Electrostatic

> **charge:** It is an intrinsic propesty of any material which contains small amount of mass.

Electron ($m_e = 9.1\times10^{-31}$ Kg) and Proton ($m_p = 1.6\times10^{-29}$ Kg) are charge carriers

> Propesties

① Same charge repulse and opposite charge attract

② They are addetive in nature $2e + 4e = 6e$

③ They are quantized $Q = \pm ne$

④ They must contains mass

⑤ They are relatinsticaly invarient quantity.

⑥ For an isolated system total charge is conserved.

> Transfer of charge

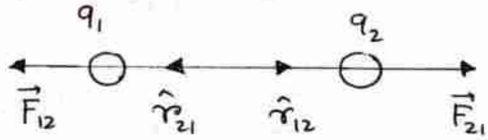
(a) Conduction of charges (Physical contact)

(b) Induction of charges (Polarization)

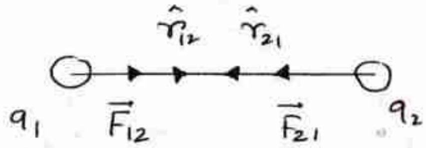
① Coulomb's Law

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12}$$

$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{21}$$



Now $\hat{r}_{12} = -\hat{r}_{21} \Rightarrow \vec{F}_{21} = -\vec{F}_{12}$

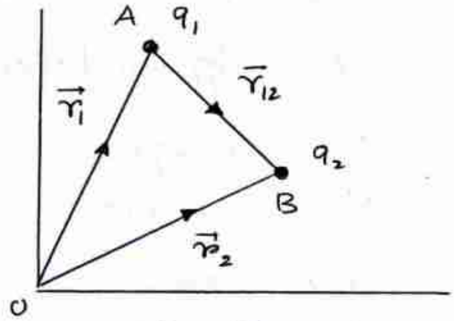


$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{12}$$

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\hat{r}_{21}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

Position vector Form



$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}_{12}|^2}\hat{r}_{12}$$

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}_2-\vec{r}_1|^2}\times\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|}$$

$$\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}_2-\vec{r}_1|^3}(\vec{r}_2-\vec{r}_1)$$

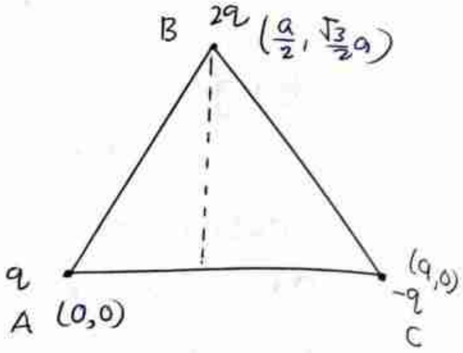
$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1-\vec{r}_2)$$

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\vec{AB} = \vec{r}_2 - \vec{r}_1 = \vec{r}_{12}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{|\vec{r}_{12}|} = \frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|}$$

① Find the net force on the charge $2q$



$$\vec{F}_{BA} = \frac{1}{4\pi\epsilon_0}\,\frac{2q^2}{a^3}\left(\frac{a}{2}\hat{i} + \frac{\sqrt{3}}{2}a\hat{j} - 0\hat{i} - 0\hat{j}\right)$$

or $$\vec{F}_{BA} = \frac{1}{4\pi\epsilon_0}\,\frac{2q^2}{a^3}\left(\frac{a}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$$

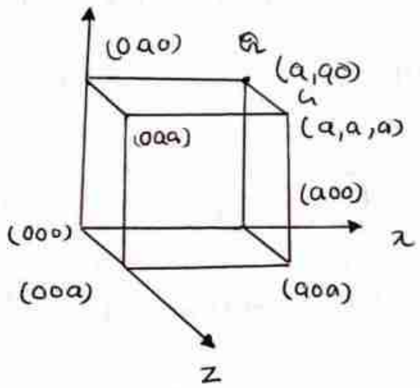
$$\vec{F}_{BC} = \frac{1}{4\pi\epsilon_0}\,\frac{-2q^2}{a^3}\left(\frac{a}{2}\hat{i} + \frac{\sqrt{3}}{2}a\hat{j} - a\hat{i} - 0\hat{j}\right)$$

$$\vec{F}_{BC} = \frac{1}{4\pi\epsilon_0}\,\frac{-2q^2}{a^3}\left(-\frac{a}{2}\hat{i} + \frac{\sqrt{3}}{2}a\hat{j}\right) = \frac{1}{4\pi\epsilon_0}\,\frac{2q^2}{a^3}\left(\frac{a}{2}\hat{i} - \frac{\sqrt{3}}{2}a\hat{j}\right)$$

So the net force $\vec{F} = \frac{1}{4\pi\epsilon_0}\,\frac{2q^2}{a^3}\,a\hat{i} = \frac{1}{4\pi\epsilon_0}\,\frac{2q^2}{a^2}\hat{i}$

21.05.2024

Find the net force on G



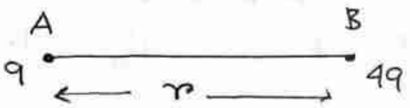
$$\Rightarrow \vec{F}_{GO} = \frac{1}{4\pi\epsilon_0}\,\frac{q^2}{(\sqrt{3}a)^3}\,(a\hat{i} + a\hat{j} + a\hat{k})$$

$$= \frac{1}{4\pi\epsilon_0}\,\frac{q^2}{3a^2}\,(\hat{i} + \hat{j} + \hat{k})$$

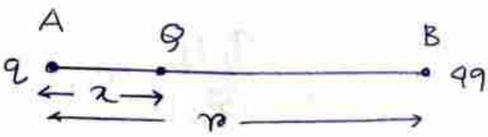
$$\vec{F}_{GC} = \frac{1}{4\pi\epsilon_0}\,\frac{q^2}{(\sqrt{2}a)^3}\,(a\hat{i} + a\hat{j} + a\hat{k} - a\hat{k})$$

Such manner Net force

Placed a charge between them such that net force became zero.



$\Rightarrow$ Q is sign independent



$$\frac{KQq}{x^2} = \frac{K4qQ}{(r-x)^2}$$

or $(r-x)^2 = 4x^2$

$r - x = 2x$

$r = 3x$

$x = r/3$

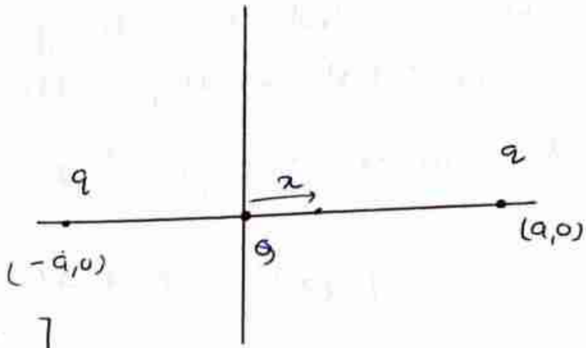
If the charge outside of them then Equilibrium is not Possible.

When charge is between them

stable $\rightarrow +Q$ unstable $\rightarrow -Q$

If the charge Q displaces than the angular frequency will be

$\Rightarrow$ Net force on the charge Q is

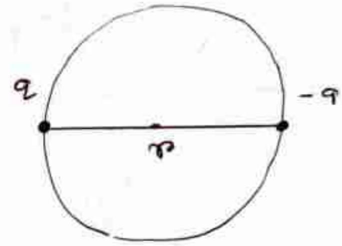


$$F = \frac{1}{4\pi\epsilon_0} Qq \left[\frac{1}{(a-x)^2} - \frac{1}{(a+x)^2} \right]$$

$$= \frac{1}{4\pi\epsilon_0} Qq \left[\frac{4ax}{(a^2-x^2)^2} \right] = \frac{Qq}{4\pi\epsilon_0} \frac{4a}{a^4} x = m\omega^2 x$$

$$\text{So } \omega^2 = \frac{Qq}{\pi\epsilon_0} \frac{1}{ma^3} \Rightarrow \omega = \sqrt{\frac{Qq}{\pi\epsilon_0 ma^3}}$$

Find the time Period of the system



$$\Rightarrow \quad F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2} \quad \text{— (1)}$$

$$\frac{mv^2}{(r/2)} = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

$$2mv^2 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r}$$

$$v^2 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{2mr}$$

$$v = \sqrt{\frac{1}{4\pi\epsilon_0} \frac{q^2}{2mr}}$$

Time Period

$$T = \frac{2\pi r/2}{v} = \frac{2\pi r/2}{\sqrt{\frac{1}{4\pi\epsilon_0} \frac{q^2}{2mr}}}$$

Because here the radius is $r/2$

22.05.2024

Linear charge density:

$$\lambda = \frac{dq}{dl} \Rightarrow q = \int \lambda \, dl$$

Surface charge density

$$\sigma = \frac{dq}{ds} \Rightarrow q = \int \sigma \, ds \qquad ds = r^2 \sin\theta \, d\theta \, d\phi$$

Volume charge density

$$dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\rho = \frac{dq}{dV} \Rightarrow q = \int \rho \, dV$$

---

① charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2} e^{-\frac{2r}{a}}$. if Q is total charge then R is

$\Rightarrow$ Total charge $Q = \int \rho\, dV$

$$Q = \int \frac{A}{r^2} e^{-\frac{2r}{a}} 4\pi r^2 dr = 4\pi A \int_0^R e^{-2r/a} dr$$

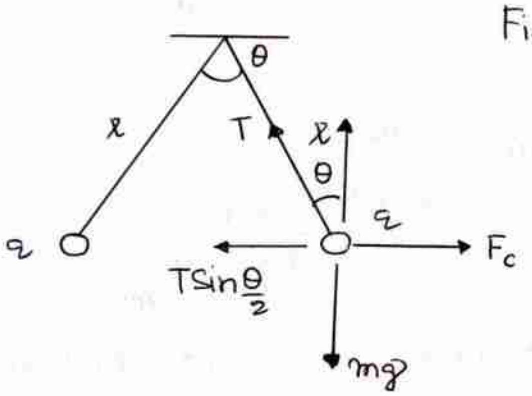
$$= 4\pi A \left. \frac{e^{-2r/a}}{-2/a} \right|_0^R = 2\pi A a \left(1 - e^{-2R/a}\right)$$

$$\frac{Q}{2\pi A a} = 1 - e^{-2R/a}$$

$$e^{-2R/a} = \left(1 - \frac{Q}{2\pi A a}\right)$$

$$-\frac{2R}{a} = \log\left(1 - \frac{Q}{2\pi A a}\right)$$

$$R = \frac{a}{2} \log\left(\frac{1}{1 - \frac{Q}{2\pi A a}}\right)$$



Find the value of charge Q

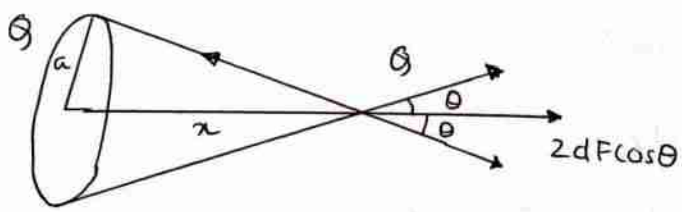
$$F_c = T \sin\frac{\theta}{2}$$

$$mg = T\cos\theta/2$$

$$F_c = mg \tan\theta/2$$

$$\frac{1}{4\pi\epsilon_0} \frac{q^2}{(2l\sin\theta/2)^2} = mg\tan\theta/2$$

$$q = \left[4\left(4l^2 mg \tan\frac{\theta}{2} \sin\frac{\theta}{2}\right)^2 \pi\epsilon_0\right]$$



What will be the angular frequency

$$dF = \frac{1}{4\pi\epsilon_0} \frac{Q\,dQ}{x^2 + a^2} \text{ and } F_{net} = 2\int dF\cos\theta$$

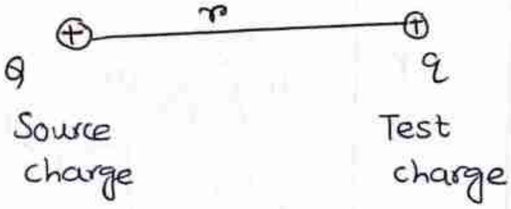
$$= \frac{1}{2\pi\epsilon_0} \frac{q^2}{(x^2+a^2)^{3/2}} x$$

$$F = \left(\frac{Q^2}{2\pi\epsilon_0 a^3}\right) x = m\omega^2 x$$

$$\omega = \left(\frac{Q^2}{2\pi\epsilon_0 a^3 m}\right)^{1/2}$$

② Electrostatic Field: 24.05.2024

Test charge has finite inertia
Source charge has infinite inertia.



$$F = \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}$$

Electrostatic field is force per unit test charge

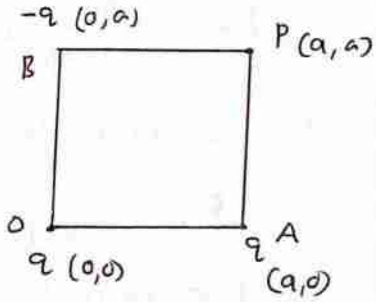
$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$ Here Q is Source charge

Vector form $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$ $\quad \vec{\nabla}\times\vec{E} = 0$

Conservative or irrotational Vector field. $\quad \vec{E} = -\vec{\nabla}\phi$

> If q_0 is Source charge and q_1 is test charge then $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_0 q_1}{|\vec{r}_0-\vec{r}_1|^3}(\vec{r}_0-\vec{r}_1) \Rightarrow \vec{E} = \frac{\vec{F}}{q_1}$

Find Electrostatic field at the Point P



⇒ Electrostatic field at P

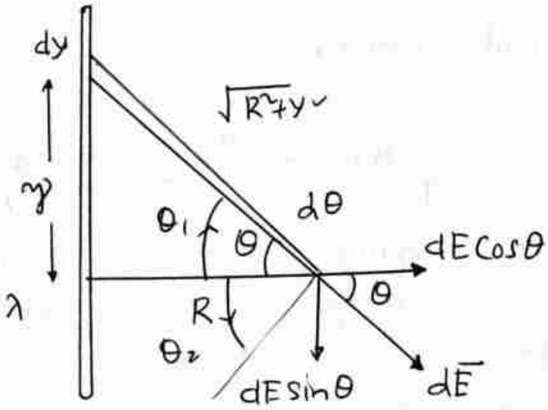
$$\vec{E}_P = \vec{E}_{PO} + \vec{E}_{PB} + \vec{E}_{PA}$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{(a\sqrt{2})^2}(a\hat{i}+a\hat{j})$$

$$+ \frac{1}{4\pi\epsilon_0}\frac{q}{a^3}(a\hat{i}+a\hat{j}-a\hat{i}) - \frac{1}{4\pi\epsilon_0}\frac{q}{a^3}(a\hat{i}+a\hat{j}-a\hat{j})$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{a^3}\left[\frac{a\hat{i}+a\hat{j}}{\sqrt{2}} + a\hat{j} - a\hat{i}\right]$$

Electrostatic field due to a Rod



$$\frac{y}{R} = \tan\theta$$

$$y = R\tan\theta$$

$$dy = R\sec^2\theta \, d\theta$$

$$dE = \frac{1}{4\pi\epsilon_0} \frac{\lambda dy}{(R^2+y^2)}$$

$$E_y = \int dE\sin\theta$$

$$= -\frac{\lambda}{4\pi\epsilon_0 R}\Big[\cos\theta\Big]_{\theta_1}^{\theta_2}$$

$$E_x = \int dE\cos\theta$$

$$= \frac{1}{4\pi\epsilon_0}\int \frac{\lambda dy}{(R^2+y^2)}\cos\theta$$

$$= \frac{\lambda}{4\pi\epsilon_0}\int \frac{R\sec^2\theta d\theta}{R^2\sec^2\theta}\cos\theta$$

$$= \frac{\lambda}{4\pi\epsilon_0 R}\Big[\sin\theta\Big]_{\theta_1}^{\theta_2}$$

$$E_y = \frac{\lambda}{4\pi\epsilon_0 R}(\cos\theta_1 - \cos\theta_2)$$

$$E_x = \frac{\lambda}{4\pi\epsilon_0 R}(\sin\theta_2 + \sin\theta_1)$$

> At Perpendicular Bisector $\theta_1 = \theta_2 = \theta$

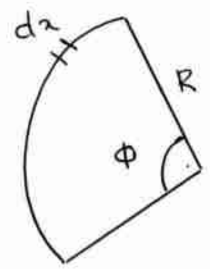
$$E_x = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{R}\sin\theta \quad , \quad E_y = 0$$

> For Infinite Rod. $E_x = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{R}$ $(\because \theta = \pi/2)$

Electric field due to an arc

length of arc $dx = \lambda R d\theta$

$dq = \lambda dx = \lambda R d\theta$



$$dE_x = 2dE\cos\theta$$

$$= 2\int_0^{\phi/2} \frac{1}{4\pi\epsilon_0}\frac{\lambda R d\theta}{R^2}\cos\theta$$

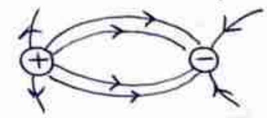
$$= \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{R}\sin\phi/2$$

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{R}\sin\phi/2}$$

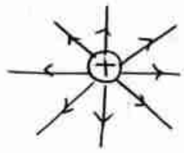
25.05.2024

Electrostatic field line:

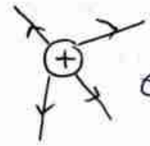
① Imaginary lines along which a positive test charge moves



② No of field lines originating or terminating on a charge gives the magnitude of the charge



Q



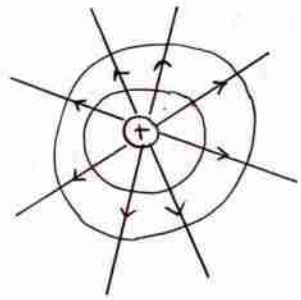
Q/2

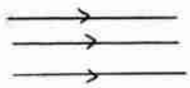
③ Electric field lines never intersect each other

④ They never form Closed loop $\vec{\nabla} \times \vec{E} = 0$

⑤ Density of field lines decreases with distance

$E \propto \frac{1}{r^2}$

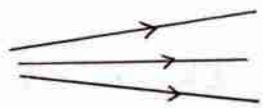




uniform

Non uniform field

A



B

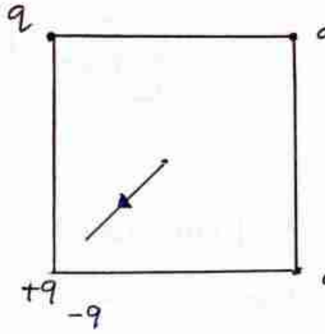
$E_A > E_B$

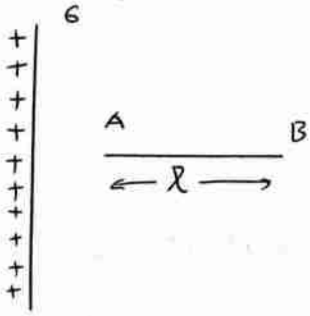
26.05.2024

Electrostatic flux is perpendicular to metalic surface.

> For a regular Polygon if charges at Corner is Same then field in the Centre is zero.

> Particle of mass m, q accure Velocity v over distance AB Surface charge density is





$F = \frac{q\sigma}{2\epsilon_0} = ma$

$a = \frac{q\sigma}{2\epsilon_0 m}$

$v^2 = 2as$

$v^2 = \frac{q\sigma}{\epsilon_0 m} l \Rightarrow \sigma = \frac{\epsilon_0 m v^2}{q l}$

$E = \frac{1}{4\pi\epsilon_0} \frac{q}{(a/\sqrt{2})^2}$

Metalic Surface

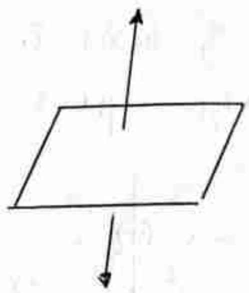
Electric flux:

No of electric field lines passing through a Surface is Known as Electric flux

> open surface

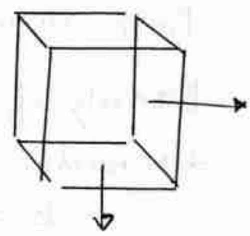
It does't Contains any volume.

Area vector may be outward or toward inward.



> Closed Surface

Contains a finite amount of volume. Area is always outward.



If field increase then flux also.

So, Electric flux $\phi_E \propto E$

$\phi_E \propto A$

$\phi_E \propto \cos\theta$

So $\phi_E = EA\cos\theta$

$\phi_E = \vec{E}\cdot\vec{A}$

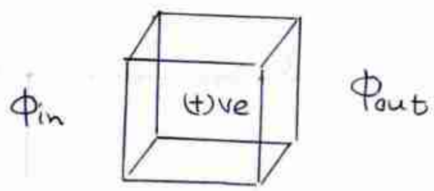
> Electric field $E = \dfrac{d\phi}{ds}$ $\Rightarrow$ Gauss Law $\phi = \dfrac{q_{en}}{\epsilon_0}$

$$\phi = \int \bar{E}\cdot d\bar{s} = \frac{\sum q_{en}}{\epsilon_0}$$

$\oint \bar{E}\cdot d\bar{s} > 0$ Source

$\oint \bar{E}\cdot d\bar{s} = 0$ Solenoidal field.

$\oint \bar{E}\cdot d\bar{s} < 0$ Sink



$\phi_{out} > \phi_{in} \Rightarrow \int \bar{E}\cdot d\bar{s} > 0$ (+q inside)

$\phi_{out} < \phi_{in} \Rightarrow \int \bar{E}\cdot d\bar{s} < 0$ (−q inside)

$\phi_{out} = \phi_{in} \Rightarrow \int E\cdot ds = 0$ (No charge ")

uniform field. $E \neq E(x,y,z)$ is Constant

Non " " $E = E(x,y,z)$ is Variable

> Gauss law is always applicable but it is not useful.

① The Gaussian Surface should be symmetric about the charge distribution

② Electrostatic field must be symmetric around the Gaussian Surface

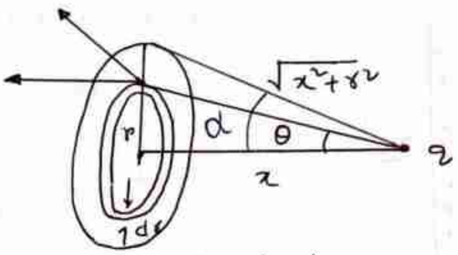
③ $\vec{E}$ and $\vec{A}$ (angle between) must be equal at all Points on the Surfaces

⑥ The Gaussian Surface should not Pass through the charge distribution.

① Find the amount of flux passing through the disc

=)

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2+r^2}$$



Flux $\phi = \int \vec{E} \cdot d\vec{s}$

$$= \int \frac{1}{4\pi\epsilon_0} \frac{q}{(x^2+r^2)} 2\pi r dr \cdot \frac{x}{\sqrt{x^2+r^2}}$$

put, $x^2+r^2 = t$

$\cancel{x dx}$ $r dr = dt/2$

$$= \frac{2\pi x q}{4\pi\epsilon_0} \int \frac{r dr}{(x^2+r^2)^{3/2}}$$

$$\phi = \frac{2\pi x q}{4\pi\epsilon_0} \int \frac{dt}{2t^{3/2}} = -\frac{2\pi x q \cdot 2}{8\pi\epsilon_0}\left[\frac{1}{\sqrt{t}}\right] = \frac{x q}{2\epsilon_0}\left[\frac{1}{\sqrt{r^2+x^2}}\right]_0^R$$

$$\phi = -\frac{xq}{2\epsilon_0}\left[\frac{1}{\sqrt{R^2+x^2}} - \frac{1}{x}\right] = \frac{q}{2\epsilon_0}\left[1 - \frac{x}{\sqrt{R^2+x^2}}\right]$$

Amount of flux $\phi = \frac{q}{2\epsilon}(1-\cos\theta)$

* In non conducting Sphere charge cannot move
> Conducting sphere charge distributed through the Surface

28.05.2024

Gauss Law for Electrostatics

Electrostatic flux. $\phi = \oint \bar{E} \cdot d\bar{s} = \frac{q_{en}}{\epsilon_0}$

when ρ, q given and find E

$$\oint \bar{E} \cdot d\bar{s} = \frac{1}{\epsilon_0} \int \rho dV$$

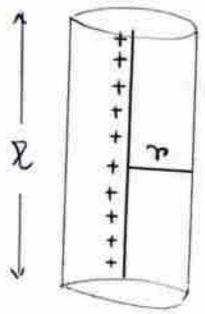
$$\int (\bar{\nabla} \cdot \bar{E}) dV = \int \frac{\rho}{\epsilon_0} dV$$

when E is given and find ρ / q

$$\int \left(\bar{\nabla} \cdot \bar{E} - \frac{\rho}{\epsilon_0} \right) dV = 0$$

so $\bar{\nabla} \cdot \bar{E} = \rho / \epsilon_0$

Electrostatic field due to Rod:



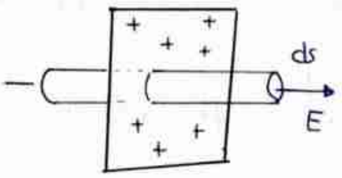
From Gauss law of electrostatics

$$\oint \bar{E} \cdot d\bar{s} = \frac{\lambda l}{\epsilon_0}$$

$$E \oint ds = \frac{\lambda l}{\epsilon_0} \quad [\because \text{E is uniform}]$$

$$E \cdot 2\pi r l = \frac{\lambda l}{\epsilon_0} \Rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r}$$

Electrostatic field due to a sheet



From Gauss law of electrostatic

$$2 \oint \bar{E} ds = \frac{\sigma ds}{\epsilon_0}$$

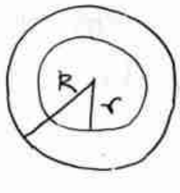
$$2 E ds = \frac{\sigma ds}{\epsilon_0} \Rightarrow E = \frac{\sigma}{2\epsilon_0}$$

Electrostatic field due to a Sphere

Inside: $\oint \bar{E} d\bar{s} = \frac{1}{\epsilon_0} \int \rho dV$

$$E \cdot 4\pi r^2 = \frac{\rho}{\epsilon_0} \frac{4}{3} \pi r^3$$

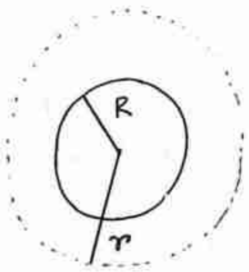
$$\bar{E} = \frac{\rho \bar{r}}{3\epsilon_0}$$



$\oint \bar{E} \cdot d\bar{s} \Rightarrow$ Till Gaussian Surface

$\int \rho dV \Rightarrow$ charge inside Gaussian surface outside

From Gauss Law of Electrostatics



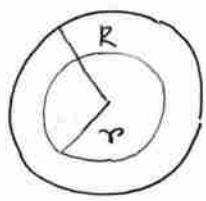
$$\oint \bar{E} \cdot d\bar{s} = \frac{1}{\epsilon_0} \int \rho dV$$

$$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} \rho \frac{4}{3}\pi R^3$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

Case① Non uniform charge density $\rho \propto r^n$

Inside



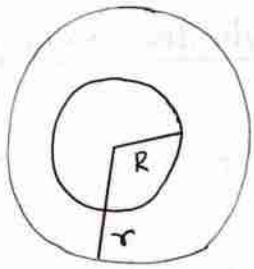
$$\oint \bar{E} \cdot d\bar{s} = \frac{1}{\epsilon_0} \int \rho dV$$

$$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} \int_0^r K r^n 4\pi r^2 dr$$

$$E \cdot 4\pi r^2 = \frac{4\pi K}{\epsilon_0} \int_0^r r^{n+2} dr$$

$$E \cdot 4\pi r^2 = \frac{4\pi K}{\epsilon} \frac{r^{n+3}}{n+3} \Rightarrow E \propto r^{n+1}$$

Outside



$$\oint \bar{E} \cdot d\bar{s} = \frac{1}{\epsilon_0} \int \rho dV$$

$$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} \int_0^R K r^n 4\pi r^2 dr$$

$$E \cdot 4\pi r^2 = \frac{4\pi K}{\epsilon_0} \int_0^R r^{n+2} dr$$

$$E = \frac{K}{\epsilon_0 r^2} R^{n+3}/(n+3) \Rightarrow E \propto \frac{1}{r^2}$$

So we get

$$\rho \propto r^n$$

$$E_{in} \propto r^{n+1}$$

$$E_{out} \propto \frac{1}{r^2}$$

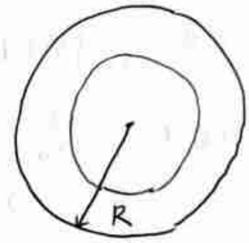
Electrostatic field due to Hollow Sphere

Inside:

Here the charge Enclosed by the Gaussian Surface is zero

So $\oint \bar{E} \cdot d\bar{s} = \frac{1}{\epsilon_0} q_{en} = 0$

So $E = 0$

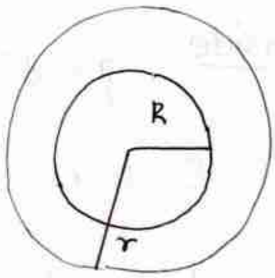


Outside:

$$\oint \bar{E} \cdot d\bar{s} = \frac{q_{en}}{\epsilon_0}$$

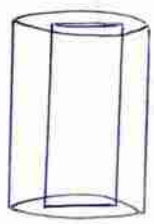
$$E \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$



> For Conducting sphere all charge distributed among the Surfaces So
$E_{in} = 0$ and $E_{out} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

Cylinder having uniform charge density



charge density is ρ = Constant

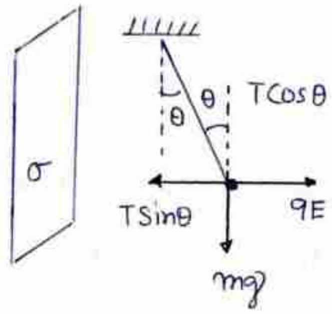
$E_{in} = \frac{\rho r}{2\epsilon_0}$ $\quad E_{out} = \frac{\rho R^2}{2\epsilon_0 r}$

Non uniform charge density $\rho \propto r^n$

$E_{in} \propto r^{n+1}$ $\quad E_{out} \propto \frac{1}{r}$

① An charged ball hangs from a silk thread which an angle θ and a layer conducting thin sheet. The surface charge density σ of the sheet is proportional to

⇒



By equillibrium condition

$$T\cos\theta = mg$$

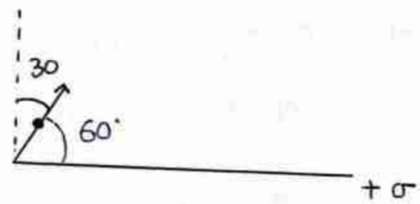
$$T\sin\theta = qE = \frac{q\sigma}{2\epsilon_0}$$

$$\tan\theta = \frac{q\sigma}{2\epsilon_0 mg} \Rightarrow \sigma \propto \tan\theta$$

② A large metal surface has an uniform charge density $+\sigma$. A charge particle of charge $-q_0$ and mass m is thrown with speed v making an angle $\theta = 30^\circ$ with the normal to the metal surface. The time particle will take to reach max height from metal surface and ignore the gravitational effect

⇒ Acting force on the charge particle $F = ma = \frac{\sigma}{2\epsilon_0} q_0$

$$a = \frac{\sigma q_0}{2\epsilon_0 m}$$



Time taken to reach max height

$$t = \frac{2V\sin\theta}{a'} \times \frac{1}{2} = \frac{V\sin 60}{a} = \frac{\sqrt{3}}{2} \times \frac{V \times 2\epsilon_0 m}{\sigma q_0} = \frac{\sqrt{3}Vm\epsilon_0}{q\sigma}$$

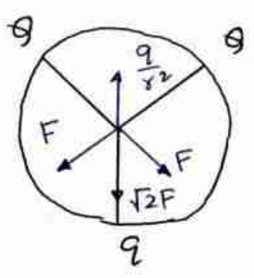
③ If $\bar{E} = E_0(4x\hat{i} - by\hat{j} + 5z\hat{k})$ represents an electrostatic field in a charge free region value of b is

⇒ From the differential form of Gauss law

$\bar{\nabla}.\bar{E} = \rho/\epsilon_0$ Since there is no charge $\rho = 0$

$$\bar{\nabla}.\bar{E} = (4-b+5) = 0 \Rightarrow b = 9$$

④



E at centre is zero then $|Q| = ?$

$$\sqrt{2}\frac{Q}{r^2} = \frac{q}{r^2}$$

$$Q = q/\sqrt{2}$$

(5) Somehow, we have imported a charge density $\rho = \rho_0 r$ on the yolk portion of an egg. Assume that the Egg has ellipsoid shape $\frac{x^2}{9R^2} + \frac{y^2}{16R^2} + \frac{z^2}{25R^2} = 1$ and yolk part has spherical shape of radius R flux through the whole Egg Surface is

$\Rightarrow$ By Gauss law of Electrostatics

$$\oint \bar{E}.d\bar{s} = \frac{1}{\epsilon_0}\int \rho dv$$

$$E \cdot 4\pi r^2 = \frac{1}{\epsilon_0}\int_0^R \rho_0 r \cdot 4\pi r^2 dr$$

$$E \cdot r^2 = \frac{1}{\epsilon_0}\rho_0 \frac{R^4}{4}$$

$$E = \frac{\rho_0 R^4}{4\epsilon_0 r^2}$$

Electrostatic flux

$$\phi = \frac{1}{\epsilon_0}\int \rho dV = \frac{\pi \rho_0 R^4}{\epsilon_0}$$

$$= \frac{1}{\epsilon_0}\int_0^R \rho_0 r\, 4\pi r^2 dr$$

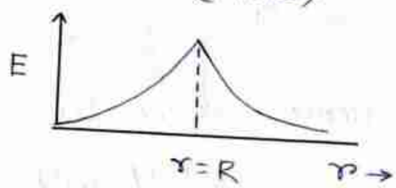
$$= \frac{\pi \rho_0 R^4}{\epsilon_0}$$

(6) For a Solid Sphere having radius R, charge density Varies as $\rho = \rho_0 r$. Variation of E and r is

$\Rightarrow$ charge density. $\rho = \rho_0 r^n$ $\qquad \rho = \rho_0 r$

Electrostatic field $E \propto r^{n+1}$ $\qquad E \propto r^2$ (inside)

At outside $E \propto \frac{1}{r^2}$



(7) A Solid sphere $\rho = \rho_0 r$. If the electric field at $r = R/2$ is E_0 and Electric field at $r = 3R/4$ is

$\Rightarrow$ Charge density $\rho = \rho_0 r^n$ $\quad E(r) = K r^{n+1} = K r^2$

$$\frac{E(\frac{R}{2})}{E(\frac{3R}{4})} = \frac{(R/2)^2}{(3R/4)^2} \Rightarrow \frac{E_0}{E(\frac{3R}{4})} = \frac{1}{4} \times \frac{16}{9} = \frac{4}{9} \Rightarrow E = \frac{9}{4}E_0$$

(8) Electric field $\vec{E} = \alpha x \hat{i}$. Cube $x = l$ to $2l$, $y, z: 0 \to l$ then total flux is

$$\Rightarrow \phi = \oint \bar{E}.d\bar{s} = \int (\bar{\nabla}.\bar{E})dV = \alpha \int_l^{2l} dx \int_0^l dy \int_0^l dz = \alpha l^3$$

Flux $\phi = \alpha l^3$

9. A Space is filled by a charge with $\rho = \rho_0 e^{-r^3}$ then

⇒ By Gauss Law of electrostatics

$$\oint \bar{E}\cdot d\bar{s} = \frac{1}{\epsilon_0}\int \rho\, dV$$

put $r^3 = t$

$3r^2 dr = dt$

$r^2 dr = \frac{1}{3} dt$

or $E\cdot 4\pi r^2 = \frac{1}{\epsilon_0}\int \rho_0 e^{-r^3} 4\pi r^2 dr$

or $E\cdot r^2 = \frac{\rho_0}{\epsilon_0}\int_0^r e^{-r^3} r^2 dr$

or $E\cdot r^2 = \frac{\rho_0}{3\epsilon_0}\int_0^r e^{-t} dt$

$E = \frac{\rho_0}{3\epsilon_0 r^2} e^{-t}\Big|_{r^3}^{0}$

$E = \frac{\rho_0}{3\epsilon_0 r^2}(1 - e^{-r^3})$

if $r >> 1$ $\quad e^{-r^3} \to 0$

$E = \frac{\rho_0}{3\epsilon_0 r^2}$

if $r << 1$ $\quad e^{-r^3} = 1 - r^3$ then $E = \frac{\rho_0}{3\epsilon_0 r^2} r^3 = \frac{\rho r}{3\epsilon_0}$

Simmilar to sphere have Constant ρ.

