A Project Report on

# Computation Through Evolution

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by


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# CERTIFICATE

This is to certify that the work presented in this report entitled ***Computation Through Evolution*** submitted by **Nawaj Sharif** and **Kousik Das** having the examination roll number 510814018 and 510814050 respectively, has been carried out under my supervision for the partial fulfilment of the degree of *Bachelor of Technology* in *Information Technology* during the session 2017-18 in the Department of Information Technology, Indian Institute of Engineering Science and Technology, Shibpur.

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# Chapter 1

# Introduction

A cellular automaton (CA) is a discrete model studied in computer science, mathematics, physics [2]. A CA is a collection of cells on a grid of specified shape that evolves through a number of discrete time steps according to a set of rules based on the states of neighboring cells. The rules are then applied iteratively for as many time steps as desired. Von Neumann was one of the first people to consider such a model.

## 1.1 Related Work

In 1985, Stephen Wolfram made a conjecture that rule 110 is capable of universal computation [3]. Obviously, this CA would be able to implement the logic gates. In [4], hazari at. al established that rule 184 can implement any logic circuits.

## 1.2 Work done

In this reported work we have studied the computational capability of one dimension two state three neighbor cellular automata. To check the computational capabilities of the CAs, first evolve the CAs and after some evolution, we find the basic logic gates (such as AND, OR, NOT, XOR, XNOR, NAND, NOR) function.

# Chapter 2

# Preliminaries

## 2.1 Cellular Automata

A cellular automata (CA) consists of an array of cells, each of which stores a binary state at time $t$. The next state of a cell is affected by its present state and the present states of its two nearest neighbors. We consider that CA use the periodic boundary condition, where the CA cells are arranged in a ring. The next state of a cell $i$ is determined as:

$$S_i^{t+1} = f(S_{i-1}^t, S_i^t, S_{i+1}^t) \tag{2.1}$$

where $f$ is the next state function and $S_{i-1}^t$, $S_i^t$ and $S_{i+1}^t$ are the present states of the left neighbor, self, and right neighbor of the $i^{th}$ cell at time $t$. The function $f : \{0,1\}^3 \mapsto \{0,1\}$ can be express as lookup table (see Table 2.1). The decimal equivalent of the next eight states is called a rule. There are $2^8(256)$ rules in two state, three neighborhood dependency. Three such rules, 30, 57 and 184 are shown in Table 2.1.

**Table 2.1:** Look-up table for rule 30, 57 and 184

| Present State | **111** | **110** | **101** | **100** | **011** | **010** | **001** | **000** | Rule |
|---|---|---|---|---|---|---|---|---|---|
| Next State | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 30 |
| Next State | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 57 |
| Next State | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 184 |

## 2.2 The Minimal Representative Rules

It is observed from the evolutions of CAs that some CAs are equivalent with respect to their dynamics. For instance, rule 170 and 240 are equivalent to each other according

**Table 2.2:** 88 equivalent rules

| Rule | Equi. rules | Rule | Equi. rules | Rule | Equi. rules | Rule | Equi. rules |
|---|---|---|---|---|---|---|---|
| 0 | 255 | 26 | 82, 167, 181 | 56 | 98, 185, 227 | 132 | 222 |
| 1 | 127 | 27 | 39, 53, 83 | 57 | 99 | 134 | 148, 158, 214 |
| 2 | 16, 191, 247 | 28 | 70, 157, 199 | 58 | 114, 163, 177 | 136 | 192, 238, 252 |
| 3 | 17, 63, 119 | 29 | 71 | 60 | 102, 153, 195 | 138 | 174, 208, 224 |
| 4 | 223 | 30 | 86, 135, 149 | 62 | 118, 131, 145 | 140 | 196, 206, 220 |
| 5 | 95 | 32 | 251 | 72 | 237 | 142 | 212 |
| 6 | 20, 159, 215 | 33 | 123 | 73 | 109 | 146 | 182 |
| 7 | 21, 31, 87 | 34 | 48, 187, 243 | 74 | 88, 173, 229 | 150 | - |
| 8 | 64, 239, 253 | 35 | 49, 59, 115 | 76 | 205 | 152 | 188, 194, 230 |
| 9 | 65, 111, 125 | 36 | 219 | 77 | - | 154 | 166, 180, 210 |
| 10 | 80, 175, 245 | 37 | 91 | 78 | 92, 141, 197 | 156 | 198 |
| 11 | 47, 81, 117 | 38 | 52, 155, 211 | 90 | 165 | 160 | 250 |
| 12 | 68, 207, 221 | 40 | 96, 235, 249 | 94 | 133 | 162 | 176, 186, 242 |
| 13 | 69, 79, 93 | 41 | 97, 107, 121 | 104 | 233 | 164 | 218 |
| 14 | 84, 143, 213 | 42 | 112, 171, 241 | 105 | - | 168 | 224, 234, 248 |
| 15 | 85 | 43 | 113 | 106 | 120, 169, 225 | 170 | 240 |
| 18 | 183 | 44 | 100, 203, 217 | 108 | 201 | 172 | 202, 216, 228 |
| 19 | 55 | 45 | 75, 89, 101 | 110 | 124, 137, 193 | 178 | - |
| 22 | 151 | 46 | 116, 139, 209 | 122 | 161 | 184 | 226 |
| 23 | - | 50 | 179 | 126 | 129 | 200 | 236 |
| 24 | 66, 189, 231 | 51 | - | 128 | 254 | 204 | - |
| 25 | 61, 67, 103 | 54 | 147 | 130 | 144, 190, 246 | 232 | - |

to their dynamics. Thus, rule 170 is considered as minimal representative rule of these two rules. All the 256 CA rules are reduced to 88 independent minimal representative rules. All these 88 minimal representative rules are listed in Table 2.2.

# Chapter 3

# Cellular Automata as a Logic Gate Generator

## 3.1 Proposed Approach

In this work, we first evolve the CAs and build the truth tables for each evolution. After that we get the output function from those truth tables. Using the output function, we can realized any logic gate.

### 3.1.1 CA as $n$-input $n$-output block

We consider $n$ cells CA as a $n$-input CA block. The initial configuration of CAs is the input to the block. A CA block maps $n$ inputs $I_1, I_2, \cdots, I_n$ to $n$ outputs $O_1, O_2, \cdots, O_n$. $n$-input $n$-output CA block is shown in Figure 3.1.

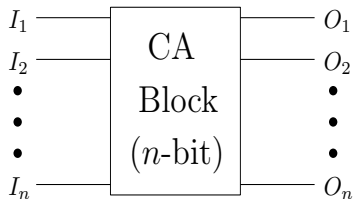


**Figure 3.1:** $n$-input $n$-output CA block

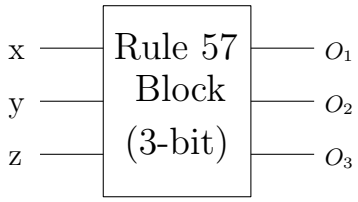


**Figure 3.2:** 3-input 3-output rule 57 block

Let us consider a 3-input 3-output rule 57 block, shown in Figure 3.2. Now we can express the output variables of 3-input 3-output rule 57 block as explicit functions of input variables. After $1^{st}$ evolution we get the truth table, which is shown in Table 3.1 and output can be written as following:

$$O_1 = xyz^{'} + x^{'}z + x^{'}y^{'}$$
$$O_2 = x^{'}yz + xy^{'} + y^{'}z^{'}$$
$$O_3 = xy^{'}z + yz^{'} + x^{'}z^{'}$$

**Table 3.1:** Truth table of rule 57 for 3-cell (After $1^{st}$ evolution)

| $\mathbf{I_1}$ | $\mathbf{I_2}$ | $\mathbf{I_3}$ | $\mathbf{O_1}$ | $\mathbf{O_2}$ | $\mathbf{O_3}$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 |
| Initial Input | | | Output (After $1^{st}$ evolution) | | |

Similarly, after $2^{nd}$ evolution again we get the truth table, which is shown in Table 3.2 and output can be written as following:

$$O_1 = xz + x^{'}y$$
$$O_2 = xy + y^{'}z$$
$$O_3 = yz + xz^{'}$$

**Table 3.2:** Truth table of rule 57 for 3-cell (After $2^{nd}$ evolution)

| $\mathbf{I_1}$ | $\mathbf{I_2}$ | $\mathbf{I_3}$ | $\mathbf{O_1}$ | $\mathbf{O_2}$ | $\mathbf{O_3}$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |
| Initial Input | | | Output (After $2^{nd}$ evolution) | | |

In this way we can find out the outputs of any $n$-input $n$-output CA block.

### 3.1.2 Implementation of logic gates using rule 57 block

To implement the logic gates using rule 57 block, we set some input in this way that after some evolution get the desire output.

