1 GROUP 2: Compare the Signed Rank Test to the 1-sample t-test.

1.1 Functions to generate 2 of the distributions

```
plus1outlier = function(n){
# Generate n-1 observations from normal, mean 0, sd 1
sample1 = rnorm(n-1, mean = 0, sd = 1)
# Generating 1 observation from normal, mean 0, sd 4
outlier = rnorm(n = 1, mean = 0, sd = 4) # Combine all observations
    sample_outlier = c(sample1, outlier)
return(sample_outlier)
}


plus20poutlier = function(n){
# Generate 80% observations from normal, mean 0, sd 1
sample80 = rnorm(0.8*n, mean = 0, sd = 1)
# Generate 20% outliers from normal, mean 0, sd 4
outliers20 = rnorm(0.2*n, mean = 0, sd = 4) # Combine all observations
    sample_outlier = c(sample80, outliers20)
return(sample_outlier)
}
```

1.2 Function to generate samples:

```
set.seed(123)

generatesample <- function(dist, n, theta) {


  if (dist == "Normal") {

    sample1 <- rnorm(n, 0, 1) + theta

  } else if (dist == "Uniform") {

    sample1 <- runif(n, min = -sqrt(3), max = sqrt(3)) + theta

  } else if (dist == "Exponential") {

    sample1 <- rexp(n, rate = 1) - 1 + theta

  } else if (dist == "Plus1outlier") {

    sample1 <- plus1outlier(n) + theta
```

```
  } else if (dist == "Plus20poutlier") {

    sample1 <- plus20poutlier(n) + theta

  }

  return(sample1)
}
```

1.3 Function to run tests and calculate power:

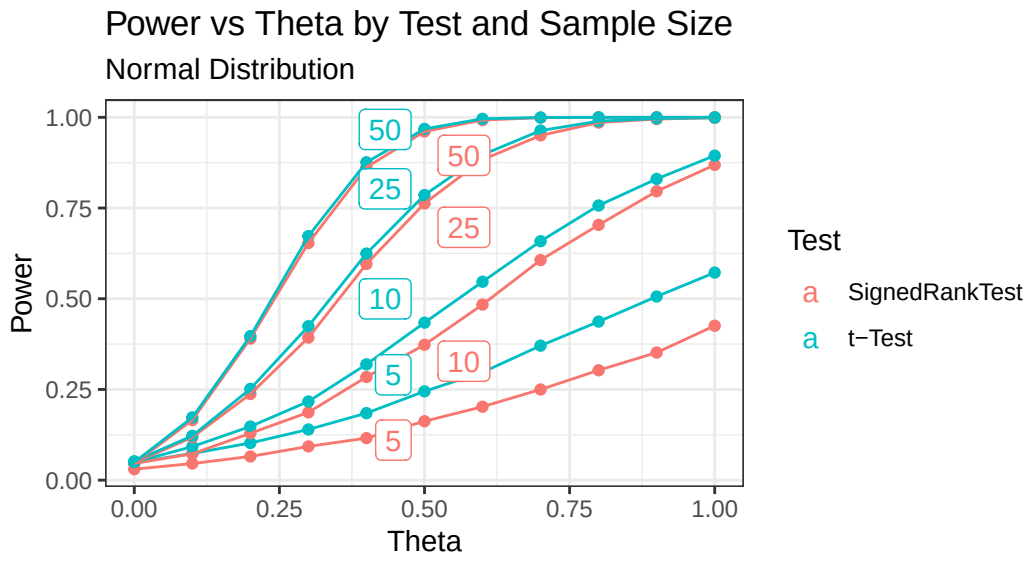
```
set.seed(123)

power <- function(test, dist, n, theta) {


  M = 10000
  p = vector(length = M)

  for (i in 1:M){

  sample1 <- generatesample(dist, n, theta)

  if (test == "SignedRankTest") {
    t <- sum(rank(abs(sample1))*(sign(sample1)>0))
    p[i] = t > qsignrank(0.95, n)
  } else if (test == "t-Test") {
    t <- mean(sample1)/(sd(sample1)/sqrt(n))
    p[i] = t > qt(0.95, df = n-1)
  }

  }

  return(sum(p)/M)

}
```