---

10 In a Certain region $\bar{E} = Kr\theta\hat{r} + mr\hat{\theta}$ where K and m are Constant then

⇒ For Electrostatic field $\bar{\nabla}\times\bar{E} = 0$

$$\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ Kr\theta & mr^2 & 0 \end{vmatrix} = r\sin\theta\hat{\phi}(2mr - Kr) = 0$$

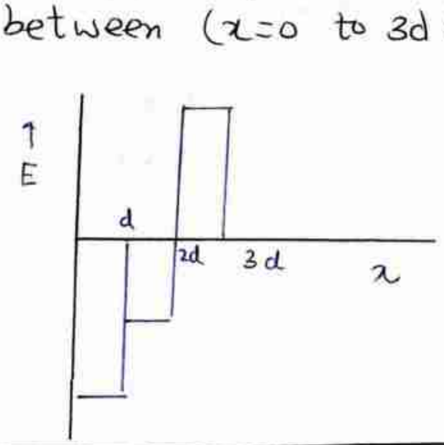
$2m - K = 0$

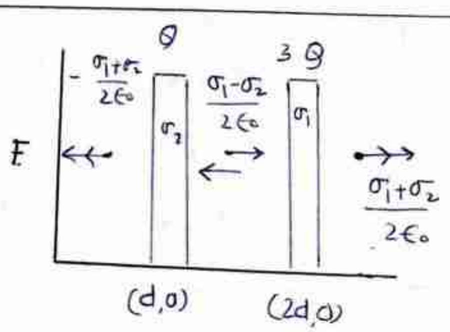
$K - 2m = 0$

---

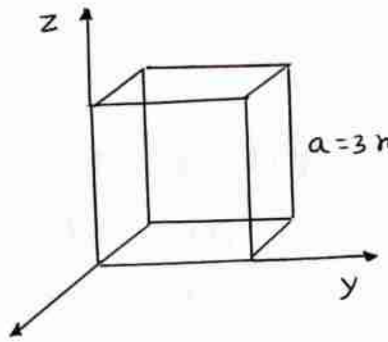
(11) The Variation of E with x in between ($x=0$ to 3d is)

⇒





12



$\bar{E} = [(2x+y)\hat{i} + 8\hat{j} + 3\hat{k}]$ N/c

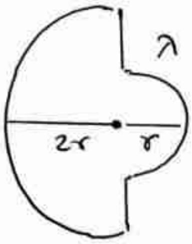
The net charge enclosed by cube is equal to $\alpha\epsilon_0$

$\Rightarrow \phi = \oint \bar{E}\cdot d\bar{s} = \int(\bar{\nabla}\cdot\bar{E})\,dV$

$= 2\int dV = 54 = \frac{q}{\epsilon_0}$

$q = 54\epsilon_0$

So Value of α is 54



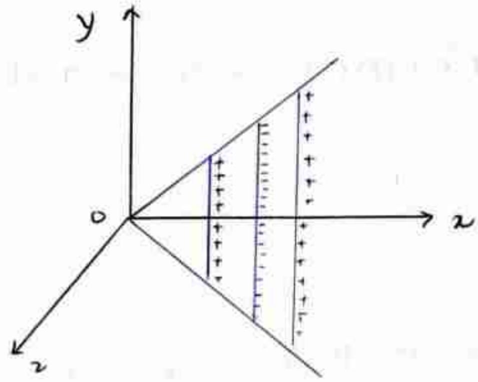
The Electric field at Common Centre is

$E_1 = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}$, $E_2 = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{2r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}$

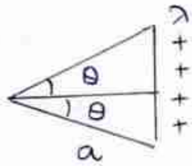
$E = E_1 - E_2 = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}$

---

(13)



Equal & opposite linear charge density λ. The net field at O is



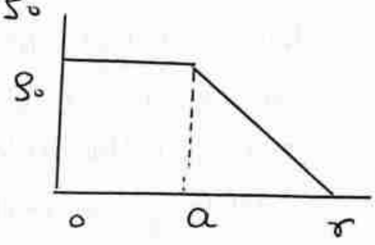
$E_1 = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}(\sin\theta_1 + \sin\theta_2)$

$= \frac{\lambda}{4\pi\epsilon_0(a\cos\theta)} \times 2\sin\theta = \frac{-\lambda\sqrt{3}}{2\pi\epsilon_0 a}(-\hat{i})$

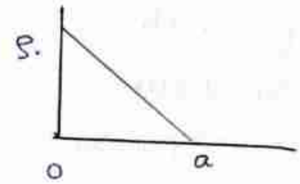
$E_2 = \frac{\lambda\sqrt{3}}{2\pi\epsilon_0 a} \times \frac{1}{2}(\hat{i})$ $\quad E_3 = \frac{\lambda\sqrt{3}}{2\pi\epsilon_0 a} \times \frac{1}{3}(-\hat{i})$

$E_{net} = \frac{-\lambda\sqrt{3}}{2\pi\epsilon_0 a}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots\right)\hat{i} = \frac{\lambda\sqrt{3}}{2\pi\epsilon_0 a}\ln 2\,(\hat{i})$

14 charge (ze) figure represent the charge density Variation with R. The $\vec{E}$ is only radial distance for a = 0, what is the Value of ρ_0



$\Rightarrow$ If a = 0 then Variation became



Intercest form

$$\frac{r}{R} + \frac{\rho}{\rho_0} = 1$$

$$\rho = \rho_0 (1 - r/R)$$

charge $\int \rho \, dv = ze$

$$\int_0^R \rho_0 \left(1 - \frac{r}{R}\right) 4\pi r^2 dr = ze$$

or $$4\pi\rho_0 \int_0^R \left(r^2 - \frac{r^3}{R}\right) dr = ze$$

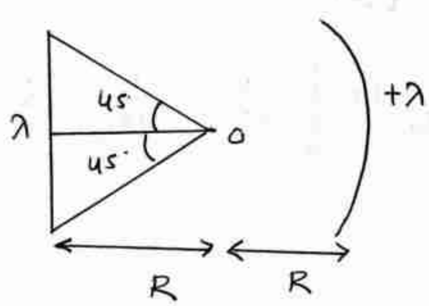
or $$\left(\frac{R^3}{3} - \frac{R^3}{4}\right) = ze/4\pi\rho_0$$

or $$\frac{R^3}{12} = \frac{ze}{4\pi\rho_0}$$

$$\rho_0 = \frac{3ze}{4\pi R^3}$$

---

(15)



Charge density $\lambda = 2\,c/m$

$R = 2m$

Find Electrostatic field

$$E_{Ring} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{R} \sin \phi/2$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2\times 2}{2} = \frac{1}{2\pi\epsilon_0}$$

E_{Rod}

$$= \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{r} \sin 45$$

$$= \frac{1}{2\pi\epsilon_0} \times \frac{1}{\sqrt{2}}$$

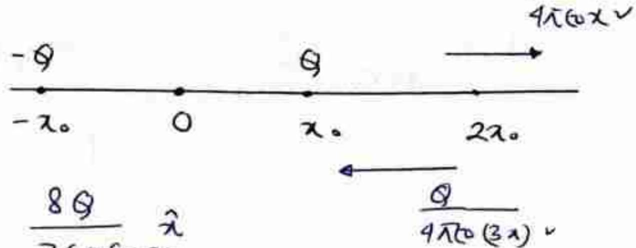
So $E_{Ring} > E_{Rod}$

---

16 A charge distribution has charge density $\rho = Q \{ \delta(x - x_0) - \delta(x + x_0) \}$. EF at $(2x_0, 0, 0)$

$\Rightarrow$ Q at x_0

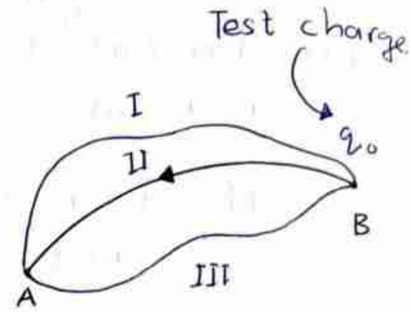
$-Q$ at $-x_0$



$$E = \frac{Q}{4\pi\epsilon_0} \frac{1}{x^2} \left(1 - \frac{1}{9}\right) = \frac{8Q}{36\pi\epsilon_0 x^2} \hat{x}$$

03.06.2024

Electrostatic Potential



The charge (q_0) should be move so slow so that there is no envolvement of Kinetic energy. workdone in Electrostatic field is path independent because $\vec{\nabla} \times \vec{E} = 0$

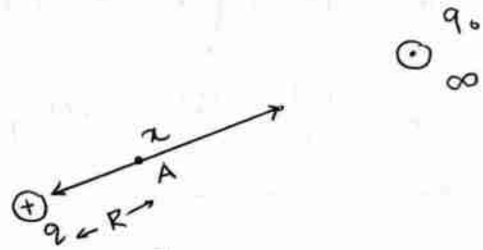
$$V_A - V_B = \frac{W_{ext}(B \to A)}{q_0}$$

$$W_I = W_{II} = W_{III}$$

(Infinite small step)

V_B will be zero if $B \to \infty$

So. $$V_A = \frac{W_{ext}(\infty \to A)}{q_0}$$



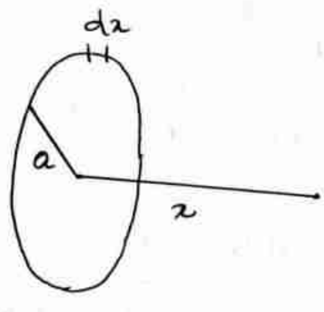
$$W_{ext} = -\int (q_0 E)\, dx$$

$$= -\int \frac{1}{4\pi\epsilon_0} \frac{q q_0}{x^2}\, dx = -\frac{1}{4\pi\epsilon_0} q q_0 \int_{\infty}^{R} \frac{dx}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{q q_0}{R}$$

$$V_A = \frac{W_{ext}}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{R}$$

Electrostatic Potential due to a ring

charge on the element
$$dq = \lambda\, dx$$



$$d\phi = \frac{1}{4\pi\epsilon_0} \frac{dq}{\sqrt{x^2+a^2}}$$

$$d\phi = \frac{1}{4\pi\epsilon_0} \frac{\lambda\, dx}{\sqrt{x^2+a^2}}$$

$$\phi = \frac{\lambda}{4\pi\epsilon_0} \int \frac{dx}{\sqrt{x^2+a^2}} = \frac{\lambda \cdot 2\pi a}{4\pi\epsilon_0 \sqrt{x^2+a^2}}$$

$$\phi = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{x^2+a^2}}$$

Electrostatic field, $\vec{E} = -\vec{\nabla}\phi$

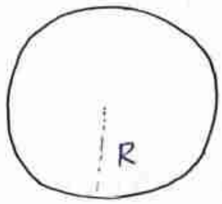
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Qx}{(x^2+a^2)^{3/2}} \hat{i}$$

Non Conducting Hollow Sphere / Conducting Solid Sphere

$r \to \infty$
$\cdot \infty$
q

Due to the charge field at outside

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$



$E_{in} = 0$

$$W_{ext}(\infty \to R) = -\int_{\infty}^{R} qE \cdot dr$$

$$= -q\int_{\infty}^{R} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\,dr$$

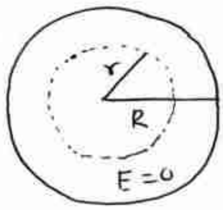
$$= \frac{1}{4\pi\epsilon_0}\frac{Qq}{R}$$

And

$$V_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

$$\text{So } V_R = \frac{W_{\infty \to R}}{q} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

$$V_r - V_R = -\int_{R}^{r} \vec{E}\cdot d\vec{r} = -\int 0\,dr$$



$$V_r - V_R = 0 \Rightarrow V_r = V_R$$

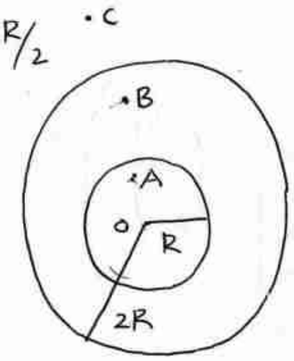
$$V(r > R) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \quad \text{(Equipotential surface)}$$

$$V(r = R) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

$$V(r \leq R) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

Find the Potential at Point A, B, C

$OA = R/2 \quad OB = 3R/2 \quad OC = 5R/2$



$\Rightarrow$

$$V_A = \frac{KQ}{R} - \frac{KQ}{2R} = \frac{KQ}{2R}$$

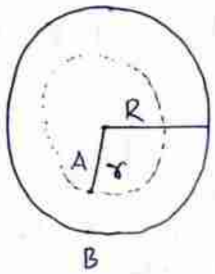
$$V_B = \frac{KQ}{\left(\frac{3R}{2}\right)} - \frac{KQ}{2R}$$

$$V_C = \frac{KQ}{5/2R} - \frac{KQ}{5/2R} = 0$$

check wheather the Points are in or out of Sphere

Solid Non-Conducting Sphere

04.06.2024



Electrostatic field at outer region of sphere $E = \frac{KQ}{r^2}$

$$V = -\int_{\infty}^{R} \frac{KQ}{r^2}\,dr = \frac{KQ}{R}$$

(outer point)

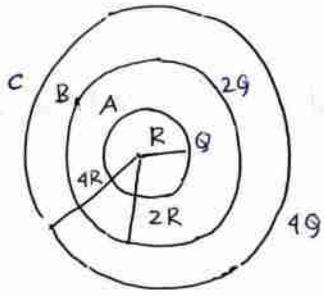
$$V_A - V_B = -\int_{R}^{r} \vec{E}\cdot d\vec{r}$$

Also charge density $\rho = \frac{3Q}{4\pi R^3}$

$$V_A - \frac{KQ}{R} = -\int_{R}^{r} \frac{\rho R}{3\epsilon_0}\,dr$$

$$V = \frac{KQ}{R} - \frac{\rho}{6\epsilon_0}(r^2 - R^2) = \frac{KQ}{2R}\left(3 - \frac{r^2}{R^2}\right)$$

Q



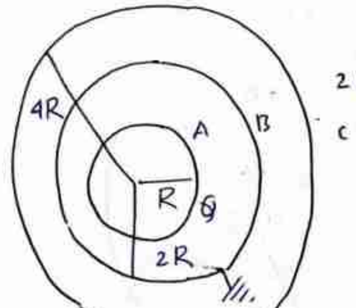
What will be the electrostatic Potential at Points A, B, C

Potential at B, V_B

$$= \frac{KQ}{2R} + \frac{K(2Q)}{2R} + \frac{K(4Q)}{4R}$$

Potential at C, $V_C = \frac{KQ}{4R} + \frac{K(2Q)}{2R} + \frac{K(4Q)}{4R}$

Q



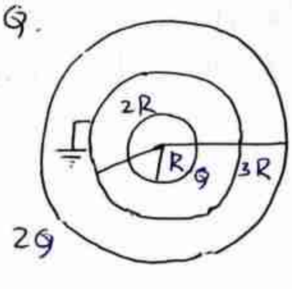
charge on the Sphere C is

$\Rightarrow$ Potential at B

$$V_B = \frac{KQ}{2R} + \frac{Kq}{2R} + \frac{K(2Q)}{4R} = 0$$

$$KQ = -\frac{Kq}{2} \Rightarrow q = -2Q$$

Q.



charge on the earthed Sphere

$$\Rightarrow \frac{KQ}{2R} + \frac{KQ'}{2R} + \frac{K(2Q)}{3R} = 0$$

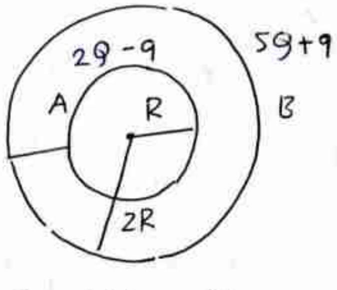
$$Q' = -\tfrac{7}{3}Q$$

Q. What will be the amount of charge

$\Rightarrow$

Potential due to the Sphere

A. $V_A = \frac{K(2Q-q)}{R} + \frac{K(5Q+q)}{2R}$



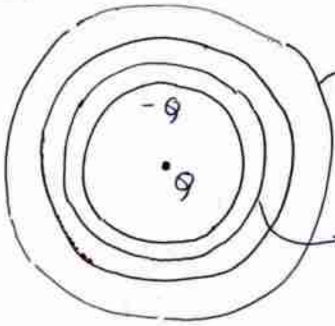
Potential due to sphere B

$V_B = \frac{K(2Q-q)}{2R} + \frac{K(5Q+q)}{2R}$

So $V_A = V_B$

$q = -2Q$

Inside the metallic Sphere



$\rightarrow Q - Q + Q = Q = Q_{en}$

$\rightarrow -Q + Q = 0$

$Q_{en} = 0$

$E = 0$

$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{en}}{\epsilon_0}$

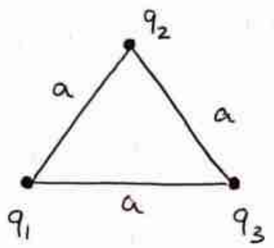
$\oint \vec{E} \cdot d\vec{s} = \frac{+Q - Q}{\epsilon_0} = 0$

$E = 0$

<u>Electrostatic Potential Energy</u> <u>05.06.2024</u>

$U_f - U_i = - W_{consev.}$

$U_f - U_i = W_{ext}$



$V_{q_1} = 0$ and $U_1 = 0$

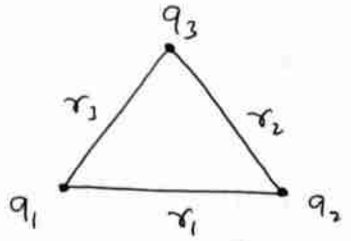
$V = \frac{Kq_1}{a}$ and $U_2 = \frac{Kq_1q_2}{a}$

$V = \frac{Kq_1}{a} + \frac{Kq_2}{a}$ $U_3 = \frac{Kq_1q_3}{a} + \frac{Kq_2q_3}{a}$

$U = \frac{K}{a}(q_1q_2 + q_1q_3 + q_2q_3)$

Electrostatic Energy

$W = U = \frac{1}{4\pi\epsilon_0}\frac{1}{a}(q_1q_2 + q_2q_3 + q_1q_3)$



Electrostatic Potential energy

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1q_2}{r_1} + \frac{q_1q_3}{r_3} + \frac{q_2q_3}{r_2}\right]$$

$$U = \frac{1}{2}\cdot\frac{1}{4\pi\epsilon_0}\left[\frac{q_1q_2}{r_1} + \frac{q_1q_2}{r_1} + \frac{q_1q_3}{r_3} + \frac{q_1q_3}{r_3} + \frac{q_2q_3}{r_2} + \frac{q_2q_3}{r_2}\right]$$

$$= \frac{1}{2}\frac{1}{4\pi\epsilon_0}\left[q_1\left\{\frac{q_2}{r_1} + \frac{q_3}{r_3}\right\} + q_2\left\{\frac{q_1}{r_1} + \frac{q_3}{r_2}\right\} + q_3\left\{\frac{q_1}{r_3} + \frac{q_2}{r_2}\right\}\right]$$

$$= \frac{1}{2}\left[q_1(V_{12}+V_{13}) + q_2(V_{21}+V_{23}) + q_3(V_{31}+V_{32})\right]$$

$$= \frac{1}{2}(q_1V_1 + q_2V_2 + q_3V_3)$$

Potential energy of system $U = \frac{1}{2}\sum q_iV_i$

> Now for continous system $w = \frac{1}{2}\int V\,dq$

$$w = \frac{1}{2}\int \rho V\,d\tau = \frac{1}{2}\epsilon_0\int(\bar{\nabla}\cdot\bar{E})V\,d\tau$$

$$w = \frac{1}{2}\epsilon_0\int\left[\bar{\nabla}\cdot(V\bar{E}) - \bar{\nabla}V\cdot\bar{E}\right]d\tau$$

$$w = \frac{1}{2}\epsilon_0\left[\int\bar{\nabla}\cdot(V\bar{E})\,d\tau - \int(\bar{\nabla}V\cdot\bar{E})\,d\tau\right]$$

$$w = \frac{1}{2}\epsilon_0\left[\oint\!\!\oint(V\cdot\bar{E})\,ds + \int E^2 d\tau\right]$$

Now at $r\to\infty$ first term diverge

$$w = \frac{1}{2}\epsilon_0\int E^2 d\tau \Rightarrow \text{Energy density} \quad u = \frac{1}{2}\epsilon_0 E^2$$

① A hollow sphere contains charge q then find electrostatic energy (conducting)

$$\Rightarrow \quad U = \frac{1}{2}\epsilon_0\int_0^\infty E^2 d\tau = \frac{1}{2}\epsilon_0\left[\int_0^R E^2 d\tau^{\nearrow 0} + \int_R^\infty E^2 d\tau\right]$$

$$= \frac{1}{2}\epsilon_0\int_R^\infty\left(\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\right)^2 4\pi r^2 dr$$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q^2}{2R}$$

② Electrostatic Energy due to a Solid Sphere

$\Rightarrow$

$$U = \frac{1}{2}\epsilon_0 \int_0^{\infty} E^2 d\tau \qquad E_{in} = \frac{1}{4\pi\epsilon_0}\frac{Q r}{a^3}$$

$$E_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$$

$$U = \frac{1}{2}\epsilon_0 \left[\int_0^{R} \left(\frac{Q}{4\pi\epsilon_0}\right)^2 \frac{r^2}{a^6}\, 4\pi r^2 dr + \int_R^{\infty} \left(\frac{Q}{4\pi\epsilon_0}\right)^2 \frac{1}{r^4}\, 4\pi r^2 dr \right]$$

$$U = \frac{1}{4\pi\epsilon_0}\, \frac{3}{5}\, \frac{Q^2}{a}$$

---

③ If the electrostatic Potential is $V = (2x + 4y)$ then Electrostatic energy density is

$\Rightarrow$ $\bar{E} = -\bar{\nabla} V = -(2\hat{i} + 4\hat{j})$

Energy density $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2} \times \epsilon_0 \times 20 = 10\epsilon_0$

---

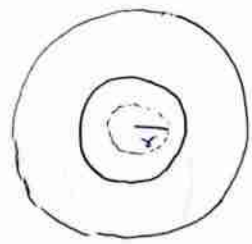
④ $$\rho(r) = \begin{cases} K/r & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$

Variation of flux through a spherical surface of radius r and centred at origin

$\Rightarrow$ In outer Point

Q is Constant

So ϕ is Constant

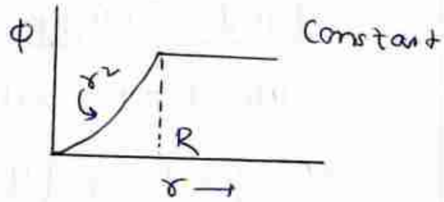


In inner Point

$$\phi = \frac{1}{\epsilon_0}\int \rho\, dV = \frac{1}{\epsilon_0}\int_0^{r} 4\pi r^2 \frac{K}{r} dr = \frac{4\pi K}{\epsilon_0}\frac{r^2}{2}$$

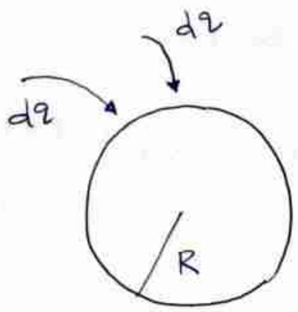
ϕ = Konstant (~~inner~~)

$\phi \propto r^2$ (inner)



Capacitor and Capacitance

For some electronic device, the initial charge should be very high so to store the charge Capacitor is required.



Metal Conductor

$V = \frac{1}{4\pi\epsilon_0}\frac{dq}{R}$

$V' = \frac{1}{4\pi\epsilon_0}\frac{2dq}{R}$

Capacitance

$C \propto \frac{1}{V}$

$C \propto Q$ $\Rightarrow C = \frac{Q}{V}$

$Q = CV$

But Capacitance is independent of Q, V and depends on dimension, material

Capacitance $C = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\epsilon_0}\frac{Q}{R}} = 4\pi\epsilon_0 R$

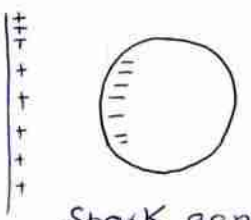
For earth its Capacitance is

$C = 4\pi\epsilon_0 R = \frac{6.4\times10^6}{9\times10^9} = 7.11\times10^{-4}$ Farad

Limitation of Spherical Capacitor

① It has very low capacitance

② It will have a Constant Capacitance and we cannot control it



Spark generate $\Rightarrow$ Corona Discharge

Charge getting loss

Stored Energy in a Capacitor:

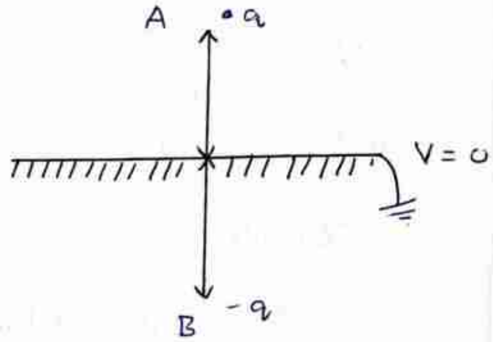
work done. $dW = Vdq = \frac{1}{C}qdq$

$W = \int dW = \frac{1}{C}\int qdq = \frac{Q^2}{2C}$ Now $Q = CV$

$W = \frac{1}{2C}\times C^2V^2 = \frac{1}{2}CV^2$ $\quad W = \frac{1}{2}CV^2$

This become stored in capacitor as the form of Energy.

Image Method



Potential due to A

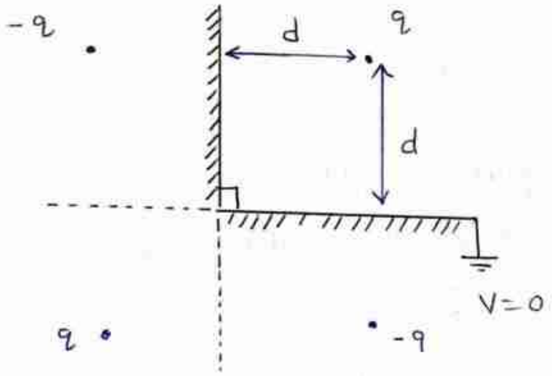
$V_A = \frac{1}{4\pi\epsilon_0} \frac{q}{d}$

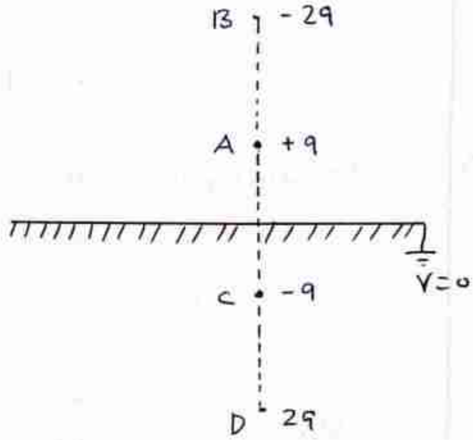
due to B $V_B = \frac{1}{4\pi\epsilon_0} \frac{-q}{d}$

$V = V_A + V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{d} - \frac{q}{d}\right) = 0$

This is image method

Electric field at Point A

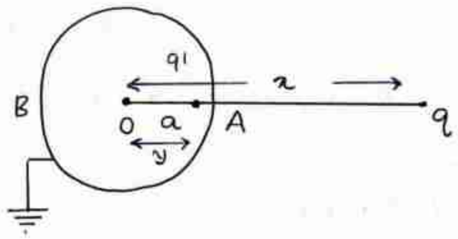




$E_A = \frac{K(2q)}{d^2} + \frac{K(2q)}{(3d)^2} - \frac{Kq}{(2d)^2}$

Spherical Object

Potential at Point A, $V_A = 0$



$V_A = \frac{Kq}{x-a} + \frac{Kq'}{a-y} = 0$

$\frac{Kq}{x-a} = \frac{Kq'}{a-y}$

$\frac{q'}{q} = -\left(\frac{a-y}{x-a}\right)$ — ①

Potential at B. $V_B = 0$

$V_B = \frac{Kq}{x+a} + \frac{Kq'}{a+y} = 0$

$\frac{q'}{q} = -\left(\frac{a+y}{x+a}\right)$ — ②

$-\left(\frac{a-y}{x-a}\right) = -\left(\frac{a+y}{x+a}\right)$

$y = a^2/x$

Value of $q' = -\frac{qa}{x}$

09.06.2024

Laplace Equation

$\bar{\nabla}.\bar{E} = \frac{\rho}{\epsilon_0}$

and $\bar{E} = -\bar{\nabla}\phi$

$-\bar{\nabla}.(\bar{\nabla}\phi) = \frac{\rho}{\epsilon_0}$

$\nabla^2\phi = -\rho/\epsilon_0$ (Poisson Eqn)

charge free region

$\rho = 0$

$\nabla^2\phi = 0$

(Laplace equation)

---

① If electrostatic Potential in a charge free region is given by $V(x,y,z) = 3x^2 + \beta y^2 + 4z^2$ the β is

$\Rightarrow \bar{E} = -\bar{\nabla}\phi = -(6x\hat{i} + 2\beta y\hat{j} + 8z\hat{k})$

$\bar{\nabla}\bar{E} = \frac{\rho}{\epsilon_0} \Rightarrow 6 + 2\beta + 8 = 0 \Rightarrow \beta = -7$

---

② Electrostatic field in a region is given by

$\bar{E} = ax^2y^2z\,\hat{i} + bx^3yz\,\hat{k} + cx^3y^2z\hat{k}$. Value a/b is

$$\Rightarrow \quad \bar{\nabla}\times\bar{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax^2y^2z & bx^3yz & cx^3y^2z \end{vmatrix} = 0$$

$$= \hat{i}\,(2cyx^3z - bx^3y) + \hat{j}\,(ax^2y^2 - 3cx^2y^2z) + \hat{k}\,(3bx^2yz - 2ax^2yz) = 0$$

$3bx^2yz - 2ax^2yz = 0 \Rightarrow \frac{a}{b} = 3/2 = 1.5$

---

③ $\phi = \phi_0(x^2+y^2+z^2)$ Then charge density will be

$\Rightarrow \bar{E} = -\bar{\nabla}\phi = -2\phi_0(x\hat{i} + y\hat{j} + z\hat{k})$

From the diff form of Gauss law

$\bar{\nabla}.\bar{E} = \frac{\rho}{\epsilon_0}$

$-2\phi_0(\bar{\nabla}.\bar{r}) = \rho/\epsilon_0$

$\Rightarrow \rho = -6\phi_0\epsilon_0$

---

(4) The electrostatic Potential inside a Solid charge carrying sphere of radius R is $V = V_0 r^3$. If the potential is zero at $r = 3R$. Potential at $r = 2R$ is