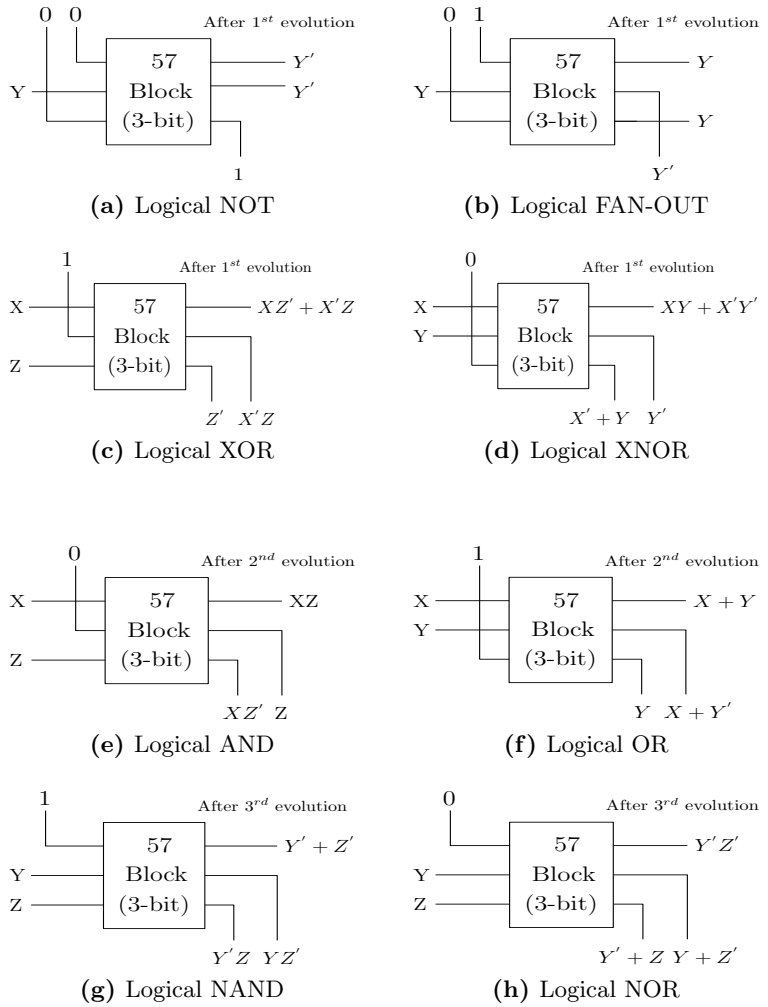


**Figure 3.3:** Implementation of logic gates using 3-input 3-output 57 block

For realizing AND, OR, NOT, XOR, XNOR, NAND and NOR, we consider 3-input 3-output 57 block, which is shown in Figure 3.2. Let us set the input X and Z with logic 0. Then after $1^{st}$ evolution, output $O_1$ results in $Y^{'}$. That is , under this setting $O_1$ is the output of NOT (NOT Y). This realization is shown in Figure 3.3a.

Similarly, if we set input Y with logic 0 then after $2^{nd}$ evolution output $O_1$ becomes $XZ$. That is, under this setting $O_1$ is the output of two input AND (X AND Z). Figure 3.3e shows this realization.

In this way, by proper selection of input and after some evolution we can realized other logic gates like OR, XOR, XNOR, NAND and NOR. Implementation of all the logic gates using 3-input 3-output 57 block are shown in Figure 3.3. In the next section, we try to realize the logic gates for all the 2-state 3-neighborhood dependency CAs.

## 3.2 Realization of basic gates

By proper selection of input, it is possible to implement AND, OR, NOT, FAN-OUT, NAND, NOR, XOR and XNOR by a $n$-input $n$-output CA block. In this selection we realized the basic gates function for all the 88 minimal equivalent CA rules.

### 3.2.1 Rule 0

In case of rule 0, we can not find any output function in term of input variable. So, we can not realized any logic gate using rule 0.

### 3.2.2 Rule 1

In 3-input 3-output rule 1 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z^{'}$$
$$O_2 = x^{'}y^{'}z^{'}$$
$$O_3 = x^{'}y^{'}z^{'}$$

- Realization of NOT gate: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x^{'}$
- Realization of NOR gate: In $O_1$, set $z = 0$ then $O_1 = x^{'}y^{'}$

In 3-input 3-output rule 1 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x + y + z$$
$$O_2 = x + y + z$$
$$O_3 = x + y + z$$

- Realization of FAN-OUT gate: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of OR gate: In $O_1$, set $z = 0$ then $O_1 = x + y$

In 4-input 4-output rule 1 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = yz + xz + xy + w$$
$$O_2 = yz + wz + wy + x$$

$$O_3 = xz + wz + wx + y$$
$$O_4 = xy + wy + wx + z$$

- Realization of AND gate: In $O_1$, set $w = 0$, $z = 0$ then $O_1 = xy$

### 3.2.3 Rule 2

In 3-input 3-output rule 2 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz^{'}$$
$$O_2 = x^{'}y^{'}z$$
$$O_3 = xy^{'}z^{'}$$

- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x^{'}$
- Realization of FAN-OUT: In $O_1$, set $x = 0$, $z = 0$ then $O_1 = y$
- Realization of NOR gate: In $O_1$, set $y = 1$ then $O_1 = x^{'}z^{'}$

### 3.2.4 Rule 3

In 3-input 3-output rule 3 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}z^{'}$$
$$O_2 = x^{'}y^{'}$$
$$O_3 = y^{'}z^{'}$$

- Realization of NOT gate: In $O_1$, set $x = 0$ then $O_1 = z^{'}$
- Realization of NOR gate: In $O_1$, set $x = 0$ then $O_1 = x^{'}z^{'}$

In 3-input 3-output rule 3 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xy + z$$
$$O_2 = yz + x$$
$$O_3 = xz + y$$

- Realization of FAN-OUT gate: In $O_1$, set $y = 0$ then $O_1 = z$
- Realization of OR gate: In $O_1$, set $y = 1$ then $O_1 = x + z$
- Realization of AND gate: In $O_1$, set $y = 0$ then $O_1 = xy$

### 3.2.5 Rule 4

In 3-input 3-output rule 4 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'}z^{'}$$
$$O_2 = x^{'}yz^{'}$$
$$O_3 = x^{'}y^{'}z$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 1$, $z = 0$ then $O_1 = y^{'}$
- Realization of NOR gate: In $O_1$, set $x = 1$ then $O_1 = y^{'}z^{'}$

### 3.2.6 Rule 5

In 3-input 3-output rule 5 block: After $1^{st}$ evolution get the following functions:

$$O_1 = y^{'}z^{'}$$
$$O_2 = x^{'}z^{'}$$
$$O_3 = x^{'}y^{'}$$

- Realization of NOT gate: In $O_1$, set $z = 0$ then $O_1 = y^{'}$
- Realization of NOR gate: $O_1 = y^{'}z^{'}$

In 3-input 3-output rule 5 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = yz + x$$
$$O_2 = xz + y$$
$$O_3 = xy + z$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$ then $O_1 = x$
- Realization of AND gate: In $O_1$, set $x = 0$ then $O_1 = yz$
- Realization of OR gate: In $O_1$, set $z = 1$ then $O_1 = y + x$

### 3.2.7 Rule 6

In 3-input 3-output rule 6 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz^{'} + xy^{'}z^{'}$$
$$O_2 = x^{'}y^{'}z + x^{'}yz^{'}$$
$$O_3 = x^{'}y^{'}z + xy^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 0$, $z = 0$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x^{'}$
- Realization of XOR gate: In $O_1$, set $z = 0$ then $O_1 = x^{'}y + xy^{'}$
- Realization of NOR gate: In $O_1$, set $y = 1$ then $O_1 = x^{'}z^{'}$

In 4-input 4-output rule 6 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}x^{'}y + w^{'}yz^{'} + x^{'}yz^{'}$$
$$O_2 = w^{'}x^{'}z + w^{'}y^{'}z + x^{'}y^{'}z$$
$$O_3 = wx^{'}y^{'} + wx^{'}z^{'} + wy^{'}z^{'}$$
$$O_4 = w^{'}xy^{'} + w^{'}xz^{'} + xy^{'}z^{'}$$

- Realization of NAND gate: In $O_1$, set $w = 0$, $y = 1$ then $O_1 = x^{'} + z^{'}$

In 5-input 5-output rule 6 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = v^{'}wx^{'}yz + v^{'}w^{'}xy + vw^{'}yz^{'} + v^{'}xz^{'}$$
$$O_2 = vw^{'}xy^{'}z + v^{'}wx^{'}z + w^{'}x^{'}yz + v^{'}w^{'}y$$
$$O_3 = vwx^{'}yz^{'} + vw^{'}xy^{'} + vx^{'}y^{'}z + w^{'}x^{'}z$$
$$O_4 = v^{'}wxy^{'}z + wx^{'}yz^{'} + vwy^{'}z^{'} + vx^{'}y^{'}$$
$$O_5 = vw^{'}xyz^{'} + v^{'}xy^{'}z + v^{'}wxz^{'} + wy^{'}z^{'}$$

