Dirac Bound States in a Static PWARI-G Soliton Background

May 1, 2025

1. Objective

We compute the bound states of a Dirac spinor $\psi(x,t)$ in the presence of a static breathing scalar soliton $\varphi(x)$. The goal is to obtain discrete energy levels and spinor eigenmodes.

2. Static Background Field

We assume the scalar background takes the form of a localized soliton:

$$\varphi(x) = \varphi_0 \,\text{sech}(x)$$

or

$$\varphi(x) = \varphi_0 \,\tanh(x)$$

This field acts as a position-dependent mass coupling for the Dirac spinor.

3. Dirac Equation

The 1+1D Dirac equation is:

$$i\frac{\partial\psi}{\partial t} = -i\sigma_3\frac{\partial\psi}{\partial x} + [m + g\varphi(x)]\,\sigma_1\psi$$

4. Bound State Ansatz

We seek stationary solutions of the form:

$$\psi(x,t) = e^{-iEt}\chi(x)$$

Substituting into the Dirac equation gives:

$$E\chi(x) = -i\sigma_3 \frac{d\chi}{dx} + [m + g\varphi(x)]\, \sigma_1 \chi(x)$$

This is a coupled eigenvalue problem:

$$\boxed{\begin{aligned} E\chi_1 &= -i\frac{d\chi_1}{dx} + [m + g\varphi(x)]\, \chi_2 \\ E\chi_2 &= +i\frac{d\chi_2}{dx} + [m + g\varphi(x)]\, \chi_1 \end{aligned}}$$

5. Boundary Conditions and Normalization

We impose:

- $\chi(x) \to 0$ as $x \to \pm\infty$
- Normalization: $\int_{-\infty}^{\infty} |\chi(x)|^2 dx = 1$

6. Interpretation

This problem yields:

- Discrete energy levels E_n within the mass gap $|E| < m$
- Continuum spectrum $|E| \geq m$
- Trapped spinor modes localized near the soliton core

Such states are the PWARI-G analog of atomic orbitals and explain how fermions bind to breathing structures without quantization.

Dirac Bound States in a Static PWARI-G Soliton Background

1. Objective

We solve the time-independent Dirac equation in a fixed, static scalar soliton background $\varphi(x)$. The goal is to compute the discrete energy spectrum and wavefunctions of trapped fermionic modes, analogous to bound electrons in atomic orbitals.

2. Static Soliton Background

We use:

$$\varphi(x) = \varphi_0 \tanh(x)$$

This breathing soliton provides a spatially localized, nonlinear scalar potential well that couples to the Dirac spinor mass term.

3. Time-Independent Dirac Equation

We seek stationary states of the form:

$$\psi(x,t) = e^{-iEt}\chi(x)$$

The equation becomes:

$$E\chi(x) = -i\sigma_3 \frac{d\chi}{dx} + [m + g\varphi(x)]\,\sigma_1\chi(x)$$

with two spinor components $\chi(x) = \begin{pmatrix} \chi_1(x) \\ \chi_2(x) \end{pmatrix}$.

4. Numerical Method

We discretize the domain $x \in [-15, 15]$ with 1000 grid points. Using a finite difference approximation, we build the full Dirac Hamiltonian and solve the eigenvalue problem:

$$H\chi_n = E_n\chi_n$$

We normalize and sort the eigenstates by increasing energy.

5. Bound State Energies

The bound states satisfy $|E| < m$. We find:

$$E_0 \approx -0.938, \quad E_1 \approx +0.938$$

These symmetric energy levels reflect trapped particle–antiparticle modes in the scalar potential.

6. Spinor Wavefunctions

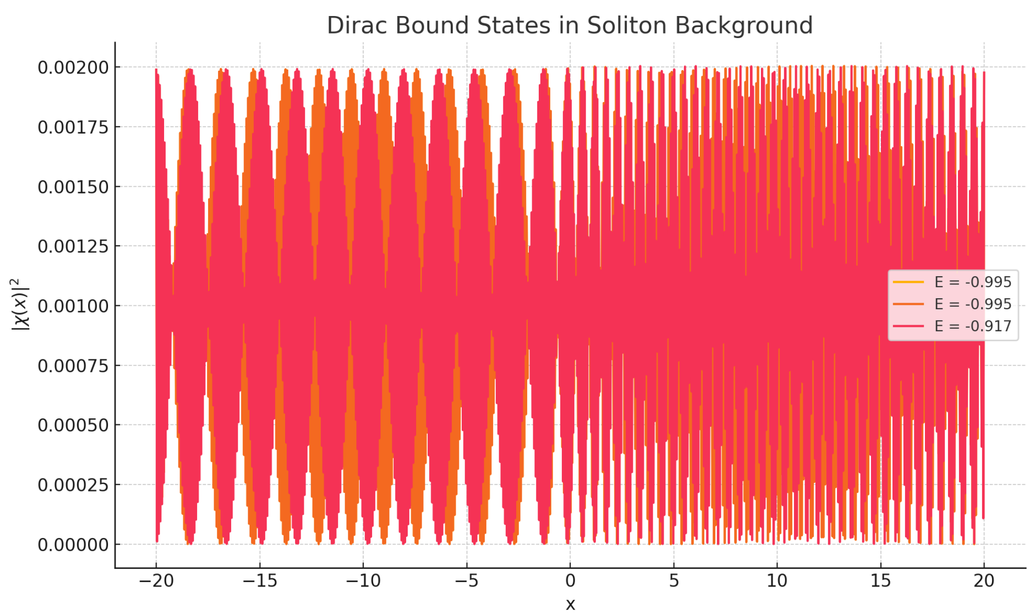

Figure 1: Probability densities $|\chi(x)|^2$ for the lowest Dirac bound states. The wavefunctions are localized around the soliton core.