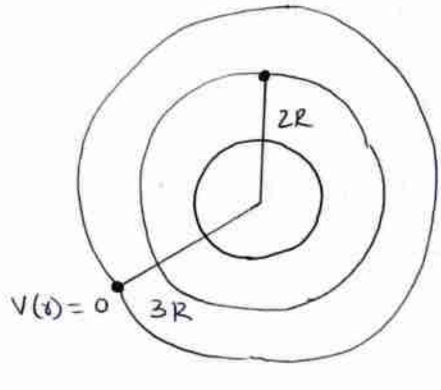
=)

$$\nabla^2 \phi = -\frac{\rho}{\epsilon_0}$$

or $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) = -\frac{\rho}{\epsilon_0}$$

or $$-\frac{\rho}{\epsilon_0} = \frac{+3V_0}{r^2} 4r^3$$

or $$\rho = -12 V_0 r \epsilon_0$$



Electrostatic field at outer Point

$$\oint \bar{E} \cdot d\bar{s} = \frac{1}{\epsilon_0}\int \rho dV$$

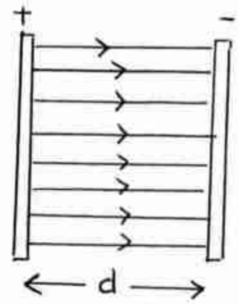
$$E \cdot 4\pi r^2 = -\frac{1}{\epsilon_0} 12 V_0 \epsilon_0 \int 4\pi r^2 r dr \Rightarrow E = -\frac{3V_0 R^4}{r^2}$$

$$V_A - V_B = -\int \bar{E} \cdot d\bar{r} = -3V_0 R^4 \int_{3R}^{2R} \frac{dr}{r^2}$$

$$V_{2R} = 3V_0 R^4 \left(\frac{1}{2R} - \frac{1}{3R}\right) = \frac{3V_0 R^3}{6 \times 2} = \frac{1}{4} V_0 R^3$$

10.06.2024

Parallel Plate Capacitor



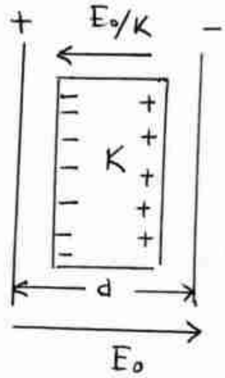
Electrostatic field between any point of them

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$$

Also field $E = \frac{V}{d} = \frac{Q}{A\epsilon_0}$

$$\frac{Q}{d} = \frac{\epsilon_0 A}{d} \Rightarrow \quad C = \frac{\epsilon_0 A}{d}$$

Insertation of dielectric between them



Let the width of the Plate inserted is t. Now the Electrostatic field $E = V/d$

$$V = E_0(d-t) + \frac{E_0}{K}t$$

$$V = E_0\left[d - t\left(1 - \frac{1}{K}\right)\right]$$

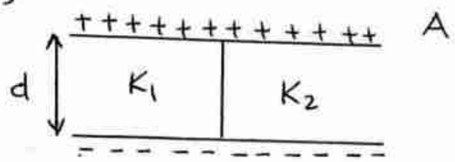
Also $E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$$V = \frac{Q}{A\epsilon_0}\left[d - t\left(1-\frac{1}{K}\right)\right] \Rightarrow \frac{Q}{V} = \frac{\epsilon_0 A}{\left[d - t\left(1-\frac{1}{K}\right)\right]}$$

$$C = \frac{\epsilon_0 A}{\left[d - t\left(1-\frac{1}{K}\right)\right]}$$

① Equivalent Capactince

(a)

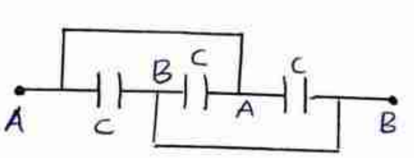


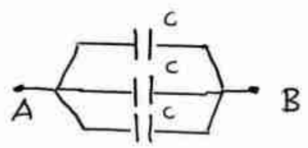
Both Positive are Connected at Same Point So Parallel Combination

$$C_1 = \frac{K_1 \epsilon_0 A/2}{d} \quad C_2 = \frac{K_2 \epsilon_0 A/2}{d}$$

$$C = C_1 + C_2 = \frac{(K_1 + K_2)\epsilon_0 A/2}{d} = \frac{\epsilon_0 A}{d}\frac{(K_1+K_2)}{2}$$

(b)

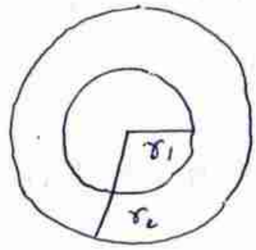




Equivalane Capacitance = C+C+C = 3C

Spherical Capacitor

Electrostatic field in any point at a distance r is given by



$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = -\frac{dV}{dr}$$

$$-\int_V^0 dV = \frac{q}{4\pi\epsilon_0}\int_{r_1}^{r_2}\frac{dr}{r^2} \Rightarrow V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$C = \frac{q}{V} = \frac{4\pi\epsilon_0 r_1 r_2}{(r_2-r_1)} \quad \text{(SI System)}$$

For dielectric capacitor $C = \dfrac{4\pi\epsilon_0 K r_1 r_2}{r_2 - r_1}$

Cylindrical Capacitor

Electrostatic field between them

$$E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r} = -\frac{dV}{dr} \Rightarrow V = \frac{\lambda}{4\pi\epsilon_0} \ln(b/a)$$

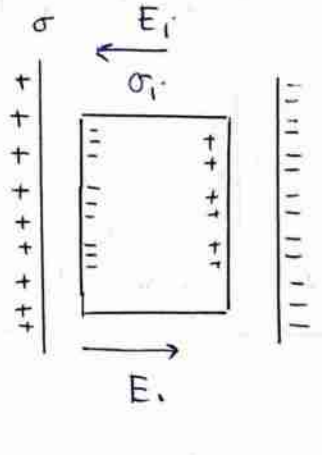
Capacitance $C = \dfrac{Q}{V} = \dfrac{2\pi\epsilon_0 l}{\ln(b/a)}$

$$E_{net} = E_0 - E_i$$

$$\frac{E_0}{K} = E_0 - E_i$$

$$\frac{\sigma}{\epsilon_0 K} = \frac{\sigma}{\epsilon_0} - \frac{\sigma_i}{\epsilon_0}$$

$$\boxed{\sigma_i = \sigma\left(1-\frac{1}{K}\right)}$$



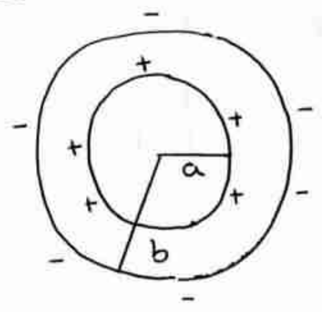
Energy stored in Capacitor

work done, $dW = V\,dq = \frac{q}{C}\,dq$

$$W = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C} = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2$$

$$U = \frac{1}{2}CV^2$$

Case



Electrostatic field between then

$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$

$$V = \int \vec{E}\cdot d\vec{r}$$

$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$V = \frac{Q}{4\pi\epsilon_0}\frac{(b-a)}{ab}$$

$$C = \frac{4\pi\epsilon_0\, ab}{(b-a)}$$

14.06.2024

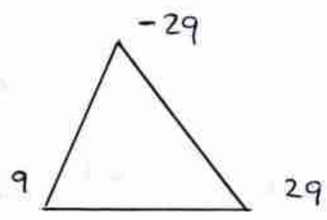
> Multipole Expansion:

① Monopole: $V(r) \propto \frac{1}{r}$ $\quad E(r) \propto \frac{1}{r^2}$ $\quad \sum q \neq 0$

② Dipole: $V(r) \propto \frac{1}{r^2}$ $\quad E(r) \propto \frac{1}{r^3}$ $\quad \sum q = 0,\ \sum qd \neq 0$

③ Quadrpole: $V(r) \propto \frac{1}{r^3}$ $\quad E(r) \propto \frac{1}{r^4}$ $\quad \sum q = 0\ \ \sum q_i d_i = 0$

> which type of Pole it is

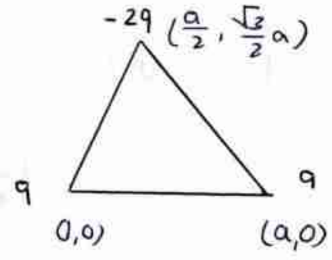


=> $\sum q_i = q + 2q - 2q = q \neq 0$

So monopole

$V(r) \propto \frac{1}{r}, \quad E \propto \frac{1}{r^2}$

> which type of pole



=> $\sum q_i = -2q + q + q = 0$

* (So we can take origin anywhere)

$\sum q_i d_i = q \times 0 + qa\hat{i}$

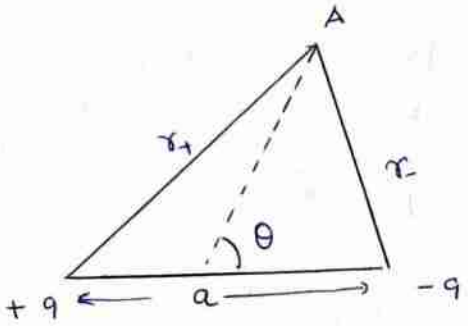
$\quad - 2q\left(\frac{a}{2}\hat{i} + \frac{\sqrt{3}}{2}a\hat{j}\right) \neq 0$

$\sum q_i = 0$ and $\sum q_i d_i \neq 0$

if $\sum q_i \neq 0$ then origin mentioned

Dipole $V \propto \frac{1}{r^2}$

$r \gg a$ $\quad E \propto \frac{1}{r^3}$



Potential at A

$$V_A = \frac{1}{4\pi\epsilon_0} q \left[\frac{1}{r_+} - \frac{1}{r_-} \right]$$

$$r_- = \sqrt{r^2 + \left(\frac{a}{2}\right)^2 - 2r\frac{a}{2}\cos\theta} \qquad r_+ = \sqrt{r^2 + \frac{a^2}{4} - 2r \times \frac{a}{2}\cos(180-\theta)}$$

$$= \sqrt{r^2 + \frac{a^2}{4} - ar\cos\theta} \qquad r_+ = \sqrt{r^2 + \frac{a^2}{4} + ar\cos\theta}$$

$$\frac{1}{r_+} = \frac{1}{\left(r^2 + \frac{a^2}{4} + ar\cos\theta\right)^{1/2}} = \frac{1}{r\left[1 + \frac{a\cos\theta}{r} + \frac{a^2}{4r^2}\right]^{1/2}} \quad (a << r)$$

$$= \frac{1}{r}\left(1 + \frac{a\cos\theta}{r}\right)^{-1/2} = \frac{1}{r}\left(1 + \frac{a\cos\theta}{2r}\right) = \frac{1}{r} + \frac{a\cos\theta}{2r^2}$$

$$\frac{1}{r_-} = \frac{1}{r}\left[1 + \frac{a\cos\theta}{2r}\right] = \frac{1}{r} + \frac{a\cos\theta}{2r^2}$$

$$\frac{1}{r_+} - \frac{1}{r_-} = \frac{1}{r}\left(1 - \frac{a\cos\theta}{2r} - 1 - \frac{a\cos\theta}{2r}\right)$$

$$= \frac{1}{r} \frac{-2a\cos\theta}{2r} = \frac{-a\cos\theta}{r^2}$$

$$V_A = -\frac{Kq}{r^2} a\cos\theta \qquad V(r) \propto \frac{1}{r^2} \quad \text{(Dipole)}$$

$$= \frac{Kp\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{p\cos\theta}{r^2}$$

$$\vec{E} = -\bar{\nabla} V = -\left[\hat{r}\frac{\partial V}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial V}{\partial \theta}\right]$$

$$= \left[\hat{r} \frac{-2p\cos\theta}{r^3} + \frac{p}{r^3}(-\sin\theta)\hat{\theta}\right] Kp$$

$$= Kp\left[\frac{2\cos\theta}{r^3}\hat{r} + \frac{\sin\theta}{r^3}\hat{\theta}\right]$$

$$|\vec{E}| = \frac{Kp}{r^3}\sqrt{3\cos^2\theta + 1}$$

Equatorial Point $\theta = 90$

Axial Point $\theta = 0$.

So electric field due to a dipole

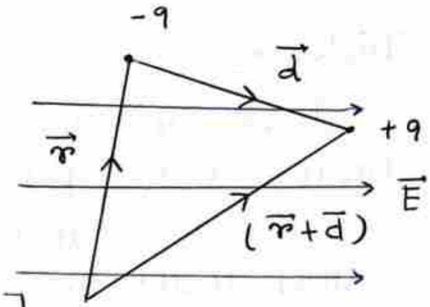
$$\vec{E}(r,\theta) = \frac{1}{4\pi\epsilon_0}\frac{P}{r^3}\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[3(\vec{P}\cdot\hat{r})\hat{r} - \vec{P}\right]$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{3(\vec{P}\cdot\hat{r})\,\hat{r}\,r^2}{r^3\,r^2} - \frac{\vec{P}}{r^3}\right] \quad \text{As } r\hat{r} = \vec{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{3(\vec{P}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{P}}{r^3}\right]$$

Potential Energy due to Dipole



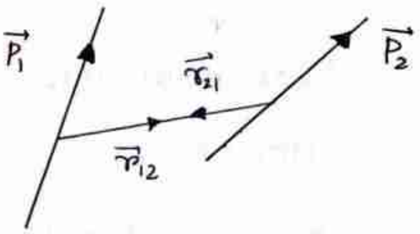
Potential Energy

$$U = -q\,\phi(\vec{r}) + q\,\phi(\vec{r}+\vec{d})$$

$$U = -q\,\phi(\vec{r}) + q\left[\phi(\vec{r}) + \frac{\partial\phi}{\partial r}d\right]$$

$$U = -q\,\phi(\vec{r}) + q\,\phi(\vec{r}) + qd\frac{\partial\phi}{\partial r} \Rightarrow \boxed{U = -\vec{P}\cdot\vec{E}}$$

Dipole-Dipole Interaction



Electric field at 2 due to dipole 1 is given by

$$\vec{E}_{21} = \frac{1}{4\pi\epsilon_0}\left[\frac{3(\vec{P_1}\cdot\vec{r}_{12})\vec{r}_{12}}{r_{12}^5} - \frac{\vec{P_1}}{r_{12}^3}\right]$$

Electric field at 1 due to dipole 1 is given by

$$\vec{E}_{12} = \frac{1}{4\pi\epsilon_0}\left[\frac{3(\vec{P_2}\cdot\vec{r}_{21})\vec{r}_{21}}{r_{21}^5} - \frac{\vec{P_2}}{r_{21}^3}\right]$$

Potential energy

$$U_{21} = -\vec{P_2}\cdot\vec{E}_{21} = -\frac{1}{4\pi\epsilon_0}\left[\frac{3(\vec{P_1}\cdot\vec{r}_{12})(\vec{P_2}\cdot\vec{r}_{12})}{r_{12}^5} - \frac{\vec{P_1}\cdot\vec{P_2}}{r_{12}^3}\right]$$

$$U_{21} = \frac{1}{4\pi\epsilon_0}\left[\frac{\vec{P_1}\cdot\vec{P_2}}{r_{12}^3} - \frac{3(\vec{P_1}\cdot\hat{r}_{12})(\vec{P_2}\cdot\hat{r}_{12})}{r_{12}^5}\right] = U_{12} = -\vec{P_1}\cdot\vec{E}_{12}$$

So $\boxed{U_{12} = U_{21}}$

① There are two dipole $\vec{P_1} = p\hat{i}$ and $\vec{P_2} = p\hat{j}$ at respective positions $(0,0,0)$ and $(d,d,0)$. What will be the interaction energy between them

=)
$$U_{12} = \frac{1}{4\pi\epsilon_0}\left[0 - \frac{3\{p\hat{i}\cdot(d\hat{i}+d\hat{j})\}\{p\hat{j}\cdot(d\hat{i}+d\hat{j})\}}{(\sqrt{2}d)^5}\right]$$

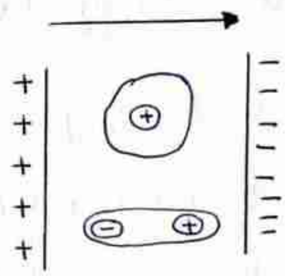
$$U_{12} = \frac{-3K}{4\sqrt{2}d^5}(Pd \cdot Pd) = \frac{-3KP^2}{4\sqrt{2}d^3}$$

Polarization 18.06.2024

Dielectric:

Insulators which get Polarized on application of Electric field is called Dielectric.

So All insulator cannot be dielectric



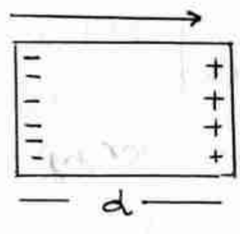
Polarization

Dielectrics

- Polar molecules
  - HCl, H_2O
- Non Polar molecules
  - H_2, N_2

Induced Dipole moment

Dipole moment produced due to supply of External Applied electric field. $\vec{P} = q\vec{d}$



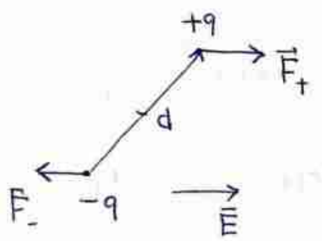
$\vec{P}_{in} \propto \vec{E}$ =) $\boxed{\vec{P}_{in} = \alpha\vec{E}}$ (α is electric Polarizibility)

Torque on a Dipole

$\vec{\tau} = (r_+ \times F_+) + (r_- \times F_-)$

$= (\frac{d}{2} \times qE) + (\frac{d}{2} \times qE)$

$= q\vec{d} \times \vec{E}$



$\vec{\tau} = \vec{p} \times \vec{E}$

Polarization Net dipole moment per unit Volume is Called Polarization

$$\vec{P} = \underset{\Delta V \to 0}{Lt} \frac{n\vec{p}}{\Delta V} = \underset{\Delta V \to 0}{Lt} \frac{1}{\Delta V} \sum_i \vec{P}_i$$

(P_i is induced dipole moment)

> Bounded charges cannot move or displace slightly but in Conductor charge move with drift velocity.

Bound charge

- Surface bound charge $\sigma = \vec{P}\cdot\hat{n}$
- Volume bound charge $\rho = -\vec{\nabla}\cdot\vec{P}$

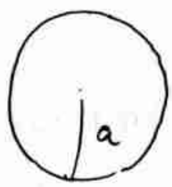
---

① For an insulator polarization vector $\bar{P} = Kr^2\hat{r}$

$\Rightarrow$ Surface bound charge

$\sigma = \bar{P}\cdot\hat{n} = Kr^2\hat{r}\cdot\hat{r} = Kr^2$

$\sigma = Ka^2$



Volume bound charge

$$\rho = -\bar{\nabla}\cdot\bar{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 K r^2) = -\frac{1}{r^3} K\cdot 4r^3 = -4Kr$$

$$Q_{surface} = Ka^2 \times 4\pi a^2 = 4\pi K a^4$$

$$Q_{vol} = -4K\int_0^a r\cdot 4\pi r^2 dr = -\frac{16\pi K a^4}{4} = -4\pi K a^4$$

$Q_{net} = Q_{surface} + Q_{vol} = 0$ Sphere is electrically neutral

---

Bound charge density

$$\rho_b = -\bar{\nabla}\cdot\bar{P}$$

$$-\int \rho_b dV = \int (\bar{\nabla}\cdot\bar{P}) dV$$

$$\oint \bar{P}\cdot\hat{n}\, ds = -q_b$$

Polarization vector $\vec{P} \propto \vec{E}$

$\vec{P} = \epsilon_0 \chi \vec{E}$ (χ is Electric Succeptibility

$\oint \vec{P}\cdot\hat{n}\,ds = \epsilon_0\chi \oint \vec{E}\cdot\hat{n}\,ds$

$-q_b = \epsilon_0 \chi \frac{q}{\epsilon_0}$

$-q_b = \chi(q_f + q_b)$

$-q_b - \chi q_b = \chi q_f$

$-q_b(1+\chi) = \chi q_f$

$\boxed{q_b = -\left(\frac{\chi}{1+\chi}\right) q_f}$

$\sigma_b = -\left(\frac{\chi}{1+\chi}\right)\sigma_f$

$\rho_b = -\left(\frac{\chi}{1+\chi}\right)\rho_f$

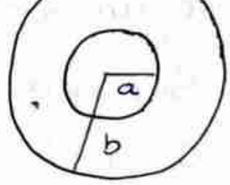
---

① $\vec{P}(r) = \frac{K}{r}\hat{r}$ No free charge in the System find the Electric field at

(a) $r < a$ $E = 0$

(b) $a < r < b$

(c) $r > b$



=)

Surface density $\sigma_a = \vec{P}\cdot\hat{n} = \frac{K}{r}\cdot\hat{r}(-\hat{r}) = -\frac{K}{r}$

$\sigma_a = -\frac{K}{a}$ $q_1 = -\frac{K}{a}4\pi a^2 = -4\pi a K$

$\rho_b = -\vec{\nabla}\cdot\vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{K}{r}\right) = -\frac{K}{r^2}$

$\int \vec{E}\cdot d\vec{s} = \frac{q_{en}}{\epsilon_0}$

or $E\cdot 4\pi r^2 = \frac{1}{\epsilon_0}\left[-4\pi a K + \int_a^r -\frac{K}{r^2}4\pi r^2 dr\right]$

• $E\cdot 4\pi r^2 = \frac{1}{\epsilon_0}\left[-4\pi a K - 4\pi K(r-a)\right]$

• $E\cdot \cancel{4\pi} r^{\cancel{2}} = -\frac{1}{\epsilon_0}\cancel{4\pi}K\cancel{r}$ $\Rightarrow E = -\frac{K}{\epsilon_0 r}$

Also $\sigma_b = \vec{P}\cdot\hat{n} = \frac{K}{r}\cdot\hat{r}(\hat{r}) = \frac{K}{r}$ $\sigma_b = K/b$

$\rho_b = -\frac{K}{r^2}$ $q_2 = \frac{K}{b}4\pi b^2 = 4\pi b K$

$Q_b = -4\pi K\int_a^b dr = -4\pi K(b-a)$ $Q_t = q_1 + q_2 + Q_b = 0$

So $E|_{r>b} = 0$

$$\oint \bar{E}\cdot\hat{n}\,ds = \frac{q}{\epsilon_0} \qquad q_b = -\oint \vec{P}\cdot\hat{n}\,ds$$

$$\oint E\,\hat{n}\,ds = \frac{1}{\epsilon_0}(q_b + q_f) \qquad \vec{D} = \epsilon\vec{E} = \epsilon_0\epsilon_r\vec{E}$$

$$\oint \epsilon_0\bar{E}\,\hat{n}\,ds = -\oint \bar{P}\cdot\hat{n}\,ds + q_f$$

$$\oint (\epsilon_0\bar{E}+\bar{P})\,\hat{n}\,ds = q_f \Rightarrow \oint \bar{D}\cdot\hat{n}\,ds = q_f$$

Displacement vector $\bar{D} = \epsilon_0\bar{E} + \bar{P}$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} \qquad \epsilon_0\epsilon_r\bar{E} = \epsilon_0(1+\chi)\vec{E}$$

$$\vec{D} = \epsilon_0\bar{E} + \epsilon_0\chi\vec{E} \qquad \boxed{\epsilon_r = 1+\chi}$$

$$\oint \vec{E}\cdot d\bar{s} = \frac{q}{\epsilon_0} \qquad \epsilon_r = 1+\chi \qquad \vec{P} = \epsilon_0\chi\vec{E}$$

$$\oint \bar{P}\cdot\hat{n}\,ds = -q_b \qquad q_b = -\left(\frac{\chi}{1+\chi}\right)q_f \qquad \bar{D} = \epsilon_0\bar{E} + \bar{P}$$

$$\sigma_b = \vec{P}\cdot\hat{n}$$

$$\oint \bar{D}\cdot\hat{n}\,ds = q_f \qquad q_b = -\left(\frac{\epsilon_r - 1}{\epsilon_r}\right)q_f \qquad \rho_b = -\bar{\nabla}\cdot\bar{P}$$

---

② A cylindrical rod of length L and radius r is placed with its axis along $+z$ direction $\vec{P} = (5z^2+7)\hat{k}$. The volume bound charge inside dielectric

$$\Rightarrow \quad \rho = -\bar{\nabla}\cdot\bar{P} = -\frac{\partial}{\partial z}(5z^2+7) = -10z$$

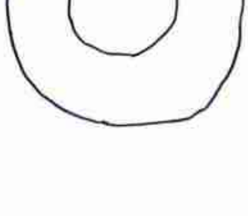
$$Q = -\int 10z\cdot r\,dr\,d\theta\,dz = -10\int_0^L z\,dz\int_0^r r\,dr\int_0^{2\pi} d\theta$$

$$= -10\times\frac{L^2}{2}\times\frac{r^2}{2}\times 2\pi$$

$$= -5\pi r^2 L^2$$

A spherical shell of inner a and outer b. Carries $\vec{P} = \beta\hat{r}$ Potential at centre

$$\Rightarrow \quad V_A - V_B = \int \bar{E}\cdot d\bar{r}$$



$$\sigma_b = \vec{P}\cdot\hat{n} = -\beta$$

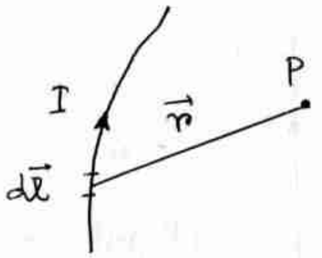
$$q_b = -\beta\, 4\pi a^2$$

$$\rho = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\beta) = -2\beta/r$$

Magnetic field

20.06.2024

Biot - Savart's Law:



Magnetic field due to $Id\vec{l}$ current element

$d\vec{B} \propto I\,d\vec{l}$

$d\vec{B} \propto \frac{1}{r^2}$

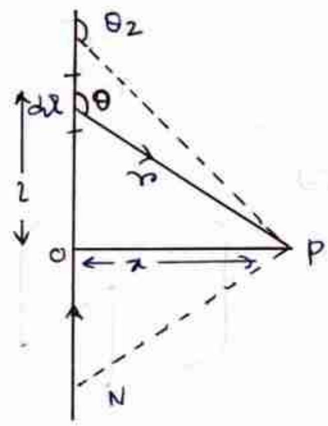
$d\vec{B} \propto \sin\theta$

$$d\vec{B} \propto \frac{I\,d\vec{l}\sin\theta}{r^2} \Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,(d\vec{l}\times\vec{r})}{r^3}$$

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\vec{r}}{r^3}$$

① Magnetic field due to straight wire



$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2}$$

$$\frac{l}{x} = \cot(\pi-\theta) = -\cot\theta$$

$$dl = +\,x\,\mathrm{cosec}^2\theta\,d\theta$$

$$\frac{r}{x} = \mathrm{cosec}(\pi-\theta) = \mathrm{cosec}\,\theta$$

$$r^2 = x^2\,\mathrm{cosec}^2\theta$$

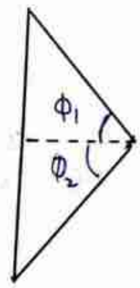
$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,x\,\mathrm{cosec}^2\theta\,d\theta\sin\theta}{x^2\,\mathrm{cosec}^2\theta}$$

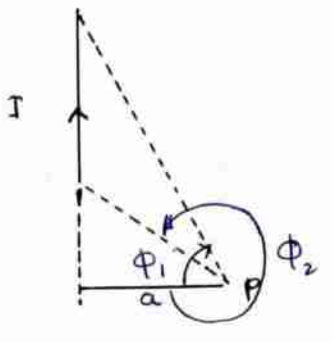
$$\int dB = \frac{\mu_0}{4\pi}\frac{i}{x}\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta \Rightarrow B = \frac{\mu_0}{4\pi}\frac{i}{x}(\cos\theta_1 - \cos\theta_2)$$

Now for infinite wire $\theta_1 = 0$ and $\theta_2 = \pi$

$$\boxed{B = \frac{\mu_0}{4\pi}\frac{2i}{x}}$$

$$B = \frac{\mu_0}{4\pi}\frac{i}{x}(\sin\phi_1 + \sin\phi_2)$$

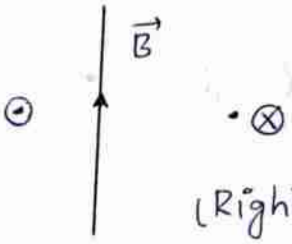




ϕ_1 should be clockwise

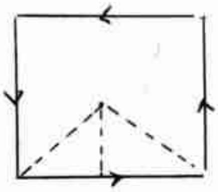
ϕ_2 will be anticlockwise

<u>Direction of B</u>



(Right hand Rule)

---

① 

Magnetic field at Centre

$$\Rightarrow \quad B = \frac{\mu_0}{4\pi}\frac{i}{x}(\sin\phi_1 + \sin\phi_2)$$

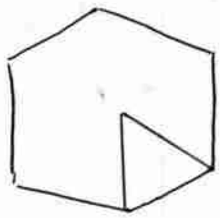
$$= \frac{\mu_0}{4\pi}\frac{i}{\left(\frac{a}{2}\right)} 2\sin 45$$

$$\frac{\mu}{4\pi}\frac{4i}{a}\frac{1}{\sqrt{2}} = \frac{\mu i}{a\sqrt{2}\pi}$$

$$B_{net} = 4B$$

$$= \frac{4\mu i}{a\sqrt{2}\pi}$$

---

② 

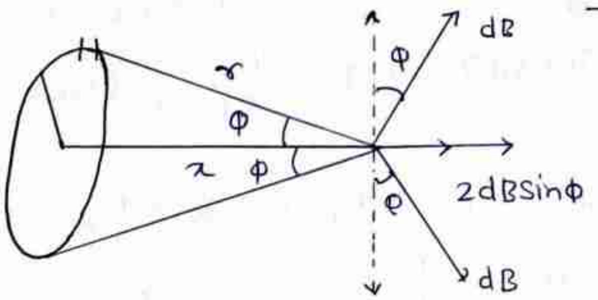
Magnetic field at centre

$$x = \frac{a\sqrt{3}}{2} \qquad \phi_1 = \phi_2 = 30$$

$$B = \frac{\mu}{4\pi}\frac{i}{\frac{a\sqrt{3}}{2}} 2\sin 30 = \frac{\mu}{4\pi}\frac{2i}{a\sqrt{3}}$$

$$B_{net} = \frac{\mu}{4\pi}\frac{12i}{a\sqrt{2}}$$

<u>Axial distance of a Coil</u>

③ 

$$B = \int 2\,dB\sin\phi$$

$$B = 2\int \frac{\mu_0}{4\pi}\frac{i\,dl\sin\left(\frac{\pi}{2}\right)}{r^2}\sin\phi$$

$$B = 2\frac{\mu_0}{4\pi}\frac{i}{r^2}\sin\phi\int_{hc} dl$$

$$B = \frac{\mu}{4\pi}\frac{2i}{r^2}\frac{a}{r}(\pi a) = \frac{\mu}{4\pi}\frac{2\pi i a^2}{(a^2+x^2)^{3/2}}$$

> At Centre $x=0$ $\quad B = \dfrac{\mu_0 I}{2a}$ $\quad$ Direction is outward.

> $x \gg a$ $\quad B = \dfrac{\mu_0 I a^2}{2x^3}$

For maximum field $\dfrac{dB}{dx} = 0$

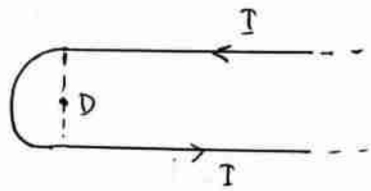
$$\frac{d}{dx}(a^2+x^2)^{-3/2} = 0$$

$$-\frac{3}{2}(a^2+x^2)^{-5/2} \cdot 2x = 0$$

$$\frac{3x}{(a^2+x^2)^{5/2}} = 0 \Rightarrow x = 0$$

$$B = \frac{\mu_0 I}{2a}$$

Due to wires

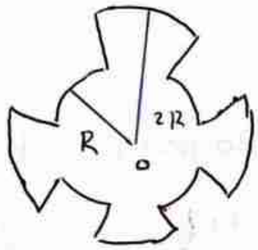


$$B_1 = \frac{\mu}{4\pi}\frac{i}{\left(\frac{D}{2}\right)} 2\sin\frac{\pi}{2}$$

$$= \frac{\mu}{4\pi}\frac{4i}{D}$$

$$B_2 = \frac{\mu_0 i}{2\times\frac{D}{2}} = \frac{\mu_0 i}{D}\times\frac{1}{2} = \frac{\mu_0 i}{2D}$$

$$B = B_1 + B_2 = \frac{\mu_0 i}{2D}\left(1+\frac{2}{\pi}\right)$$



$$B_R = \frac{\mu_0 i}{2R}\times\frac{1}{2}$$

$$B_{2R} = \frac{\mu_0 i}{4R}\times\frac{1}{2}$$

$$B_0 = \frac{\mu_0 i}{R}\left(\frac{1}{4}+\frac{1}{8}\right) = \frac{\mu_0 i}{R}\,\frac{2+1}{8} = \frac{3\mu_0 i}{8R}$$

Magnetic field due to an arc

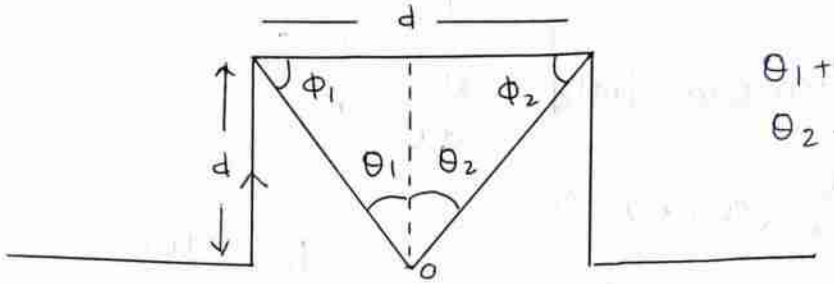
$$B = \frac{\mu_0 i}{2R}\left(\frac{\phi}{2\pi}\right)$$

* For a regular polygon

$$B = \frac{\mu_0}{4\pi}\frac{2i}{R}\, n\sin\left(\frac{\pi}{n}\right)$$

21.06.2024

Magnetic field at O will be



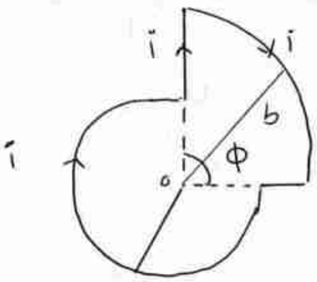
$\theta_1 + \phi_1 = 90$

$\theta_2 + \phi_2 = 90$

Magnetic field

$$B = \frac{\mu_0}{4\pi}\frac{I}{d}(\sin\theta_1 + \sin\theta_2)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{d}(\cos\phi_1 + \cos\phi_2)$$

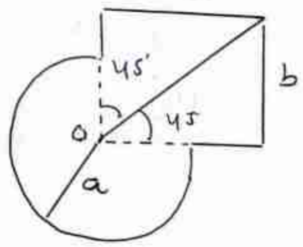


Magnetic field at O

$$\left(\frac{2\pi-\phi}{2\pi}\right)\frac{\mu_0 i}{2a} + \frac{\phi}{2\pi}\frac{\mu_0 i}{2b}$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{2\pi-\phi}{2a} + \frac{\phi}{4b}\right]$$

Magnetic field at O



$\Rightarrow$ Magnetic field due to Circle

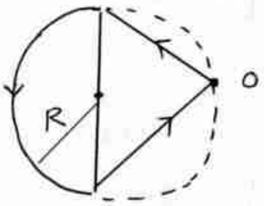
$$B_1 = \frac{\mu_0 i}{2a} \times \frac{3}{4}$$

due to $B_2 = \frac{\mu_0}{4\pi}\frac{i}{d}(\sin 0 + \sin 45) \times 2$

$$\frac{\mu_0}{4\pi}\frac{i}{b}\sqrt{2}$$

$$B = B_1 + B_2 = \frac{\mu_0 i}{4\pi}\left(\frac{\sqrt{2}}{b} + \frac{3\pi}{2a}\right)$$

Magnetic field at o.