- Realization of AND gate: In $O_1$, set $v = 0$, $w = 0$, $z = 1$ then $O_1 = xy$

### 3.2.8 Rule 7

In 3-input 3-output rule 7 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}z^{'} + y^{'}z^{'}$$
$$O_2 = x^{'}y^{'} + x^{'}z^{'}$$
$$O_3 = x^{'}y^{'} + y^{'}z^{'}$$

- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x^{'}$
- Realization of NOR gate: In $O_1$, set $y = 1$ then $O_1 = x^{'}z^{'}$
- Realization of NAND gate: In $O_1$, set $z = 1$ then $O_1 = x^{'} + y^{'}$

In 3-input 3-output rule 7 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = yz + xz + xy$$
$$O_2 = yz + xz + xy$$
$$O_3 = yz + xz + xy$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x$
- Realization of AND gate: In $O_1$, set $z = 0$ then $O_1 = xy$
- Realization of OR gate: In $O_1$, set $z = 1$ then $O_1 = y + x$

### 3.2.9 Rule 8

In 3-input 3-output rule 8 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz^{'}$$
$$O_2 = x^{'}yz$$
$$O_3 = xy^{'}z$$

- Realization of FANOUT: In $O_1$, Set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $x = 1$, $y = 1$ then $O_1 = z^{'}$
- Realization of AND: In $O_1$, Set $z = 0$ then $O_1 = xy$

### 3.2.10 Rule 9

In 3-input 3-output rule 9 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz^{'} + x^{'}y^{'}z^{'}$$
$$O_2 = x^{'}yz + x^{'}y^{'}z^{'}$$
$$O_3 = xy^{'}z + x^{'}y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x^{'}$
- Realization of XNOR gate: In $O_1$, set $z = 0$ then $O_1 = xy + x^{'}y^{'}$

In 3-input 3-output rule 9 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xyz + x^{'}y^{'}z + x^{'}yz^{'} + xy^{'}z^{'}$$
$$O_2 = xyz + x^{'}y^{'}z + x^{'}yz^{'} + xy^{'}z^{'}$$
$$O_3 = xyz + x^{'}y^{'}z + x^{'}yz^{'} + xy^{'}z^{'}$$

- Realization of XOR gate: In $O_1$, set $z = 0$ then $O_1 = y^{'}z + yz^{'}$

In 4-input 4-output rule 9 block: After $1^{st}$ evolution get the following functions:

$$O_1 = wxz^{'} + w^{'}x^{'}z^{'}$$
$$O_2 = w^{'}xy + w^{'}x^{'}y^{'}$$

$$O_3 = x^{'}yz + x^{'}y^{'}z^{'}$$
$$O_4 = wy^{'}z + w^{'}y^{'}z^{'}$$

- Realization of NOR gate: In $O_1$, set $x = 1$, $y = 0$ then $O_1 = x^{'}z^{'}$

In 4-input 4-output rule 9 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}xy^{'}z + wyz + x^{'}yz + wx^{'}z^{'}$$
$$O_2 = wx^{'}yz^{'} + wxz + wy^{'}z + w^{'}xy^{'}$$
$$O_3 = w^{'}xy^{'}z + wxy + wxz^{'} + x^{'}yz^{'}$$
$$O_4 = wx^{'}yz^{'} + wxz + wy^{'}z + w^{'}xy^{'}$$

- Realization of AND gate: In $O_1$, set $w = 0$, $y = 0$ then $O_1 = xz$

### 3.2.11 Rule 10

In 3-input 3-output rule 10 block: After $1^{st}$ evolution get the following functions:

$$O_1 = yz^{'}$$
$$O_2 = x^{'}z$$
$$O_3 = xy^{'}$$

- Realization of FAN-OUT: In $O_1$, set $z = 0$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 1$ then $O_1 = z^{'}$

### 3.2.12 Rule 11

In 3-input 3-output rule 11 block: After $1^{st}$ evolution get the following functions:

$$O_1 = yz^{'} + x^{'}z^{'}$$
$$O_2 = x^{'}z + x^{'}y^{'}$$
$$O_3 = xy^{'} + y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 1$, $z = 1$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x^{'}$
- Realization of NOR gate: In $O_1$, set $y = 0$ then $O_1 = x^{'}z^{'}$

In 3-input 3-output rule 11 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = yz + x^{'}z$$
$$O_2 = xz + xy^{'}$$
$$O_3 = xy + yz^{'}$$

- Realization of AND gate: In $O_1$, set $x = 1$ then $O_1 = yz$

In 5-input 5-output rule 11 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wxz + vwx + vw^{'}z + v^{'}yz^{'} + w^{'}yz^{'}$$
$$O_2 = wxy + vxy + v^{'}w^{'}z + vwx^{'} + v^{'}x^{'}z$$
$$O_3 = xyz + wyz + vw^{'}x^{'} + wxy^{'} + vw^{'}x^{'}$$
$$O_4 = vwy + vxz + wx^{'}y^{'} + xyz^{'} + wx^{'}z^{'}$$
$$O_5 = vwz + vwy + v^{'}yz + v^{'}xz^{'} + xy^{'}z^{'}$$

- Realization of OR gate: In $O_1$, set $w = 1$, $x = 1$, $y = 0$ then $O_1 = z + v$

### 3.2.13 Rule 12

In 3-input 3-output rule 12 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xz^{'}$$
$$O_2 = x^{'}y$$
$$O_3 = y^{'}z$$

- Realization of FAN-OUT: In $O_1$, set $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 1$ then $O_1 = z^{'}$

### 3.2.14 Rule 13

In 3-input 3-output rule 13 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xz^{'} + y^{'}z^{'}$$
$$O_2 = x^{'}y + x^{'}z^{'}$$
$$O_3 = y^{'}z + x^{'}y^{'}$$

- Realization of FAN-OUT: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 1$, $y = 1$ then $O_1 = z^{'}$
- Realization of NOR gate: In $O_1$, set $x = 0$ then $O_1 = y^{'}z^{'}$

In 3-input 3-output rule 13 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xy + xz^{'}$$
$$O_2 = yz + x^{'}y$$
$$O_3 = xz + y^{'}z$$

- Realization of AND gate: In $O_1$, set $z = 1$ then $O_1 = xy$

In 4-input 4-output rule 13 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wy + x^{'}y + wz^{'}$$
$$O_2 = xz + w^{'}x + y^{'}z$$
$$O_3 = wy + x^{'}y + wz^{'}$$
$$O_4 = xz + w^{'}x + y^{'}z$$

- Realization of OR gate: In $O_1$, set $x = 0$, $z = 0$ then $O_1 = y + w$

In 4-input 4-output rule 13 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = x^{'}y + wz^{'} + x^{'}z^{'}$$
$$O_2 = w^{'}x + y^{'}z + w^{'}y^{'}$$
$$O_3 = x^{'}y + wz^{'} + x^{'}z^{'}$$
$$O_4 = w^{'}x + y^{'}z + w^{'}y^{'}$$

- Realization of NAND gate: In $O_1$, set $w = 1$, $y = 1$ then $O_1 = x^{'} + z^{'}$

### 3.2.15 Rule 14

In 3-input 3-output rule 14 block: After $1^{st}$ evolution get the following functions:

$$O_1 = yz^{'} + xz^{'}$$
$$O_2 = x^{'}z + x^{'}y$$
$$O_3 = y^{'}z + xy^{'}$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of OR gate: In $O_1$, set $z = 0$ then $O_1 = x + y$

In 3-input 3-output rule 14 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}y + yz^{'}$$
$$O_2 = x^{'}z + y^{'}z$$
$$O_3 = xy^{'} + xz^{'}$$

- Realization of NAND gate: In $O_1$, set $y = 1$ then $O_1 = x^{'} + z^{'}$

In 4-input 4-output rule 14 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}xz^{'} + w^{'}y + yz^{'}$$
$$O_2 = w^{'}x^{'}y + w^{'}z + x^{'}z$$
$$O_3 = x^{'}y^{'}z + wx^{'} + wy^{'}$$
$$O_4 = wy^{'}z^{'} + xy^{'} + xz^{'}$$

- Realization of NOR gate: In $O_1$, set $x = 1$, $y = 0$ then $O_1 = w^{'}z^{'}$

In 4-input 4-output rule 14 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = w^{'}yz + x^{'}z + x^{'}y$$
$$O_2 = wx^{'}z + y^{'}z + wy^{'}$$
$$O_3 = wxy^{'} + xz^{'} + wz^{'}$$
$$O_4 = xyz^{'} + w^{'}y + w^{'}x$$

