31.07.2024
class: 01

> periodic Motion:

Particle repeats its motion after a regular time interval then it is known as Periodic motion

If T be the period of the function f(t)

then $f(t+T) = f(t)$

$f(t+2T) = f(t)$

> Simple Harmonic Motion

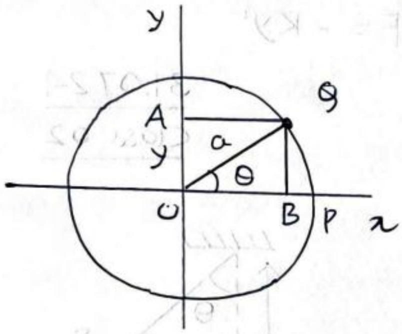
There should be a TO and FRO motion about a fixed Point is Called SHM

Harmonic motion

⇓

$F \propto -x^n$

(n is odd number)



All SHM are periodic motion but every periodic motion is not SHM.

> Motion of earth around sun is periodic but not SHM

> If an object is rotating in Circular Path, foot of Perpendicular make SHM

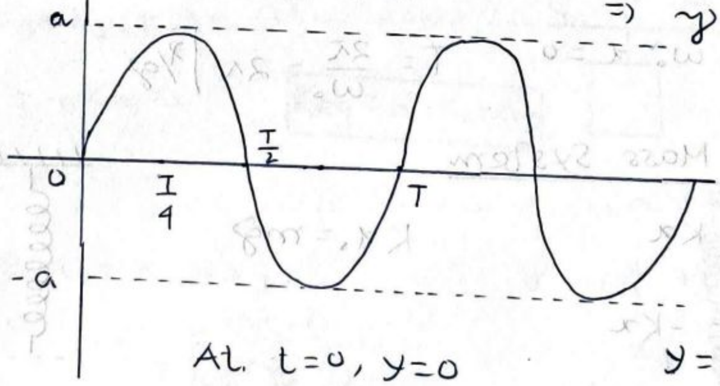
Particle moves with angular velocity ω

$BQ = OA = y$

$\theta = \omega t$

$\frac{QB}{OQ} = \sin\theta \Rightarrow y = OB\sin\omega t$

$\Rightarrow y = a\sin\omega t$



At. $t=0$, $y=0$ $\qquad y = a\sin\omega t$

Displacement $y = a\sin\omega t$ $\quad y = y_{max},\ \omega t = \frac{\pi}{2}$

$$v = \frac{dy}{dt} = a\omega\cos\omega t \qquad \therefore y_{max} = a$$

$$v = a\omega\sqrt{1 - y^2/a^2} \qquad V = V_{max} = a\omega \text{ at } y = 0$$

$$\text{or } v = \omega\sqrt{a^2 - y^2} \qquad \text{at } t = 0,\ V = V_{max}$$

Acceleration $a = -a\omega^2\cos\omega t$

$$\frac{d^2y}{dt^2} = -\omega^2 y \Rightarrow \text{acceleration} \propto -y$$

$$\frac{d^2y}{dt^2} + \omega^2 y = 0 \Rightarrow \text{Differential Eq}^n \text{ of SHM}$$

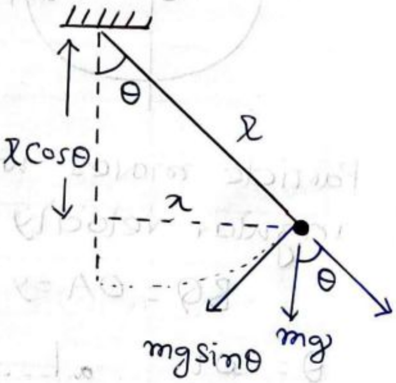
Negative sign: force is restoring force.

$$m\frac{d^2y}{dt^2} = -m\omega^2 y$$

$$F = -Ky'$$

31.07.24
Class: 02

> Simple Pendulum



$$F = -mg\sin\theta$$

$$m\frac{d^2x}{dt^2} = -mg\frac{x}{l}$$

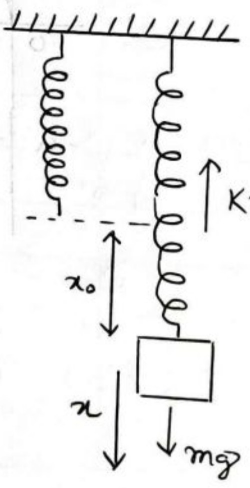
$$\text{or } \frac{d^2x}{dt^2} + \frac{g}{l}x = 0$$

$$\text{or } \frac{d^2x}{dt^2} + \omega_0^2 x = 0$$

Time Period

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{l/g}$$

> Spring Mass System



$$F = -Kx \qquad Kx_0 = mg$$

$$m\frac{d^2x}{dt^2} = -Kx$$

$$\frac{d^2x}{dt^2} + \frac{K}{m}x = 0 \Rightarrow \frac{d^2x}{dt^2} + \omega_0^2 x = 0$$

$$\omega_0 = \sqrt{K/m} \qquad T = 2\pi\sqrt{\frac{m}{K}}$$

Kinetic Energy in SHM

Kinetic Energy. $KE = \frac{1}{2} m v^2$

$v = a\omega \cos \omega t$ $\quad KE = \frac{1}{2} m \omega^2 a^2 \cos^2 \omega t$

$KE = \frac{1}{2} m \omega^2 (a^2 - y^2)$

Potential Energy in SHM

Potential Energy. $dw = F dy = -m\omega^2 y\, dy$

$w = \int dw = -m\omega^2 \int y\, dy = -\frac{1}{2} m \omega^2 y^2$

As $y = a \sin \omega t$, $\quad PE = \frac{1}{2} m \omega^2 a^2 \sin^2 \omega t$

$PE = \frac{1}{2} m \omega^2 y^2$

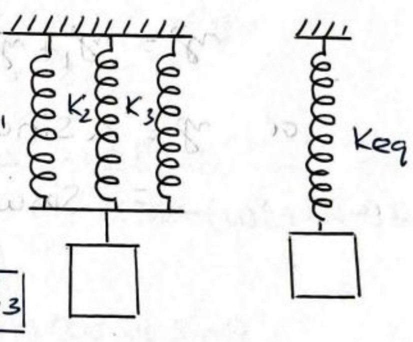
For any Conservative System $w = -\Delta PE$

So total Energy. $E = \frac{1}{2} m \omega^2 a^2 = \text{constant}$

$\langle KE \rangle = \frac{1}{4} m \omega^2 a^2$ $\quad \langle E \rangle = \frac{1}{4} m \omega^2 a^2 + \frac{1}{4} m \omega^2 a^2$

$\langle PE \rangle = \frac{1}{4} m \omega^2 a^2$ $\quad = \frac{1}{2} m \omega^2 a^2$

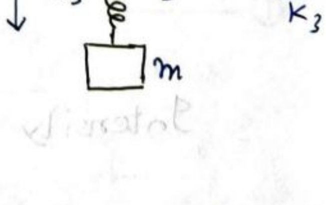
Combination of Springs



- $F = -K_1 x - K_2 x - K_3 x$

  $F = -(K_1 + K_2 + K_3) x$

  Also $F = -K_{eq} x$ $\quad \boxed{K_{eq} = K_1 + K_2 + K_3}$

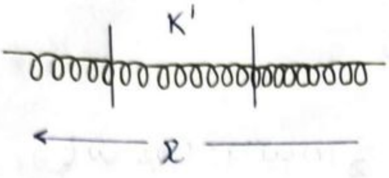
- 

  As $x = x_1 + x_2 + x_3$

  $-\frac{F}{K_{eq}} = -\left(\frac{F}{K_1} + \frac{F}{K_2} + \frac{F}{K_3}\right)$

  $$\boxed{\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}}$$

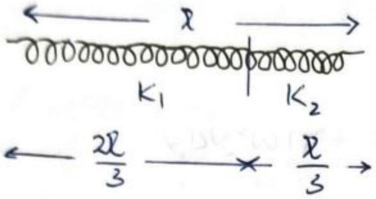
Cutting of a Spring



In series combination

$$\frac{1}{K_1}+\frac{1}{K_2}+\frac{1}{K_3}=\frac{1}{K'}$$

$$\frac{3}{K'}=\frac{1}{K} \Rightarrow K'=\frac{K'}{3}$$



$$K\cdot\ell = K_1\times\frac{2\ell}{3} = K_2\,\frac{\ell}{3}$$

$$K_2 = 3K \qquad K_1 = \frac{3}{2}K$$

$$\frac{1}{K_1}+\frac{1}{K_2}=\frac{1}{K} \text{ (Verified)}$$

01.08.2023
Class : 03

① Superposition of Linear SHM

Here we have $y_1 = a\sin\omega t$

$y_2 = b\sin(\omega t+\theta)$

As they are Oscillating in same direction, Linear SHM

(i) They have different amplitude
(ii) Same angular velocity
(iii) Constant phase difference.

$$y = y_1 + y_2 = a\sin\omega t + b\sin(\omega t+\theta)$$

or $y = a\sin\omega t + b\sin\omega t\cos\theta + b\cos\omega t\sin\theta$

$= \sin\omega t\,(a+b\cos\theta) + \cos\omega t\,(b\sin\theta)$

put, $a+b\cos\theta = A\cos\phi$

$b\sin\theta = A\sin\phi$

$y = A\sin\omega t\cos\phi + A\cos\omega t\sin\phi$

$y = A\sin(\omega t+\phi)$ ⇒ Resultant is also SHM having diff amplitude

Amplitude $A = (a^2+b^2+2ab\cos\theta)^{1/2}$

$\phi = \tan^{-1}\left(\frac{b\sin\theta}{a+b\cos\theta}\right)$

Intensity $I \propto a^2$

$I = Ka^2$

Now, $A^2 = (a^2+b^2+2ab\cos\theta)$

or, $KA^2 = Ka^2 + Kb^2 + 2\sqrt{K}a\sqrt{K}b\cos\theta$

or $I_R = I_a + I_b + 2\sqrt{I_a}\sqrt{I_b}\cos\theta$

$A_{max} = \sqrt{(a^2+b^2+2ab)} = (a+b)$ at $\cos\theta = 1$

$A_{min} = \sqrt{a^2+b^2-2ab} = (a-b)$ at $\cos\theta = -1$

$I_{max} = (\sqrt{I_1}+\sqrt{I_2})^2 \quad I_{min} = (\sqrt{I_1}-\sqrt{I_2})^2$

$$\frac{I_{min}}{I_{max}} = \left(\frac{\sqrt{I_1}-\sqrt{I_2}}{\sqrt{I_1}+\sqrt{I_2}}\right)^2 = \text{visibility}$$

② <u>Superposition of N linear SHM</u>

$y_1 = a\sin\omega t$

$y_2 = a\sin(\omega t+\theta)$

$y_3 = a\sin(\omega t+2\theta)$

$\vdots$

$y_n = a\sin(\omega t+(N-1)\theta)$

They are linear and same amplitude. Phase difference between any two consecutive SHM will be θ.

Resultant $y_R = y_1 + y_2 + y_3 + \dots + y_N$

$y_R = a\sin\omega t + a\sin(\omega t+\theta) + a\sin(\omega t+2\theta) + \dots + a\sin(\omega t+(N-1)\theta)$

or $y_R = a\sin\omega t + a\sin\omega t\cos\theta + a\cos\omega t\sin\theta + a\sin\omega t\cos 2\theta + a\cos\omega t\sin 2\theta \dots + \dots a\sin\omega t\cos(N-1)\theta + a\cos\omega t\sin(N-1)\theta$

or $y_R = a\sin\omega t\,(1+\cos\theta+\cos 2\theta + \dots \cos(N-1)\theta) + a\cos\omega t\,(\sin\theta + \sin 2\theta + \dots \sin(N-1)\theta)$ ———— (i)

$$y_R = a\sin\omega t\left[1+\cos\frac{N\theta}{2}\,\frac{\sin\frac{(N-1)\theta}{2}}{\sin\theta/2}\right] + a\cos\omega t\left[\sin\frac{N\theta}{2}\,\frac{\sin(N-1)\theta/2}{\sin\theta/2}\right]$$

$$e^{i\theta}+e^{2i\theta}+e^{3i\theta}+\cdots\cdots+e^{i(N-1)\theta} = \frac{e^{i\theta}(e^{iN\theta-i\theta}-1)}{(e^{i\theta}-1)}$$

$$= \frac{e^{i\theta}(e^{i(N-1)\theta}-1)}{e^{i\theta}-1}$$

$$\frac{a(r^n-1)}{r-1} = GP$$

$$= \frac{e^{i\theta}\cdot e^{\frac{i(N-1)\theta}{2}}\left[e^{\frac{i(N-1)\theta}{2}}-e^{-\frac{i(N-1)\theta}{2}}\right]}{e^{i\theta/2}\left[e^{i\theta/2}-e^{-i\theta/2}\right]}$$

$$= e^{i\theta/2}e^{\frac{i(N-1)\theta}{2}}\frac{\sin(N-1)\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= e^{iN\theta/2}\frac{\sin(N-1)\theta/2}{\sin\theta/2}$$

$$= \left(\cos\frac{N\theta}{2}+i\sin\frac{N\theta}{2}\right)\frac{\sin\frac{(N-1)\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= \cos\frac{N\theta}{2}\frac{\sin\frac{(N-1)\theta}{2}}{\sin\frac{\theta}{2}} + i\sin\frac{N\theta}{2}\frac{\sin\frac{(N-1)\theta}{2}}{\sin\frac{\theta}{2}}$$

$$= e^{i\theta}+e^{2i\theta}+e^{3i\theta}+\cdots+e^{i(N-1)\theta}$$

$$= [\cos\theta+\cos 2\theta+\cos 3\theta+\cdots\cos(N-1)\theta]$$
$$+ i[\sin\theta+\sin 2\theta+\cdots+\sin(N-1)\theta]$$

$$\cos\theta+\cos 2\theta+\cdots\cos(N-1)\theta = \cos\frac{N\theta}{2}\frac{\sin\frac{(N-1)\theta}{2}}{\sin\frac{\theta}{2}}$$

$$\sin\theta+\sin 2\theta+\cdots\sin(N-1)\theta = \sin\frac{N\theta}{2}\frac{\sin(N-1)\theta/2}{\sin\theta/2}$$

$$a\left(1+\cos\frac{N\theta}{2}\frac{\sin(N-1)\frac{\theta}{2}}{\sin\theta/2}\right) = A\cos\phi$$

$$a\sin\frac{N\theta}{2}\frac{\sin(N-1)\theta/2}{\sin\theta/2} = A\sin\phi$$

$$\boxed{y_R = A\sin(\omega t+\phi)}$$ Resultant is also SHM

So,
$$A^2 = a^2\cos^2\frac{N\theta}{2}\,\frac{\sin^2(N-1)\theta/2}{\sin^2\theta/2} + 2a^2\cos\frac{N\theta}{2}\,\frac{\sin(N-1)\theta/2}{\sin\theta/2}$$
$$+\, a^2\sin^2\frac{N\theta}{2}\,\frac{\sin^2(N-1)\theta/2}{\sin^2\theta/2}$$

$$A^2 = a^2 + a^2\,\frac{\sin^2(N-1)\theta/2}{\sin^2\theta/2} + 2a^2\cos\frac{N\theta}{2}\,\frac{\sin(N-1)\theta/2}{\sin^2\theta/2}$$

③ Superposition of Perpnd SHM

02.08.24
class:04

Two SHMs are

$$x = a\sin\omega t$$
$$y = b\sin(\omega t+\delta)$$

or $y = b\sin\omega t\cos\delta + b\cos\omega t\sin\delta$

or $\frac{y}{b} = \sin\omega t\cos\delta + \cos\omega t\sin\delta$

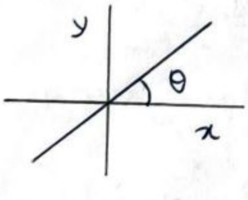
or $\frac{y}{b} = \frac{x}{a}\cos\delta + \sqrt{1-\frac{x^2}{a^2}}\,\sin\delta$

$\therefore \left(\frac{y}{b} - \frac{x}{a}\cos\delta\right)^2 = \left(1-\frac{x^2}{a^2}\right)\sin^2\delta$

or $\frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta + \frac{x^2}{a^2}\cos^2\delta = \sin^2\delta - \frac{x^2}{a^2}\sin^2\delta$

$\therefore$ or $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b}\cos\delta = \sin^2\delta}$

Case①: $\delta = 0$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{x}{a}\frac{y}{b} = 0$$

Linear Polarization

$$\Rightarrow \left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \Rightarrow y = \frac{b}{a}x$$

St Line

$$\theta = \tan^{-1}(b/a)$$

if $a = b$ then $\theta = \pi/4$

Case ② : $\delta = \pi/2$

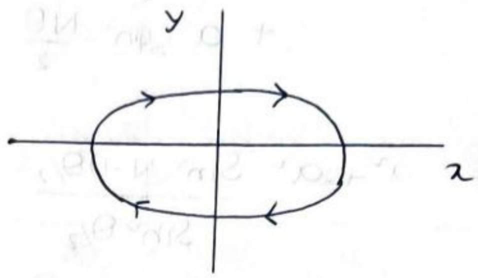
$x = a \sin\omega t$

$y = b \sin(\omega t + \pi/2)$

ωt	x	y
0	0	b
$\frac{\pi}{2}$	a	0
π	0	-b

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

(Elliptical Polarization)



Right handed Elliptical Polarization

If $x = a = a = y$
Right handed Circular Polarization. $x^2 + y^2 = a^2$

Case ③ $\delta = \pi$

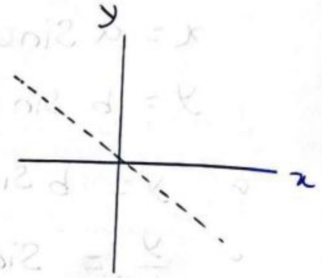
$x = a \sin\omega t$

$y = b \sin(\omega t + \pi)$

$y = -b \sin\omega t$

$$\frac{x}{a} = -\frac{y}{b}$$

$$y = -\frac{b}{a}x$$



Linear with negative slope

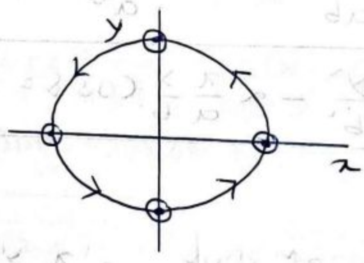
Case ④ $\delta = \frac{3\pi}{2}$

$x = a \sin\omega t$

$y = b \sin(\omega t + \frac{3\pi}{2})$

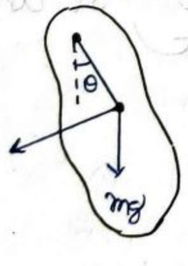
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$$

ωt	x	y
0	0	-b
$\frac{\pi}{2}$	a	0
π	0	b



Left handed Elliptical Polarization

Compound Pendulum



$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

$$\tau = -mgx\sin\theta$$

$$T = 2\pi\sqrt{\frac{I}{mgx}}$$

$$I\frac{d^2\theta}{dt^2} + mgx\sin\theta = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{mgx}{I}\theta = 0$$

I is moment of inertia about fixed Point.

Time Period of the Rod

$\Rightarrow$ Mass, m

Length l

Time Period

$$I = \frac{1}{3} m l^2$$

$$x = \frac{l}{2}$$

$$T = 2\pi \sqrt{\frac{I}{mgx}}$$

$$= 2\pi \sqrt{\frac{\frac{1}{3} m l^2}{mg\, l/2}} = 2\pi \sqrt{\frac{2l}{3g}}$$

$$L = \frac{2}{3} l \text{ (Equivalent length)} : 2\pi \sqrt{\frac{L}{g}}$$

④ <u>Damped Harmonic Motion</u>

<u>07.08.24</u>

Viscous force

$$F = 6\pi \eta r \dot{x}$$

$$F \propto \dot{x}$$

$$m\ddot{x} = -Kx - \beta \dot{x}$$

or $m\ddot{x} = -(Kx + \beta\dot{x})$

$$\frac{K}{m} = \omega_0^2 \qquad \frac{\beta}{m} = 2b$$

or $\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{K}{m}x = 0$

β is damping Coefficent

or $\ddot{x} + 2b\dot{x} + \omega_0^2 x = 0$

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0 \Rightarrow \text{Equation of damped Harmonic motion}$$

Now $m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + Kx = 0$

$\beta - R$

or $m\frac{dv}{dt} + \beta v + Kx = 0$

$\beta - R$ - damping coeff (Ns/m)

or $mv\frac{dv}{dt} + Kx\frac{dx}{dt} = -\beta v^2$

$b = \frac{\beta}{2m} = \frac{R}{2m}$: damping factor

or $\frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}Kx^2\right) = -\beta v^2$

or $\frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}Kx^2\right) = \frac{-\beta v^2}{2m} \times 2m$

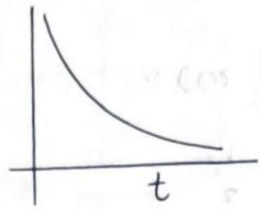
or $\frac{d}{dt}(E) = -\frac{2\beta}{m}\left(\frac{1}{2}mv^2\right)$

or $\frac{dE}{dt} = -\frac{2\beta}{m}\left(\frac{E}{2}\right)$

or, $\frac{d\bar{E}}{dt} = -\frac{\beta}{m}(\bar{E})$

Energy is decaying exponentially after a particular time

or, $\int \frac{d\bar{E}}{\bar{E}} = -\frac{\beta}{m}\int dt$



or $\log E = \log E_0 - \frac{\beta}{m}t$

or $E = E_0 e^{-\frac{\beta}{m}t}$

Now. $\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0$

Let the Solution is $x = e^{mt}$

$\frac{dx}{dt} = me^{mt}$ $\quad \frac{d^2x}{dt^2} = m^2 e^{mt}$

$(m^2 + 2bm + \omega_0^2)e^{mt} = 0$

As $e^{mt} \neq 0$, $\quad m^2 + 2bm + \omega_0^2 = 0$

$m = \frac{-2b \pm \sqrt{4b^2 - 4\omega_0^2}}{2}$

$m = -b \pm \sqrt{b^2 - \omega_0^2}$

$x(t) = A_1 e^{(-b \pm \sqrt{b^2-\omega_0^2})t} + A_2 e^{(-b - \sqrt{b^2-\omega_0^2})t}$

or $x(t) = e^{-bt}\left(A_1 e^{\sqrt{b^2-\omega_0^2}\,t} + A_2 e^{-\sqrt{b^2-\omega_0^2}\,t}\right)$

① b > ω₀ (Damping is large): Dead beat Motion

② b < ω₀ (Damping is small):

$\sqrt{b^2 - \omega_0^2} = \sqrt{-(\omega_0^2 - b^2)} = i\sqrt{\omega_0^2 - b^2} = i\omega'$

$x(t) = e^{-bt}\left[A_1 e^{i\omega' t} + A_2 e^{-i\omega' t}\right]$

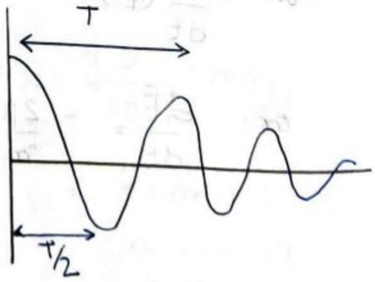
$= e^{-bt} R\cos\omega t$

Time Period

$T = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\omega_0^2 - b^2}}$

$x(t) = e^{-bt} R\cos(\omega' t - \phi)$

Amplitude oscillating and value decrease exponentially



$t=0, \quad x(t)=R$

$t=\frac{T}{2}, \quad x(t) = -Re^{-bT/2}$ $\qquad \left|\frac{x_0}{x_1}\right| = e^{bT/2}$

$t=T \quad x(t) = -Re^{-bT}$ $\qquad \left|\frac{x_1}{x_2}\right| = e^{bT/2}$

$t=\frac{3T}{2} \quad x(t) = -Re^{-\frac{3}{2}bT}$ $\qquad \left|\frac{x_2}{x_3}\right| = e^{bT/2}$

$t=2T \quad x(t) = -Re^{-2bT}$

$$\Delta = 2\ln\left|\frac{x_n}{x_{n+1}}\right| = 2\ln(e^{bT/2}) = bT = b/\nu$$

lograthimic decrement $\Delta = b/\nu = \frac{2\beta}{2m}\cdot\frac{2\pi}{\omega_1} = \frac{4\beta\pi}{2m\omega_1}$

⇓

Decrese of amplitude per cycle

$\cancel{\Delta = 2\beta\pi}$ So $\Delta = \frac{b}{\nu} = \frac{\beta}{2m}\cdot\frac{2\pi}{\omega'}$

$$\Delta = \frac{\beta\pi}{m\sqrt{\omega^2-\frac{\beta^2}{4m^2}}} = \frac{2\pi\beta}{\sqrt{4\omega^2m^2-\beta^2}}$$

$$\Delta = \frac{2\pi\beta}{\sqrt{4mK-\beta^2}}$$

$x(t) = Re^{-bt}\cos(\omega t-\phi) \qquad \omega=\sqrt{\omega_0^2-b^2}$

or $\dot{x}(t) = Re^{-bt}(-\omega)\sin(\omega t+\phi) + R(-b)e^{-bt}\cos(\omega t-\phi)$

or $\dot{x}(t) = Re^{-bt}\left[-\omega\sin(\omega t+\phi) - b\cos(\omega t-\phi)\right]$

or $\dot{x}^2(t) = R^2e^{-2bt}\left[\omega^2\sin^2(\omega t+\phi) + b^2\cos^2(\omega t-\phi)\right.$
$\left. + 2b\omega\sin(\omega t+\phi)\cos(\omega t-\phi)\right]$

Energy. $E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}Kx^2$

$\langle E\rangle = \frac{1}{2}mR^2e^{-2bt}\left[\langle\sin^2(\omega t-\phi)\rangle\omega^2 + \langle\cos^2(\omega t-\phi)\rangle b^2\right.$
$\left. + 2b\omega\langle\sin(\omega t-\phi)\cos(\omega t-\phi)\rangle\right] + \frac{1}{2}Kx^2e^{-2bt}$

or $\langle E\rangle = \frac{1}{4}mR^2\omega^2e^{-2bt} + \frac{1}{4}mR^2b^2e^{-2bt} + \frac{1}{4}KR^2e^{-2bt}$

$\langle E\rangle = \frac{1}{4}mR^2e^{-2bt}(\omega^2+b^2) + \frac{1}{4}KR^2e^{-2bt}$

or $\langle E\rangle = \frac{1}{4}R^2e^{-2bt}(m\omega^2+K+mb^2) \quad$ (as $K=m\omega^2$)

$\langle E\rangle = \frac{1}{4}R^2e^{-2bt}(2m\omega^2+mb^2) = \frac{1}{2}m\omega^2R^2e^{-2bT}$

$\omega >> b$

Energy stored in SHM

Also we have

$$\frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2+\frac{1}{2}Kx^2\right) = -\beta v^2$$

or, $-\left(\frac{dE}{dt}\right)dt = \beta\dot{x}^2 dt$

This energy is due to dampping

Now energy dissipated per cycle $= \beta\dot{x}^2 T$

Average energy dissipated per cycle is

$$\beta\dot{x}^2_{avg}T = \beta\left(\frac{R^2e^{-2bT}}{2}\omega^2\right)T$$

Now $\dot{x} = -Re^{-bt}\omega\sin(\omega t-\theta) - Re^{-bt}b\cos(\omega t-\theta)$

$$\dot{x}^2 = R^2\left[e^{-2bt}\omega^2\sin^2(\omega t-\theta) + e^{-2bt}b\cos^2(\omega t-\theta) + 2e^{-2bt}\omega b\sin(\omega t-\theta)\cos(\omega t-\theta)\right]$$

or, $\langle\dot{x}^2\rangle = R^2\left[e^{-2bt}\omega^2 + e^{-2bt}\cdot b^2\right]\frac{1}{2}$

$$= \frac{R^2e^{-2bt}}{2}(\omega^2+b^2)$$

Now if $\omega >> b$ then $\langle\dot{x}^2\rangle = \frac{R^2e^{-2bt}}{2}\cdot\omega^2$

Quality Factor:

$$Q = \frac{2\pi \times \text{avg energy stored per cycle}}{\text{Energy dissipeted per cycle}} = \frac{\omega}{2b}$$

A spring mass system has undamped natural frequency $\omega_0 = 100\,rad/s$ The Solution $x(t) = x_0(1+\omega_0 t)e^{-\omega_0 t}$ The system experience the maxm dampins

$\Rightarrow$ $x = x_0(1+\omega_0 t)e^{-\omega_0 t}$

$$\frac{dx}{dt} = x_0\left[\omega_0 e^{-\omega_0 t} + (1+\omega_0 t)(-\omega_0)\right]$$

$$= x_0\left[\omega_0 e^{-\omega_0 t} - \omega_0 e^{-\omega_0 t} - \omega_0^2 te^{-\omega_0 t}\right]$$

$$= -x_0\omega_0^2 te^{-\omega_0 t}$$

damping force.

$$f = -b\dot{x} = -b\left[-x_0 \omega_0^2 t e^{-\omega_0 t}\right]$$

$$f = b x_0 \omega_0^2 \left[t e^{-\omega_0 t}\right]$$

$$\frac{df}{dt} = b x_0 \omega_0^2 \left[e^{-\omega t} - \omega t e^{-\omega t}\right] \Rightarrow 0$$

$$t = \frac{1}{\omega} \qquad t = \frac{1}{\omega} = \frac{1}{100} = 0.01 \text{ sec.}$$

