waves and internal strain release.

Energy difference:

$$\Delta E = E_{\text{init}} - E_{\text{final}} > 0, \tag{7}$$

which reflects twist emission, strain relief, and nonlinear consolidation into a lower-energy breathing mode.

2 Twist Mode Quantization

The twist field evolves according to:

$$\ddot{\theta} = \nabla^2\theta - \phi^2(x)\theta. \tag{8}$$

With ansatz $\theta(x,t) = u(r)e^{-i\omega t}$, this becomes:

$$\left[-\nabla^2 + \phi^2(r)\right] u(r) = \omega^2 u(r). \tag{9}$$

Assuming $\phi^2(r) = A^2 e^{-2r^2/R^2}$ and changing to dimensionless units $x = r/R$, we approximate the potential:

$$V(x) \approx V_0 R^2 - 2V_0 R^2 x^2, \tag{10}$$

yielding harmonic oscillator energy levels:

$$\omega_n^2 \approx \frac{V_0 R^2 + (2n+1)\sqrt{2V_0 R^2}}{R^2}. \tag{11}$$

The energy spacing between levels scales as:

$$\Delta\omega \propto \frac{\sqrt{V_0}}{R}. \tag{12}$$

Deeper ϕ wells and tighter cores increase level spacing.

3 Pauli Exclusion from Twist Interference

Two identical twist modes:

$$\theta_1(x,t) = u_1(x)\cos(\omega t),\ \theta_2(x,t) = u_1(x)\sin(\omega t), \tag{13}$$

are orthogonal and stable. A third mode leads to destructive interference:

$$\theta_3(x,t) = u_1(x)\cos(\omega t + \delta), \quad \delta = \pi \Rightarrow \text{cancellation}. \tag{14}$$

Thus the 1s shell is limited to two orthogonal modes — a natural origin for Pauli exclusion.

4 Angular Momentum Orbitals

Using spherical harmonics:

$$\theta(x,t) = u_{n\ell}(r) Y_{\ell m}(\theta,\varphi)\cos(\omega t), \tag{15}$$

we find angular modes (p, d, f) arise from nonzero ℓ. Radial equation becomes:

$$-\frac{d^2u}{dr^2} - \frac{2}{r}\frac{du}{dr} + \left[\frac{\ell(\ell+1)}{r^2} + \phi^2(r)\right] u = \omega^2 u. \tag{16}$$

The twist angular momentum is:

$$L_z = \int (x\partial_y\theta - y\partial_x\theta)\dot{\theta}, d^3x \propto m, \tag{17}$$

where m corresponds to azimuthal structure of the twist field.

5 Lithium: 7-Soliton Core and 2s Shell

A merged 7-soliton ϕ-core allows an additional twist eigenmode:

- 1s: two orthogonal twist modes (closed)
- 2s: one radial node, delocalized twist, supports third electron

The structure naturally explains:

- Shell closure via twist interference
- Shell opening from eigenmode capacity
- Periodic structure from soliton merge geometry and field depth

6 Carbon: 12-Soliton Core and Emergence of 2p Orbitals

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7 Argon: 18-Soliton Core and Completion of the Third Shell

Argon (Z = 18) consists of 18 protons and 22 neutrons, totaling 40 nucleons. In PWARI-G, we interpret this as a 40-soliton merged ϕ-core. This deeper, broader core supports multiple radial and angular twist eigenmodes, enabling full filling of three shells.

Expected Twist Structure

- 1s: two orthogonal twist modes (closed)
- 2s: two orthogonal twist modes (closed)
- 2p: three angular twist modes $\times$ two phase polarizations = six electrons (closed)
- 3s: two orthogonal twist modes
- 3p: three angular twist modes $\times$ two phase polarizations = six electrons

Total electron support: $2 + 2 + 6 + 2 + 6 = 18$ electrons, matching argon's configuration $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$.

Shell Structure and Field Mode Completion

Each shell corresponds to radial and angular twist modes supported by the soliton core:

- Twist eigenmodes increase with ϕ^2 depth and spatial confinement
- Higher shells arise from nodes and angular gradients in twist standing waves
- Full shells occur when no additional orthogonal twist fields can occupy a given mode

Stability and Noble Gas Completion

The 18-soliton argon core provides:

- Stable, phase-balanced twist interference
- Complete exhaustion of all eigenmodes up to 3p
- Inert behavior explained by lack of available field slots for bonding twist fields

This framework explains argon's noble gas closure without invoking quantum orbitals: it arises from deterministic, orthogonal field saturation.

8 Twist Mode Periodic Table

To summarize the PWARI-G atomic structure across elements, we construct a twist mode periodic table showing how soliton core size determines orbital structure.

Element	**Z**	**Solitons**	**Twist Orbitals Filled**	**Configuration**
Hydrogen	1	1	$1s^1$	$1s^1$
Helium	2	4	$1s^2$	3
Lithium	7	$1s^2$, $2s^1$	$1s^2$ $2s^1$	
Beryllium	4	9	$1s^2$, $2s^2$	$1s^2$ $2s^2$
Boron	5	11	$1s^2$, $2s^2$, $2p^1$	$1s^2$ $2s^2$ $2p^1$
Carbon	12	$1s^2$, $2s^2$, $2p^2$	$1s^2$ $2s^2$ $2p^2$	
Nitrogen	7	14	$1s^2$, $2s^2$, $2p^3$	$1s^2$ $2s^2$ $2p^3$
Oxygen	8	16	$1s^2$, $2s^2$, $2p^4$	$1s^2$ $2s^2$ $2p^4$
Fluorine	17	$1s^2$, $2s^2$, $2p^5$	$1s^2$ $2s^2$ $2p^5$	
Neon	10	18	$1s^2$, $2s^2$, $2p^6$	$1s^2$ $2s^2$ $2p^6$
Sodium	11	23	$1s^2$, $2s^2$, $2p^6$, $3s^1$	$1s^2$ $2s^2$ $2p^6$ $3s^1$
Magnesium	24	$1s^2$, $2s^2$, $2p^6$, $3s^2$	$1s^2$ $2s^2$ $2p^6$ $3s^2$	
Aluminum	13	27	$1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^1$	$1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^1$
Silicon	14	28	$1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^2$	$1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^2$
Phosphorus	31	$1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^3$	$1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^3$	
Sulfur	16	32	$1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^4$	$1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^4$
Chlorine	17	35	$1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^5$	$1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^5$
Argon	40	$1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^6$	$1s^2$ $2s^2$ $2p^6$ $3s^2$ $3p^6$	

Each entry reflects the number of twist eigenmodes supported by the soliton core. As Z increases, more radial and angular modes emerge, forming a twist-mode-based periodic structure that matches observed electron configurations.

Conclusion

We have derived atomic orbital structure in PWARI-G from first principles, using soliton core mergers and twist wave quantization. This framework reproduces known features of helium, lithium, carbon, and argon, and explains Pauli exclusion, shell structure, angular momentum, and orbital degeneracy without postulates. All properties arise from field interference and soliton geometry, providing a predictive foundation for the periodic structure of matter.