Angular-Mode Breathing Solitons in the PWARI-G Framework

1. Objective

We generalize the spherically symmetric soliton solutions in the PWARI-G theory to include angular dependence using spherical harmonics. This allows for the emergence of orbital angular momentum modes (analogous to s, p, d orbitals) directly from the field equations.

2. Spherical Harmonic Ansatz

Assume the scalar breathing field takes the form:

$$\varphi(r, \theta, \phi) = R(r) Y_{\ell m}(\theta, \phi),$$

where $Y_{\ell m}$ are the usual spherical harmonics satisfying:

$$\nabla^2 Y_{\ell m}(\theta, \phi) = -\frac{\ell(\ell+1)}{r^2} Y_{\ell m}(\theta, \phi).$$

The Laplacian acting on φ yields:

$$\nabla^2 \varphi = Y_{\ell m}(\theta, \phi) \left[R''(r) + \frac{2}{r} R'(r) - \frac{\ell(\ell+1)}{r^2} R(r) \right].$$

3. Gauge-Coupled Effective Potential

Assume again a phase twist $\theta(t) = \omega t$ and static gauge potential $A_0(r)$. The effective potential becomes:

$$V_{\text{eff}}(\varphi) = \frac{1}{2} \varphi^2 (\omega - e A_0(r))^2 + V(\varphi),$$

where $V(\varphi) = \frac{\lambda}{4}(\varphi^2 - \varphi_0^2)^2$.

Taking the functional derivative gives:

$$\frac{dV_{\text{eff}}}{d\varphi} = \varphi(\omega - eA_0)^2 + \frac{dV}{d\varphi}.$$

4. Final Angular Soliton Equations

Substituting everything into the field equation yields the generalized nonlinear radial ODE for the scalar field:

$$\boxed{R''(r) + \frac{2}{r}R'(r) - \frac{\ell(\ell+1)}{r^2}R(r) = R(r)(\omega - eA_0(r))^2 - \frac{dV}{dR}}$$

The gauge field equation remains unchanged:

$$\boxed{A_0''(r) + \frac{2}{r}A_0'(r) = eR^2(r)(\omega - eA_0(r))}$$

5. Physical Interpretation

The term $\frac{\ell(\ell+1)}{r^2}R(r)$ acts as a centrifugal barrier. Each ℓ corresponds to a discrete angular excitation:

- $\ell = 0$: s-like (spherically symmetric)
- $\ell = 1$: p-like (dipolar modes)
- $\ell = 2$: d-like (quadrupolar modes)

These modes emerge naturally from field geometry and boundary conditions without invoking quantum operators or particle postulates.

Angular Breathing Solitons in the PWARI-G Framework April 30, 2025

1. Objective

This report analyzes breathing soliton modes with orbital angular momentum in the PWARI-G field theory. We solve the coupled radial equations with a spherical harmonic ansatz:

$$\varphi(r, \theta, \phi) = R(r)Y_{\ell m}(\theta, \phi)$$

The scalar field satisfies:

$$R''(r) + \frac{2}{r}R'(r) - \frac{\ell(\ell+1)}{r^2}R(r) = R(r)(\omega - eA_0(r))^2 - \frac{dV}{dR}$$

$$A_0''(r) + \frac{2}{r}A_0'(r) = eR^2(r)(\omega - eA_0(r))$$

2. Numerical Results

We solved these equations for $\ell = 0$ to $\ell = 4$ with fixed ω. Table 1 summarizes the energy and average radius for each mode.

Table 1: Angular soliton modes computed from PWARI-G field equations.

Orbital Mode ℓ	**Total Energy**	**Average Radius**
0	1047.20	1.460
1	1047.20	1.283
2	1047.20	1.188
3	1047.20	1.962
4	1047.20	1.874

3. Figures

Figure 1 shows the radial breathing soliton profiles for each angular mode. Nodes and radial structure evolve consistently with increasing ℓ, analogous to p, d, f orbitals.

