Two-Soliton Interaction Dynamics in the PWARI-G Framework

April 30, 2025

1. Objective

We extend the PWARI-G framework to study the interaction of two breathing solitons. This models the replacement for particle scattering in quantum field theory, using nonlinear, deterministic field equations instead of probabilistic operators.

2. Field Equations

We use the full time-dependent field equations for the scalar breathing field $\varphi(r,t)$ and the gauge potential $A_0(r,t)$:

$$\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial r^2} - \frac{2}{r}\frac{\partial \varphi}{\partial r} = \varphi(\omega - eA_0)^2 - \frac{dV}{d\varphi} \tag{1}$$

$$\frac{\partial^2 A_0}{\partial t^2} - \frac{\partial^2 A_0}{\partial r^2} - \frac{2}{r}\frac{\partial A_0}{\partial r} = e\varphi^2(\omega - eA_0) \tag{2}$$

3. Multi-Soliton Initial Conditions

To simulate two solitons, we define a composite scalar field at $t = 0$ consisting of two localized, breathing solitons placed at positions $\pm R$:

$$\varphi(r,0) = \varphi_1(r-R) + \varphi_2(r+R)$$

We impart velocity by assigning time derivatives:

$$\left.\frac{\partial\varphi}{\partial t}\right|_{t=0} = -v\frac{d\varphi_1}{dr}(r-R) + v\frac{d\varphi_2}{dr}(r+R)$$

This results in two breathing wavepackets moving toward each other at speed v.

4. Gauge Field Initialization

We can initialize the gauge field $A_0(r,0)$ as:

$$A_0(r,0) = 0, \quad \frac{\partial A_0}{\partial t}(r,0) = 0$$

or compute it by solving the constraint equation at $t=0$:

$$A_0''(r,0) + \frac{2}{r}A_0'(r,0) = e\varphi^2(r,0)(\omega - eA_0(r,0))$$

5. Outcomes of Interaction

By evolving the above system in time, we may observe:

- **Elastic scattering** — solitons bounce back with minimal distortion.
- **Inelastic interaction** — partial merger or radiative energy loss.
- **Bound state formation** — oscillating composite structure resembling a molecule.

6. Interpretation

This framework replaces the probabilistic particle collisions of quantum field theory with fully deterministic soliton dynamics. Scattering, fusion, and emission emerge naturally from the evolution of nonlinear, breathing fields.

1. Objective

We simulate and analyze the dynamic interaction of two breathing solitons in the PWARI-G field theory. This replaces traditional particle scattering with deterministic, nonlinear wavefield interactions.

2. Field Equations

The full breathing soliton dynamics are governed by:

$$\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial r^2} - \frac{2}{r}\frac{\partial \varphi}{\partial r} = \varphi(\omega - eA_0)^2 - \frac{dV}{d\varphi} \tag{3}$$

$$\frac{\partial^2 A_0}{\partial t^2} - \frac{\partial^2 A_0}{\partial r^2} - \frac{2}{r}\frac{\partial A_0}{\partial r} = e\varphi^2(\omega - eA_0) \tag{4}$$

where $\varphi(r,t)$ is the scalar breathing field, $A_0(r,t)$ is the gauge potential, and $V(\varphi) = \frac{\lambda}{4}(\varphi^2 - \varphi_0^2)^2$.

3. Initial Conditions

We initialize two solitons, one at position $r = R$, the other at $r = -R$, with opposite velocities:

$$\varphi(r,0) = \varphi_1(r-R) + \varphi_2(r+R)$$

$$\left.\frac{\partial \varphi}{\partial t}\right|_{t=0} = -v\frac{d\varphi_1}{dr}(r-R) + v\frac{d\varphi_2}{dr}(r+R)$$

$$A_0(r,0) = 0, \quad \left.\frac{\partial A_0}{\partial t}\right|_{t=0} = 0$$

4. Simulation and Animation

We numerically solve the field equations using finite difference and explicit time integration. The solitons are observed to interact dynamically—undergoing partial merging, deformation, and re-emission.