<u>09.08.2024</u>

⑤ <u>Forced Oscillation</u>

Equation of forced vibration

$$m\ddot{x} + \beta\dot{x} + Kx = F\sin pt$$

External Periodic force

or. $$\ddot{x} + \frac{\beta}{m}\dot{x} + \frac{K}{m}x = \frac{F}{m}\sin pt$$

or, $$\ddot{x} + 2b\dot{x} + \omega_0^2 x = f\sin pt$$

$$\ddot{x} + 2b\dot{x} + \omega_0^2 x = 0 \quad \text{(Homogenous part)}$$

or, $$m^2 + 2bm + \omega_0^2 = 0$$

$$m = \frac{-2b \pm \sqrt{4b^2 - 4\omega_0^2}}{2} = -b \pm \sqrt{b^2 - \omega_0^2}$$

$$CF = \left[c_1 e^{\sqrt{b^2-\omega_0^2}\,t} + c_2 e^{-\sqrt{b^2-\omega_0^2}\,t}\right] e^{-bt}$$

$$PI = \frac{1}{D^2 + 2bD + \omega_0^2} f\sin pt = f\frac{1}{-p^2 + 2bD + \omega_0^2}\sin pt$$

$$= f\frac{1}{(\omega_0^2 - p^2) + 2bD}\sin pt$$

$$= f\frac{(\omega_0^2 - p^2) - 2bD}{(\omega_0^2 - p^2)^2 - 4b^2D^2}\sin pt$$

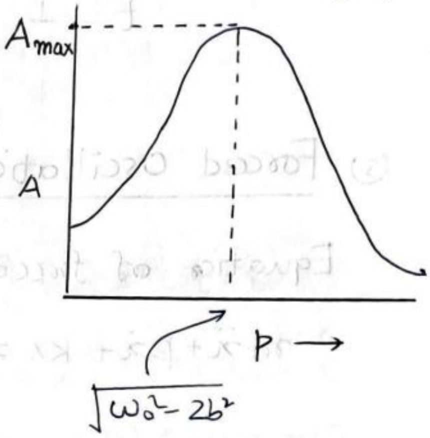
$$= f\frac{(\omega_0^2 - p^2) - 2bD}{(\omega_0^2 - p^2)^2 + 4b^2p^2}\sin pt$$

$$= f\frac{(\omega_0^2 - p^2)\sin pt - 2bp\cos pt}{(\omega_0^2 - p^2)^2 + 4b^2p^2}$$

$$PI = f\left[\frac{(\omega_0^2-p^2)}{(\omega_0^2-p^2)^2+4b^2p^2}\sin pt - \frac{2bp}{(\omega_0^2-p^2)^2+4b^2p^2}\cos pt\right]$$

$$= \frac{f}{\sqrt{(\omega_0^2-b^2)^2+4b^2p^2}}\left[\cos\alpha \sin pt - \sin\alpha \cos pt\right]$$

$$= \underbrace{\frac{f}{\sqrt{(\omega_0^2-p^2)^2+4b^2p^2}}}_{\text{Amplitude}} \underbrace{\sin(pt-\alpha)}_{\text{Periodic Part}} \qquad \tan\alpha = \frac{2bp}{\omega_0^2-p^2}$$



$$A = \frac{f}{\sqrt{(\omega_0^2-p^2)^2+4b^2p^2}} = \frac{f}{y}$$

$$y = (\omega_0^2-p^2)^2+4b^2p^2$$

$A = A_{max}$ at $y = y_{min}$

$$\frac{dy}{dp} = 2(\omega_0^2-p^2)(-2p) + 8b^2p$$

$$= -4p(\omega_0^2-p^2) + 8b^2p = 0$$

$$4(\omega_0^2-p^2) = 8b^2 \Rightarrow (\omega_0^2-p^2) = 2b^2$$

$$\Rightarrow p = \sqrt{\omega_0^2-2b^2}$$

$$\boxed{A_{max} = \frac{f}{2b\sqrt{\omega_0^2-b^2}}}$$

So, $$x = \frac{f}{\sqrt{(\omega_0^2-p^2)^2+4b^2p^2}}\sin(pt-\alpha)$$

$$v = \frac{dx}{dt} = \frac{pf}{\sqrt{(\omega_0^2-p^2)^2+4b^2p^2}}\cos(pt-\alpha)$$

or $$v_a = \frac{fp}{\sqrt{(\omega_0^2-p^2)^2+4p^2b^2}} \Rightarrow \text{Amplitude of Velocity}$$

For $v_a = (v_a)_{max}$, $\frac{dv_a}{dp} = 0$

$$\frac{dv_a}{dp} = \frac{[(\omega_0^2-p^2)^2+4p^2b^2] - p[2(\omega_0^2-p^2)(-2p)+8pb^2]}{[(\omega_0^2-p^2)^2+4b^2p^2]^2} = 0$$

$$(\omega_0^2-p^2)^2+4b^2p^2 = -2(\omega_0^2-p^2)(2p^2)+8p^2b^2$$

$$(\omega_0^2-p^2)^2+4b^2p^2 = -4p^2(\omega_0^2-p^2)+8p^2b^2$$

$$(\omega_0^2-p^2)^2+4p^2(\omega_0^2-p^2) = 4p^2b^2$$

So $V_a = \dfrac{fp}{\sqrt{(\omega_0^2-p^2)^2+4b^2p^2}}$ $\quad \dfrac{dV_a}{dp} = 0 \quad \omega_0 = p$

$$(V_a)_{max} = \frac{f}{2b}$$

we have, $p = \sqrt{\omega_0^2 - 2b^2}$ $\quad p \sim \omega_0$

10.08.2024

⑥ Work done by Forced Vibration

work done for displacement dx,

$$dw = F_0 \sin pt \, dx$$

Total work done. $w = \int_0^T F_0 \sin pt \frac{dx}{dt} dt$

Average work done $= \frac{1}{T}\int_0^T F_0 \sin pt \frac{dx}{dt} dt$

$$\langle W \rangle = \frac{1}{T}\int_0^T F_0 \sin pt \; AP\cos(pt-\alpha)\, dt$$

$$= \frac{1}{T}\int_0^T F_0 AP \sin pt \; (\cos pt \cos\alpha + \sin pt \sin\alpha) dt$$

$$= \frac{1}{T}\int_0^T F_0 AP \left[\langle \sin pt \cancel{\cos pt} \rangle^{0} \cos\alpha + \langle \sin^2 pt \rangle \sin\alpha \right] dt$$

$$= \frac{1}{T}\int_0^T F_0 AP \, \frac{1}{2}\sin\alpha \, dt = \frac{1}{2} F_0 AP \sin\alpha$$

⑦ Dissipation in Energy:

$$x = A\sin(pt-\alpha)$$

$$\frac{dx}{dt} = AP\cos(pt-\alpha)$$

$$E = \beta\int\left(\frac{dx}{dt}\right)^2 dt$$

$$E = \beta\int A^2p^2\cos^2(pt-\alpha)\,dt$$

~~Eve~~

$$E = \frac{1}{2}\beta A^2p^2 T.$$

Average energy dissipated per cycle

$$E_{av} = \frac{1}{T}\left(\frac{1}{2}\beta A^2p^2 T\right)$$

$$E_{av} = \frac{1}{2}\beta A^2p^2$$

So Energy dissipated per cycle is $\frac{1}{2}\beta p^2A^2$

Energy provided externally $\frac{1}{2}FAP\sin\alpha$

$$\frac{1}{2}\beta p^2A^2 = \frac{1}{2}FAP\sin\alpha$$

or
$$\sin\alpha = \frac{\beta PA}{F}$$

Now.
$$P = \frac{1}{2}\beta p^2A^2$$

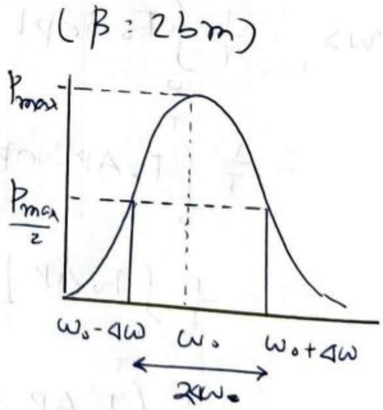
$$= \frac{1}{2}\beta p^2 \frac{f^2}{(\omega_0^2-p^2)^2+4b^2p^2}$$

$$= \frac{1}{2}\frac{\beta f^2}{\left(\frac{\omega_0^2}{p}-p\right)^2+4b^2}$$

For maximum Power. $\frac{\omega_0^2}{p}-p=0 \Rightarrow \omega_0=p$

$$P_{max} = \frac{\beta f^2}{8b^2} \quad (\beta = 2bm)$$

$$P_{max} = \frac{mf^2}{4b}$$



Half Power Point decides bandwidth and sharpness of resonance.

Sharpness of resonance $= \frac{\omega_0}{2\Delta\omega}$

ω_0 is frequency at resonance

$2\Delta\omega$ is full width of half maxima

$$p = \frac{mbf^2}{\left(\frac{\omega_0^2}{p} - p\right)^2 + 4b^2}$$

$$p = \frac{mbf^2}{\omega_0^2\left[\frac{\omega_0}{p} - \frac{p}{\omega_0}\right]^2 + 4b^2}$$

$p = \omega_0 + \Delta\omega$

$P = \frac{P_{max}}{2} = \frac{mf^2}{8b}$

$$\frac{mf^2}{8b} = \frac{mbf^2}{\omega_0^2\left[\frac{\omega_0}{\omega_0+\Delta\omega} - \frac{\omega_0+\Delta\omega}{\omega_0}\right]^2 + 4b^2}$$

$$8b^2 = \omega_0^2\left[\frac{\omega_0}{\omega_0+\Delta\omega} - \frac{\omega_0+\Delta\omega}{\omega_0}\right]^2 + 4b^2$$

$$4b^2 = \omega_0^2\left[\frac{\omega_0}{\omega_0+\Delta\omega} - \frac{\omega_0+\Delta\omega}{\omega_0}\right]^2$$

$$2b = \omega_0\left[\frac{\omega_0}{\omega_0+\Delta\omega} - \frac{\Delta\omega+\omega_0}{\omega_0}\right]$$

$$2b = \omega_0\left[\frac{\omega_0^2 - (\omega_0+\Delta\omega)^2}{\omega_0(\omega_0+\Delta\omega)}\right]$$

$$2b = \omega_0\left[\frac{\omega_0^2 - \omega_0^2 - 2\omega_0\Delta\omega + (\Delta\omega)^2}{\omega_0(\omega_0+\Delta\omega)}\right]$$

$$2b = -\frac{\Delta\omega(\Delta\omega + 2\omega_0)}{(\omega_0+\Delta\omega)}$$

$$2b = -\frac{2\omega_0\Delta\omega}{\omega_0+\Delta\omega}$$

$$2b = -2\Delta\omega\left(1 - \frac{\Delta\omega}{\omega_0}\right)$$

$$\frac{2b}{\omega_0} = \frac{-2\Delta\omega}{\omega_0}$$

Sharpness of Resonance $S = \frac{\omega_0}{2b}$

we have,
$$2b = \frac{-\Delta\omega\,[\Delta\omega + 2\omega_0]}{(\omega_0 + \Delta\omega)}$$

$$E_{avg} = \frac{1}{4} m A^2 (\omega^2 + p^2)$$

$$E_{dis} = \frac{1}{2} \beta A^2 p^2$$

Quality factor.

$$Q = \frac{2\pi \times \text{Average Energy stored Per Cycle}}{\text{Average Energy dissipated Per Cycle}}$$

or, $$Q = \frac{2\pi \times \frac{1}{4} m A^2 (\omega^2 + p^2)}{\frac{1}{2}\beta A^2 p^2 T}$$

or $$Q = \frac{\pi}{2b} \cancel{m} \left(\frac{\omega^2}{p^2} + 1\right) = \frac{p}{4b}\left(\frac{\omega^2}{p^2} + 1\right)$$

or. $$Q_{min} = \frac{\omega_0}{2b}$$

⑧ <u>Bandwidth:</u>

At half Power frequncies we have $\langle P \rangle = \frac{P_{max}}{2}$

$$\cancel{\frac{1}{2}} \frac{\cancel{\beta} p^2 \cancel{f^2}}{(\omega_0^2 - p^2)^2 + 4b^2p^2} = \cancel{\frac{1}{2}} \times \frac{\cancel{\beta f^2}}{8b^2}$$

or $(\omega_0^2 - p^2)^2 + 4b^2p^2 = 8b^2p^2$

or $(\omega_0^2 - p^2)^2 = 4b^2p^2$

or $\omega_0^2 - p^2 = \pm 2bP$

$p^2 \pm 2bP - \omega_0^2 = 0$

$$p = \frac{\mp 2b \pm \sqrt{4b^2 + 4\omega_0^2}}{2} = \mp b \pm \sqrt{b^2 + \omega_0^2}$$

$P_1 = b + \sqrt{b^2 + \omega_0^2}$

$P_2 = -b + \sqrt{b^2 + \omega_0^2}$

$BW = \Delta P = P_1 - P_2 = \cancel{2P}$
$= 2b$

Also quality factor
$$Q = \frac{\omega_0}{BW} = \frac{\omega_0}{2b}$$

① A particle of mass m executes SHM, the KE_{avr} during it's motion from the position of Eq bm to end

$\Rightarrow$ Kinetic Energy. $KE = \frac{1}{2} m\omega^2 a^2 \cos^2 \omega t$

$$\langle KE \rangle = \frac{\frac{1}{2} m\omega^2 a^2 \int_0^{T/4} \cos^2 \omega t \, dt}{\int_0^{T/4} dt} = \frac{1}{4} m\omega^2 a^2$$

$$= \pi^2 m a^2 \nu^2$$

$T = \frac{2\pi}{\omega} = \frac{1}{\nu}$

$\omega = 2\pi\nu$

---

② The differential equation of damped motion is $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$ at $x(0) = 0$, $\dot{x}(0) = 1$. The displacement $x(t)$ attains maximum value when $t =$ — sec

$\Rightarrow$ $m^2 + 2m + 1 = 0$ $\qquad x = (C_1 + C_2 t) e^{-t}$

or $(m+1)^2 = 0$

$m = -1, -1$ $\qquad \frac{dx}{dt} = C_2 e^{-t} - (C_1 + C_2 t) e^{-t}$

$x(0) = 0$ $\qquad C_2 = 1$ $\qquad \frac{dx}{dt} = e^{-t} - t e^{-t} = 0$

$C_1 = 0$

$t = 1$ sec

---

③ An undamped oscillator has a period $\tau_0 = 1$ sec after adding in the system the period changes to $\tau_1 = 1.001$ sec. The damping factor is

$\Rightarrow$ $\tau_0 = \frac{2\pi}{\omega} = 1$ $\qquad \tau_1 = \frac{2\pi}{\sqrt{\omega^2 - b^2}} = 1.001$

$(1.001)^2 = \frac{4\pi^2}{(2\pi)^2 - b^2}$

$b = 2\pi\sqrt{1 - \frac{1}{(1.001)^2}}$ $\qquad 4\pi^2 - b^2 = \frac{4\pi^2}{(1.001)^2}$

$b = 0.281$

$b^2 = 4\pi^2\left(1 - \frac{1}{(1.001)^2}\right)$

---

③ A thin rod of length L suspends from a point on its length such that it can oscillate about a horizontal axis, through point of suspension. If time period of oscillation is $2\pi\sqrt{7L/12g}$ The distance of a point of suspension from centre of rod is

⇒
$$T = 2\pi\sqrt{\frac{I}{mgx}} = 2\pi\sqrt{\frac{7L}{12g}} \qquad I = \frac{ML^2}{12} + Mx^2$$

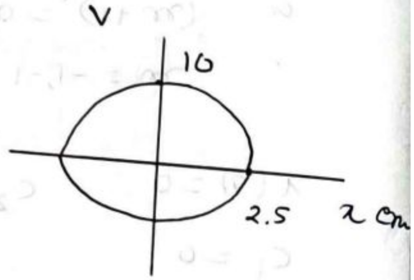
$$2\pi\sqrt{\frac{\frac{ML^2}{12} + Mx^2}{mgx}} = 2\pi\sqrt{\frac{7L}{12g}}$$

$$\frac{7L}{12} = \frac{L^2}{x12} + x \quad \text{or} \quad 7L = \frac{L^2}{x} + 12x$$

Length is $\qquad L^2 + 12x^2 - 7Lx = 0$

$\frac{L}{3}$ and $\frac{L}{4}$ $\qquad x = \frac{L}{4}, \frac{L}{3}$

④ A particle is performing SHM
Find, T, a_{max},



⇒
$$E = \frac{1}{2}Kx^2 + \frac{1}{2}mv^2$$

or
$$\frac{x^2}{\left(\sqrt{\frac{2E}{K}}\right)^2} + \frac{v^2}{\left(\sqrt{\frac{2E}{m}}\right)^2} = 1$$

$$\sqrt{\frac{2E}{K}} = 2.5 \qquad \sqrt{\frac{2E}{m}} = 10$$

$$\omega = \sqrt{\frac{K}{m}} = \frac{10}{2.5} = 4 \text{ rad/s} \qquad T = \frac{2\pi}{\omega} = 1.57 \text{ sec}$$

$$a_{max} = \omega^2 A = 4^2 \times 2.5 = 40 \text{ cm/s}$$

Wave Motion

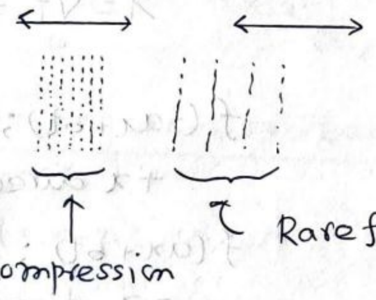
14.08.2024

Form of energy transfer without the actual movement of constituent Particles.

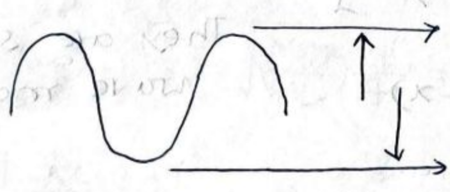
wave →
- → Mechanical waves (Medium required)
  water wave, Sound wave
- → Non Mechanical wave (No need of medium)
  (Electromagnetic wave)

Waves

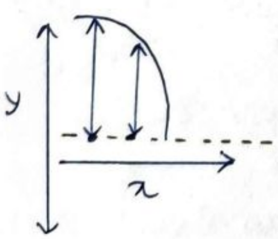
Longitudinal waves	Transverse waves
⇓	⇓
Direction of Particle Vibration & wave motion is in same direction	Direction of Particle vibration is $\perp$ to wave motion
⇓	⇓
Sound waves	EMWs.



Longitudal



Transverse



$y = f(ax \pm bt)$

① where is the Particle

② which time

① General Equation wave Motion

$$y = f(ax \pm bt)$$
$$= f(x \pm vt)$$

$$y = Ae^{i(\omega t \pm Kx)}$$

$$v = \omega / K$$

Velocity of wave $= \dfrac{\text{coeff of } t}{\text{coeff of } x}$

$y = f(x+vt)$ $\qquad \dfrac{dy}{dt}$ is Particle velocity

or $\dfrac{dy}{dt} = v f'(x+vt)$ $\qquad v\dfrac{dy}{dx} = \dfrac{dy}{dt}$

or $\dfrac{dy}{dx} = f'(x+vt)$

Now. $\dfrac{d^2y}{dt^2} = v^2 f''(x+vt)$ $\qquad \dfrac{d^2y}{dx^2} = f''(x+vt)$

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \Rightarrow \boxed{\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}}$$ Differential Eqn of wave motion

$$y = A\sin(\omega t + Kx)$$
$$= A\cos(\omega t + Kx)$$
$$= Ae^{i(\omega t + Kx)}$$

$\lambda = vT \Rightarrow \boxed{v = \lambda\nu}$

$f(-ax+bt)$; $f(ax-bt)$
$+x$ direction

$f(ax+bt)$; $f(-ax-bt)$
$-x$ direction

$$y = A\sin(\omega t - Kx)$$

$$y = A\sin\left[\frac{2\pi}{\lambda}vt - \frac{2\pi}{\lambda}x\right]$$

$$y = A\sin\frac{2\pi}{\lambda}(vt - x)$$

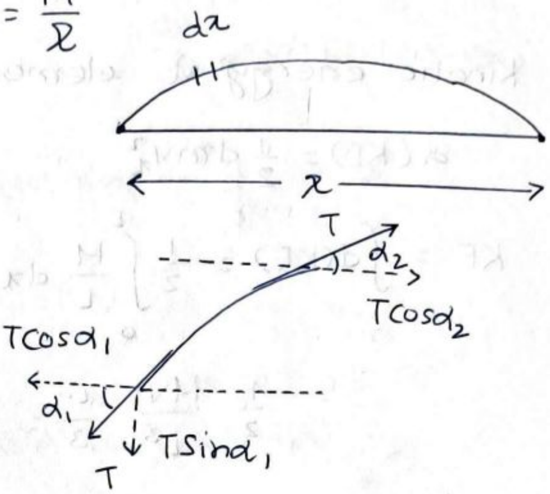
They are SHM, wave motion

$$y = \frac{1}{1+(x-vt)^2} \Rightarrow$$ Also wave motion but not Simple Harmonic.

15.08.24

② Velocity of transverse wave in a stretched string:

Mass per unit length. $\lambda = \frac{M}{l}$



Here α_1 and α_2 are very small an equal,

$\cos\alpha_1 \simeq \cos\alpha_2 = 1$

Net force

$= T\sin\alpha_2 - T\sin\alpha_1$

$= T(\tan\alpha_2 - \tan\alpha_1)$

$= T\left[\left(\frac{dy}{dx}\right)_{x+\delta x} - \left(\frac{dy}{dx}\right)_x\right]$

As α_1, α_2 are small

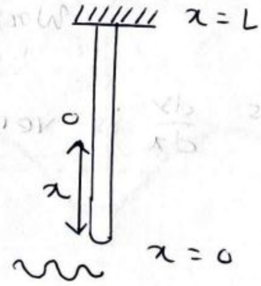
$\sin\alpha_2 \simeq \sin\alpha_1 \simeq \tan\alpha_1 \simeq \tan\alpha_2$

$$= \frac{T\left[\left(\frac{dy}{dx}\right)_{x+\delta x} - \left(\frac{dy}{dx}\right)_x\right]}{dx}\, dx = T\left(\frac{d^2y}{dx^2}\right)dx$$

Also force $= (\lambda dx)\frac{d^2y}{dt^2}$ so $\lambda dx \frac{d^2y}{dt^2} = T\frac{d^2y}{dx^2}dx$

$$\frac{d^2y}{dt^2} = \frac{T}{\lambda}\frac{d^2y}{dx^2} \Rightarrow \frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2} \Rightarrow v = \sqrt{\frac{T}{\lambda}}$$

Time required to reach the rod.



Tension at o: $T = \frac{Mgx}{L}$

$\lambda = \frac{M}{l}$

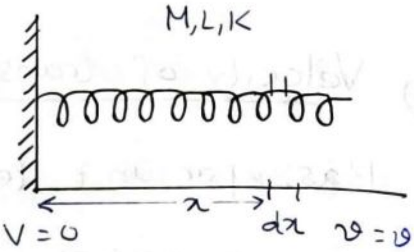
Velocity $v = \sqrt{\frac{T}{\lambda}} = \sqrt{\frac{Mgx}{L}\cdot\frac{L}{M}} = \sqrt{gx} = \frac{dx}{dt}$

$$\int_0^L \frac{dx}{\sqrt{x}} = \sqrt{g}\int_0^t dt \qquad t = \; = 2\sqrt{L/g}$$

$+\frac{1}{2}x^{+3/2}\Big|_0^L = t$

After time $2\sqrt{L/g}$ the wave will reached to the top.

③ Massive Spring:



Kinetic energy of element

$$d(KE) = \frac{1}{2} dm v_x^2$$

$$KE = \int d(KE) = \frac{1}{2}\int_0^L \frac{M}{L}\cdot dx \cdot \left(\frac{Vx}{L}\right)^2 = \frac{1}{2}\cdot\frac{M}{L}\cdot\frac{V^2}{L^2}\int_0^L x^2 dx$$

$$= \frac{1}{2}\,\frac{MV^2}{L^3}\cdot\frac{L^3}{3} = \frac{1}{6}MV^2 = \frac{1}{2}\left(\frac{M}{3}\right)V^2 = \frac{1}{2}M_e V^2$$

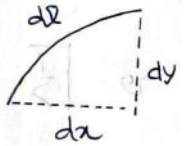
Here effective mass is $M_e = \frac{M}{3}$ used everywhe

Total mass of spring never contribute only 1/3 of total mass is taken

> For a massive spring time Period $T = 2\pi\sqrt{\frac{m + M_s/3}{K}}$

④ Energy & Power Calculation

16.08.2024



Net displacement on the string

$$= (dl - dx)$$

Work done, $dW = T(dl - dx)$

As $\frac{dy}{dx}$ is very small

$$= T\left(\sqrt{(dx)^2 + (dy)^2} - dx\right)$$

$$= T\,dx\left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2} - 1\right)$$

$$dW = T\,dx\left[1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2 - 1\right] = \frac{1}{2}T\left(\frac{dy}{dx}\right)^2 dx$$

$y = A\sin(\omega t - Kx)$

$\frac{dy}{dx} = -KA\cos(\omega t - Kx)$

$V = \frac{\omega}{K} = \sqrt{\frac{T}{\lambda}}$

$\lambda \frac{\omega^2}{K^2} = T$

$$\frac{dW}{dx} = \frac{1}{2}T\left(\frac{dy}{dx}\right)^2$$

$$\frac{dW}{dx} = \frac{T}{2}K^2A^2\cos^2(\omega t - Kx)$$

$$\frac{dW}{dx} = \frac{\lambda\omega^2}{2K^2}\cdot K^2A^2\cos^2(\omega t - Kx)$$

$$\frac{dW}{dx} = \frac{dU}{dx} = \frac{1}{2}\lambda\omega^2A^2\cos^2(\omega t - Kx)$$

Kinetic energy in a small length

$d\,KE = \frac{1}{2}(dm)v_p^2$

$y = A\sin(\omega t - Kx)$

$\frac{dy}{dt} = \omega A\cos(\omega t - Kx)$

$d\,KE = \frac{1}{2}(dm)\,\omega^2A^2\cos^2(\omega t - Kx)$

or $\frac{dKE}{dx} = \frac{1}{2}\lambda\,dx\,\omega^2A^2\cos^2(\omega t - Kx)$

$\frac{d(KE)}{dx} = \frac{1}{2}\lambda\omega^2A^2\cos^2(\omega t - Kx)$

Now $\frac{d}{dx}(U + KE) = \frac{dE}{dx} = \lambda\omega^2A^2\cos^2(\omega t - Kx)$

$\frac{dE}{dt} = \frac{dE}{dx}\cdot\frac{dx}{dt} = v\frac{dE}{dx} = \lambda\omega^2A^2v\cos^2(\omega t - Kx) = \text{Power}$

Now, Average power $= \frac{1}{2}\lambda\omega^2A^2v = P_{avg}$

$P_{avg} = \frac{1}{2}\lambda\omega^2A^2v$ (α is area)

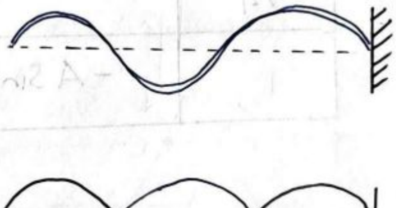
$\frac{P_{avg}}{Area} = \frac{E}{\alpha t} = I = \frac{1}{2}\lambda\omega^2A^2\frac{v}{\alpha}$

$= \frac{1}{2}\frac{M}{x}\omega^2A^2\frac{v}{\alpha} = \frac{1}{2}\frac{M}{x\alpha}\omega^2A^2v$

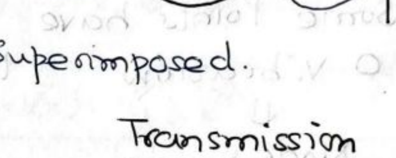
$= \frac{1}{2}\frac{M}{V}\omega^2A^2v = \frac{1}{2}\rho\omega^2A^2v$

$\boxed{I = \frac{1}{2}\rho\omega^2A^2v}$

Stationary waves

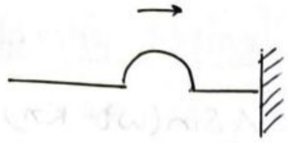


> Same Amplitude
> Same frequency
> opposite direction
> Incoming & outgoing will be superimposed.