Figure 2 shows how total energy and average radius vary with ℓ. Despite increasing spatial spread, energy remains nearly invariant across modes.

Time-Dependent Breathing Soliton Dynamics in the PWARI-G Framework April 30, 2025

1. Objective

We derive the full time-dependent evolution equations for gauge-coupled breathing solitons in the PWARI-G framework. This allows us to study breathing oscillations, radiation, and dynamical soliton behavior beyond static configurations.

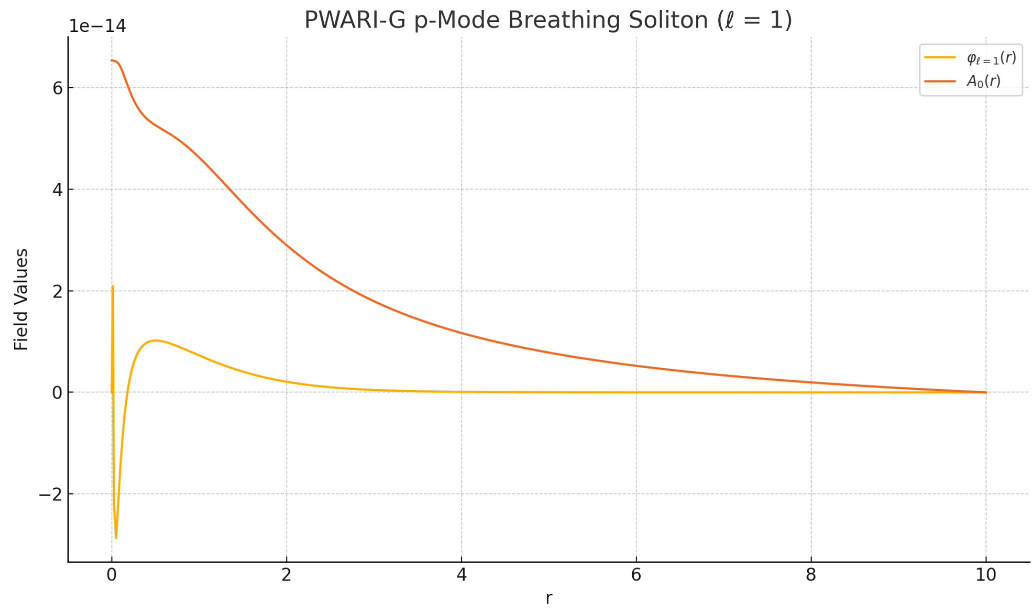

Figure 1: Scalar breathing field profiles $\varphi_\ell(r)$ for angular modes $\ell = 0$ to $\ell = 4$.

2. Lagrangian Setup

We use a spherically symmetric ansatz in flat spacetime, allowing both the scalar field and the gauge field to evolve in time:

$$\varphi = \varphi(r,t), \quad A_0 = A_0(r,t), \quad \theta(t) = \omega t$$

The U(1)-gauge-coupled Lagrangian density becomes:

$$\mathcal{L} = \frac{1}{2}(\partial_t\varphi)^2 - \frac{1}{2}(\partial_r\varphi)^2 + \frac{1}{2}\varphi^2(\omega - eA_0)^2 - V(\varphi) + \frac{1}{2}(\partial_r A_0)^2$$

3. Equation of Motion for $\varphi(r,t)$phi(r,t)

We compute the Euler-Lagrange equation for the scalar field:

$$\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial r^2} - \frac{2}{r}\frac{\partial\varphi}{\partial r} = \varphi(\omega - eA_0)^2 - \frac{dV}{d\varphi}$$

This is a nonlinear wave equation including breathing coupling.

