2024_“ShuWei Cup”

Problem A：Frequency Estimation Problems in Aircraft Laser Velocimetry

(I) Background of the Problem

Airspeed, the velocity of an aircraft relative to the air, is a critical parameter that needs to be monitored during flight. Airspeed is closely linked to flight states such as angle of attack and sideslip angle. If airspeed data is abnormal, it can easily lead to accidents like stalling. Therefore, accurate measurement of airspeed is of great significance.

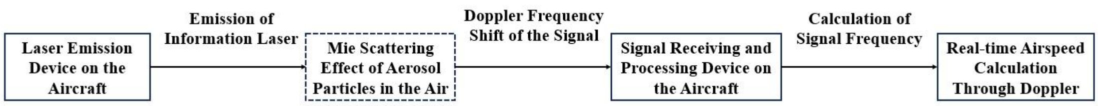

Figure 1: Schematic Diagram of Laser Velocimetry for Airspeed

Laser velocimetry is a feasible method for measuring airspeed. The principle, as shown in Figure 1, involves emitting a laser at a fixed frequency, then obtaining the signal light with Doppler frequency shift through the Mie scattering effect on aerosol particles in the air. Using the principle of coherent interference, a signal containing Doppler shift information is obtained, and the frequency of this signal is estimated. By using the estimated Doppler shift information, airspeed can be calculated. A critical step in airspeed measurement is estimating the frequency information of the time series signal, where the signal satisfies the following expression:

$$x(t) = A\sin(2\pi f_0 t + \varphi) + z(t)$$

where A represents the amplitude of the signal, f_0 is the signal's frequency, φ is the signal's phase, and $z(t)$ denotes noise information. Due to interference from factors such as air pressure and temperature, the signal received by the aircraft is mixed with substantial noise information $z(t)$, making the frequency estimation problem for the signal $x(t)$ extremely challenging.

Currently, there is an example of an aircraft flying in space with a sampling interval of T_s=2×10^{-9} seconds, receiving multiple Doppler-shifted signals, with actual received data provided in Attachment 1. To ensure information security, different flight periods may result in variations in the amplitude, frequency, and phase of the laser signals received by the aircraft, and the environmental noise may differ. Therefore, the characteristics of the received laser signal, including frequency and phase, may not be the same in different flight periods. Moreover, environmental noise also differs, i.e., the noise characteristics in Attachment 1 are not the same across different sub-tables.

(II) Data for the Problem

Please refer to Attachment 1.

(III) Problems to Be Solved

1. Analyzing the noise characteristics of the actual received signal can aid in the design of a signal frequency estimation algorithm. In Flight Period 1 of Attachment 1, the known amplitude of the non-noise part of the received signal is 4, the frequency is 30×10^6 Hz, and the phase is 45°. Please analyze the noise $z(t)$ characteristics of the received data in Flight Period 1.

2. In practical scenarios, the frequency of the non-noise part of the received signal is unknown and needs to be estimated. In Flight Period 2 of Attachment 1, the amplitude of the real received signal is known to be 2, and the phase is 0°. Please design a method to estimate the frequency of the non-noise part of the received signal in Flight Period 2. (Note: The noise characteristics in Flight Period 2 may differ from those in Flight Period 1.)

3. In practice, it is often impossible to know the amplitude and phase information of the non-noise part of the received signal in advance, yet the frequency still needs to be estimated. Based on the data from Flight Period 3 in Attachment 1, please design a method to estimate the frequency of the received signal in Flight Period 3. (Note: The

noise characteristics in Flight Period 3 may differ from those in Flight Periods 1 and 2.)

4. To avoid interference between signals, intermittent reception is used in practice, limiting the amount of available information. Refer to the data from Flight Period 4 in Attachment 1. Based on this data, please analyze the pattern of intermittent reception and design a method to estimate the frequency of the received signal in Flight Period 4.