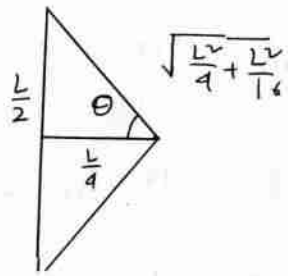


$$B = \frac{\mu_0 i}{2R} \times \frac{\phi}{2\pi}$$

$$= \frac{\mu_0 i}{2R} \times \frac{\pi}{2\pi} = \frac{\mu_0 i}{4R}$$

A current i is flowing in a straight conductor of length L The magnetic induction at a point distant L/4 from the centre is

$\Rightarrow$

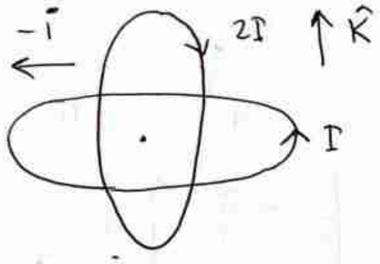


$$\sin\theta = \frac{\frac{L}{2}}{L\sqrt{\frac{4+1}{16}}} = \frac{1}{2\sqrt{\frac{5}{16}}} = \frac{2}{\sqrt{5}}$$

$$B = \frac{\mu_0}{4\pi}\frac{i}{d}(\sin\theta_1 + \sin\theta_2)$$

$$= \frac{\mu_0}{4\pi}\frac{4i}{L} \times 2 \times \frac{2}{\sqrt{5}} = \frac{4\mu_0 i}{\sqrt{5}\pi L}$$

Magnetic field at o

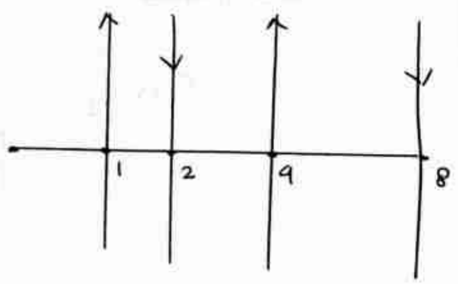


$\Rightarrow$

$$B_1 = \frac{\mu_0 i}{2R}\hat{k}$$

$$B_2 = \frac{\mu_0 2i}{2R}(-\hat{i}) \qquad B = \frac{\mu_0 i}{2R}(-2\hat{i} + \hat{k})$$

Each carry current 1A.
Magnetic field at origin $\alpha \times 10^{-7}\, T\hat{k}$



$$\Rightarrow \quad B = \frac{\mu_0}{4\pi} 2i\left(1 - \frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right)$$

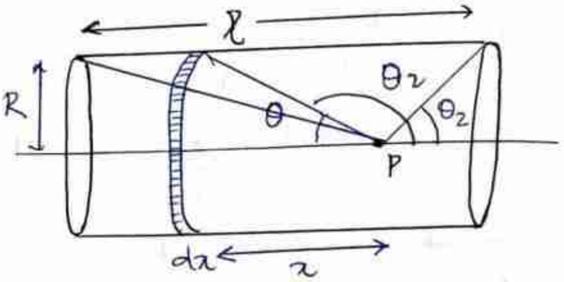
$$= (2\times10^{-7})\frac{1}{1-(-\frac{1}{2})} = \frac{4}{3}\times10^{-7}\, T\hat{k}$$

$\alpha = 1.33$

Magnetic field due to Solenoid

No of turns per unit length = $\frac{N}{\ell}$

Here turn = $\frac{N}{\ell} dx$



$$dB = \frac{\mu_0 i R^2}{2(x^2+R^2)^{3/2}} \cdot \frac{N}{\ell} dx$$

$$\sin\theta = \frac{R}{\sqrt{R^2+x^2}}$$

$$dB = \frac{\mu_0 i R^2}{2(R^2 \operatorname{cosec}^2\theta)^{3/2}} \frac{N}{\ell}(-R \operatorname{cosec}^2\theta d\theta)$$

$$\cos\theta d\theta = \frac{-R}{(R^2+x^2)^{3/2}} x dx$$

$$dB = \frac{\mu_0 i R^2}{2R^3 \operatorname{cosec}^3\theta} \frac{N}{\ell}(-R)\operatorname{cosec}^2\theta d\theta$$

$$\frac{x}{R} = \cot\theta$$

$$x = R\cot\theta$$

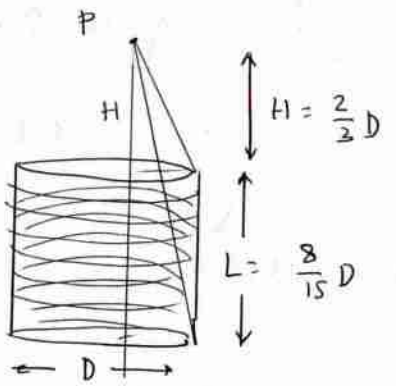
$$dx = -R\operatorname{cosec}^2\theta d\theta$$

$$dB = -\frac{\mu_0 i}{2} n \sin\theta d\theta$$

$$B = -\frac{\mu_0 i n}{2}\left[\cos\theta\right]_{\theta_1}^{\theta_2} = \frac{\mu_0 i n}{2}(\cos\theta_1 - \cos\theta_2)$$

For infinite Solenoid $\theta_1 = 0$ $\theta_2 = 180$

Magnetic field at P



$H = \frac{2}{3}D$

$L = \frac{8}{15}D$

$$\cos\theta_1 = \frac{H}{\sqrt{H^2+\frac{D^2}{2}}} = \frac{\frac{2}{3}D}{D\sqrt{\frac{4}{9}+\frac{1}{2}}}$$

$$\cos\theta_1 = \frac{\frac{2}{3}}{\sqrt{\frac{8+9}{18}}} = \frac{2}{3} \times \frac{3\sqrt{2}}{17}$$

$$\cos\theta_1 = \frac{2\sqrt{2}}{17}$$

$$\cos\theta_2 = \frac{\frac{6}{5}D}{\sqrt{(\frac{6}{5}D)^2 + D^2}}$$

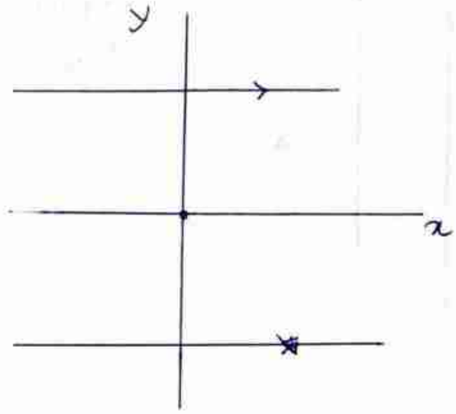
$$H + L = \left(\frac{2}{3}+\frac{8}{15}\right)D$$

$$\frac{\frac{6}{5}}{\sqrt{\frac{36+25}{25}}} = \frac{6}{\sqrt{61}}$$

$$= \frac{10+8}{15} = D$$

$$= \frac{6}{5}D$$

① Two infinitly long wires carrying current I_0 lie on xy plane and parallel to x axis. pass through the points $(0,a,0)$ and $(0,-a,0)$ The max magnetic field on the z axis is



$\Rightarrow$ Maximum magnetic field at origin

$$B = \frac{\mu_0}{4\pi}\frac{2I_0}{a} \times 2 = \frac{\mu_0 I_0}{\pi a}$$

(Maximum)

24.06.2024

Ampere's Circuital Law

$$\oint_c \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$\Downarrow$

when B has to be find & current given

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

$\Downarrow$

when B given and i, J have to calculated

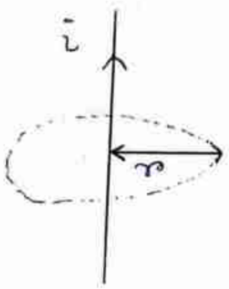
So. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$

$$\oint(\vec{\nabla}\times\vec{B})\, d\bar{s} = \mu_0\int\vec{J}\cdot d\vec{s}$$

$$\int(\vec{\nabla}\times\vec{B} - \mu_0\vec{J})\, d\bar{s} = 0$$

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J}}$$

① Straight wire



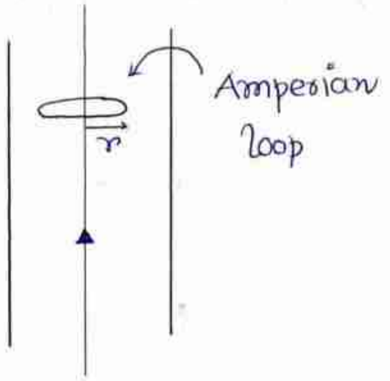
$$\oint \bar{B}\cdot d\bar{l} = \mu_0 i$$

$$B\cdot 2\pi r = \mu_0 i$$

$$B = \frac{\mu_0}{4\pi}\frac{2i}{r}$$

<u>Case:</u> Current density

$J \propto r^n$



<u>Inside</u>

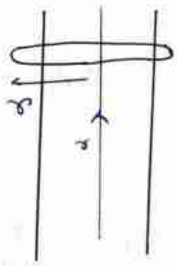
$$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \bar{J}\cdot d\bar{s}$$

$$= \mu_0 \int K r^n \, 2\pi r \, dr$$

$$B \cdot 2\pi r = \mu_0 K 2\pi \int_0^r r^{n+1} dr$$

$$B = \frac{\mu_0 K}{r} \frac{r^{n+2}}{n+2} \Rightarrow B_{in} \propto r^{n+1}$$

<u>outside</u>



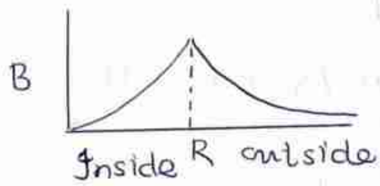
$$\oint \vec{B}\cdot d\bar{l} = \mu_0 \int \bar{J}\cdot d\bar{s}$$

$$B \cdot 2\pi r = \mu_0 \int_0^R K r^n \, 2\pi r \, dr$$

$$B \cdot 2\pi r = 2\pi \mu_0 K \frac{R^{n+2}}{n+2} \Rightarrow B_{out} \propto \frac{1}{r}$$

<u>when</u> $n = 1$

$J \propto r$



Force in magnetic field <u>27.06.2024</u>

$$\vec{F} = q(\vec{v}\times\vec{B}) \quad |\vec{F}| = qvB\sin\theta = qv_\perp B = qvB_\perp$$

If $\vec{v} \perp \vec{B}$ then $\sin\theta = \sin 90 = 1$ $F = qvB$

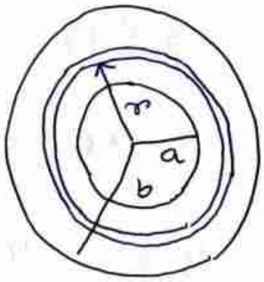
So Circular Path $qvB = \frac{mv^2}{r}$

$$\Rightarrow qBr = v \Rightarrow r = \frac{mv}{qB}$$

Time Period $T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

Kinetic Energy $= \frac{p^2}{2m} = \frac{q^2B^2r^2}{2m}$

① A non Conducting disc of inner radius a and outer b Q charge is distributed on the disc. Disc is rotating about its axis with ω. Magnetic field at Centre



$$da = 2\pi r dr$$

$$dq = \sigma da = \frac{Q}{(b^2-a^2)} 2\pi r dr$$

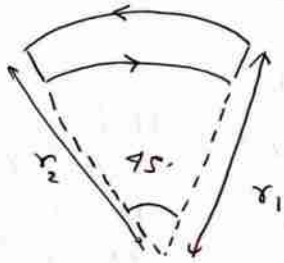
$$di = \frac{dQ}{T} = \frac{dQ}{2\pi}.\omega$$

$$= \frac{Q}{(b^2-a^2)} 2\pi r dr.\frac{\omega}{2\pi} = \frac{Q\omega}{(b^2-a^2)} r dr$$

$$dB = \frac{\mu_0}{2r} di = \int_a^b \frac{\mu_0}{2r} \frac{Q\omega}{(b^2-a^2)} r dr = \frac{\mu_0 \omega Q}{2\pi(b+a)}$$

---

② Magnetic field at O is



due to angle ϕ

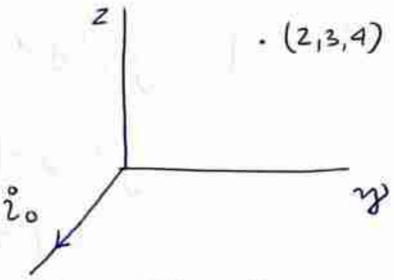
$$B = \frac{\mu_0 i}{2r} \frac{\phi}{2\pi}$$

$$= \frac{\mu_0 i}{2r} \frac{\pi}{8\pi} \cdot \frac{\mu_0 i}{2\times 8 r}$$

$$B_{net} = \frac{\mu_0 i}{16}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

---

③ An infinitly long wire passing through origin is Carrying current i_0 along x axis. The Magnetic field at (2, 3, 4) is



$\Rightarrow$ $\vec{ix} = \hat{i}$ $\quad \vec{r} = 2\hat{i}+3\hat{j}+4\hat{k}$

$i(\vec{x}\times\vec{r}) \Rightarrow$

$\vec{r} = 2\hat{i}+3\hat{j}+4\hat{k}$

$|\vec{r}| = \sqrt{29}$

$$B = \frac{\mu_0}{4\pi} \frac{2i}{r} (d\vec{l}\times\vec{r})$$

$$= \frac{\mu_0}{4\pi} \frac{2i}{r} \frac{\hat{x}\times(2\hat{x}+3\hat{y}+4\hat{z})}{\sqrt{29}}$$

$$= \frac{\mu_0}{4\pi} \frac{2i}{29} (3\hat{z} - 4\hat{y})$$

---

(4) Volume current density due to a Cylindrical conductor is $\vec{J} = J_0 (1 - r/R)\hat{z}$. The Value of r for which magnetic field is maximum

$\Rightarrow$

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 \int \vec{J} \cdot d\vec{a}$$

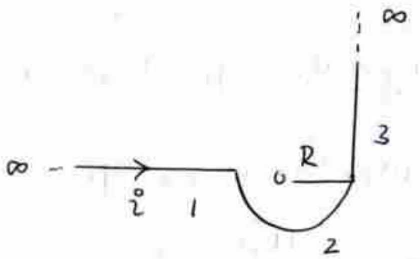
$$B \cdot 2\pi r = \mu_0 \int_0^r J_0 \left(1 - \frac{r}{R}\right) 2\pi r \, dr$$

$$B \cdot r = \mu_0 J \int_0^r \left(r - \frac{r^2}{R}\right) dr$$

$$B = \frac{\mu_0 J}{r}\left(\frac{r^2}{2} - \frac{r^3}{3R}\right) \Rightarrow B = \mu_0 J \left(\frac{r}{2} - \frac{r^2}{3R}\right)$$

$$\frac{dB}{dr} = \frac{1}{2} - \frac{2r}{3R} = 0 \Rightarrow \frac{2r}{3R} = \frac{1}{2} \Rightarrow r = 3R/4$$

(5) Magnetic field at O



$B_1 = 0$

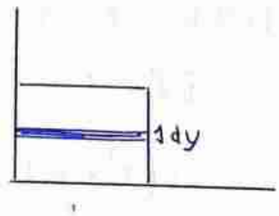
$B_2 = \frac{\mu_0 i_0}{2R} \times \frac{1}{2}$

$B_3 = \frac{\mu_0}{4\pi} \frac{2 i_0}{R} \times \frac{1}{2}$ $\quad$ $B_{net} = B_1 + B_2 + B_3$

$$= \frac{\mu_0 i_0}{4\pi}\left(1 + \frac{1}{\pi}\right)$$

(6) A square loop of wire of side a lies in the first region of xy plane with a corner at origin. In this region of a non uniform time varying field $\vec{B} = B_0 y^3 t \hat{k}$. Induced emf is

$\Rightarrow$



$$\phi = \int \vec{B} \cdot d\vec{s}$$

$$= \int B_0 y^3 t \cdot a \, dy$$

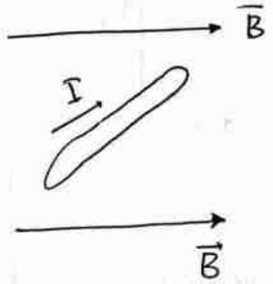
$$= B_0 t a \frac{y^4}{4}\Big|_0^a = \frac{B_0 t a^5}{4}$$

$$e = -\frac{d\phi}{dt} = \frac{B_0 a^5}{4}$$

01.07.2024

Force on a Current Carrying Conductor in a MF

Force on the rod



$\vec{F} = q(\vec{v} \times \vec{B})$

$\vec{F} = q(\vec{v}_d \times \vec{B})$

$\vec{F} = e v_d B \sin\theta$

For n number of electron $F = N e v_d B \sin\theta$

Number of the e^-

$N = n l A$

n is number density
l is length of segment
A is Area of cross section

So $dF = n\, dl A e v_d B \sin\theta$

$dF = (neAv_d)\, dl\, B \sin\theta$

$dF = i\, dl\, B \sin\theta$

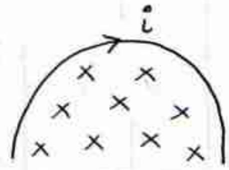
$F = \int dF = \int i(d\vec{l} \times \vec{B})$

For uniform length

$\vec{F} = i\, \vec{l} \times \vec{B}$

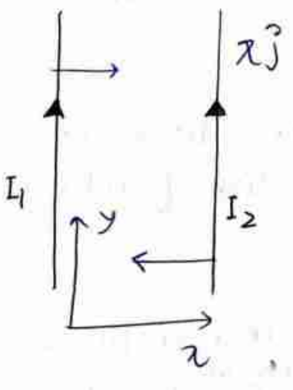
($d\vec{l}$ is the displacement vector of current element)

Force closed loop displacement vector $\vec{l} = 0$, $\vec{F} = 0$



$F = Bil$

$F = 2RiB$



$B_{21} = \dfrac{\mu_0 I_1}{2\pi d}(-\hat{k})$

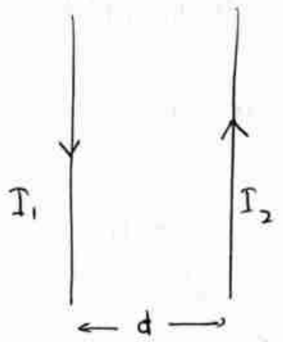
$\vec{F}_{21} = I_2\, l\hat{j} \times \dfrac{\mu_0 I_1}{2\pi d}(-\hat{k})$

$= \dfrac{\mu_0 I_1 I_2 l}{2\pi d}(-\hat{i})$

$\dfrac{\vec{F}_{21}}{l} = \dfrac{\mu_0 I I_2}{2\pi d}(-\hat{i})$

They are attractive in nature.

$\dfrac{\vec{F}_{12}}{l} = \dfrac{\mu_0 I_1 I_2}{2\pi d}(\hat{i})$



Here they will oppose each other

Torque on a loop.

Torque is defined as

$$\bar{\tau} = \vec{r} \times \vec{F} = rF\sin\theta$$

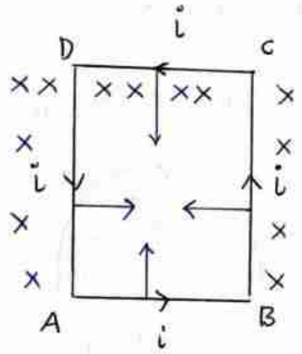
or $\bar{\tau} = r\,(Bi\ell)\sin\theta$

or $\vec{\tau} = Bi\,(r\ell)\sin\theta$ (Area $A = r\ell$)

or $\bar{\tau} = BiA\sin\theta$

or $\bar{\tau} = mB\sin\theta$ ($m = iA$)

or $\boxed{\bar{\tau} = \vec{m} \times \vec{B}}$

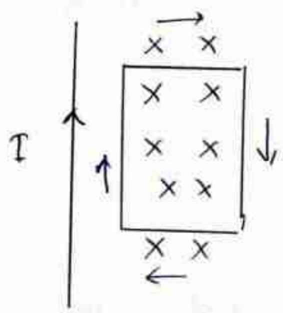


Force $\vec{F}_{AB} = i \times (\ell\hat{i}) \times (-B\hat{k})$

$= Bi\ell\,\hat{j}$

$F_{net} = 0$ so torque. $\bar{\tau} = 0$

But torque is Present



If the current on wire decrease with time then direction of induced current will be

Direction of magnetic ~~mag~~ field is inside and decrese with time so the loop will try to increase magnetic field so direction of magnetic field will be clockwise

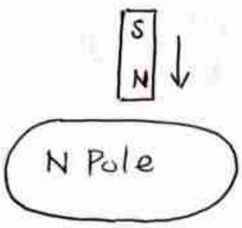
① An electric field as a function of radial co-ordinate r has the form $E(r) = \frac{\alpha e^{r^2}}{r}\hat{r}$. The Electric flux through a sphere of radius $\sqrt{2}$ centred at origin is ϕ. The Value of $\phi/2\pi\alpha$ is

$\Rightarrow$ Flux. $\phi = \iint \vec{E}\cdot d\vec{s} = \int \frac{\alpha e^{r^2}}{r} r^2 \sin\theta \, d\theta \, d\phi$

$$= \frac{\alpha e^{R^2}}{R} R^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi = 4\pi\alpha e^{R^2} R = \phi$$

$$\phi = 4\pi\alpha e^2\sqrt{2} \Rightarrow \frac{\phi}{2\pi\alpha} = 2\sqrt{2}e^2 = 20.89.$$

② If the magnet falls then the acceleration of the magnet will be



$\Rightarrow$ So it oppose the North pole

So $a < g$

③ Magnetic flux in a closed circuit of $R = 10\,\Omega$ Varies with time as $\phi = 6t^2 - 5t + 1$. Magnetude of induced current at $t = 0.25$ sec be

$\Rightarrow$ $\phi = -(6t^2 - 5t + 1)$ $\qquad \varepsilon = -(12\times 0.25 - 5)$

$\varepsilon = \frac{d\phi}{dt} = -(12t - 5)$ $\qquad \varepsilon = -(3-5) = +2$ volt

$$I = \frac{\varepsilon}{R} = \frac{2}{10} = 0.2\ A$$

④ Induced emf in a uniform field depends on

no of turns in the Coil

on the magnetic field

Area of Coil and Speed of Rotation

02.07.2024

Faraday's Law:

① **First Law:**

Whenever there is a change in magnetic flux across a Conductor an EMF induced if the Conductor forms a closed loop a current will flows through the Circuit.

Magnetic flux

$\phi = \int \vec{B} \cdot d\vec{s}$

$\phi = \vec{B} \cdot \vec{A}$

$\phi = BA\cos\theta$

unit of Magnetic flux $T \cdot m^2$ or Weber

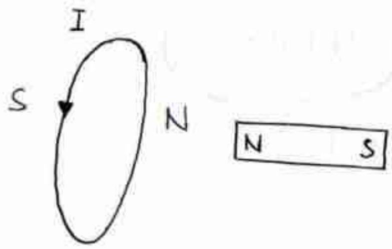
$\phi = \oint \vec{A} \cdot d\vec{l}$

$(\vec{B} = \vec{\nabla} \times \vec{A})$

② **Second Law**

Induced EMF is equal to rate of change in magnetic flux

$\varepsilon = \frac{d\phi}{dt}$ | It will be further edited by Lenz Law.



$$B = \frac{\mu_0}{4\pi} \frac{2\pi I A}{(a^2+x^2)^{3/2}}$$

$$B = \frac{\mu}{4\pi} \frac{2m}{x^3} \quad x >> a$$

$$\varepsilon = -\frac{d\phi}{dt}$$

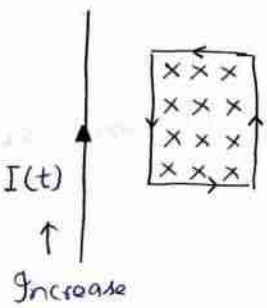
Induced Emf. $\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt}$

$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{s}$

$\iint (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = -\frac{d}{dt} \iint \vec{B} \cdot d\vec{s}$

$\iint \left(\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t}\right) \cdot d\vec{s} = 0$

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$



Magnetic field toward inward increase So flux across loop increase.

The Current on the loop will decrease Magnetic field

$\Downarrow$

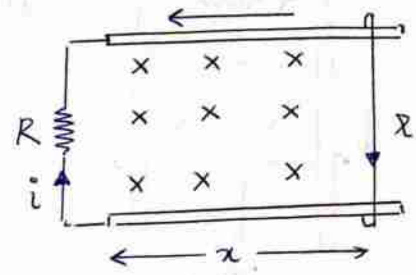
Lenz Law

Opposite the Cause of generate thats why (−)ve sign used.

Faraday Law Based on Conservation of Energy)

03.07.2024

Motional EMF



Current

$$i = \frac{\varepsilon}{R}$$

$$i = \frac{Blv}{R}$$

Flux $\phi = \vec{B} \cdot \vec{A}$

$$\phi = Blx$$

$$-\frac{\partial \phi}{\partial t} = -Bl\frac{dx}{dt}$$

$$\varepsilon = Blv$$

Induced EMF

$$\varepsilon = Blv$$

From Lentz law direction of current is clockwise

$$i = \frac{Blv}{R}$$

Swipe rod leftward A & ϕ decrease so increase Magnetic field clockwise

Power $P = i^2R = \left(\frac{Blv}{R}\right)^2 R$

$$P = \frac{B^2l^2v^2}{R}$$

Generated Power due to current

Direction of force $\vec{F} = i\,\vec{l} \times \vec{B}$

$$= \frac{Blv}{R}(-l\hat{j}) \times (-B\hat{k}) = \frac{B^2l^2}{R} v\,\hat{i}$$

Also Mechanical Power. $P = Fv = \frac{B^2l^2v^2}{R}$

So force $F = \frac{B^2l^2v}{R}$

$$ma = \frac{B^2l^2v}{R}$$

Acceleration

$$a = \frac{B^2l^2v}{mR}$$

$$\frac{dv}{dt} = \frac{B^2l^2v}{mR}$$

$$\Rightarrow \int_{v_0}^{v} \frac{dv}{v} = \frac{B^2l^2}{mR}\int_0^t dt$$

$$\Rightarrow \ln\left(\frac{v}{v_0}\right) = \frac{B^2l^2}{mR}t$$

$$\Rightarrow v = v_0 e^{-\frac{B^2l^2}{mR}t}$$

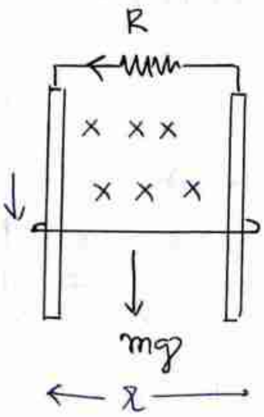
Velocity decrease with time exponentially

$$v\frac{dv}{dx} = -\frac{B^2l^2v}{mR}$$

$$\int_{v_0}^{v} dv = -\frac{B^2l^2}{mR}\int_0^x dx$$

$$v = v_0 - \frac{B^2l^2}{mR}x$$

(I) Flux, EMF, Current



Flux, $\phi = \bar{B}\cdot\bar{A} = Blx$

Emf $\varepsilon = \frac{d\phi}{dt} = -Bl\frac{dx}{dt} = Blv$

Current $i = \frac{\varepsilon}{R} = \frac{Blv}{R}$

Power $P = i^2 R = \frac{B^2l^2v^2}{R}$

(II) Force

Force $\vec{F} = i(\vec{l}\times\vec{B})$

$= i(-l\hat{j}\times(-B\hat{k})) = Bli\,\hat{i}$

Now

$mg - F = ma$

$mg - \frac{B^2l^2v}{R} = ma \Rightarrow a = g - \frac{B^2l^2v}{mR}$

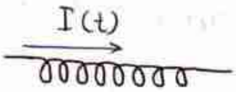
Now for terminal velocity Acceleration will be zero

$g - \frac{B^2l^2v}{mR} = 0 \Rightarrow g = \frac{B^2l^2v}{mR}$

$\Rightarrow v_T = \frac{mgR}{B^2l^2}$

> Self Inductance:

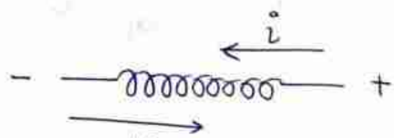
For Constant current



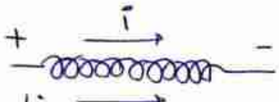
it simply behave like conductor. But for time varying current Magnetic flux

$\phi \propto i \Rightarrow \phi = Li$ L is Coefficient of self inductance

Induced Emf $\varepsilon = -\frac{d\phi}{dt}$ $\boxed{\varepsilon = -L\frac{di}{dt}}$

- 

$\frac{di}{dt} = +2A/sec$

- 

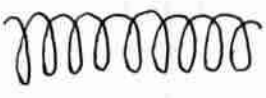
$\frac{di}{dt} = -2A/sec$

Direction of induced current is such that it oppose the growing current and support decaying current

Based on Lenz law

$n l = N$

Solenoid



$N\Phi = Li$

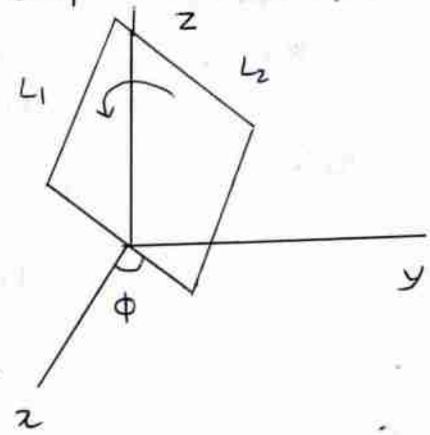
$N(\mu_0 n i)\pi R^2 = Li$

$N(\mu_0 \frac{N}{l} i)\pi R^2 = Li$

$\frac{\mu_0 N^2 \pi R^2}{l} = L$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

① Loop is free to rotate about z axis $\frac{dB}{dt} = K$ if the resistance of the loop is R, torque require to prevent the loop from Rotating



$\Rightarrow$

$\phi = BL_1L_2\cos\phi$

$\mathcal{E} = \frac{d\phi}{dt} = L_1L_2\cos\phi \frac{dB}{dt}$

$\mathcal{E} = KL_1L_2\cos\phi$

$i = \frac{\mathcal{E}}{R} = \frac{KL_1L_2}{R}\cos\phi$

Magnetic moment

$\vec{m} = i\vec{A}$

$= \frac{K(L_1L_2)^2}{R}\cos\phi$

Torque $\vec{\tau} = \vec{m} \times \vec{B}$

$= mB\sin(180-\phi)$

$= \frac{KB(L_1L_2)^2}{R}\sin\phi\cos\phi\,\hat{z}$

② A Circular loop of radius 1m Spins with Constant angular velocity $\omega = 20\pi$ rad/sec in a magnetic field $B = 3T$. The resistance of the loop is $10\,\Omega$ then Power dissipated Per cycle

$\Rightarrow$ $\phi = \vec{B}\cdot\vec{A} = BA\cos\theta = BA\cos\omega t = B\pi a^2\cos\omega t$

$\mathcal{E} = -\omega B\pi a^2\sin\omega t$

$P = \frac{\mathcal{E}^2}{R}$

$\langle P \rangle = \frac{\omega^2 B^2 \pi^2 a^4}{R}\langle\sin^2\omega t\rangle$

$= \frac{\omega^2 B^2 \pi^2 a^4}{2R}$

$= \frac{(20\pi)^2 \times 3^2 \times \pi^2}{2 \times 10}$ Answer

$$\oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{s} = -\frac{\partial}{\partial t}\int \vec{B}\,(x\,dl)$$

$$\oint \bar{E}\cdot d\bar{l} = -\vec{B}\int \frac{\partial}{\partial t}(\vec{x}\,d\vec{l}) = -\vec{B}\cdot\int \frac{\partial \bar{x}}{\partial t}\times d\bar{l}$$

$$\oint \bar{E}\cdot d\bar{l} = -B\int (\vec{v}\times d\bar{l}) \Rightarrow \oint \vec{E}\cdot d\vec{l} = -\int (\vec{v}\times\vec{B})\,d\bar{l}$$

