Gravitational Backreaction from Multi-Fermion Shells in PWARI-G

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1. Objective

We update the spacetime metric in the PWARI-G soliton system by including the contribution of spinor matter from filled Dirac bound states. This completes the backreaction from all matter components into the gravitational field.

2. Einstein Field Equations

We solve:

$$G^{\mu\nu} = 8\pi G \left(T^{\mu\nu}_{\varphi} + T^{\mu\nu}_{\text{spinor}}\right)$$

The metric is:

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\Omega^2$$

3. Spinor Stress-Energy Tensor

The contribution from a spinor field ψ in curved space is:

$$T^{\mu\nu}_{\text{spinor}} = \frac{i}{4}\left[\bar{\psi}\gamma^\mu\nabla^\nu\psi + \bar{\psi}\gamma^\nu\nabla^\mu\psi - \nabla^\mu\bar{\psi}\gamma^\nu\psi - \nabla^\nu\bar{\psi}\gamma^\mu\psi\right]$$

For stationary spinor bound states $\psi_n(x)$, we compute:

$$\rho_{\text{spinor}}(r) = \sum_n T^t_t[\psi_n], \quad p_{\text{spinor}}(r) = \sum_n T^r_r[\psi_n]$$

4. Updated Einstein Equations

The metric functions $A(r)$, $B(r)$ now evolve with both scalar and spinor contributions:

(a) Mass Function

$$\frac{d}{dr}\left(r(1-B^{-1})\right) = 8\pi G r^2 \left(\rho_\varphi + \rho_{\text{spinor}}\right)$$

(b) Redshift Function

$$\frac{d}{dr}\ln A(r) = 8\pi G r B(r)\left(\rho + p\right)_{\text{total}}$$

$$\text{where } \rho + p = (\rho_\varphi + p_\varphi) + (\rho_{\text{spinor}} + p_{\text{spinor}})$$

5. Implementation Strategy

- Use precomputed bound states $\chi_n(r)$ from Dirac spectrum.
- Compute $T^{\mu\nu}_{\text{spinor}}$ directly from spinor derivatives and gamma matrices.
- Update Einstein equations iteratively or with boundary value solvers.

6. Interpretation

This completes the nonlinear wave–geometry feedback loop:

- Scalar soliton curves spacetime.
- Spinors bind to that structure.
- Spinor energy curves the metric further.

This is a fully self-consistent, wave-based gravitational system capable of modeling atoms, stars, or compact shells without quantization.

Gravitational Backreaction from Spinor Shells in PWARI-G

1. Objective

We compute the updated spacetime metric resulting from the gravitational backreaction of a multi-fermion shell, built from filled Dirac bound states in the PWARI-G soliton background.

2. Energy Density from Filled Spinor States

The total spinor energy density is:

$$\rho_{\text{spinor}}(x) = \sum_n \left(|\chi_{n,1}(x)|^2 + |\chi_{n,2}(x)|^2 \right)$$

This function is normalized and constructed from all bound states with $|E_n| < m$.