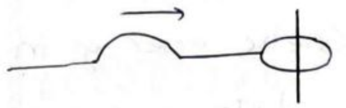


Properties	Reflection	Transmission
v	Same	change
f, ω	Same	Same
$\lambda, K = \frac{2\pi}{\lambda}$	Same	change

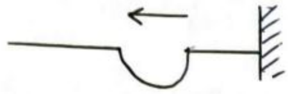
Phase diff: Rarer to denser (180°)
denser to Rarer (No change)



$y = A\sin(\omega t - Kx)$



$y_i = A\sin(\omega t - Kx)$



$y = -A\sin(\omega t + Kx)$

$y = A\sin(\omega t + Kx + \pi)$

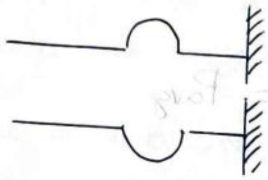
(Rigid)

$y_R = A\sin(\omega t + Kx)$

(Non rigid)

(No phase difference)

⑤ Reflection through a Rigid Support: 17.08.2024



$y_i = A\sin(\omega t - Kx)$

$y_R = -A\sin(\omega t + Kx)$

t	$y_i = A\sin(\omega t - Kx)$	$y_R = -A\sin(\omega t + Kx)$	y_t
0	$-A\sin Kx$	$-A\sin Kx$	$-2A\sin Kx$
$\frac{T}{4}$	$A\cos Kx$	$-A\cos Kx$	0
$T/2$	$A\sin Kx$	$A\sin Kx$	$2A\sin Kx$
$3T/4$			0
T	$-A\sin Kx$	$-A\sin Kx$	$-2A\sin Kx$

Some Points have 0 Vibration

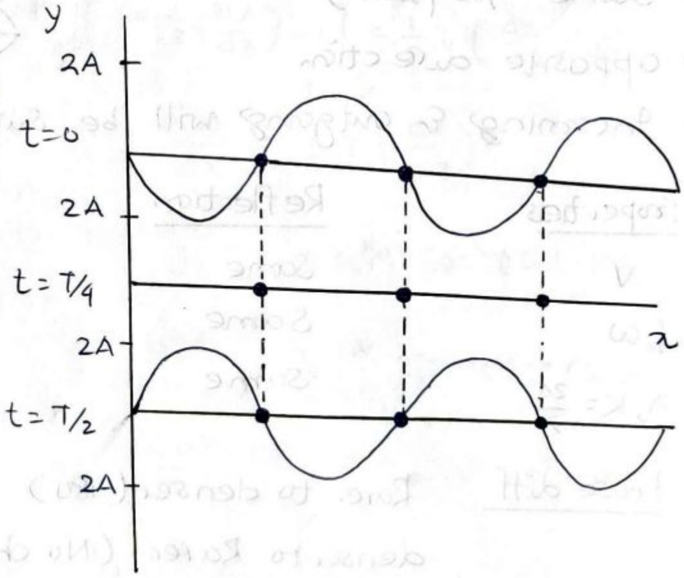
⇓

Nodes

Some Point Vibrate with double Amplitude

⇓

Antinode



Points have zero displacement is Node

Points have 2A displacement is Antinode

> Distance between two succesive node is $\lambda/2$

succesive antinode is $\lambda/2$

For nodes, $\sin Kx = 0$

$$y = y_i + y_R = A\sin(\omega t - Kx) - A\sin(\omega t + Kx)$$

$$y = 2A\sin\left(\frac{\omega t - Kx - \omega t - Kx}{2}\right)\cos\left(\frac{\omega t - Kx + \omega t + Kx}{2}\right)$$

$$y = -2A\sin Kx\cos\omega t$$

For nodes. $y = 0$

$\sin Kx = 0$

$Kx = n\pi$

or $\frac{2\pi}{\lambda}x = n\pi$

or $x = n\frac{\lambda}{2}$

$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \dots$

For antinode.

$\sin Kx = 1$

$\sin Kx = \sin(2n+1)\frac{\pi}{2}$

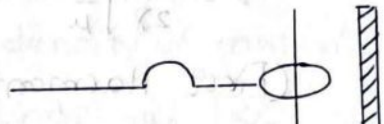
$\frac{2\pi}{\lambda}x = (2n+1)\frac{\pi}{2}$

$x = (2n+1)\frac{\lambda}{4}$

$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \dots$

⑥ Reflection through a Free End:

Resultant waves



$y_i = A\sin(\omega t - Kx)$

$$y = y_i + y_R$$

$$= A\sin(\omega t - Kx) + A\sin(\omega t + Kx)$$

$$= 2A\sin\omega t\cos Kx$$

$y_R = A\sin(\omega t + Kx)$

t	$y_i = A\sin(\omega t - Kx)$	$y_R = A\sin(\omega t + Kx)$	y_{Rt}
0	$-A\sin Kx$	$A\sin Kx$	0
T/4	$A\cos Kx$	$A\cos Kx$	$2A\cos Kx$

For Nodes,

$$\cos kx = 0$$

$$\cos kx = \cos(2n+1)\frac{\pi}{2}$$

$$\frac{2\pi}{\lambda}x = (2n+1)\frac{\pi}{2}$$

$$x = (2n+1)\frac{\lambda}{4} \quad \left(x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots\right)$$

Antinodes

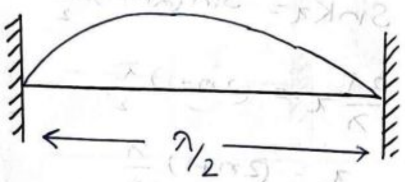
$$\cos kx = \pm 1$$

$$\cos kx = \cos n\pi$$

$$\frac{2\pi}{\lambda}\cdot x = \pi \cdot n$$

$$x = n\frac{\lambda}{2} \quad \left(x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2} \dots\right)$$

⑦ String Fixed at Both Ends:



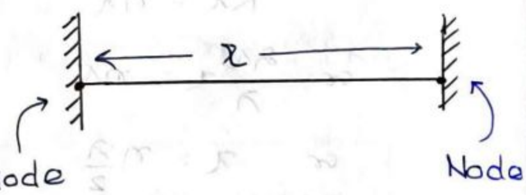
Fundamental Mode

Here $l = \lambda/2$

$$\lambda = 2l$$

$$\nu = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

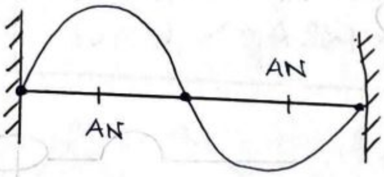
(First Harmonic)



$$v = \sqrt{\frac{T}{\mu}}$$

$$v = \nu\lambda$$

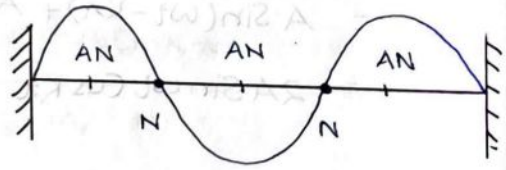
$$\nu = \frac{v}{\lambda}$$



Here, $l = \lambda$

$$\nu = \frac{1}{l}\sqrt{\frac{T}{\mu}}$$

(First overtone/
Second Harmonic)



Here, $\frac{3\lambda}{2} = l$

$$\lambda = \frac{2}{3}l$$

$$\nu = \frac{3}{2l}\sqrt{\frac{T}{\mu}}$$

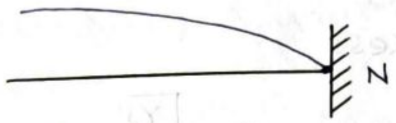
(Second overtone/
Third harmonic)

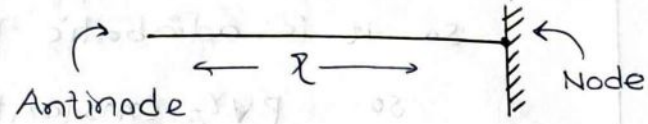
For nth Harmonic or
$(n-1)$th overtone

$$\boxed{\nu_n = \frac{n}{2l}\sqrt{\frac{T}{\mu}}}$$

T is tension
μ is mass density

(8) only one End is fixed:

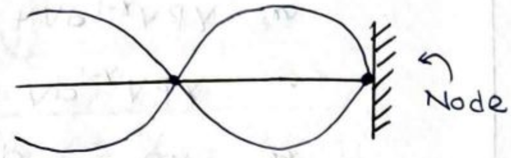




Here $l = \frac{\lambda}{4}$

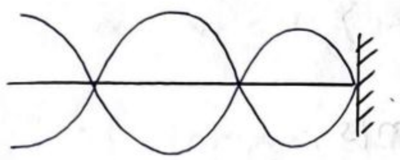
$\lambda = 4l$

$\nu = \frac{1}{4l}\sqrt{\frac{T}{\mu}}$



$l = \frac{3\lambda}{4} \Rightarrow \lambda = \frac{4l}{3}$

$\nu = \frac{3}{4l}\sqrt{\frac{T}{\mu}}$



$\frac{5\lambda}{4} = l$ $\quad \nu = \frac{5}{4l}\sqrt{\frac{T}{\mu}}$

$\lambda = \frac{4l}{5}$

only odd Harmonic are presents, evens are absent

$\nu_n = n\left(\frac{1}{4l}\sqrt{\frac{T}{\mu}}\right)$

(n is odd number)

Longitudinal waves: 18.08.2024

> Velocity of longitudinal wave in any medium is given by $v = \sqrt{\frac{K}{\rho}}$ $\quad$ K is Bulk modulus, ρ is density of material

> Newton's theory of Velocity of Sound. $v = \sqrt{K/\rho}$

$PV = nRT = \text{Constant}$

$PdV + VdP = 0$

or $PdV = -VdP$

or $P = -\frac{dP}{\left(\frac{dV}{V}\right)} = K$

$v = \sqrt{P/\rho}$

Bulk Modulus $\approx$ Pressure when T is constant

For air. $P = 1.1 \times 10^5$ Pa

$\rho = 1.29$ Kg/m³

$v = \sqrt{\frac{P}{\rho}} = 292$ m/s

But at room temperature Velocity of Sound $= 330$ m/s

Laplace Correction:

The Process of travelling of Sound is very fast

So it is adiabatic Process

so $PV^{\gamma} = \text{Constant}$ $\qquad v = \sqrt{\frac{\gamma P}{\rho}}$

or $\gamma P V^{\gamma-1} dV + V^{\gamma} dP = 0$

or $\gamma P V^{\gamma-1} dV = -V^{\gamma} dP$

or $\gamma P = -\frac{dP}{(dV/V)} = K$ = Bulk Modulus

So Velocity of Sound, $v = \sqrt{\frac{\gamma P}{\rho}}$

$P = 1.1 \times 10^5$ Pa

$\rho = 2.93$ Kg/m³ $\qquad v = 332$ m/s

$\gamma = 1.4$

$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

$PV = nRT$

$PV = \frac{w}{M} RT$

$P = \frac{\rho RT}{M} \Rightarrow \frac{P}{\rho} = \frac{RT}{M}$

Beats:

$y_1 = A_1 \sin 2\pi\nu_1 t$ $\qquad \nu_1 > \nu_2$ are very close to each other

$y_2 = A_2 \sin 2\pi\nu_2 t$ $\qquad \nu_1 > \nu_2$

$= A_2 \sin 2\pi[\nu_1 - (\nu_1 - \nu_2)]t$

$y_2 = A_2 \sin 2\pi\nu_1 t \cos 2\pi(\nu_1-\nu_2)t - A_2 \sin 2\pi(\nu_1-\nu_2)t \cos 2\pi\nu_1 t$

$y = y_1 + y_2 = [A_1 + A_2 \cos 2\pi(\nu_1-\nu_2)t] \sin 2\pi\nu_1 t$

$\qquad - \cos 2\pi\nu_1 t \, (A_2 \sin 2\pi(\nu_1-\nu_2)t)$

$y = A\cos\theta \sin 2\pi\nu_1 t - A\sin\theta \cos 2\pi\nu_1 t$

$y = A\sin(2\pi\nu_1 t - \theta)$

Now $A = \sqrt{[A_1 + A_2\cos 2\pi(\nu_1-\nu_2)t]^2 + [A_2 \sin 2\pi(\nu_1-\nu_2)t]^2}$

$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos 2\pi(\nu_1-\nu_2)t}$

$A_{max} = (A_1 + A_2)$ $\qquad$ Here, $(\nu_1 - \nu_2)$ is called Beat frequency.

$\cos 2\pi(\nu_1-\nu_2)t = 1$

$t = \frac{n}{\nu_1 - \nu_2}$

19.08.24

Factors on which effect the Velocity of Sound:

① Velocity of Sound. $V = \sqrt{\frac{\gamma RT}{M}} \Rightarrow V \propto \sqrt{T}$

At temperature $t°C = (t+273)K$

So $\frac{V_t}{V_0} = \sqrt{\frac{t+273}{273}}$

or $\frac{V_t}{V_0} = \left(1+\frac{t}{273}\right)^{1/2}$

or $V_t = V_0\left(1+\frac{t}{546}\right)^{1/2}$

$V_t = V_0 + \left(\frac{332}{546}\right)t$

$\boxed{V_{t°C} = V_{0°C} + 0.61t}$

Temperature is taken in °C scale. (t shold be Small)

> If temperature increase 1°C. change in velocity of Sound is 0.61 m/s

② Velocity. $V = \sqrt{\frac{\gamma P}{\rho}}$ if $P\uparrow, V\downarrow, \rho\uparrow$

So $\frac{P}{\rho}$ = Constant

Independent of Pressure

③ Density of Humid air $\rho_{Humid} < \rho_{dry\ air}$

$(V_s)_{dry\ air} < (V_s)_{Humid}$ Velocity increase with Humid.

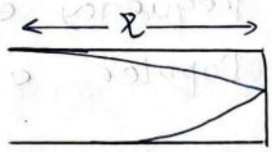
Organ Pipe:

Basically it is two types -

(a) open end organ Pipe (Both open)

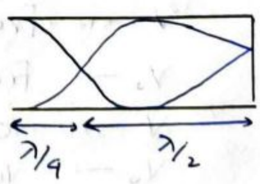
(b) closed end organ Pipe (one open)

closed end organ pipe:



$\lambda = 4l$ $\quad \nu = \frac{V}{\lambda}$ First Harmonic/

$\nu_0 = \frac{V}{4l}$ — ① Fundamental freq



$l = \frac{3\lambda}{4}$ $\quad \nu_1 = 3\left(\frac{V}{4l}\right)$

$\lambda = \frac{4}{3}l$ $\quad \boxed{\nu_1 = 3\nu_0}$

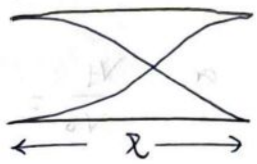
Third Harmonic or first overtone.

For nth frequency, $\nu_n = (2n+1)\nu_0$

$$\nu_n = (2n+1)\frac{v}{4l}$$

All Even harmonics are absent here.

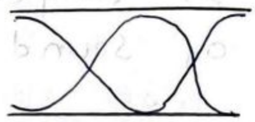
Open end organ Pipe:



So here $\frac{\lambda}{2} = l$

$\lambda = 2l$

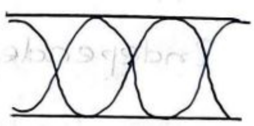
Fundamental frequency, $\nu_1 = \frac{v}{2l}$



So here $\lambda = l$

frequency, $\nu_2 = 2\left(\frac{v}{2l}\right) = 2\nu_1$

First overtone or second Harmonic



So here $\frac{3\lambda}{2} = l \Rightarrow \lambda = \frac{2}{3}l$

$\nu_3 = 3\left(\frac{v}{2l}\right) = 3\nu_1$

Second overtone or third Harmonic

If both end is open then all Harmonics are possible

Doppler Effect:

whenever there is a relative motion between an observer and Source the frequency received by observer is different as compared to the frequency emitted by source. This effect is called Doppler effect

$$\nu = \nu_0 \left[\frac{V \mp V_o}{V \pm V_s}\right]$$

ν → Freq received by observer
ν_0 → Freq emitted by source
V → Velocity of Sound
V_o → Velocity of observer
V_s → Velocity of Source.

Case ①:

when observer is stationary and source is moving away from observer.

So Basically, $V_o = 0$. $\qquad \nu = \nu_o \left(\frac{V}{V+V_s} \right)$

Case ② Source is stationary and observer is moving away from Source

so $V_s = 0 \qquad \nu = \nu_o \left(\frac{V-V_o}{V} \right)$

Case ③ Source is stationary and observer is moving toward the source

$V_s = 0 \qquad \nu = \nu_o \left(\frac{V+V_o}{V} \right)$

Case ④ observer is stationary and source is moving toward the observer

$V_o = 0 \qquad \nu = \nu_o \left(\frac{V}{V-V_s} \right)$

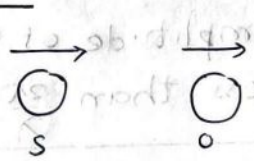
Case ⑤ both are moving toward each other

$$\nu = \nu_o \left(\frac{V+V_o}{V-V_s} \right)$$

Case ⑥ Both moving away from that

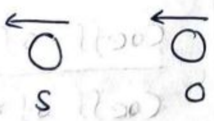
$$\nu = \nu_o \left(\frac{V-V_o}{V+V_s} \right)$$

Case ⑦



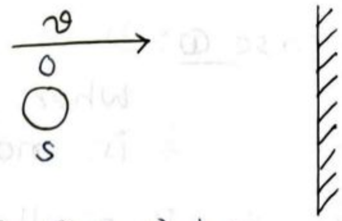
$$\nu = \nu_o \left(\frac{V-V_o}{V-V_s} \right)$$

Case ⑧



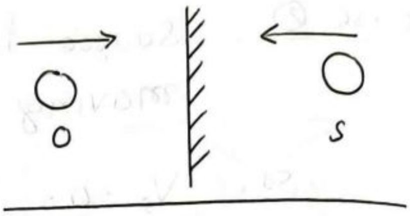
$$\nu = \nu_o \left(\frac{V+V_o}{V+V_s} \right)$$

The Source emit the Signal and that reflected on wall and



it also receive the signal. So it Cannot be say Source.

Image will behave like Source. Both try to increase frequency so apparent frequency



$$\nu = \nu_0 \left(\frac{V+V_0}{V-V_s}\right)$$

---

① The plane wave represented by an equation of the form $y = f(x-vt)$ implies the Propagation along positive x axis without change of shape with constant velocity

$\Rightarrow$ $y = f(x-vt)$

$$\frac{dy}{dx} = f'(x-vt)$$

$$\frac{dy}{dt} = -vf'(x-vt)$$

$$\frac{dy}{dt} = -v\frac{dy}{dx}$$

$$\frac{d^2y}{dt^2} = v^2 f''(x-vt)$$

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

---

② In a plane progressive wave particle is less than the wave speed if

$\Rightarrow$ $y = a\sin\frac{2\pi}{\lambda}(vt-x)$

particle speed, $= \left(\frac{2\pi}{\lambda}av\right)$

wave speed $= v$

$$\frac{2\pi}{\lambda}av < v \Rightarrow a < \frac{\lambda}{2\pi}$$

Amplitude of wave is less than $\frac{2\pi}{\lambda}$

---

③ A transverse wave is described by $y = y_0\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$ the maximum particle velocity is equal to 4 times the wave velocity if

$\Rightarrow$ $y = y_0\sin 2\pi\left(ft - \frac{x}{\lambda}\right)$

wave Velocity $= \frac{\text{Coeff. of } t}{\text{Coeff. of } x} = \lambda f$

$$\frac{dy}{dt} = 2\pi f y_0$$

$$2\pi f y_0 = 4\lambda f$$

$$\lambda = \frac{\pi y_0}{2}$$

Amplitude of waves in positive x axis $y = \frac{1}{1+x^2}$ at $t = 0$ and $y = \frac{1}{1+(x-1)^2}$ at $t = 2$ sec. Velocity of wave in m/s is

$\Rightarrow$ $$y = \frac{1}{1+(x-vt)^2} \qquad \text{At } t = 2 \quad y = \frac{1}{1+(x-2v)^2} = \frac{1}{1+(x-1)^2}$$

so $2v = 1 \Rightarrow v = 1/2$ m/s

---

5) A wave pulse along x-axis is given by, $y = \frac{0.5}{1+(\pi x-10t)^2}$ m

Speed of the pulse

$\Rightarrow$ $$y = \frac{1}{1+f(ax-bt)^2} \qquad \text{Speed} = \frac{\text{Coeff } t}{\text{Coeff of } x} = \frac{10}{\pi} \text{ m/s}$$

---

6) String 1 has twice the length, twice the radius, twice the tension and twice the density of other string 2. The relation between the fundamental frequencies

$\Rightarrow$ Fundamental frequencies. $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$ $\qquad f_2 = 4f_1$

$$f = \frac{1}{2l}\sqrt{\frac{lT}{M}} = \frac{1}{2l}\sqrt{\frac{lT}{\rho \times \frac{4}{3}\pi R^3}} \Rightarrow f = \sqrt{\frac{T}{\rho \times R^3 \times l}}$$

$$\frac{f_1}{f_2} = \left(\frac{T_1}{T_2} \times \frac{\rho_2}{\rho_1} \times \frac{R_2^3}{R_1^3} \times \frac{l_2}{l_1}\right)^{1/2} = \left(2 \times \frac{1}{2} \times \frac{1}{8} \times \frac{1}{2}\right)^{1/2} = \frac{1}{4}$$

---

7) A stationary wave is given by $\frac{d^2y}{dt^2} = 11.56 \times 10^4 \frac{d^2y}{dx^2}$ is established in $L = 17$m long pipe filled with gas grid closed at both ends. The Permissible frequencies are

$\Rightarrow$ Velocity $V = \sqrt{11.56 \times 10^4} = 340$ m/s $\qquad L = 17$m, $\lambda = 34$m

$$\nu_0 = \frac{V}{2l} = \frac{340}{34} = 10 \text{ Hz}$$

Allowed freques are. $\nu = n\nu_0 = 10, 20 \ldots 100 \quad 250$ Hz

8) Polarization Cannot occur at sound waves.

---

(9) The equation $y = A\cos^2(2\pi ft - \frac{2\pi x}{\lambda})$ represents a wave

$\Rightarrow$ $y = \frac{1}{2}A\left(1 + \cos\left(4\pi ft - \frac{4\pi x}{\lambda}\right)\right)$

$y = \frac{A}{2} + \frac{1}{2}A\cos 4\pi\left(ft - \frac{x}{\lambda}\right)$ $\qquad \omega = 4\pi f = 2\pi f_0$

$f_0 = 2f$

Amplitude is $\frac{A}{2}$ $\qquad$ wavelength is $\lambda/2$

Frequency is $2f$

---

(10) The extension in a string obeying Hooke's law is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$. The speed of sound will be

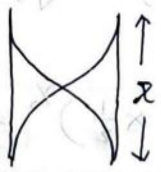
$\Rightarrow$ Tension. $T \propto x$ $\qquad \frac{T_2}{T_1} = \frac{1.5x}{x} = 1.5$

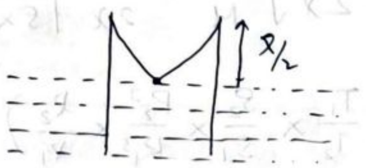
$v = \sqrt{\frac{T}{\mu}}$ $\qquad \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow v' = \sqrt{1.5}\, v = 1.22v$

---

(11) A cylindrical tube open at both ends has a fundamental frequency f in air. The tube is vertically dipped in water so that half of it is in water. The fundamental frequency of air column is

$\Rightarrow$





$l = \lambda/2$ $\qquad \frac{l}{2} = \frac{\lambda}{4} \Rightarrow l = \lambda/2$

$\nu = \frac{v}{2l}$ — (1) $\qquad \nu = v/2l$ — (11)

So here frequency will remain same (f).

---

(12) The speed of sound through oxygen at TK (300 m/s) when the temperature is increased to 3T. the molecule dissociate into oxygen atom. how the speed of sound will be

$\Rightarrow$ $V_1 = \sqrt{\frac{\gamma_1 RT_1}{M_1}} = \sqrt{\frac{1.4 \times R \times T}{M}} = \sqrt{1.4}\sqrt{\frac{RT}{M}}$

$V_2 = \sqrt{\frac{\gamma_2 RT_2}{M_2}} = \sqrt{\frac{1.67 \times R \times 3T}{\frac{M}{2}}} = \sqrt{10.02}\,\frac{300}{\sqrt{1.4}} = 800\ m/s$

13. Two air planes A and B are approaching each other and their velocities are 108 Km/hr and 144 Km/hr. The frequency of a note emitted by A as heard by the Passengers in B is 1170 Hz. Calculate the frequency of the note as heard by the Passengers in A ($V_s = 350 m/s$)

=) 30 m/s $\longrightarrow$ A (s) 40 m/s $\longleftarrow$ B (o)

$$\nu = \nu_o \left(\frac{V + V_o}{V - V_s} \right)$$

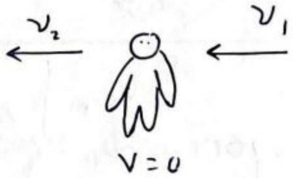
$$1170 = \nu_o \left(\frac{350 + 40}{350 - 30} \right)$$

$$\text{or } \nu_o = \frac{1170 \times 320}{390} \Rightarrow 960 \text{ Hz}$$

---

14. An observer on a railway platform observed that as a train passed through the station at 108 Km/hr, the frequency of the whistle appeared to drop by 300 Hz. Find the frequency of whistle ($V = 350 m/s$)

=) $\nu_1 - \nu_2 = 300$



$$\nu_o \left(\frac{V}{V - V_s} \right) - \nu_o \left(\frac{V}{V + V_s} \right) = 300$$

$$\nu_o \left[\frac{350}{350 - 30} - \frac{350}{350 + 30} \right] = 300 \Rightarrow \nu_o = 1734 \text{ Hz}$$

---

15. when a Source of sound of frequency crosses a frequency observer with a speed V ($<< V_s$). the apparent change in frequency ΔF is

=) $$\Delta F = \nu_1 - \nu_2 = f \left[\frac{V_s}{V_s - V} - \frac{V_s}{V_s + V} \right]$$

$$= f \left[\frac{V_s}{V_s \left(1 - \frac{V}{V_s}\right)} - \frac{V_s}{V_s \left(1 + \frac{V}{V_s}\right)} \right]$$

$$= f \left[\left(1 - \frac{V}{V_s}\right)^{-1} - \left(1 + \frac{V}{V_s}\right)^{-1} \right]$$

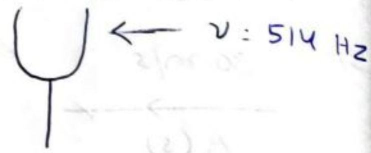
$$= f \left(1 + \frac{V}{V_s} - 1 + \frac{V}{V_s} \right) = \frac{2Vf}{V_s}$$

---

(16) An accurate and reliable audio oscillator is used to standardise a tuning fork. when the oscillator reading is 514, two beats are heard per second. when the oscillator reading is 510 the beat frequency is 6Hz. The frequency of tuning fork is

=)

Beat frequency



$\Delta\nu = \nu_1 - \nu_2 = 2$

$\nu = 514\ Hz, \quad \nu_1 = 516\ Hz$

$\nu_2 = 512\ Hz$

Now also $\nu_1 - \nu_2 = 6 \qquad \nu_1 = 516\ Hz$

$\nu = 510 Hz \qquad \nu_2 = 504\ Hz$

As 516 Hz is common, frequency is 516Hz

Light It is that Part of EM spectrum which enables us to see the objects from which it is comming.

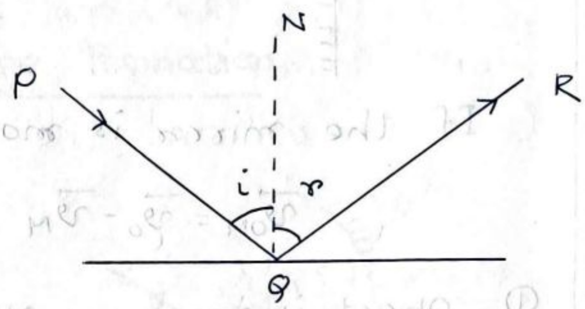
> optics has two major Part:

① Ray optics (Here apparature size is very large as compare to wavelength)

② wave optics (Apparature size is comparable to the wavelength)

Beam: Bundle of light rays travelling in a straight line. Path of it is always straight line

① **Laws of Reflection:**

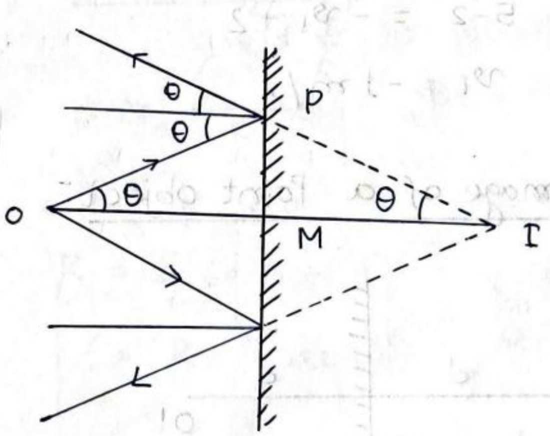


PQ is incident ray
QR is Reflected ray
QN is Normal

(a) Incident ray, reflected ray and the normal at the Point of incidince all lie in the same plane

(b) Incident Ray = Reflected Ray $(i = r)$

② **Image Formation by plane mirror**



From the triangles

$\triangle OPM$ & $\triangle IPM$ we have

PM = Common side

$\angle OPM = \angle IPM$

$\angle PMO = \angle PMI$

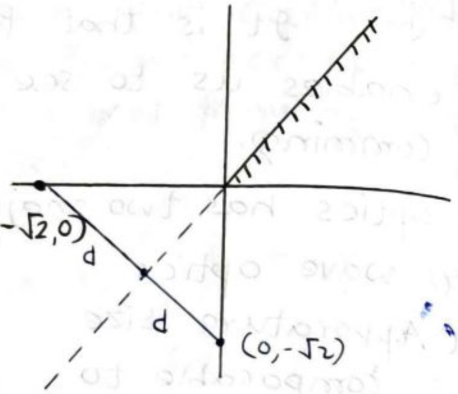
So $\triangle OPM \cong \triangle IPM$

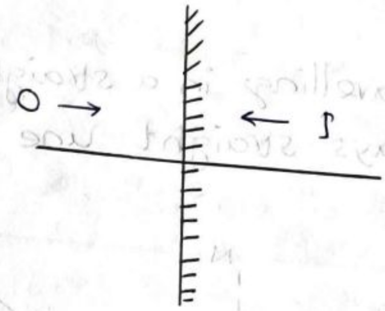
$\therefore OM = MI$

object and image is at same distance from the mirror.