- Realization of AND gate: In $O_1$, set $w = 0$, $x = 1$ then $O_1 = yz$

### 3.2.16 Rule 15

In 3-input 3-output rule 15 block: After $1^{st}$ evolution get the following functions:

$$O_1 = z^{'}$$
$$O_2 = x^{'}$$
$$O_3 = y^{'}$$

- Realization of NOT gate: $O_1 = z^{'}$

In 3-input 3-output rule 15 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = y$$
$$O_2 = z$$
$$O_3 = x$$

- Realization of FAN-OUT: $O_1 = y$

### 3.2.17 Rule 18

In 3-input 3-output rule 18 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z + x^{'}yz^{'}$$
$$O_2 = x^{'}y^{'}z + xy^{'}z^{'}$$
$$O_3 = x^{'}yz^{'} + xy^{'}z^{'}$$

- Realization of FAN-OUT gate: In $O_1$, set $x = 0$, $y = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $z = 1$, $y = 0$ then $O_1 = y^{'}$
- Realization of NOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y^{'}$
- Realization of XOR gate: In $O_1$, set $x = 0$ then $O_1 = y^{'}z + yz^{'}$

### 3.2.18 Rule 19

In 3-input 3-output rule 19 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'} + x^{'}z^{'}$$
$$O_2 = x^{'}y^{'} + y^{'}z^{'}$$
$$O_3 = x^{'}z^{'} + y^{'}z^{'}$$

- Realization of NOT gate: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x^{'}$
- Realization of NOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y^{'}$
- Realization of NAND gate: In $O_1$, set $x = 0$ then $O_1 = y^{'} + z^{'}$

In 3-input 3-output rule 19 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = yz + xz + xy$$
$$O_2 = yz + xz + xy$$
$$O_3 = yz + xz + xy$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x$
- Realization of AND gate: In $O_1$, set $z = 0$ then $O_1 = xy$
- Realization of OR gate: In $O_1$, set $z = 1$ then $O_1 = y + x$

### 3.2.19 Rule 22

In 3-input 3-output rule 22 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z + x^{'}yz^{'} + xy^{'}z^{'}$$
$$O_2 = x^{'}y^{'}z + x^{'}yz^{'} + xy^{'}z^{'}$$
$$O_3 = x^{'}y^{'}z + x^{'}yz^{'} + xy^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 0$, $x = 0$ then $O_1 = z$
- Realization of NOT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x^{'}$

- Realization of NOR: In $O_1$, Set $z = 1$ then $O_1 = x^{'}y^{'}$
- Realization of XOR: In $O_1$, Set $x = 0$ then $O_1 = y^{'}z + yz^{'}$

In 5-input 5-output rule 22 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = vw^{'}xyz^{'} + v^{'}x^{'}yz + vw^{'}x^{'}z + v^{'}w^{'}x^{'}y + v^{'}wxy^{'} + vwy^{'}z^{'} + v^{'}xy^{'}z^{'}$$
$$O_2 = v^{'}wx^{'}yz + vw^{'}y^{'}z + vwx^{'}y^{'} + w^{'}x^{'}y^{'}z + v^{'}wxz^{'} + w^{'}xyz^{'} + v^{'}w^{'}yz^{'}$$
$$O_3 = vw^{'}xy^{'}z + v^{'}w^{'}xy + v^{'}x^{'}yz + v^{'}w^{'}x^{'}z + vwx^{'}z^{'} + wxy^{'}z^{'} + vx^{'}y^{'}z^{'}$$
$$O_4 = vwx^{'}yz^{'} + w^{'}x^{'}yz + v^{'}wxy^{'} + vw^{'}x^{'}y^{'} + vw^{'}y^{'}z + v^{'}xyz^{'} + v^{'}wy^{'}z^{'}$$
$$O_5 = v^{'}wxy^{'}z + v^{'}w^{'}yz + vx^{'}y^{'}z + v^{'}w^{'}xz^{'} + w^{'}xyz^{'} + vwx^{'}z^{'} + wx^{'}y^{'}z^{'}$$

- Realization of AND: In $O_1$, Set $v = 1$, $w = 0$, $z = 0$ then $O_1 = xy$
- Realization of NAND: In $O_1$, Set $x = 0$, $y = 1$, $z = 1$ then $O_1 = v^{'} + w^{'}$

### 3.2.20 Rule 23

In 3-input 3-output rule 23 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'} + x^{'}z^{'} + y^{'}z^{'}$$
$$O_2 = x^{'}y^{'} + x^{'}z^{'} + y^{'}z^{'}$$
$$O_3 = x^{'}z^{'} + x^{'}z^{'} + y^{'}z^{'}$$

- Realization of NOT gate: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of NOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y^{'}$
- Realization of NAND gate: In $O_1$, set $x = 0$ then $O_1 = y^{'} + z^{'}$

In 3-input 3-output rule 23 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = yz + xz + xy$$
$$O_2 = yz + xz + xy$$
$$O_3 = yz + xz + xy$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x$
- Realization of AND gate: In $O_1$, set $z = 0$ then $O_1 = xy$
- Realization of OR gate: In $O_1$, set $z = 1$ then $O_1 = y + x$

### 3.2.21 Rule 24

In 3-input 3-output rule 24 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z + xyz^{'}$$
$$O_2 = x^{'}yz + xy^{'}z^{'}$$
$$O_3 = xy^{'}z + x^{'}yz^{'}$$

- Realization of AND gate: In $O_1$, set $z = 0$ then $O_1 = xy$
- Realization of FAN-OUT: In $O_1$, set $x = 0$, $y = 0$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of NOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y^{'}$

In 6-input 6-output rule 24 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = uwy^{'}z + uvy^{'}z + u^{'}v^{'}yz^{'} + u^{'}w^{'}yz^{'}$$
$$O_2 = u^{'}v^{'}w^{'}z + u^{'}v^{'}x^{'}z + uvwz^{'} + uvxz^{'}$$
$$O_3 = u^{'}vwy + u^{'}vwx + uv^{'}w^{'}y^{'} + uv^{'}w^{'}x^{'}$$
$$O_4 = v^{'}wxz + v^{'}wxy + vw^{'}x^{'}y^{'} + vw^{'}x^{'}z^{'}$$
$$O_5 = w^{'}xyz + uw^{'}xy + u^{'}wx^{'}y^{'} + wx^{'}y^{'}z^{'}$$
$$O_6 = vx^{'}yz + ux^{'}yz + u^{'}xy^{'}z^{'} + v^{'}xy^{'}z^{'}$$

- Realization of OR gate: In $O_1$, set $u = 1$, $y = 0$, $z = 1$ then $O_1 = w + v$
- Realization of NAND gate: In $O_1$, set $u = 0$, $v = 0$, $z = 1$ then $O_1 = w^{'} + x^{'}$

### 3.2.22 Rule 25

In 3-input 3-output rule 25 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz^{'} + x^{'}y^{'}$$
$$O_2 = x^{'}yz + y^{'}z^{'}$$
$$O_3 = xy^{'}z + y^{'}z^{'}$$

- Realization of NOT gate: In $O_1$, set $y = 0$ then $O_1 = x^{'}$
- Realization of FAN-OUT gate: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x$

In 3-input 3-output rule 25 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}yz^{'} + xz$$
$$O_2 = x^{'}y^{'}z + xy$$
$$O_3 = xy^{'}z^{'} + yz$$

- Realization of AND gate: In $O_1$, set $y = 0$ then $O_1 = xz$

### 3.2.23 Rule 26

In 3-input 3-output rule 26 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z + yz^{'}$$
$$O_2 = xy^{'}z^{'} + x^{'}z$$
$$O_1 = x^{'}yz^{'} + xy^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $z = 0$ then $O_1 = y$
- Realization of NOT: In $O_1$, Set $y = 1$ then $O_1 = z^{'}$
- Realization of XOR: In $O_1$, Set $x = 0$ then $O_1 = y^{'}z + yz^{'}$
- Realization of NOR: In $O_1$, Set $z = 1$ then $O_1 = x^{'}y^{'}$

In 3-input 3-output rule 26 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}z + y^{'}z$$
$$O_2 = xy^{'} + xz^{'}$$
$$O_1 = x^{'}y + yz^{'}$$

- Realization of NAND: In $O_1$, Set $z = 1$ then $O_1 = x^{'} + y^{'}$

In 4-input 4-output rule 26 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wxy^{'}z + w^{'}yz$$
$$O_2 = wxyz^{'} + wx^{'}z$$
$$O_3 = w^{'}xyz + wxy^{'}$$
$$O_4 = wx^{'}yz + xyz^{'}$$