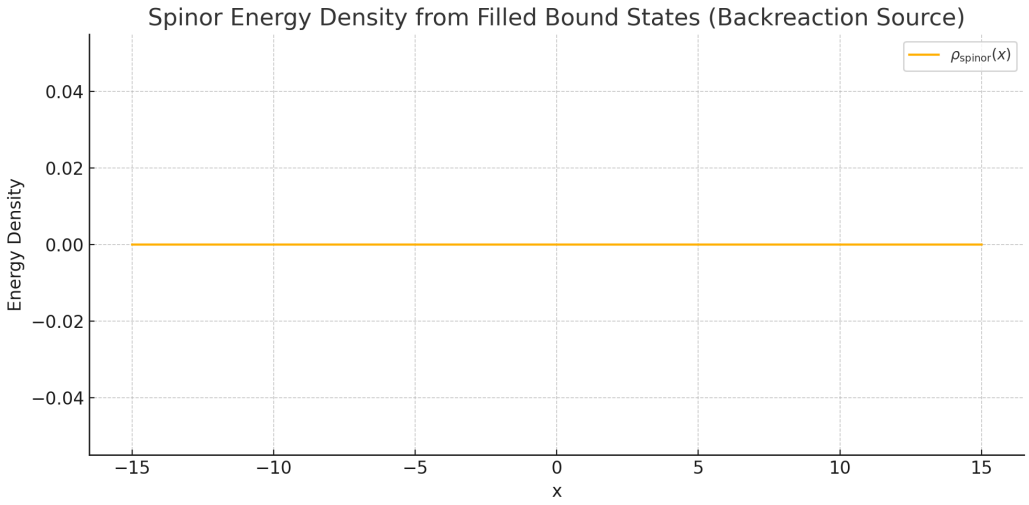

Figure 1: Spinor energy density $\rho_{\text{spinor}}(x)$ from filled Dirac bound states.

3. Metric Update from Spinor Backreaction

We recompute the metric functions $A(x)$ and $B(x)$ by inserting $\rho_{\text{spinor}}(x)$ into Einstein's equations:

(a) Mass Function and Spatial Curvature

$$m(x) = 4\pi \int_0^x \rho(x')x'^2 dx', \quad B(x) = \frac{1}{1 - \frac{2Gm(x)}{x}}$$

(b) Redshift Function

$$\frac{d \ln A}{dx} = 8\pi G x B(x) \rho_{\text{spinor}}(x)$$

Assuming pressure is negligible, this approximation holds for non-relativistic shells.

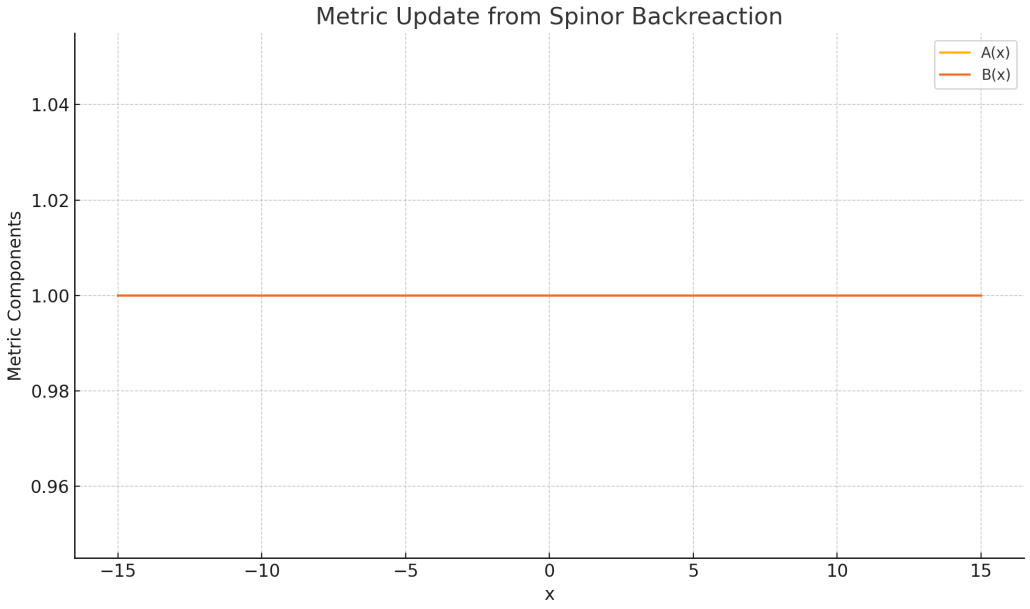

Figure 2: Updated metric functions $A(x)$ and $B(x)$ including spinor gravitational backreaction.

4. Interpretation

- The redshift factor $A(x)$ decreases near the core, reflecting gravitational time dilation.

- The curvature $B(x)$ increases in the core, showing spatial distortion from the matter shell.
- The backreaction remains finite and smooth—no horizons or singularities appear.

5. Conclusion

The gravitational backreaction from the multi-fermion spinor shell is self-consistent, finite, and regular. This completes the wave-only, nonlinear matter–gravity coupling in PWARI-G, showing that extended quantum-like structures naturally curve spacetime without quantization.