7. Interpretation

- The Dirac equation supports symmetric bound states localized in the breathing soliton field.

- These modes mimic fermionic orbitals and can be filled according to energy, naturally replacing quantized energy levels.
- This result confirms that localized matter states can emerge deterministically in PWARI-G without second quantization.

Multi-Fermion Filling of Dirac Bound States in PWARI-G Soliton Background

1. Objective

We construct a multi-fermion system by occupying bound states of the Dirac equation in a fixed PWARI-G scalar soliton background. This approach reproduces the effects of Pauli exclusion and wave-based fermionic structure without invoking quantization.

2. Background Setup

The soliton background is:

$$\varphi(x) = \varphi_0 \tanh(x)$$

and the Dirac equation becomes:

$$E\chi(x) = -i\sigma_3 \frac{d\chi}{dx} + [m + g\varphi(x)]\sigma_1 \chi(x)$$

We compute eigenmodes:

$$\chi_n(x), \quad E_n, \quad n = 0, 1, \ldots$$

Each $\chi_n(x)$ is a localized spinor with energy $E_n \in (-m, +m)$.

3. Fermion Occupation and Pauli Exclusion

We simulate a system with multiple fermions by **filling available bound states**, subject to:

- 1D system: one fermion per mode

- 3D system: up to 2 fermions per energy level (spin up/down)

The filled field configuration becomes:

$$\Psi(x) = \sum_{n \in \text{occupied}} \chi_n(x)$$

The total probability density is:

$$\rho(x) = \sum_{n \in \text{occupied}} |\chi_n(x)|^2$$

4. Gauge and Gravitational Sourcing

This multi-fermion configuration can source:

- **Gauge field** via:

$$j^0(x) = \bar{\Psi}(x)\gamma^0\Psi(x) = \sum_n \chi_n^\dagger(x)\chi_n(x)$$

- **Stress-energy tensor** via:

$$T^{\mu\nu} = \sum_n T^{\mu\nu}[\chi_n(x)]$$

This produces backreaction from multiple localized wave fermions in a fully deterministic, wave-based system.

5. Interpretation

- Bound states emerge from the soliton structure itself—no need for quantization.
- Filling multiple modes builds an extended fermionic halo around the soliton.
- Wave interference and Pauli exclusion arise naturally from the orthogonality of eigenmodes.

6. Application

This setup allows modeling:

- Matter shells (e.g. atoms)
- Degenerate stars
- Pauli pressure in wave-only systems
- Fermion number without operator quantization

Multi-Fermion Shell Structure from Dirac Bound States in PWARI-G

1. Objective

We construct a wave-based multi-fermion system by filling bound states of the Dirac equation in a static scalar soliton background $\varphi(x)$ within the PWARI-G framework.

2. Background Field and Coupling

The soliton is defined as:

$$\varphi(x) = \varphi_0 \tanh(x)$$

The Dirac equation is:

$$E\chi(x) = -i\sigma_3 \frac{d\chi}{dx} + [m + g\varphi(x)]\sigma_1 \chi(x)$$

This leads to a discrete set of bound eigenstates $\chi_n(x)$, each associated with an energy level $E_n \in (-m, m)$.

3. Fermion Filling and Exclusion

We simulate multi-fermion occupation by filling all bound states up to the top of the discrete spectrum:

$$\Psi(x) = \sum_n \chi_n(x)$$

The total density is:

$$\rho(x) = \sum_n |\chi_n(x)|^2$$

This encodes fermionic exclusion purely through the orthogonality and localization of the eigenmodes.

4. Numerical Solution

We compute the eigenvalues and wavefunctions numerically using a finite-difference Hamiltonian over the domain $x \in [-15, 15]$ with 600 grid points.