> Find the Coordinate of image

⇒ Extend the mirror
draw foot of perpendicular
So the Coordinate of the image will be
$(0, -\sqrt{2})$





$\vec{x}_o = -\vec{x}_i$

$\Rightarrow \frac{d\vec{x}_o}{dt} = -\frac{d\vec{x}_i}{dt}$ (Mirror is at rest)

$\Rightarrow \vec{v}_o = -\vec{v}_i \quad \vec{v}_{OM} = -\vec{v}_{IM}$

If the mirror is moving then,

$\vec{v}_{OM} = \vec{v}_o - \vec{v}_M$ and $\vec{v}_{IM} = \vec{v}_I - \vec{v}_M$

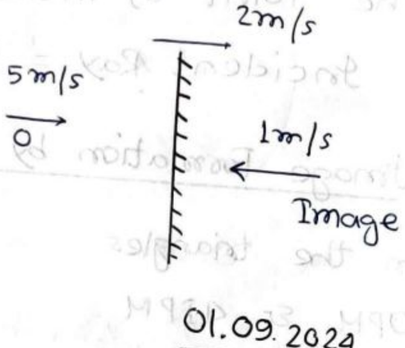
① Object is moving with 5m/s and mirror with 2m/s then the velocity of image will be

$\vec{v}_{OM} = -\vec{v}_{IM}$

or $\vec{v}_o - \vec{v}_M = -\vec{v}_I + \vec{v}_M$

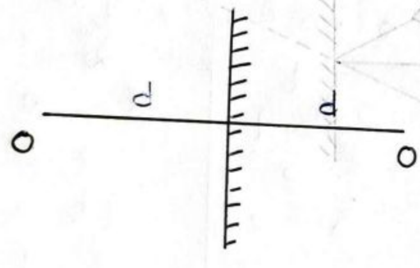
or $5-2 = -v_I + 2$

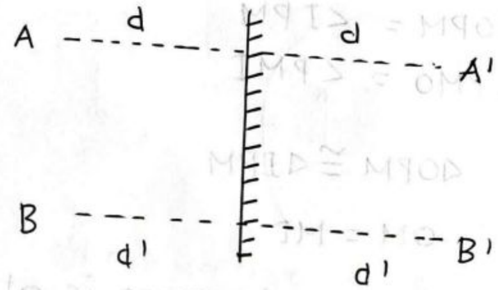
or $v_I = -1\ m/s$

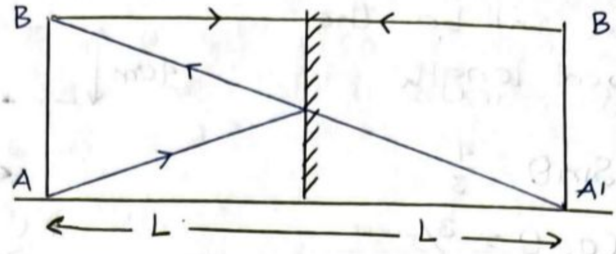


01.09.2024

Image of a Point object:







As $\triangle BPQ \sim \triangle BB'A'$

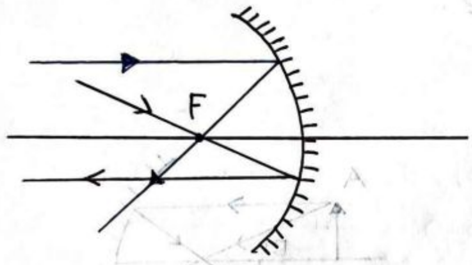
then $\frac{OP}{BB'} = \frac{PQ}{B'A'}$

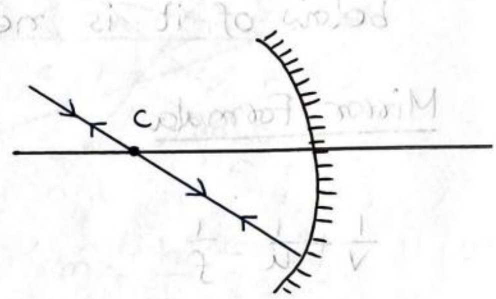
$\frac{L}{2L} = \frac{H_M}{H_o}$

Height of mirror should be half of the height of the object

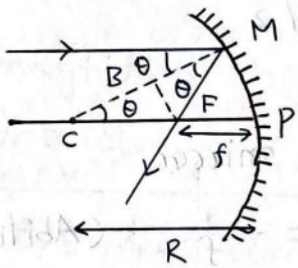
Spherical Mirror

① General Theory of Image Formation:





Relantion b/w R and f



From $\triangle CFM$

$CF = FM$

$BC = R/2$

$\cos\theta = \frac{BC}{CF}$

$CF = \frac{R}{2}\sec\theta$

$CP = CF + FP$

$R = \frac{R}{2}\sec\theta + f$

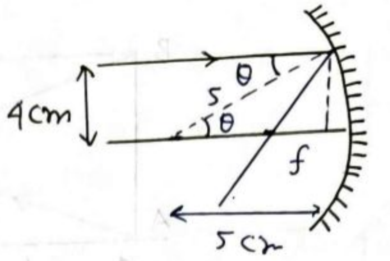
$f = (R - \frac{R}{2}\sec\theta)$

Para axial Rays:

when incident angle is almost Parallel to Principal axis is called Paraxial Rays.

θ is small, $\sec\theta \sim 1$ $f = R - \frac{R}{2} = \frac{R}{2}$

What will be the focal length



R = 5 cm

$\Rightarrow$

$\sin\theta = \frac{4}{5}$

$\cos\theta = 3/5$

$\sec\theta = 5/3$

$$f = R - \frac{R}{2}\sec\theta$$

$$= 5 - \frac{5}{2}\times\frac{5}{3} = \frac{5}{6}\ \text{cm}$$

Sign Convention

① All the distances are measured from the Pole.

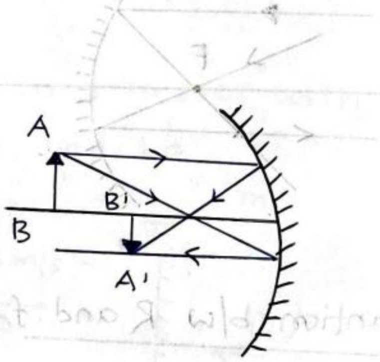
② Distances measured in the direction of propagation of light (+ve) and in the opposite direction it is taken as (−ve).

③ Distance above the Principal axis are (+)ve and below of it is negative.

Mirror Formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{u} = 1\Big/\frac{R}{2}$$

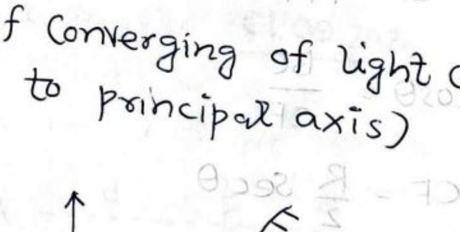


power of mirror.

03.09.2024

$P_m = -\frac{1}{f}$ (Ability of Converging of light Coming parallel to principal axis)

Object	Image
∞	f
∞ – c	f – c
c	c
c – f	c – ∞
p – f	Behind mirror ⇓ Virtual image



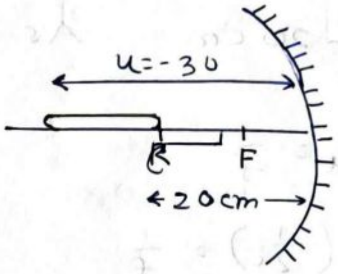
Here image is formed behind the mirror and Virtual image is formed.

① A rod of length 10 cm is along the Principal axis of a Concave mirror of focal length 10 cm is such a way that the end closer to the Pole is 20 cm away from it. length of the image of rod is

⇒

$u = -30$ cm

$f = -10$ cm



$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{30} = -\frac{1}{10}$

$\frac{1}{v} = \frac{1}{30} - \frac{1}{10}$

$\frac{1}{v} = \frac{1-3}{30}$

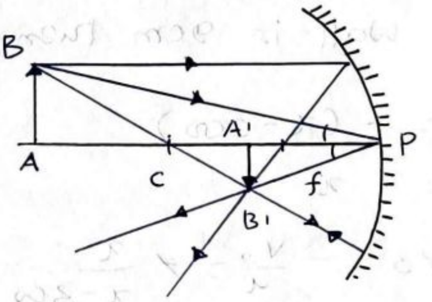
$\frac{1}{v} = -\frac{2}{30}$

$v = -15$ cm

Length of the image is

$(20-15) = 5$ cm

Magnification:



$\frac{AB}{A'B'} = \frac{AP}{A'P}$

$\frac{AB}{A'B'} = \frac{PA}{PA'}$

$\frac{AB}{A'B'} = \frac{-u}{-v} = \frac{h_o}{-h_i}$

$m = \frac{h_i}{h_o} = \frac{-v}{u}$

$|m| > 1$ Image will be magnified $h_i > h_o$

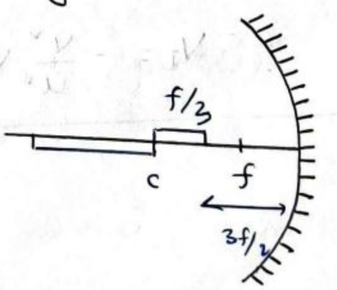
$|m| < 1$ Image wibe contracted $h_i < h_o$

---

② A thin rod of length $f/3$ lies along x-axis of a concave mirror of focal length f. one end of its magnified image touches one end of the rod. what is the length of image

⇒

$u = \frac{3f}{2}$ $f + \frac{2f}{3} = \frac{5f}{3}$



$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{3}{5f} = \frac{1}{f}$

$\frac{1}{v} = \frac{8}{5}f$

③ A Concave mirror gives an image three times as large as the object placed at a distance of 20 cm from it. For the image to be real. the focal length should be

⇒ $u = -20$ cm As real image is formed

$m = -3 = -\frac{v}{u}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$v = 3u = -60$ cm

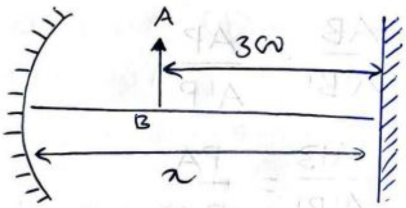
$-\frac{1}{60} + \left(-\frac{1}{20}\right) = \frac{1}{f}$

$\frac{1+3}{60} = -\frac{1}{f}$ ⇒ $f = -15$ cm

---

④ A candle flame of 3cm is placed at 300cm from a wall. A concave mirror is kept at a distance x from the wall in such a way that image of the flame on the wall is 9cm then x is

⇒



$u = -(x - 300)$

$v = -x$

$m = \cancel{+}3 = -\frac{v}{u} = \cancel{+}\frac{x}{x-300}$

$x = 3x - 900$

$2x = 900$

$x = 450$ cm

---

⑤ The rear view mirror of a car is convex

⑤ When an object is kept at a distance of 30 cm from a concave mirror., the image is formed at a distance of 10 cm from the mirror. If the object is moved is moved with a speed of 9 cm/s. The speed (cm/s) with which image moves at the instant is

⇒ $V_I = -\frac{v^2}{u^2} V_o = -\left(\frac{10}{30}\right)^2 \times 9 = -1$ cm/s

From mirror's formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

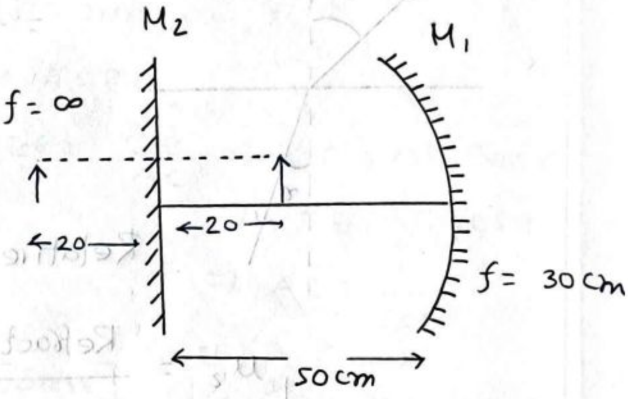
or $-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$

or $-\frac{1}{v^2}V_I - \frac{1}{u^2}V_0 = 0$

Velocity of image. $v_I = -\frac{v^2}{u^2}V_0$

Combination of Mirrors

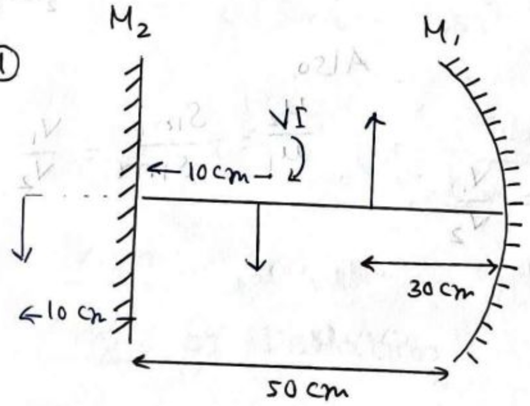
Let first reflection by M_2 mirror — Virtual image is formed behind M_2 - that's all



If the reflection done by M_1 then

$u = -30$ cm
$f = -30$ cm
$v = -\infty$

Plane mirror make image at infinity right side Virtual image is formed.

① 

$f = -20$ cm. First reflection is done by M_1

$u = -30$ cm
$f = -20$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{30} = -\frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$

$v = -60$ cm

As the image is Virtual. so no further reflection is done by M_1

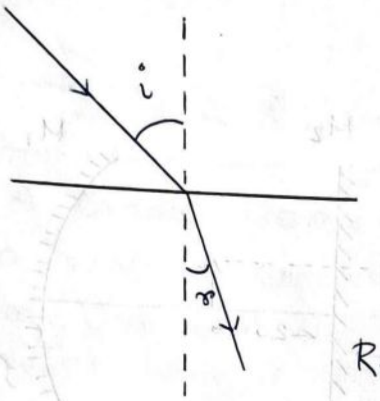
Refraction

05.09.2024

① Laws of Refraction

(a) Incident ray, refracted Ray and the normal all lie in the same plane

(b) $\dfrac{\sin i}{\sin r} = \dfrac{\mu_2}{\mu_1} \Rightarrow$ Snell's law



Refractive index (μ)

$= \dfrac{\text{Velocity of light in Space}}{\text{Velocity of light medium}} = \dfrac{c}{v}$

= Absolute refractive index

Relative refractive index,

${}_1\mu_2 = \dfrac{\text{Refractive index of 2 wrt 1}}{}$

${}_2\mu_1$ = Refractive index of 1 wrt 2 $= \dfrac{\mu_1}{\mu_2}$

So ${}_2\mu_1 = \dfrac{\mu_1}{\mu_2}$ and ${}_1\mu_2 = \dfrac{\mu_2}{\mu_1} \Rightarrow {}_1\mu_2 = \dfrac{1}{{}_2\mu_1}$

$$\boxed{{}_2\mu_1 \times {}_1\mu_2 = 1}$$

Now, ${}_1\mu_2 = \dfrac{\mu_2}{\mu_1} = \dfrac{c/v_2}{c/v_1} = \dfrac{v_1}{v_2}$

Also, $\dfrac{\mu_2}{\mu_1} = \dfrac{\sin i}{\sin r} = \dfrac{v_1}{v_2}$

$$\boxed{\dfrac{\mu_2}{\mu_1} = \dfrac{\sin i}{\sin r} = \dfrac{v_1}{v_2}}$$

NOTE

${}_a\mu_g = 1.5 = \dfrac{\mu_g}{\mu_a}$

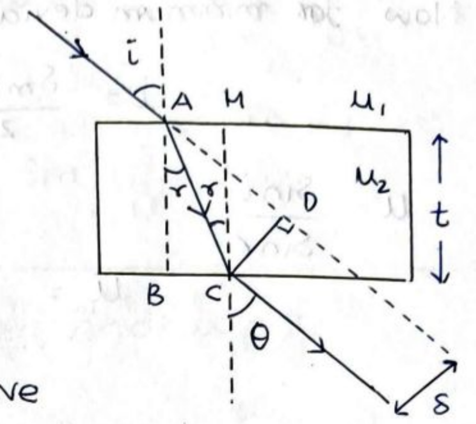
${}_a\mu_w = 1.33 = \dfrac{\mu_w}{\mu_a}$

${}_w\mu_g = \dfrac{\mu_g}{\mu_w} = \dfrac{\mu_g}{\mu_a} \times \dfrac{\mu_a}{\mu_w}$

${}_w\mu_g = 1.5 \times \dfrac{1}{1.33} = 1.13$

Lateral Shift

06.09.2024



By Snell's Law, we have

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{— ①}$$

$$\frac{\sin r}{\sin \theta} = \frac{\mu_1}{\mu_2} \quad \text{— ②}$$

By this two equation we have

$$\sin\theta = \sin i \Rightarrow i = \theta$$

$$\sin \angle CAD = \frac{CD}{AC}$$

From $\triangle ABC$

$$\sin(i-r) = \frac{CD}{AC} \qquad \cos r = \frac{AB}{AC} \Rightarrow AC = AB\sec r$$

$$\Rightarrow t = AC\cos r$$

$$\delta = AC\sin(i-r) \qquad \Rightarrow AC = \frac{t}{\cos r}$$

$$\delta = AC\left[\sin i\cos r - \cos i\sin r\right]$$

Also we have

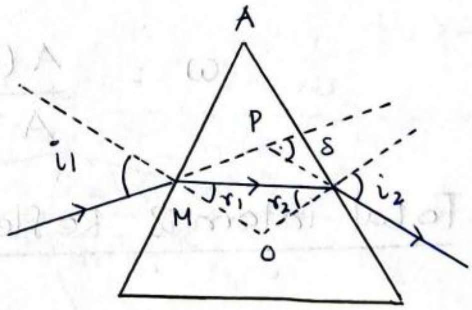
$$\delta = \frac{t}{\cos r}\left[\sin i\cos r - \cos i\sin r\right] \qquad \frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = {}_1\mu_2$$

$$\delta = t\left(\sin i - \cos i\,\frac{\sin r}{\cos r}\right)$$

$$\delta = t\left(\sin i - \cos i\,\frac{\sin i}{{}_1\mu_2}\times\frac{{}_1\mu_2}{\sqrt{{}_1\mu_2^2-\sin^2 i}}\right)$$

$$\delta = t\sin i\left(1-\frac{\cos i}{\sqrt{\mu^2-\sin^2 i}}\right)$$

Prism:



Angle of Deviation

From $\triangle MON$

$$r_1 + r_2 + \angle MON = 180$$

From $\square AMON$ $\quad \angle AMO = 90$, $\angle ANO = 90$

$$A + \angle MON = 180 \quad \text{so} \quad A = r_1 + r_2 \quad \text{— ①}$$

$$\angle PMN = i_1 - r_1 \qquad \delta = (i_1+i_2) - (r_1+r_2)$$

$$\angle PNM = i_2 - r_2 \qquad \delta = i_1 + i_2 - A$$

Deviation in the Prism $\delta = i_1 + i_2 - A$

Now for minimum deviation $i_1 = i_2 = i$. $\delta_m = 2i - A$

$$i = \frac{\delta_m + A}{2}$$

$r_1 + r_2 = A$

$2r = A$

$r = A/2$

$$\mu = \frac{\sin i}{\sin r}$$

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$ Prism formula

If A and δ_m are small. $\mu = \frac{A+\delta m/2}{A/2}$

$$\boxed{\delta_m = A(\mu - 1)}$$

$$\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4} + \cdots$$

Cauchy's formula

07.09.2024

$\delta_m = A(\mu - 1)$

$(\delta_m)_V = A(\mu_V - 1)$

$(\delta_m)_R = A(\mu_R - 1)$

$(\delta m)_V - (\delta m)_R = \theta$

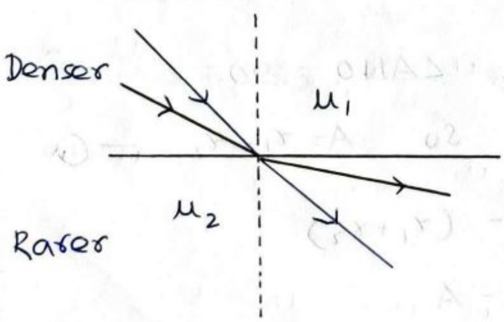
$\boxed{\theta = A(\mu_V - \mu_R)}$ Angular dispersion

Mean deviation, $(\delta m)_y = A(\mu_y - 1)$

Dispersive power. $\omega = \frac{\text{Angular dispersion}}{\text{Mean deviation}}$

$$\omega = \frac{A(\mu_V - \mu_R)}{A(\mu_y - 1)} = \frac{\mu_V - \mu_R}{\mu_y - 1}$$

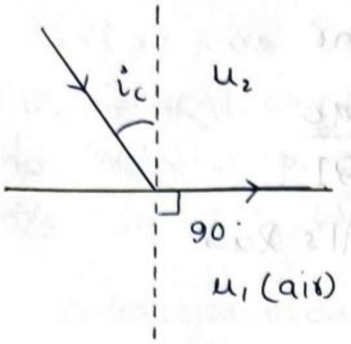
Total internal Reflection:



The value of i for which angle of reflection $r = 90$

⇓

Critical angle



$$\frac{\sin i_c}{\sin r} = \frac{u_1}{u_2}$$

$$\frac{\sin i_c}{\sin 90} = {}_2\mu_1$$

$$i_c = \sin^{-1}({}_2\mu_1)$$

---

① For diamond. $i_c = 24.2°$ then find μ = ?

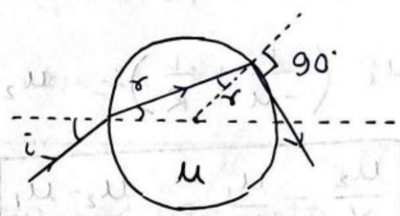
=> $${}_2\mu_1 = \sin i_c = \sin 24.2 = 0.409 = \frac{\mu_1}{\mu_2} = \frac{1}{\mu_2}$$

$$\mu_2 = \frac{1}{0.409} = 2.44$$

---

② What should be the incident angle so that light not comes out.

So basically,



$$\frac{\sin r}{\sin 90} = \frac{1}{\mu}$$

or $\sin r = \frac{1}{\mu}$

Also $$\frac{\sin i}{\sin r} = \frac{\mu}{1}$$

$\sin i = 1$

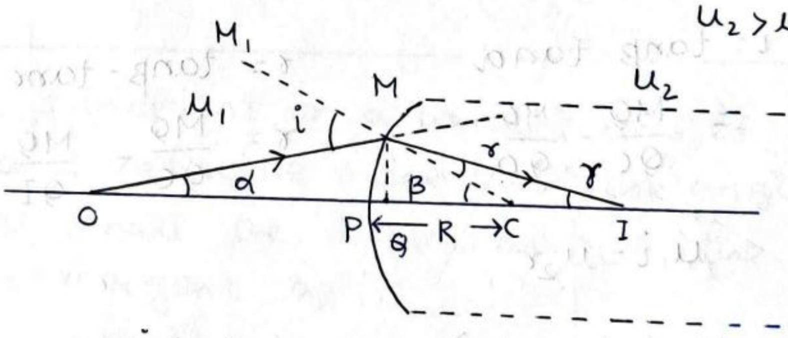
$i = 90$

$$\sin r = \frac{\sin i}{\mu} = \frac{1}{\mu} \Rightarrow \sin i = 1$$

Incident angle should be 90°

<u>Refraction through Spherical Surfaces:</u>

$u_2 > \mu_1$



From ∆MCI:

$\beta = r + \gamma$

$r = \beta - \gamma$ — (i)

From ∆MOC

$i = \alpha + \beta$

$i \sim \tan i \sim \sin i$

$\alpha \sim \tan \alpha$

$\gamma \sim \tan \gamma$

As these angles are very small

So $i = \alpha + \beta$

$i = \tan\alpha + \tan\beta$

$i = \tan\alpha + \tan\beta$ $\qquad$ $r = \tan\beta - \tan\gamma$

$i = \frac{MQ}{QO} + \frac{MQ}{QC}$ $\qquad$ $r = \frac{MQ}{QC} - \frac{MQ}{QI}$

As i, r are very small, from Snell's Law

$$\mu_1 i = \mu_2 r$$

$$\mu_1\left(\frac{MQ}{QO} + \frac{MQ}{QC}\right) = \mu_2\left(\frac{MQ}{QC} - \frac{MQ}{QI}\right)$$

or $\mu_1\left(\frac{1}{QO} + \frac{1}{QC}\right) = \mu_2\left(\frac{1}{QC} - \frac{1}{QI}\right)$

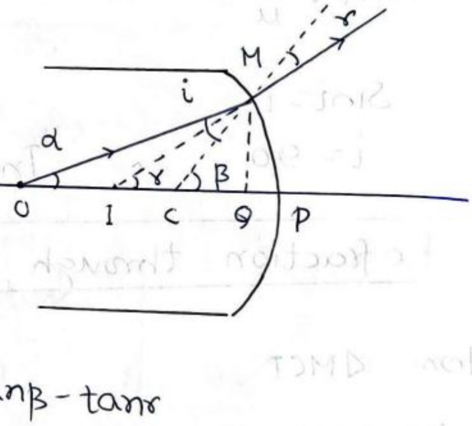
As the Points Q, P coincides we have

or $\mu_1\left(\frac{1}{PO} + \frac{1}{PC}\right) = \mu_2\left(\frac{1}{PC} - \frac{1}{PI}\right)$

or $\mu_1\left(\frac{1}{-u} + \frac{1}{R}\right) = \mu_2\left(\frac{1}{R} - \frac{1}{v}\right)$

or, $\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$

>



From $\triangle OMC$

$\beta = i + \alpha \Rightarrow i = \beta - \alpha$ — ①

From $\triangle IMC$

$\beta = \gamma + r \Rightarrow r = \beta - \gamma$ — ②

$i = \tan\beta - \tan\alpha$ $\qquad$ $r = \tan\beta - \tan\gamma$

$= \frac{MQ}{QC} - \frac{MQ}{QO}$ $\qquad$ $r = \frac{MQ}{QC} - \frac{MQ}{QI}$

$\mu_1 i = \mu_2 r$

or $\mu_1\left(\frac{1}{PC} - \frac{1}{PO}\right) = \mu_2\left(\frac{1}{PC} - \frac{1}{PI}\right)$

or $\mu_1\left(\frac{1}{-R} - \frac{1}{-u}\right) = \mu_2\left(\frac{1}{-R} - \frac{1}{-v}\right)$

$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$

08.09.2024

1) The Critical angle of a medium for a Specific wavelength has relative permittivity 3 and relative permeability 4/3 for the wavelength will be

⇒ Refractive index, $\mu = \frac{c}{\sqrt{\mu_r \epsilon_r}} = \sqrt{3 \times 4/3} = 2$

$i_c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

---

2) The wavelength of light in Vaccum is 6000 Å and in a medium it is 4000 Å. Refractive index be

⇒ $\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\nu\lambda_1}{\nu\lambda_2} = \frac{\lambda_1}{\lambda_2}$ No change in the frequency.

$\frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{6000}{4000} = \frac{\mu}{1} \Rightarrow \mu = 1.5$

---

3) A ray of light passes from Vaccum into a medium of Refractive index n. If the angle of incidence is twice the angle of refraction, the angle of incidence

⇒ $\frac{\sin i}{\sin r} = n$ also $i = 2r$

$\frac{\sin i}{\sin i/2} = n$ $\quad \frac{2\sin i/2 \cos i/2}{\sin i/2} = n$

$i = 2\cos^{-1}(n/2)$

---

4) A ray of light is incident on a parallel slab of thickness t and refractive index μ. If the angle of incidence is small, the displacement in the incident and emergent ray is

⇒ Lateral shift.

