2.7 $f(n) = 3n^2 - n + 4$ $\qquad g(n) = n\log n + 5$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = \lim_{n\to\infty} \frac{n\log n + 5}{3n^2 - n + 4} = \lim_{n\to\infty} \frac{\cancel{n^2}\left(\frac{\log n}{n} + \frac{5}{n^2}\right)}{\cancel{n^2}\left(3 - \frac{1}{n} + \frac{4}{n^2}\right)} = 0$$

Отже, $f(n) + g(n) = O(f) = O(n^2)$ ✓

2.13 (a, d, g, h)

a) $T(n) = \begin{cases} O(1), & n = 0 \\ T(n-1) + O(1), & n \geq 1 \end{cases}$

$T(n) \leq T(n-1) + C \leq T(n-2) + 2C \leq T(n-n) + nC = O(n)$

d) $T(n) = \begin{cases} O(1), & n \leq a,\ a > 1 \\ aT(n-a) + O(1), & n > a \end{cases}$

$T(n) \leq aT(n-a) + C \leq a(aT(n-2a) + C) + C = a^2T(n-2a) + Ca + C$

~~$\leq a(a^2T(n-3a) + C) + Ca + C = a^3T(n-3a) + C(2a+1)$~~

$\leq a^2(aT(n-3a) + C) + Ca + C = a^3T(n-3a) + a^2C + aC + C \leq$

$\leq a^{\frac{n}{a}} T\left(n - \frac{na}{a}\right) + C\sum_{i=0}^{\frac{n}{a}-1} a^i \leq a^{\frac{n}{a}} C + \frac{a^{\frac{n}{a}} - 1}{a - 1} C$

Оскільки рахуємо оцінку, основу степеня можна замінити на 2

$T(n) = O(2^{\frac{n}{a}})$

g) $T(n) = \begin{cases} O(1), & n = 1 \\ \cancel{T(n-a)+}\ aT([n/a]) + O(1), & n \geq 2,\ a \geq 2 \end{cases}$

$$n = a^m \Rightarrow m = \log_a n$$

$$T(n) \leq aT\left(\left[\frac{n}{a}\right]\right) + C = aT\left(a^{m-1}\right) + C \leq a\left(aT\left(a^{m-2}\right) + C\right) + C =$$

$$= a^2T\left(a^{m-2}\right) + aC + C \leq a^2\left(aT\left(a^{m-3}\right) + C\right) + aC + C =$$

$$= a^3T\left(a^{m-3}\right) + a^2C + aC + C \leq a^mT\left(a^{m-m}\right) + C\sum_{i=0}^{m-1} a^i =$$

$$= a^m C + C\frac{a^m - 1}{a - 1} = nC + \frac{n-1}{a-1}C = O(n)$$

h) $T(n) = \begin{cases} O(1), & n = 1 \\ aT\left(\left[\frac{n}{a}\right]\right) + O(n), & n \geq 2,\ a \geq 2 \end{cases}$

$$n = a^m \Rightarrow m = \log_a n$$

$$T(n) \leq aT\left(a^{m-1}\right) + a^m \cdot C \leq a\left(aT\left(a^{m-2}\right) + a^{m-1}C\right) + a^m \cdot C =$$

$$= a^2T\left(a^{m-2}\right) + a^mC + a^mC \leq a^mT\left(a^{m-m}\right) + ma^m \cdot C =$$

$$= n \cdot C + n\log_a n \cdot C \leq O(n \log n)$$