- Realization of AND: In $O_1$, Set $w = 0$ then $O_1 = yz$

In 5-input 5-output rule 26 block: After $8^{th}$ evolution get the following functions:

$$O_1 = vw^{'}xy + vwx^{'}z + vwxz^{'} + v^{'}x^{'}y + yz^{'}$$
$$O_2 = v^{'}wxy + wx^{'}yz + vwxy^{'} + w^{'}y^{'}z + vz^{'}$$
$$O_3 = w^{'}xyz + vxy^{'}z + wxyz^{'} + vx^{'}z^{'} + vw^{'}$$
$$O_4 = v^{'}xyz + vx^{'}yz + vwyz^{'} + v^{'}wy^{'} + wx^{'}$$
$$O_5 = v^{'}wxz + vw^{'}yz + vwy^{'}z + w^{'}xz^{'} + xy^{'}$$

- Realization of OR: In $O_1$, Set $w = 1$, $x = 1$, $z = 0$ then $O_1 = v + y$

In 5-input 5-output rule 26 block: After $10^{th}$ evolution get the following functions:

$$O_1 = v^{'}wx^{'}z^{'} + vy^{'}z + vxy^{'} + vw^{'}$$
$$O_2 = v^{'}w^{'}xy^{'} + wyz^{'} + vwz^{'} + wx^{'}$$
$$O_3 = w^{'}x^{'}yz^{'} + v^{'}xz + v^{'}wx + xy^{'}$$
$$O_4 = v^{'}x^{'}y^{'}z + w^{'}xy + vw^{'}y + yz^{'}$$
$$O_5 = vw^{'}y^{'}z^{'} + x^{'}yz + wx^{'}z + v^{'}z$$

- Realization of XNOR: In $O_1$, Set $y = 0$, $z = 0$, $w = 1$ then $O_1 = vx + v^{'}x^{'}$

### 3.2.24 Rule 27

In 3-input 3-output rule 27 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'} + yz^{'}$$
$$O_2 = x^{'}z + y^{'}z^{'}$$
$$O_3 = xy^{'} + y^{'}z^{'}$$

- Realization of FAN-OUT gate: In $O_1$, set $x = 1$, $z = 0$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 1$, $x = 1$ then $O_1 = z^{'}$
- Realization of NOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y^{'}$

In 4-input 4-output rule 27 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wx^{'}y + xz + w^{'}z$$
$$O_2 = xy^{'}z + wy + wx^{'}$$
$$O_3 = wyz^{'} + xz + xy^{'}$$
$$O_4 = w^{'}xz + wy + yz^{'}$$

- Realization of AND gate: In $O_1$, set $y = 0$, $w = 1$ then $O_1 = xz$

### 3.2.25 Rule 28

In 3-input 3-output rule 28 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z + xz^{'}$$
$$O_2 = xy^{'}z^{'} + x^{'}y$$
$$O_3 = x^{'}yz^{'} + y^{'}z$$

- Realization of FAN-OUT gate: In $O_1$, set $x = 0$, $y = 0$ then $O_1 = z$
- Realization of NOT gate: In $O_1$, set $x = 1$ then $O_1 = z^{'}$

- Realization of NOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y^{'}$

In 4-input 4-output rule 28 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}x^{'}y + wxy^{'} + wz^{'}$$
$$O_2 = x^{'}y^{'}z + xyz^{'} + w^{'}x$$
$$O_3 = w^{'}yz + wy^{'}z^{'} + x^{'}y$$
$$O_4 = wx^{'}z + w^{'}xz^{'} + y^{'}z$$

- Realization of AND gate: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = wx$

### 3.2.26 Rule 29

In 3-input 3-output rule 29 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'} + xz^{'}$$
$$O_2 = x^{'}y + y^{'}z^{'}$$
$$O_3 = y^{'}z + x^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 0$ then $O_1 = y^{'}$
- Realization of NOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y^{'}$

In 4-input 4-output rule 29 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wx + x^{'}yz + wz^{'}$$
$$O_2 = wy^{'}z + xy + w^{'}x$$
$$O_3 = wxz^{'} + yz + x^{'}y$$
$$O_4 = w^{'}xy + wz + y^{'}z$$

- Realization of AND gate: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = wx$

In 5-input 5-output rule 29 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = v^{'}w^{'}yz + vx + vw + vz^{'}$$
$$O_2 = vw^{'}x^{'}z + wy + wx + v^{'}w$$
$$O_3 = vwx^{'}y^{'} + xz + xy + w^{'}x$$
$$O_4 = wxy^{'}z^{'} + yz + vy + x^{'}y$$
$$O_5 = v^{'}xyz^{'} + wz + vz + y^{'}z$$

- Realization of OR gate: In $O_1$, set $v = 1$, $z = 1$ then $O_1 = x + w$
- Realization of XNOR gate: In $O_1$, set $x = 0$, $y = 1$, $z = 1$ then $O_1 = vw + v^{'}w^{'}$

### 3.2.27 Rule 30

In 3-input 3-output rule 30 block: After $1^{st}$ evolution get the following functions:

$$O1 = x'y'z + yz' + xz'$$
$$O2 = xy'z' + x'z + x'y$$
$$O3 = x'yz' + y'z + xy'$$

- Realization of OR gate: In $O_1$, set $z = 0$ then $O_1 = x + y$
- Realization of FAN-OUT: In $O_1$, set $x = 0$, $z = 0$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of NOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y^{'}$

In 3-input 3-output rule 30 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}yz + xy^{'}z + xyz^{'}$$
$$O_2 = x^{'}yz + xy^{'}z + xyz^{'}$$
$$O_3 = x^{'}yz + xy^{'}z + xyz^{'}$$

- Realization of AND gate: In $O_1$, set $x = 0$ then $O_1 = yz$
- Realization of XOR gate: In $O_1$, set $y = 1$ then $O_1 = x^{'}z + xz^{'}$

In 4-input 4-output rule 30 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wxy^{'}z + w^{'}xy^{'}z^{'} + w^{'}yz + wyz^{'}$$
$$O_2 = wxyz^{'} + w^{'}x^{'}yz^{'} + w^{'}xz + wx^{'}z$$
$$O_3 = w^{'}xyz + w^{'}x^{'}y^{'}z + wx^{'}y + wxy^{'}$$
$$O_4 = wx^{'}yz + wx^{'}y^{'}z^{'} + xy^{'}z + xyz^{'}$$

- Realization of XNOR gate: In $O_1$, set $w = 0$, $x = 1$ then $O_1 = y^{'}z^{'} + yz$

### 3.2.28 Rule 32

In 3-input 3-output rule 32 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz$$
$$O_2 = xy^{'}z$$
$$O_3 = xyz^{'}$$

- Realization of FANOUT: In $O_1$, Set $y = 1$, $x = 0$ then $O_1 = z$
- Realization of NOT: In $O_1$, Set $z = 1$, $y = 1$ then $O_1 = x^{'}$
- Realization of AND: In $O_1$, Set $x = 0$ then $O_1 = yz$

In 4-input 4-output rule 32 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wx^{'}yz^{'}$$
$$O_2 = w^{'}xy^{'}z$$
$$O_3 = wx^{'}yz^{'}$$
$$O_4 = w^{'}xy^{'}z$$

- Realization of NOR: In $O_1$, Set $w = 1$, $y = 1$ then $O_1 = x^{'}z'$

### 3.2.29 Rule 33

In 3-input 3-output rule 33 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + x^{'}y^{'}z^{'}$$
$$O_2 = xy^{'}z + x^{'}y^{'}z^{'}$$
$$O_3 = xyz^{'} + x^{'}y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 0$, $z = 1$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of XNOR gate: In $O_1$, set $x = 0$ then $O_1 = yz + y^{'}z^{'}$
- Realization of NOR gate: In $O_1$, set $z = 0$ then $O_1 = x^{'}y^{'}$

In 4-input 4-output rule 33 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}x^{'}yz + w^{'}xyz^{'} + wxz + wx^{'}z^{'} + wy^{'}$$
$$O_2 = w^{'}x^{'}yz + wx^{'}y^{'}z + wxy + w^{'}xy^{'} + xz^{'}$$
$$O_3 = wx^{'}y^{'}z + wxy^{'}z^{'} + xyz + x^{'}yz^{'} + w^{'}y$$
$$O_4 = w^{'}xyz^{'} + wxy^{'}z^{'} + wyz + w^{'}y^{'}z + x^{'}z$$

- Realization of AND gate: In $O_1$, set $w = 0$, $x = 0$ then $O_1 = yz$
- Realization of NAND gate: In $O_1$, set $w = 1$, $z = 0$ then $O_1 = x^{'} + y^{'}$

### 3.2.30 Rule 34

In 3-input 3-output rule 34 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y$$
$$O_2 = y^{'}z$$
$$O_3 = xz^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 0$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 1$ then $O_1 = x^{'}$

### 3.2.31 Rule 35

In 3-input 3-output rule 35 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y + x^{'}z^{'}$$
$$O_2 = y^{'}z + x^{'}y^{'}$$
$$O_3 = xz^{'} + y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 0$, $z = 1$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of NOR gate: In $O_1$, set $y = 0$ then $O_1 = x^{'}z^{'}$