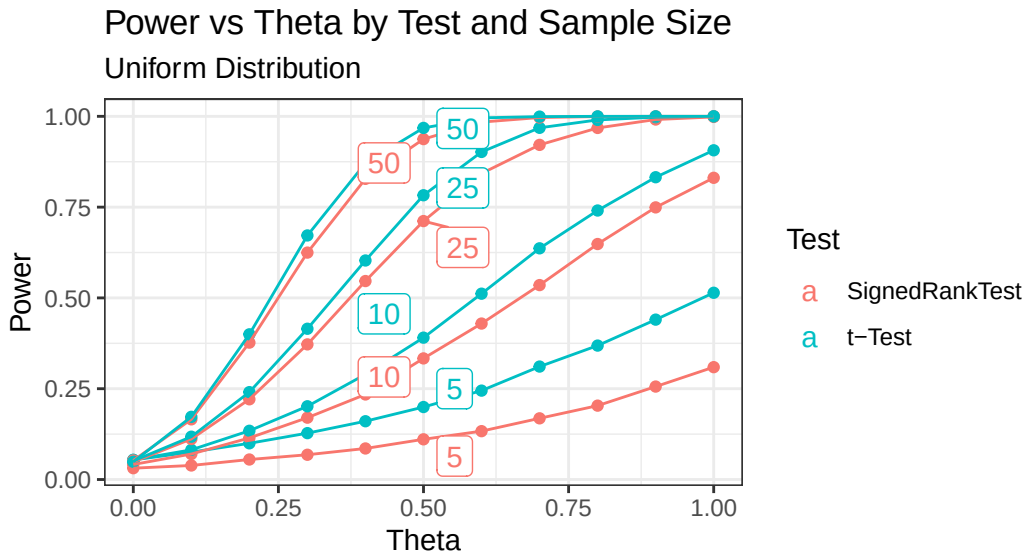
1.4 Running the simulation and generating data

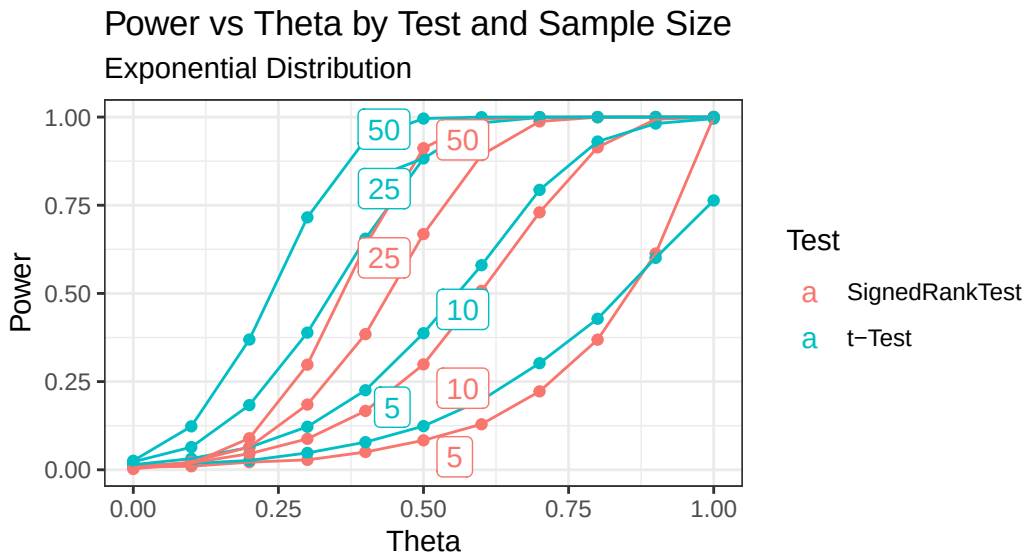
```
testn <- c(5, 10, 25, 50)
testtheta <- seq(from = 0, to = 1, by = 0.1)
testtest <- c("SignedRankTest", "t-Test")
```