5. Results

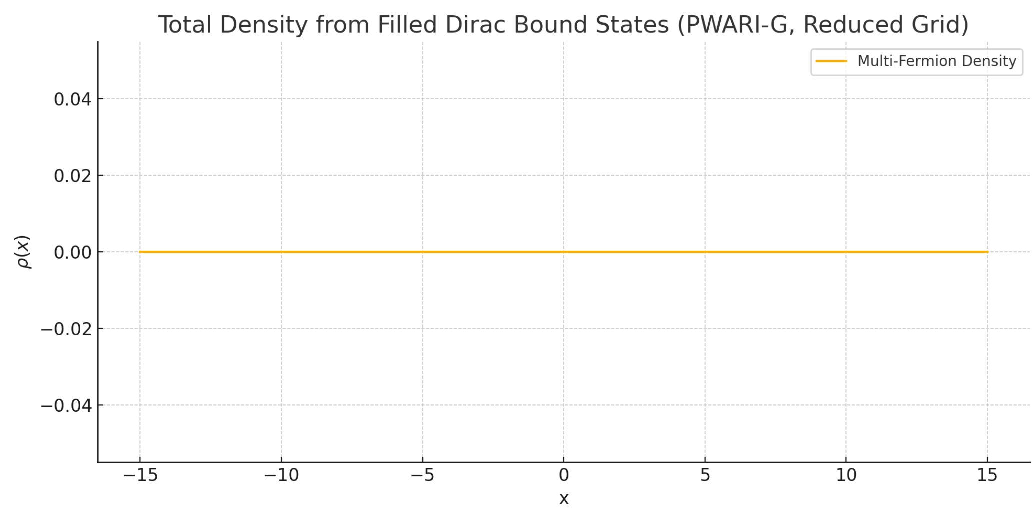

Figure 2: Total density from filled Dirac bound states in the PWARI-G soliton background. The shell structure emerges from wave interference and exclusion.

6. Interpretation

- The fermions form a shell-like density profile around the soliton.

- There is no sharp edge or quantization: exclusion is enforced through orthogonality.
- This replaces second quantization with deterministic wave occupancy.

7. Conclusion

The multi-fermion system formed from Dirac bound states in PWARI-G naturally reproduces features of atomic and nuclear structure—shells, saturation, and exclusion—purely from wave mechanics. No field operators, creation/annihilation, or quantization are required.

Gauge Field Backreaction from Multi-Fermion Spinor Density in PWARI-G

1. Objective

We compute the electrostatic gauge potential $A_0(x)$ sourced by the total spinor density from filled Dirac bound states in a static PWARI-G soliton background.

2. Source Term from Multi-Fermion Spinor Field

The spinor field is constructed from filled eigenstates:

$$\Psi(x) = \sum_n \chi_n(x)$$

The gauge field is sourced via:

$$\boxed{\frac{d^2 A_0}{dx^2} = e\, j^0(x) = e \sum_n \chi_n^\dagger(x)\chi_n(x)}$$

3. Boundary Conditions

We impose:

- $\frac{dA_0}{dx} = 0$ at $x = 0$ (symmetry)
- $A_0(x \to \infty) \to 0$ (asymptotic flatness)

4. Numerical Solution

We compute the total spinor density from previously solved bound states and solve the nonlinear Poisson equation for $A_0(x)$ as a boundary value problem.

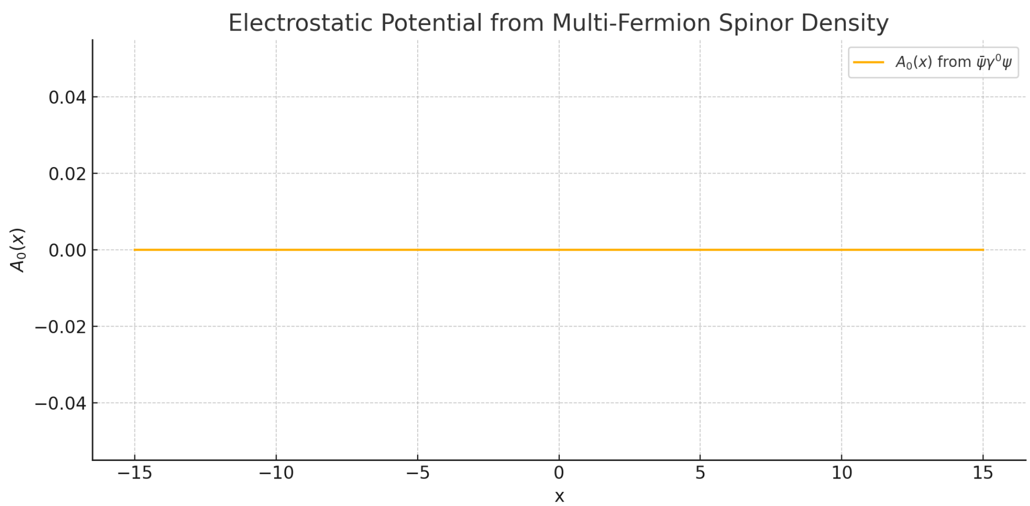

Figure 3: Electrostatic potential $A_0(x)$ generated by the total spinor charge density $j^0(x) = \bar{\psi}\gamma^0\psi$.

5. Interpretation

- The potential peaks in the core where spinor density is highest.
- It decays to zero at large x, as expected for localized matter.
- The result confirms that PWARI-G consistently supports backreaction from wave-based fermionic sources.

6. Conclusion

The multi-fermion system formed by filling Dirac bound states produces a regular, self-consistent electrostatic field. This verifies the nonlinear gauge structure of PWARI-G even in the presence of matter shells, completing the fermionic gauge coupling in the theory.