

# Thesis Simulation Results Analysis for Chapter 4

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2024-04-03

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This file is intended to synthesize and analyze the results from the simulation.

## Reading in the Result Data Sets

### Logistic Regression

The results data set has 400 observations for each of the simulation trials, 100 from each sample size:  
 $n = 500, 1000, 2500, 5000$ .

```
lr_500 <- read.csv("/home/dasienga24/Statistics-Senior-Honors-Thesis/R/Simulation/LogisticRegression/Re  
lr_1000 <- read.csv("/home/dasienga24/Statistics-Senior-Honors-Thesis/R/Simulation/LogisticRegression/R  
lr_2500 <- read.csv("/home/dasienga24/Statistics-Senior-Honors-Thesis/R/Simulation/LogisticRegression/R
```

```
lr_5000 <- read.csv("/home/dasienga24/Statistics-Senior-Honors-Thesis/R/Simulation/LogisticRegression/R")

lr_500 <- lr_500 |>
  mutate(sample_size = 500) |>
  dplyr::select(-X)

lr_1000 <- lr_1000 |>
  mutate(sample_size = 1000) |>
  dplyr::select(-X)

lr_2500 <- lr_2500 |>
  mutate(sample_size = 2500) |>
  dplyr::select(-X)

lr_5000 <- lr_5000 |>
  mutate(sample_size = 5000) |>
  dplyr::select(-X)

logistic_results <- rbind(lr_500, lr_1000, lr_2500, lr_5000)
glimpse(logistic_results)
```

```
## Rows: 400
## Columns: 5
## $ dataset_id      <int> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 1~
## $ lr_convergence  <int> 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1~
## $ lr_accuracy     <dbl> 0.790, 0.738, 0.778, 0.758, 0.788, 0.822, 0.776, 0.7~
## $ lr_discrimination <dbl> 0.2020, 0.2402, 0.2730, 0.3123, 0.2663, 0.2171, 0.31~
## $ sample_size     <dbl> 500, 500, 500, 500, 500, 500, 500, 500, 500, 50~
```

## Seldonian Solutions

The results data set has 200 observations for each of the simulation trials, 50 from each sample size:  
 $n = 500, 1000, 2500, 5000$ .

```
seldonian_results <- read.csv("/home/dasienga24/Statistics-Senior-Honors-Thesis/Python/COMPAS Simulation")
#seldonian_results <- read.csv("/home/dasienga24/Statistics-Senior-Honors-Thesis/Data Sets/seldonian_si")
glimpse(seldonian_results)
```

```
## Rows: 400
## Columns: 14
## $ sample_size     <int> 1000, 1000, 1000, 1000, 1000, 1000, 5000, 1000, 1000~
## $ dataset_id      <int> 100, 61, 4, 41, 42, 56, 58, 52, 48, 81, 55, 60, 22, ~
## $ passed_safety_02 <chr> "True", "True", "True", "True", "True", "True", "True", "Tru~
## $ passed_safety_01 <chr> "True", "True", "True", "True", "True", "True", "True", "Tru~
## $ passed_safety_005 <chr> "True", "True", "True", "True", "True", "True", "True", "Tru~
## $ passed_safety_001 <chr> "True", "True", "True", "True", "True", "True", "True", "Tru~
## $ sa_02_accuracy   <dbl> 0.5690, 0.6260, 0.4730, 0.7140, 0.7410, 0.5220, 0.58~
## $ sa_01_accuracy   <dbl> 0.5220, 0.4890, 0.4730, 0.6610, 0.7550, 0.5220, 0.72~
## $ sa_005_accuracy  <dbl> 0.5220, 0.5110, 0.4730, 0.5150, 0.4850, 0.5220, 0.59~
## $ sa_001_accuracy  <dbl> 0.5220, 0.5110, 0.4730, 0.4880, 0.4850, 0.5220, 0.51~
## $ sa_02_disc_stat  <dbl> 0.0776, 0.0789, 0.0000, 0.2059, 0.2840, 0.0000, 0.04~
## $ sa_01_disc_stat  <dbl> 0.0000, 0.0000, 0.0000, 0.1681, 0.1406, 0.0000, 0.19~
## $ sa_005_disc_stat <dbl> 0.0000, 0.0000, 0.0000, 0.0398, 0.0000, 0.0000, 0.08~
## $ sa_001_disc_stat <dbl> 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.00~
```

## Combining the Data Sets from Both Simulations

```
sim_results <- inner_join(logistic_results, seldonian_results,
                          by = c("sample_size", "dataset_id"))

glimpse(sim_results)

## Rows: 400
## Columns: 17
## $ dataset_id      <int> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 1~
## $ lr_convergence  <int> 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1~
## $ lr_accuracy     <dbl> 0.790, 0.738, 0.778, 0.758, 0.788, 0.822, 0.776, 0.7~
## $ lr_discrimination <dbl> 0.2020, 0.2402, 0.2730, 0.3123, 0.2663, 0.2171, 0.31~
## $ sample_size     <dbl> 500, 500, 500, 500, 500, 500, 500, 500, 500, 500, 50~
## $ passed_safety_02 <chr> "True", "True", "True", "True", "True", "True", "Tru~
## $ passed_safety_01 <chr> "True", "True", "False", "True", "True", "False", "T~
## $ passed_safety_005 <chr> "True", "True", "False", "True", "True", "False", "T~
## $ passed_safety_001 <chr> "True", "True", "False", "True", "True", "False", "T~
## $ sa_02_accuracy  <dbl> 0.526, 0.510, 0.528, 0.520, 0.512, 0.508, 0.608, 0.5~
## $ sa_01_accuracy  <dbl> 0.522, 0.510, 0.528, 0.520, 0.496, 0.508, 0.482, 0.5~
## $ sa_005_accuracy <dbl> 0.522, 0.510, 0.528, 0.520, 0.496, 0.508, 0.756, 0.5~
## $ sa_001_accuracy <dbl> 0.522, 0.510, 0.528, 0.520, 0.496, 0.508, 0.482, 0.5~
## $ sa_02_disc_stat <dbl> NA, 0.0000, 0.0000, 0.0000, 0.0429, 0.0000, 0.0106, ~
## $ sa_01_disc_stat <dbl> 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.00~
## $ sa_005_disc_stat <dbl> 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.13~
## $ sa_001_disc_stat <dbl> 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.0000, 0.00~
```

## Probability of a Solution

### Table

This section assesses what proportion of the trials returned a solution. It is expected that all logistic regression trials will return a solution. However, for the Seldonian algorithms, while all trials will return a candidate solution (based on the logistic regression as a starting point), it is expected that not all candidate solutions will pass the safety test. The table below records the number of Seldonian solutions that passed the safety test in each sample size.

```
reps <- nrow(sim_results)/4

sim_results |>
  group_by(sample_size) |>
  summarise(LR = 100*count(lr_convergence == 1)/reps,
            `SA (0.2)` = 100*count(passed_safety_02 == "True")/reps,
            `SA (0.1)` = 100*count(passed_safety_01 == "True")/reps,
            `SA (0.05)` = 100*count(passed_safety_005 == "True")/reps,
            `SA (0.01)` = 100*count(passed_safety_001 == "True")/reps) |>
  rename("Sample Size" = sample_size) |>
  kable(caption = "Probability of a Solution")
```

Table 1: Probability of a Solution

Sample Size	LR	SA (0.2)	SA (0.1)	SA (0.05)	SA (0.01)
500	100	100	92	88	72

Sample Size	LR	SA (0.2)	SA (0.1)	SA (0.05)	SA (0.01)
1000	100	100	98	93	84
2500	100	100	100	97	54
5000	100	100	100	95	29

## Accuracy

```
# filter accuracy data out for only seldonian solutions that converged
sim_results_converged_accuracy <- sim_results |>
  mutate(sa_02_accuracy = ifelse(passed_safety_02 == "True", sa_02_accuracy, NA),
         sa_01_accuracy = ifelse(passed_safety_01 == "True", sa_01_accuracy, NA),
         sa_005_accuracy = ifelse(passed_safety_005 == "True", sa_005_accuracy, NA),
         sa_001_accuracy = ifelse(passed_safety_001 == "True", sa_001_accuracy, NA))
```

## Table

```
# table
sim_results_converged_accuracy |>
  group_by(sample_size) |>
  summarise(LR = round(100*mean(lr_accuracy, na.rm = TRUE),2),
            sd_lr = round(sd(lr_accuracy, na.rm = TRUE),2),
            `SA (0.2)` = round(100*mean(sa_02_accuracy, na.rm = TRUE),2),
            sd_02 = round(sd(sa_02_accuracy, na.rm = TRUE),2),
            `SA (0.1)` = round(100*mean(sa_01_accuracy, na.rm = TRUE),2),
            sd_01 = round(sd(sa_01_accuracy, na.rm = TRUE),2),
            `SA (0.05)` = round(100*mean(sa_005_accuracy, na.rm = TRUE),2),
            sd_005 = round(sd(sa_005_accuracy, na.rm = TRUE),2),
            `SA (0.01)` = round(100*mean(sa_001_accuracy, na.rm = TRUE),2),
            sd_001 = round(sd(sa_001_accuracy, na.rm = TRUE),2)) |>
  rename("Sample Size" = sample_size,
         "sd" = sd_lr,
         "sd " = sd_02,
         "sd  " = sd_01,
         "sd   " = sd_005,
         "sd    " = sd_001) |>
  kable(caption = "Accuracy of Convergent Solutions")
```