Motional EMF $\quad \varepsilon = \int (\vec{B}\times\vec{v})\cdot d\bar{l}$

Energy store in an Inductor

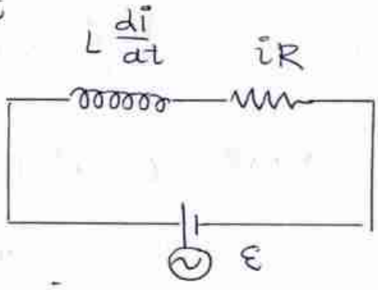
Power $p = \frac{dw}{dt}$ $\qquad P = \varepsilon\, i$

$$= L\frac{di}{dt}\cdot i$$

$$w = \int P\,dt = \int_0^I L\,i\,di = \frac{1}{2}LI^2$$

Magnetic Energy $\rightarrow$ $\boxed{U = \frac{1}{2}LI^2}$

Growth and decay of Current



$$\varepsilon = L\frac{di}{dt} + iR$$

$$\frac{\varepsilon - iR}{L} = \frac{di}{dt}$$

$$\int_0^I \frac{di}{\varepsilon - iR} = \frac{1}{L}\int_0^t dt$$

$t = 0,\ i = 0$

$t = \infty\ \ i = \varepsilon/R = i_0$

$$-\frac{1}{R}\ln(\varepsilon - iR) = \frac{t}{L} + c$$

$$\ln(\varepsilon - iR) = \frac{-Rt}{L} + c$$

$$\boxed{i = \frac{\varepsilon}{R}\left(1 - e^{-Rt/L}\right)}$$

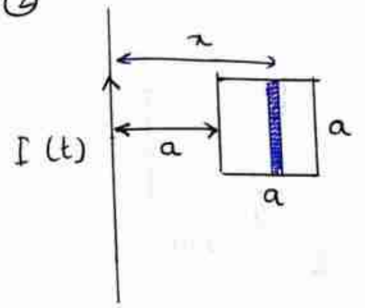
① $\vec{A} = yz\,\hat{i} + zx\,\hat{j} + xy\,\hat{k}$ and c is the circle of unit radius in plane $z = 1$. $\oint \bar{A}\cdot d\bar{l} = ?$

$\Rightarrow$ $\oint \vec{A}\cdot d\bar{l} = \int_s (\bar{\nabla}\times\bar{A})\,d\bar{s}$

$$\bar{\nabla}\times\bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix} = 0$$

$$\oint \bar{A}\cdot d\bar{l} = 0$$

②



$I(t) = I_0 \cos\omega t$

If the resistance of the loop is R then amplitude of induced current will be

$\Rightarrow$ Magnetic field at the distance $B = \frac{\mu_0 i}{2\pi x}$

$$d\phi = \frac{\mu_0 i}{2\pi x} a\,dx \Rightarrow \phi = \frac{\mu_0 i a}{2\pi}\int_a^{2a} \frac{dx}{x}$$

$$\Rightarrow \phi = \frac{\mu_0 i a}{2\pi} \ln 2$$

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\Rightarrow \phi = \frac{\mu_0 i_0 \cos\omega t\, a}{2\pi} \ln 2$$

$$\varepsilon = \frac{\omega \mu_0 i_0 a \sin\omega t}{2\pi} \ln 2$$

$$i = \frac{\varepsilon}{R} = \frac{\mu_0 i a}{2\pi R} \sin\omega t \ln 2$$

Alternating Current

$$\langle \varepsilon \rangle = \frac{\int_0^{2\pi/\omega} \varepsilon\, dt}{\int_0^{2\pi/\omega} dt} = 0$$

Voltage is

$\varepsilon = \varepsilon_0 \sin\omega t$

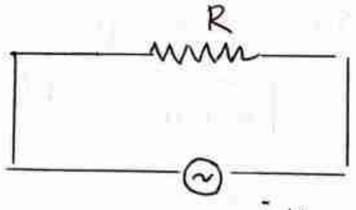
$\varepsilon = \varepsilon_0 \cos\omega t$

$\varepsilon = \varepsilon_0 e^{j\omega t}$

$$\langle \varepsilon^2 \rangle = \frac{\int_0^{2\pi/\omega} \varepsilon^2 dt}{\int_0^{2\pi} dt} = \frac{\int \varepsilon_0^2 \sin^2\omega t\, dt}{\int dt} = \frac{\varepsilon_0^2}{2}$$

$$\varepsilon_{rms} = \sqrt{\langle \varepsilon^2 \rangle} = \frac{\varepsilon_0}{\sqrt{2}}$$

① Pure Resistive Circuit



$\varepsilon = \varepsilon_0 e^{j\omega t}$

From Kirchoff's law

$\varepsilon - iR = 0$

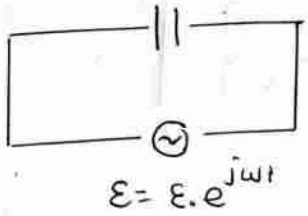
$\varepsilon = iR$

$i = \left(\frac{\varepsilon_0}{R}\right) e^{j\omega t}$

$i = i_0 e^{j\omega t}$

Voltage & Current is in same phase

① pure Capacitor Circuit



charge. $q = CV$

$q = C\varepsilon_0 e^{j\omega t}$

$i = \frac{dq}{dt} = C\varepsilon_0 \omega j e^{j\omega t} \Rightarrow i = \frac{\varepsilon_0}{\left(\frac{1}{\omega C}\right)} j e^{j\omega t}$

$\Rightarrow i = \frac{\varepsilon_0}{\left(\frac{1}{\omega C}\right)} e^{j\pi/2} e^{j\omega t}$

$i_0 = \frac{\varepsilon_0}{\left(\frac{1}{\omega C}\right)}$

$\Rightarrow i = \frac{\varepsilon_0}{\left(\frac{1}{\omega C}\right)} e^{j(\omega t + \frac{\pi}{2})}$

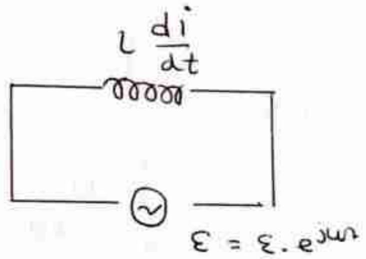
$i_0 = \frac{\varepsilon_0}{X_C}$

$X_C = \frac{1}{\omega C}$ $\Rightarrow i = i_0 e^{j(\omega t + \pi/2)}$

Capacitor Reactance

Current leads by an angle $\pi/2$ with the Voltage

② Pure Inductive Circuit



$\varepsilon = L \frac{di}{dt}$

or $\frac{\varepsilon}{L} dt = di$

$\cdot \frac{1}{L} \varepsilon_0 \int e^{j\omega t} dt = \int di$

$\cdot \frac{\varepsilon_0}{(\omega L j)} e^{j\omega t} = i$

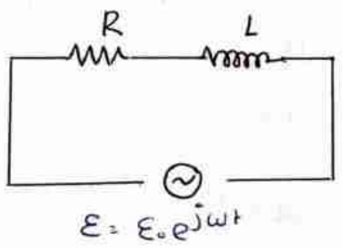
$\cdot \frac{\varepsilon_0}{\omega L} \frac{e^{j\omega t}}{e^{j\pi/2}} = i$

$i = \frac{\varepsilon_0}{\omega L} e^{j(\omega t - \pi/2)}$

$i = i_0 e^{j(\omega t - \pi/2)}$ $\quad X_L = \omega L$

Current lags by an angle $\pi/2$ with the voltage

③ LR Circuit



Impedance $Z = R + j\omega L$

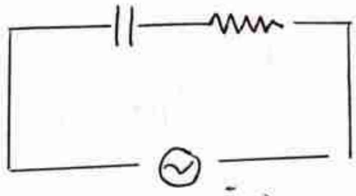
$Z = \sqrt{R^2 + \omega^2 L^2}\, e^{j\left(\tan^{-1}\frac{\omega L}{R}\right)}$

$i = \frac{\varepsilon}{Z} = \frac{\varepsilon_0 e^{j\omega t}}{\sqrt{R^2 + \omega^2 L^2} \cdot e^{j\tan^{-1}\frac{\omega L}{R}}}$

$i = \frac{\varepsilon_0}{\sqrt{R^2 + \omega^2 L^2}} e^{j\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)}$

current lag ~~s~~ Inductive circuit

④ CR Circuit



$$\mathcal{E} = \mathcal{E}_0 e^{j\omega t}$$

Impedance

$$Z = R + \frac{1}{j\omega C}$$

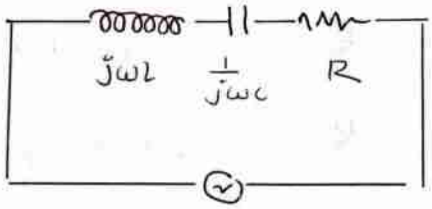
$$|Z| = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}\; e^{j\tan^{-1}\left(\frac{-1}{\omega C R}\right)}$$

$$i = \frac{\mathcal{E}_0 e^{j\omega t}}{|Z|} = \frac{\mathcal{E}_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}\; e^{j\left(\omega t + \tan^{-1}\frac{1}{\omega C R}\right)}$$

If current leads by certain angle then it is called Capacitive Circuit.

If angle is $\pi/2$ then Pure Capacitive Circuit.

⑤ LCR Series Circuit



Supplied emf

$$\mathcal{E} = \mathcal{E}_0 e^{j\omega t}$$

$$i = \frac{\mathcal{E}}{|Z|} = \frac{\mathcal{E}_0 e^{j\omega t}}{\left| R + j\left(\omega L - \frac{1}{\omega C}\right) \right|}$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}\; e^{i\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)}$$

$$i = \frac{\mathcal{E}_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}\; e^{j\left[\omega t - \tan^{-1}\frac{\left(\omega L - \frac{1}{\omega C}\right)}{R}\right]}$$

If $\omega L > \frac{1}{\omega C}$ Current will lag (Inductive Circuit)

$\omega L < \frac{1}{\omega C}$ Current will lead (Capacitive Circuit)

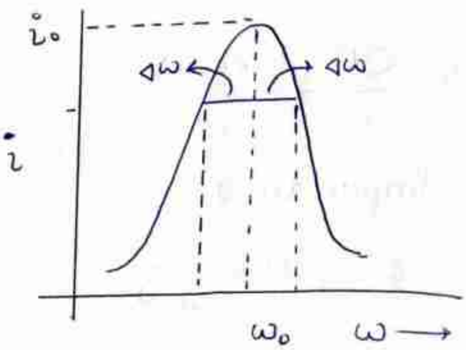
If $\omega L = \frac{1}{\omega C}$ then $i = \frac{\mathcal{E}_0}{R} e^{j\omega t}$

$i = i_{max} = \frac{\mathcal{E}_0}{R}$ Resonance Condition

$\omega_0 = \frac{1}{\sqrt{LC}}$ ⇒ Resonance frequency

So current

$$i = \frac{\varepsilon_0}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$



$$\frac{i_0}{\sqrt{2}} = \frac{\varepsilon_0}{\sqrt{R^2 + \left\{(\omega_0+\Delta\omega)L - \frac{1}{(\omega_0+\Delta\omega)C}\right\}^2}}$$

$$\frac{\varepsilon_0}{R\sqrt{2}} = \frac{\varepsilon_0}{\sqrt{R^2 + \left\{(\omega_0+\Delta\omega)L - \frac{1}{(\omega_0+\Delta\omega)C}\right\}^2}}$$

$$2R^2 = R^2 + \left\{(\omega_0+\Delta\omega)L - \frac{1}{(\omega_0+\Delta\omega)C}\right\}^2$$

$$R^2 = \left\{(\omega_0+\Delta\omega)L - \frac{1}{(\omega_0+\Delta\omega)C}\right\}^2$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$\Downarrow$

sharpness of Resonance

$$R = (\omega_0+\Delta\omega)L - \frac{1}{(\omega_0+\Delta\omega)C}$$

$$R = \omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right) - \frac{1}{\omega_0 C\left(1+\frac{\Delta\omega}{\omega_0}\right)}$$

$$R = \cancel{\omega_0 L} + (\omega_0 L)\frac{\Delta\omega}{\omega_0} - \cancel{\frac{1}{\omega_0 C}} + \frac{1}{\omega_0 C}\frac{\Delta\omega}{\omega_0}$$

$$R = \left(\omega_0 L + \frac{1}{\omega_0 C}\right)\frac{\Delta\omega}{\omega_0}$$

$$R = 2\omega_0 L\frac{\Delta\omega}{\omega_0} \Rightarrow \frac{R}{\omega_0 L} = \frac{2\Delta\omega}{\omega_0}$$

$$\frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R} = Q = \frac{1}{\sqrt{LC}}\frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

<u>Average Power</u>

$\varepsilon = \varepsilon_0 \sin\omega t$

$i = i_0 \sin(\omega t + \phi)$

Power $P = \varepsilon i$

$= \varepsilon_0 \sin\omega t * i_0 \sin(\omega t + \phi)$

$= \varepsilon_0 i_0 \sin\omega t(\sin\omega t\cos\phi + \cos\omega t\sin\phi$

$P = \varepsilon_0 i_0(\sin^2\omega t\cos\phi + \sin\omega t\cos\omega t\sin\phi$

$\langle P\rangle = \frac{1}{2}\varepsilon_0 i_0 \cos\phi$

$\langle P\rangle = \frac{\varepsilon_0}{\sqrt{2}}\frac{i_0}{\sqrt{2}}\cos\phi \Rightarrow \langle P\rangle = \varepsilon_{rms} i_{rms}\cos\phi$

Power factor $\cos\phi = \frac{R}{|Z|}$

$\cos\phi = \frac{R}{\sqrt{R^2+\omega^2L^2}}$ $\quad \cos\phi = \frac{R}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}$

08.07.2024

Energy stored in an inductor

$$W = \frac{1}{2}Li^2 = \frac{1}{2}i(Li) = \frac{1}{2}i\phi$$

$$W = \frac{1}{2}i\oint \bar{B}\,d\bar{s} = \frac{1}{2}i\oint \bar{A}\cdot d\bar{l}$$

$$i = \int \bar{J}\cdot ds$$

$$W = \frac{1}{2}\int \bar{J}\,d\bar{a}\oint \bar{A}\cdot d\bar{l}$$

$$\bar{\nabla}\times\bar{B} = \mu_0\bar{J}$$

$$W = \frac{1}{2}\int (\bar{J}\cdot\bar{A})\,d\tau$$

$$\bar{\nabla}\cdot(\bar{A}\times\bar{B}) = \bar{B}\cdot(\bar{\nabla}\times\bar{A}) - \bar{A}\cdot(\bar{\nabla}\times\bar{B})$$

$$W = \frac{1}{2}\int \frac{\bar{\nabla}\times\bar{B}}{\mu_0}\cdot\bar{A}\,d\tau$$

$$W = \frac{1}{2\mu_0}\int(\bar{\nabla}\times\bar{B})\cdot\bar{A}\,d\tau$$

$$W = \frac{1}{2\mu_0}\int\left[\bar{B}\cdot(\bar{\nabla}\times\bar{A}) - \bar{\nabla}\cdot(\bar{A}\times\bar{B})\right]d\tau$$

$$W = \frac{1}{2\mu_0}\left[\int\bar{B}\cdot(\bar{\nabla}\times\bar{A})\,d\tau - \int\bar{\nabla}\cdot(\bar{A}\times\bar{B})\,d\tau\right]$$

diverged

$$W = \frac{1}{2\mu_0}\left[\int\bar{B}\cdot(\bar{\nabla}\times\bar{A})\,d\tau - \cancel{\int(\vec{A}\times\bar{B})\cdot d\bar{s}}\right]$$

$$W = \frac{1}{2\mu_0}\int\vec{B}\,(\bar{\nabla}\times\bar{A})\,d\tau \Rightarrow u_B = \frac{1}{2\mu_0}\int B^2\,d\tau$$

Magnetic Energy density $u_B = \frac{B^2}{2\mu_0}$

Current $i = \frac{dq}{dt}$

$$\int J\cdot ds = \frac{\partial}{\partial t}\int \rho\,dV$$

$$\int(\bar{\nabla}\cdot\bar{J})\,dV = -\int\frac{\partial\rho}{\partial t}\,dV$$

$$\int\left(\bar{\nabla}\cdot\bar{J} + \frac{\partial\rho}{\partial t}\right)dV = 0$$

$$\bar{\nabla}\cdot\bar{J} + \frac{\partial\rho}{\partial t} = 0$$

Equation of Continuity

$$\bar{\nabla}\cdot\bar{J} + \frac{\partial\rho}{\partial t} = 0$$

From Gauss Law

$$\bar{\nabla}.\bar{E} = \rho/\epsilon_0$$

$$\frac{1}{\sigma}(\bar{\nabla}.\bar{J}) = \frac{\rho}{\epsilon_0}$$

$$\bar{\nabla}.\bar{J} = \frac{\rho\sigma}{\epsilon_0}$$

$$\bar{\nabla}.\bar{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial \rho}{\partial t} = -\frac{\rho\sigma}{\epsilon_0}$$

$$\boxed{\rho = \rho_0 e^{-\frac{\sigma}{\epsilon_0}t}}$$

For metal / conductor

σ is very high

$\rho \to 0$ with a few time Also

$\epsilon_0 = 8.85 \times 10^{-12}$

$\Downarrow$

conductor has no Volume charge density

Maxwell's Eq^n

$$\bar{\nabla}.\bar{E} = \rho/\epsilon_0$$

$$\bar{\nabla}.\bar{B} = 0$$

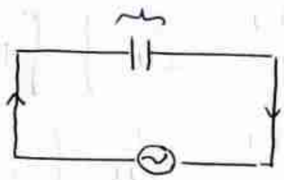
$$\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla}\times\bar{B} = \mu_0\bar{J} + \epsilon_0\frac{\partial E}{\partial t}$$

Displacement Current

Here current cannot flow but still B is present

due to current B is present



$I_{en} = 0$

So $\oint \bar{B}\cdot d\bar{l} = \mu_0 I$

$B = 0$

(Should)

According to Maxwell

$$\oint \bar{B}\cdot d\bar{l} = \mu_0(i + i_d)$$

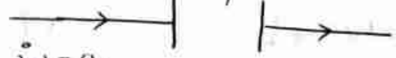
i is Conducting Current

i_d is displacement Current

$q = EA\epsilon_0$ Electric flux

$$\frac{dq}{dt} = \epsilon_0\frac{d\phi_E}{dt}$$

$$i = \epsilon_0\frac{d\phi_E}{dt} \Rightarrow \text{Displacement Current}$$



$i_d = 0$, $i_c \neq 0$

$i_c = 0$, $i_d \neq 0$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 i$$

$$\Downarrow$$

$$\oint \bar{B}\cdot d\bar{l} = \mu_0(i + i_d)$$

$$\oint(\bar{\nabla}\times\bar{B})\, d\bar{s} = \mu_0\left[\left(\bar{J} + \epsilon_0\frac{\partial \bar{E}}{\partial t}\right) d\bar{s}\right]$$

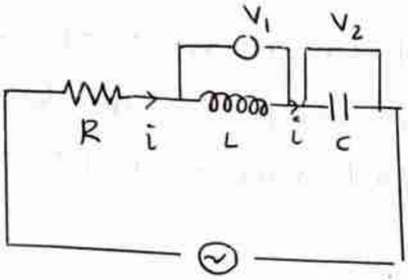
$$\bar{\nabla}\times\bar{B} = \mu_0\bar{J} + \mu_0\epsilon_0\frac{\partial \bar{E}}{\partial t}$$

$$i_d = \epsilon_0 A\frac{dE}{dt}$$

$$\frac{i_d}{A} = \epsilon_0\frac{dE}{dt}$$

$$\vec{J}_d = \epsilon_0\frac{\partial \bar{E}}{\partial t}$$

09.07.2024



$R = 100\,\Omega$, $L = \frac{2}{\pi}$ H

$C = \frac{8}{\pi}\ \mu F$ $V_0 = 200V$

and $V_1 = V_2$

$\Rightarrow$ Since Voltage and Current (series) is same so we have $X_L = X_c$ $V_1 = iX_L = i\omega L$

so $V_1 = V_2$ $V_2 = \frac{i}{\omega C}$

$\omega L = \frac{1}{\omega C}$ $\omega = \frac{1}{\sqrt{\frac{2}{\pi} \times \frac{8}{\pi} \times 10^{-6}}} = \frac{10^3}{4/\pi}$

$\omega = \frac{1}{\sqrt{LC}}$ $f = \frac{\omega}{2\pi} = \frac{10^3}{8} = 125$ Hz

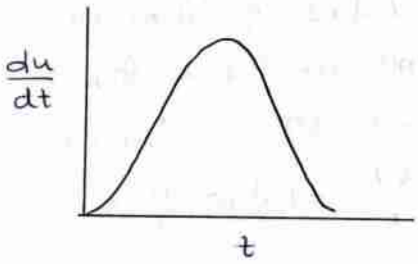
$V_1 = iX_L = \frac{V_0}{R} \times \omega L = \frac{200}{100} \times \frac{10^3\pi}{4} \times \frac{2}{\pi} = 250V$

② Rate of incroment of energy in an inductor with time in series LR Circuit getting change with battery of emf ε is

$\Rightarrow$ $i = i_0(1 - e^{-Rt/L})$ $u = \frac{1}{2}Li^2$

$\frac{di}{dt} = \frac{R}{L} i_0 e^{-Rt/L}$ $\frac{du}{dt} = Li\frac{di}{dt}$

$\frac{du}{dt} = \frac{R}{L} L i_0^2 (1 - e^{-Rt/L}) e^{-Rt/L}$



$\frac{du}{dt} = Ri_0^2 (e^{-Rt/L} - e^{-2Rt/L})$

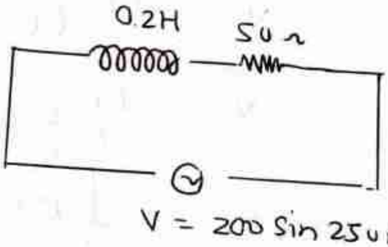
At $t \to \infty$ $\frac{du}{dt} = 0$

$t \to 0$ $\frac{du}{dt} = 0$

③ Average power delivered in the Circuit is

$\Rightarrow$ $i_0 = \frac{\varepsilon}{\sqrt{R^2 + (\omega L)^2}}$

$= \frac{200}{\sqrt{50^2 + (0.2 \times 250)^2}} = \frac{200}{70.71}$



$\cos\phi = \frac{50}{50\sqrt{2}} = \frac{1}{\sqrt{2}}$ $\langle P \rangle = \frac{1}{2} V_0 i_0 \cos\phi$

$= \frac{1}{2} \times \frac{200}{70.71} \times 200 \times \frac{1}{\sqrt{2}}$

$= 200W$

④ A Series LCR is designed to resonate at an angular frequency $\omega_0 = 10^5$ rad/sec The Circuit drawn 16W power from 120V. Source at Resonance Value of R will be

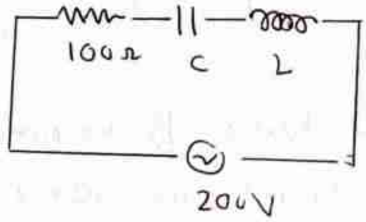
$\Rightarrow$ $V = 120\,V$ $\quad P = \frac{1}{2} V_0 i_0 \cos\phi = 16$

At resonance Cond^n $\cos\phi = 1$

$R = \frac{V_0}{i_0}$ $\quad i_0 = \frac{32}{V_0} = \frac{32}{120}$

$R = \frac{120}{32} \times 120 = \cancel{375}\,\Omega \quad 450\,\Omega$

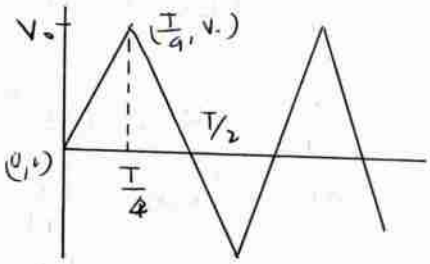
⑤ If L is removed then the current leads the Supply Voltage by 30° If only C is removed the Current lags the voltage by 60. The resonant frequency $\frac{50x}{\sqrt{3}\pi}$ Hz value of x



$\Rightarrow$ $\tan\phi = \frac{1}{\omega RC}$ $\quad \tan\phi = \frac{\omega L}{R}$

$\tan 30 = \frac{1}{\omega RC}$ $\quad \tan 60 = \frac{\omega L}{100}$ $\quad$ Data insufficent

$\frac{100}{\sqrt{3}} = \frac{1}{RC}$ $\quad \omega L = 100\sqrt{3}$

⑥ 

The Value of V_{rms} in time interval from $t=0$ to $t = T/4$ is $\frac{\sqrt{3}V}{x}$. Value of x is

$\Rightarrow$ $V - 0 = \frac{V_0 - 0}{\frac{T}{4} - 0}(t - \cancel{0})$

$V = \frac{4V_0}{T}(t - \cancel{0})$

$V = \frac{4V_0}{T} t - \cancel{V_0} \quad (0 \le t \le T/4) \Rightarrow V = \frac{4V_0}{T} t$

$$\langle V^2 \rangle = \frac{\int_0^{T/4} V^2 dt}{\int_0^{T/4} dt} = \frac{\frac{16V_0^2}{\cancel{T^2}} \times \frac{1}{3} \times \frac{\cancel{T^3}}{64}\frac{T}{4}}{\frac{T}{4}} = \frac{V_0^2 T}{12} \times \frac{4}{T} : V_0^2$$

$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{3}\,V \qquad x = 1$

⑦ The time required for a 50 HZ AC to increase from 0 to 70.7% of its peak value is

$\Rightarrow \quad V = V_0 \sin\omega t$

$\frac{V_0}{\sqrt{2}} = V_0 \sin\omega t$

$\sin\omega t = \frac{1}{\sqrt{2}}$

$\omega t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

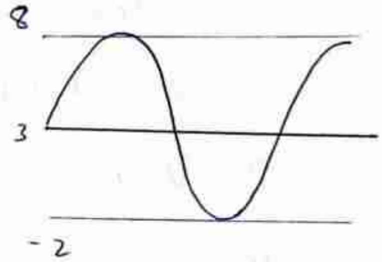
$\omega t = \frac{\pi}{4}$

$50t = \frac{1}{8}$

$t = \frac{1}{400} = 2.5\,ms$

⑧ The Power dissipated across the resistor is

$R = 1\Omega$



$\Rightarrow$ AC Power $= \frac{V_{rms}^2}{2}$

$= \frac{5^2}{2\times 1} = \frac{25}{2}$

DC Power $= \frac{3^2}{1} = 9$

Power dissipated

$= \frac{25}{2} + 9 = 21.9$

⑨ Two small metallic objects are embedded in weakly conductivity medium of conductivity σ and dielectric constant ϵ. A battery connected between them leads to a potential difference V_0. It is subsequently disconnected at $t=0$. The pot. diff at a later time t is

$\Rightarrow$ Capacitance $C = \frac{q(t)}{V(t)} \Rightarrow V(t) = \frac{q(t)}{C}$

$\Rightarrow V(t) = \frac{q(t)d}{\epsilon_0 A}$

$i = \int \bar{J}\cdot d\bar{a} = \sigma \int \bar{E}\cdot d\bar{a}$

$i = \frac{q\sigma}{\epsilon_0}$

$-\frac{dq}{dt} = \frac{q\sigma}{\epsilon_0}$

$\frac{dq}{q} = -\frac{1}{\epsilon_0}dt$

$q = q_0 e^{-t\sigma/\epsilon_0}$

$\Rightarrow V(t) = \frac{q_0 e^{-\frac{t\sigma}{\epsilon}}}{C}$

$\Rightarrow V(t) = V_0 e^{-t\sigma/\epsilon_0}$

11.07.24

Electro-Magnetic Waves

Maxwell's Equation

$$\bar{\nabla}.\bar{E} = \frac{\rho}{\epsilon_0}$$

$$\bar{\nabla}.\bar{B} = 0$$

$$\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla}\times\bar{B} = \mu_0\left[\bar{J} + \epsilon_0\frac{\partial \bar{E}}{\partial t}\right]$$

In vaccum. $\rho = 0$, $\bar{J} = 0$

$$\vec{\nabla}.\vec{E} = 0$$

$$\bar{\nabla}.\bar{B} = 0$$

$$\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla}\times\bar{B} = \mu_0\epsilon_0\frac{\partial \bar{E}}{\partial t}$$

Now $\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$$\bar{\nabla}\times(\bar{\nabla}\times\bar{E}) = -\bar{\nabla}\times\frac{\partial \bar{B}}{\partial t}$$

or $\bar{\nabla}(\bar{\nabla}.\bar{E}) - \nabla^2\bar{E} = -\frac{\partial}{\partial t}(\bar{\nabla}\times\bar{B})$

or $-\nabla^2\bar{E} = -\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial \bar{E}}{\partial t}\right)$

or $-\nabla^2\bar{E} = -\mu_0\epsilon_0\frac{\partial^2 \bar{E}}{\partial t^2}$

or $\nabla^2\bar{E} = (\mu_0\epsilon_0)\frac{\partial^2 \bar{E}}{\partial t^2} \Rightarrow \nabla^2\bar{E} = \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2}$

($c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$ is speed of light in vaccum)

Also $\bar{\nabla}\times\bar{B} = \mu_0\epsilon_0\frac{\partial \bar{E}}{\partial t}$

$$\bar{\nabla}\times(\bar{\nabla}\times\bar{B}) = \mu_0\epsilon_0\frac{\partial}{\partial t}(\bar{\nabla}\times\bar{E})$$

or $\bar{\nabla}(\bar{\nabla}.\bar{B}) - \nabla^2\bar{B} = -\mu_0\epsilon_0\frac{\partial^2 \bar{B}}{\partial t^2}$

or $\nabla^2\bar{B} = \mu_0\epsilon_0\frac{\partial^2 \bar{B}}{\partial t^2} \Rightarrow \nabla^2\bar{B} = \frac{1}{c^2}\frac{\partial^2 \bar{B}}{\partial t^2}$

wave Equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

Solution $y = y_0 e^{i(kx-\omega t)}$

$y = y_0 \sin(kx-\omega t)$ $\qquad K = \frac{2\pi}{\lambda}$

$y = y_0 \cos(kx-\omega t)$ $\qquad \omega = \frac{2\pi}{T}$

The Solution of the equations will be

$$\vec{E} = E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)} \quad \text{and} \quad \vec{B} = B_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}$$

($\vec{K}$ is propagation vector)

$$(\vec{K} = K_x\hat{i} + K_y\hat{j} + K_z\hat{k} \;;\; \vec{r} = x\hat{i} + y\hat{j} + z\hat{k})$$

$$\vec{E} = E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}$$

$$\frac{\partial}{\partial t} = -i\omega$$

$$\bar{\nabla} = iK$$

$$\frac{\partial \bar{E}}{\partial t} = (-i\omega) E_0 e^{i(\bar{K}\cdot\bar{r}-\omega t)}$$

$$\frac{\partial \bar{E}}{\partial t} = -i\omega E \Rightarrow \frac{\partial}{\partial t} \equiv -i\omega$$

$$\bar{\nabla}\cdot\bar{E} = iK E_0 e^{i(\bar{K}\cdot\bar{r}-\omega t)} = iK\vec{E} \Rightarrow \bar{\nabla} \equiv iK$$

Now $\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$$i\,\vec{K}\times\bar{E} = -(-i\omega)\bar{B} \qquad \vec{B} = \frac{\vec{K}\times\vec{E}}{\omega}$$

$$\Rightarrow \vec{B} = \frac{\vec{K}\times\vec{E}}{\omega} \qquad \vec{E} = -\left(\frac{\bar{K}\times\bar{B}}{\omega}\right)c^2$$

And $\bar{\nabla}\times\bar{B} = \mu_0\epsilon_0 \frac{\partial \bar{E}}{\partial t}$

$$i\,\frac{\bar{K}\times\bar{B}}{\mu_0\epsilon_0} = (-i\omega)\vec{E} \Rightarrow \bar{E} = -\frac{(\bar{K}\times\bar{B})}{\omega}c^2$$

$$\bar{\nabla}\cdot\bar{E} = 0$$

$$i\,\bar{K}\cdot\bar{E} = 0 \Rightarrow \bar{K}\cdot\bar{E} = 0 \Rightarrow \bar{K}\perp\vec{E}$$

Simmilarly

$$\bar{K}\cdot\bar{B} = 0$$

$$\Rightarrow \bar{K}\perp\vec{B}$$

> $\vec{E} = E_0 e^{i(2x+3y-\omega t)}$ $\qquad \bar{K}\cdot\bar{r} = 2x+3y$

$= E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}$ $\qquad \vec{K} = (2\hat{i}+3\hat{j})$

12.07.2024

Maxwell's Equation in dielectric Medium

$$\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0}$$

$$\bar{\nabla}\cdot\bar{B} = 0$$

$$\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{\nabla}\times\bar{B} = \mu_0\bar{J} + \mu_0\epsilon_0\frac{\partial \bar{E}}{\partial t}$$

(Vacum)

In medium $\epsilon = \epsilon_0\epsilon_r$, $\mu = \mu_0\mu_r$

$$\bar{\nabla}\cdot\bar{E} = \rho/\epsilon$$

$$\bar{\nabla}\times\bar{B} = \mu\bar{J} + \mu\epsilon\frac{\partial \bar{E}}{\partial t}$$

we take a dielectric so that no charge density

$$\bar{\nabla}\cdot\bar{E} = 0 \qquad \bar{\nabla}\cdot\bar{B} = 0$$

$$\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \bar{\nabla}\times\bar{B} = \epsilon\mu\frac{\partial \bar{E}}{\partial t}$$

$$\bar{\nabla}\times(\bar{\nabla}\times\bar{E}) = -\bar{\nabla}\times\frac{\partial\bar{B}}{\partial t}$$

or $$\bar{\nabla}(\bar{\nabla}.\bar{E}) - \nabla^2\bar{E} = -\frac{\partial}{\partial t}(\bar{\nabla}\times\bar{B})$$

or $$-\nabla^2\bar{E} = -\mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} \Rightarrow \nabla^2\bar{E} = \frac{1}{v^2}\frac{\partial^2\bar{E}}{\partial t^2}$$

$$\nabla^2\vec{B} = (\mu\epsilon)\frac{\partial^2\bar{B}}{\partial t^2} \qquad v^2 = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\epsilon_0\epsilon_r}}$$

$$\nabla^2\bar{E} = (\mu\epsilon)\frac{\partial^2\bar{E}}{\partial t^2} \qquad = \frac{1}{\sqrt{\mu_0\epsilon_0}}\frac{1}{\sqrt{\mu_r\epsilon_r}} = \frac{c}{\sqrt{\mu_r\epsilon_r}}$$

