Material Motion $n°1$:

$$\boldsymbol{\psi} = \psi_\beta(\boldsymbol{\chi}, t) = \begin{pmatrix} \chi_1 + \beta \sin(2\pi\chi_1)\sin(\pi\chi_2/3)\sin(\pi t/T) \\ \chi_2 + 5\beta\sin(2\pi\chi_1)\sin(\pi\chi_2/3)\sin(2\pi t/T) \\ \chi_3 \end{pmatrix}, \tag{1}$$

with $T = 2$ and β an amplitude parameter defined for each simulation.

Material Motion $n°2$:

$$\boldsymbol{\psi} = \psi_\beta(\boldsymbol{\chi}, t) = \begin{pmatrix} \chi_1 + \beta \sin^2(2\pi\chi_1)\sin^2(\pi\chi_2/3)\sin(\pi t/T) \\ \chi_2 + 5\beta\sin^2(2\pi\chi_1)\sin^2(\pi\chi_2/3)\sin(2\pi t/T) \\ \chi_3 \end{pmatrix}, \tag{2}$$

$$\boldsymbol{w} = \begin{pmatrix} \frac{\beta\pi}{T}\sin^2(2\pi\chi_1)\sin^2(2\pi\chi_1)\cos(\pi t/T) \\ \frac{10.0\beta\pi}{T}\sin^2(2\pi\chi_1)\sin^2(\pi Y/3.0)\cos(2\pi t/T) \\ 0.0 \end{pmatrix}, \tag{3}$$

$$\boldsymbol{F}_\psi = \begin{pmatrix} 1 + 4\beta\pi\sin(2\pi X)\cos(2\pi X)\sin(\pi Y/3.0)\sin(\pi Y/3.0)\sin(\pi t/T) & 2\beta\pi/3.0\sin(2\pi X)\sin \\ 20.0\beta\pi\cos(2\pi X)\sin(2\pi X)\sin(\pi Y/3.0)\sin(\pi Y/3.0)\sin(2\pi t/T) & 1 + 10.0\beta\pi/3.0\sin(2\pi X) \\ 0 & \end{pmatrix} \tag{4}$$

0.1 Conservation Laws

0.1.1 Neo Hookean model

$$p = \kappa \left(det(\boldsymbol{trueF}) - 1 \right), \quad \text{with } \boldsymbol{trueF} = \boldsymbol{F}\boldsymbol{F}_{\psi}^{-1} \tag{5}$$

0.1.2 Set of conservation laws

$$\frac{\partial \boldsymbol{F}}{\partial t} = \mathrm{DIV}_{\boldsymbol{\chi}} \left(\hat{\boldsymbol{v}} \otimes \boldsymbol{I} \right) \tag{6a}$$

$$\frac{\partial J}{\partial t} = \boldsymbol{H}_{\psi} : \mathrm{GRAD}_{\boldsymbol{\chi}} \left(\boldsymbol{w} \right) \tag{6b}$$

$$\frac{\partial \boldsymbol{F}_{\psi}}{} = \mathrm{DIV}_{\boldsymbol{\chi}} \left(\boldsymbol{w} \otimes \boldsymbol{I} \right) \tag{6c}$$

$$\frac{\partial \tilde{\boldsymbol{p}}}{\partial t} = \mathrm{DIV}_{\boldsymbol{\chi}} \left(\boldsymbol{P}\boldsymbol{H}_{\psi} \right) \ + \ \mathrm{GRAD}_{\boldsymbol{\chi}} \left(\boldsymbol{p}_R \right) \left(\boldsymbol{H}_{\psi}^T \boldsymbol{w} \right) \ + \ \boldsymbol{p}_R \left[\boldsymbol{H}_{\psi} : \mathrm{GRAD}_{\boldsymbol{\chi}} \left(\boldsymbol{w} \right) \right] \tag{6d}$$

0.2 Finite Volume Method spatial discretization

0.2.1 Discretised set of conservation laws

$$\Omega_{\boldsymbol{\chi}}^a \frac{\partial \boldsymbol{F}^a}{\partial t} = \sum_{b \in \Lambda_a} \hat{\boldsymbol{v}}^{Ave} \otimes \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{ab} \ + \ \sum_{\gamma \in \Lambda_a^B} \hat{\boldsymbol{v}}_a^{\gamma} \otimes \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{\gamma} \tag{7a}$$

$$\Omega_{\boldsymbol{\chi}}^a \frac{\partial J^a}{\partial t} = \boldsymbol{H}_{\psi} : \left(\sum_{b \in \Lambda_a} \boldsymbol{w}^{Ave} \otimes \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{ab} \right) \tag{7b}$$

$$\Omega_{\boldsymbol{\chi}}^a \frac{\partial \boldsymbol{F}_{\psi}^a}{\partial t} = \sum_{b \in \Lambda_a} \boldsymbol{w}^{Ave} \otimes \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{ab} \ + \ \sum_{\gamma \in \Lambda_a^B} \boldsymbol{w}_a^{\gamma} \otimes \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{\gamma} \tag{7c}$$

