Proceedings, Bicycle and Motorcycle Dynamics 2016
Symposium on the Dynamics and Control of Single Track Vehicles,
21–23 September 2016, Milwaukee, Wisconsin, USA

Buckling and dynamic collapse of the bicycle wheel

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ABSTRACT

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Keywords: bicycle wheel, buckling, bifurcation.

1 INTRODUCTION

The bicycle wheel is a prestressed structure and is susceptible to buckling under internal forces. As the spokes are tightened uniformly, the rim deforms radially to accommodate the spoke strain. At a critical tension, the system reaches a bifurcation point and the rim buckles out of its initial plane. The post-buckling configuration is generally stable and the original shape of the wheel can be recovered by reducing the tension. Buckling can also be triggered by external forces in an otherwise stable wheel leading to a release of strain energy. This is an unstable process which generally leads to catastrophic failure.

Despite its implications for wheel strength, the buckling problem has never received a rigorous treatment to our knowledge. Jobst Brandt alludes to buckling in his practical manual for wheelbuilders[3]:

> "If the wheel becomes untrue in two large waves during stress relieving, the maximum, safe tension has been exceeded. Approach this tension carefully to avoid major rim distortions. When the wheel loses alignment from stress relieving, loosen all spokes a half turn before retruing the wheel."

He did not discuss the problem further, but suggested that wheel failure commonly occurs due to a loss of lateral stability caused by spoke buckling. Pippard and Francis[7] derived a model for lateral stiffness based on the elastic foundation model, but did not discuss stability and neglected any effects of spoke tension.

Flexural-torsional buckling of rings can be treated as a special case of buckling of arches, where the included angle is allowed to go to 2π. Timoshenko and Gere[8] gave a formula for the critical load for a ring with doubly-symmetric cross-section subjected to radial loads. The theory of flexural-torsional buckling of monosymmetric arches (bicycle rims have only one plane of symmetry) was broadly formalized by Trahair and Papangelis[9] using the virtual work approach to derive the equilibrium and stability equations. Their theory has been extended to treat arches restrained by continuous[6] or discrete[2] elastic supports and elastic end restraints[4].

The problem of the prestressed bicycle wheel is unique for a number of reasons. First, the buckling load is internal to the structure. Second, the spokes act both as elastic restraints resisting buckling and as prestressing elements causing buckling. Third, the lateral, radial, and torsional restraining actions of the spokes are coupled—i.e. lateral deflection at a spoke may produce lateral, radial, and torsional reactions. These considerations extend to other structural systems. Large observation wheels such as the London Eye[5] and the Singapore Flyer[1] resemble large bicycle wheels and achieve lateral stability due to the bracing angle of prestressed cables, and must be designed against flexural-torsional buckling.

Here we derive a general theory for buckling of a spoked bicycle wheel under self-tension and investigate several special cases. Furthermore, we analyze large deformation post-buckling behavior and show how local buckling of individual spokes can lead to collapse of an otherwise sub-critical wheel.

2 DEFORMATION OF THE WHEEL

2.1 Rim equations

The rim is modeled as an initially circular beam with a constant cross-section.

2.2 Spoke equations

In this paper we consider bicycle wheels with slender prestressed spokes. We model a single spoke as an elastic bar pinned at each end. This ensures that the spoke force is always collinear with the spoke axis. The linearized elongation of a spoke is given by

$$\Delta l = (\boldsymbol{u}_2 - \boldsymbol{u}_1) \cdot \hat{\boldsymbol{n}}_0 \tag{1}$$

where $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ are the displacement vectors of the two nodes of the spoke and $\hat{\boldsymbol{n}}_0$ is a unit vector pointing from node 1 to node 2.

2.3 Figures and tables

All figures should be clearly readable and relevant to the presented text. Use of at least 300 dpi resolution for pictures and 600 dpi for line art is required, 1 px wide lines in figures should be

Table 1. Example of a table with a short caption.

	x	y	z
x'	α_1	β_1	γ_1
y'	α_2	β_2	γ_2
z'	α_3	β_3	γ_3

avoided as they may become invisible in print. There is no limit on the amount of figures as long as they do not dominate the text and the total length of the paper is within the specified limits. Both figures and tables should be centred on the page.