$v = \frac{c}{n}$ $\qquad v = \frac{c}{\sqrt{\epsilon_r}} = \frac{c}{n}$ $\qquad \mu_r \simeq 1$

(n is Refractive index) $\qquad n = \sqrt{\epsilon_r}$

Q A plane electromagnetic wave is propagating in a lossless dielectric $\bar{E} = E_0(\hat{x}+A\hat{z})\exp[ik.\{-ct+(x+\sqrt{3}z)\}]$

dielectric constants are ϵ_r and A

$\Rightarrow$ $E = (\hat{x}+A\hat{z})$ $\quad \bar{K} = (\hat{i}+\sqrt{3}\hat{k})K$. $\quad \bar{K}.\bar{E} = 0$

$1+\sqrt{3}A = 0 \Rightarrow A = -1/\sqrt{3}$

$$v = \frac{\omega}{|\bar{K}|} = \frac{cK_0}{K_0 2} = \frac{c}{2} = \frac{c}{\sqrt{\epsilon_r}} \qquad n^2 = 4 = \epsilon_r$$

$\vec{E} = E_0 e^{i(\omega t+\bar{K}.\bar{r})}$ $\qquad \bar{E} = E_0 e^{i(-\omega t+\bar{K}.\bar{r})}$

$\bar{E} = E_0 e^{i(-\omega t-\bar{K}.\bar{r})}$ $\qquad \bar{E} = E_0 e^{i(\omega t-\bar{K}.\bar{r})}$ (Propagation

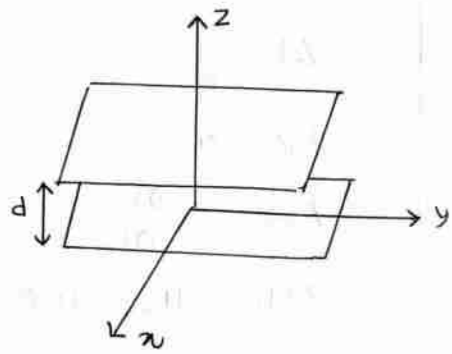
wave along $-\vec{r}$ direction $\qquad$ wave along $+\bar{r}$ directions direction)

---

1 A charged capacitor is placed in uniform field $\vec{B} = B_0\hat{x}$, $\bar{E} = E_0\hat{z}$. The electromagnetic momentum stored in space between the plates is

$\Rightarrow$ Momentum density

$$\boxed{P = \frac{1}{c^2}(\bar{E}\times\bar{H})}$$



$$P = \frac{1}{c^2}\frac{E_0B_0}{\mu_0}\hat{y}$$

$$P = E_0B_0\epsilon_0\hat{y}$$

Momentum $\quad PV = AdE_0B_0\epsilon_0\hat{y}$

Maxwell's Equation in a Conducting Medium

13.07.24

In conductive medium $\rho \neq 0$

$$\bar{\nabla}.\bar{E} = 0 \quad \text{--- (1)} \qquad \bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t} \quad \text{--- (3)}$$

$$\bar{\nabla}.\bar{B} = 0 \quad \text{--- (2)} \qquad \bar{\nabla}\times\bar{B} = \mu_{0}\sigma\bar{E} + \mu_{0}\epsilon_{0}\frac{\partial \bar{E}}{\partial t} \quad (4)$$

$$\bar{\nabla}\times(\bar{\nabla}\times\bar{E}) = -\frac{\partial}{\partial t}(\bar{\nabla}\times\bar{B})$$

$$\bar{\nabla}(\bar{\nabla}.\bar{E}) - \nabla^2 E = -\mu\sigma\frac{\partial \bar{E}}{\partial t} - \mu\epsilon_{0}\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2\vec{E} = \mu\sigma\frac{\partial \bar{E}}{\partial t} + \mu.\epsilon\frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\nabla^2\vec{E} - \mu\sigma\frac{\partial \bar{E}}{\partial t} - \mu\epsilon\frac{\partial^2 E}{\partial t^2} = 0$$

$$\nabla^2\bar{B} - \mu\sigma\frac{\partial \bar{B}}{\partial t} - \mu\epsilon\frac{\partial^2 B}{\partial t^2} = 0$$

$\mu\sigma\frac{\partial \bar{E}}{\partial t}$ is damping term

Let the Solution of the differential Eqn is

$$\vec{E} = E_0 e^{i(\vec{K}^*\vec{r} - \omega t)}$$

$$\frac{\partial \bar{E}}{\partial t} = -i\omega E_0 e^{i(\vec{K}^*\vec{r} - \omega t)}$$

$$\frac{\partial^2 \bar{E}}{\partial t^2} = -\omega^2 E_0 e^{i(\vec{K}^*\vec{r} - \omega t)}$$

And $\nabla^2\vec{E} = -\bar{K}^{*2} E_0 e^{i(\vec{K}^*\vec{r} - \omega t)}$

$$(-\bar{K}^{*2} + i\mu\sigma\omega + \mu\epsilon\omega^2)E_0 e^{i(\vec{K}^*\vec{r} - \omega t)} = 0$$

so $\boxed{K^{*2} = \mu\epsilon\omega^2 + i\omega\sigma\mu}$

<u>NOTE</u> $z = x+iy$ then $\sqrt{z+iy} = a+ib$

$$a = \sqrt{\frac{\sqrt{x^2+y^2}+x}{2}} \qquad b = \sqrt{\frac{\sqrt{x^2+y^2}-x}{2}}$$

(Square root of Complex number)

$$K^{*2} = \mu\epsilon\omega^2 + i\omega\sigma\mu$$

$$K^* = \sqrt{\frac{\sqrt{\mu^2\epsilon^2\omega^4+\omega^2\sigma^2\mu^2}}{2} + \frac{\mu\epsilon\omega^2}{2}} + i\sqrt{\frac{\sqrt{\mu^2\epsilon^2\omega^4+\omega^2\sigma^2\mu^2}}{2} + \frac{\mu\epsilon\omega^2}{2}}$$

$$K^* = \alpha + i\beta$$

$$\alpha = \left[\frac{\sqrt{\mu^2\epsilon^2\omega^4+\omega^2\sigma^2\mu^2}}{2} + \frac{\mu\epsilon\omega^2}{2}\right]^{1/2}$$

$$= \sqrt{\frac{\mu\epsilon\omega^2}{2}}\left[\left(1+\frac{\omega^2\sigma^2\mu^2}{\mu^2\epsilon^2\omega^4}\right)^{1/2}+1\right]^{1/2}$$

$$= \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\left(1+\frac{\sigma^2}{\epsilon^2\omega^2}\right)^{1/2}+1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\left(1+\frac{\sigma^2}{\epsilon^2\omega^2}\right)^{1/2}-1\right]^{1/2}$$

For a metal have high conductivity $\sigma \gg \omega\epsilon$

$$1+\left(\frac{\sigma}{\omega\epsilon}\right)^2 \approx \left(\frac{\sigma}{\omega\epsilon}\right)^2$$

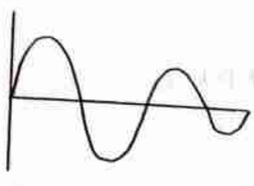
$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\sqrt{\frac{\sigma}{\omega\epsilon}} = \sqrt{\frac{\mu\omega\sigma}{2}} \quad , \quad \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$E = E_0 e^{i(\vec{K}^*\cdot\vec{r}-\omega t)}$$

$$E = E_0 e^{i\{(\alpha+i\beta)r-\omega t\}}$$

$$E = E_0 e^{-\beta r}\left[e^{(i\alpha r - i\omega t)}\right]$$

$$E = \underbrace{E_0 e^{-\beta r}}_{\substack{\Downarrow \\ \text{Amplitude decrease} \\ \text{with distance}}}\Big[\underbrace{e^{i(\alpha r-\omega t)}}_{\substack{\Downarrow \\ \text{oscillating} \\ \text{term}}}\Big]$$



At a distance

$$E = \frac{E_0}{e}$$

$$E_0 e^{-\beta r} = \frac{E_0}{e}$$

$$r = \frac{1}{\beta} = \sqrt{\frac{2}{\omega\sigma\mu}}$$

$$\boxed{\delta = \sqrt{\frac{2}{\omega\sigma\mu}}}$$

skin depth

15.07.24

$\vec{E} = E_0 \hat{i} e^{i(kz-\omega t)}$

Vaccum $\vec{B} = \frac{E_0}{c} \hat{j} e^{i(kz-\omega t)}$ $B_0 = \frac{E_0}{c}$ (Amplitudes)

$u_E = \frac{1}{2}\epsilon_0 \bar{E}^2$

$\langle u_E \rangle = \frac{1}{2}\epsilon_0 \langle \bar{E}^2 \rangle$

$\boxed{\langle u_E \rangle = \frac{1}{4}\epsilon_0 E_0^2}$

Magnetostatic Energy

$\langle u_m \rangle = \frac{1}{4}\frac{B_0^2}{\mu_0}$

$\langle u_m \rangle = \frac{1}{4} \times \frac{1}{\mu_0} \times \frac{E_0^2}{c^2} = \frac{1}{4}\epsilon_0 E_0^2$

$\boxed{\langle u_B \rangle = \langle u_E \rangle = \frac{1}{4}\epsilon_0 E_0^2}$

Total energy density $\boxed{\langle U \rangle = \frac{1}{2}\epsilon_0 E_0^2}$

$\langle U \rangle = \langle U_E \rangle + \langle U_B \rangle = \frac{1}{4}\epsilon_0 E_0^2 + \frac{1}{4}\epsilon_0 E_0^2 = \frac{1}{2}\epsilon_0 E_0^2$

Poyenting Vector $\boxed{\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B})}$

If $\bar{E} = E_0 \hat{i}\cos(\omega t - kz)$

$\vec{B} = B_0 \hat{j}\cos(\omega t - kz)$

$\vec{S} = \frac{E_0^2}{\mu_0 c}\cos^2(\omega t - kz)\hat{k}$

$\langle \vec{S} \rangle = \frac{E_0^2}{2\mu_0 c}\hat{k}$

$\langle \vec{S} \rangle = \frac{1}{2}\frac{E_0^2}{\mu_0 c} \times \frac{c}{c}\hat{k}$

$\langle \vec{S} \rangle = \frac{1}{2}\frac{E_0^2}{\mu_0} \times \mu_0\epsilon_0 c\hat{k}$

$\boxed{\langle \bar{S} \rangle = \langle U \rangle c\hat{k}}$

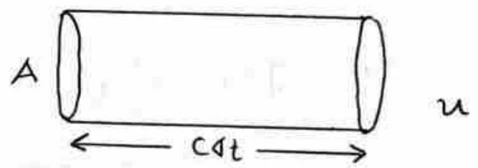
$\langle \vec{S} \rangle = \frac{1}{2}\epsilon_0 E_0^2 c\hat{k}$

(u is energy density)

1. $\vec{S}$ is parallel to Propagation vector ($\bar{k}$)
2. $\vec{S}$ Carry energy density. $\langle \vec{S} \rangle = c\langle U \rangle \hat{k}$
3.

Carried Energy

$= uAc\Delta t$



Momentum. $p = \frac{E}{c} = \frac{uA\Delta t c}{c}$

Momentum density $= \frac{p}{Ac\Delta t} = \frac{u}{c}$ $\vec{p} = \frac{u}{c}\hat{z}$

$\langle S \rangle = \frac{1}{2} c \epsilon_0 E_0^2 = I$

pressure $p = \frac{F}{A} = \frac{1}{A} \frac{\Delta p}{\Delta t} = \frac{I}{c} = \frac{\langle S \rangle}{c}$

Radiation Pressure $\boxed{p = \frac{\langle S \rangle}{c}}$

Polarization

(Only valid for Transverse wave)

$x = a \sin\omega t$

$y = a \sin(\omega t + \delta)$

or $y = a \sin\omega t \cos\delta + a \cos\omega t \sin\delta$

or $y = a \cdot \frac{x}{a} \cos\delta + a \sqrt{1 - \frac{x^2}{a^2}} \sin\delta$

or $y = x \cos\delta + \sqrt{a^2 - x^2} \sin\delta$

or $(y - x\cos\delta) = \sqrt{a^2 - x^2} \sin\delta$

or $y^2 - 2xy\cos\delta + x^2\cos^2\delta = a^2\sin^2\delta - x^2\sin^2\delta$

or $x^2 + y^2 - 2xy\cos\delta = a^2\sin^2\delta$

Case: $x = a\sin\omega t$

$y = b\sin(\omega t + \delta)$

$\left(y - \frac{b}{a} x\cos\delta\right)^2 = \left(\frac{b}{a}\sqrt{a^2 - x^2}\sin\delta\right)^2$

or $y^2 + \frac{b^2}{a^2} x^2\cos^2\delta - 2xy\frac{b}{a}\cos\delta = \frac{b^2}{a^2}(a^2 - x^2)\sin^2\delta$

or $y^2 + \frac{b^2}{a^2} x^2(\cos^2\delta + \sin^2\delta) - \frac{2xy\,b}{a}\cos\delta = b^2\sin^2\delta$

or $y^2 + \frac{b^2}{a^2} x^2 - 2xy\frac{b}{a}\cos\delta = b^2\sin^2\delta$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b}\cos\delta = \sin^2\delta}$$

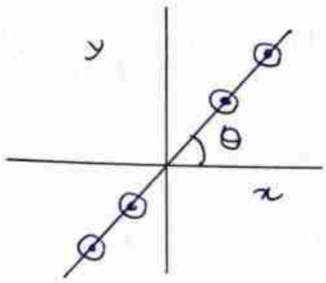
(i) $\underline{\delta = 0}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2xy\left(\frac{1}{ab}\right) = 0 \Rightarrow \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0$$

$$\Rightarrow y = \frac{b}{a}x$$

<u>Linear Polarization</u>

ωt	x	y
0	0	0
$\frac{\pi}{4}$	$a/\sqrt{2}$	$b/\sqrt{2}$
$\frac{\pi}{2}$	a	b



$\theta = \tan^{-1}(b/a)$

(ii) $\underline{\delta = \frac{\pi}{2}}$

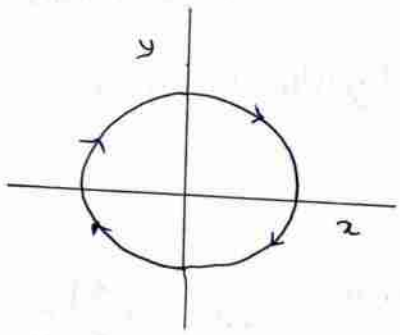
$x = a\sin\omega t$

$y = b\sin(\omega t + \frac{\pi}{2}) = b\cos\omega t$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

<u>Elliptical Polarization</u>

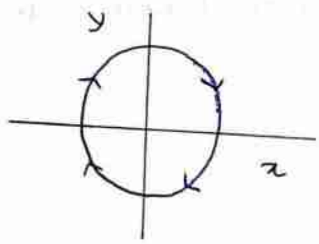
ωt	x	y
0	0	b
$\pi/4$	$a/\sqrt{2}$	$b/\sqrt{2}$
$\frac{\pi}{2}$	a	0



oppose: Right handed
Elliptical Polarization

for $\underline{a=b}$ $\quad x^2+y^2 = a^2$

Right handed
<u>Circular Polarization</u>



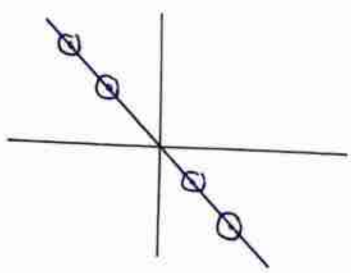
(iii) $\underline{\delta = \pi}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

$$\left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0$$

$$y = -\frac{b}{a}x$$

(Linear Polarization)



(N) $\delta = 3\pi/2$

left handed Elliptical Polarization

> Electromagnetic wave

$$\vec{E} = \hat{i} E_0 e^{i(kz-\omega t)} + \hat{j}\, 2E_0 e^{i(kz-\omega t-\pi/2)}$$

$$E_x = E_0 e^{i(kz-\omega t)}$$

$$E_y = 2E_0 e^{i(kz-\omega t-\pi/2)}$$

Phase difference $\pi/2$

Elliptical Polarization

It can't possible to get Right/left handed as it is exponential

---

(I) $\bar{E} = \dfrac{A\cos\omega t}{2\pi\epsilon_0 r}\hat{\theta}$ and $\hat{H} = \dfrac{B\cos\omega t}{\mu_0 r}\hat{\phi}$ 35/-1

Time average power radiated by the source is

=> Poynting vector $\bar{S} = \bar{E}\times\bar{H}$

$$= \frac{AB\cos^2\omega t}{2\pi\epsilon_0\mu_0 r^2}$$

and $\langle\vec{S}\rangle = \dfrac{AB}{2\pi\epsilon_0\mu_0 r^2}\times\dfrac{1}{2} = \dfrac{AB}{4\pi\epsilon_0\mu_0 r^2}$

Radiated Power $P = \int\langle\bar{S}\rangle\, da$

$$= \int\frac{AB}{4\pi\epsilon_0\mu_0 r^2} r^2 \sin\theta\, d\theta\, d\phi$$

(3.5)

$$= \frac{AB}{4\pi\epsilon_0\mu_0}\int\sin\theta\, d\theta\int d\phi$$

$$= \frac{AB}{\mu_0\epsilon_0} = ABc^2$$

---

① In a certain region of space through which EM waves is propagating the Poynting vector is given by $\vec{S} = A\hat{x}\sin^2(kx-\omega t)$. The time average power carried by the wave through square of side a on the plane $y+2x=3$ is $\frac{Aa^2}{\sqrt{n}}$. value of n is

$\Rightarrow$ Area vector

$$\phi = 2x+y-3$$

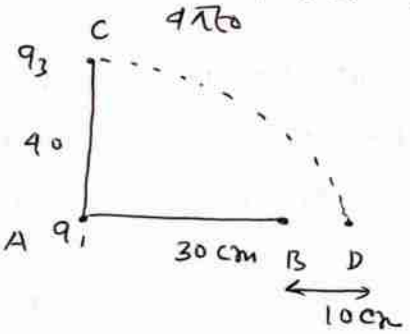
$$\bar{\nabla}\phi = 2\hat{i}+\hat{j}$$

$$\vec{A} = a^2 \frac{\bar{\nabla}\phi}{|\bar{\nabla}\phi|} = a^2 \frac{2\hat{i}+\hat{j}}{\sqrt{5}}$$

$$\langle P \rangle = \int A\hat{x}\sin^2(kx-\omega t)dt \cdot a^2\left(\frac{2\hat{i}+\hat{j}}{\sqrt{5}}\right) = \frac{A}{2}\frac{2a^2}{\sqrt{5}} = \frac{Aa^2}{\sqrt{5}}$$

Two charges q_1 and q_2 are placed 30 cm apart. A third charge q_3 is moved along the arc of a circle of radius 40cm from C to D. The change in potential energy of the system is $\frac{Kq_3}{4\pi\epsilon_0}$. value of K

$\Rightarrow$



$$PE_1 = \frac{Kq_1q_2}{30} + \frac{Kq_1q_3}{40} + \frac{Kq_2q_3}{50}$$

$$PE_2 = \frac{Kq_1q_2}{30} + \frac{Kq_1q_3}{40} + \frac{Kq_2q_3}{10}$$

$$\Delta PE = Kq_2q_3\left(\frac{1}{50} - \frac{1}{10}\right) = \frac{4}{50}(Kq_2q_3)$$

---

③ $\vec{H} = \hat{y}\, 0.5 \sin(6z * 2\times10^8 t)$. A/m dielectric constant of the medium will be

$$\Rightarrow \quad V = \frac{c}{\sqrt{\epsilon_r}} = \frac{c}{\mu}$$

$$\omega = 2\times10^8$$

$$K = 6$$

$$V = \frac{\omega}{K} = \frac{2\times10^8}{6}$$

$$V = \frac{1}{3}\times10^8$$

$$\frac{1}{3}\times10^8 = \frac{3\times10^8}{\sqrt{\epsilon_r}} \Rightarrow \sqrt{\epsilon_r} = 9$$

$$\Rightarrow \epsilon_r = 81$$

The magnitude field in an empty space is described by $B = B_0 \exp(ax) \sin(Ky - \omega t)\,\hat{z}$. y-component of electric field will be

=) $\bar{E} = -\dfrac{(\vec{K}\times\vec{B})}{\omega} c^2$ $\vec{K} = \hat{j}$, $\vec{B} = \hat{k}$

$= -\dfrac{(\hat{j}\times\hat{k})}{\omega} c^2\, K B_0 \exp(ax) \sin(Ky - \omega t)$

$= -\dfrac{B_0}{c} e^{ax} \sin(Ky - \omega t)\,\hat{i}$ $E_y = 0$

---

(13) An oscillating voltage $V(t) = V_0 \cos\omega t$ is applied across a parallel plate capacitor have speration d The displacement current density is

=) $E = \dfrac{V(t)}{d} = \dfrac{V_0 \cos\omega t}{d}$ $J_d = \epsilon_0 \dfrac{\partial E}{\partial t}$

$\dfrac{\partial E}{\partial t} = -\dfrac{\omega V_0 \sin\omega t}{d}$ $= -\dfrac{\omega V_0 \epsilon_0 \sin\omega t}{d}$

---

(14) The Power radiated by Sun is 3.6×10^{26} W and radius 7.5×10^5 Km. Magnitude of Poynting vector

=) Magnitude of Poynting vector

$S = I = \dfrac{P}{A} = \dfrac{P}{4\pi R^2} = \dfrac{3.6\times10^{26}}{4\pi\times(7.5\times10^8)^2}$

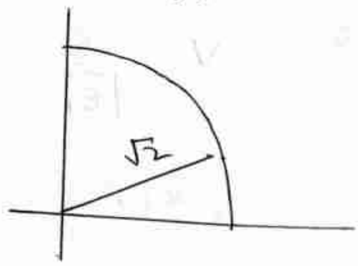
$= 5.1\times10^7$ W/m²

---

(15) 1W Point Source at origin emits light uniformly in all directions. If the units for both the axes are measured in cm. The Poynting vector at (1,1,0)

=> $S = \dfrac{P}{A} = \dfrac{1}{4\pi r^2} = \dfrac{1}{4\pi(\sqrt{2})^2} = \dfrac{1}{8\pi}$

$\bar{S} = \dfrac{1}{8\pi}\dfrac{\hat{i}+\hat{j}}{\sqrt{2}} = \dfrac{1}{8\pi\sqrt{2}}(\hat{e}_x + \hat{e}_y)$



① A time dependent magnetic field $\vec{B}(t)$ is produced in a circular region of space, infinitely long and of radius R. The magnetic field is $\vec{B} = B_0 t\,\hat{z}$ and is zero for $r > R$. The field at $r = 2R$ is

$\Rightarrow$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\int (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

$$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

$$E \cdot 2\pi r = -B_0 \pi R^2$$

$$E = -\frac{B_0 R^2}{2r}$$

$$E = -\frac{B_0 R^2}{2 \cdot 2R}$$

$$\vec{E} = -\frac{B_0 R}{4}\hat{\theta}$$

---

① Consider an electrostatic field E in a region of space

$\Rightarrow$ The Potential difference between any two points in the regions is always zero. $\Rightarrow$ Incorrect.

---

⑬ The current through a series RL circuit subjected to emf ε. obeys $L\frac{di}{dt} + iR = \varepsilon$ Let $L = 1\,mH$, $R = 1\,K\Omega$ and $\varepsilon = 1V$. The initial condition $i(0) = 0$. At $t = 1\,\mu s$. the current

$\Rightarrow$

$$L\frac{di}{dt} + R = \varepsilon$$

$$L\frac{di}{dt} = \varepsilon - iR$$

$$\frac{di}{\varepsilon - iR} = \frac{1}{L}dt$$

$$\frac{1}{R}\ln(\varepsilon - iR) = -\frac{t}{L} + c$$

$$i(0) = 0$$

$$\frac{1}{R}\ln\varepsilon = c$$

$$\frac{1}{R}\ln(\varepsilon - iR) = \frac{1}{R}\ln\varepsilon - \frac{t}{L}$$

$$\ln(\varepsilon - iR) = \ln\varepsilon - \frac{Rt}{L}$$

$$e^{\ln\left(\frac{\varepsilon - iR}{\varepsilon}\right)} = e^{-Rt/L}$$

$$i = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$$

$$i = \frac{1}{1}\left(1 - e^{-\frac{10^3 \times 10^{-6}}{10^{-3}}}\right)$$

$$i = (1 - e^{-1}) = \left(1 - \frac{1}{e}\right)$$

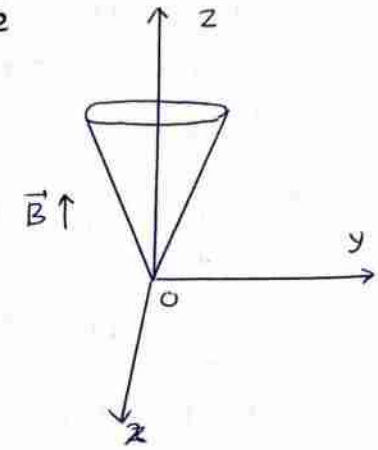
㉛ The electric field associated with an electromagnetic radiation is given by $E = a(1 + \cos\omega_1 t)\cos\omega_2 t$. which frequencies are present here

$\Rightarrow$ $E = a(\cos\omega_2 t + \cos\omega_2 t \cos\omega_1 t)$

$$= a\cos\omega_2 t + \frac{a}{2}\cos(\omega_1 + \omega_2)t + \frac{a}{2}\cos(\omega_1 - \omega_2)t$$

$\omega_1 + \omega_2$; $|\omega_1 - \omega_2|$; ω_2 are present

A magnetic field $\vec{B} = B_0 \hat{z}$ exist everywhere. The magnetic flux through the base (ϕ_b) and that through the curve surface of the cone (ϕ_c) are



$\Rightarrow$ Flux through the base

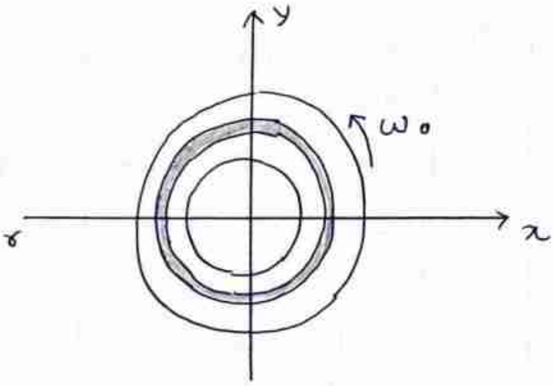
$$\phi_b = \vec{B} \cdot \vec{A}$$

$$= B_0 \hat{z} \cdot \pi R^2 \hat{z} = B_0 \pi R^2$$

Since $\phi_{net} = \phi_b + \phi_c = 0$

$$\phi_c = -\phi_b = -B_0 \pi R^2$$

) If the sheets carries a uniform surface-charge density σ and spins about the origin O with a constant angular velocity $\vec{\omega} = \omega_0 \hat{z}$. then the current flow on the sheet is



$\Rightarrow$ Current. $i = \int J ds$

$$i = \int (\sigma v) ds = \int \sigma \cdot \omega r \, 2\pi r dr$$

$$= 2\pi \omega_0 \sigma \int r^2 dr$$

$$= \frac{2}{3} \pi \omega_0 \sigma (R_2^3 - R_1^3)$$

---

i) $\vec{B}(\rho, \phi, z, t) = K \rho^3 t^3 \hat{z}$. The magnitude of induced emf

$$\Rightarrow \quad \phi = \int \vec{B} \cdot d\vec{s} = \int K \rho^3 t^3 \cdot \rho d\rho d\phi = \int K t^3 \rho^4 d\rho \int d\phi$$

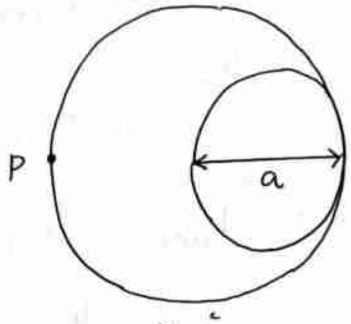
$$= 2\pi t^3 K \int \rho^4 d\rho$$

$$\phi_B = 2\pi t^3 K \int_0^R \rho^4 d\rho = \frac{2}{5} \pi t^3 K R^5$$

induced emf. $e = \frac{d\phi_B}{dt} = \frac{6}{5} \pi t^2 K R^5$

---

① A Cylindrical Cavity of diameter a exists inside a Cylinder of diameter 2a Both the Cylinder and the Cavity are infinitely long. A uniform current density J flows along the length. Magnitude of magnetic field at P is given by



$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 i_{en}$

$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{s}$

$\Rightarrow B\cdot 2\pi a = \mu_0 \frac{i}{\pi a^2}\cdot \pi a^2$

$\Rightarrow B_1 = \frac{\mu_0 i}{2\pi a}$

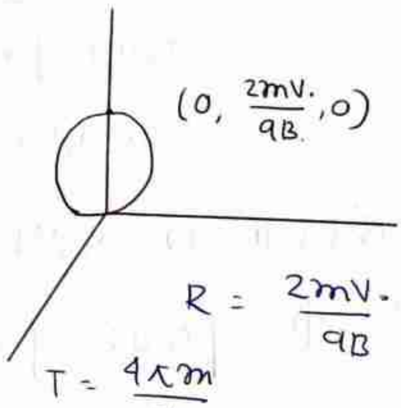
$B_2 = -\frac{\mu_0 i}{2\pi\left(\frac{3a}{2}\right)}$

$= -\frac{\mu_0 i}{3\pi a}$

$B = \frac{\mu_0 i}{\pi a}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{\mu_0 i}{6\pi a}$

---

② A uniform magnetic field $\vec{B} = B_0\hat{k}$ exist in space Now a particle of charge q and mass m thrown from the point $(0, \frac{2mV_0}{qB_0}, 0)$ with initial velocity $V_0\hat{i}$, The min time it will take to reach origin is



$\Rightarrow \vec{B} = B_0\hat{k}$ $\quad \vec{F} = q(\vec{v}\times\vec{B}) =$

$\vec{v} = v_0\hat{i}$ $\quad T = \frac{2\pi R}{V}$

time taken $\frac{T}{2} = \frac{\pi R}{V}$

$R = \frac{2mV_0}{qB}$

$T = \frac{2\pi}{V}\times\frac{2mV}{qB}$

$T = \frac{4\pi m}{qB}$

---

③ Two events separated by a spatial distance 9×10^9 m, are simultaneous in one inertial frame. The time interval between these two events in a frame moving with a Constant speed 0.8c is

JRF-2012

$\Rightarrow t' = \gamma\left(t - \frac{vx}{c^2}\right)$

$\Delta t' = -\gamma \frac{v}{c^2}\Delta x$

$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-0.64}} = \frac{1}{0.6} = \frac{5}{3}$

$= \frac{5}{3}\times\frac{0.8c}{c^2}\times 9\times10^9 = \frac{5}{3}\times 0.8\times 3\times 10 = 40\text{sec}$

26 Infinite charges of magnitude q and −q are placed along x-axis alternatively at $x=1, x=2, x=4, x=8 \ldots \infty$ The Electric field at the origin will be

=)

$$F = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{1^2} - \frac{q}{2^2} + \frac{q}{4^2} - \frac{q}{8^2} + \cdots\right]$$

$$F = \frac{q}{4\pi\epsilon_0}\left[1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \cdots\right]$$

$r = -1/4$

$$F = Kq \frac{a}{1-r} \qquad a = 1$$

$$F = \frac{q}{4\pi\epsilon_0} \frac{1}{1+1/4} = \frac{4}{5}\frac{q}{4\pi\epsilon_0}$$

---

27 charge density $\rho(r) = \frac{A}{r^2}e^{-2r/a}$, then find the radius R

=) $$Q = \int \rho \, dV = \int^R \frac{A}{r^2} e^{-2r/a} \, 4\pi r^2 dr$$

$$= 4\pi A \int_0^R e^{-2r/a} dr = \frac{4\pi A a}{2}\left[e^{-2r/a}\right]_R^0$$

or $$Q = 2\pi A a \left(1 - e^{-2R/a}\right)$$

$$\frac{Q}{2\pi A a} = 1 - e^{-2R/a}$$

or $$e^{-2R/a} = 1 - \frac{Q}{2\pi A a}$$

$$-\frac{2R}{a} = \log\left(1 - \frac{Q}{2\pi A a}\right)$$

$$R = \frac{a}{2}\log\left(\frac{1}{1 - Q/2\pi A a}\right)$$

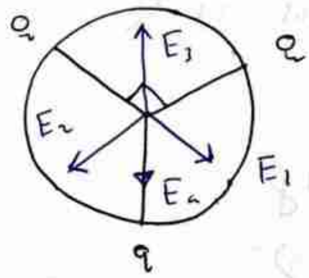
28 In a certain region $\vec{E} = Kr\theta\,\hat{r} + mr\,\hat{\theta}$.