$\delta = t\sin i\left(1 - \frac{\cos i}{\sqrt{\mu^2 - \sin^2 i}}\right) = t\theta\left(1 - \frac{1}{\sqrt{\mu^2 - \theta^2}}\right)$

$\delta = t\theta\left(1 - \frac{1}{n}\right)$

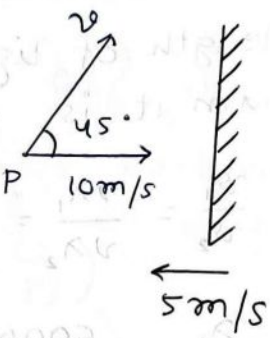
5) Find the angle of refraction in a medium ($\mu = 2$) is light is incident in Vaccum making angle equal to twice the Critical angle

$\Rightarrow$

$$i_c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30$$

---

5) Plane mirror moving with Velocity 5 m/s along (-)ve x direction. An observer O is moving with Velocity $10\hat{i}$ m/s



$v \cos 45 = 10$

$v = 10\sqrt{2}$

$v_y = v \sin 45 = 10$ m/s

There will be no change in $v_y = 10\hat{j}$ m/s

$\vec{v}_{OM} = -\vec{v}_{IM}$

$\vec{v}_0 - \vec{v}_M = -(\vec{v}_I - \vec{v}_M)$

$\vec{v}_0 - \vec{v}_M = -\vec{v}_I + \vec{v}_M$

$\vec{v}_I = 2\vec{v}_M - \vec{v}_0$

$\vec{v}_I = 2(-5) - 10$

$\vec{v}_I = -20\hat{i}$ m/s

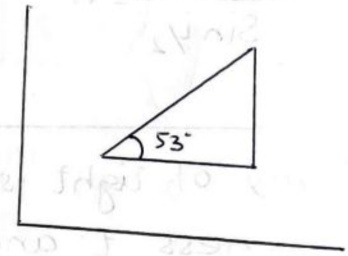
Velocity of image $-20\hat{i} + 10\hat{j}$

---

7) Graph of $\sin i$ vs $\sin r$ on interface is given by

$\Rightarrow$

$$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$



$\sin i = {}_1\mu_2 \sin r$

$$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} = \frac{1}{\tan 53} = 0.753 = \frac{3}{4} \quad \mu_1 > \mu_2$$

(Denser to rarer medium)

${}_1\mu_2 = \frac{3}{4}$ Critical angle

$$i_c = \sin^{-1}(3/4) = 48.59$$

---

when a ray of light of frequency 6×10^{14} Hz travells from water of refractive index 4/3 to glass of $\mu = 8/5$ its

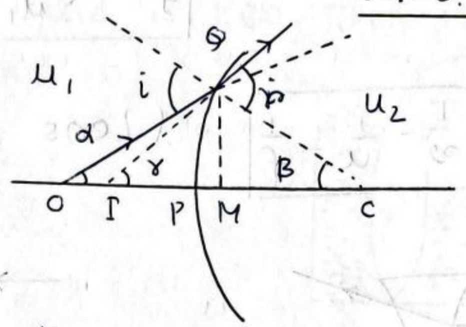
$\Rightarrow$ $$\frac{\mu_w}{\mu_g} = \frac{\frac{c}{c_w}}{\frac{c}{V_g}} = \frac{V_g}{V_w} = \frac{8}{5}\times\frac{3}{4} = \frac{6}{5} \Rightarrow V_w = \frac{5}{6}V_g$$

Speed decreased by factor 5/6

No change in frequency. $V \propto \lambda$

09.09.2024

when $\mu_2 < \mu_1$



Here virtual image is formed

$\triangle QIC$

$r = \gamma + \beta$

$\tan\gamma + \tan\beta = r$

$$\frac{QM}{IM} + \frac{QM}{MC} = r$$

From $\triangle QOC$ we have

$i = \alpha + \beta$

$i = \tan\alpha + \tan\beta$

$$i = \frac{QM}{OM} + \frac{QM}{MC}$$

From Snell's law

$\mu_1 i = \mu_2 r$

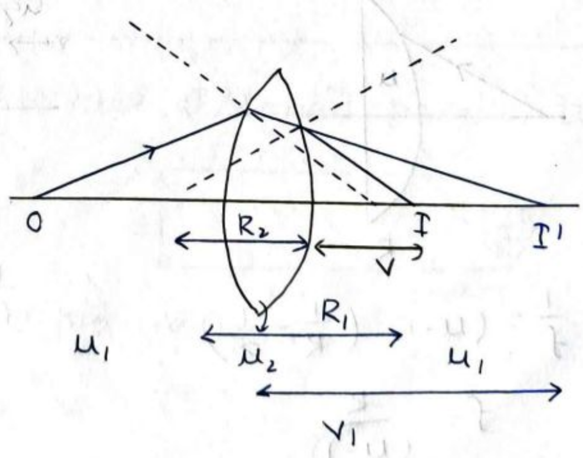
$$\mu_1\left(\frac{1}{OP} + \frac{1}{MC}\right) = \mu_2\left(\frac{1}{PI} + \frac{1}{PC}\right)$$

$$\boxed{\frac{\mu_2}{V} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}}$$

Lenses:

$$\frac{\mu_2}{V_1} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R_1} \quad -(1)$$

$$\frac{\mu_1}{V} - \frac{\mu_2}{V_1} = \frac{\mu_1-\mu_2}{R_2} \quad -(2)$$



By adding these two equations we have

$$\frac{\mu_1}{v} - \frac{\mu_1}{u} = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or

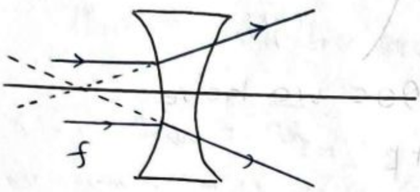
$$\boxed{\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

If $u = -\infty$
$v = f$

$$\frac{1}{f} - \frac{1}{(-\infty)} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

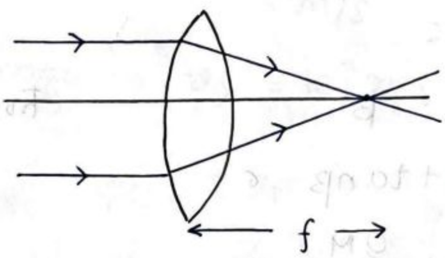
$$\boxed{\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

$$\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$ (Lens formula)



$f = -ve$

Con Cave lens



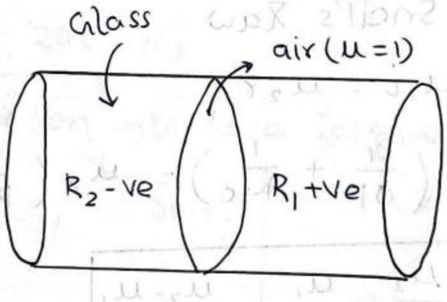
$f = +ve$

Convex lens

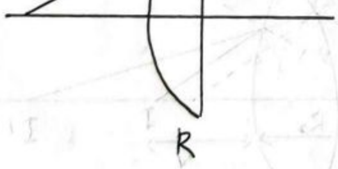
$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

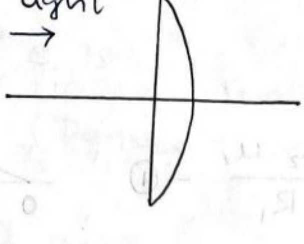
$$\frac{1}{f} = \left(\frac{1}{1.5} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$f = -ve$



It is a con cave lense





$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

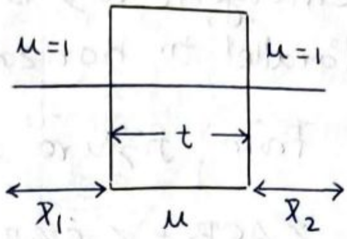
$$f = \frac{R}{(\mu - 1)}$$

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{\infty} - \left(-\frac{1}{R}\right)\right]$$

$$f = \frac{R}{(\mu - 1)}$$

10.09.2024

Optical Path



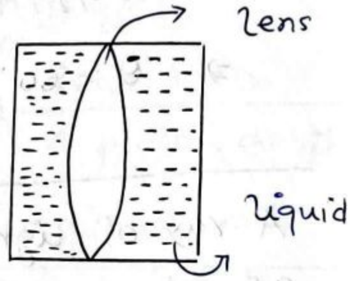
Optical Path

$= (1\times l_1) + \mu t + (1\times l_2)$

$= (l_1 + l_2) + \mu t$

---

① The lens has focal length 20 cm when it in air and its material has refractive index 1.5. If the refractive index of liquid is 1.60. The focal length of the system is.



$\Rightarrow$ $$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

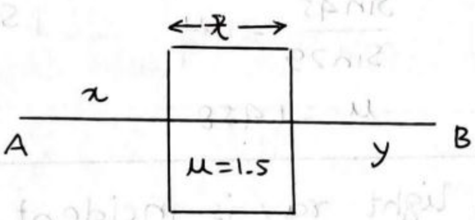
$$\frac{1}{20} = (1.5-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$\frac{1}{R_1}+\frac{1}{R_2} = \frac{1}{20\times 0.5}$ So $\frac{1}{R_1}+\frac{1}{R_2} = 0.1$ cm

$$\frac{1}{f'} = \left(\frac{\mu_2}{\mu_1}-1\right)\times 0.1 = \left(\frac{1.5}{1.6}-1\right)\times 0.1 \Rightarrow f' = -160\,cm$$

---

② A slab of thickness t and refractive index 1.5 is placed between A and B. optical path is



$\Rightarrow$ optical path

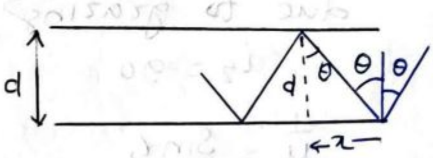
$= \mu\times$ geometric Path

$= (x\times 1) + 1.5t + (y\times 1)$

$= (x + y + \frac{3}{2}t)$

---

③ Two plane mirror P and Q are alingned parallel to each other



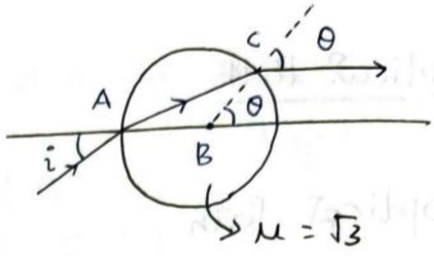
$\frac{x}{d} = \tan\theta$

$x = d\tan\theta$

No of reflection

$$n = \frac{l}{x} = \frac{l}{d\tan\theta}$$

④ The Value of i for which emergent ray is Parallel to horizontal axis



$\Rightarrow$ From figure $AB = BC$ and $\angle ACB = \angle CAB$

$\angle ACB + \angle CAB = \theta$

The angles are $\theta/2$

$\frac{\sin i}{\sin r} = \frac{1}{\sqrt{3}}$

$2\cos\theta/2 = \sqrt{3}$

$\cos\theta/2 = \frac{\sqrt{3}}{2}$

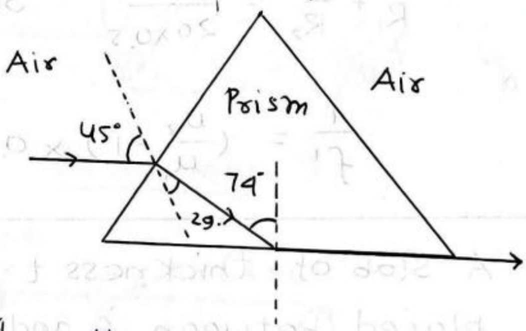
$\Rightarrow \theta = 60^\circ$

$\frac{\sin\theta/2}{\sin\theta} = \frac{1}{\sqrt{3}}$

$\frac{\sin i}{\sin 30} = \sqrt{3} \Rightarrow \sin i = \frac{\sqrt{3}}{2} \Rightarrow i = 60^\circ$

---

⑤ A ray of light is incident on a right angle prism as shown. The lower surface of this prism is Coated with a gel. If the incident ray makes angle. (in degree) refractive index of gel is



$\Rightarrow \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$

$\frac{\sin 45}{\sin 29} = \frac{\mu}{1}$

$\mu = 1.458$

$\frac{\sin 74}{\sin 90} = \frac{\mu_2}{1.458}$

$\mu_2 = 1.403$

Refractive index of gel is 1.403.

---

⑥ A light ray is incident normally on one of the refracting surfaces of a Prism of known angle of Prism. The emergent rays gazes the other surface what is the refractive index of material of Prism

$\Rightarrow$ due to grazing

$i_2 = 90$

$r_2 + r_1 = A$

$r_2 = A$

$r_1 = 0. \; i_1 = 0$

$\frac{1}{\mu} = \frac{\sin r_2}{\sin i_2} \Rightarrow \sin A = \frac{1}{\mu}$

$\Rightarrow \mu = \text{Cosec} A$

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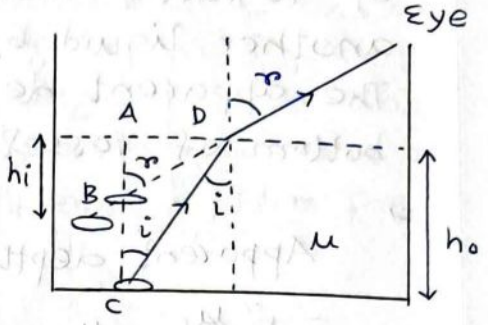


$\tan r = \dfrac{AD}{AB}$ If the angle i, r are very small

$\tan i = \dfrac{AD}{AC}$

$\sin r = \dfrac{AD}{AB}$ $\sin i = \dfrac{AD}{AC}$

From Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

$\mu \sin i = \sin r$

$\mu = \dfrac{\sin r}{\sin i}$

$\mu = \dfrac{\text{Real depth}}{\text{Apparant depth}}$

$\mu = \dfrac{AD}{AB} \times \dfrac{AC}{AD} = \dfrac{AC}{AB} = \dfrac{h_o}{h_i}$ $\mu = h_o/h_i$

---

① A beam of unpolarized light is incident on a glass plate at an angle of 60° from normal the reflected light is completly plane polarized. If angle of incidence is 45°, angle of refraction i

$\Rightarrow$

$i_p = \tan^{-1}\mu$

$\mu = \tan 60 = \sqrt{3}$ $\dfrac{\sin i}{\sin r} = \sqrt{3}$

$r = \sin^{-1}\sqrt{\dfrac{2}{3}}$ $\dfrac{1}{\sqrt{2}\sin r} = \sqrt{3} \Rightarrow \sin r = \sqrt{\dfrac{2}{3}}$

---

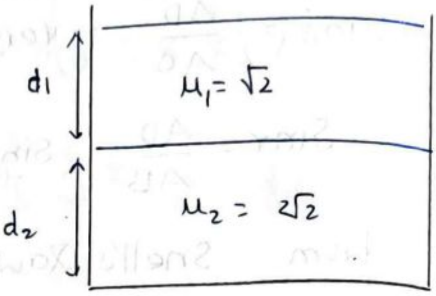
① A Vessel of depth 2h is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid by RI $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be

⇒

Apparent depth

$= \cancel{h}\left(\cancel{\frac{\mu_1}{d_1}} + \cancel{\frac{\mu_2}{d_2}}\right)$

$= \cancel{h}/\sqrt{2}$



$= \left(\frac{d_1}{\mu_1} + \frac{d_2}{\mu_2}\right) = \left(\frac{h}{\sqrt{2}} + \frac{h}{2\sqrt{2}}\right) = \frac{h}{\sqrt{2}} \times \frac{3}{2} = \frac{3h\sqrt{2}}{4}$

---

② An equiconvex lens of focal length 10cm (at t=0) and refractive index (Hg = 1.5) is placed in a liquid $\mu(t) = 1.0 + \frac{1}{10}t$. If the lens was placed in the liquid at t=0, after what time lens will act as concave lens of focal length 20 cm

⇒

$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

At. t=0 $\quad \frac{1}{10} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$\frac{1}{10 \times 0.5} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \Rightarrow \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5}$

$\frac{1}{-20} = \left(\frac{1.5}{1+0.1t} - 1\right) \times \frac{1}{5}$

$-\frac{1}{4} = \frac{1.5}{1+0.1t} - 1$

$\frac{1.5}{1+0.1t} = \frac{3}{4} \Rightarrow$

$6 = 3 + 0.3t$

$0.3t = 3$

$t = 3/0.3$

$t = 10$ sec

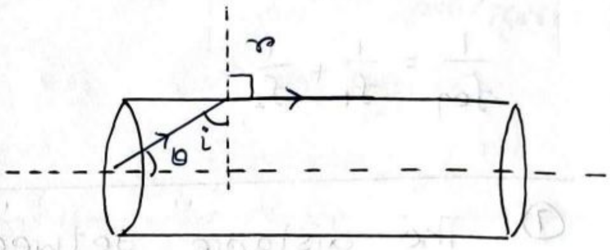
---

③ Find the angle of refraction in a medium ($\mu = 2$) if light is incident in vaccum making angle equal to twice the critical angle

⇒ $\theta_c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30 \qquad i = 2\theta_c = 60$

$\frac{\sin i}{\sin r} = \mu \Rightarrow \frac{\sin 60}{\sin r} = 2 \Rightarrow \sin r = \frac{\sqrt{3}}{4} \Rightarrow r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

④ Consider a point source of light kept at distance outside a transparent, solid cylinder on its axis and near the end face (Base). The refractive index of the material is 1.12. The max angle with the axis of the cylinder of incident ray at which none of the ray entering the base will emerge from the curved surface is



⇒

$$\frac{\sin i}{\sin r} = \frac{1}{\mu}$$

$\sin i = \frac{1}{1.12} \Rightarrow i = 63.23°$ The angle is

$\theta = 90 - 63.23 = 27°$

13.09.2024

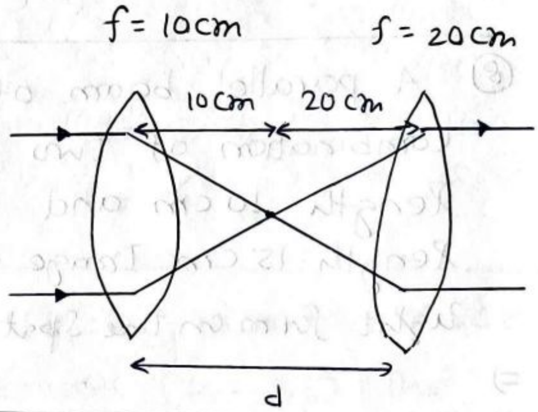
⑤ Find the value of d for which the final rays are also parallel

⇒

The value of d will be

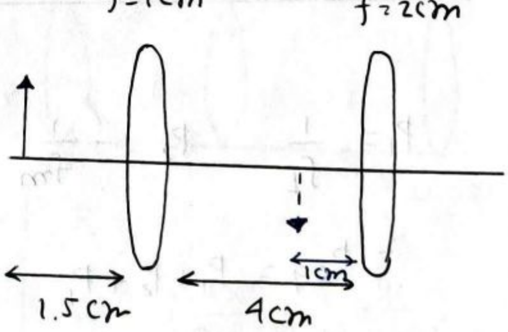
$(10 + 20)$

$= 30$ cm



---

⑥ Two thin convex lenses L_1 and L_2 with focal lengths 1 cm and 2cm respectively are seperated by a distance of 4cm along their axis. The ratio of final image to object size



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{1.5} = 1$$

$$\frac{1}{v} = 1 + \left(-\frac{1}{1.5}\right) \Rightarrow v = 3 \text{ cm}$$

$u = -1$ cm

$v = ?$

$f = 2$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + 1 = \frac{1}{2}$$

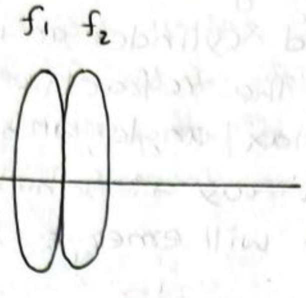
$v = -2$ cm

$m_1 = \frac{h'}{h_0}$ $m_2 = \frac{h_i}{h'}$

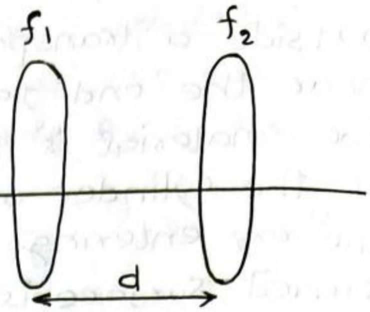
$m = \frac{h_i}{h_0} = m_1 m_2$

$m = \frac{3}{-1.5} \times \frac{-2}{-1} = -4$

Combination of Lens



$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$



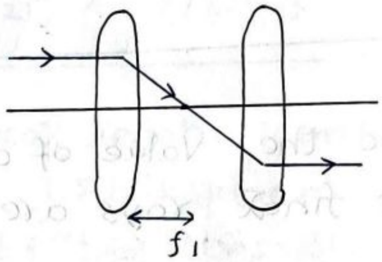
$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

---

⑦ The distance between L_1 and L_2 are.

⇒

The distance between L_1 and L_2 is $(f_1 + f_2)$



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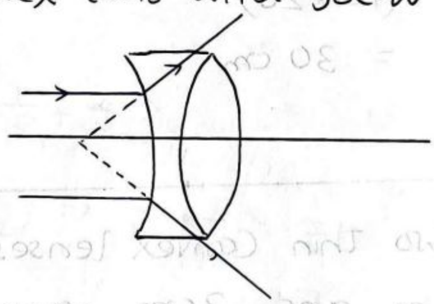
⑧ A parallel beam of light is incident on a Combination of two lens. Concave lens with focal length 10 cm and convex lens with focal length 15 cm Image of light form on the Spot at

⇒



$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

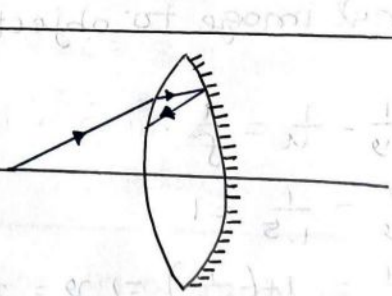
$$\frac{1}{f} = \frac{1}{-10} + \frac{1}{15} \qquad \frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$f = -30 \text{ cm}$$

$$P_1 = \frac{1}{f_L} \qquad P_2 = -\frac{1}{f_m}$$

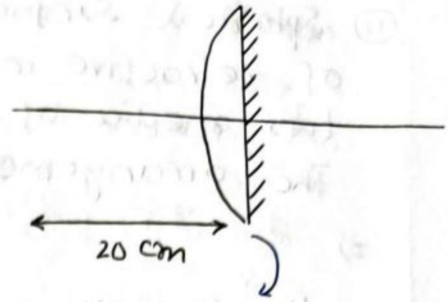
$$-P_{eq} = P_1 + P_2 + P_1$$

$$\Rightarrow \; -\frac{1}{f_{eq}} = \frac{2}{f_L} - \frac{1}{f_m}$$



$$P_{eq} = P_1 + P_2$$

⑨ If $f_L = 15$ cm then $v = ?$



$\Rightarrow$

$$-\frac{1}{f_e} = \frac{2}{f_L} - \frac{1}{f_m}$$

$$-\frac{1}{f_e} = \frac{2}{15} \Rightarrow f_e = -\frac{15}{2} \text{ cm}$$

$u = -20$ cm

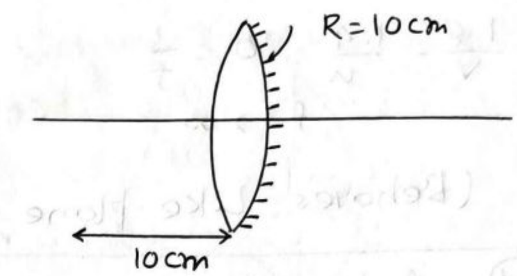
$f = -7.5$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{(Mirror formula)}$$

$$\frac{1}{v} - \frac{1}{20} = -\frac{1}{7.5} \Rightarrow v = -12 \text{ cm}$$

---

⑩ Find v



$\Rightarrow$

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{10} + \frac{1}{10}\right)$$

$f_L = 10$ cm

$$-\frac{1}{f_{eq}} = \frac{2}{f_L} - \frac{1}{f_m}$$

$$-\frac{1}{f_{eq}} = \frac{2}{10} - \frac{1}{-5} = \frac{2}{5}$$

$f_{eq} = -2.5$ cm

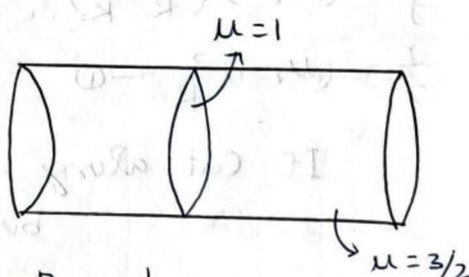
(Behave like concave mirror)

---

⑪ An equiconvex air lens ($R_1 = R_2 = 10$ cm) is made in an extended glass medium ($\mu = 3/2$). The refractive index of the material to be filled in to that the power of the lens changes without change in the magnitude

$\Rightarrow$ $R_1 = R_2 = 10$ cm

$\mu = 3/2$



$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{2}{3} - 1\right)\left(\frac{1}{10} + \frac{1}{10}\right) = -\frac{1}{15}$$

$P_1 = -\frac{1}{15}$

$P_2 = \frac{1}{15}$

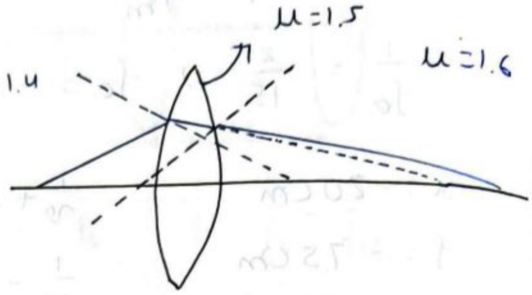
$$\frac{1}{15} = \left(\frac{\mu \times 2}{3} - 1\right)\left(\frac{1}{10} + \frac{1}{10}\right)$$

$$\frac{1}{15} = \left(\frac{2\mu}{3} - 1\right) \times \frac{2}{10}$$

$\mu = 2$

(12) Spherical surface of thin equi convex lens of glass of refractive index $\mu_2 = 1.5$ acts as interface between two media of refractive index $\mu_1 = 1.4$ and $\mu_3 = 1.6$. The arrangement will behave like

⇒



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}$$

$$\frac{1.5}{v_1} - \frac{1.4}{u} = \frac{0.1}{R_1}$$

$$\frac{1.6}{v} - \frac{1.5}{u} = \frac{1.5-1.6}{R_2}$$

$$\frac{1.6}{v} - \frac{1.5}{v_1} = -\frac{0.1}{R_2}$$

$$\frac{1.6}{v} - \frac{1.4}{u} = 0 = \frac{1}{f}$$

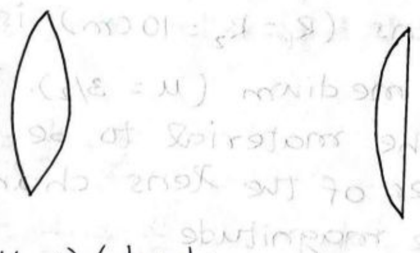
$$f \to \infty$$

(Behaves like plane glass plate)

---

(13) A lens is formed with two curved surfaced having radius of curvature 20 cm and 40 cm. medium lens has $\mu = 1.5$. An object is placed on a distance 80 cm from the lens. Find the distance of image when lens is cut half along x-axis

⇒



$$\frac{1}{f_1} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f_1} = ({}_1\mu_2 - 1)\frac{1}{R}$$

$$\frac{1}{f} = ({}_1\mu_2 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$f_1 = 2f$$

$$\frac{1}{f} = ({}_1\mu_2 - 1)\frac{2}{R} \quad -①$$

If cut along x axis. f remain same but intensity will decrease

(14) A Concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75 it will behave as

$R_1 = -ve$

$R_2 = +ve$

$\Rightarrow$ $\frac{1}{f} = (\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= (1.5-1)\left(-\frac{2}{R}\right)$

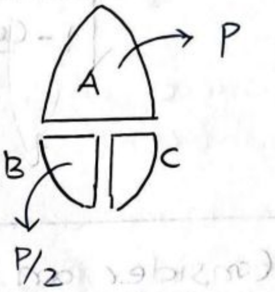
$\frac{1}{f'} = \left(\frac{1.5}{1.75} - 1\right)\left(-\frac{2}{R}\right)$

$f' = 3.5R$

Convergent line of focal lens 3.5R

---

(15) A thin symmetric double convex lens by power P. is cut as shown in figure



$\Rightarrow$

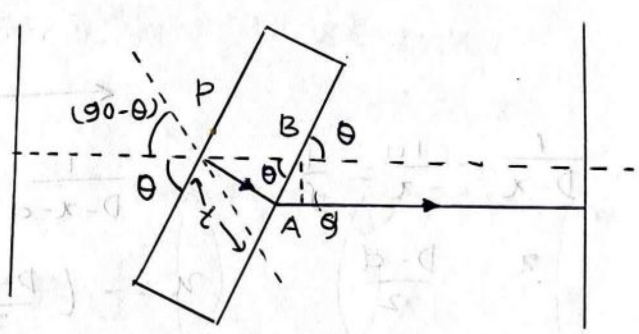
power of part B is $P/2$

Power of A is P

---

(16) A ray of light passing through the origin O along x-axis reaches the Point A on the screen S, placed perpendicular to x-axis. A glass plate of thickness t and refractive index μ is introduced in the path making an angle θ with x-axis. The extra optical path that the ray has to travel to reach A' on screen

$\Rightarrow$



$\cos r = \frac{t}{PA}$

$PA = \frac{t}{\cos r}$

$\frac{\sin(90-\theta)}{\sin r} = \frac{\mu}{1}$

$\frac{\cos\theta}{\sin r} = \mu$

$\sin r = \frac{\cos\theta}{\mu}$

$\cos r = \sqrt{1 - \frac{\cos^2\theta}{\mu^2}}$

$\sin\theta = \frac{t}{PB}$

$PB = \frac{t}{\sin\theta}$

$\frac{AQ}{AB} = \cos\theta$

$AQ = AB\cos\theta$

$AB = BC - AC$

$\tan r = \frac{AC}{t}$

$AC = t \cdot \tan r$

$$\tan\theta = \frac{t}{BC}$$

$$BC = \frac{t}{\tan\theta}$$

$$AB = BC - AC = \frac{t}{\tan\theta} - t\cdot\tan r$$

optical Path difference

$$= \mu(PA) + AQ - PB$$

$$= \frac{\mu t}{\cos r} + AB\cos\theta - \frac{t}{\sin\theta}$$

$$= \frac{\mu t}{\cos r} + \left(\frac{t}{\tan\theta} - t\cdot\tan r\right)\cos\theta - \frac{t}{\sin\theta}$$

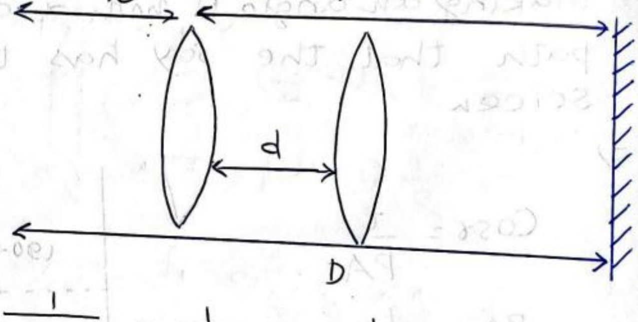
$$= \frac{\mu t}{\sqrt{1-\frac{\cos^2\theta}{\mu^2}}} + \left(\frac{t}{\frac{\cos\theta}{\mu}}\cdot\sqrt{1-\frac{\cos^2\theta}{\mu^2}} - t\right) - \frac{t}{\sin\theta}$$

$$= t\left[(\mu^2 - \sin^2\theta)^{1/2} - \cos\theta\right]$$

---

(17) Consider an object placed at a fixed distance D from a screen. Real image are formed on the screen for two positions of a lens, seperated by a distance d. The ratio between the size of two images is given by

$\Rightarrow$



$$\frac{1}{D-x} - \frac{1}{-x} = \frac{1}{f}$$

$$x = \frac{D-d}{2}$$

$$\frac{h_I}{h_o} = \frac{D+d}{D-d}$$

$$\frac{1}{D-x-d} - \frac{1}{-(x+d)} = \frac{1}{f}$$

$$u = -\left(\frac{D-d}{2}\right)$$

$$V = D - \frac{D-d}{2} = \frac{D+d}{2}$$

$$u = D - d - \frac{D}{2} + \frac{d}{2} = \frac{D}{2} - \frac{d}{2} = \frac{D-d}{2}$$

$$v = x + d = \frac{D}{2} - \frac{d}{2} + d = \frac{D+d}{2}$$

$$\frac{h_I}{h_o} = \frac{D-d}{D+d}$$

Ratio is

$$\left(\frac{D+d}{D-d}\right)^2$$

15 th Sept

Interference:

$y_1 = a \sin \omega t$

$y_2 = b \sin(\omega t + \delta)$

$y = y_1 + y_2$

$= a \sin \omega t + b \sin(\omega t + \delta)$

or $y = a \sin \omega t + b \sin \omega t \cos\delta + b \cos \omega t \sin \delta$

or $y = (a + b\cos\delta) \sin \omega t + (b \sin\delta) \cos \omega t$

or $y = A\cos\theta \sin\omega t + A \sin\theta \cos\omega t$

or $y = A \sin(\omega t + \theta)$

so, $A\cos\theta = a + b\cos\delta$

$A \sin\theta = b \sin\delta$

$A = \sqrt{a^2 + b^2 + 2ab\cos\delta}$

$A_{max} = (a+b)$ $\qquad A_{min} = (a-b)$

$KA^2 = Ka^2 + Kb^2 + 2\sqrt{K}a\sqrt{K}b\cos\delta$

$\Rightarrow I_R = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\delta$

$\Rightarrow I_R = (\sqrt{I_1} + \sqrt{I_2})^2$ when $\cos\delta = 1$

$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$\cos\delta = 1$

If $I_1 = I_2 = I_0$, $I_{max} = 4I_0$

$\delta = 2n\pi$

$(n = 0, \pm 1, \pm 2 ..)$

[path difference $= \left(\frac{\lambda}{2\pi}\right)$ Phase difference

$\delta = \left(\frac{2\pi}{\lambda}\right) \times \Delta x$]

Now, $\delta = 2n\pi$

$\frac{2\pi}{\lambda} \cdot \Delta x = 2n\pi$

$\boxed{\Delta x = n\lambda}$

(Constructive Interference)

(Bright fringes)

For minimum. (Destructive interference)

$$I_{min} = (\sqrt{I_1}-\sqrt{I_2})^2$$

(Dark fringe)

If $I_1 = I_2$ then $I_{min} = 0$

$$\cos\delta = -1$$

or $\cos\delta = \cos(2n+1)\frac{\pi}{2}$

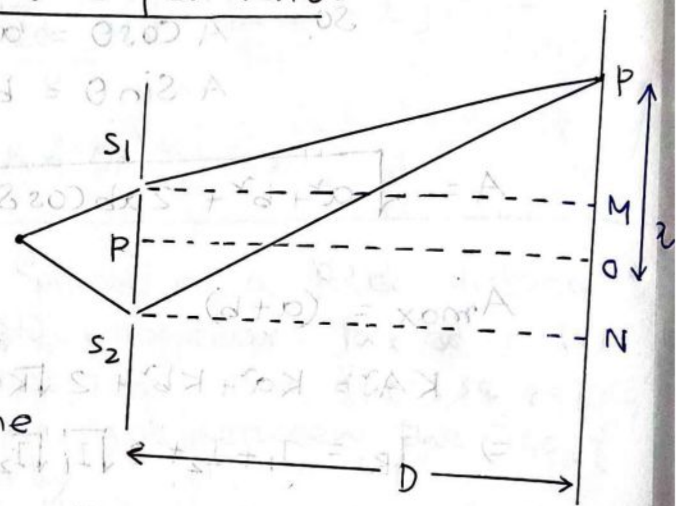
or $\boxed{\Delta x = (2n+1)\frac{\lambda}{2}}$

young's Double Slit Experiment:

① Source should be Coherent.

