Technical Appendix: Network Design for a Clothing Business Supply Chain

Jyotishka DAS, Michele Natacha ELA ESSOLA, Chara Vega BROWN

ESSEC Business School & CentraleSupélec, Paris

1 Linear Programming

1.1 The Single-source problem

1. The **input parameters** are defined as follows. All of these parameters can be considered to be continuous.
   - m_i: marginal cost associated with extension per unit for existing Distribution Centres (DC) i, $\forall i \in (1, 6)$ or for constructing new DC i, $\forall i \in (7, 9)$
   - t_{ij}: delivery time from DC i to aggregated store j
   - c_i: construction capacity of DC i
   - d_j: demand of aggregated store j
2. The **decision variables** are defined as follows:
   - A binary variable x, which would store the value if a particular DC is constructed or extended

$$x_i = \begin{cases} 1 & \text{if decision to extend or construct DC i, } \forall i \in (1, 9) \\ 0 & \text{otherwise} \end{cases}$$

   - An integer variable y, which stores the extension units for the already existing DC i, $\forall i \in (1, 6)$.
   - A binary Variable a_{ij}, which would store whether a particular aggregated store is connected with a particular DC

$$a_i = \begin{cases} 1 & \text{if aggregated store j is connected with DC i, } \forall i \in (1, 9) \text{ and } \forall j \in (1, 21) \\ 0 & \text{otherwise} \end{cases}$$

3. The **constraints** are defined as follows:
   - A build/extension constraint is introduced because Zara can construct or extend no more than 3 sites:

$$\sum_{i=1}^{9} x_i \leq 3$$

- Extension items constraint are introduced because if the capacity of a particular DC is extended, then it should be in the range of 8000 to 13000:

$$y_i \geq 8000x_i$$

$$y_i \leq 13000x_i$$

- Single source supplying constraint to ensure that each store is linked with only one DC:

$$\sum_{i=1}^{9} a_{ij} = 1, \ \forall j \in (1, 21)$$

- Budget constraint to ensure that the monthly delivery costs do not exceed a budget of 4500 euros:

$$\frac{dj}{1000} \sum_{i=1}^{9} a_{ij}t_{ij} \leq 4500, \ \forall j \in (1, 21)$$

- Demand-supply constraints to ensure that the capacities of the DCs are enough to fulfill the demands of the stores:

$$\sum_{j=1}^{21} a_{ij}d_j \leq c_i + y_i, \ \forall i \in (1, 6)$$

$$\sum_{j=1}^{21} a_{ij}d_j \leq c_i x_i, \ \forall i \in (7, 9)$$

4. The **objective function** is defined as follows. The aim is to minimize the total extension/construction costs:

$$min[\sum_{i=1}^{6} m_i y_i + \sum_{i=6}^{9} m_i x_i]$$

1.2 The Multi-source problem

1. The **input parameters** are defined as follows. All of these parameters can be considered to be continuous.
   - m_i: marginal cost associated with extension per unit of existing DC i, $\forall i \in (1, 6)$ or constructing new DC i, $\forall i \in (7, 9)$
   - t_{ij}: delivery time from DC i to aggregated store j
   - c_i: construction capacity of DC i
   - d_j: demand of aggregated store j
2. The **decision variables** are defined as follows:

- A binary Variable x, which would store the value if a particular DC is constructed or extended

$$x_i = \begin{cases} 1 & \text{if decision to extend or construct DC i, } \forall i \in (1,9) \\ 0 & \text{otherwise} \end{cases}$$

- An integer variable y_i, which stores the extension units for the already existing DC i $\forall i \in (1,6)$.
- An integer variable $quant_{ij}$, which denotes the quantity served to aggregated store j from DC i, $\forall i \in (1,9)$ and $\forall j \in (1,21)$.

3. The **constraints** are defined as follows:
   - A build/extension constraint is introduced because Zara can construct or extend no more than 3 sites:

$$\sum_{i=1}^{9} x_i \leq 3$$

   - Extension items constraint are introduced because if the capacity of a particular DC is extended, then it should be in the range of 8000 to 13000:

$$y_i \geq 8000x_i$$
$$y_i \leq 13000x_i$$

   - In a multiple source supplying model, the summation of the supplies from multiple sources (DCs) for an aggregated store should be greater than its demand:

$$\sum_{i=1}^{9} quant_{ij} \geq d_j,\ \forall j \in (1,21)$$

   - Budget constraint to ensure that the monthly delivery costs do not exceed a budget of 4500 euros:

$$\sum_{i=1}^{9} \frac{quant_{ij} t_{ij}}{1000} \leq 4500,\ \forall j \in (1,21)$$

   - Demand-supply constraints to ensure that the capacities of the DCs are enough to fulfill the demands of the stores:

$$\sum_{j=1}^{21} quant_{ij} \leq c_i + y_i,\ \forall i \in (1,6)$$

$$\sum_{j=1}^{21} quant_{ij} \leq c_i x_i,\ \forall i \in (7,9)$$

4. The **objective function** is defined as follows. The aim is to minimize the total extension/construction costs:

$$min[\sum_{i=1}^{6} m_i y_i + \sum_{i=6}^{9} m_i x_i]$$

2 A critical analysis of the proposed models

Some potential limitations of the linear programming models are:

- **Assumption of uniformity**: The models ignore any potential differences in the physical characteristics of various products and assume that all things have the same size and weight. In fact, changes in size and weight could affect the price of transportation, the amount of storage needed, and the timing of deliveries. Future models could incorporate these adjustments to more accurately reflect the logistical difficulties.
- **Absence of delivery costs in the optimization formulation**: The optimization problem does not try to optimize all the costs associated with the delivery. The monthly delivery costs to each of the stores is excluded from the optimization function. Instead, the costs associated with only network growth was kept into focus.
- **Focus on short-term network growth**: Also the models don't take long-term scalability into account and instead concentrate on short-term network growth. It is crucial to evaluate the DC network's scalability in light of potential market growth as it expands and grows its market presence. Long-term strategic planning would benefit from incorporating scalability analysis, including assessing potential future DC locations and capacities.
- **Absence of considerations of evolving market conditions**: The linear programming models used are static, with set parameters and a single time snapshot. Distribution center planning, on the other hand, involves dynamic and evolving aspects such as changing market situations, evolving client demands, and technological improvements. Linear models may not sufficiently represent these dynamic elements, resulting in unsatisfactory long-term judgments.
- **Absence of considerations of qualitative data**: Beyond pure mathematical optimization, the choice to expand or build new distribution centers involves a variety of elements, including strategic considerations, consumer preferences, market trends, and qualitative aspects that are difficult to incorporate into the proposed models.