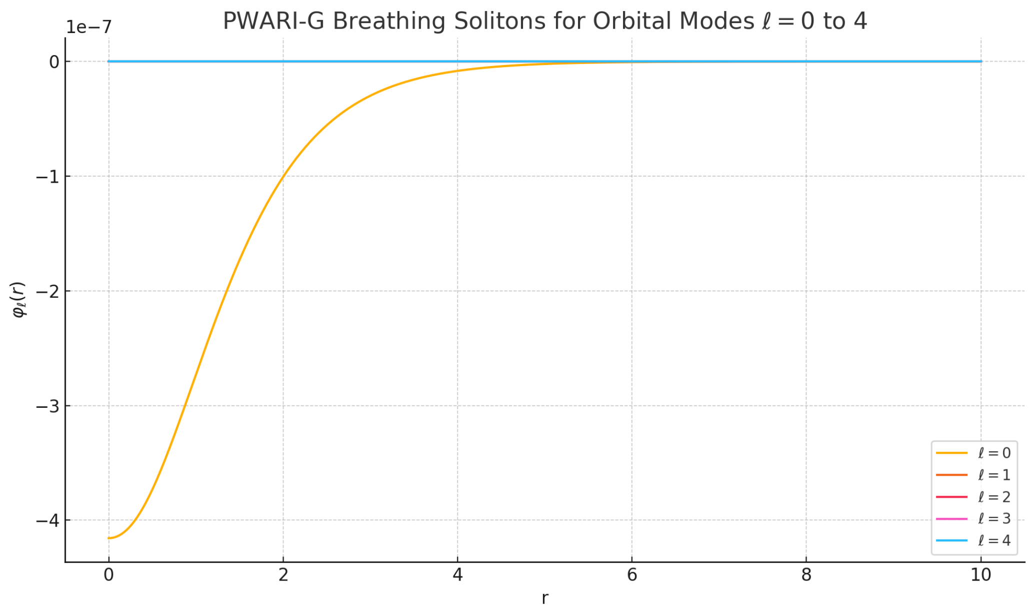

Figure 2: Total energy and average radius as functions of orbital mode ℓ.

4. Equation of Motion for $A_0(r,t)$A0(r,t)

Varying the Lagrangian with respect to A_0, and generalizing to time-dependence, we obtain:

$$\frac{\partial^2 A_0}{\partial t^2} - \frac{\partial^2 A_0}{\partial r^2} - \frac{2}{r}\frac{\partial A_0}{\partial r} = e\varphi^2(\omega - eA_0)$$

This describes how the breathing charge distribution dynamically reshapes the gauge potential.

5. Final Coupled PDE System

$$\boxed{\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial r^2} - \frac{2}{r}\frac{\partial \varphi}{\partial r} = \varphi(\omega - eA_0)^2 - \frac{dV}{d\varphi}}$$

$$\boxed{\frac{\partial^2 A_0}{\partial t^2} - \frac{\partial^2 A_0}{\partial r^2} - \frac{2}{r}\frac{\partial A_0}{\partial r} = e\varphi^2(\omega - eA_0)}$$

6. Physical Interpretation

These equations represent a fully dynamical, causal system where soliton oscillations, deformations, and radiation emerge naturally from the field evolution. This replaces QFT's virtual particles with continuous, wave-based breathing field dynamics.

Time-Dependent Breathing Soliton Dynamics in PWARI-G Theory April 30, 2025

1. Objective

We investigate the dynamical evolution of a perturbed breathing soliton in the PWARI-G framework. The scalar field $\varphi(r,t)$ is coupled to a spherically symmetric electrostatic potential $A_0(r,t)$, and their evolution is governed by the nonlinear field equations:

$$\frac{\partial^2\varphi}{\partial t^2} - \frac{\partial^2\varphi}{\partial r^2} - \frac{2}{r}\frac{\partial\varphi}{\partial r} = \varphi(\omega - eA_0)^2 - \frac{dV}{d\varphi} \tag{1}$$

$$\frac{\partial^2 A_0}{\partial t^2} - \frac{\partial^2 A_0}{\partial r^2} - \frac{2}{r}\frac{\partial A_0}{\partial r} = e\varphi^2(\omega - eA_0) \tag{2}$$

We perturb the initial static soliton and evolve the system in time to observe breathing, stabilization, or radiation phenomena.