Table 2: Accuracy of Convergent Solutions

Sample Size	LR	sd	SA (0.2)	sd	SA (0.1)	sd	SA (0.05)	sd	SA (0.01)	sd
500	78.20	0.02	54.19	0.06	52.43	0.07	52.62	0.06	52.18	0.06
1000	77.99	0.01	59.24	0.07	53.28	0.06	52.47	0.06	51.43	0.04
2500	78.02	0.01	71.71	0.07	57.07	0.06	51.10	0.03	50.28	0.02
5000	78.01	0.00	68.84	0.08	64.75	0.10	55.48	0.06	50.34	0.01

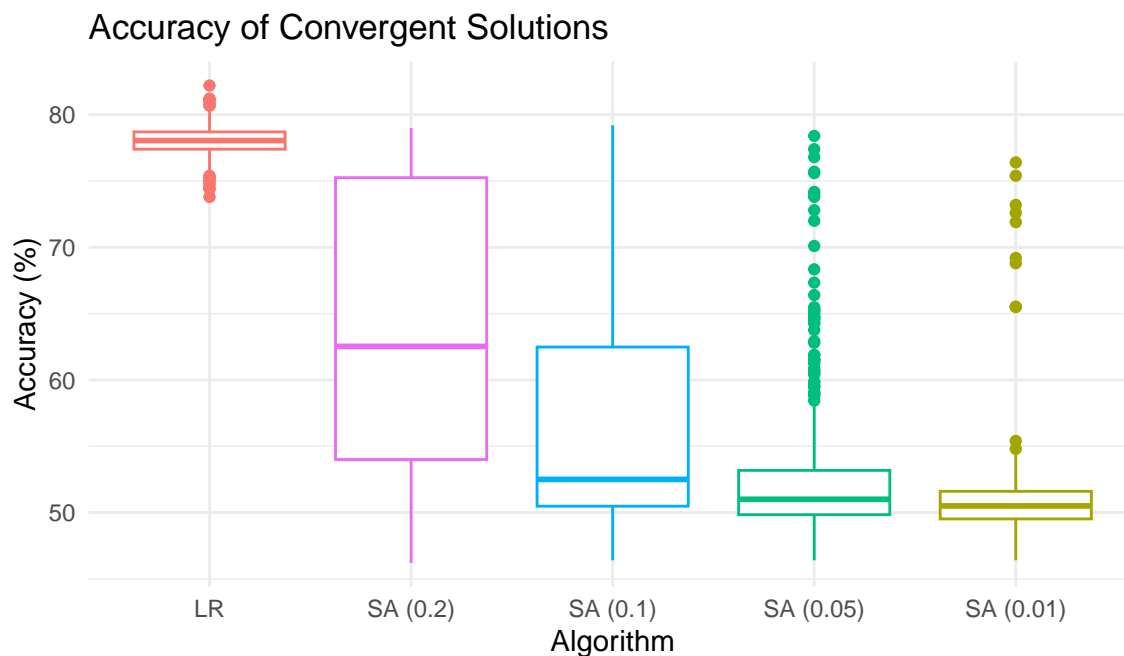
## Visualizations

```
# visualization
sim_results_converged_accuracy |>
```

```

dplyr::select(c(lr_accuracy, sa_02_accuracy, sa_01_accuracy, sa_005_accuracy,
               sa_001_accuracy, sample_size)) |>
rename("Sample Size" = sample_size,
       "LR" = lr_accuracy,
       "SA (0.2)" = sa_02_accuracy,
       "SA (0.1)" = sa_01_accuracy,
       "SA (0.05)" = sa_005_accuracy,
       "SA (0.01)" = sa_001_accuracy) |>
pivot_longer(cols = -c(`Sample Size`),
             names_to = "model",
             values_to = "accuracy") |>
mutate(accuracy = 100*accuracy) |>
ggplot(mapping = aes(x = model, y = accuracy, color = model)) +
  geom_boxplot() +
  scale_x_discrete(limits = c("LR", "SA (0.2)", "SA (0.1)", "SA (0.05)", "SA (0.01)")) +
  theme_minimal() +
  guides(color = "none") +
  labs(x = "Algorithm",
       y = "Accuracy (%)",
       title = "Accuracy of Convergent Solutions")

```



```

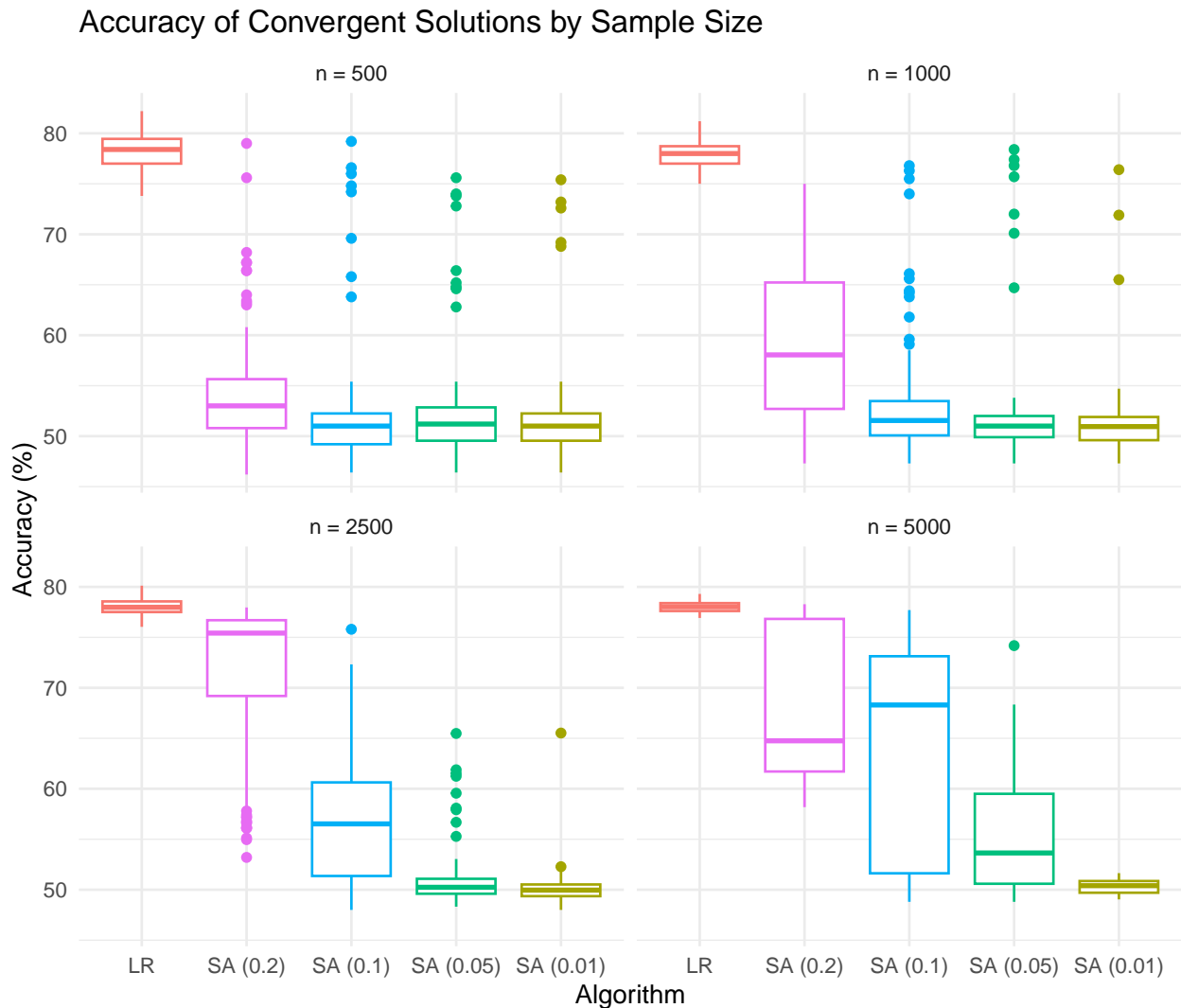
# visualization
sim_results_converged_accuracy |>
dplyr::select(c(lr_accuracy, sa_02_accuracy, sa_01_accuracy, sa_005_accuracy,
               sa_001_accuracy, sample_size)) |>
mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
                                     sample_size == 1000 ~ "n = 1000",
                                     sample_size == 2500 ~ "n = 2500",
                                     sample_size == 5000 ~ "n = 5000"
                                     ),
       levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000"))) |>
rename("Sample Size" = sample_size,
       "LR" = lr_accuracy,

```

```

"SA (0.2)" = sa_02_accuracy,
"SA (0.1)" = sa_01_accuracy,
"SA (0.05)" = sa_005_accuracy,
"SA (0.01)" = sa_001_accuracy) |>
pivot_longer(cols = -c(`Sample Size`),
  names_to = "model",
  values_to = "accuracy") |>
mutate(accuracy = 100*accuracy) |>
ggplot(mapping = aes(x = model, y = accuracy, color = model)) +
  geom_boxplot() +
  scale_x_discrete(limits = c("LR", "SA (0.2)", "SA (0.1)", "SA (0.05)", "SA (0.01)")) +
  theme_minimal() +
  guides(color = "none") +
  facet_wrap(~`Sample Size`) +
  labs(x = "Algorithm",
    y = "Accuracy (%)",
    title = "Accuracy of Convergent Solutions by Sample Size")

```



## Discrimination

```
# filter discrimination data out for only seldonian solutions that converged
sim_results_converged_disc <- sim_results |>
  mutate(sa_02_disc_stat = ifelse(passed_safety_02 == "True", sa_02_disc_stat, NA),
         sa_01_disc_stat = ifelse(passed_safety_01 == "True", sa_01_disc_stat, NA),
         sa_005_disc_stat = ifelse(passed_safety_005 == "True", sa_005_disc_stat, NA),
         sa_001_disc_stat = ifelse(passed_safety_001 == "True", sa_001_disc_stat, NA))
```

