1.4 a) $T(n) = \begin{cases} 1, & n \le a,\ a > 0 \\ T(n-a)+1, & n > a \end{cases}$

$n - \frac{n\cancel{a}}{\cancel{a}} \le a > 0$

$$T(n) = T(n-a)+1 = T(n-2a)+2 = T\left(n - \frac{n\cancel{a}}{\cancel{a}}\right) + \frac{n}{a} = \frac{n}{a} + 1 = \underline{ceil\left(\frac{n}{a}\right)+1}$$

b) $T(n) = \begin{cases} 1, & n=0 \\ T(n-1) + 2^n, & n \ge 1 \end{cases}$

$$T(n) = T(n-1) + 2^n = T(n-2) + 2^{n-1} + 2^n = \cancel{T(n-2)} + T(n-n) + 2^1 + 2^2 + \dots 2^{n-1} + 2^n =$$
$$= 1 + (2^{n+1} - 2) = \underline{2^{n+1} - 1}$$

c) $T(n) = \begin{cases} 1, & n=1 \\ 2T([\frac{n}{2}])+1, & n \ge 2 \end{cases}$

$$T(n) = 2T([\tfrac{n}{2}]) + 1 = 2\left(2T([\tfrac{n}{4}]) + 1\right) + 1 = 4T([\tfrac{n}{4}]) + 2 + 1 =$$
$$= 4\left(2T([\tfrac{n}{8}]) + 1\right) + 2 + 1 = 8T([\tfrac{n}{8}]) + 4 + 2 + 1 = \left| \begin{matrix} n = 2^m \\ m = \log n \end{matrix} \right| = 8T(2^{m-3}) + 4 + 2 + 1 =$$
$$= 2^m T(2^{m-m}) + \sum_{i=0}^{m-1} 2^i = 2^m + 2^m - 1 = 2^{m+1} - 1 = 2^{\log n} \cdot 2 - 1 = \underline{2n-1}$$

d) $T(n) = \begin{cases} 1, & n=1 \\ aT([n/a]) + n, & n \ge 2,\ a \ge 2 \end{cases}$

$n = a^m \Rightarrow m = \log_a n$

$$T(n) = aT(a^{m-1}) + a^m = a\left(aT(a^{m-2}) + a^{m-1}\right) + n = a^2 T(a^{m-2}) + a^m + a^m =$$
$$= a^2\left(aT(a^{m-3}) + a^{m-2}\right) + a^m + a^m = a^3 T(a^{m-3}) + a^m + a^m + a^m = a^m T(a^{m-m}) + m a^m =$$
$$= n + \log_a n \cdot n = \underline{n + n\log_a n}$$