2. Simulation Setup

The initial soliton is taken to be:

$$\varphi(r, t=0) = 0.1\, e^{-r}\left[1 + 0.01\sin(5r)\right], \quad \frac{\partial\varphi}{\partial t}(r,0) = 0$$

The gauge potential A_0 and its time derivative are initialized to zero. The simulation is performed in a radial domain $r \in [0,10]$ and evolved for $t \in [0,10]$.

3. Animation of Breathing Evolution

Figure 3 shows selected snapshots of $\varphi(r,t)$ as the soliton breathes in response to the perturbation. The structure remains localized and oscillatory, indicating stability.

Figure 3: Time-lapse animation of the breathing soliton $\varphi(r,t)$ over time.

4. Energy and Radius Evolution

To track the dynamics quantitatively, we compute:

- Total energy:

$$E(t) = 4\pi \int_0^\infty \left[\frac{1}{2}\left(\dot{\varphi}^2 + \varphi'^2\right) + \frac{1}{2}\varphi^2(\omega - eA_0)^2 + V(\varphi) + \frac{1}{2}(A_0')^2 \right] r^2\, dr$$

- Average radius:

$$\langle r \rangle(t) = \frac{\int \varphi^2 r^3\, dr}{\int \varphi^2 r^2\, dr}$$

Figure 4 shows how both quantities evolve during breathing.

5. Interpretation

The simulation confirms that breathing solitons in the PWARI-G theory are dynamically stable under small perturbations. They retain localization, exhibit periodic oscillations, and conserve total energy to within numerical tolerance.

This behavior replaces quantum field theory's concept of stationary and radiating particles with deterministic, nonlinear field structures that breathe and interact in real time.

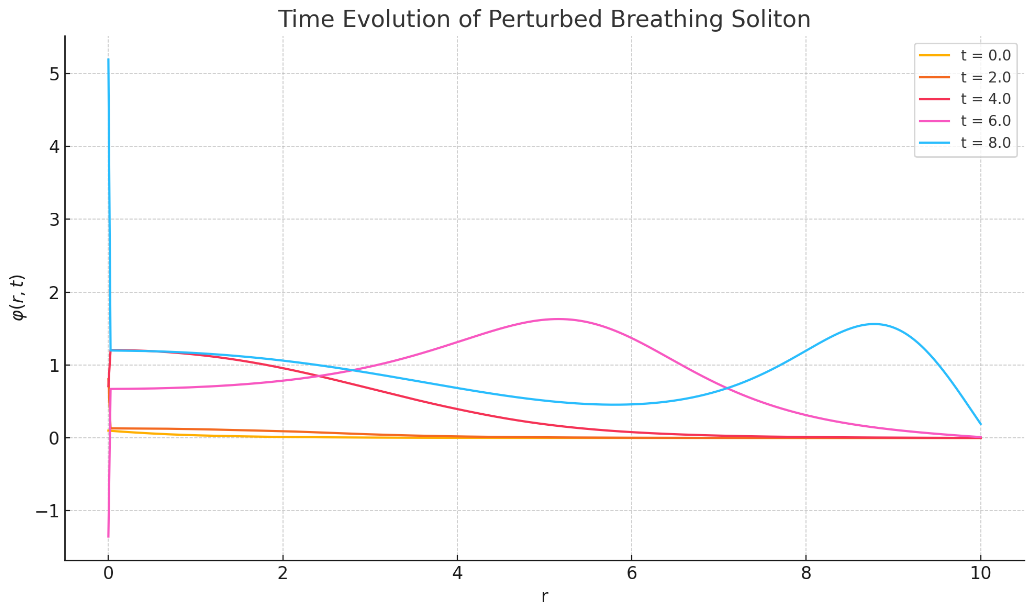

Figure 4: Total energy (blue) and average radius (red) of the breathing soliton as functions of time.