## Tables

```
# table
sim_results_converged_disc |>
  group_by(sample_size) |>
  summarise(LR = round(mean(lr_discrimination, na.rm = TRUE),2),
            sd_lr = round(sd(lr_discrimination, na.rm = TRUE),2),
            `SA (0.2)` = round(mean(sa_02_disc_stat, na.rm = TRUE),2),
            sd_02 = round(sd(sa_02_disc_stat, na.rm = TRUE),2),
            `SA (0.1)` = round(mean(sa_01_disc_stat, na.rm = TRUE),2),
            sd_01 = round(sd(sa_01_disc_stat, na.rm = TRUE),2),
            `SA (0.05)` = round(mean(sa_005_disc_stat, na.rm = TRUE),2),
            sd_005 = round(sd(sa_005_disc_stat, na.rm = TRUE),2),
            `SA (0.01)` = round(mean(sa_001_disc_stat, na.rm = TRUE),2),
            sd_001 = round(sd(sa_001_disc_stat, na.rm = TRUE),2)) |>
  rename("Sample Size" = sample_size,
         "sd" = sd_lr,
         "sd " = sd_02,
         "sd  " = sd_01,
         "sd   " = sd_005,
         "sd    " = sd_001) |>
  kable(caption = "Discrimination of Convergent Solutions")
```

Table 3: Discrimination of Convergent Solutions

Sample Size	LR	sd	SA (0.2)	sd	SA (0.1)	sd	SA (0.05)	sd	SA (0.01)	sd
500	0.25	0.08	0.05	0.06	0.03	0.08	0.02	0.06	0.02	0.07
1000	0.23	0.05	0.10	0.07	0.03	0.06	0.02	0.06	0.01	0.04
2500	0.24	0.03	0.17	0.07	0.07	0.06	0.02	0.03	0.00	0.02
5000	0.25	0.02	0.12	0.07	0.13	0.08	0.05	0.05	0.00	0.00

```
# table
sim_results_converged_disc |>
  group_by(sample_size) |>
  summarise(`SA (0.2)` = round(100*count(sa_02_disc_stat <= 0.2)/count(passed_safety_02 == "True"),2),
            `SA (0.1)` = round(100*count(sa_01_disc_stat <= 0.1)/count(passed_safety_01 == "True"),2),
            `SA (0.05)` = round(100*count(sa_005_disc_stat <= 0.05)/count(passed_safety_005 == "True"),2),
            `SA (0.01)` = round(100*count(sa_001_disc_stat <= 0.01)/count(passed_safety_001 == "True"),2),
  rename("Sample Size" = sample_size) |>
  kable(caption = "Satisfaction of the Behavioral Constraint by Convergent Solutions")
```

Table 4: Satisfaction of the Behavioral Constraint by Convergent Solutions

Sample Size	SA (0.2)	SA (0.1)	SA (0.05)	SA (0.01)
500	82	85.87	87.50	90.28
1000	82	76.53	82.80	91.67
2500	61	71.00	76.29	96.30
5000	83	36.00	54.74	100.00

```
# what % returned a solution and also met the constraint
sim_results |>
  group_by(sample_size) |>
  summarise(`SA (0.2)` = 100*count(passed_safety_02 == "True" & sa_02_disc_stat <= 0.2)/reps,
            `SA (0.1)` = 100*count(passed_safety_01 == "True" & sa_01_disc_stat <= 0.1)/reps,
            `SA (0.05)` = 100*count(passed_safety_005 == "True" & sa_005_disc_stat <= 0.05)/reps,
            `SA (0.01)` = 100*count(passed_safety_001 == "True" & sa_001_disc_stat <= 0.01)/reps) |>
  rename("Sample Size" = sample_size) |>
  kable(caption = "Probability of a Returning a Solution and Meeting the Satisfied Constraint")
```

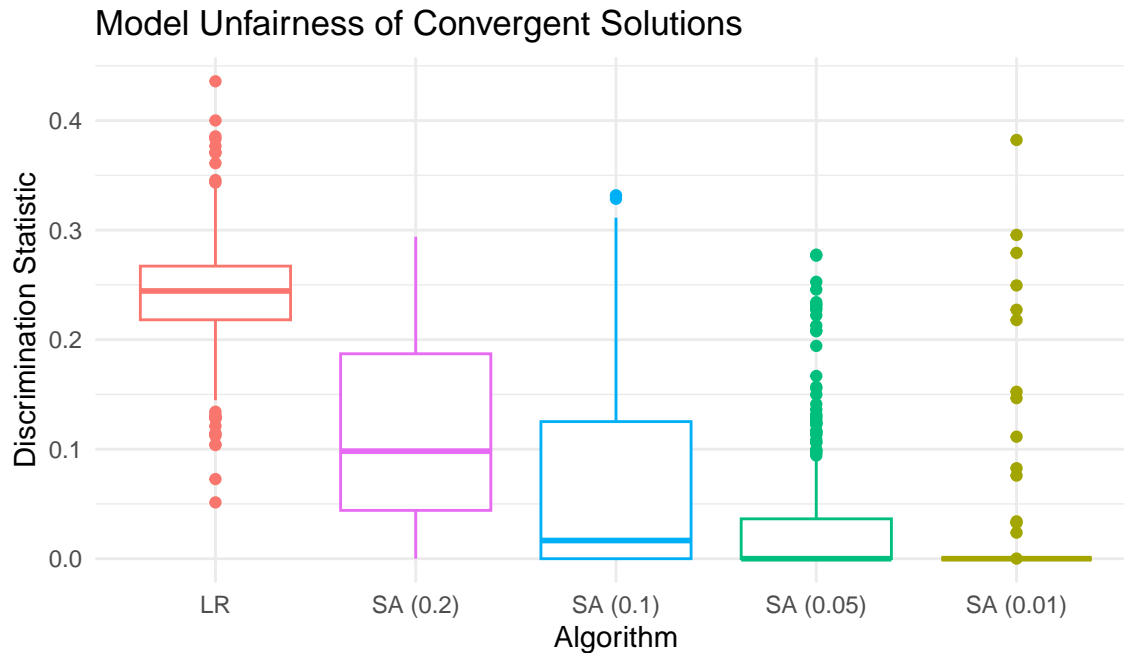
Table 5: Probability of a Returning a Solution and Meeting the Satisfied Constraint

Sample Size	SA (0.2)	SA (0.1)	SA (0.05)	SA (0.01)
500	82	79	77	65
1000	82	75	77	77
2500	61	71	74	52
5000	83	36	52	29

## Visualizations

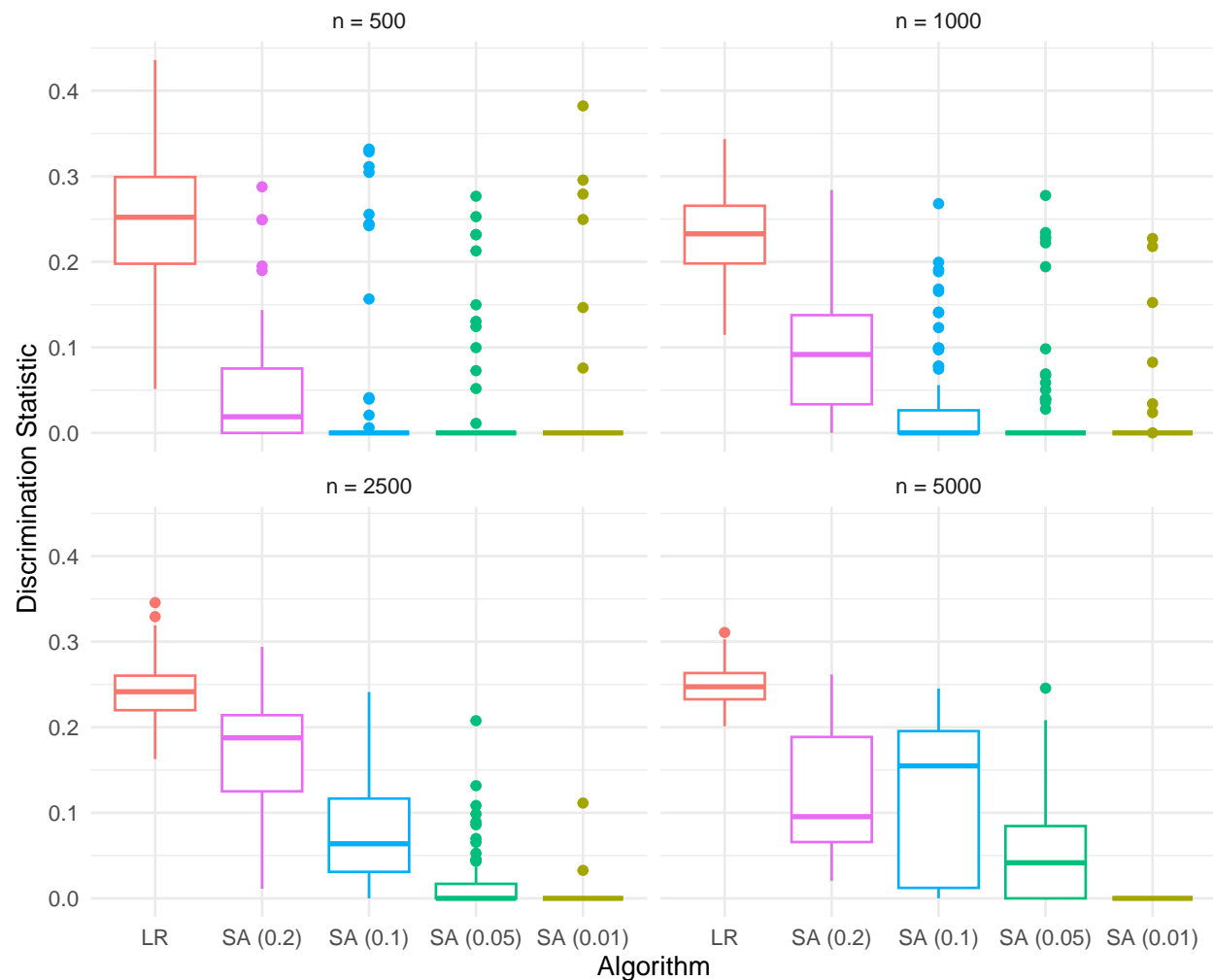
```
# visualization
sim_results_converged_disc |>
  dplyr::select(c(lr_discrimination, sa_02_disc_stat, sa_01_disc_stat, sa_005_disc_stat,
                  sa_001_disc_stat, sample_size)) |>
  rename("Sample Size" = sample_size,
         "LR" = lr_discrimination,
         "SA (0.2)" = sa_02_disc_stat,
         "SA (0.1)" = sa_01_disc_stat,
         "SA (0.05)" = sa_005_disc_stat,
         "SA (0.01)" = sa_001_disc_stat) |>
  pivot_longer(cols = -c(`Sample Size`),
               names_to = "model",
               values_to = "discrimination") |>
  ggplot(mapping = aes(x = model, y = discrimination, color = model)) +
  geom_boxplot() +
  scale_x_discrete(limits = c("LR", "SA (0.2)", "SA (0.1)", "SA (0.05)", "SA (0.01)")) +
  theme_minimal() +
  guides(color = "none") +
  labs(x = "Algorithm",
       y = "Discrimination Statistic",
       title = "Model Unfairness of Convergent Solutions")
```





```
# visualization
sim_results_converged_disc |>
  dplyr::select(c(lr_discrimination, sa_02_disc_stat, sa_01_disc_stat, sa_005_disc_stat,
                  sa_001_disc_stat, sample_size)) |>
  mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
                                         sample_size == 1000 ~ "n = 1000",
                                         sample_size == 2500 ~ "n = 2500",
                                         sample_size == 5000 ~ "n = 5000"
                                     ),
         levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000"))) |>
  rename("Sample Size" = sample_size,
        "LR" = lr_discrimination,
        "SA (0.2)" = sa_02_disc_stat,
        "SA (0.1)" = sa_01_disc_stat,
        "SA (0.05)" = sa_005_disc_stat,
        "SA (0.01)" = sa_001_disc_stat) |>
  pivot_longer(cols = -c(`Sample Size`),
               names_to = "model",
               values_to = "discrimination") |>
  ggplot(mapping = aes(x = model, y = discrimination, color = model)) +
  geom_boxplot() +
  scale_x_discrete(limits = c("LR", "SA (0.2)", "SA (0.1)", "SA (0.05)", "SA (0.01)")) +
  theme_minimal() +
  guides(color = "none") +
  facet_wrap(~`Sample Size`) +
  labs(x = "Algorithm",
       y = "Discrimination Statistic",
       title = "Model Unfairness of Convergent Solutions by Sample Size")
```