Amplitude same
Constant Phase difference

Distance between the Sources are d.



Now, $(S_2P)^2 = (S_2N)^2 + (PN)^2$

$$(S_2P)^2 = D^2 + (x + d/2)^2$$

$$(S_1P)^2 = D^2 + (x - d/2)^2$$

$$(S_2P)^2 - (S_1P)^2 = (x+d/2)^2 - (x-d/2)^2 = 2xd$$

or $(S_2P + S_1P)(S_2P - S_1P) = 2xd$

or $\Delta x = \frac{2xd}{(S_2P+S_1P)}$

or $\Delta x = \frac{2xd}{2D}$

or $\Delta x = \frac{xd}{D}$

Constructive

$\Delta x = n\lambda$

$\frac{xd}{D} = n\lambda$

$x = \frac{n\lambda D}{d}$ – Bright

$\boxed{\text{Central}}$, $n=0$ $x=0$

(Central Maxima)

Destructive

$$\Delta x = (2n-1)\frac{\lambda}{2}$$

$$\frac{xd}{D} = (2n-1)\frac{\lambda}{2}$$

$$x = (2n-1)\frac{\lambda D}{2d}$$

First minima

$n = 1$

$$x_1 = \frac{\lambda D}{2d}$$

$$x_2 = \frac{3\lambda D}{2d}$$

$x = 0$ Bright

$x = \frac{\lambda D}{2d}$ Dark

$x = \frac{\lambda D}{d}$ Bright

Fringe width

$$\beta = \frac{\lambda D}{d}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

if $I_1 = I_2$

$$I = I + I + 2I\cos\delta$$

$$I = 2I(1+\cos\delta)$$

$$\Rightarrow I_R = 4I\cos^2\delta/2$$

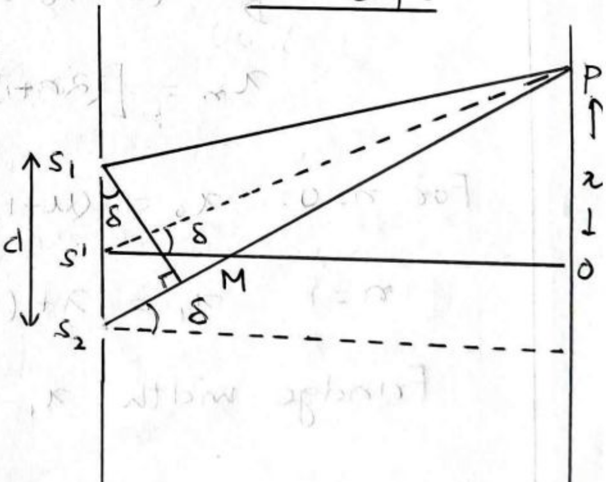
So basically

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

Visibility. $V = \frac{I_{min}}{I_{max}}$

16 th sept



Path difference

$\Delta = S_2P - S_1P = S_2M$

$\Delta = S_2M = d\sin\delta$

$\Delta x = d\sin\delta$

Phase difference

$$= \frac{2\pi}{\lambda}(\text{Path diff})$$

$$= \frac{2\pi}{\lambda}(d\sin\delta) = \frac{2\pi}{\lambda}\frac{xd}{D} = \phi$$

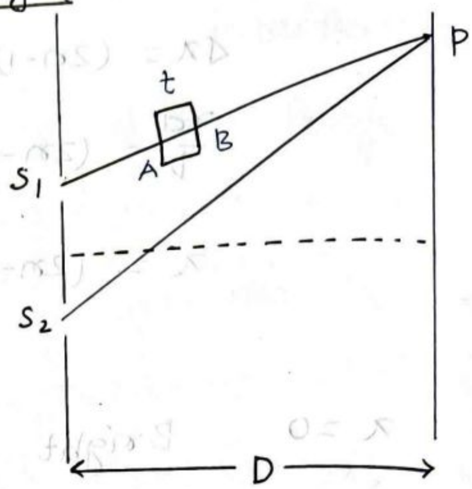
$\sin\delta \simeq \tan\delta = \frac{x}{D}$

$\phi = 2n\pi$

(Bright)

17th sept

Lateral Displacement of Fringes



Here the transparent sheet has thickness t

$$S_1P = S_1A + \mu(AB) + BP$$
$$= S_1A + \mu t + BP$$
$$= S_1A + t - t + \mu t + BP$$
$$= S_1P + (\mu-1)t$$

Path difference was

$$S_2P - S_1P = \frac{xd}{D}$$

Now path difference

$$S_2P - [S_1P + (\mu-1)t] = \frac{xd}{D} - (\mu-1)t$$

For Bright frindge.

$$\frac{xd}{D} - (\mu-1)t = n\lambda$$

$$x_n = [n\lambda + (\mu-1)t]\frac{D}{d}$$

For Dark Frindge

$$\frac{xd}{D} - (\mu-1)t = (2n+1)\frac{\lambda}{2}$$

$$x_n = \left[(2n+1)\frac{\lambda}{2} + (\mu-1)t\right]\frac{D}{d}$$

NOTE:

Thickness of the sheet

$$t = \frac{xd}{D(\mu-1)}$$

> Central maxima Shifted to upward.

For $n=0$: $x_0 = (\mu-1)t\left(\frac{D}{d}\right)$

$n=1$. $x_1 = [\lambda + (\mu-1)t]\frac{D}{d}$

For Bridget Fringes

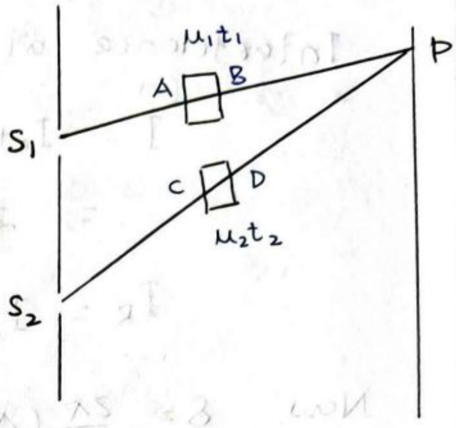
Frindge width $x_1 - x_0 = \frac{\lambda D}{d}$

For $n=0$ $x_0 = \left[(\mu-1)t - \frac{\lambda}{2}\right]\frac{D}{d}$

$n=1$ $x_1 = \left[(\mu-1)t + \frac{\lambda}{2}\right]\frac{D}{d}$ $\beta = \frac{\lambda D}{d}$

Frindge width remain the same

when two transperent sheet is introduced



After indroducing the optical Path

$$(S_1P)_g = S_1A + \mu_1(AB) + BP$$
$$= S_1A + \mu_1 t_1 + t_1 - t_1 + BP$$
$$= S_1A + t_1 + BP + (\mu_1 - 1)t_1$$

so. $(S_1P)_g = S_1P + (\mu_1 - 1)t_1$

Simmilarly. $(S_2P)_g = S_2P + (\mu_2 - 1)t_2$

Path difference. $(S_2P)_g - (S_1P)_g$

$$\Delta = (S_2P - S_1P) + [(\mu_2 - 1)t_2 - (\mu_1 - 1)t_1]$$

or $\Delta = \frac{xd}{D} + [(\mu_2 - 1)t_2 - (\mu_1 - 1)t_1]$

Note if both sheet are identical $[\mu_1 = \mu_2 ; t_1 = t_2]$ then $\Delta = \frac{xd}{D}$. there will not be any change.

18.09.2024

Diffraction

$y_1 = a \sin\omega t$

$y_2 = a \sin(\omega t + \delta)$

$y_3 = a \sin(\omega t + 2\delta)$

$\vdots$

$y_N = a \sin[\omega t + (N-1)\delta]$

Phase difference b/w consecutive SHM is δ, a constant

Resultant of them

$$y_R = a\sin\omega t + a\sin(\omega t+\delta) + a\sin(\omega t + 2\delta) + \cdots + a\sin(\omega t + (N-1)\delta)$$

or

$$y_R = a\sin\omega t\,(1 + \cos\delta + \cos 2\delta + \cos 3\delta + \cdots + \cos(N-1)\delta) + a\cos\omega t\,(\sin\delta + \sin 2\delta + \cdots + \sin(N-1)\delta) \quad \text{— (1)}$$

$$1 + e^{i\delta} + e^{2i\delta} + \cdots + e^{i(N-1)\delta} = \frac{1\,(e^{i\delta N} - 1)}{(e^{i\delta} - 1)}$$

$$= \frac{e^{\frac{iN\delta}{2}}\left(e^{\frac{iN\delta}{2}} - e^{-\frac{iN\delta}{2}}\right)}{e^{\frac{i\delta}{2}}\left(e^{\frac{i\delta}{2}} - e^{-\frac{i\delta}{2}}\right)} = e^{i(N-1)\frac{\delta}{2}}\left(\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}\right)$$

$$= \left(\cos\frac{(N-1)\delta}{2} + i\sin\frac{(N-1)\delta}{2}\right)\left(\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}\right)$$

$$= \cos\frac{(N-1)\delta}{2}\,\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}} + i\sin\frac{(N-1)\delta}{2}\,\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}} \quad \text{— (2)}$$

$$1 + e^{i\delta} + e^{2i\delta} + \cdots + e^{i(N-1)\delta}$$

$$= 1 + (\cos\delta + i\sin\delta) + (\cos 2\delta + i\sin 2\delta) + \cdots + (\cos(N-1)\delta + i\sin(N-1)\delta)$$

$$= [1 + \cos\delta + \cos 2\delta + \cdots + \cos(N-1)\delta] + i[\sin\delta + \cdots + \sin(N-1)\delta]$$

$$= \cos\frac{(N-1)\delta}{2}\,\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}} + i\sin\frac{(N-1)\delta}{2}\,\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}$$

$$y_R = a\sin\omega t\,(1+\cos\delta+\cos 2\delta+ \cdots + \cos(N-1)\delta)$$
$$+ a\cos\omega t\,(\sin\delta+\sin 2\delta+ \cdots + \sin(N-1)\delta)$$

or $$y_R = a\sin\omega t \cos\frac{(N-1)\delta}{2}\,\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}$$
$$+ a\cos\omega t \sin\frac{(N-1)\delta}{2}\,\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}$$

put $$a\cos\frac{(N-1)\delta}{2}\,\frac{\sin\frac{N\delta}{2}}{\sin\delta/2} = A\cos\phi$$

$$a\sin\frac{(N-1)\delta}{2}\,\frac{\sin\frac{N\delta}{2}}{\sin\delta/2} = A\sin\phi$$

or $y_R = A\sin\omega t\cos\phi + A\cos\omega t\sin\phi$

or $y_R = A\sin(\omega t+\phi)$

$$\phi = \frac{(N-1)\delta}{2}$$

Here,

(N is no of Source) $$A = a\,\frac{\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}}$$

In interference, $\Delta = n\lambda$ gives Bright

$\Delta = (2n+1)\frac{\lambda}{2}$ gives Dark

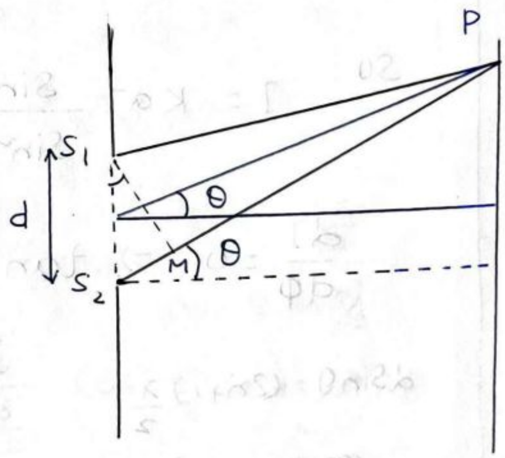
In Diffraction $\Delta = n\lambda$ gives Dark

$\Delta = (2n-1)\frac{\lambda}{2}$ gives Bright

Here path difference

$S_2M = d\sin\theta$

Let there be N no of Source. The phase difference b/w S_1 and S_2 is $\delta = \frac{2\pi}{\lambda} d\sin\theta$



Phase difference between consecutive sources

$$\delta = \frac{2\pi}{\lambda}\left(\frac{d\sin\theta}{N}\right)$$

Resultant amplitude. $A = \dfrac{a\sin\frac{N\delta}{2}}{\sin\delta/2}$

$$A = \frac{a\sin\left(\frac{N\cdot\frac{2\pi}{\lambda}\frac{d\sin\theta}{N}}{2}\right)}{\sin\left(\frac{2\pi}{\lambda}\frac{d\sin\theta}{2N}\right)}$$

or $A = \dfrac{a\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi d\sin\theta}{N\lambda}\right)}$ Let $\phi = \dfrac{\pi d\sin\theta}{\lambda}$

or $A = \dfrac{a\sin\phi}{\sin(\phi/N)}$

Intensity

$I = KA^2$

$I = a^2\dfrac{\sin^2\phi}{\sin^2(\phi/N)}K$

If $N \to$ large

$\frac{\phi}{N} \to$ small $\sin^2\frac{\phi}{N} = \frac{\phi^2}{N^2}$

$$I = N^2Ka^2\left(\frac{\sin^2\phi}{\phi^2}\right)$$

For minima, $I = 0$ $\sin\phi = 0 \Rightarrow \phi = n\pi$

$\Delta = d\sin\theta = n\lambda$ give dark fringe

$\dfrac{\pi d\sin\theta}{\lambda} = n\pi$

$d\sin\theta = n\lambda$

So $I = Ka^2\dfrac{\sin^2\phi}{\sin^2(\phi/N)}$ Maxima at $\phi = 0$

Primary maxima

$\dfrac{dI}{d\phi} = 0 \Rightarrow \tan\phi = \phi$ $\phi = \frac{3\pi}{2}$

$d\sin\theta = (2n+1)\frac{\lambda}{2}$

$(n \neq 0,\ n = 1, 2, 3.)$

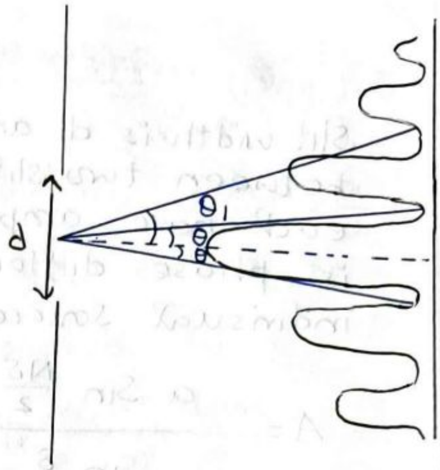
$I = KN^2a^2\dfrac{\sin^2\phi}{\phi^2} = KN^2a^2\dfrac{\sin^2\frac{3\pi}{2}}{\left(\frac{3\pi}{2}\right)^2}$

$I = I_0/22$

when, $\phi = \frac{5\pi}{2}$. $I = \frac{I_0}{64}$

$$\phi = \frac{\pi d \sin\theta}{\lambda} = \frac{3\pi}{2}$$

$$d\sin\theta = \frac{3\lambda}{2}$$



First minima,

$$d\sin\theta = \lambda \Rightarrow \theta = \sin^{-1}(\lambda/d)$$

$$d\sin\theta_1 = 2\lambda \Rightarrow \theta_1 = \sin^{-1}(2\lambda/d)$$

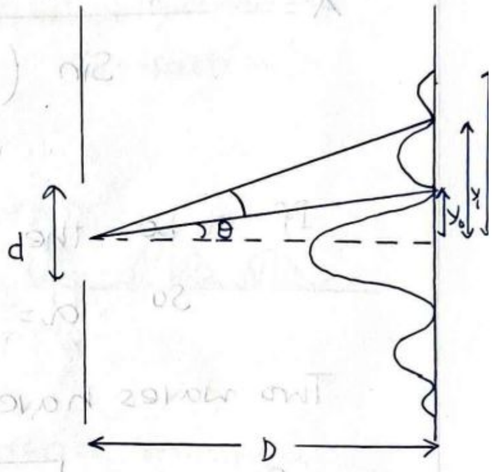
Angular seperation $\theta_1 - \theta = \sin^{-1}(2\lambda/d) - \sin^{-1}(\lambda/d)$

$d\sin\theta = \lambda$ $\quad$ $d\sin\theta = 2\lambda$

$d\cdot\frac{y_0}{D} = \lambda$ $\quad$ $d\cdot\frac{y_1}{D} = 2\lambda$

$y_0 = \frac{\lambda D}{d}$ $\quad$ $y_1 = \frac{2\lambda D}{d}$

$$y_1 - y_0 = \frac{\lambda D}{d}$$



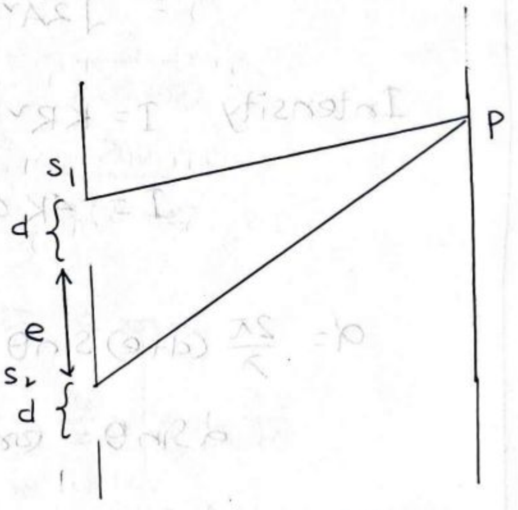
Diffraction through Double slit

Path difference

$$\Delta = S_2P - S_1P = (d+e)\sin\theta$$

Phase difference

$$\beta = \frac{2\pi}{\lambda}(d+e)\sin\theta$$



Both source has the amplitude A where phase difference is β the resultant amplitude

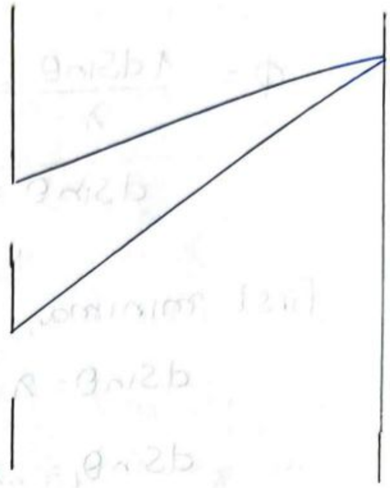
$$A = \frac{a\sin\frac{N\delta}{2}}{\sin\delta/2}$$

$$R = 4A^2\cos^2\beta/2 = 4a^2\frac{\sin^2\frac{N\delta}{2}}{\sin^2\frac{\delta}{2}}\cos^2\frac{\pi}{\lambda}(d+e)\sin\theta$$

$\delta = \frac{2\pi d\sin\theta}{\lambda}$ $\quad$ Diffraction $\quad$ Interferene

20.09.2024

Slit width is d and distance between two slit is e and each have amplitude A. δ is phase difference between indivisual source so



$$A = \frac{a \sin \frac{N\delta}{2}}{\sin \frac{\delta}{2}}$$

$$\delta = \frac{2\pi}{\lambda}\left(\frac{d\sin\theta}{N}\right)$$

$$A = \frac{a \sin\left(\frac{N}{2}\frac{2\pi}{\lambda}\frac{d\sin\theta}{N}\right)}{\sin\left(\frac{\pi d \sin\theta}{N\lambda}\right)} = \frac{a\sin\phi}{\sin(\phi/N)}$$

Here, $\phi = \frac{\pi d \sin\theta}{\lambda}$

If α be the phase differce between two source

so $\alpha = \frac{2\pi}{\lambda}(d+e)\sin\theta$

Two waves have amplitude A, phase differen α

so $R = \sqrt{A^2+A^2+2A^2\cos\alpha}$

$= \sqrt{2A^2(1+\cos\alpha)} = \sqrt{4A^2\cos^2\alpha/2}$

Intensity $I = KR^2 = 4KA^2\cos^2\alpha/2$

$$I = 4Ka^2 \frac{\sin^2\phi}{\sin^2\phi/N}\cos^2\alpha/2$$

$\alpha = \frac{2\pi}{\lambda}(d+e)\sin\theta$; $\phi = \frac{\pi}{\lambda} d\sin\theta$

$d\sin\theta = (2n+1)\frac{\lambda}{2}$ (Maxima) (Diffraction) ($n=1,2,3$)

$d\sin\theta = n\lambda$ (Diffraction min)

Interference,

Maxima

$\cos\frac{\alpha}{2} = \pm 1$

$\alpha = 2m\pi$

Minima

$\cos\frac{\alpha}{2} = 0$

$\alpha = (2m+1)\pi$ ($m = 0, 1, 2 \ldots$)

$$\frac{\alpha}{2} = m\pi$$

$$\frac{2\pi}{\lambda}\frac{(d+e)\sin\theta}{2} = m\pi$$

$$(d+e)\sin\theta = m\lambda$$

$$\frac{2\pi}{\lambda}\frac{(d+e)\sin\theta}{2} = (2m+1)\frac{\pi}{2}$$

$$(d+e)\sin\theta = (2m+1)\frac{\lambda}{2}$$

So Interference

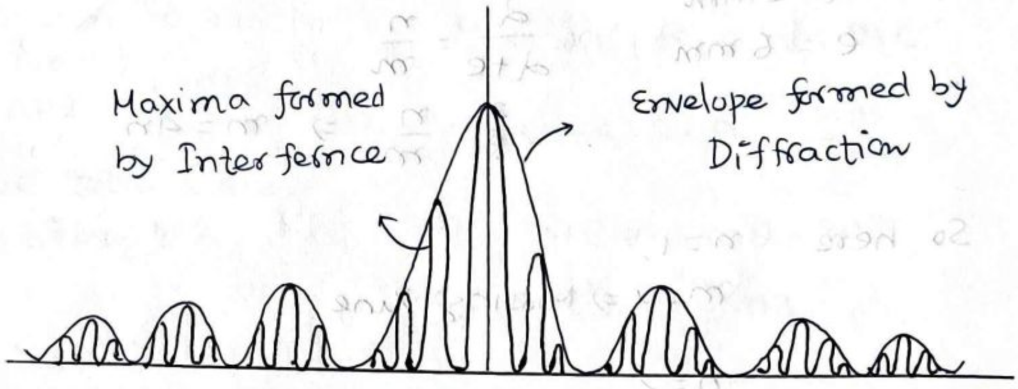
Maxima: $(d+e)\sin\theta = m\lambda$

Minima: $(d+e)\sin\theta = (2m+1)\frac{\lambda}{2}$

Diffraction

Maxima: $d\sin\theta = (2n+1)\lambda/2$

Minima: $d\sin\theta = n\lambda$



For $n=1$ diffraction minima

$$\sin\theta = \frac{\lambda}{d}$$

$$\frac{y}{D} = \frac{\lambda}{d}$$

$$y = \frac{\lambda D}{d}$$

Interference minima

$$\sin\theta = \frac{\lambda}{2(d+e)}$$

$$\frac{y}{D} = \frac{\lambda}{2(d+e)}$$

$$y = \frac{\lambda D}{2(d+e)}$$

Here if $d = 1mm$. e goes to 4.mm

So $y = \lambda D$ $\qquad$ $y = \frac{\lambda D}{2}\times\frac{1}{5} = \frac{\lambda D}{10}$

Here always diffraction dominate and work like envelope. Interference change about it. One diffraction maxima has multiple interference maxima $d+e > d$

If interference maxima and diffraction minima over laps then diffraction dominate and dark fringes will occur

$(d+e)\sin\theta_1 = m\lambda$ $\qquad$ $d\sin\theta_2 = n\lambda$

But here $\theta_1 = \theta_2$ $\qquad$ n = order of diff min

So $\dfrac{d}{d+e} = \dfrac{n}{m}$ $\qquad$ m = order of inter. max

$$\boxed{\frac{d+e}{d} = \frac{m}{n}}$$ (Condition for missing order)

Let $d = 2mm$
$e = 6mm$

$$\frac{d}{d+e} = \frac{n}{m}$$

$$\frac{2}{8} = \frac{n}{m} \Rightarrow m = 4n$$

So here $n = 1$,
$m = 4 \Rightarrow$ Missing line

$n = 2$
$m = 8 \Rightarrow$ Missing line

23.09.24

Diffraction through N-slits: Diffraction Grating:

If the phase difference b/w to consecutive Source is δ

$$\delta = \frac{2\pi}{\lambda}\left(\frac{d\sin\theta}{N}\right)$$

Phase difference between to Source is γ

$$\gamma = \frac{2\pi}{\lambda}(d+e)\sin\theta$$

$$A = \frac{a\sin\frac{N\delta}{2}}{\sin\frac{\delta}{2}} = \frac{a\sin\frac{\pi}{\lambda}d\sin\theta}{\sin\frac{\pi}{\lambda}\frac{d\sin\theta}{N}}$$

Resultant amplitude due to N source

$$R = A\,\frac{\sin\frac{N\gamma}{2}}{\sin\gamma/2}$$

So $R = \dfrac{a \sin\left(\frac{\pi}{\lambda} d \sin\theta\right)}{\sin\left(\frac{\pi}{\lambda} \frac{d \sin\theta}{N}\right)} \quad \dfrac{\sin\left[M \frac{\pi}{\lambda}(d+e)\sin\theta\right]}{\sin\left[\frac{\pi}{\lambda}(d+e)\sin\theta\right]}$

Intensity. $I = Ka^2 \dfrac{\sin^2\phi}{\sin^2(\phi/N)} \quad \dfrac{\sin^2(\frac{M\gamma}{2})}{\sin^2 M/2}$

As we already have $\dfrac{\sin^2\phi}{\sin^2(\phi/N)}$

Now For minima, $\sin\frac{M\gamma}{2} = 0 = \sin m\pi$

(M is No of Source)

$\frac{M}{2} \times \frac{2\pi}{\lambda}(d+e)\sin\theta = m\pi$

$(d+e)\sin\theta = \frac{\lambda m}{M}$

(M(d+e) total width of grating)

$\Rightarrow M(d+e)\sin\theta = m\lambda$

$\Rightarrow D\sin\theta = m\lambda$

Minima

For maxima. $\frac{dI}{d\gamma} = 0$

$$\frac{d}{d\gamma}\left[\frac{\sin^2(\frac{M\gamma}{2})}{\sin^2\frac{\gamma}{2}}\right] = 0$$

or $$\frac{\sin^2\frac{\gamma}{2} \cdot 2\sin\frac{M\gamma}{2}\cos\frac{\gamma}{2} \cdot \frac{M}{2} - \sin^2\left(\frac{M\gamma}{2}\right) \cdot 2\sin\frac{\gamma}{2}\cos\frac{\gamma}{2} \cdot 2}{\sin^4\frac{\gamma}{2}} = 0$$

or $M \tan\frac{M\gamma}{2} = \tan\frac{M\gamma}{2}$

or $\frac{M\gamma}{2} = 3\frac{\pi}{2}, 5\frac{\pi}{2}, 7\frac{\pi}{2} \dots$

$\frac{M\gamma}{2} = (2m+1)\frac{\pi}{2}$

$\frac{M}{2} \cdot \frac{2\pi}{\lambda}(d+e)\sin\theta = (2m+1)\frac{\pi}{2}$

$M(d+e)\sin\theta = (2m+1)\frac{\lambda}{2}$

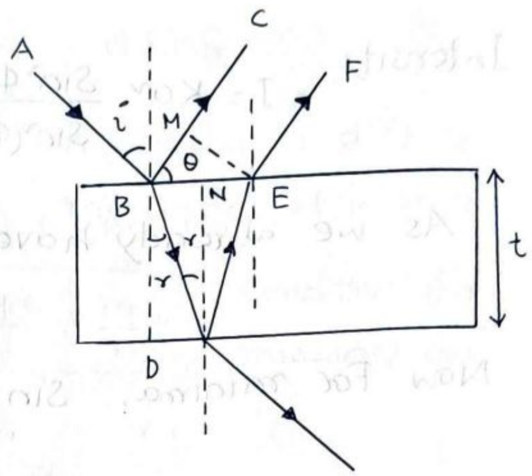
$\Rightarrow D = (2m+1)\frac{\lambda}{2}$

Maxima

26.09.24

Thin film Interference:

The path difference between two outgoing rays are given by.



