

Material Motion:

$$\psi = \psi_\beta(\chi, t) = \begin{pmatrix} \chi_1 + \beta \sin(2\pi\chi_1) \sin(\pi\chi_2/3) \sin(\pi t/T) \\ \chi_2 + 5\beta \sin(2\pi\chi_1) \sin(\pi\chi_2/3) \sin(2\pi t/T) \\ \chi_3 \end{pmatrix}, \quad (1)$$

with  $T = 2$  and  $\beta$  an amplitude parameter defined for each simulation.

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**Algorithm 1:** Computation of Right Hand Sides.

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**Data:**  $c_R^p, c_R^s$ .  
**Result:**  $\text{rhsLm}, \text{rhsF}$ .  
1 Update of  $\mathbf{w}, \mathbf{F}_\psi, J_\psi, \mathbf{H}_\psi$   
2  $\mathbf{F} = \mathbf{F}_\phi \mathbf{F}_\psi^{-1}$   
3  $\mathbf{P} = \mathbf{P}(\mathbf{F})$   
4  $\mathbf{v} = J_\psi^{-1} \mathbf{p}_x / \rho$   
5  $\hat{\mathbf{v}} = \mathbf{v} + [(\mathbf{F}_\phi \mathbf{F}_\psi^{-1}) \mathbf{w}]$   
6  $\hat{\mathbf{P}} = [\mathbf{P} + ((J_\psi^{-1} \mathbf{p}_x) \otimes \mathbf{w})] \mathbf{H}_\psi$   
7  $\mathbf{vC}$ , point to edge interpolation of  $\hat{\mathbf{v}}$   
8  $\Lambda_{\mathbf{H}_\phi}^2 = (\mathbf{H}_\psi^{Ave} \mathbf{N}_\chi) \cdot (\mathbf{H}_\psi^{Ave} \mathbf{N}_\chi)$   
9  $c_x^p = J_\psi^{-1} [\Lambda_{\mathbf{H}_\phi} c_R^p - \mathbf{w} \cdot (\mathbf{H}_\psi \mathbf{N}_\chi)]$   
10  $c_x^s = J_\psi^{-1} [\Lambda_{\mathbf{H}_\phi} c_R^s - \mathbf{w} \cdot (\mathbf{H}_\psi \mathbf{N}_\chi)]$   
11  $\mathbf{tC} = \hat{\mathbf{P}}^{Ave} \mathbf{N}_x + \frac{1}{2} \text{Smat}(c_x^p, c_x^s) (\mathbf{p}_x^+ - \mathbf{p}_x^-)$   
12  $\text{rhsLm} = \sum_{b \in \Lambda_a} (\mathbf{tC} \|C_{ab}^x\|)$   
13  $\text{rhsF} = \sum_{b \in \Lambda_a} (\mathbf{vC} \otimes \mathbf{C}_{ab}^x)$

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**Algorithm 2:** Use of Right Hand Sides.

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**Data:**  $\text{rhsLm}, \text{rhsF}$ .  
**Result:**  $\mathbf{p}_x^n, \mathbf{F}_\phi^n, \mathbf{u}^n, \mathbf{u}_w^n$   
1  $\mathbf{v} = J_\psi^{-1} \mathbf{p}_x / \rho$   
2  $\hat{\mathbf{v}} = \mathbf{v} + [(\mathbf{F}_\phi \mathbf{F}_\psi^{-1}) \cdot \mathbf{w}]$   
3  $\mathbf{x} += \Delta t \hat{\mathbf{v}}$   
4  $\mathbf{x}_w += \Delta t \mathbf{w}$   
5  $\mathbf{p}_x += \Delta t \text{rhsLm}$   
6  $\mathbf{F} += \Delta t \text{rhsF}$   
7  $\mathbf{p}_x^n = \frac{1}{2} (\mathbf{p}_x^{n-1} + \mathbf{p}_x^n)$   
8  $\mathbf{F}_\phi^n = \frac{1}{2} (\mathbf{F}_\phi^{n-1} + \mathbf{F}_\phi^n)$   
9  $\mathbf{x}^n = \frac{1}{2} (\mathbf{x}^{n-1} + \mathbf{x}^n)$   
10  $\mathbf{x}_w^n = \frac{1}{2} (\mathbf{x}_w^{n-1} + \mathbf{x}_w^n)$   
11  $\mathbf{u}^n = \mathbf{x}^n - \mathbf{X}$   
12  $\mathbf{u}_w^n = \mathbf{x}_w^n - \mathbf{X}$

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