## Model Unfairness of Convergent Solutions by Sample Size



## Accuracy-Discrimination Trade-Off

This section aims to visualize the accuracy-discrimination trade-off for each of the models, by sample-size.

```
# get long discrimination data set for plotting (1000 rows)
sim_results_converged_disc_long <- sim_results_converged_disc |>
  dplyr::select(c(lr_discrimination, sa_02_disc_stat, sa_01_disc_stat, sa_005_disc_stat,
                  sa_001_disc_stat, sample_size, dataset_id)) |>
  mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
                                         sample_size == 1000 ~ "n = 1000",
                                         sample_size == 2500 ~ "n = 2500",
                                         sample_size == 5000 ~ "n = 5000"
                                     ),
          levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000"))) |>
  rename("Sample Size" = sample_size,
        "LR" = lr_discrimination,
        "SA (0.2)" = sa_02_disc_stat,
        "SA (0.1)" = sa_01_disc_stat,
        "SA (0.05)" = sa_005_disc_stat,
```

```

      "SA (0.01)" = sa_001_disc_stat) |>
pivot_longer(cols = -c(`Sample Size`, dataset_id),
             names_to = "model",
             values_to = "discrimination")

# get long accuracy data set for plotting (1000 rows)
sim_results_converged_accuracy_long <- sim_results_converged_accuracy |>
dplyr::select(c(lr_accuracy, sa_02_accuracy, sa_01_accuracy, sa_005_accuracy,
               sa_001_accuracy, sample_size, dataset_id)) |>
mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
                                     sample_size == 1000 ~ "n = 1000",
                                     sample_size == 2500 ~ "n = 2500",
                                     sample_size == 5000 ~ "n = 5000"
                                     ),
      levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000"))) |>
rename("Sample Size" = sample_size,
      "LR" = lr_accuracy,
      "SA (0.2)" = sa_02_accuracy,
      "SA (0.1)" = sa_01_accuracy,
      "SA (0.05)" = sa_005_accuracy,
      "SA (0.01)" = sa_001_accuracy) |>
pivot_longer(cols = -c(`Sample Size`, dataset_id),
             names_to = "model",
             values_to = "accuracy")

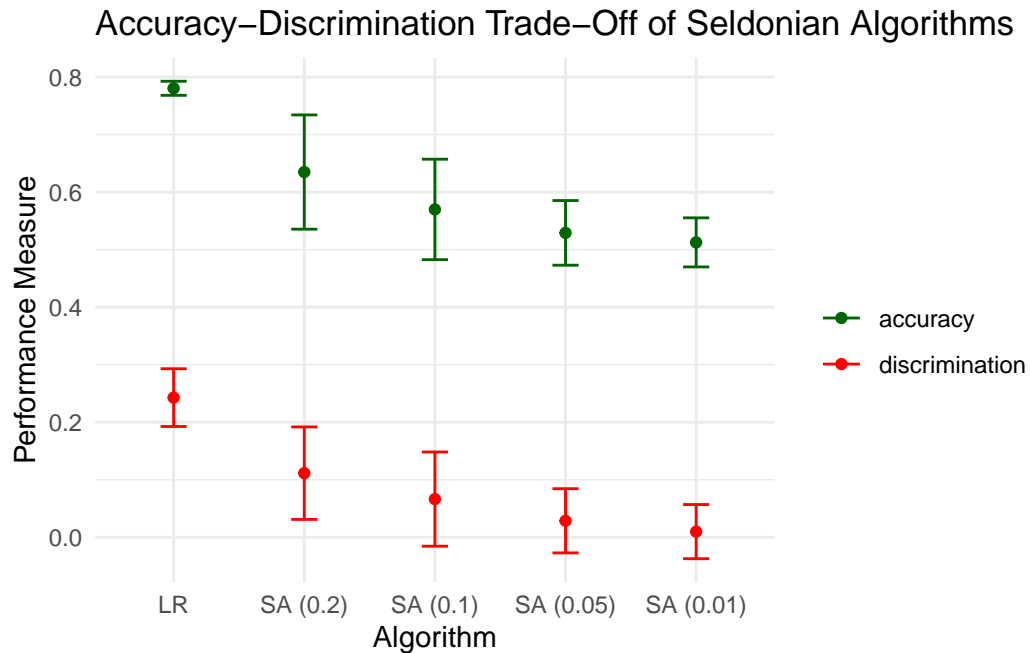
# join both data sets
accuracy_disc <- inner_join(sim_results_converged_disc_long, sim_results_converged_accuracy_long,
                           by = c("dataset_id", "Sample Size", "model")) |>
pivot_longer(cols = -c(`Sample Size`, dataset_id, model),
             names_to = "statistic",
             values_to = "value")

# plot error-bars
color <- c("darkgreen", "red")

accuracy_disc |>
group_by(model, statistic) |>
summarise(avg = mean(value, na.rm = TRUE),
          se = sd(value, na.rm = TRUE)) |>
ggplot(mapping = aes(x = model, y = avg, color = statistic)) +
geom_point() +
geom_errorbar(mapping = aes(ymin = avg - se,
                           ymax = avg + se,
                           width = 0.2)) +

theme_minimal() +
scale_color_manual(values = c("accuracy" = color[1],
                             "discrimination" = color[2])) +
scale_x_discrete(limits = c("LR", "SA (0.2)", "SA (0.1)", "SA (0.05)", "SA (0.01)")) +
labs(x = "Algorithm",
     y = "Performance Measure",
     title = "Accuracy-Discrimination Trade-Off of Seldonian Algorithms",
     color = " ")

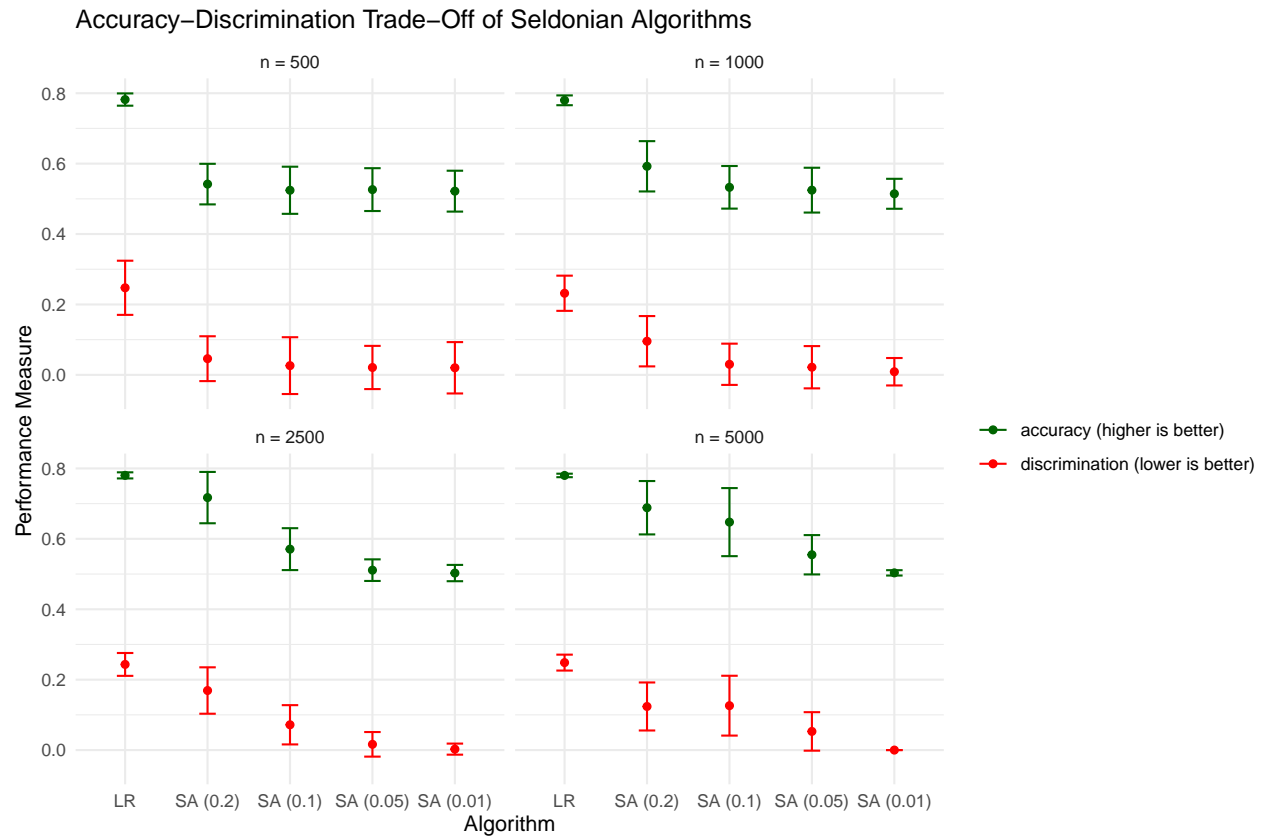
```



```
# plot error-bars by sample size
color <- c("darkgreen", "red")

accuracy_disc |>
  group_by(model, statistic, `Sample Size`) |>
  summarise(avg = mean(value, na.rm = TRUE),
            se = sd(value, na.rm = TRUE)) |>
  mutate(statistic = ifelse(statistic == "accuracy", "accuracy (higher is better)", "discrimination (lower is better)")) |>
  ggplot(mapping = aes(x = model, y = avg, color = statistic)) +
  geom_point() +
  geom_errorbar(mapping = aes(ymin = avg - se,
                             ymax = avg + se,
                             width = 0.2)) +

  theme_minimal() +
  scale_color_manual(values = c("accuracy (higher is better)" = color[1],
                                "discrimination (lower is better)" = color[2])) +
  scale_x_discrete(limits = c("LR", "SA (0.2)", "SA (0.1)", "SA (0.05)", "SA (0.01)")) +
  facet_wrap(~`Sample Size`) +
  labs(x = "Algorithm",
       y = "Performance Measure",
       title = "Accuracy-Discrimination Trade-Off of Seldonian Algorithms",
       color = " ")
```