$\Rightarrow$

$$\vec{\nabla}\times\vec{E} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ Kr\theta & mr^2 & 0 \end{vmatrix} = 0$$

$$\frac{1}{r}(2mr - Kr) = 0 \Rightarrow 2m - K = 0$$

---

29 

E at center is 0. then $|Q| = ?$

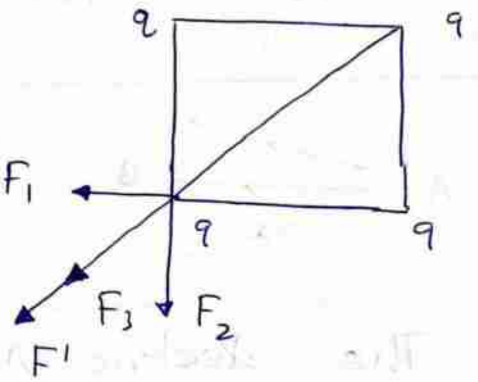
$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$$

$$E_4 = \sqrt{2}E_1 = \frac{\sqrt{2}}{4\pi\epsilon_0}\frac{Q}{R^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$$

$$Q = q/\sqrt{2}$$

30/27 Four identical point charges each of magnitude are placed at the Corners of a Square of Side a. The force on any of the charge due to other three is

$\Rightarrow$



$$F_1 = F_2 = K\frac{q^2}{a^2}$$

$$F' = \sqrt{2}F_1 = \sqrt{2}K\frac{q^2}{a^2}$$

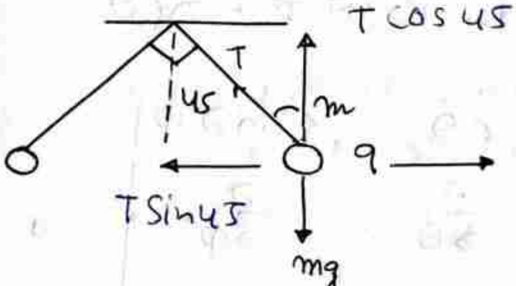
$$F_3 = K\frac{q^2}{(\sqrt{2}a)^2}$$

$$F_3 = \frac{1}{2}K\frac{q^2}{a^2} \qquad F_{net} = \frac{Kq^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$$

31/53 SI unit of flux is Volt m

32/50 $\vec{E} = 10\hat{j} + 7\hat{i}$ V/m flux through $1m^2$ are in yz plan

$$= \quad \phi = \bar{E}\cdot\vec{S} = (10\hat{j} + 7\hat{i})\cdot\hat{i} = 7\,\text{Vm}$$



The value of q is

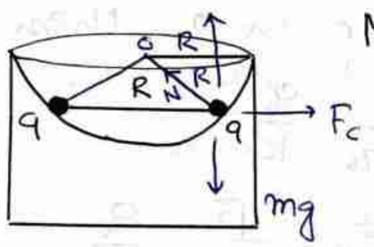
$T\cos 45 = mg$

$T = \sqrt{2}\,mg$

$T\sin 45 = \frac{1}{4\pi\epsilon_0}\frac{q^2}{(\sqrt{2}l)^2}$

$\frac{\sqrt{2}}{\sqrt{2}}mg = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2l^2}$

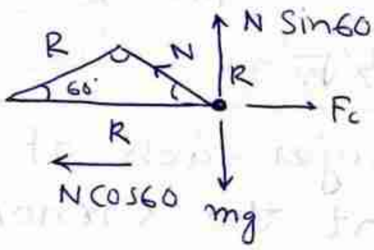
$q^2 = l\sqrt{8\pi\epsilon_0 mg}$



Mass is m, radius is R then charge

$N\sin 60 = mg$

$N \times \frac{\sqrt{3}}{2} = mg$



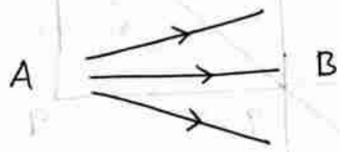
$N = 2mg/\sqrt{3}$

$N\cos 60 = K\frac{q^2}{R^2}$

$\frac{2}{\sqrt{3}}mg\,\frac{1}{2} = K\frac{q^2}{R^2}$

$\frac{mg}{\sqrt{3}} = K\frac{q^2}{R^2}$

$q = \left(\frac{mg\,R^2}{\sqrt{3}\,K}\right)^{1/2}$



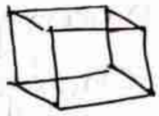
Electric field at A is more stronger than B

The electric field in a sphere of space is spherically symmetric and directly proportional to r^2 then ρ is

=) If $\rho \propto r^n$ =) $\rho \propto r$ directly proportional to 1

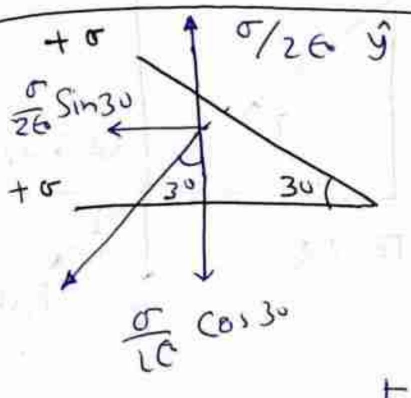
$E \propto r^{n+1}$ $n=1$

$\vec{E} = (2x+4)\,i + 8j + 3\hat{k}$ N/C. The net charge enclosed by it is $(a=3)$



$\Rightarrow \quad \Phi = \int_s \vec{E}\cdot d\vec{s} = \int (\vec{\nabla}\cdot\vec{E})\, dV$

$= 2\int dV = 2\times 27 = 54$



The electric field between them is

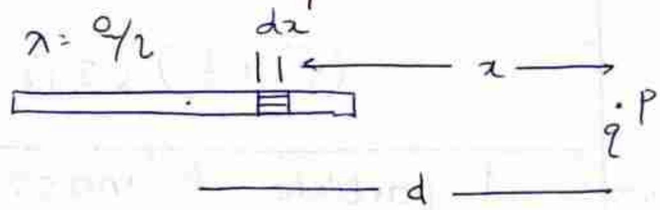
$$-\frac{\sigma}{2\epsilon_0}\frac{\hat{x}}{2} + \left(\frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0}\frac{\sqrt{3}}{2}\right)\hat{y}$$

$$E = \frac{\sigma}{2\epsilon_0}\left[\left(1+\frac{\sqrt{3}}{2}\right)\hat{y} - \frac{\hat{x}}{2}\right]$$

---

29 The force between a uniformly charged rod of length l with charge Q and a point charge q placed at a distance d from the center of rod at axial position

$\Rightarrow$



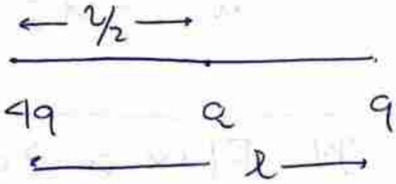
charge on the segment $\lambda\, dx$

$$dF = \frac{1}{4\pi\epsilon_0}\frac{\lambda q\, dx}{x^2} \Rightarrow F = \frac{\lambda q}{4\pi\epsilon_0}\left[-\frac{1}{x}\right]_{d-l/2}^{d+l/2}$$

$$F = \frac{qQ}{4\pi\epsilon_0 (d^2 - l^2/4)}$$

---

31. Three charges $4q$, Q and q are placed in a st line of length l at points distances $0, l/2, l$. the net force on q is zero



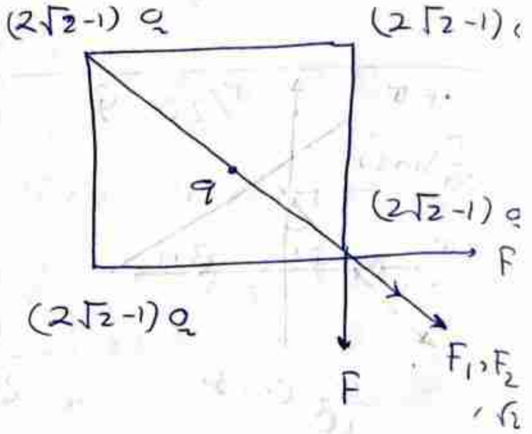
$\Rightarrow$

$$\frac{\cancel{4}Qq}{(l/2)^2} + \frac{4q^2}{l^2} = 0$$

$$\frac{4q}{l^2}\left(\frac{Q}{\cancel{4}/4} + q\right) = 0 \Rightarrow Q = -q$$

34 Four particles with charges $(2\sqrt{2}-1)Q$ are arranged at corners of a square. Another charge q is placed at the centre of the square. Resultant force on each is zero, if

$\Rightarrow$



$$F = K\frac{(2\sqrt{2}-1)^2Q^2}{a^2}$$

$$F_1 = K\frac{(2\sqrt{2}-1)^2Q^2}{(\sqrt{2}a)^2}$$

$$F_2 = K\frac{(2\sqrt{2}-1)Qq}{(a/\sqrt{2})^2}$$

$$F_{net} = K\left[\frac{(2\sqrt{2}-1)^2Q^2}{a^2}\sqrt{2} + \frac{(2\sqrt{2}-1)^2Q^2}{2a^2} + \frac{(2\sqrt{2}-1)Qq}{\frac{1}{2}a^2}\right]$$

$$\frac{(2\sqrt{2}-1)Q}{a^2}\left[\sqrt{2}\frac{(2\sqrt{2}-1)Q}{1} + \frac{(2\sqrt{2}-1)Q}{2} + 2q\right] = 0$$

$$\frac{3}{2}(2\sqrt{2}-1)Q = -2q$$

$$\left(\sqrt{2}+\frac{1}{2}\right)(2\sqrt{2}-1)Q = -2q \Rightarrow q = -\frac{7}{4}Q$$

---

36 A particle of mass 2 kg and charge 1 mC is projected vertically with a velocity 10 m/s. If uniform horizontal field is 10^4 N/C

$$a_x = \frac{qE}{m} = \frac{10^{-3}\times 10^4}{2} = 5\,m/s$$

$$S_x = u_xT + \frac{1}{2}a_xT^2$$

$$S_y = u_yt + \frac{1}{2}a_yt^2$$

$$0 = 10T - \frac{1}{2}10T^2 \Rightarrow T = 2s$$

$$S_x = \frac{1}{2}\times 5\times 2^2 = 10\,m$$

---

56 If flux is zero $\phi = \oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\epsilon_0}$

$E = 0$, $q_{in} = 0$

If a charge q is given to a neutral pendulum then time period be came

$\downarrow g$

$\Rightarrow$

$$g_{eff} = g - qE/m \qquad \uparrow\uparrow\uparrow \ \frac{qE}{m}$$

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad T' = 2\pi\sqrt{\frac{l}{g - qE/m}}$$

So time period increase

A semicircular ring of radius 0.5m is uniformally charged with a total charge of 1.4×10^{-9} c. Field at the center

$\Rightarrow$

$$E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}\sin\phi/2 \qquad \phi = \pi$$

$$\lambda = \frac{1.4\times10^{-9}}{\pi\times0.5}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{2\lambda}{r}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{2\times1.4\times10^{-9}}{\pi\times(0.5)^2} = 32 \text{ V/m}$$

A long string with charge density λ passes through an imaginary cube of length l. The maximum flux of field through the Cube is

$\Rightarrow$

$$\phi = \frac{q}{\epsilon_0} = \frac{\sqrt{3}l\lambda}{\epsilon_0}$$

Electric flux entering and leaving is ϕ_2, ϕ_1. The magnitude of charge enclosed is

$\Rightarrow$

Enclosed charge $q_{en} = \epsilon_0 \Delta\phi$

$$= \epsilon_0(\phi_2 - \phi_1)$$

---

52 If the radius of Gaussian Surface is doubled

$\phi = q/\epsilon$ ϕ remains Same

58. The electric field inside a sphere which carries a volume charge density $\rho = ar$. u

$\Rightarrow \quad E_{in} = \frac{a}{\epsilon_0} \cdot \frac{r^{n+1}}{(n+3)} = \frac{ar^2}{3\epsilon_0}$

$\rho = ar \Rightarrow n = 1$

---

61. A charge particle is free to move in a electric field. it will travel always along a line of force

---

62. Electric field in a region $\vec{E} = ax\hat{i}$. the flux bounded $l < x < 2l$; $0 < y < l$; $0 < z < l$

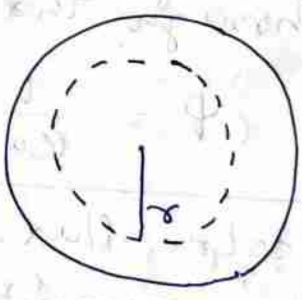
$\Rightarrow \quad \Phi = \iint_S \vec{E} \cdot d\vec{S} = \int_V (\vec{\nabla} \cdot \vec{E})\, dV$

$= a \int_l^{2l} \int_0^l \int_0^l dx\,dy\,dz = al^3$

---

63. Volume charge density $\rho = \rho_0 e^{-ar^3}$ find magnitude of electric field

$\Rightarrow \quad q_{en} = \int_0^r \rho\,(4\pi r^2 dr)$



$= 4\pi \rho_0 \left[\frac{-e^{-\alpha r^3}}{3a} \right]_0^r$

$= 4\pi \epsilon_0 \frac{1 - e^{-\alpha r^3}}{3\alpha}$

$\Phi = \oint \vec{E} \cdot ds = E\, 4\pi r^2 = \frac{4\pi}{3\epsilon_0} (1 - e^{-\alpha r^3})$

$E = \frac{\rho_0}{3\alpha \epsilon_0 r^2} (1 - e^{-\alpha r^3})$

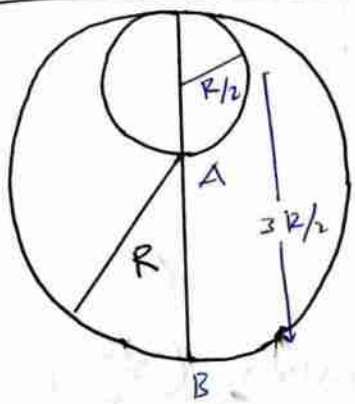
$E = E_0(4x\hat{i} - by\hat{j} + 5z\hat{k})$ represents a electrostatic field in a charge free region. $b = $ —

Since electrostatic field is irrotational

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 4x & -by & 5z \end{vmatrix} = 0$$

for any value

a $\hat{i}(0) + \hat{j}(0) + \hat{k}(0) = 0$



find $\left|\frac{\vec{E}_A}{E_B}\right|$

$$|E_A| = \frac{\rho R/2}{3\epsilon_0} = \frac{\rho R}{6\epsilon_0} \quad —①$$

(Small)

(For Big Sphere $E_A = 0$ as $r = 0$)

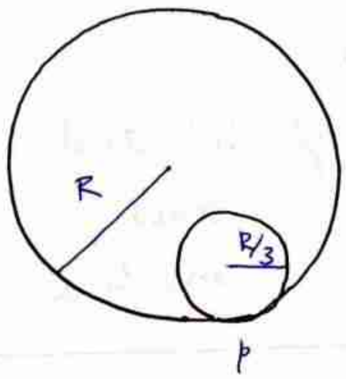
$$|\vec{E}_B| = \frac{\rho R}{3\epsilon_0} - \frac{\rho R}{54\epsilon_0} \qquad Q = -\rho\times\frac{4}{3}\pi\times\frac{R^3}{8}$$

$$= -\frac{\rho}{6}\pi R^3$$

$$\frac{Q}{4\pi\epsilon_0\left(\frac{3R}{2}\right)^2} = \frac{-\rho R}{54\epsilon_0} \qquad \left|\frac{\vec{E}_A}{E_B}\right| = \frac{\rho R/6\epsilon_0}{\frac{17\rho R}{54\epsilon_0}}$$

$$\frac{17\rho R}{54\epsilon_0}$$

Sphere having charge density ρ $E_P = ?$



$$\frac{\rho R}{3\epsilon_0} - \frac{\rho R}{9\epsilon_0}$$

$$= \frac{2\rho R}{9\epsilon_0}$$

The charge contained within a sphere of radius R is $\vec{E} = a\,(1-e^{-r/R})\,\frac{\hat{r}}{r^2}$

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{r^2}\,a\,(1-e^{-r/R})\right)$$

$$= \frac{a}{r^2}\frac{\partial}{\partial r}\left(1-e^{-r/R}\right)$$

$$= \left(\frac{a}{Rr^2}e^{-r/R}\right)\epsilon_0$$

Charge $Q = \int \rho\, dV = \frac{\alpha\epsilon_0}{R}\int_0^R \frac{1}{r^2}e^{-r/R}\,4\pi r^2\,dr$

$$= \frac{4\pi\epsilon_0\alpha}{R}\left[-Re^{-r/R}\right]_0^R$$

$$= -4\pi\epsilon_0\alpha\left[e^{-r/R}\right]_0^R$$

$$= -4\pi\epsilon_0 a\left[e^{-1}-1\right] = 4\pi\epsilon_0 a\left(\frac{e-1}{e}\right)$$

$V(r,\theta,\phi) = f(r)\cos\theta$ in a charge free region $f(r)$ has the form

$\Rightarrow$ $\nabla^2\phi = 0$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) = 0$$

$$\frac{\cos\theta}{r^2}\frac{\partial}{\partial r}\left(r^2 f'(r)\right) + \frac{f(r)}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(-\sin^2\theta\right) = 0$$

$$\cancel{\cos\theta}\frac{\partial}{\partial r}\left(r^2 f'(r)\right) = \frac{f(r)}{\cancel{\sin\theta}}\,2\cancel{\sin\theta}\cancel{\cos\theta}$$

$$2r f'(r) + r^2 f''(r) = f(r)$$

$$r^2\frac{\partial^2 f}{\partial r^2} +$$

using hit and trail method

Consider a parallel plate Capacitor of $\varepsilon(x) = \dfrac{\varepsilon_0}{(1-\frac{x^2}{3d^2})}$ Seperation is d then the Capacitance is

=)

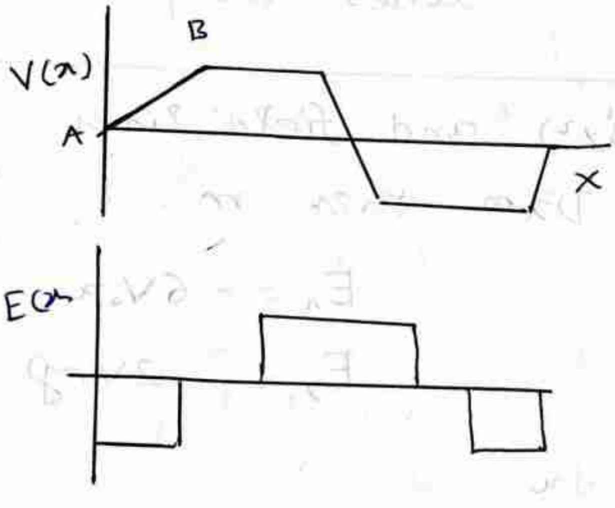
$$E = \frac{\sigma}{\epsilon_0} = \frac{\sigma}{\varepsilon_0}\left(1-x^2/3d^2\right)$$

$$V = \int_0^d E\,dx = \frac{\sigma}{\epsilon_0}\int_0^d \left(1-\frac{x^2}{3d^2}\right)dx$$

$$V = \frac{\sigma}{\epsilon}\left(x-\frac{x^3}{9d^2}\right)_0^d$$

$$V = \frac{\sigma}{\epsilon_0}\left(d-\frac{d^3}{9d^2}\right) = \frac{8}{9}\frac{\sigma d}{\epsilon_0} = \frac{8}{9}\frac{Qd}{A\epsilon_0}$$

$$C = \frac{Q}{V} = \frac{Q}{\frac{8}{9}\frac{Qd}{A\epsilon_0}} = \frac{9A\epsilon_0}{8d}$$



AB region

$E = -\dfrac{dV}{dx}$

V is positive Slope

E is neg Constant

---

88 The Spherical portion has been remove from a Solid Sphere having a charge distributed uniformly in its volume Field inside the emptied Space is

=)

Non zero and uniform

The potential due to an charge distribution $V(r) = \frac{q}{4\pi\epsilon_0} \frac{e^{-ar}}{r}$. The net charge within a sphere centred at the origin and of radius $1/\alpha$ is

$$\Rightarrow \quad E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{q}{4\pi\epsilon_0}\frac{e^{-ar}}{r}\right)$$

$$E = -\frac{q}{4\pi\epsilon_0}\left(\frac{-\alpha r e^{-\alpha r} - e^{-\alpha r}}{r^2}\right)$$

$$E = -\frac{q}{4\pi\epsilon_0} e^{-\alpha r}\left(\frac{\alpha r + 1}{r^2}\right)$$

$$E = \frac{q}{4\pi\epsilon_0} e^{-\alpha \times \frac{1}{\alpha}} \frac{(\alpha \times \frac{1}{\alpha} + 1)}{1/\alpha^2} = \frac{(q/e)}{4\pi\epsilon_0} 2\alpha^2$$

$$\phi = \int E\,dA = \frac{q/e}{4\pi\epsilon_0} 2\alpha^2 \, 4\pi \left(\frac{1}{\alpha}\right)^2 = \frac{2q}{\epsilon_0 e}$$

I. Three concentric spherical metallic shells A, B, C have $\sigma, -\sigma, \sigma$. The potential of the middle sphere is

$$\Rightarrow \quad V_B = \frac{1}{4\pi\epsilon_0}\left[\frac{(+\sigma)4\pi a^2}{b} + \frac{(-\sigma)4\pi b^2}{b} + \frac{(+\sigma)4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

---

(2) If the electric potential is given by $\phi = cxy$. The field is given by

$$\Rightarrow \quad \vec{E} = -\vec{\nabla}\phi = -\hat{i}\frac{\partial}{\partial x}(cxy) - \hat{j}\frac{\partial}{\partial y}(cxy)$$

$$= c(-y\hat{i} - x\hat{j})$$

$$= -c(y\hat{i} + x\hat{j})$$

---

111. Find the charge density and the total charge of the System which gives rise to the electric field $\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{e^{-ar}}{r^2} \hat{r}$.

$\Rightarrow$ $$\vec{\nabla}.\vec{E} = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{q}{4\pi\epsilon_0} \frac{e^{-ar}}{r^2}\right)$$

$$\vec{\nabla}.\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} \frac{d}{dr}(e^{-ar})$$

$$\vec{\nabla}.\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{1}{r^2} e^{-ar}(-a) = \frac{\rho}{\epsilon_0}$$

charge density, $\rho = -\frac{qa}{4\pi\epsilon_0 r^2}$

---

114. A charge Q is distributed over two concentr hollow Spheres of radii r and $R > r$ Such that the Surface densities are equal. The potential at the Common center is

$\Rightarrow$ $Q = Q_1 + Q_2$

$$\frac{Q_1}{Q_2} = \frac{r^2}{R^2} \Rightarrow \frac{Q_1+Q_2}{Q_2} = \frac{r^2+R^2}{R^2}$$

$$\frac{Q_2}{Q} = \frac{R^2}{r^2+R^2} \Rightarrow Q_2 = \frac{QR^2}{R^2+r^2}$$

$$Q_1 = \frac{Qr^2}{R^2+r^2}$$

$$V = \frac{Q_1}{r} + \frac{Q_2}{R}$$

$$V = \frac{Q(r+R)}{r^2+R^2}$$

---

118. A and B are two Concentric Spherical shells. If a is given a charge $+q$ while B is earthed

$q_B = 0$, $\quad q_B = -q_A$.

A solid Sphere of radius R is charged uniformly. At what distance from its Surface is the electrostatic potential half of the potential at centre!

$$V_c = \frac{3}{2}\frac{KQ}{R}$$

$$V = \frac{KQ}{x}$$

$\Rightarrow$ $$\frac{KQ}{x} = \frac{1}{2}\left(\frac{3}{2}\frac{KQ}{R}\right)$$

$$x = 4R/3$$

distance from Surface is $x - R = R/3$

Time Constant of an RC Circuit is $\frac{2}{\ln 2}$ sec. Capacitor is discharged at $t=0$. The ratio of charge on capacitor at time $t = 2$ sec and $t = 6$ is

$\Rightarrow$ $$\frac{q_2}{q_6} = \frac{e^{-2/2 \ln 2}}{e^{-6/2 \ln 2}} = \frac{e^{-\ln 2}}{e^{-3\ln 2}} = \frac{2^3}{2} = \frac{4}{1}$$

---

2) A parallel plate Capacitor of capacitance $2\mu F$ is connected with a battery of 20V. Now the the plates of the Capacitor is drawn slowly to double the distance then work done

$$C_1 = \frac{\epsilon_0 A}{d}$$

$$C_2 = \frac{\epsilon_0 A}{2d} = \frac{C_1}{2}$$

$\Rightarrow$ $$W = \frac{1}{2}C_1V^2 - \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}V^2(C_1 - C_2)$$

$$= \frac{1}{2}V^2\left(C_1 - \frac{C_1}{2}\right) = \frac{1}{4}C_1V^2$$

$$= \frac{1}{4} \times 2 \times (20)^2 = 200\ \mu J$$

3P If a Gaussian surface encloses no charge. then both E and V both zero

---

124 The dielectric of thickness $d/2$ and electrical constant K is placed between the plates of a parallel plate capacitor. The energy

$\Rightarrow$

$$C = \frac{\epsilon_0 A}{d - t(1-\frac{1}{K})} = \frac{\epsilon_0 A}{d - \frac{d}{2}(1-\frac{1}{K})}$$

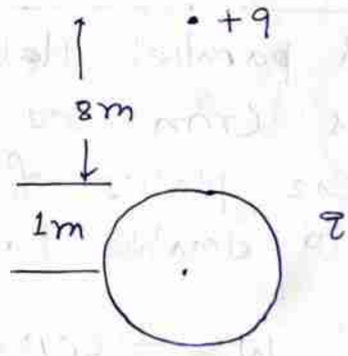
$$C = \frac{\epsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{\epsilon_0 A}{\frac{d}{2}(1+\frac{1}{K})} = \frac{2K\epsilon_0 A}{d(1+K)}$$

$$\text{Energy } E = \frac{1}{2}\frac{2K\epsilon_0 A}{d(1+K)}V^2 = \frac{K\epsilon_0 A V^2}{d(1+K)}$$

---

125 A very small sphere of mass 80g (charge q) held at a height 9m above centre of a sphere having radius 1m (q). When relased it falls untill is repelled before it comes in contact with spher. q = —

: By energy conservation



$$\frac{1}{4\pi\epsilon_0}\frac{q^2}{9} + mg \times 9$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{1} + mg \times 1$$

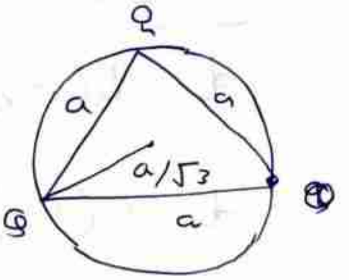
$$\cancel{8}mg = \frac{1}{4\pi\epsilon_0} q^2 (1 - \frac{1}{9}) = \frac{1}{4\pi\epsilon_0}\frac{\cancel{8}q^2}{9}$$

$$q^2 = 9mg \times 4\pi\epsilon_0$$

$$q = 6\sqrt{mg\pi\epsilon_0} = 35\,\mu C$$

Three equal charges Q are placed at the 3 vertices of an equilateral traingle. what should be the value of a charge then when placed at the centeroid interaction energy became zero

$\Rightarrow$ Interaction energy after bring the charge q is



$$K\frac{3Q^2}{a} + K\frac{3Qq}{a/\sqrt{3}} = 0$$

$$\frac{3Q^2}{a} = -\frac{3Qq}{a/\sqrt{3}} \Rightarrow q = -Q/\sqrt{3}$$

A Conducting Sphere of radius R has charge Q on its Surface. If charge on Sphere is doubled and radius is halved the Energy

$\Rightarrow$
$$E_i = \frac{1}{4\pi\epsilon_0}\,\frac{3}{5}\,\frac{Q^2}{R}$$

$$E_f = \frac{1}{4\pi\epsilon_0}\,\frac{3}{5}\,\frac{(2Q)^2}{(R/2)} = \frac{1}{4\pi\epsilon_0}\,\frac{3}{5}\,8\,\frac{Q^2}{R}$$

Energy becam 8 time from previous

The Capacitance of parallel plate Capacitor with air as medium is $3\mu F$. with the introduction of a dielectric medium between the plate, Capacitance betwee $15\mu F$ Then permitivity becam

$$C' = \frac{K\epsilon_0 A}{d}$$

$\Rightarrow$ $C' = CK \Rightarrow K = \frac{C'}{C} = 5$

$$C = \frac{\epsilon_0 A}{d}$$

$$K = \frac{\epsilon}{\epsilon_0} \Rightarrow \epsilon = K\epsilon_0 = 5 \times 8.8 \times 10^{-12}$$

$$= 0.4 \times 10^{-10}\ C^2 N^{-1} m^{-2}$$

---

131 In order to increase capacitance 66% a dielectric slab of medium constant 5 is introduced between the plates. The thickness

$\Rightarrow$ $$c' = 1.66c$$

$$\frac{\epsilon_0 A}{d - t(1-\frac{1}{K})} = \frac{\epsilon_0 A}{d} \times 1.66$$

$$d - t\left(1-\frac{1}{5}\right) = \frac{d}{1.66}$$

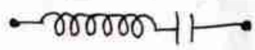
$$d - t\frac{5}{9} = \frac{d}{1.66}$$

$$0.4d = 0.25t$$

$$d = 2t$$

$$t = d/2$$

134 Find the extension of the spring



$\Rightarrow$ $$F = Kx \Rightarrow \frac{\sigma^2}{2\epsilon_0} A = Kx$$

$$x = \frac{\sigma^2 A}{2\epsilon_0 K} = \left(\frac{Q}{A}\right)^2 \frac{A}{2\epsilon_0 K} = \frac{Q^2}{2\epsilon_0 AK}$$

135 A particle with specific charges q/m moves due to field $E = E_0 - ax$ —

① distance covers till rest

$$\frac{dv}{dt} = \frac{q}{m}(E_0 - ax) = v\frac{dv}{dx}$$

$$v\,dv = \frac{q}{m}(E_0 - ax)dx$$

$$\frac{v^2}{2} - \frac{q}{m}\left(E_0 x - \frac{ax^2}{2}\right) = \text{Constant}$$

$$v^2 = \frac{2q}{m}\left(E_0 x - \frac{1}{2}ax^2\right)$$

$v = 0$ at $x = 0$

At $v = 0$, $x = \frac{2E_0}{a}$

Accleration $$\left(\frac{dv}{dt}\right) = \frac{q}{m}(E_0 - 2E_0) = -\frac{qE_0}{m}$$

The Conductors in a 0.75 Km long two-wire transmission line are seperated by a centre to centre distance of 0.2m. Each diameter is 4 cm. the Capacitance

$\Rightarrow$ $$C = \frac{\pi \epsilon_0}{\ln d/a} \cdot l = \frac{3.14 \times 8.8 \times 10^{-12}}{\ln\left(\frac{2 \times 0.2}{4 \times 10^{-2}}\right)} \times 0.75 \times 10^{3}$$

$$C = 80.11 \times 10^{-9} = 88.5 nF$$

A Coaxial Cable of uniform cross section contains an insultaing material of dEC 3.5. The radius of central wire 0.01 m and that of sheath is 0.02 m. The Capacitance per Km of the Cable

$\Rightarrow$ Capacitance per km $$\frac{2\pi\epsilon_0 K}{\ln(b/a)}$$

$$= \frac{2 \times 3.14 \times 8.8 \times 10^{-12} \times 10^{3} \times 3.5}{\ln\left(\frac{0.02}{0.01}\right)} = 280 nF/Km$$

Potential satifies Laplace Eqn then find the value of b

$$V = A\left(bx^2 + \frac{y^2}{2} - 2z\right)$$

$\nabla^2 V = 0 \Rightarrow$ $$\frac{d}{dx}(2bx) + \frac{1}{2}\frac{d}{dy}(2y) - \frac{d}{dz}(2z) = 0$$

$$2b + 1 - 2 = 0$$

$$2b = 1 \Rightarrow b = 1/2$$

A cube has a Constant field potential V on Surface If there are no charges inside Cube then potential at Centre

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon} = 0 \Rightarrow E = 0 \therefore \frac{dV}{dr} = 0 \therefore V = \text{constant} = V$$

14. Potential distribution in a region $\phi = \phi_0 e^{-ax^2}$

The charge density

$\Rightarrow$ $\rho = \epsilon_0 (-\nabla^2 \phi)$

$$= -\epsilon_0 \left[\frac{d^2}{dx^2} (\phi_0 e^{-ax^2}) \right.$$

$$\rho = -\phi_0 \epsilon_0 \left[\frac{d}{dx} (-2ax e^{-ax^2}) \right.$$

$$\rho = 2a\phi_0 \epsilon_0 \frac{d}{dx} (x e^{-ax^2})$$

$$\rho = 2a\phi_0 \epsilon_0 \left[e^{-ax^2} + x(-2ax) e^{-ax^2} \right]$$

$$\rho = 2a\phi_0 \epsilon_0 e^{-ax^2} (1 - 2ax^2)$$

$$\rho = 2a\epsilon_0 (1 - 2ax^2) \phi$$

---

14. Potential $V(r) = Ar^3 + B$. Charge enclosed by a Sphere of radius R is centered at origin is Q. Then enclosed by $2R$ is

$\Rightarrow$

$$\nabla^2 V = -\rho/\epsilon_0 \qquad \frac{\partial V}{\partial r} = 3Ar^2$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) = -\rho/\epsilon_0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} (3Ar^4) = -\rho/\epsilon_0$$

$$\frac{1}{r^2} 12Ar^3 = -\rho/\epsilon_0 \Rightarrow \rho = -12A\epsilon_0 r$$

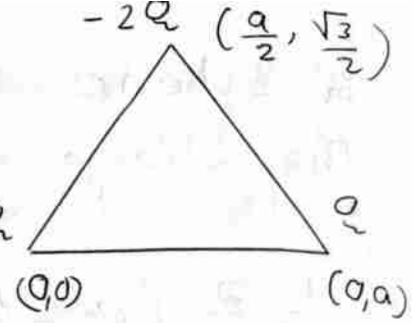
$$Q = \int \rho \, dV = -12A\epsilon_0 4\pi \int_0^R r^3 dr = K \frac{R^4}{4}$$

$$Q' = K \int_0^{2R} r^3 dr = K \frac{(2R)^4}{4} = 16 \frac{KR^4}{4}$$

$$Q' = Q \times 16 \Rightarrow Q' = 16Q$$

---

18 Find the dipole moment



$\sum q_i = -2q + q + q = 0$

$\sum q_i \vec{r_i} = (q \times 0) + aq\hat{j} - 2q\left(\frac{\sqrt{3}}{2} j + \frac{a}{2} i\right)$

$= aq\hat{j} - \sqrt{3} q j - qa j = -\sqrt{3} q \hat{j}$

---

19 $\rho = \rho_0 \left(\frac{r_0}{r}\right)^2 e^{-r/r_0} \cos^2\phi$. The radial dipole moment due to charge distribu

$\Rightarrow p = \int r \rho \, dV = \int r \, \rho_0 \frac{r_0^2}{r^2} e^{-r/r_0} \cos^2\phi \, dv$

$= \rho_0 r_0^2 \int \frac{1}{r} e^{-r/r_0} \cos^2\phi \, r^2 \sin\theta \, dr \, d\theta \, d\phi$

$= \rho_0 r_0^2 \int_0^r e^{-r/r_0} r \, dr \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} \cos^2\phi \, d$

An insulating sphere of radius a carries a charge density $\rho(r) = \rho_0 (a^2 - r^2) \cos\theta$. field is proportional to

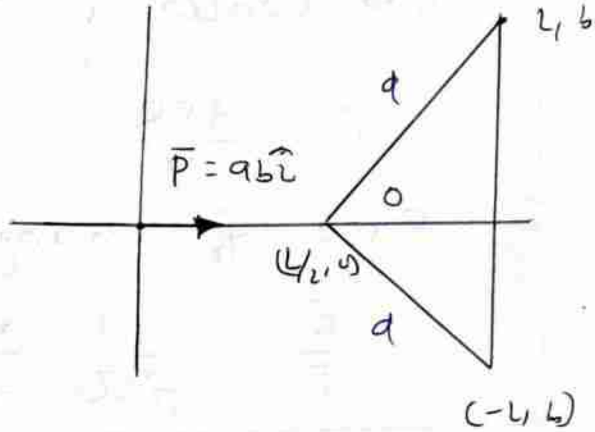
$p = \int \rho r \, dV = \int \rho_0 (a^2 - r^2) \cos\theta \, r \, r^2 \sin\theta \, dr d\theta$

$p = \rho_0 \int_0^a (a^2 - r^2) r^3 \, dr \int_0^\pi \cos\theta \sin\theta \, d\theta \int_0^{2\pi} d\phi$

Two particle each of mass m are moving in a circle of radius a. magnitude on each particle is a their speed is

$\frac{mv^2}{r} = \frac{Kq^2}{(2r)^2} \Rightarrow K = \sqrt{\frac{Kq^2}{4ma}}$

An electric dipole of dipole moment $\vec{p} = qb\,i$ of two charges $q\,(L, b)$ and $-q\,(L, -b)$. The potential at $(L/2, 0)$ is



$\Rightarrow$

Potential at o

is $V = 0$

$$V = \frac{1}{4\pi\epsilon_0} \frac{P\cos\theta}{(L/2)^2}$$

$$V = \frac{qb}{\pi\epsilon_0 L^2}$$

---

12 A charge q is kept at distance $2R$ from the center of a grounded conducting sphere of radius R. The image charge and distance of image charge

$\Rightarrow$ distance $d' = \frac{R^2}{d} = \frac{R^2}{2R} = \frac{R}{2}$

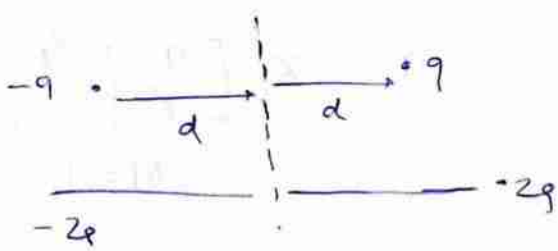
Image charge $q' = -q\frac{R}{d} = -q\frac{R}{2R} = -q/2$

---

13. A charge q is placed in front of an infinite grounded conducting material plate at a distance d. If instead a charge $2q$ were to experience the same force at what distance it will be equal

$\Rightarrow$

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{(4d^2)}$$



$$F = \frac{1}{4\pi\epsilon_0} \frac{4q^2}{x^2}$$

$$\frac{4q^2}{x^2} = \frac{q^2}{4d^2} \Rightarrow \frac{2q}{x} = \frac{q}{2d} \quad x = 4d \Rightarrow \text{distance } 2d$$

302 A particle is moving in a circural path of radius R with constant Speed v. magnitude of rate of change of electric field at the centre is

$$T = \frac{2\pi R}{V} \qquad E = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}$$

Rate of change of electric field

$$\frac{E}{T} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\frac{V}{2\pi R} = \frac{qv}{8\pi^2 R^3 \epsilon_0}$$

304 An electron (m,e) gets accelerated by a constant electric field E. the rate of change of de-Broglie wavelength

⇒ de-Bröglie wavelength

$$\lambda = \frac{h}{mV} \qquad V = at$$

$$\lambda = \frac{h}{eEt} \qquad V = \frac{eE}{m}t$$

$$mV = eEt$$

$$\frac{d\lambda}{dt} = -\frac{h}{eEt^2}$$

311 Four charges each of $+q$ are rigidly fixed at the four corners of a square planar soap film of side a. the surface tension of the soap is γ. $a = K\left[\frac{q^2}{\gamma}\right]$.