In 3-input 3-output rule 35 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xz + y^{'}z$$
$$O_2 = xy + xz^{'}$$
$$O_3 = yz + x^{'}y$$

- Realization of AND gate: In $O_1$, set $y = 1$ then $O_1 = xz$

In 4-input 4-output rule 35 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wx^{'}y + wz + x^{'}z$$
$$O_2 = xy^{'}z + wx + wy^{'}$$
$$O_3 = wyz^{'} + xy + xz^{'}$$
$$O_4 = w^{'}xz + yz + w^{'}y$$

- Realization of OR gate: In $O_1$, set $w = 1$, $x = 0$ then $O_1 = y + z$

In 4-input 4-output rule 35 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = w^{'}xy^{'} + wxz^{'} + y^{'}z^{'}$$
$$O_2 = w^{'}xy + x^{'}yz^{'} + w^{'}z^{'}$$
$$O_3 = x^{'}yz + w^{'}y^{'}z + w^{'}x^{'}$$
$$O_4 = wy^{'}z + wx^{'}z^{'} + x^{'}y^{'}$$

- Realization of NAND gate: In $O_1$, set $x = 1$, $y = 0$ then $O_1 = w^{'} + z^{'}$

### 3.2.32 Rule 36

In 3-input 3-output rule 36 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + xy^{'}z^{'}$$
$$O_2 = xy^{'}z + x^{'}yz^{'}$$
$$O_3 = x^{'}y^{'}z + xyz^{'}$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of AND gate: In $O_1$, set $x = 0$ then $O_1 = yz$
- Realization of NOR gate: In $O_1$, set $x = 1$ then $O_1 = y^{'}z^{'}$

### 3.2.33 Rule 37

In 3-input 3-output rule 37 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + y^{'}z^{'}$$
$$O_2 = xy^{'}z + x^{'}z^{'}$$
$$O_3 = xyz^{'} + x^{'}y^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 0$, $z = 1$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of XNOR gate: In $O_1$, set $x = 0$ then $O_1 = yz + y^{'}z^{'}$
- Realization of NOR gate: In $O_1$, set $z = 0$ then $O_1 = x^{'}y^{'}$

In 3-input 3-output rule 37 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xy^{'}z^{'} + yz$$
$$O_2 = x^{'}yz^{'} + xz$$
$$O_3 = x^{'}y^{'}z + xy$$

- Realization of AND gate: In $O_1$, set $x = 0$ then $O_1 = yz$

In 4-input 4-output rule 37 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wxz + x^{'}y^{'}z + xy^{'}z^{'} + w^{'}y + wy^{'}$$
$$O_2 = wxy + w^{'}yz^{'} + wy^{'}z^{'} + x^{'}z + xz^{'}$$
$$O_3 = wxz + x^{'}y^{'}z + xy^{'}z^{'} + w^{'}y + wy^{'}$$
$$O_4 = wxy + w^{'}yz^{'} + wy^{'}z^{'} + x^{'}z + xz^{'}$$

- Realization of XOR gate: In $O_1$, set $x = 0$, $z = 0$ then $O_1 = w^{'}y + wy^{'}$
- Realization of OR gate: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x + w$

In 4-input 4-output rule 37 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = w^{'}x^{'}y^{'}z^{'} + w^{'}xz + wx^{'}y + xy^{'}z + wyz^{'}$$
$$O_2 = w^{'}x^{'}y^{'}z^{'} + w^{'}xz + wx^{'}y + xy^{'}z + wyz^{'}$$
$$O_3 = w^{'}x^{'}y^{'}z^{'} + w^{'}xz + wx^{'}y + xy^{'}z + wyz^{'}$$
$$O_4 = w^{'}x^{'}y^{'}z^{'} + w^{'}xz + wx^{'}y + xy^{'}z + wyz^{'}$$

- Realization of NAND gate: In $O_1$, set $X = 1$, $z = 1$ then $O_1 = w^{'} + y^{'}$

### 3.2.34 Rule 38

In 3-input 3-output rule 38 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'}z^{'} + x^{'}y$$
$$O_2 = x^{'}yz^{'} + y^{'}z$$
$$O_3 = x^{'}y^{'}z + xz^{'}$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $y = 1$ then $O_1 = x^{'}$
- Realization of NOR gate: In $O_1$, set $x = 1$ then $O_1 = y^{'}z^{'}$
- Realization of XOR gate: In $O_1$, set $z = 0$ then $O_1 = xy^{'} + x^{'}y$

In 3-input 3-output rule 38 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}z + y^{'}z$$
$$O_2 = xy^{'} + xz^{'}$$
$$O_3 = x^{'}y + yz^{'}$$

- Realization of NAND gate: In $O_1$, set $z = 1$ then $O_1 = x^{'} + y^{'}$

### 3.2.35 Rule 40

In 3-input 3-output rule 40 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + xyz^{'}$$
$$O_2 = x^{'}yz + xy^{'}z$$
$$O_3 = xy^{'}z + xyz^{'}$$

- Realization of FANOUT: In $O_1$, Set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $z = 1$, $y = 1$ then $O_1 = x^{'}$
- Realization of AND: In $O_1$, Set $x = 0$ then $O_1 = yz$
- Realization of XOR: In $O_1$, Set $y = 1$ then $O_1 = x^{'}z + xz^{'}$

In 4-input 4-output rule 40 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}xyz + wx^{'}yz^{'}$$
$$O_2 = wx^{'}yz + w^{'}xy^{'}z$$
$$O_3 = wxy^{'}z + wx^{'}yz^{'}$$
$$O_1 = w^{'}xy^{'}z + wxyz^{'}$$

- Realization of NOR: In $O_1$, Set $w = 1$, $y = 1$ then $O_1 = x^{'}z'$

### 3.2.36 Rule 41

In 3-input 3-output rule 41 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + xyz^{'} + x^{'}y^{'}z^{'}$$
$$O_1 = x^{'}yz + xy^{'}z + x^{'}y^{'}z^{'}$$
$$O_1 = xy^{'}z + xyz^{'} + x^{'}y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of XNOR: In $O_1$, Set $x = 0$ then $O_1 = yz + y^{'}z^{'}$
- Realization of XOR: In $O_1$, Set $y = 1$ then $O_1 = x^{'}z + xz^{'}$

In 3-input 3-output rule 41 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}y + xy^{'} + z$$
$$O_2 = y^{'}z + yz^{'} + x$$
$$O_3 = x^{'}z + xz^{'} + y$$

- Realization of OR: In $O_1$, Set $y = 0$ then $O_1 = x + z$

In 4-input 4-output rule 41 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xyz + w^{'}yz + wx^{'}z^{'}$$
$$O_2 = wyz + wx^{'}z + w^{'}xy^{'}$$
$$O_1 = wxz + wxy^{'} + x^{'}yz^{'}$$
$$O_1 = wxy + xyz^{'} + w^{'}y^{'}z$$

- Realization of AND: In $O_1$, Set $w = 1$, $z = 1$ then $O_1 = xy$

In 5-input 5-output rule 41 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = vw^{'}xyz^{'} + vw^{'}x^{'}y^{'}z^{'} + vwyz + vx^{'}yz + v^{'}wxy^{'} + w^{'}xy^{'}z + v^{'}x^{'}yz^{'} + v^{'}xz$$
$$O_2 = v^{'}wx^{'}yz + v^{'}wx^{'}y^{'}z^{'} + vwxz + vwy^{'}z + v^{'}w^{'}y^{'}z + w^{'}xyz^{'} + vx^{'}yz^{'} + vw^{'}y$$
$$O_3 = vw^{'}xy^{'}z + v^{'}w^{'}xy^{'}z^{'} + vwxy + v^{'}x^{'}yz + v^{'}wy^{'}z + vwxz^{'} + vw^{'}x^{'}z^{'} + wx^{'}z$$
$$O_4 = vwx^{'}yz^{'} + v^{'}w^{'}x^{'}yz^{'} + vwxz + v^{'}wxy + vw^{'}y^{'}z + v^{'}wx^{'}y^{'} + vw^{'}xz^{'} + vxy^{'}$$
$$O_5 = v^{'}wxy^{'}z + v^{'}w^{'}x^{'}y^{'}z + vwxy + w^{'}xyz + v^{'}wx^{'}y + vwx^{'}z^{'} + w^{'}xy^{'}z^{'} + wyz^{'}$$

- Realization of NOR: In $O_1$, Set $v = 1$, $y = 0$, $z = 0$ then $O_1 = w^{'}x^{'}$
- Realization of NAND: In $O_1$, Set $x = 1$, $z = 1$, $w = 0$ then $O_1 = v^{'} + y^{'}$