## Convergence–Discrimination Trade–Off

This section examines whether there is a trade-off in the proportion of solutions that pass the safety test and the proportion of Seldonian solutions that satisfy the fairness constraint. I need to brainstorm better ways of visualizing this relationship.

```
# get long failure data for plotting
sim_failure_long <- sim_results |>
  group_by(sample_size) |>
  summarise(`SA (0.2)` = 100*count(passed_safety_02 == "False")/reps,
            `SA (0.1)` = 100*count(passed_safety_01 == "False")/reps,
            `SA (0.05)` = 100*count(passed_safety_005 == "False")/reps,
            `SA (0.01)` = 100*count(passed_safety_001 == "False")/reps) |>
  pivot_longer(cols = -c(sample_size),
               names_to = "model",
               values_to = "prop_failed_safety")

# get long convergence data for plotting
sim_converged_long <- sim_results |>
  group_by(sample_size) |>
  summarise(`SA (0.2)` = 100*count(passed_safety_02 == "True")/reps,
            `SA (0.1)` = 100*count(passed_safety_01 == "True")/reps,
            `SA (0.05)` = 100*count(passed_safety_005 == "True")/reps,
            `SA (0.01)` = 100*count(passed_safety_001 == "True")/reps) |>
  pivot_longer(cols = -c(sample_size),
               names_to = "model",
```

```

        values_to = "prop_passed_safety")

# get long discrimination data for plotting
sim_satisfied_cstr_long <- sim_results |>
  group_by(sample_size) |>
  summarise(`SA (0.2)` = 100*count(passed_safety_02 == "True" & sa_02_disc_stat <= 0.2)/reps,
            `SA (0.1)` = 100*count(passed_safety_01 == "True" & sa_01_disc_stat <= 0.1)/reps,
            `SA (0.05)` = 100*count(passed_safety_005 == "True" & sa_005_disc_stat <= 0.05)/reps,
            `SA (0.01)` = 100*count(passed_safety_001 == "True" & sa_001_disc_stat <= 0.01)/reps) |>
  pivot_longer(cols = -c(sample_size),
               names_to = "model",
               values_to = "prop_passed_safety_satisfied_cstr")

# join both data sets
conv_disc <- inner_join(sim_converged_long, sim_satisfied_cstr_long,
                       by = c("sample_size", "model")) |>
  mutate(prop_passed_safety = prop_passed_safety - prop_passed_safety_satisfied_cstr) |>
  rename("prop_passed_safety_failed_cstr" = prop_passed_safety) |>
  inner_join(sim_failure_long,
            by = c("sample_size", "model")) |>
  rename("failed safety test" = prop_failed_safety,
        "passed safety test, didn't satisfy constraint" = prop_passed_safety_failed_cstr,
        "passed safety test, satisfied constraint" = prop_passed_safety_satisfied_cstr) |>
  pivot_longer(cols = -c(sample_size, model),
               names_to = "statistic",
               values_to = "value") |>
  mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
                                         sample_size == 1000 ~ "n = 1000",
                                         sample_size == 2500 ~ "n = 2500",
                                         sample_size == 5000 ~ "n = 5000"
                                         ),
         levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000")))

# plot

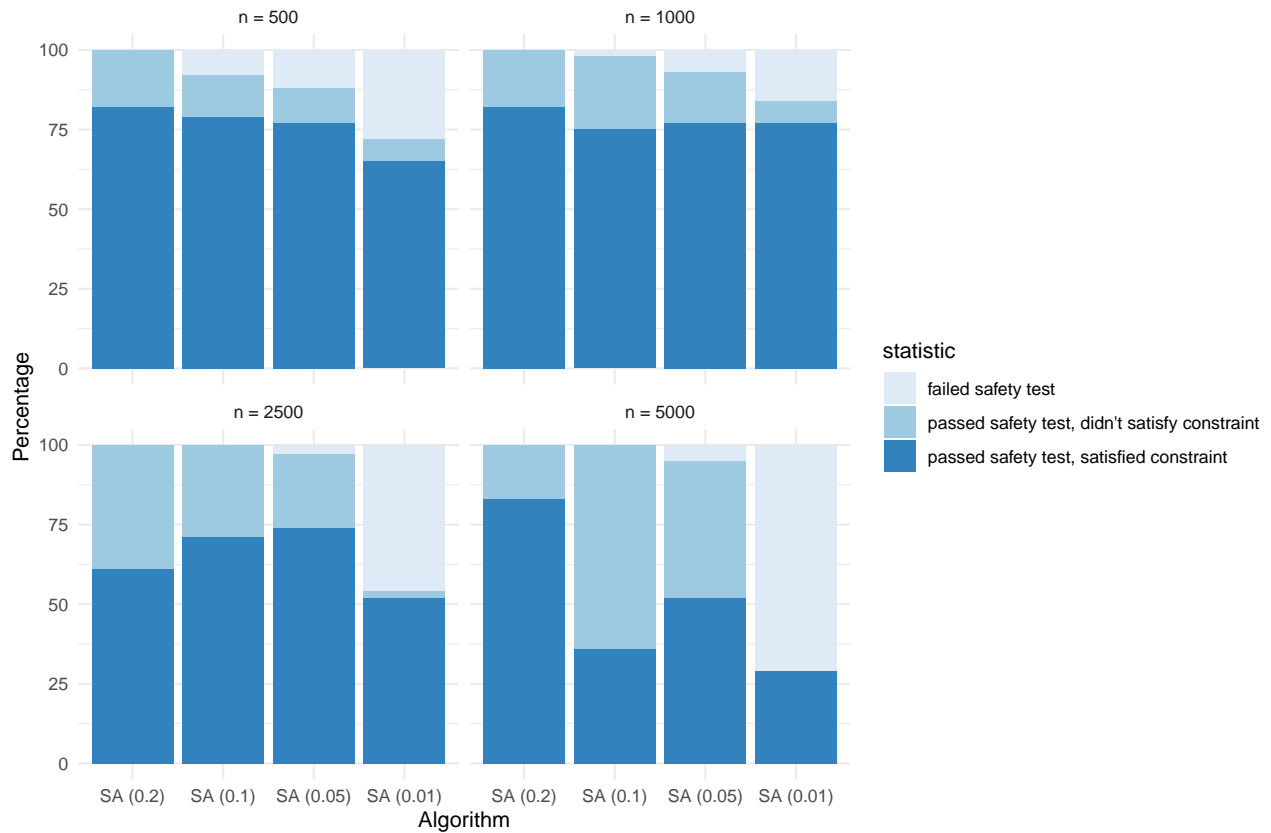
conv_disc |>
  mutate(model = factor(model,
                        levels = c("SA (0.2)", "SA (0.1)", "SA (0.05)", "SA (0.01)"))) |>
  group_by(model, statistic) |>
  summarise(avg = mean(value, na.rm = TRUE)) |>
  ggplot(mapping = aes(x = model, y = avg, fill = statistic)) +
  geom_col() +
  theme_minimal() +
  scale_fill_brewer(palette = "Blues") +
  labs(y = "Percentage",
       x = "Algorithm",
       title = "Probability of Returning a Solution and Satisfying the Constraint",
       fill = " ")

```

## Probability of Returning a Solution and Satisfying the Constraint



```
# plot  
  
conv_disc |>  
  mutate(model = factor(model,  
                          levels = c("SA (0.2)", "SA (0.1)", "SA (0.05)", "SA (0.01)")))|>  
  group_by(model, statistic, sample_size) |>  
  summarise(avg = mean(value, na.rm = TRUE)) |>  
  ggplot(mapping = aes(x = model, y = avg, fill = statistic)) +  
  geom_col() +  
  theme_minimal() +  
  scale_fill_brewer(palette = "Blues") +  
  facet_wrap(~sample_size) +  
  labs(y = "Percentage",  
       x = "Algorithm")
```



## NSF

Finally, this section will assess the candidate solutions of the Seldonian algorithm that didn't pass the safety test.