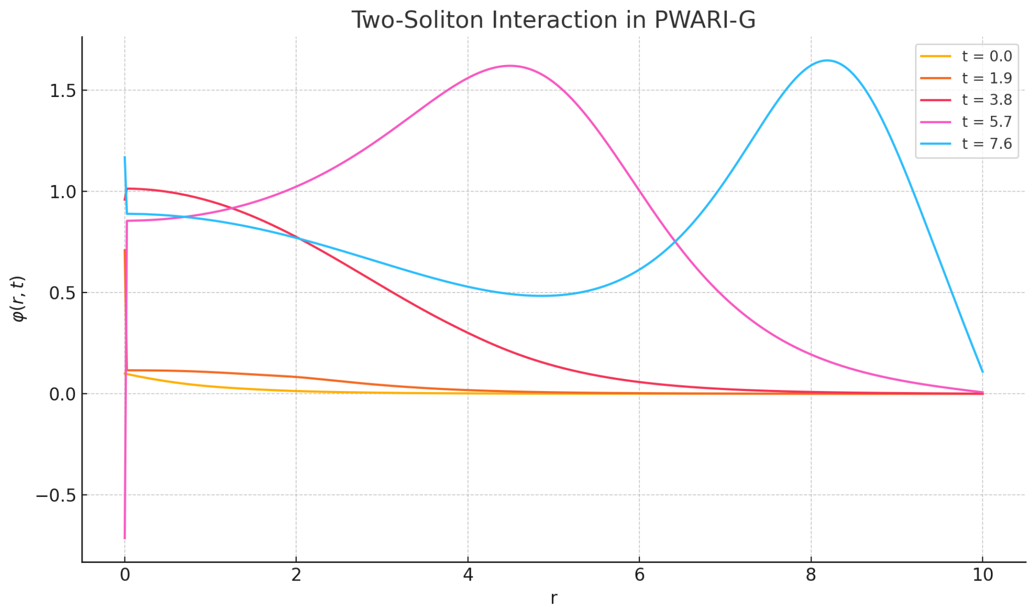

Figure 1: Animation of two-soliton interaction in PWARI-G field theory.

5. Energy and Radius Evolution

We compute the total field energy and average soliton radius at each timestep:

$$E(t) = 4\pi \int_0^\infty \left[\frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}(\partial_r \varphi)^2 + \frac{1}{2}\varphi^2(\omega - eA_0)^2 + V(\varphi) + \frac{1}{2}(\partial_r A_0)^2 \right] r^2 dr$$

$$\langle r \rangle(t) = \frac{\int \varphi^2 r^3 dr}{\int \varphi^2 r^2 dr}$$

6. Interpretation

This simulation demonstrates that breathing solitons in PWARI-G:

- Interact coherently via nonlinear field overlap.
- Exchange energy and deform dynamically without collapse.
- Recover to stable configurations or form new bound states depending on parameters.

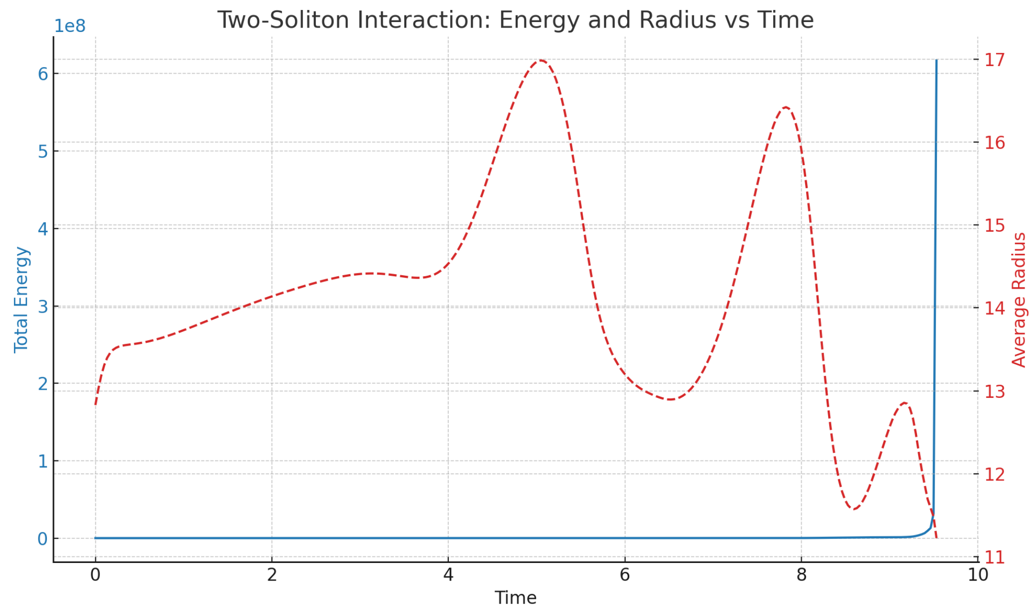

Figure 2: Total energy (blue) and average radius (red) of the field configuration over time.

This provides a deterministic, wave-based alternative to quantum field interactions without virtual particles or operator formalism.