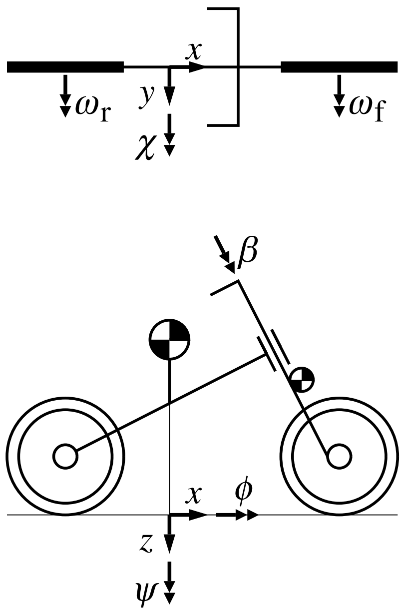

Figure 1. An example of a figure caption. Use 10 pt Times New Roman. For long captions, use a text width of 13 cm. Use the same style for the tables.

Figures, graphs and tables must be included in the same style as shown for Figure 1 and Table 1.

2.4 References

Bibliographical citations should be written in the order in which they are cited, see the References section below, where Reference [**?**] exemplifies the case of a textbook, while Reference [**?**] is an article in conference proceedings and Reference [**?**] is an article in a journal. Use can be made of a pre-existing bibtex style which uses a similar style.

3 SUBMISSION OF PAPER

The paper should be submitted by e-mail to jbrendelson@bmd2016mke.org with the email subject containing the phrase *"BMD 2016 paper submission - submitting author last name"*. The deadline is September 1st, 2016. The file of the paper must be in the Portable Document Format (PDF) and the file size should be within the limit of 10 MB. Exceptionally, other file formats can be accepted if the symposium organizers have been informed beforehand.

Please make sure that the final version of the paper complies with the standards set here. Please note slightly different formatting requirements of the abstract and the final paper.

4 CONCLUSIONS

We very much look forward to welcoming you in Milwaukee! Best wishes and the warmest regards from the Organizing Committee of Bicycle and Motorcycle Dynamics 2016.

REFERENCES

[1] ALLSOP, A., DALLARD, P., AND MCNIVEN, B. The Singapore Flyer and Design of Giant Observation Wheels, Singapore. Struct. Eng. Int. 19, 1 (feb 2009), 12–16.

[2] BRADFORD, M. A., AND PI, Y.-L. Elastic Flexural-Torsional Buckling of Discretely Restrained Arches. J. Struct. Eng. 128, 6 (2002), 719–727.

[3] BRANDT, J. The Bicycle Wheel, 3rd ed. Avocet, Palo Alto, CA, 1993.

[4] GUO, Y. L., ZHAO, S. Y., DOU, C., AND PI, Y. L. Out-of-Plane Elastic Buckling of Circular Arches with Elastic End Restraints. J. Struct. Eng. 140, 10 (2014), 9.

[5] MANN, A., THOMPSON, N., AND SMITS, M. Building the British Airways London Eye. Proc. Inst. Civ. Eng. - Civ. Eng. 144, 2 (2001), 60–72.

[6] PI, Y.-L., AND BRADFORD, M. Elastic flexural-torsional buckling of continuously restrained arches. Int. J. Solids Struct. 39, 8 (apr 2002), 2299–2322.

[7] PIPPARD, A. S., AND FRANCIS, W. The Stresses in a Wire Wheel under Side Loads on the Rim. Philos. Mag. 14, 91 (1932), 436–445.

[8] TIMOSHENKO, S. P., AND GERE, J. M. Theory of Elastic Stability, 2nd ed. McGraw-Hill, New York, NY, 1961.

[9] TRAHAIR, N. S., AND PAPANGELIS, J. P. Flexural-torsional buckling of monosymmetric arches. J. Struct. Eng. 113, 10 (1987), 2271–2288.