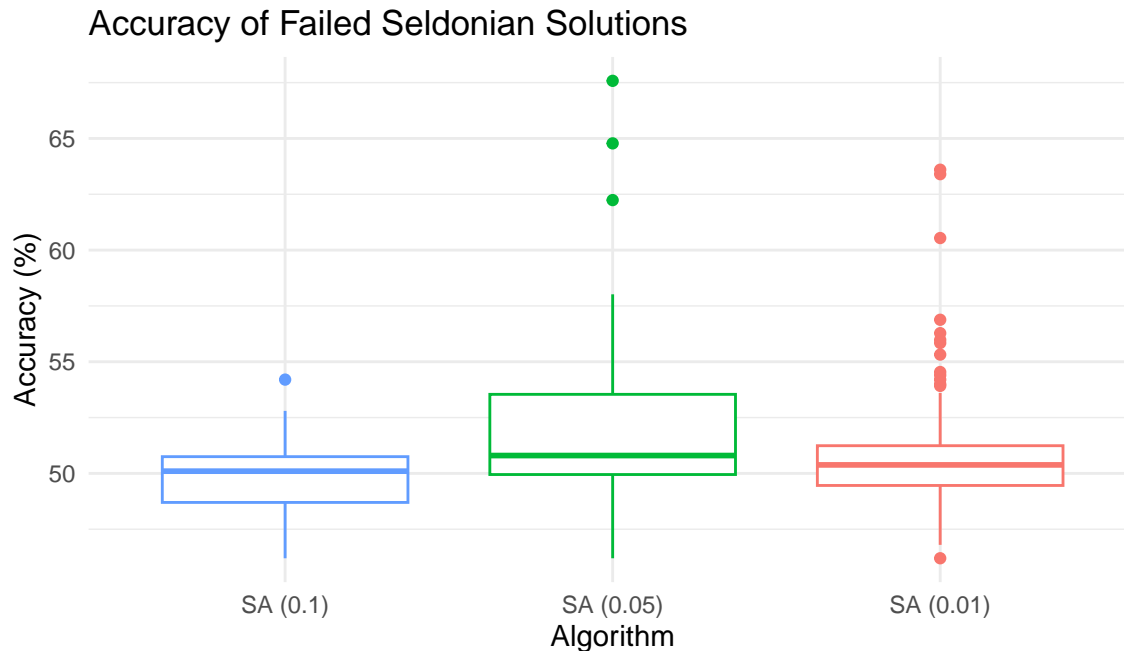
## Accuracy

```
# filter accuracy data out for only seldonian solutions that failed the safety test
sim_results_failed_accuracy <- sim_results |>
  mutate(sa_02_accuracy = ifelse(passed_safety_02 == "False", sa_02_accuracy, NA),
         sa_01_accuracy = ifelse(passed_safety_01 == "False", sa_01_accuracy, NA),
         sa_005_accuracy = ifelse(passed_safety_005 == "False", sa_005_accuracy, NA),
         sa_001_accuracy = ifelse(passed_safety_001 == "False", sa_001_accuracy, NA))
```

```
# visualization
sim_results_failed_accuracy |>
  dplyr::select(c(sa_02_accuracy, sa_01_accuracy, sa_005_accuracy,
                 sa_001_accuracy, sample_size)) |>
  rename("Sample Size" = sample_size,
         "SA (0.2)" = sa_02_accuracy,
         "SA (0.1)" = sa_01_accuracy,
         "SA (0.05)" = sa_005_accuracy,
         "SA (0.01)" = sa_001_accuracy) |>
  pivot_longer(cols = -c(`Sample Size`),
               names_to = "model",
               values_to = "accuracy") |>
```

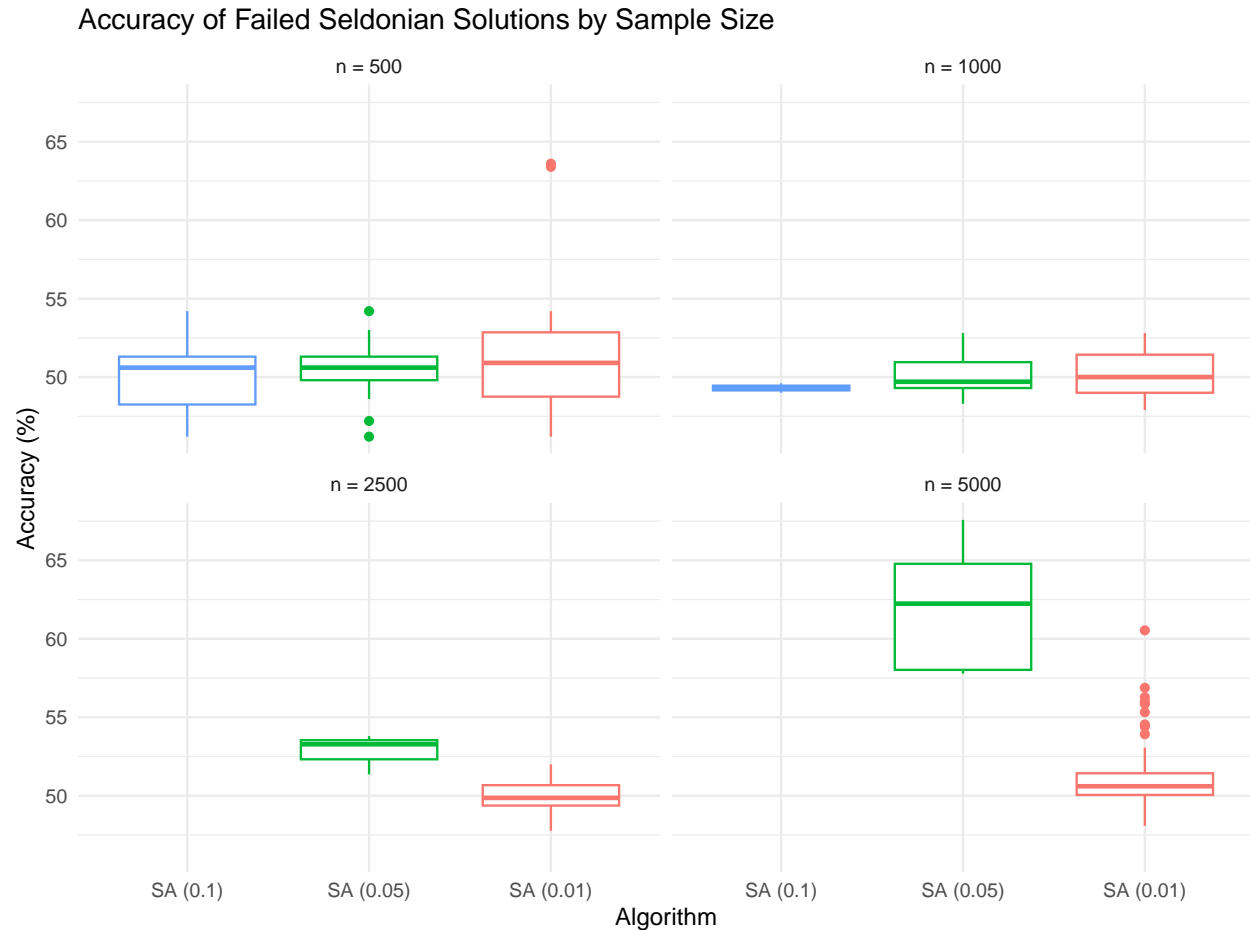


```
mutate(accuracy = 100*accuracy) |>
ggplot(mapping = aes(x = model, y = accuracy, color = model)) +
geom_boxplot() +
scale_x_discrete(limits = c("SA (0.1)", "SA (0.05)", "SA (0.01)")) +
theme_minimal() +
guides(color = "none") +
labs(x = "Algorithm",
y = "Accuracy (%)",
title = "Accuracy of Failed Seldonian Solutions")
```



```
# visualization
sim_results_failed_accuracy |>
dplyr::select(c(sa_02_accuracy, sa_01_accuracy, sa_005_accuracy,
sa_001_accuracy, sample_size)) |>
mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
sample_size == 1000 ~ "n = 1000",
sample_size == 2500 ~ "n = 2500",
sample_size == 5000 ~ "n = 5000"
),
levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000"))) |>
rename("Sample Size" = sample_size,
"SA (0.2)" = sa_02_accuracy,
"SA (0.1)" = sa_01_accuracy,
"SA (0.05)" = sa_005_accuracy,
"SA (0.01)" = sa_001_accuracy) |>
pivot_longer(cols = -c("Sample Size"),
names_to = "model",
values_to = "accuracy") |>
mutate(accuracy = 100*accuracy) |>
ggplot(mapping = aes(x = model, y = accuracy, color = model)) +
geom_boxplot() +
scale_x_discrete(limits = c("SA (0.1)", "SA (0.05)", "SA (0.01)")) +
theme_minimal() +
```

```
guides(color = "none") +
facet_wrap(~`Sample Size`) +
labs(x = "Algorithm",
     y = "Accuracy (%)",
     title = "Accuracy of Failed Seldonian Solutions by Sample Size")
```