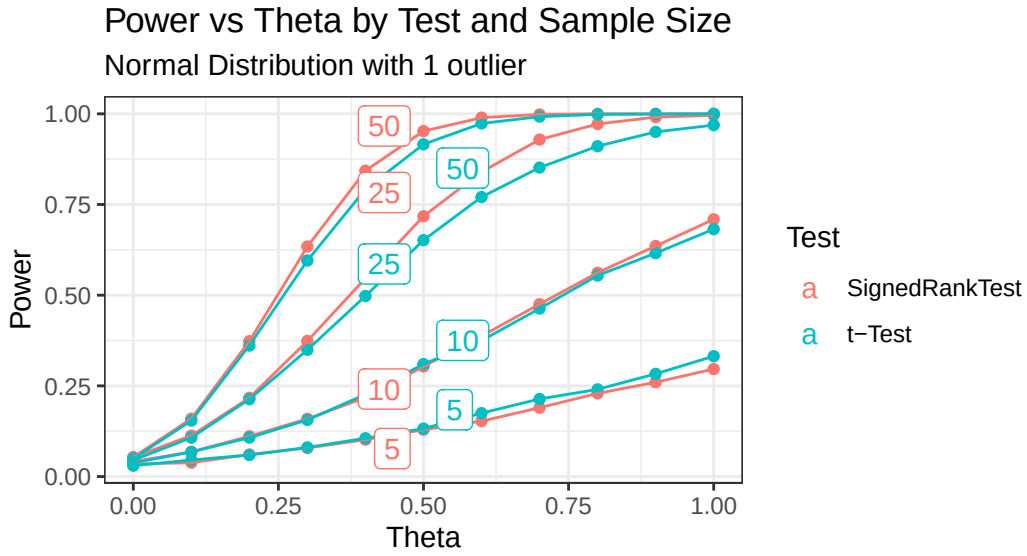
```r
sim_data_normal <- expand.grid(Test = testtest, N = testn, Theta = testtheta) %>%
  rowwise() %>%
  mutate(power = power(test = Test, dist = "Normal", n = N, theta = Theta))

sim_data_uniform <- expand.grid(Test = testtest, N = testn, Theta = testtheta) %>%
  rowwise() %>%
  mutate(power = power(test = Test, dist = "Uniform", n = N, theta = Theta))

sim_data_exponential <- expand.grid(Test = testtest, N = testn, Theta = testtheta) %>%
  rowwise() %>%
  mutate(power = power(test = Test, dist = "Exponential", n = N, theta = Theta))

sim_data_plus1outlier <- expand.grid(Test = testtest, N = testn, Theta = testtheta) %>%
  rowwise() %>%
  mutate(power = power(test = Test, dist = "Plus1outlier", n = N, theta = Theta))

sim_data_plus20poutlier <- expand.grid(Test = testtest, N = testn, Theta = testtheta) %>%
  rowwise() %>%
  mutate(power = power(test = Test, dist = "Plus20poutlier", n = N, theta = Theta))
```

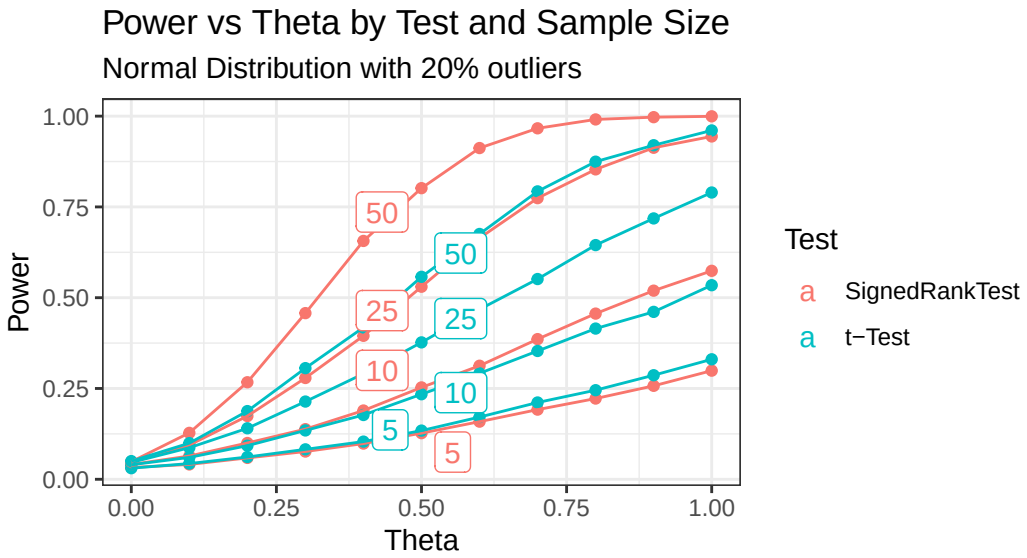
1.5 Plots




Power vs Theta by Test and Sample Size
Uniform Distribution
Power
Theta
0.00
0.25
0.50
0.75
1.00
50
50
25
25
10
10
5
5
Test
a SignedRankTest
a t−Test