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{a^2}\left[\frac{1}{2}+\sqrt{2}\right] = \frac{q^2}{a^2}\times \text{constant}$$

$$a = K\left[\frac{q^2}{\gamma}\right]^{1/3} \qquad \frac{q^2}{a^2}\times \text{Constant} = \gamma a \cdot \text{con}$$

$$N = 3$$

Four dipoles each one having magnitude of charges $\pm Q$ are placed inside a sphere. The total flux of $\bar{E}$ coming out sphere

$\Rightarrow$ Net charge $= 0$ So flux is zero

Flux $\phi = \frac{q_{in}}{\epsilon_0} = 0$

A conducting sphere of radius R carrying charge Q lies inside an uncharged conducting shell of radius 2R. If they joined by a metal wire

Q amount of charge will flow.

potential $\phi = K\frac{Q}{2R}$

Energy $W = \frac{1}{2}\int V\sigma ds = K\frac{Q}{4R}\int \sigma ds$

$= \frac{KQ^2}{4R}$

The electric field at the centre of a hemispherical surface having uniform surface charge density is σ $\sigma/2\epsilon_0$

---

279 Let the electric field in a certain region of space be given by $\vec{E} = \frac{C\vec{r}}{\epsilon_0 a^3}$ Then $\rho(r)$

$\Rightarrow$ Charge density $\rho = \epsilon_0(\vec{\nabla}.\vec{E})$

$\rho = \vec{\nabla}.\left(\frac{C\vec{r}}{a^3}\right) = \frac{3C}{a^3}$

---

277 An field $\vec{E} = ax^3\,\hat{i}$. Total flux thru a sphere of radius R at centre

$\phi = \int \vec{E}.d\bar{s} = \int ax^3\, 4\pi x^2 dx$

$\frac{4\pi a R^6}{6}$

28. Electric field $\bar{E} = \frac{q}{r^2} e^{-4r} \hat{r}$ find total q

$\Rightarrow \quad \vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$

$= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{q}{r^2} e^{-4r} \right) = \rho/\epsilon_0$

$\frac{q}{r^2} \frac{\partial}{\partial r} (e^{-4r}) = \rho/\epsilon_0$

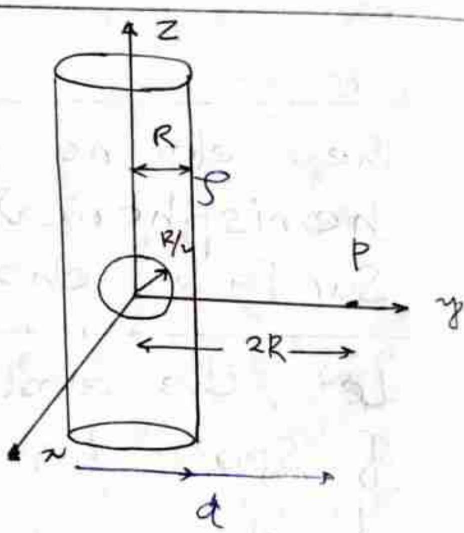
$\frac{q}{r^2} (-4) e^{-4r} = \rho/\epsilon_0$

$\rho = (-4q\epsilon_0) \frac{e^{-4r}}{r^2}$

$Q = \int \rho \, dV = 4\pi \int_0^\infty (-4q\epsilon_0) e^{-4r}$

$= -4q \int_0^\infty e^{-4r} = -\frac{4q}{-4} \left[e^{-4r} \right]_0^\infty = q$

If the electric field at P is $\frac{23\rho R}{16 K \epsilon_0}$ then the value of K



$\Rightarrow E = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{d}$

$\lambda = \rho A = \rho \pi R^2$

$E_{cy} = \frac{1}{4\pi\epsilon_0} \frac{2\rho\pi R^2}{2R}$

$E_{sp} = \frac{1}{4\pi\epsilon_0} \frac{Q}{d^2}$

charge on Sphere $Q = -\rho V = -\frac{4}{3}\pi \left(\frac{R}{2}\right)^3 \rho$

$E_{sp} = -\frac{4\pi (R/2)^3 \rho}{3 \times 4\pi\epsilon_0 (2R)^2} = -\frac{\rho R}{96\epsilon_0}$

$E = \frac{\rho R}{\epsilon_0} \left(\frac{1}{4} - \frac{1}{96} \right) = \frac{23\rho R}{96\epsilon_0} \Rightarrow K = 6$

The total charge on the cylinder 100

$\rho = 16xyz$ $\quad 0 < x = y = z < 1$

$\Rightarrow$ $Q = \int \rho \, dV = \int 16xyz \, dx\,dy\,dz = 8$

$= 16 \int_0^1 x\,dx \int_0^1 y\,dy \int_0^1 z\,dz = \frac{16}{8} = 2$

A semi circular arc of radius a is charged uniformly and the charge per unit length is λ the field at centre

$\Rightarrow$ $E = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{R} \sin\frac{\phi}{2}$ $\quad \phi = \pi$

$E = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{R} \sin\frac{\pi}{2} = \frac{\lambda}{2\pi\epsilon_0 R}$

---

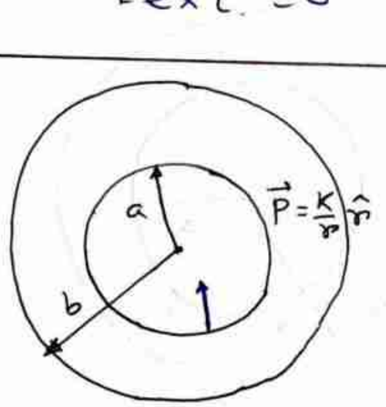
① The polarization of the dielectric $\vec{P} = K\vec{r}$ Then ρ_b, σ_b, E_{out} = ?

$\Rightarrow$ $\rho_b = -\vec{\nabla}\cdot\vec{P} = -\vec{\nabla}\cdot(K\vec{r}) = -3K$

$\sigma_b = \vec{P}\cdot\hat{n} = K\vec{r}\cdot\frac{\vec{r}}{r} = Kr = KR$

Total charge $= -4K\pi R^3 + KR\, 4\pi R^2 = 0$

$E_{ext} = 0$ Since $\oint \vec{E}\cdot d\vec{s} = q/\epsilon_0$



find the Value of σ_b, ρ_b, Q, E

$(\sigma_b)_1 = \frac{K}{r}\hat{r}\cdot\hat{r} = -\frac{K}{r} = -\frac{K}{a}$

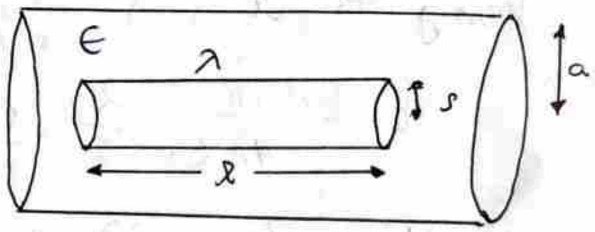
$(\sigma_b)_2 = +\frac{K}{r}\hat{r}\cdot = \frac{K}{r} = \frac{K}{b}$

$\rho_b = -\vec{\nabla}\cdot\vec{P} = -K/r^2$

$$Q_{total} = -\frac{K}{a} 4\pi a^2 + \frac{K}{b} \times 4\pi b^2 - \int_a^b \frac{K}{r^2} 4\pi b r$$

$$= -4\pi a K + 4\pi K b + 4\pi a K - 4\pi K b = 0$$

A long straight wire carrying a line charge λ is surrounded by a rubber insulation find $\vec{E}$ and $\vec{D}$ inside and outside the rubber insulator



Sn

$$\oint \vec{D} \cdot d\vec{s} = q_f \qquad \oint \vec{E} \cdot d\vec{s} = \frac{q_{en}}{\epsilon}$$

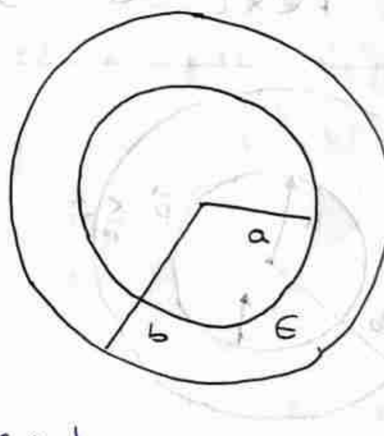
$$D \cdot 2\pi r l = \lambda l \qquad E = \frac{\lambda}{2\pi r} \frac{1}{\epsilon_0 \epsilon_r}$$

$$D = \frac{\lambda}{2\pi r}$$

outside $\quad D = \frac{\lambda}{2\pi r} \quad E = \frac{\lambda}{2\pi \epsilon_0 r}$

A metalic sphere have radius a (Q) is surrounded by an insulator radius b then find $\vec{P}$, ρ_b, $\vec{D}$, $\vec{E}$, V at centre

$\Rightarrow$ $r < a$



$Q_{en} = 0 \quad \vec{D} = 0$

$\vec{E} = 0$, $\vec{P} = 0$

$a < r < b$

$$D \cdot 4\pi r^2 = \frac{Q}{1}$$

$$D = \frac{Q}{4\pi r^2}$$

$$E = \frac{Q}{4\pi \epsilon_0 \epsilon_r r^2}$$

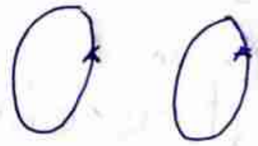
$r > b$

$$D \cdot 4\pi r^2 = Q$$

$$D = \frac{Q}{4\pi r^2}, \quad E = \frac{Q}{4\pi \epsilon_0 r^2}$$

① Two circular rings carrying current i in anti wise direction in both of radius R are kept R distance apart have N turns The magnetic field at midway is

$\Rightarrow$ $B = \frac{\mu_0}{4\pi} \frac{2\pi N i R^2}{(R^2+x^2)^{3/2}}$

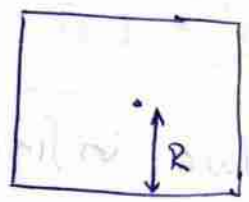


$$B = \frac{\mu_0}{4\pi} \frac{2\pi N i R^2}{\left(R^2 + \frac{R^2}{4}\right)^{3/2}} = \frac{\mu_0}{4\pi} \frac{2\pi N i R^2}{R^3 \frac{5^{3/2}}{8}}$$

$$B_{net} = 2B = \frac{8 N \mu_0 i}{5^{3/2} R}$$

② Find the magnetic field at the center of a square loop which carries a steady current, R be the distance from centre to side

$\Rightarrow$



$$B = \frac{\mu_0}{4\pi} \frac{i}{R} (\sin\theta_1 + \sin\theta_2)$$

$$B = \frac{\mu_0}{4\pi} \frac{i}{R} \sqrt{2} \quad \text{, At centre}$$

$$B_{net} = \frac{\mu_0}{4\pi} \frac{i}{R} 4\sqrt{2}$$

③ A wire carrying current i and other $2i$ in same direction produce magnetic field B at mid point what will if $2i$ is off

$$\frac{\mu_0 (2i)}{2\pi R} - \frac{\mu_0 i}{2\pi R} = \frac{\mu_0 i}{2\pi R} \odot$$

So field remain same but the direction become opposite

4

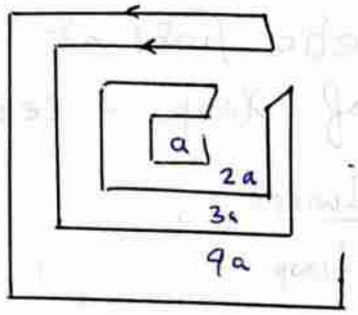
Find magnetic field at o

$$B_1 = \frac{\mu_0}{4\pi}\frac{2i}{r_1}\alpha \odot$$

$$B_2 = \frac{\mu_0}{4\pi}\frac{2i}{r_2}\alpha \otimes$$

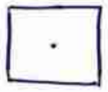
$$\vec{B} = B_1 - B_2 = \frac{\mu_0 i}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \odot$$

5



.. ∞

Find the magnetic field at the Centre



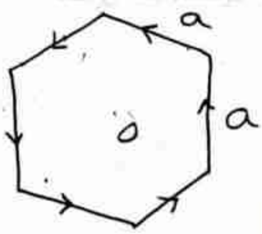
$$4\left[\frac{\mu_0}{4\pi}\frac{i}{\frac{a}{2}} \times 2\sin 45\right] = 2\sqrt{2}\,\frac{\mu_0 i}{\pi a}$$

$$B = 2\sqrt{2}\frac{\mu_0 i}{\pi a} - \frac{2\sqrt{2}\,\mu_0 i}{\pi(2a)} + \frac{2\sqrt{2}\,\mu_0 i}{\pi(3a)} - \cdots$$

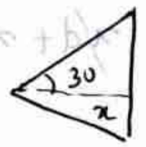
$$B = 2\sqrt{2}\,\frac{\mu_0 i}{\pi a}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots\right)$$

$$B = 2\sqrt{2}\,\frac{\mu_0 i}{\pi a}\ln 2$$

7



Magnetic field at o is



$$B = \frac{\mu_0}{4\pi}\frac{2i}{r}\,n\sin\left(\frac{\pi}{n}\right)$$

$$\frac{a}{2x} = \frac{1}{\sqrt{3}}$$

$$x = \frac{\sqrt{3}a}{2}$$

$$B = \frac{\mu_0}{4\pi}\frac{2i}{\frac{\sqrt{3}a}{2}}\,6\sin 30 = \frac{\sqrt{3}\,\mu_0 i}{\pi a}$$

(14) A long straight wire of radius a carries a steady current i. The current is uniformly distributed across cross section. The ratio of the magnetic field at $a/2$ and $2a$ is

$\because$ $B_{a/2} = \frac{\mu_0}{4\pi}\frac{2ir}{R^2} = \frac{\mu_0}{4\pi}\frac{i}{a}$

$B_{2a} = \frac{\mu_0}{4\pi}\frac{2i}{r} = \frac{\mu_0}{4\pi}\frac{2i}{2a} = \frac{\mu_0}{4\pi}\frac{i}{a}$

$\therefore$ Ratio of magnetic field will be 1

---

(15) The volume current density through a long cylindrical conduction is given to be $\vec{J} = J_0\hat{z}\left(1-\frac{r^2}{R^2}\right)$. Value of r where the magnetic field will be maximum

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \int \vec{J}\cdot d\vec{s}$

$B \cdot 2\pi r = \mu_0 J \int_0^r \left(1-\frac{r^2}{R^2}\right) 2\pi r\, dr$

$B_{in} = \mu_0 J \left[\frac{r}{2} - \frac{r^3}{4R^2}\right]$ — ①

$B_{out}\, \cancel{2\pi r} = \mu_0 J \left[\frac{R^2}{2} - \frac{R^2}{4}\right] = \frac{\mu_0 J}{r}\frac{R^2}{4}$

$B_{out} = \frac{\mu_0 J}{r}\frac{R^2}{4}$ — ② $B_{out} \propto \frac{1}{r}$ (not max)

Magnetic field will be max^m

$\frac{dB}{dr} = 0$. $\frac{1}{2} - \frac{3r^2}{4R^2} = 0$

$r = \sqrt{\frac{2}{3}}R$

(12) For a long Cylinder $B \propto r$. If radius of the Cable is R then

$J \propto r$

$J = \alpha r$

$\Rightarrow$

$$I = \int J ds$$

$$I = \int_0^R \alpha r \cdot 2\pi r dr = 2\pi\alpha \frac{R^3}{3}$$

$$\alpha = \frac{3I}{2\pi R^3}$$

$$J = \alpha r = \frac{3Ir}{2\pi R^3}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{s}$$

$$B \cdot 2\pi r = \mu_0 \cdot \frac{3I}{2\pi R^3} \int_0^r r \cdot r \, dr \, 2\pi$$

$$B = \frac{\mu_0 i}{2\pi R^3} r^2$$

(13) In a Conducting wire of radius a having Current density that Varies as $J = J_0 \frac{r}{a}$. The total Current

$$i = \int J ds = \int_0^a J_0 \frac{r}{a} 2\pi r dr = J_0 \frac{2}{3} \pi a^2$$

(14) The steady Current density gives rise to a magnetic field $\vec{B} = K(\hat{i}y - \hat{j}x)$.

$\Rightarrow$

$$\vec{\nabla} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix}$$

$$= \hat{k}(-1-1) = -2\hat{k} = \mu_0 \vec{J}$$

$$\vec{J} = -\frac{2}{\mu_0} \hat{k}$$

The charge on a parallel plate capacitor varies as $q = q_0 \cos 2\pi\nu t$. I_d

$\Rightarrow \quad I_d = \frac{dq}{dt} = -2\pi\nu q_0 \cancel{\cos} \sin 2\pi\nu t$

The displacement current density when the electric flux density is $20 \sin 0.5t$

$\Rightarrow$ displacement current:

$$I_d = \frac{dD}{dt} = 0.5 \times 20 \cos 0.5t = 10 \cos 0.5t$$

Electric field and magnetic field is given by $\vec{E} = \hat{i} E_0 \sin(Kz - \omega t)$

$\vec{B} = \hat{j} B_0 \sin(Kz - \omega t)$

$$\Rightarrow \quad \vec{B} = \frac{\vec{K} \times \vec{E}}{\omega}$$

$$\hat{j} B_0 \sin(Kz - \omega t) = \frac{K E_0 \hat{j} E_0 \sin(Kz - \omega t)}{\omega}$$

$$B_0 = \frac{K E_0}{\omega}$$

The electric field $\vec{E}$ corresponding to

$$\vec{B} = B_0 \sin\left[(x+y)\frac{K}{\sqrt{2}} + \omega t\right] \hat{n}$$

$$\Rightarrow \quad \vec{K} = \frac{K\hat{i} + K\hat{j}}{\sqrt{2}}$$

$$\vec{E} = -\frac{c^2}{\omega}(\vec{K} \times \vec{B})$$

Electrostatic potential $V = (2x + 4y)$ Volts. The energy density $\vec{E} = -\vec{\nabla} V = -2\hat{x}\hat{i} - 4\hat{y}$

$|\vec{E}| = \sqrt{4+16} = \sqrt{20}$ V/m

$u_e = \frac{1}{2} \epsilon_0 |\vec{E}|^2 = \frac{1}{2} \times \epsilon_0 \times 20$

$= 10\epsilon_0$ J/m3

---

(11) $A = 0.01\,m^2$. $N = 40$. $R = 20\Omega$ when it is pulled out of the magnetic field, a total charge of $Q = 2\times10^{-5}$ C flows. The magnetude of magnetic field

$\Rightarrow$ $\phi = NBA$

$$e = -\frac{d\phi}{dt} \Rightarrow i = \frac{e}{R} = -\frac{1}{R}\frac{d\phi}{dt} = \frac{dQ}{dt}$$

$$dQ = -\frac{1}{R}d\phi = -\frac{1}{R}(NBA)$$

$$2\times10^{-5} = -\frac{1}{20}(40\times B\times 0.01) \Rightarrow B = 10^{-3}\,T$$

---

(12) A metalic sphere of radius R is held at electrostatic potential V. It is enclosed in a shell 2R at potentia 2V. If the potential at the distance $\frac{3}{2}R$ from the centre is fV then f ___

$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) = 0 \Rightarrow r^2\frac{\partial\phi}{\partial r} = C_1$$

$$\phi = -\frac{C_1}{r} + C_2$$

$$V = -\frac{C_1}{R} + C_2 \qquad 2V = -\frac{C_1}{2R} + C_2$$

$$2V - V = \frac{C_1}{2R} \Rightarrow C_1 = 2VR$$

$$V = -2V + C_2 \Rightarrow C_2 = 3V$$

$$\phi = -\frac{2VR}{r} + 3V$$

$$\phi = -\frac{2VR}{\frac{3}{2}R} + 3V = -\frac{4}{3}V + 3V = \frac{5}{3}V$$

$$f = 1.67$$

---

dipole AC, Maxwell Pointing, Reflecti

A unit vector $\hat{n}$ on the x-y plane is making an angle of 120° w r t $\hat{i}$. The angle between $\vec{U} = a\hat{i} + b\hat{n}$ and $\vec{V} = a\hat{n} + b\hat{i}$ will be 60°. The relation between b and a will be

$$\Rightarrow \vec{U} \cdot \vec{V} = UV\cos\theta$$

$$(a\hat{i}+b\hat{n}) \cdot (a\hat{n}+b\hat{i}) = \left[a^2+b^2+2ab\left(-\tfrac{1}{2}\right)\right] \times \tfrac{1}{2}$$

$$(a^2\cos 120 + ab + ba + b^2\cos 120) = (a^2+b^2-ab) \times \tfrac{1}{2}$$

$$4ab - (a^2+b^2) = a^2+b^2-ab \Rightarrow b = 4a, a$$

Find the area of the Parallelogram whose adjacent sides are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} + 4\hat{k}$

$$\Rightarrow \text{Area} = |\vec{A} \times \vec{B}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & 1 & 4 \end{vmatrix}$$

$$= |\hat{i}(-8-3) + \hat{j}(6-4) + \hat{k}(1+4)|$$

$$= |-11\hat{i} + 2\hat{j} + 5\hat{k}| = \sqrt{150} = 5\sqrt{6}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$$\vec{b} = \hat{i} \times (\hat{a} \times \hat{i}) + \hat{j} \times (\hat{a} \times \hat{j}) + \hat{k} \times (\hat{a} \times \hat{k})$$

$$\hat{i} \times (\hat{a} \times \hat{i}) = (\hat{i} \cdot \hat{i})\hat{a} - (\hat{i} \cdot \hat{a})\hat{i} = \hat{a} - (\hat{a} \cdot \hat{i})\hat{i}$$

$$\hat{j} \times (\hat{a} \times \hat{j}) = \hat{a} - (\hat{a} \cdot \hat{j})\hat{j} \quad \hat{k} \times (\hat{a} \times \hat{k}) = \hat{a} - (\hat{a} \cdot \hat{k})\hat{n}$$

$$\vec{b} = 3\hat{a} - [(\hat{a} \cdot \hat{i})\hat{i} + (\hat{a} \cdot \hat{j})\hat{j} + (\hat{a} \cdot \hat{k})\hat{k}] = 3\vec{a} - \vec{a} = 2\vec{a}$$

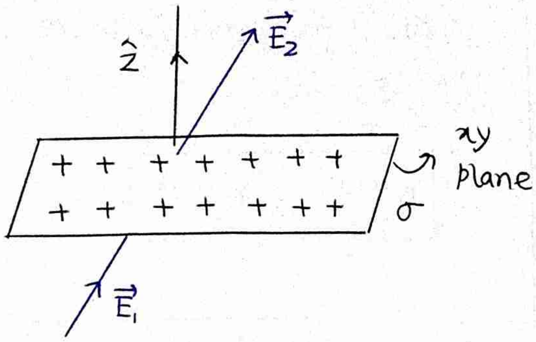
Electrostatic Boundary Conditions

The electric field

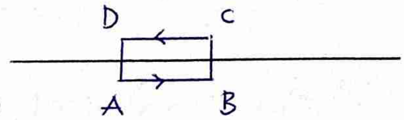
$\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$

$\vec{E}_{\parallel} = E_x \hat{i} + E_y \hat{j}$

$\vec{E}_{\perp} = E_z \hat{k}$



$\oint \vec{E} \cdot d\vec{l} = 0$



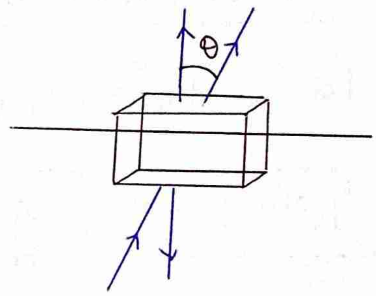
or $\int_{AB} \vec{E} \cdot d\vec{l} + \cancel{\int_{BC} \vec{E} \cdot d\vec{l}}^{\,0} + \int_{CD} \vec{E} \cdot d\vec{l} + \cancel{\int_{DA} \vec{E} \cdot d\vec{l}}^{\,0} = 0$

or, $E \cdot AB \cos 0^\circ + E\, CD \cos 180^\circ = 0$

or, $E_{\parallel}^{below} l - E_{\parallel}^{above} l = 0 \Rightarrow$

$$\boxed{E_{\parallel}^{above} = E_{\parallel}^{below}}$$

$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$



or $E_{above}^{\perp} S \cos\theta + E_{below}^{\perp} S \cos(180 - \theta) = \frac{\sigma S}{\epsilon_0}$

$$\boxed{E_{\perp}^{above} - E_{\perp}^{below} = \frac{\sigma}{\epsilon_0}}$$

Parallel component is always continuous.

perpendicular component has discontinuity $\frac{\sigma}{\epsilon_0}$ amount

$$\boxed{D_{\perp}^{above} - D_{\perp}^{below} = \sigma_f}$$

Displacement vector

$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \Rightarrow \vec{E} = \frac{1}{\epsilon_0}(\vec{D} - \vec{P})$

$\oint \vec{E} \cdot d\vec{l} = \frac{1}{\epsilon_0} \oint (\vec{D} - \vec{P}) \cdot d\vec{l} = 0$

$$\boxed{D_{\parallel}^{above} - D_{\parallel}^{below} = P_{\parallel}^{above} - P_{\parallel}^{below}}$$

Air Dielectric Interface.

No Conductor is here.

so $\sigma_f = 0$

air

dielectric

$$\boxed{D_\perp^{above} = D_\perp^{below}}$$

$$\sigma = \sigma_b + \sigma_f = \sigma_b \neq 0$$

$$\boxed{E_\perp^{above} - E_\perp^{below} = \sigma/\epsilon} \qquad \boxed{E_{\parallel}^{above} = E_{\parallel}^{below}}$$

Dielectric-Dielectric Interface:

As dielectric so no free charge only bound charge $\sigma_f = 0$

ϵ_{r_1}

ϵ_{r_2}

$$\boxed{E_{\parallel}^{above} = E_{\parallel}^{below}} \qquad \boxed{D_\perp^{above} = D_\perp^{below}}$$

Air Conductor interface

For Conductor $\sigma_f \neq 0$, $\sigma_b = 0$

$$\boxed{D_\perp^{above} - D_\perp^{below} = \sigma_f} \qquad E_{\parallel}^{above} = E_{\parallel}^{below}$$

$$\boxed{E_\perp^{above} - E_\perp^{below} = \frac{\sigma_f}{\epsilon_0}} \qquad \text{as } \sigma = \sigma_f + \sigma_b = \sigma_f$$

But in Conductor EF $= 0$ $\quad \boxed{E_\perp^{above} = \sigma_f/\epsilon_0}$

$E_{\parallel}^{below} = 0$ so. $\boxed{E_{\parallel}^{above} = 0}$

Dielectric- Conductor Interface

Dielectric

Conductor

$$\sigma = \sigma_f + \sigma_b$$

$$E_\perp^{above} - E_\perp^{below} = \frac{\sigma}{\epsilon_0}$$

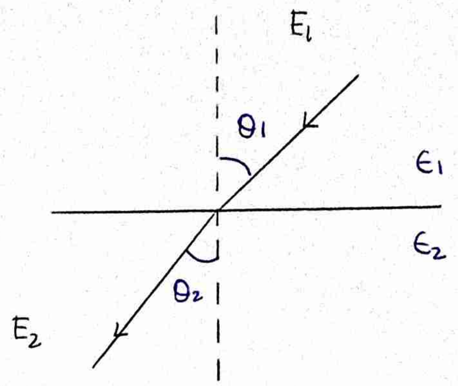
$$E_\perp^{above} = \frac{\sigma}{\epsilon_0} \qquad E_{\parallel}^{below} = 0$$

$$D_{\parallel}^{above} - D_{\parallel}^{below} = P_{\parallel}^{above} - P_{\parallel}^{below}$$

$$D^\perp_{above} - D^\perp_{below} = \sigma_f \qquad \sigma_b = 0 \quad D = 0 \quad P = 0$$

$$E_1^{\parallel} = E_2^{\parallel}$$

$$D_\perp^1 = D_\perp^2$$



$$E_1 \sin\theta_1 = E_2 \sin\theta_2$$

$$\frac{E_1}{E_2} = \frac{\sin\theta_2}{\sin\theta_1} \quad \text{— ①}$$

$$\epsilon_1 E_1 \cos\theta_1 = \epsilon_2 E_2 \cos\theta_2$$

$$\frac{E_1}{E_2} = \frac{\epsilon_2 \cos\theta_2}{\epsilon_1 \cos\theta_1} \quad \text{— ②}$$

From eqn ① and ②

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\epsilon_2 \cos\theta_2}{\epsilon_1 \cos\theta_1}$$

---

① Electric field at region 1 is $\vec{E_1} = 5\hat{i} - 2\hat{j} + 3\hat{k}$

$\Rightarrow$ $E_{\parallel}^1 = 5\hat{i} - 2\hat{j}$

$E_{\parallel}^2 = 5\hat{i} - 2\hat{j}$

$\epsilon_1 = 4$ ①

$\epsilon_2 = 3$ ②

$D_1^{\perp} = D_2^{\perp}$

$\epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp}$

$4 \times 3 = 3 \times E_2^{\perp} \Rightarrow E_2^{\perp} = 4\hat{k}$

$\vec{E_2} = 5\hat{i} - 2\hat{j} + 4\hat{k}$