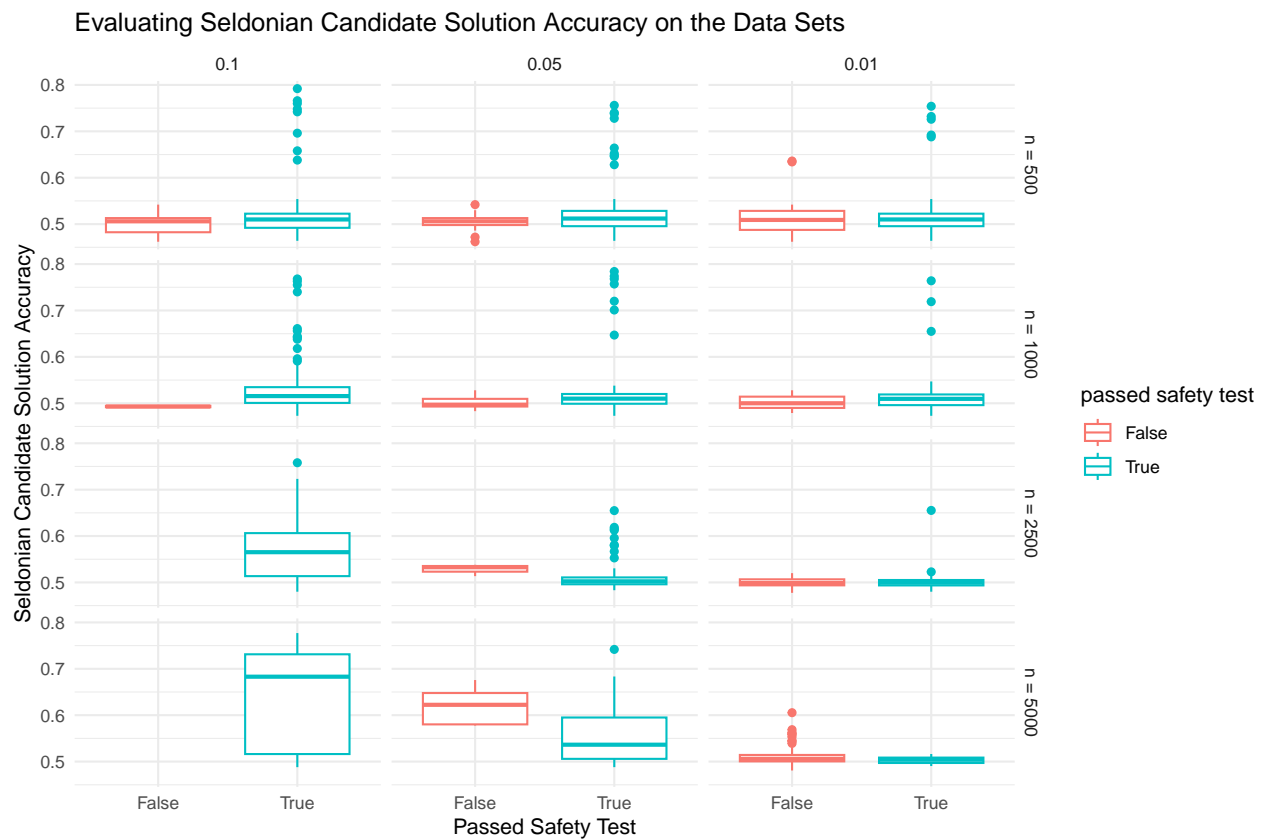
```
sa_accuracy <- sim_results |>
  dplyr::select(c(lr_accuracy, sa_01_accuracy, sa_005_accuracy,
                 sa_001_accuracy, sample_size, dataset_id)) |>
  pivot_longer(cols = -c(dataset_id, sample_size, lr_accuracy),
               names_to = "epsilon",
               values_to = "accuracy") |>
  mutate(epsilon = case_when(epsilon == "sa_01_accuracy" ~ 0.1,
                             epsilon == "sa_005_accuracy" ~ 0.05,
                             epsilon == "sa_001_accuracy" ~ 0.01))

sim_results |>
  dplyr::select(c(lr_accuracy, passed_safety_01, passed_safety_005,
                 passed_safety_001, sample_size, dataset_id)) |>
  pivot_longer(cols = -c(dataset_id, sample_size, lr_accuracy),
               names_to = "epsilon",
               values_to = "passed_safety") |>
  mutate(epsilon = case_when(epsilon == "passed_safety_01" ~ 0.1,
                             epsilon == "passed_safety_005" ~ 0.05,
```

```

    epsilon == "passed_safety_001" ~ 0.01)) |>
inner_join(sa_accuracy, by = c("lr_accuracy", "epsilon", "sample_size", "dataset_id")) |>
mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
    sample_size == 1000 ~ "n = 1000",
    sample_size == 2500 ~ "n = 2500",
    sample_size == 5000 ~ "n = 5000"
    ),
    levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000")),
    epsilon = factor(epsilon, levels = c(0.1, 0.05, 0.01))) |>
ggplot(mapping = aes(x = passed_safety, y = accuracy, color = passed_safety)) +
geom_boxplot() +
theme_minimal() +
facet_grid(rows = vars(sample_size), cols = vars(epsilon)) +
labs(x = "Passed Safety Test",
    y = "Seldonian Candidate Solution Accuracy",
    title = "Evaluating Seldonian Candidate Solution Accuracy on the Data Sets",
    color = "passed safety test")

```



## Accuracy of Logistic Regression in Failed Seldonian Solutions

```

sim_results |>
dplyr::select(c(lr_accuracy, passed_safety_01, passed_safety_005,
    passed_safety_001, sample_size)) |>
pivot_longer(cols = -c(sample_size, lr_accuracy),
    names_to = "epsilon",
    values_to = "passed_safety") |>

```

```

mutate(epsilon = case_when(epsilon == "passed_safety_01" ~ 0.1,
                           epsilon == "passed_safety_005" ~ 0.05,
                           epsilon == "passed_safety_001" ~ 0.01)) |>
mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
                                       sample_size == 1000 ~ "n = 1000",
                                       sample_size == 2500 ~ "n = 2500",
                                       sample_size == 5000 ~ "n = 5000"
                                       ),
        levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000")),
epsilon = factor(epsilon, levels = c(0.1, 0.05, 0.01)) |>
ggplot(mapping = aes(x = passed_safety, y = lr_accuracy, color = passed_safety)) +
geom_boxplot() +
theme_minimal() +
facet_grid(rows = vars(sample_size), cols = vars(epsilon)) +
labs(x = "Passed Safety Test",
     y = "Logistic Regression Accuracy",
     title = "Evaluating Logistic Regression Accuracy on the Data Sets by Seldonian Solutions",
     color = "passed safety test")