Power vs Theta by Test and Sample Size
Exponential Distribution
Power
Theta
0.00
0.25
0.50
0.75
1.00
50
50
25
25
10
10
5
5
Test
a SignedRankTest
a t−Test


Power vs Theta by Test and Sample Size
Normal Distribution with 1 outlier
Power
Theta
0.00
0.25
0.50
0.75
1.00
50
50
25
25
10
10
5
5
Test
a SignedRankTest
a t−Test



1.6 Tables

Table 1: Normal Distribution

Test	N	$\theta = 0$	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.7$	$\theta = 0.8$	$\theta = 0.9$	$\theta = 1$
SignedRankTest	5	0.0302	0.0458	0.0653	0.0930	0.1158	0.1623	0.2024	0.2499	0.3028	0.3516	0.4256
t-Test	5	0.0483	0.0735	0.1024	0.1397	0.1847	0.2448	0.2973	0.3706	0.4370	0.5061	0.5720
SignedRankTest	10	0.0460	0.0726	0.1292	0.1869	0.2843	0.3731	0.4839	0.6066	0.7035	0.7962	0.8688
t-Test	10	0.0494	0.0924	0.1478	0.2168	0.3192	0.4340	0.5469	0.6586	0.7570	0.8302	0.8944
SignedRankTest	25	0.0476	0.1182	0.2373	0.3929	0.5958	0.7626	0.8833	0.9501	0.9855	0.9953	0.9988
t-Test	25	0.0521	0.1221	0.2515	0.4243	0.6245	0.7857	0.8968	0.9634	0.9894	0.9966	0.9989
SignedRankTest	50	0.0462	0.1657	0.3902	0.6535	0.8621	0.9610	0.9925	0.9990	0.9999	1.0000	1.0000
t-Test	50	0.0494	0.1729	0.3970	0.6726	0.8761	0.9680	0.9960	0.9995	1.0000	1.0000	1.0000

Table 2: Uniform Distribution

Test	N	$\theta = 0$	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.7$	$\theta = 0.8$	$\theta = 0.9$	$\theta = 1$
SignedRankTest	5	0.0312	0.0387	0.0551	0.0682	0.0855	0.1107	0.1332	0.1685	0.2034	0.2559	0.3094
t-Test	5	0.0539	0.0755	0.0997	0.1277	0.1605	0.1996	0.2448	0.3110	0.3688	0.4402	0.5142
SignedRankTest	10	0.0414	0.0702	0.1145	0.1703	0.2342	0.3335	0.4294	0.5353	0.6484	0.7493	0.8307
t-Test	10	0.0542	0.0817	0.1342	0.2017	0.2895	0.3906	0.5117	0.6366	0.7407	0.8323	0.9065
SignedRankTest	25	0.0484	0.1116	0.2209	0.3721	0.5465	0.7116	0.8426	0.9214	0.9680	0.9912	0.9983
t-Test	25	0.0512	0.1179	0.2406	0.4151	0.6031	0.7827	0.9019	0.9684	0.9901	0.9977	1.0000
SignedRankTest	50	0.0525	0.1657	0.3769	0.6248	0.8279	0.9373	0.9841	0.9967	0.9994	1.0000	1.0000
t-Test	50	0.0489	0.1726	0.3998	0.6720	0.8713	0.9681	0.9957	0.9992	0.9999	1.0000	1.0000

Table 3: Exponential Distribution

Test	N	$\theta = 0$	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.7$	$\theta = 0.8$	$\theta = 0.9$	$\theta = 1$
SignedRankTest	5	0.0074	0.0099	0.0216	0.0281	0.0503	0.0835	0.1291	0.2223	0.3689	0.6131	1.0000
t-Test	5	0.0116	0.0179	0.0264	0.0478	0.0783	0.1239	0.1973	0.3022	0.4281	0.6008	0.7636
SignedRankTest	10	0.0089	0.0195	0.0455	0.0877	0.1668	0.2989	0.5072	0.7299	0.9142	0.9937	1.0000
t-Test	10	0.0144	0.0316	0.0635	0.1221	0.2255	0.3872	0.5800	0.7936	0.9306	0.9816	0.9956
SignedRankTest	25	0.0038	0.0223	0.0644	0.1849	0.3846	0.6683	0.8923	0.9876	0.9996	1.0000	1.0000
t-Test	25	0.0226	0.0643	0.1834	0.3889	0.6552	0.8828	0.9835	0.9989	0.9999	1.0000	1.0000
SignedRankTest	50	0.0024	0.0179	0.0894	0.2980	0.6353	0.9117	0.9940	0.9999	1.0000	1.0000	1.0000
t-Test	50	0.0256	0.1231	0.3692	0.7156	0.9360	0.9958	0.9998	1.0000	1.0000	1.0000	1.0000