### 3.2.37 Rule 42

In 3-input 3-output rule 42 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y + yz^{'}$$
$$O_2 = y^{'}z + x^{'}z$$
$$O_3 = xy^{'} + xz^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 0$, $z = 1$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of NAND gate: In $O_1$, set $y = 1$ then $O_1 = x^{'} + z^{'}$

### 3.2.38 Rule 43

In 3-input 3-output rule 43 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y + yz^{'} + x^{'}z^{'}$$
$$O_2 = x^{'}z + y^{'}z + x^{'}y^{'}$$
$$O_3 = xy^{'} + xz^{'} + y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $x = 0$, $z = 1$ then $O_1 = y$
- Realization of NOT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of NOR: In $O_1$, Set $y = 0$ then $O_1 = x^{'}z^{'}$

In 4-input 4-output rule 43 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}x^{'}z + yz + w^{'}y + x^{'}y$$
$$O_2 = wx^{'}y^{'} + wz + x^{'}z + y^{'}z$$
$$O_3 = xy^{'}z^{'} + wx + wy^{'} + wz^{'}$$
$$O_4 = w^{'}yz^{'} + xy + wz^{'} + xz^{'}$$

- Realization of AND gate: In $O_1$, Set $w = 1$, $x = 1$ then $O_1 = yz$

In 5-input 5-output rule 43 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = vyz + v^{'}xz + v^{'}xy + vw^{'}x + v^{'}w^{'}z + v^{'}w^{'}y$$
$$O_2 = vwz + w^{'}yz + vw^{'}y + wx^{'}y + w^{'}x^{'}z + vw^{'}x^{'}$$
$$O_3 = vwx + wx^{'}z + vx^{'}z + xy^{'}z + wx^{'}y^{'} + vx^{'}y^{'}$$
$$O_4 = wxy + vxy^{'} + vwy^{'} + vyz^{'} + xy^{'}z^{'} + wy^{'}z^{'}$$
$$O_5 = xyz + v^{'}wz + wyz^{'} + wxz^{'} + v^{'}yz^{'} + v^{'}xz^{'}$$

- Realization of OR: In $O_1$, Set $v = 1$, $w = 0$, $z = 1$ then $O_1 = x + y$

In 5-input 5-output rule 43 block: After $5^{th}$ evolution get the following functions:

$$O_1 = vx^{'}yz^{'} + v^{'}xy + vw^{'}x + v^{'}w^{'}x^{'} + v^{'}xz^{'}$$
$$O_2 = v^{'}wy^{'}z + w^{'}yz + v^{'}w^{'}y + wx^{'}y + w^{'}x^{'}y^{'}$$
$$O_3 = vw^{'}xz^{'} + vx^{'}z + xy^{'}z + x^{'}y^{'}z^{'} + w^{'}x^{'}z$$
$$O_4 = v^{'}wx^{'}y + vyz^{'} + vwy^{'} + v^{'}y^{'}z^{'} + vx^{'}y^{'}$$
$$O_5 = w^{'}xy^{'}z + v^{'}wz + vxz^{'} + v^{'}w^{'}z^{'} + wy^{'}z^{'}$$

- Realization of XOR: In $O_1$, Set $y = 1$, $w = 1$, $z = 0$ then $O_1 = vx^{'} + v^{'}x$
- Realization of XNOR: In $O_1$, Set $y = 0$, $w = 0$, $z = 1$ then $O_1 = vx + v^{'}x^{'}$

### 3.2.39 Rule 44

In 3-input 3-output rule 44 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + xz^{'}$$
$$O_2 = xy^{'}z + x^{'}y$$
$$O_3 = xyz^{'} + y^{'}z$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of AND gate: In $O_1$, set $x = 0$ then $O_1 = yz$
- Realization of XOR gate: In $O_1$, set $y = 1$ then $O_1 = x^{'}z + xz^{'}$

### 3.2.40 Rule 46

In 3-input 3-output rule 46 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + xz^{'} + y^{'}z^{'}$$
$$O_2 = xy^{'}z + x^{'}y + x^{'}z^{'}$$
$$O_3 = xyz^{'} + y^{'}z + x^{'}y^{'}$$

- Realization of FAN-OUT: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 0$, $y = 0$ then $O_1 = z^{'}$
- Realization of XNOR gate: In $O_1$, set $x = 0$ then $O_1 = yz + y^{'}z^{'}$
- Realization of XOR gate: In $O_1$, set $y = 1$ then $O_1 = x^{'}z + xz^{'}$

In 3-input 3-output rule 46 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = yz + xy^{'}$$
$$O_2 = xz + yz^{'}$$
$$O_3 = xy + x^{'}y$$

- Realization of AND gate: In $O_1$, set $x = 0$ then $O_1 = yz$

In 4-input 4-output rule 46 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}yz + w^{'}xz^{'} + w^{'}y$$
$$O_2 = w^{'}x^{'}y + wy^{'}z + x^{'}z$$
$$O_3 = x^{'}y^{'}z + wxz^{'} + wy^{'}$$
$$O_4 = w^{'}xy + wy^{'}z^{'} + xz^{'}$$

- Realization of OR gate: In $O_1$, set $w = 0$, $z = 0$ then $O_1 = x + y$
- Realization of NAND gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x^{'} + w^{'}$

### 3.2.41 Rule 50

In 3-input 3-output rule 50 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}z + x^{'}y$$
$$O_2 = y^{'}z + xy^{'}$$
$$O_3 = yz^{'} + xz^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $x = 0$, $z = 0$ then $O_1 = y$
- Realization of NOT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of OR: In $O_1$, Set $x = 0$ then $O_1 = z + y$

In 4-input 4-output rule 50 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}yz^{'} + wx^{'} + wz^{'}$$
$$O_2 = w^{'}y^{'}z + w^{'}x + xy^{'}$$
$$O_3 = wx^{'}z^{'} + x^{'}y + yz^{'}$$
$$O_4 = w^{'}xy^{'} + w^{'}z + y^{'}z$$

- Realization of NAND: In $O_1$, Set $y = 0$, $w = 1$ then $O_1 = x^{'} + z^{'}$
- Realization of NOR: In $O_1$, Set $w = 0$, $y = 1$ then $O_1 = x^{'}z^{'}$

In 4-input 4-output rule 50 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = xy^{'}z + w^{'}z + w^{'}x$$
$$O_2 = wyz^{'} + x^{'}y + wx^{'}$$
$$O_3 = w^{'}xz + xy^{'} + y^{'}z$$
$$O_4 = wx^{'}y + yz^{'} + wz^{'}$$

- Realization of AND: In $O_1$, Set $w = 1$, $y = 0$ then $O_1 = xz$

### 3.2.42 Rule 51

In 3-input 3-output rule 51 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}$$
$$O_2 = y^{'}$$
$$O_3 = z^{'}$$

- Realization of NOT: $O_1 = x^{'}$

In 3-input 3-output rule 51 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x$$
$$O_2 = y$$
$$O_3 = z$$

- Realization of FAN-OUT: $O_1 = x$

### 3.2.43 Rule 54

In 3-input 3-output rule 54 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'}z^{'} + x^{'}z + x^{'}y$$
$$O_2 = x^{'}yz^{'} + y^{'}z + xy^{'}$$
$$O_3 = x^{'}y^{'}z + yz^{'} + xz^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of NOR: In $O_1$, Set $x = 1$ then $O_1 = y^{'}z^{'}$
- Realization of XOR: In $O_1$, Set y=0 then $O_1 = x^{'}z + xz^{'}$

In 3-input 3-output rule 54 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}yz + xy^{'}z + xyz^{'}$$
$$O_2 = x^{'}yz + xy^{'}z + xyz^{'}$$
$$O_3 = x^{'}yz + xy^{'}z + xyz^{'}$$

- Realization of AND: In $O_1$, Set $z = 0$ then $O_1 = xy$

In 4-input 4-output rule 54 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}xyz + w^{'}x^{'}yz^{'} + wx^{'}z + wxz^{'}$$
$$O_2 = wx^{'}yz + w^{'}x^{'}y^{'}z + w^{'}xy + wxy^{'}$$
$$O_3 = wxy^{'}z + wx^{'}y^{'}z^{'} + x^{'}yz + xyz^{'}$$
$$O_4 = wxyz^{'} + w^{'}xy^{'}z^{'} + w^{'}yz + wy^{'}z$$

- Realization of XNOR: In $O_1$, Set $z = 1$, $y = 1$ then $O_1 = wx + w^{'}x^{'}$

In 4-input 4-output rule 54 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = wxy^{'}z + w^{'}x^{'}z + w^{'}yz^{'} + w^{'}xz^{'}$$
$$O_2 = wxyz^{'} + w^{'}x^{'}y + x^{'}y^{'}z + wx^{'}y^{'}$$
$$O_3 = w^{'}xyz + x^{'}y^{'}z + xy^{'}z^{'} + wy^{'}z^{'}$$
$$O_4 = wx^{'}yz + w^{'}yz^{'} + xy^{'}z^{'} + wy^{'}z^{'}$$