$\delta = \mu(BD+DE) - BM$

$\cos r = \frac{t}{BD}$

or $t = BD \cos r$

or $BD = t/\cos r$

$\frac{NE}{t} = \tan r$

$NE = t \tan r$

$\frac{\sin i}{\sin r} = \mu$

$BE = 2NE = 2t \tan r$

$\cos(90-i) = \frac{BM}{BE}$

$\sin i = BM/BE \Rightarrow BM = 2t \cdot \tan r \sin i$

Path difference,

$$\Delta x = \mu \left[\frac{2t}{\cos r}\right] - 2t \cdot \tan r \sin i$$

$$= \frac{2\mu t}{\cos r} - \frac{2t \sin r \sin i}{\cos r}$$

$$= \frac{2t}{\cos r}(\mu - \sin i \sin r)$$

$$= \frac{2t}{\cos r}\left(\mu - \frac{\sin^2 i}{\mu}\right)$$

$$= \frac{2t}{\sqrt{1-\sin^2 r}}\left(\mu - \frac{\sin^2 i}{\mu}\right) = 2\mu t \cos r$$

Net path difference $= 2\mu t \cos r + \frac{\lambda}{2} = \Delta x$

$\Delta x = n\lambda$ (Constructive interference)

$2\mu t \cos r + \frac{\lambda}{2} = n\lambda$

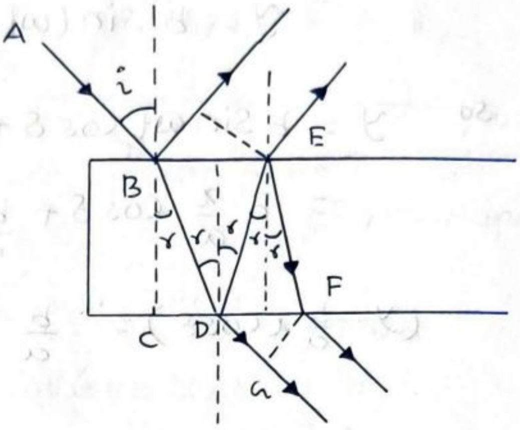
$2\mu t \cos r = (2n-1)\lambda/2$

$\Delta x = (2n+1)\frac{\lambda}{2}$ (Destructive interference)

$2\mu t\cos r + \frac{\lambda}{2} = n\lambda + \frac{\lambda}{2}$ Dark fringes are formed

$2\mu t\cos r = n\lambda$

Refracted Ray



Here the path difference

$\Delta x = \mu(DE+EF) - DG$

So Path difference

$\Delta x = 2\mu t\cos r$

$2\mu t\cos r = (2n-1)\frac{\lambda}{2}$
(Destructive)

$2\mu t\cos r = n\lambda$
(Constructive)

Wedge shaped thin film:

Polarization of Light

There are two simple Harmonic motion

$$x = a \sin\omega t$$
$$y = b \sin(\omega t + \delta)$$

So $\quad y = b\sin\omega t\cos\delta + b\cos\omega t\sin\delta$

$$= b\cdot\frac{x}{a}\cos\delta + \frac{b}{a}\sqrt{a^2-x^2}\sin\delta$$

$$\left(y - \frac{b}{a}x\cos\delta\right) = \frac{b}{a}\sqrt{a^2-x^2}\sin\delta$$

or $\quad y^2 - 2\frac{b}{a}xy\cos\delta + \frac{b^2}{a^2}x^2\cos^2\delta = \frac{b^2}{a^2}a^2\sin^2\delta - \frac{b^2}{a^2}x^2\sin^2\delta$

or $\quad x^2\cdot\frac{b^2}{a^2} - \frac{2b}{a}xy\cos\delta + y^2 = b^2\sin^2\delta$

or $\quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{xy}{ab}\cos\delta = \sin^2\delta$

① **If $\delta = 0$**

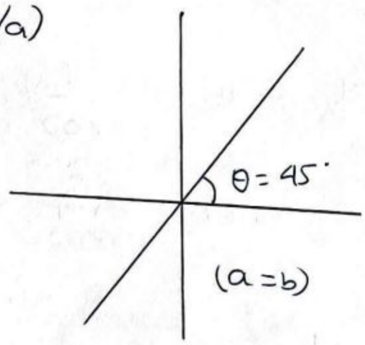
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} = 0$$

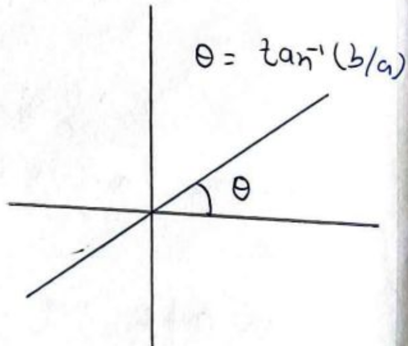
$$\left(\frac{x}{a} - \frac{y}{b}\right)^2 = 0 \quad \Rightarrow \quad y = \frac{b}{a}x$$

$m = \tan\theta$ $\qquad\qquad \Rightarrow y = mx$

$\theta = \tan^{-1}(b/a)$

(linearly polarized)





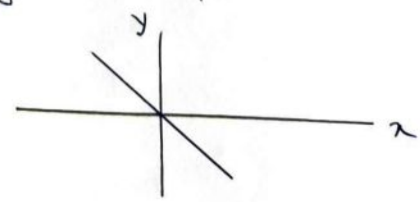
② **If $\delta = \pi$**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2xy}{ab} = 0$$

linearly polarized with negative slope

$$\left(\frac{x}{a} + \frac{y}{b}\right)^2 = 0$$

$$y = -\frac{b}{a}x$$

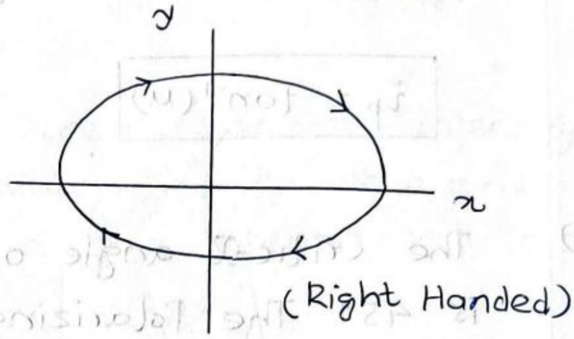


③ If $\delta = \pi/2$

$x = a\sin\omega t$
$y = b\sin(\omega t + \frac{\pi}{2})$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Elliptical Polarization)

ωt	x	y
0	0	b
$\pi/2$	a	0
π	0	$-b$



(Right Handed)

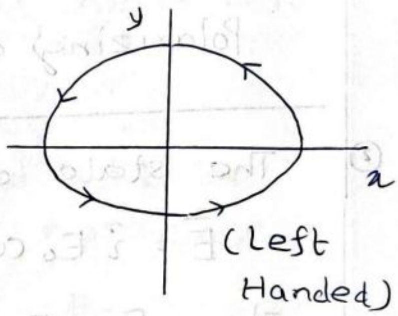
If $x^2 + y^2 = a$ (as $a = b$)
(Circular Polarization)

④ If $\delta = 3\pi/2$

$x = a\sin\omega t$
$y = b\sin(\omega t + \frac{3\pi}{2})$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = -1$

ωt	x	y
0	0	$-b$
$\pi/2$	a	0
π	0	b

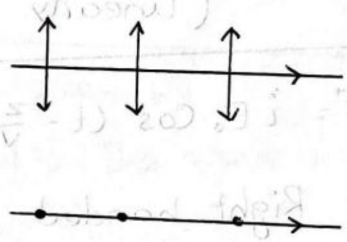


(Left Handed)

(a = b make left handed Circular Polarization)

Polarization is only shown by tranverse wave, that's why in Sound waves it not Possible

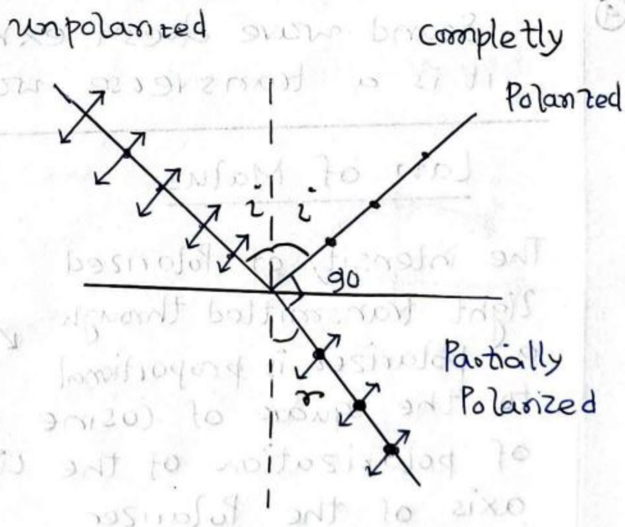
Plane Polarized light



Angle between reflected and refracted light is 90°

⇓

Brewest's law



$i + r + 90 = 180$ $\quad \frac{\sin i}{\sin r} = \mu$

$i + r = 90$

$$\frac{\sin i}{\sin(90 - i)} = \mu \Rightarrow \tan i = \mu$$

$$i_p = \tan^{-1}(\mu)$$

$$\boxed{i_p = \tan^{-1}(\mu)}$$

① The Critical angle of light in a certain substance is 45°. The Polarizing angle is

$\Rightarrow$

Critical angle. $i_c = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1} 45 = \frac{1}{\mu}$

$$\mu = \sqrt{2}$$

Polarizing angle $i_p = \tan^{-1}(\mu) = \tan^{-1}\sqrt{2} = 54.7$

---

② The state of Polarization

$$\vec{E} = \hat{i} E_0 \cos(Kx - \omega t) - \hat{j} E_0 \cos(Kz - \omega t)$$

$\Rightarrow$

$E_x = E_0 \cos(Kx - \omega t)$ $\quad \frac{E_y}{E_x} = -1 \Rightarrow \frac{y}{x} = -1$

$E_y = -E_0 \cos(Kz - \omega t)$

(linearly polarized at angle 45° to x axis)

---

③ $\vec{E} = \hat{i} E_0 \cos\left(t - \frac{z}{v}\right) + \hat{j} E_0 \cos\left\{\left(\omega t - \frac{\omega z}{v}\right) - \frac{5\pi}{4}\right\}$

$\Rightarrow$ Right handed Elliptical Polarization

④ Sound wave doesn't exhibit polarization because it is a transverse wave

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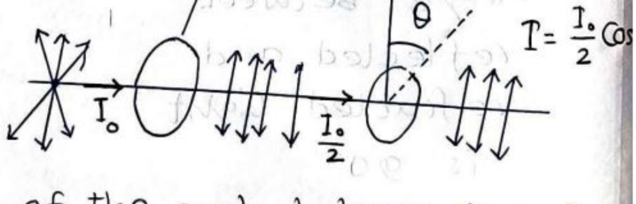
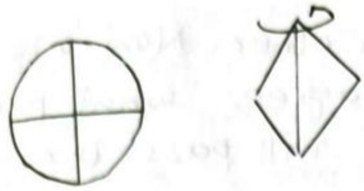
Law of Malus:



The intensity of Polarized light transmitted through a polarizer is proportional to the square of cosine of the angle between the plane of polarization of the light and the transmisson axis of the Polarizer

$$I = I_0/2 \cos^2\theta$$

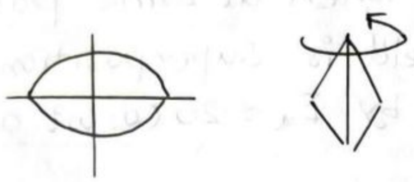


I_0 Intensity no change $I_0/2$

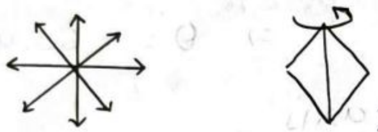
For Circular polariz^n no change of intensity during the rotation of Polarizer

I_0 $I_0/2$

For linear polarization intensity increase and decrease and at a time intensity will zero.



For Elliptical Polarization, intensity became increase and decrease but never zero



I_0 $I_0/2$

For unpolarized light the intensity has no change

$$I = \frac{P}{A}$$

$$E = \int I A\,dt$$

The electric field on an EMW is given by

$\vec{E} = 3\sin(Kz-\omega t)\hat{i} + 4\cos(Kz-\omega t)\hat{j}$ The wave is

$\Rightarrow$ $y = 3\sin(Kz-\omega t)$

$x = 4\sin(Kz-\omega t+\frac{\pi}{2})$

$$\frac{x^2}{4^2}+\frac{y^2}{3^2} = 1$$

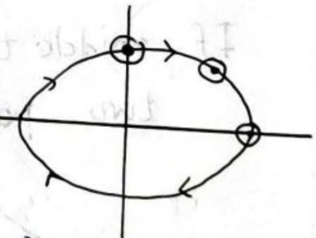
(Elliptical)

ωt	x	y
0	0	4
$\frac{\pi}{4}$	$\frac{3}{\sqrt{2}}$	$\frac{4}{\sqrt{2}}$
$\frac{\pi}{2}$	3	0

$x = 3\sin(Kz-\omega t)$

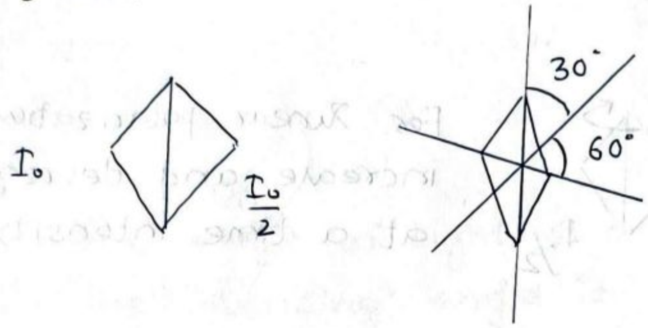
$y = 4\cos(Kz-\omega t)$

(Right handed Elliptical Polarization)



⑥ Two nicols are crossed to each other. Now one of them is rotated through 60 degrees. what percentage of incident unpolarized light will pass through the system

⇒



$I = \frac{I_0}{2} \cos^2 30$

$= \frac{I_0}{2} \cos^2 30$

$= 0.375 I_0$

---

⑦ The direction of Polarization when at some point in the resultant electric field is superposition of two electric fields is given by $E_x = 20\cos\omega t$ and $E_y = 40\cos(\omega t + \pi)$ is

$E_x = 20\cos\omega t$

$E_y = -40\cos\omega t$

$\frac{E_y}{E_x} = -2 \Rightarrow \tan\theta = -2$

$\Rightarrow \theta = -63.43°$

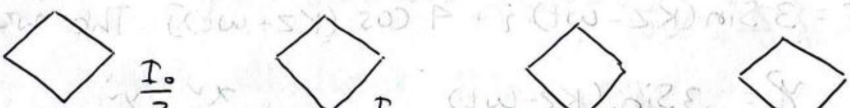
$+63.43°$ below the x axis

---

⑧ Four perfect Polarizers are stacked together such that the transmission axis of consecutive polarizers are seperated by 30°. If intensity of the incident unpolarized light is $2 mw/m^2$. Then

⇒

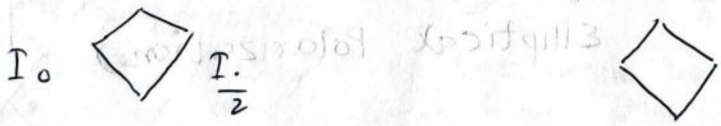
I_0



$\frac{I_0}{2}$ $\quad$ $\frac{I_0}{2}\cos^2 30$ $\quad$ $\frac{I_0}{2}\cos^4 30$ $\quad$ $\frac{I_0}{2}\cos^6 30$

$2mw/m^2$ $\quad$ $1mw/m^2$ $\quad$ $0.75\ mw/m^2$ $\quad$ $\frac{27}{64}\ mw/m^2$ $\quad$ $0.42 mw/m^2$

If middle two polarizer is removed. angle b/w remaining two polarizer is 90 $I = 0$



$\frac{I_0}{2}\cos^2 90 = 0$

unpolarized light is incident on a glass plate having refractive index 1.5. The angle of incidence at which the Plane polarized light obtained

=) $\theta = \tan^{-1}\mu = \tan^{-1}(1.5) = 56.3°$

$E_x = A_x \cos(\omega_x t + \phi_x)$ and $E_y = A_y \sin(\omega_y t + \phi_y)$. Their superposition will result in a plane polarized light if

=)

$E_x = A_x \cos(\omega_x t + \phi_x)$ For plane polarized

$E_y = A_y \sin(\omega_y t + \phi_y)$ δ should be 0 or π

$\phi_x = \pi/2$. $\phi_y = \pi$ $\omega_x = \omega_y$

---

when light passing through rotating nicol prism is observed., no change in intensity is observed. What inference can be drawn

=) (c) the incident light is unpolarized or Circularly polarized

(d) The incident light is unpolarized or Circularly polarized or Combination of both

---

Consider these waves $E_1 = E_0 \sin(\omega t - Kx + \pi/2)\hat{i}$; $E_2 = E_0 \sin(\omega t - Kx + \pi)\hat{i}$, $E_3 = E_0 \cos(\omega t - Kx)\hat{j}$ and $E_4 = E_0 \sin(\omega t - Kx + \pi/2)\hat{k}$. if these wave superimpose pairwise, which superposition will lead to interference

=) The Condition for interferce is -

(1) Same amplitude

(2) Constant phase differce

(3) Same direction

So E_1, E_2 exhibits interference.

(13) A light wave is represented by $E_x = E_o \sin 65 \cos(\omega t - kx)$ and $E_y = E_o \cos(\omega t - kx)$ then the state of the polarization is

$\Rightarrow$ $E_x = E_o \sin 65$ $\qquad \frac{E_y}{E_x} = \frac{1}{\sin 65} = 1.11$

$E_y = E_o$

$\theta = \tan^{-1}(1.11)$

linearly polarized at 45° with x axis

---

(14) which of the following is not true for circularly polarized light

$\Rightarrow$ Magnitude of electric field is constant but direction varies.

---

(15) A circularly polarized light is incident on a glass plate at Brewester angle then

$\Rightarrow$ Reflected light is plane polarized light transmitted is the combination of CPL and PPL

---

(16) An EM wave is obtained by superposition of two waves having the same amplitude and right and left hand circular polarization. If the initial phase between the wave is 60°. What is the characteristic of the resultant wave

$\Rightarrow$ $E_y = E_o \sin(\omega t + \frac{\pi}{2})$

$E_x = E_o \sin(\omega t)$

---

(17) An unpolarized plane light wave of intensity $10 mW/m^2$ passes through two nicols with their principal sections at 30°. intensity of transmitted waves

$\Rightarrow$ $I = I_o/2 \cos^2\theta = \frac{10}{2} \cos^2 30$

$= 3.75 mW/m^2$

A beam of plane polarized light falls on a polarizer which rotates about the axis of the ray with angular velocity 21 rad/sec. The energy of light passing through the polarizer per revolution if the flux of energy of the incident ray is equal to 4.0 mW

$\Rightarrow$ $I = I_0 \cos^2\theta$ $\qquad \theta = \omega t$

$$\frac{dE}{dt} = I_0 \cos^2 \omega t \Rightarrow E = I_0 \int_0^{2\pi/\omega} \cos^2 \omega t \, dt$$

$$E = \frac{I_0 \pi}{\omega} = \frac{4 \times 3.14}{21} = 0.6 \, mJ$$

A given calcite plate behaves as a half wave plate for a particular wavelength λ, If the variation of refractive index with λ is negligible, then for a light of wavelength 2λ, the given plate be

$\Rightarrow$ $\Delta x = \frac{\lambda}{2}$

$(n_e - n_o)t = \frac{\lambda}{2}$ — ①

For 2λ the plate is quarter wave plate $(\lambda/4)$

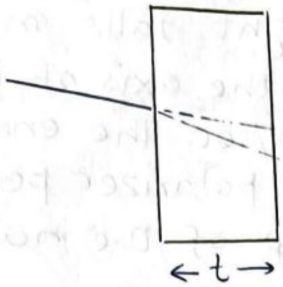
If t is minimum thickness of a quarter wave plate needed to convert plane polarized light of wave-length 480 nm into circular polarized light, then the corresponding thickness of a quarter wave plate for wavelength 600 nm

$\Rightarrow$ $(\mu_e - \mu_o) = \frac{\lambda_1}{4t_1}$ $\qquad \frac{\lambda_1}{t_1} = \frac{\lambda_2}{t_2}$ $\qquad t_2 = \frac{600}{480} t$

$(\mu_e - \mu_1) = \frac{\lambda_2}{4t_2}$ $\qquad \frac{480}{t} = \frac{600}{t_2}$ $\qquad t_2 = 1.25 t$

Two polarizing sheets have their polarizing directn parallel, so that the intensity of the transmitted light is maximum. If the intensity drops by one half, Then either of the sheets must be turned by an angle

$\Rightarrow$ $I = I_0 \cos^2\theta = \frac{I_0}{2} \Rightarrow \theta = 45$,



(ordinary & extra-ordinary are PL)

The outgoing rays are ordinary (μ_o) and extra-ordinary (μ_e)

Optical Path difference

$$(\mu_e - \mu_o)t = \Delta x$$

phase difference

$$\delta = \frac{2\pi}{\lambda} t(\mu_e - \mu_o)$$

Case ①:

if $\delta = \frac{\lambda}{4}$, $\frac{2\pi}{\lambda} |\mu_e - \mu_o| t = \frac{\lambda}{4} \cdot \frac{2\pi}{\lambda}$

Plate is Called quater wave plate

Case ②

if $\Delta x = \frac{\lambda}{2}$, $\delta = \pi$ Plate is Half wave plate

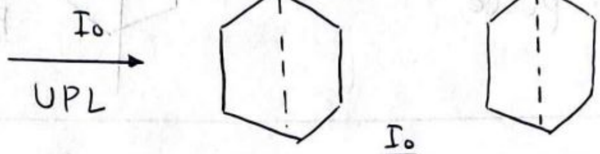
$(\mu_e - \mu_o)t = \frac{\lambda}{2}$

PPL → Quater wave plate → $\delta = \pi/2$ CPL/EPL	PPL → HWP → PPL Direction changes
CPL (LH) → QWP → PPL	CPL (LH) → HWP → RHCPL

㉒ A linearly polarized light is incident on a quarter plate. The emerge wave will be Ellipticaly polarized

A linear beam of unpolarized light passes through two plane polarizer. the plates of which are perpendicular to the direction of propagation of the beam. The first polarizer rotates around the direction with an angular velocity of 20π radius per second. If the initial intensity is I_0, then the intensity when it leaves the second polarizer is

⇒



Frequency

$I_0/2$

$$f = \frac{\omega}{2\pi} = \frac{20\pi}{2\pi} = 10\,Hz$$

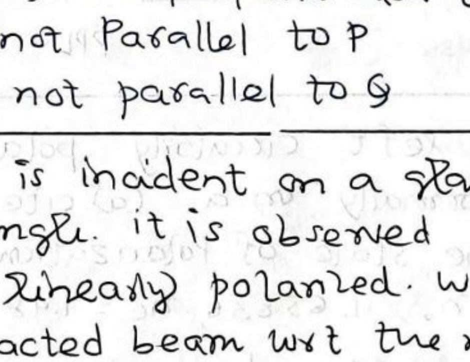
$$I_{max} = \frac{I_0}{2}\cos^2 0 = \frac{I_0}{2}$$

periodic frequency is $2f = 20Hz$

$$I_{min} = \frac{I_0}{2}\cos^2 90 = 0$$

In an optical arrangement two polarizing sheets P and Q are orinted such that no light is detected. Now when a third polarizing sheet R is placed between P and Q and then light is detected. Which is true

⇒



Polarization axis of P and Q are perpendicular to each other
Polarization axes of R is not Parallel to P
Polarization axes of R is not parallel to Q

When unpolarized light is incident on a glass plate at a particular angle. it is observed that the reflected beam is linearly polarized. What is the angle of the refracted beam wrt the normal Refractive index is 1.52

⇒ $i_p = \tan^{-1}(\mu) = \tan^{-1}(1.52) = 56.65$

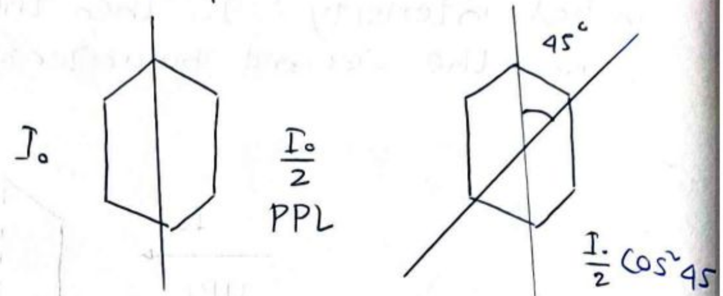
$i_p + r = 90 \Rightarrow r = 90 - 56.65 = 33.4°$

---

(26) Consider a beam of light of wavelength λ incident on a System of a polarizer and an analyzer. The analyzer is oriented at 45° to polarizer. when an optical Component is introduced between them. the output intensity zero. The optical component is

=>

An ordinary glass plate



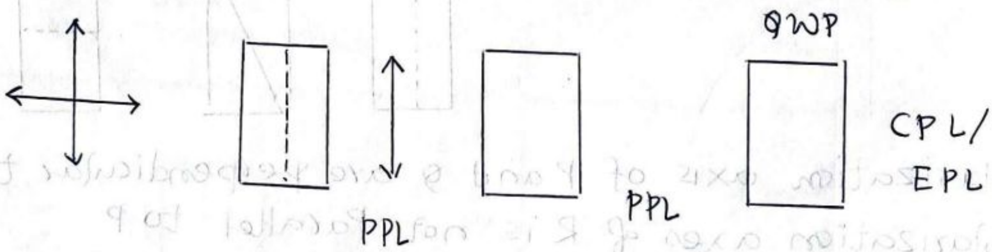
(27) A quarter wave plate is designed for a wavelength of 600nm. The difference in refractive index of the electric Components along fast and slow axes is 0.2. The geometrical thickness of the plate will be

=> $(\mu_e - \mu_o)t = \frac{\lambda}{4}$ $\qquad t = 750\ nm$

$0.2 \times t = \frac{600 \times 10^{-9}}{4}$

(28) Consider the arrangement of Polarizer P_1 and P_2 and QWP one after other. If an unpolarized light is incident then the nature of output light through QWP is

=>



(29) A left circularly polarized beam (5893Å) is incident normally on a Calcite Crystal of thickness 0.005141mm. The state of Polarization on emergent beam will be ($n_o = 1.65836$, $n_e = 1.48644$) (RHCPL)

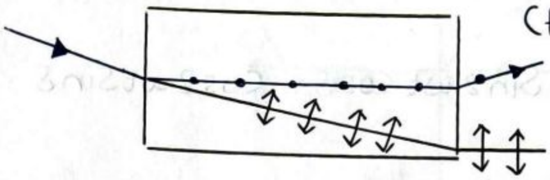
=> $4x = |n_o - n_e|\, t = (0.005141 \times 10^{-3})(1.65836 - 1.48644)$