Table 4: Normal Distribution with 1 Outlier

Test	N	$\theta = 0$	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.7$	$\theta = 0.8$	$\theta = 0.9$	$\theta = 1$
SignedRankTest	5	0.0346	0.0383	0.0607	0.0788	0.1017	0.1292	0.1528	0.1897	0.2293	0.2599	0.2964
t-Test	5	0.0300	0.0456	0.0595	0.0803	0.1057	0.1322	0.1749	0.2138	0.2404	0.2832	0.3318
SignedRankTest	10	0.0412	0.0682	0.1112	0.1592	0.2178	0.3035	0.3873	0.4753	0.5619	0.6356	0.7090
t-Test	10	0.0378	0.0677	0.1073	0.1561	0.2265	0.3099	0.3749	0.4631	0.5542	0.6159	0.6820
SignedRankTest	25	0.0533	0.1129	0.2171	0.3738	0.5467	0.7176	0.8398	0.9290	0.9720	0.9908	0.9961
t-Test	25	0.0453	0.1073	0.2133	0.3494	0.4977	0.6516	0.7706	0.8518	0.9107	0.9501	0.9689
SignedRankTest	50	0.0533	0.1596	0.3732	0.6341	0.8432	0.9523	0.9895	0.9982	1.0000	1.0000	1.0000
t-Test	50	0.0496	0.1542	0.3612	0.5956	0.7921	0.9160	0.9734	0.9922	0.9985	0.9993	0.9999

Table 5: Normal Distribution with 20 Percent Outliers

Test	N	$\theta = 0$	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.7$	$\theta = 0.8$	$\theta = 0.9$	$\theta = 1$
SignedRankTest	5	0.0324	0.0408	0.0588	0.0764	0.0983	0.1270	0.1587	0.1918	0.2223	0.2573	0.2991
t-Test	5	0.0305	0.0432	0.0613	0.0824	0.1037	0.1340	0.1713	0.2114	0.2453	0.2865	0.3304
SignedRankTest	10	0.0407	0.0640	0.0999	0.1380	0.1893	0.2529	0.3126	0.3859	0.4562	0.5194	0.5740
t-Test	10	0.0400	0.0602	0.0925	0.1345	0.1770	0.2342	0.2918	0.3533	0.4150	0.4610	0.5344
SignedRankTest	25	0.0470	0.0942	0.1741	0.2789	0.3950	0.5302	0.6643	0.7741	0.8534	0.9129	0.9441
t-Test	25	0.0464	0.0869	0.1404	0.2140	0.2906	0.3770	0.4679	0.5514	0.6449	0.7183	0.7897
SignedRankTest	50	0.0483	0.1278	0.2672	0.4575	0.6563	0.8017	0.9121	0.9665	0.9909	0.9972	0.9995
t-Test	50	0.0500	0.0995	0.1877	0.3060	0.4180	0.5575	0.6755	0.7928	0.8746	0.9199	0.9606

1.7 Observations and Conclusions

Our group was assigned to investigate power, $(1 - B)$, for the Signed Rank Test (a non-parametric test) and the 1-sample t-test (a parametric test) as n (sample size) and θ (a location shift) changes. To do so, we carried out 10,000 iterations for every combination of 4 factors: test performed, n, θ, and type of distribution. There were 5 distributions, namely the normal distribution, the uniform distribution, the exponential distribution, the normal distribution with 1 outlier, and the Normal distribution with 20% outliers. We noticed that power generally increases as n increases across all distributions, holding θ constant, except when θ is 1 in some cases. Similarly, power generally increases as θ increases across all distributions, holding n constant, although at different rates. In the Normal, Uniform, and Exponential distributions, the t-test appears to be more powerful, even though the power for both tests seem to converge more as n and θ increase. For the Normal distribution with 1 outlier, there doesn't appear to be a clear, discernible pattern, yet the Signed Rank Test seems to be slightly more powerful for larger values of n. Finally, for the Normal

distribution with 20% outliers, the Signed Rank Test is remarkably more powerful for larger values of n. The tables and graphs help to visualize this.