$$\begin{aligned} \Omega_{\boldsymbol{\chi}}^a \frac{\partial \tilde{\boldsymbol{p}}^a}{\partial t} &= \sum_{b \in \Lambda_a} \left(\boldsymbol{P}\boldsymbol{H}_{\boldsymbol{\psi}} \right)^{Ave} \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{ab} \ + \ \sum_{\gamma \in \Lambda_a^B} \boldsymbol{t}_{\boldsymbol{\chi},a}^{\gamma} ||\boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{\gamma}|| + \sum_{b \in \Lambda_a} \boldsymbol{\mathcal{D}}_{\boldsymbol{\chi},ab} \\ &\quad + \ \left(\sum_{b \in \Lambda_a} \boldsymbol{p}_R^{Ave} \otimes \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{ab} \right) \boldsymbol{H}_{\psi}^T \boldsymbol{w} \ + \ \left(\sum_{\gamma \in \Lambda_a} \boldsymbol{p}_{R,\gamma} \otimes \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{\gamma} \right) \boldsymbol{H}_{\psi}^T \boldsymbol{w} \\ &\quad + \ \boldsymbol{p}_R \left[\boldsymbol{H}_{\psi} : \left(\sum_{b \in \Lambda_a} \boldsymbol{w}^{Ave} \otimes \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{ab} \right) \right] \ + \ \boldsymbol{p}_R \left[\boldsymbol{H}_{\psi} : \left(\sum_{\gamma \in \Lambda_a^B} \boldsymbol{w}_a^{\gamma} \otimes \boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{\gamma} \right) \right] \end{aligned} \tag{7d}$$

0.2.2 Algorithms

Algorithm 1: Computation of the right hand side of the linear momentum conservation law for the first Runge-Kutta stage.

Result: rhsLm, **rhsF**.

1. $\boldsymbol{p}_R^n = J_\psi^{-1} \boldsymbol{p}_{\boldsymbol{\chi}}^n$

2. $\boldsymbol{rhsLm}_a^* = \left(\sum_{b \in \Lambda_a} \boldsymbol{tC}_b^n ||\boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{ab}|| \right) + \left(\sum_{b \in \Lambda_a} \boldsymbol{tC2}_b^n ||\boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{ab}|| \right) \boldsymbol{H}_\psi^T \boldsymbol{w} +$
$\boldsymbol{p}_R^n \left(\boldsymbol{H}_\psi : \left[\sum_{b \in \Lambda_a} \boldsymbol{tC3}_b^n ||\boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^{ab}|| \right] \right)$

3. Boundary value of the first RHS term. $\boldsymbol{tC}_{loc}^\gamma = LocalAverage(\hat{\boldsymbol{P}} \boldsymbol{\mathcal{N}}_{\boldsymbol{\chi}}^\gamma)$ or apply the prescribed traction boundary condition. The nodal value is updated as $\boldsymbol{rhsLm}_\gamma^* = \boldsymbol{rhsLm}_\gamma^n + \boldsymbol{tC}_{loc}^\gamma ||\boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^\gamma||$.

4. Boundary value of the second RHS term. $\boldsymbol{tC2}_{loc}^\gamma = LocalAverage(\boldsymbol{p}_R \boldsymbol{\mathcal{N}}_{\boldsymbol{\chi}}^\gamma)$. The nodal value is updated as
$\boldsymbol{rhsLm}_\gamma^* = \boldsymbol{rhsLm}_\gamma^n + \left(\boldsymbol{tC2}_{loc}^\gamma ||\boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^\gamma|| \right) \boldsymbol{H}_{\psi,\gamma}^T \boldsymbol{w}_\gamma$.

5. Boundary value of the third RHS term. $\boldsymbol{tC3}_{loc}^\gamma = LocalAverage(\boldsymbol{w} \boldsymbol{\mathcal{N}}_{\boldsymbol{\chi}}^\gamma)$. The nodal value is updated as
$\boldsymbol{rhsLm}_\gamma^* = \boldsymbol{rhsLm}_\gamma^n + \boldsymbol{p}_{R,\gamma} \left(\boldsymbol{H}_{\psi,\gamma} : (\boldsymbol{tC3}_{loc}^\gamma ||\boldsymbol{\mathcal{C}}_{\boldsymbol{\chi}}^\gamma||) \right)$.

6. Volume integration: $\boldsymbol{rhsLm}_a^* = \boldsymbol{rhsLm}_a^* / \Omega_a$.

7. Update of displacements $\boldsymbol{x}^* = \boldsymbol{x}^n + \Delta t \hat{\boldsymbol{v}}^n$ and linear momentum $\boldsymbol{p}_{\boldsymbol{\chi}}^* = \boldsymbol{p}_{\boldsymbol{\chi}}^n + J_\psi J^{-1} \Delta t \; \boldsymbol{rhsLm}^*$.

8. Application of boundary conditions for the linear momentum.

9. Update of variables:

$$
\begin{aligned}
\boldsymbol{trueF}^* &= \boldsymbol{F}^* \boldsymbol{F}_\psi^{-1,*} \\
\boldsymbol{P}^* &= \boldsymbol{P}^*(\boldsymbol{trueF^*}) \\
\boldsymbol{p}_R^* &= J_\psi^{-1} \boldsymbol{p}^* \\
\boldsymbol{v}^* &= \boldsymbol{p}_R^* / \rho \\
\hat{\boldsymbol{v}}^* &= \boldsymbol{v}^* + \boldsymbol{trueF}^* \boldsymbol{w}^n \\
\hat{\boldsymbol{P}}^* &= \boldsymbol{P}^* \boldsymbol{H}_\psi
\end{aligned}
$$

10. Computation of coefficient Λ_H^n and update of U_p^n and U_s^n.

11. Update of the averaged values for computing the right hand sides

$$
\begin{aligned}
\boldsymbol{tC}^* &= \hat{\boldsymbol{P}}^{Ave,*} \boldsymbol{\mathcal{N}} + 0.5 S^* [\boldsymbol{v}^*] \\
\boldsymbol{tC2}^* &= \boldsymbol{p}_R^{Ave,*} \otimes \boldsymbol{\mathcal{N}} \\
\boldsymbol{tC3}^* &= \boldsymbol{w}^{,Ave,n} \otimes \boldsymbol{\mathcal{N}}
\end{aligned}
$$