$= \cancel{888}\ 8838\text{A} \doteq 3\lambda/2$

$\frac{3\lambda}{2} = \lambda + \frac{\lambda}{2}$ $\qquad$ LHCPL $\xrightarrow{\lambda}$ LHCPL $\xrightarrow{\lambda/2}$ RHCPL

Polarization by Double Refraction

unpolarized light



Ordinary Ray (Follow Snell's law)

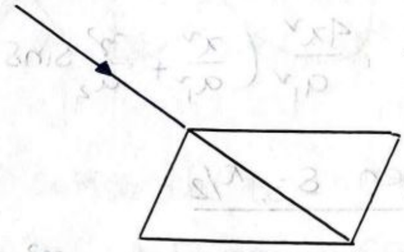
The Emergent rays are polarized light

Calcite Crystal

Extra-ordinary Ray (Not follow the laws of Refraction)

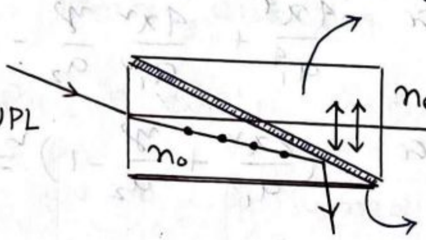
> Optic Axis:

This is an axis that exsist in a Crystal along which no double refraction occur. Here. $v_1 = v_2$



No E-ray / O-ray is formed

> Nicol Prism:



$\boxed{n_e < n_{CB} < n_o}$

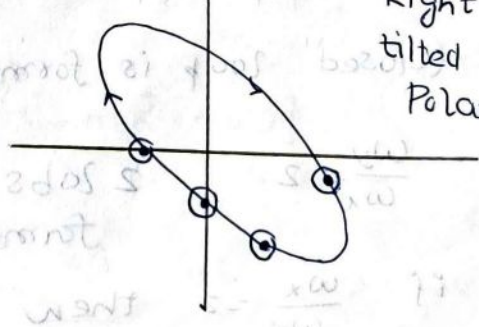
Suppose we have electromagnetic wave

$E = \frac{\sqrt{3}}{2} \cos(\omega t - Kz)\hat{i} + \frac{1}{2}\cos(\omega t - Kz + \frac{3\pi}{4})\hat{j}$ represents

$\Rightarrow$ $E_x = \frac{\sqrt{3}}{2}\cos(\omega t - \cancel{Kz}) = 0.86 \cos \omega t$

$E_y = \frac{1}{2}\cos(\omega t - \cancel{Kz} + \frac{3\pi}{4}) = 0.5 \cos(\omega t + \frac{3\pi}{4})$

ωt	E_x	E_y
0	0.86	-0.35
$\frac{\pi}{4}$	0.60	-0.5
$\frac{\pi}{2}$	0	-0.35
$\frac{3\pi}{4}$	-0.60	0



Right handed tilted Elliptical Polarization

Lissajous Figure

$x = a_1 \sin\omega t$

$y = a_2 \sin(2\omega t + \delta)$

So $\frac{y}{a_2} = \sin 2\omega t \cos\delta + \cos 2\omega t \sin\delta$

$\frac{y}{a_2} = 2\sin\omega t \cos\omega t \cos\delta + (1 - 2\sin^2\omega t)\sin^2\delta$

$\frac{y}{a_2} = 2\frac{x}{a_1}\sqrt{1 - x^2/a_1^2}\cos\delta + (1 - 2\frac{x^2}{a_1^2})\sin\delta$

$\frac{4x^2}{a_1^2}\left(\frac{x^2}{a_1^2} + \frac{y}{a_2}\sin\delta - 1\right) + \left(\frac{y}{a_2} - \sin\delta\right)^2 = 0$

<u>when $\delta = \pi/2$</u>

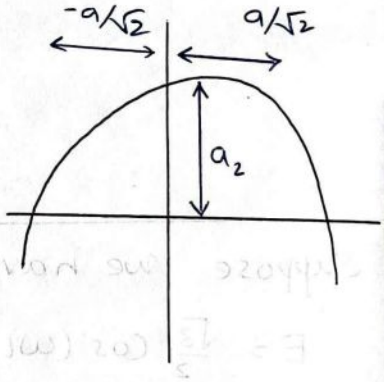
$\frac{4x^2}{a_1^2}\left(\frac{x^2}{a_1^2} + \frac{y}{a_2} - 1\right) + \left(\frac{y}{a_2} - 1\right)^2 = 0$

or $\frac{4x^4}{a_1^4} + \frac{4x^2}{a_1^2}\frac{y}{a_2} - \frac{4x^2}{a_1^2} + \frac{y^2}{a_2^2} - 2\frac{y}{a_2} + 1 = 0$

or $\left(\frac{2x^2}{a_1^2} + \frac{y}{a_2} - 1\right)^2 = 0$

Now $\frac{2x^2}{a_1^2} + \frac{y}{a_2} - 1 = 0$

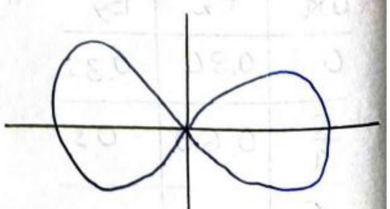
$\frac{2x^2}{a_1^2} = 1 - \frac{y}{a_2}$



<u>when $\delta = 0$</u>

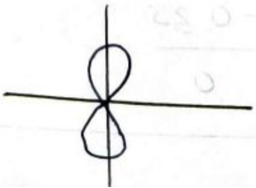
$\frac{4x^2}{a_1^2}\left(\frac{x^2}{a_1^2} - 1\right) + \left(\frac{y}{a_2}\right)^2 = 0$

(Closed loop is formed)



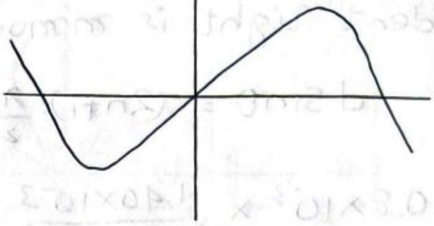
$\frac{\omega_y}{\omega_x} = 2$ 2 lobs are formed

if $\frac{\omega_x}{\omega_y} = 2$ then

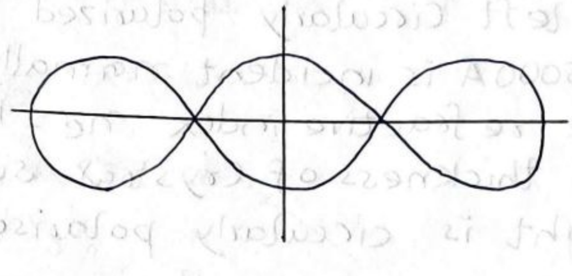


$x = a \sin \omega t$
$y = b \sin(3\omega t + \delta)$

when $\delta = 0$



when $\delta = \pi/2$



> Lissajeous figure is highly sensetive to –
(a) Frequencies (b) Amplitude (c) Phase difference

> $x = 2 \sin 4\omega t$ $\quad \frac{\omega_x}{\omega_y}$ is decides no of lobs in which direction
$y = 4 \sin(\omega t + \phi)$

Amplitude decides height & width Ratio

> $\frac{\omega_x}{\omega_y} \to$ Rational no then closed loop formed

$\frac{\omega_x}{\omega_y} \to$ Irrational no then closed loop is not formed

> $\frac{\omega_x}{\omega_y} = \frac{5}{4}$ — 5 → No of Horizontal lobes; 4 → No of Vertical lobes

> $\frac{a}{b} = \frac{3}{4}$ — 3 → width; 4 → Heigh

> ϕ is angle of rotation

---

which of following an elliptically polarized light

$\Rightarrow$ $(E_1 \hat{x} + i E_2 \hat{y}) e^{i(\bar{K}\cdot\bar{r} - \omega t)}$

$(E_1 \hat{x} + i E_2 \hat{y}) [\cos(\bar{K}\cdot\bar{r} - \omega t) + i \sin(\vec{K}\cdot\bar{r} - \omega t)]$

(31) In a single-slit diffraction, the second-order bright fringe is at a distance 1.40 mm from the centre of the central maximum. The screen is 80.0 cm from a slit of width 0.8 mm. Assuming that the incident light is monochromatic the approx wavelength

$\Rightarrow$ $d\sin\theta = (2n+1)\frac{\lambda}{2}$ $(2n+1) = 5$

$$0.8\times10^{-3} \times \frac{1.40\times10^{-3}}{80\times10^{-2}} \times \frac{1}{2.5} = \lambda \Rightarrow \lambda = 5.6\times10^{-7}\,m$$

---

(32) A left circularly polarized EMW with wavelength $\lambda = 5000\,Å$ is incident normally on a calcite crystal. The refractive index $n_e = 1.554$ and $n_o = 1.544$. The thickness of crystal so that emergent light is circularly polarised is

$\Rightarrow$ Path differece $\Delta x = t(n_e - n_o) = 0.01t$

$$\delta = \pi = \frac{2\pi}{\lambda}\times 0.01t$$

$$\lambda = 2\times0.01t \Rightarrow 0.02t = 5000\times10^{-10}$$

$$\Rightarrow t = 25\,\mu m$$

---

(33) A plane electromagnetic wave is propagating in non-magnetic, isotropic, dielectric medium is

$$\vec{E} = (A\hat{x}+\hat{z})\cos(10^9 t - 4x + 4\sqrt{3}z)$$

The refractive index of the medium

$\Rightarrow$ $\omega = 10^9$ $\vec{K} = 4\hat{i} - 4\sqrt{3}\hat{k}$

$$v = \frac{\omega}{|K|} = \frac{10^9}{8} \qquad |\vec{K}| = \sqrt{16+48} = 8$$

$$\mu = \frac{c}{v} = \frac{3\times10^8}{10^9}\times 8 = 2.4$$

---

(34) Light of wavelength 530 nm falls normally on diffraction grating with period 1.5 µm. The angle relative to normal at which the Franhofer maximum of highest order is observed is

$\Rightarrow$ $n\lambda = d\sin\theta$ $n=1$

$d\sin\theta = \lambda$

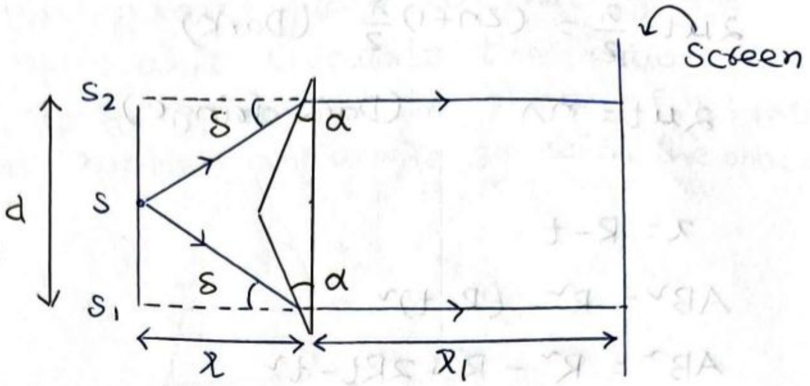
$1.5\times10^{-6}\sin\theta = 530\times10^{-9} \Rightarrow 20.67$

$2\lambda = d\sin\theta$

$\theta = 45$.

09.10.24

Fresnel Biprism



Deviation of the Prism, $\delta = (\mu-1)\alpha$

$$\delta = \frac{d}{2x} \Rightarrow \frac{d}{2x} = (\mu-1)\alpha$$

$$\Rightarrow d = 2x(\mu-1)\alpha$$

Llyod Mirror

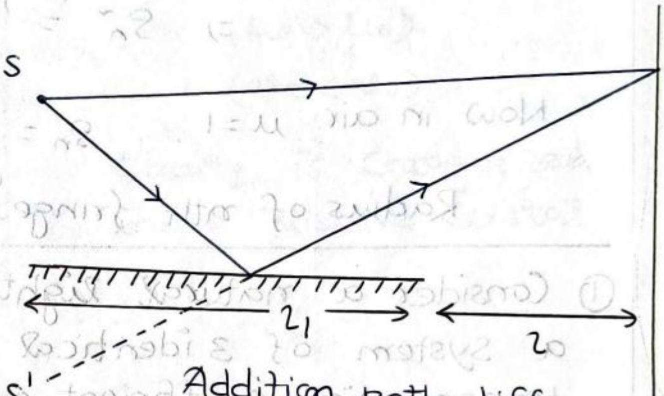
$\Delta x = n\lambda + \frac{\lambda}{2}$

(For Bright frindge)

Central maxima will be dark

$\Delta x = (2n+1)\frac{\lambda}{2} + \frac{\lambda}{2}$

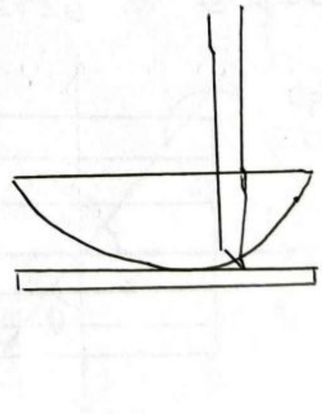
(Dark fringe)



Addition path difference for the reflection of denser to rarer medium

Newton's Ring:

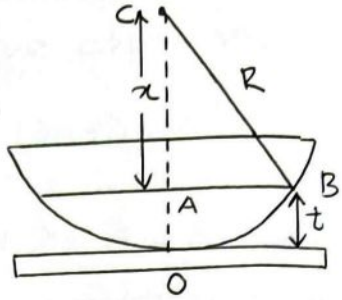
Here interference fringe is observed. But they are circular fringe. Fringe width is not vary. it depends on other factors.



Here

$$2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2} \quad \text{(Dark)}$$

$$2\mu t = n\lambda \quad \text{(Dark fringe)}$$



$$x = R - t$$

$$AB^2 = R^2 - (R-t)^2$$

$$AB^2 = R^2 - R^2 + 2Rt - t^2$$

$$\Rightarrow AB^2 = 2Rt \quad (\text{As } t^2 << 2Rt)$$

Now put $AB = \rho$ then $\rho^2 = 2Rt$

$$t = \rho^2/2R$$

$$\Rightarrow \frac{n\lambda}{2\mu} = \frac{\rho_n^2}{2R}$$

$$\Rightarrow \rho_n^2 = \frac{n\lambda R}{\mu}$$

Now in air. $\mu = 1$: $\rho_n = \sqrt{n\lambda R}$

Radius of nth fringe. $\rho_n = \sqrt{n\lambda R}$

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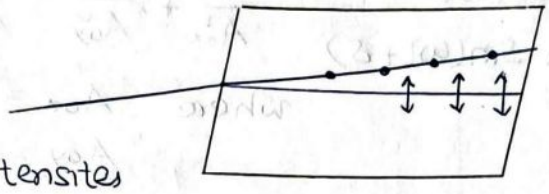
① Consider a natural light of intensity I_0 falling on a system of 3 identical inline polarizers (the max transmission coefficient of each polarizer is T) with the principal direction of the middle polarizer forming an angle ϕ with the other two polarizers. The intensity

② Plane polarized light of wavelength 6000 Å is incident of a thin quartz plate cut with faces parallel to the optic axis. Calculate the ratio of intensities of e-ray and O-ray if the phase of vibrations of incident light makes an angle 30° with the optic axis

⇒



$$\frac{A_{ox}}{A_{oy}} = \frac{A\cos\theta}{A\sin\theta}$$

$$\frac{A_{ox}}{A_{oy}} = \tan 30 = \sqrt{3}$$

Ratio of intensites

$$= \frac{\tilde{A_{ox}}}{\tilde{A_{oy}}} = \frac{3}{1}$$

Negative Crystal	Positive Crystal
$(\mu_e < \mu_o)$	$(\mu_e > \mu_o)$
$v_e > v_o$	$(v_e < v_o)$
Calcite is under the Category of Negative Crystal	Quartz is catagorized under Positive Crystal

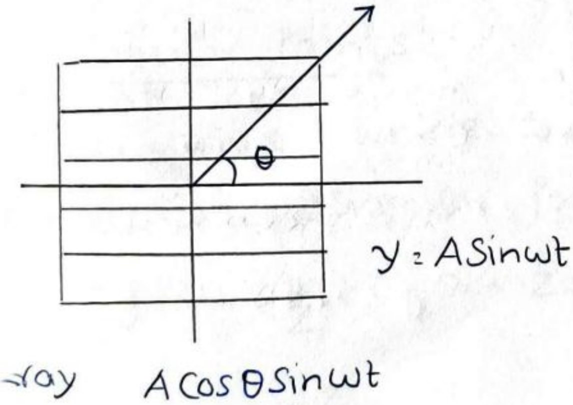
Velocity of e-ray vary with direction and is minimum in the direction of optic axis and maximum in the direction perpendicular to the optic axis.

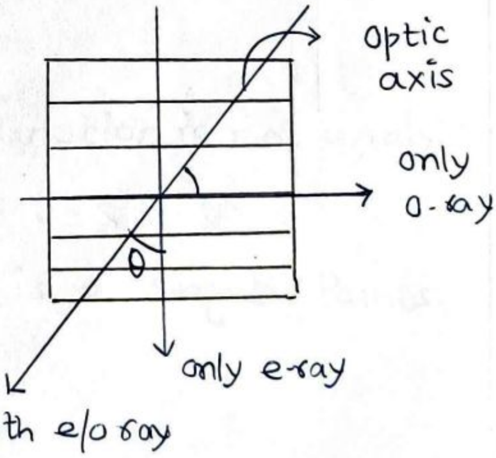
(For - Negative Crystal) ↗

$\hat{o} \cdot \hat{k} = 0$

But for Positive Crystal is reverse

$\hat{e} \cdot \hat{k} = 0$





O-ray $A\cos\theta \sin\omega t$

E-ray $A\sin\theta \sin\omega t$

Distance travells by e and o ray in the Crystal will be different.

$$\delta = \frac{2\pi}{\lambda} t (\mu_e - \mu_o)$$

$$\left.\begin{aligned} A_y &= A_{oy} \sin \omega t \\ A_x &= A_{ox} \sin(\omega t + \delta) \end{aligned}\right\}$$ Due to phase difference they generate.

$y = A_{oy} \sin \omega t$

$x = A_{ox} \sin(\omega t + \delta)$

$$\frac{x^2}{A_{ox}^2} + \frac{y^2}{A_{oy}^2} - \frac{2xy}{A_{ox}A_{oy}} \cos\delta = \sin^2\delta$$

where $A_{ox} = A\cos\theta$

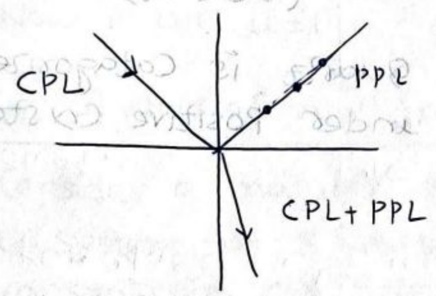
$A_{oy} = A\sin\theta$

For Elliptical $\delta = 90°, 270°$. $\theta \neq 45°$

For Circular $\delta = 90°, 270°$ $\theta = 45°$

PPL: (O-ray) $\delta = 90, 270$ $\theta = 0°, 180$

(e-ray) $\delta = 90, 270$ $\theta = 90°, 270$



Interference · Assignment

① Interference at screen

$$I = I_0 + I_0 + 2\sqrt{I_0 I_0}\cos\delta$$

$$= 2I_0(1+\cos\delta) = 4I_0\cos^2\frac{\delta}{2}$$

$$I_R = I_0\cos^2\delta/2$$

Now $\delta = \frac{2\pi}{\lambda}(\Delta) = \frac{2\pi}{\lambda}\left(\frac{yd}{D}\right) = \frac{2\pi y}{(\lambda D/d)} = \frac{2\pi y}{\beta}$

$$I_R = I_0\cos^2\frac{\delta}{2} = I_0\cos^2\frac{\pi y}{\beta}$$

③ Intensity $I_1 = 81 I_0$, $I_2 = I_0$

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = \frac{25}{16}$$

④ If the slit width are same. the intensity,

$$I_{max} = I_0 + I_0 + 2\sqrt{I_0 I_0} = 4I_0$$

$$I_{min} = I_0 + I_0 - 2\sqrt{I_0 I_0} = 0$$

when slit width be came double intensity doubled

$$I'_{max} = 2I_0 + I_0 + 2\sqrt{2}\,I_0 = 5.83 I_0$$

$$I'_{min} = 2I_0 + I_0 - 2\sqrt{2}\,I_0 = 0.17 I_0$$

So both intensities of maxima and minima increase

⑤ If μ be the refractive index, and μ be the

$$\beta' = \frac{\beta}{\mu} \Rightarrow \frac{\lambda D}{d} : \frac{\lambda D}{2d\mu} \Rightarrow$$

$$\Rightarrow \alpha = \frac{\lambda D}{d} \qquad \text{Now } \alpha' = \frac{\lambda D}{2\mu d}$$

$$\alpha' = \frac{\alpha}{2\mu} = \frac{\alpha}{3}$$

The distance between two Sucessive intensity maxima became $\alpha/3$

(6) Intitial phase difference between them $\Delta\phi_1 = \frac{\pi}{2}$

Now path difference, $\Delta x = 1.5\lambda$

$$\delta = \frac{2\pi}{\lambda}(\Delta x) = 3\pi$$

$$\Delta\phi_2 = 3\pi$$

Phase difference between the waves $S_1 \cdot S_2$ at P is

$$\Delta\phi_{net} = 3\pi - \frac{\pi}{2} = \frac{5\pi}{2}$$

(8) As the intensity remain unchanged

$(\mu-1)t = n\lambda$ For Central maxima $n = 1$

$$t = \frac{\lambda}{(\mu-1)} = \frac{\lambda}{(1.5-1)} = 2\lambda$$

(9) For Coherent source they should have same amplitude and constant phase difference

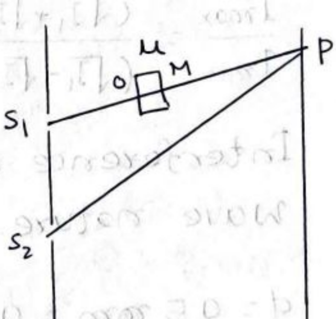
(10) The Contrast in the fringes in any interference pattern depends upon intensity ratio.

(11) Frindge width $\beta = \frac{\lambda}{n}$ as Screen is Same

$$\lambda = n_1\beta_1 = n_2\beta_2$$

$$12 \times 600 = n_2 \times 400 \Rightarrow n_2 = 18$$

(12) Optical Path.



$$(S_1P)' = S_1O + \mu_r t + MP$$
$$= S_1O + t + MP + (\mu_r - 1)t$$
$$= S_1P + (\mu_r - 1)t$$

$$\Delta x = (S_1P)' - S_2P = (S_1P - S_2P) + (\mu_r - 1)t$$

Optical Path difference $= \left(\frac{\mu_2}{\mu_1} - 1\right)t$

(13) $I_R = I + 4I + 2\sqrt{4I^2}\cos\frac{\pi}{2} = 5I$

$I_R' = I + 4I + 2\sqrt{4I^2}\cos\pi = 5I - 4I = I$

Difference $= 4I$

(14) As maximum intensity is I_0 then

$$I = I_0 \cos^2\frac{\delta}{2} \qquad I = \frac{I_0}{4}$$

or $\frac{1}{4} = \cos^2\frac{\delta}{2}$ Now phase difference

or $\frac{1}{2} = \cos\frac{\delta}{2}$ $\phi = \frac{2\pi}{\lambda}(d\sin\theta) = \frac{2\pi}{3}$

or $\frac{\pi}{3} = \frac{\delta}{2}$ $\sin\theta = \frac{\lambda}{3d}$

or $\delta = \frac{2\pi}{3}$ $\theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$

(15) If transperent sheet is introduced. their will be no change in fringe width

(16) Now the 1st dark fringe occurs at $x = d/2$

So. $\frac{xd}{D} = (2n-1)\frac{\lambda}{2} = \frac{\lambda}{2}$

$\frac{d^2}{2D} = \frac{\lambda}{2} \Rightarrow \lambda = d^2/D$

(17) $\lambda = 7\times10^{-7}m$ For the shift

$\mu = 1.5$ $(\mu-1)t = 5\lambda$

$t = \frac{5\times7\times10^{-7}}{0.5} = 7\times10^{-6}m$

(19) $\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1}+\sqrt{I_2})^2}{(\sqrt{I_1}-\sqrt{I_2})^2} = \frac{(2+1)^2}{(2-1)^2} = \frac{9}{1}$

(20) Interference and diffraction of light supports wave nature.

(23) $d = 0.5\,mm = 0.5\times10^{-3}m$ The distance between the

$D = 100\,cm = 1m$ Position of first intensity

$\lambda = 632\times10^{-9}m$ minimum wrt central

maximum. $\frac{\beta}{2} = \frac{\lambda D}{2d} = \frac{632\times10^{-9}\times\frac{1}{0.5\times10^{-3}}}{2} = 632\,\mu m$

(25) $I_1 = I$ $I_{max} = (\sqrt{I_2}+\sqrt{I_1})^2 = (2I+I)^2 = 9I$

$I_2 = 4I$ $I_{min} = (\sqrt{I_2}-\sqrt{I_1}) = (2\sqrt{I}-\sqrt{I})^2 = I$

$\frac{I_{max}}{I_{min}} = \frac{9}{1}$

(27) If the monochromatic source of light in YDSE is replaced by a white light source then there will be a central bright white fringe surrounded by a few coloured fringes

(28) In YDSE nature of fringes patters are Hyperbolic.

(31) $y_1 = a\sin\omega t$

$y_R = y_1 + y_2 + y_3 + y_4 = a\sin\omega t + a\sin(\omega t + 15)$
$+ a\sin(\omega t + 30) + a\sin(\omega t + 45)$

$= a\sin\omega t + a\sin\omega t\cos 15 + a\cos\omega t\sin 15$
$+ a\sin\omega t\cos 30 + a\cos\omega t\sin 30 + a\sin\omega t\cos 45 + a\cos\omega t\sin 45$

$= a\sin\omega t(1 + \cos 15 + \cos 30 + \cos 45)$
$+ a\cos\omega t(\sin 15 + \sin 30 + \sin 45)$

$= (a\sin\omega t) \times 2.539 + a\cos\omega t(1.465)$

$= a\sin\omega t\cos\theta + a\cos\omega t\sin\theta = A\sin(\omega t + \theta)$

(32) $\lambda = 600\,nm$ For 3rd order Bright fringe
$d = 6\,mm$

$\cancel{\frac{\lambda D}{d}}$ $\frac{xd}{D} = 3\lambda$

$\delta = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right) = \frac{2\pi}{\lambda} \times 3\lambda = 6\pi$

Also path difference is $d\sin\theta = n\lambda = 3\lambda$

$6 \times 10^{-3} \times \sin\theta = 3 \times 600 \times 10^{-9}$

$\sin\theta = 3 \times 10^{-4} \approx \theta$ $\theta = 3 \times 10^{-4}$ rad

(33) $d\sin\theta = n\lambda$

or $d \times 0.004 = n \times 589 \times 10^{-9}$

or $d = 1.4725 \times 10^{-4}$

$d\sin\theta' = n\lambda'$

$1.4725 \times 10^{-4} \times 4.4 \times 10^{-3} = n\lambda'$

$\theta' = \theta \times \frac{110}{100}$

$= 0.004 \times \frac{11}{10}$

$= 4.4 \times 10^{-3}$

$\lambda' = 6.479 \times 10^{-7}\,m$

$\lambda' = 647.9\,nm$

(35)

$\lambda = 589\,mm = 589 \times 10^{-9}\,m$

$\theta = 0.2°$

$$d \sin\theta = \lambda$$

$$d \times 3.49 \times 10^{-3} = 5.89 \times 10^{-9}$$

$$d = 1.68 \times 10^{-6}$$

$d \sin\theta = \lambda' = \frac{\lambda}{\mu}$ $\qquad$ $\sin\theta = 0.2337$

$\theta = 13.515$

$1.68 \times 10^{-6} \times \sin\theta = \frac{589 \times 10^{-9}}{1.5}$

(37)

After inserting sheet nth bright frindge

$$x_n = [\cancel{n\lambda} + (\mu - 1)t]\frac{D}{d} = \frac{6\lambda D}{d}$$

$$(\mu - 1)t = 6\lambda$$

$$t = \frac{6\lambda}{(\mu - 1)} = \frac{6 \times 580 \times 10^{-9}}{0.58} = 6 \times 10^{-6} = 6\mu F$$

(38)

Phase differnce, $d \sin\theta = n\lambda$

$$\sin\theta \times 32 \times 10^{-5} = n \times 500 \times 10^{-9}$$

$$\frac{\sin 30 \times 32 \times 10^{-5}}{500 \times 10^{-9}} = n \Rightarrow n = 320$$

Total maxima $-30 < \theta < 30$ is 641

(44)

$I = I_0 \cos^2 \frac{\delta}{2}$ $\qquad$ $\delta = \frac{2\pi}{\lambda}(d \sin\theta) = \frac{2\pi}{\lambda}\lambda = 2\pi$

$I = I_0 \cos^2 \pi = I_0$

(46)

$$(\mu_2 - 1)t_2 + (\mu_1 - 1)t_1 = n\lambda$$

$$(\mu_2 - \mu_1)t = n\lambda$$

$$(\mu_2 - \mu_1) = \frac{n\lambda}{t} = \frac{7 \times 550 \times 10^{-9}}{6 \times 10^{-6}} = 0.64$$

(47)

$\lambda = 600 \times 10^{-9}\,m$

$d = 0.15 \times 10^{-3}\,m$

$D = 1.5\,m$

Frindge width

$$\beta = \frac{\lambda D}{d} = \frac{600 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}}$$

$$\beta = 6 \times 10^{-3} = 6mm$$