```



## Discrimination

```

# filter accuracy data out for only seldonian solutions that failed the safety test
sim_results_failed_disc <- sim_results |>
mutate(sa_02_disc_stat = ifelse(passed_safety_02 == "False", sa_02_disc_stat, NA),
       sa_01_disc_stat = ifelse(passed_safety_01 == "False", sa_01_disc_stat, NA),
       sa_005_disc_stat = ifelse(passed_safety_005 == "False", sa_005_disc_stat, NA),

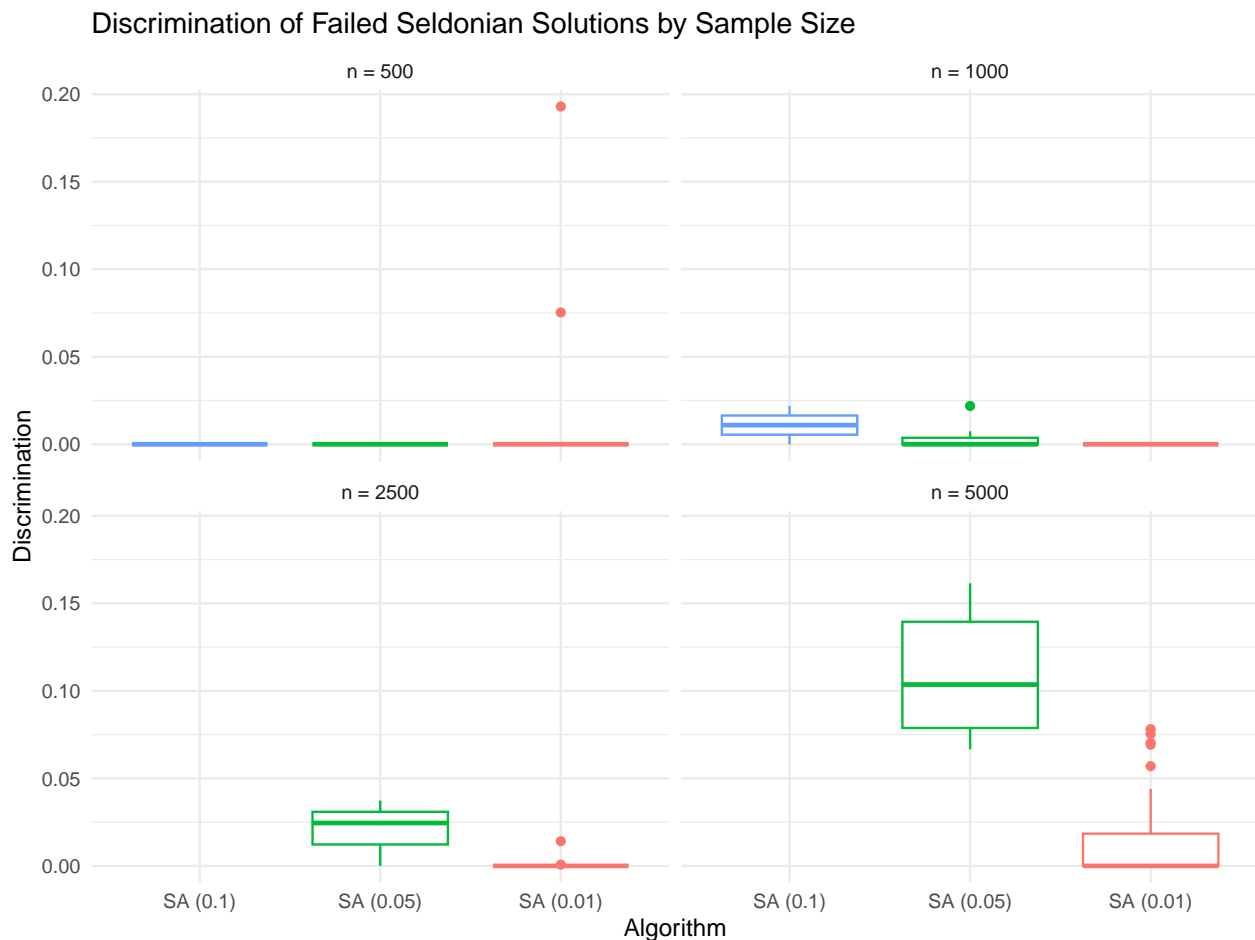
```



```

rename("Sample Size" = sample_size,
       "SA (0.2)" = sa_02_disc_stat,
       "SA (0.1)" = sa_01_disc_stat,
       "SA (0.05)" = sa_005_disc_stat,
       "SA (0.01)" = sa_001_disc_stat) |>
pivot_longer(cols = -c(`Sample Size`),
             names_to = "model",
             values_to = "discrimination") |>
ggplot(mapping = aes(x = model, y = discrimination, color = model)) +
  geom_boxplot() +
  scale_x_discrete(limits = c("SA (0.1)", "SA (0.05)", "SA (0.01)")) +
  theme_minimal() +
  guides(color = "none") +
  facet_wrap(~`Sample Size`) +
  labs(x = "Algorithm",
       y = "Discrimination",
       title = "Discrimination of Failed Seldonian Solutions by Sample Size")

```



```

sa_disc_stat <- sim_results |>
dplyr::select(c(lr_discrimination, sa_01_disc_stat, sa_005_disc_stat,
               sa_001_disc_stat, sample_size, dataset_id)) |>
pivot_longer(cols = -c(dataset_id, sample_size, lr_discrimination),
             names_to = "epsilon",
             values_to = "disc_stat") |>

```

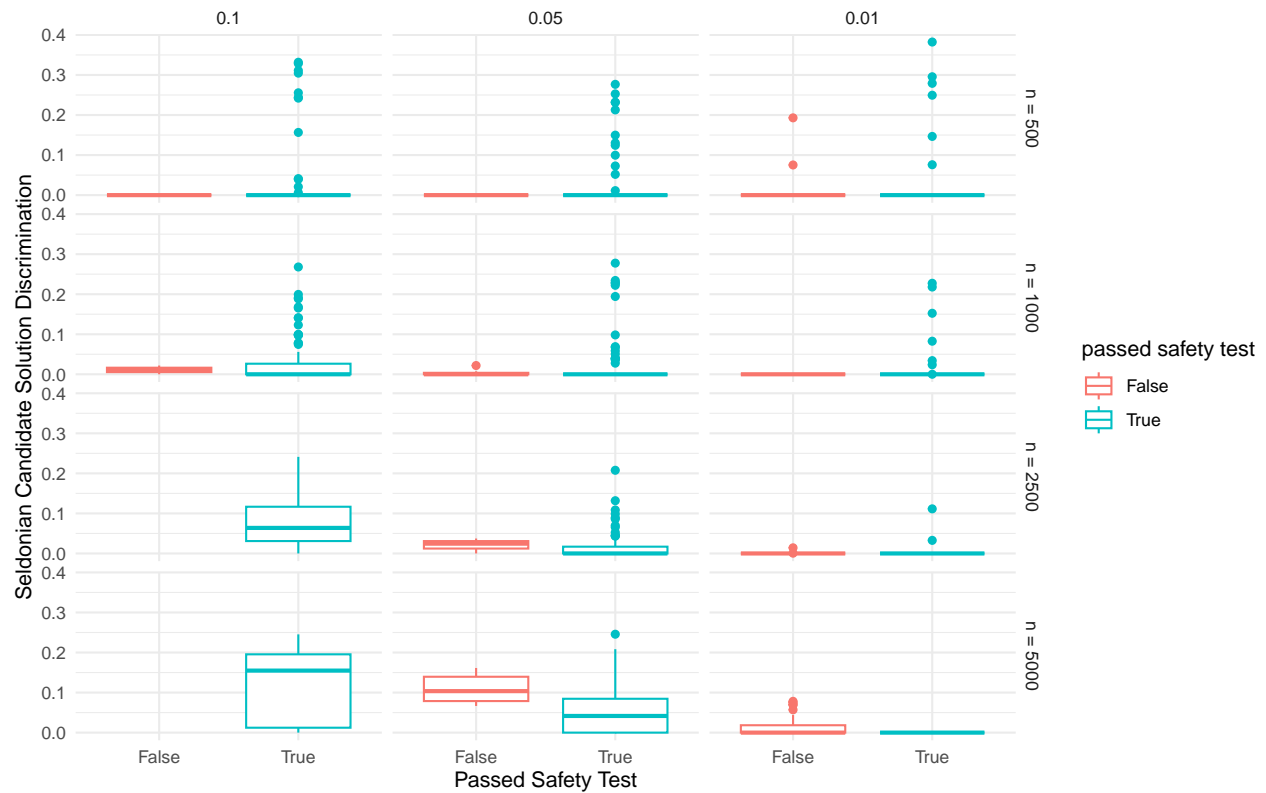
```

mutate(epsilon = case_when(epsilon == "sa_01_disc_stat" ~ 0.1,
                           epsilon == "sa_005_disc_stat" ~ 0.05,
                           epsilon == "sa_001_disc_stat" ~ 0.01))

sim_results |>
  dplyr::select(c(lr_discrimination, passed_safety_01, passed_safety_005,
                 passed_safety_001, sample_size, dataset_id)) |>
  pivot_longer(cols = -c(dataset_id, sample_size, lr_discrimination),
               names_to = "epsilon",
               values_to = "passed_safety") |>
  mutate(epsilon = case_when(epsilon == "passed_safety_01" ~ 0.1,
                             epsilon == "passed_safety_005" ~ 0.05,
                             epsilon == "passed_safety_001" ~ 0.01)) |>
  inner_join(sa_disc_stat, by = c("lr_discrimination", "epsilon", "sample_size", "dataset_id")) |>
  mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
                                         sample_size == 1000 ~ "n = 1000",
                                         sample_size == 2500 ~ "n = 2500",
                                         sample_size == 5000 ~ "n = 5000"
                                         ),
          levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000")),
         epsilon = factor(epsilon, levels = c(0.1, 0.05, 0.01))) |>
  ggplot(mapping = aes(x = passed_safety, y = disc_stat, color = passed_safety)) +
  geom_boxplot() +
  theme_minimal() +
  facet_grid(rows = vars(sample_size), cols = vars(epsilon)) +
  labs(x = "Passed Safety Test",
       y = "Seldonian Candidate Solution Discrimination",
       title = "Evaluating Seldonian Candidate Solution Discrimination on the Data Sets",
       color = "passed safety test")

```

## Evaluating Seldonian Candidate Solution Discrimination on the Data Sets



## Discrimination of Logistic Regression in Failed Seldonian Solutions

```
sim_results |>
  dplyr::select(c(lr_discrimination, passed_safety_01, passed_safety_005,
    passed_safety_001, sample_size)) |>
  pivot_longer(cols = -c(sample_size, lr_discrimination),
    names_to = "epsilon",
    values_to = "passed_safety") |>
  mutate(epsilon = case_when(epsilon == "passed_safety_01" ~ 0.1,
    epsilon == "passed_safety_005" ~ 0.05,
    epsilon == "passed_safety_001" ~ 0.01)) |>
  mutate(sample_size = factor(case_when(sample_size == 500 ~ "n = 500",
    sample_size == 1000 ~ "n = 1000",
    sample_size == 2500 ~ "n = 2500",
    sample_size == 5000 ~ "n = 5000"
  )),
    levels = c("n = 500", "n = 1000", "n = 2500", "n = 5000")),
    epsilon = factor(epsilon, levels = c(0.1, 0.05, 0.01))) |>
  ggplot(mapping = aes(x = passed_safety, y = lr_discrimination, color = passed_safety)) +
  geom_boxplot() +
  theme_minimal() +
  facet_grid(rows = vars(sample_size), cols = vars(epsilon)) +
  labs(x = "Passed Safety Test",
    y = "Logistic Regression Discrimination",
    title = "Evaluating Logistic Regression Accuracy on the Data Sets by Seldonian Solutions",
    color = "passed safety test")
```



# Evaluating Logistic Regression Accuracy on the Data Sets by Seldonian Solutions