- Realization of OR: In $O_1$, Set $w = 0$, $z = 0$ then $O_1 = x + y$

In 4-input 4-output rule 54 block: After $4^{th}$ evolution get the following functions:

$$O_1 = wx^{'}z + wxz^{'} + wy^{'}$$
$$O_2 = w^{'}xy + wxy^{'} + xz^{'}$$
$$O_1 = x^{'}yz + xyz^{'} + w^{'}y$$
$$O_1 = w^{'}yz + wy^{'}z + x^{'}z$$

- Realization of NAND: In $O_1$, Set $z = 1$, $w = 1$ then $O_1 = x^{'} + y^{'}$

### 3.2.44 Rule 56

In 3-input 3-output rule 56 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz^{'} + x^{'}z$$
$$O_2 = x^{'}yz + xy^{'}$$
$$O_3 = xy^{'}z + yz^{'}$$

- Realization of AND gate: In $O_1$, set $z = 0$ then $O_1 = xy$
- Realization of FAN-OUT: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of XOR gate: In $O_1$, set $y = 1$ then $O_1 = xz^{'} + x^{'}z$

In 3-input 3-output rule 56 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = xy^{'} + xz^{'}$$
$$O_2 = x^{'}y + yz^{'}$$
$$O_3 = x^{'}z + y^{'}z$$

- Realization of NAND gate: In $O_1$, set $x = 1$ the $O_1 = y^{'} + z^{'}$

In 4-input 4-output rule 56 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}xy + wy^{'}z + x^{'}yz^{'}$$
$$O_2 = x^{'}yz + w^{'}y^{'}z + wxz^{'}$$
$$O_3 = w^{'}xy + wy^{'}z + wx^{'}z^{'}$$
$$O_4 = x^{'}yz + w^{'}xy^{'} + wxz^{'}$$

- Realization of NOR gate: In $O_1$, set $y = 1$, $w = 1$ then $O_1 = x^{'}z^{'}$

### 3.2.45 Rule 57

In 3-input 3-output rule 57 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz^{'} + x^{'}z + x^{'}y^{'}$$
$$O_2 = x^{'}yz + xy^{'} + y^{'}z^{'}$$
$$O_3 = xy^{'}z + yz^{'} + x^{'}z^{'}$$

- Realization of NOT gate: In $O_1$, set $x = 0$, $z = 0$ then $O_1 = y^{'}$
- Realization of FAN-OUT: In $O_1$, set $x = 1$, $z = 0$ then $O_1 = y$
- Realization of XOR gate: In $O_1$, set $y = 1$ then $O_1 = xz^{'} + x^{'}z$
- Realization of XNOR gate: In $O_1$, set $z = 0$ then $O_1 = xy + x^{'}y^{'}$

In 3-input 3-output rule 57 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xz + x^{'}y$$
$$O_2 = xy + y^{'}z$$
$$O_3 = yz + xz^{'}$$

- Realization of AND gate: In $O_1$, set $y = 0$ then $O_1 = xz$
- Realization of OR gate: In $O_1$, set $z = 1$ then $O_1 = x + x^{'}y = x + y$

In 3-input 3-output rule 57 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = xy^{'} + xz^{'} + y^{'}z^{'}$$
$$O_2 = x^{'}y + yz^{'} + x^{'}z^{'}$$
$$O_3 = x^{'}z + y^{'}z + x^{'}y^{'}$$

- Realization of NAND gate: In $O_1$, set $x = 1$ then $O_1 = y^{'} + z^{'} + y^{'}z^{'} = y^{'} + z^{'}$
- Realization of NOR gate: In $O_1$, set $x = 0$ then $O_1 = y^{'}z^{'}$

### 3.2.46 Rule 58

In 3-input 3-output rule 58 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}z + yz^{'}$$
$$O_2 = x^{'}z + xy^{'}$$
$$O_3 = xy^{'} + yz^{'}$$

- Realization of NOT gate: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x^{'}$
- Realization of FAN-OUT: In $O_1$, set $x = 1$, $z = 0$ then $O_1 = y$
- Realization of OR gate:In $O_1$, set $x = 0$ then $O_1 = z + y$

In 4-input 4-output rule 58 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}yz + wy^{'}z + x^{'}y + wx^{'}$$
$$O_2 = wx^{'}z + wxz^{'} + y^{'}z + xy^{'}$$
$$O_3 = w^{'}xy + wxy^{'} + yz^{'} + wz^{'}$$
$$O_4 = x^{'}yz + xyz^{'} + w^{'}z + w^{'}x$$

- Realization of AND gate: In $O_1$, set $x = 1$, $w = 0$ then $O_1 = yz$
- Realization of NAND gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = w^{'} + x^{'}$
- Realization of XOR gate: In $O_1$, set $x = 1$, $z = 1$ then $O_1 = w^{'}y + wy^{'}$

In 4-input 4-output rule 58 block: After $4^{th}$ evolution get the following functions:

$$O_1 = wx^{'}z^{'} + w^{'}y + x^{'}y + yz^{'}$$
$$O_2 = w^{'}xy^{'} + w^{'}z + x^{'}z + y^{'}z$$
$$O_3 = x^{'}yz^{'} + wx^{'} + wy^{'} + wz^{'}$$
$$O_4 = w^{'}y^{'}z + w^{'}x + xy^{'} + xz^{'}$$

- Realization of NOR gate: In $O_1$, set $w = 1$, $y = 0$ then $O_1 = x^{'}z^{'}$

In 5-input 5-output rule 58 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = vw^{'}xy + v^{'}x^{'}z + vwz^{'} + wx^{'} + yz^{'}$$
$$O_2 = wx^{'}yz + v^{'}wx + vw^{'}y^{'} + v^{'}z + xy^{'}$$
$$O_3 = vxy^{'}z + w^{'}xy + wx^{'}z^{'} + vw^{'} + yz^{'}$$
$$O_4 = vwyz^{'} + x^{'}yz + v^{'}xy^{'} + v^{'}z + wx^{'}$$
$$O_5 = v^{'}wxz + vy^{'}z + w^{'}yz^{'} + vw^{'} + xy^{'}$$

- Realization of XNOR gate: In $O_1$, set $w = 0$, $y = 1$, $z = 1$ then $O_1 = vx + v^{'}x^{'}$

### 3.2.47 Rule 60

In 3-input 3-output rule 60 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}z + xz^{'}$$
$$O_2 = x^{'}y + xy^{'}$$
$$O_1 = y^{'}z + yz^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $z = 0$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $z = 1$ then $O_1 = x^{'}$
- Realization of XOR: In $O_1$, $O_1 = x^{'}z + xz^{'}$

In 4-input 4-output rule 60 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}xyz + wx^{'}yz + wxy^{'}z + wxyz^{'} + w^{'}x^{'}y^{'}z + w^{'}x^{'}yz^{'} + w^{'}xy^{'}z^{'} + wx^{'}y^{'}z^{'}$$
$$O_2 = w^{'}xyz + wx^{'}yz + wxy^{'}z + wxyz^{'} + w^{'}x^{'}y^{'}z + w^{'}x^{'}yz^{'} + w^{'}xy^{'}z^{'} + wx^{'}y^{'}z^{'}$$
$$O_3 = w^{'}xyz + wx^{'}yz + wxy^{'}z + wxyz^{'} + w^{'}x^{'}y^{'}z + w^{'}x^{'}yz^{'} + w^{'}xy^{'}z^{'} + wx^{'}y^{'}z^{'}$$
$$O_4 = w^{'}xyz + wx^{'}yz + wxy^{'}z + wxyz^{'} + w^{'}x^{'}y^{'}z + w^{'}x^{'}yz^{'} + w^{'}xy^{'}z^{'} + wx^{'}y^{'}z^{'}$$

- Realization of XNOR: In $O_1$, Set $w = 0$, $z = 1$ then $O_1 = xy + x^{'}y^{'}$

### 3.2.48 Rule 72

In 3-input 3-output rule 72 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'}z + xyz^{'}$$
$$O_2 = x^{'}yz + xyz^{'}$$
$$O_3 = x^{'}yz + xy^{'}z$$

- Realization of FAN-OUT: In $O_1$ set $y = 0$, $z = 1$ then $O_1 = x$
- Realization of NOT gate: In $O_1$ set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of AND gate: In $O_1$ set $y = 0$ then $O_1 = xz$
- Realization of XOR gate: In $O_1$ set $x = 1$ then $O_1 = y^{'}z + yz^{'}$

### 3.2.49 Rule 73

In 3-input 3-output rule 73 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'}z + xyz^{'} + x^{'}y^{'}z^{'}$$
$$O_2 = x^{'}yz + xyz^{'} + x^{'}y^{'}z^{'}$$
$$O_3 = x^{'}yz + xy^{'}z + x^{'}y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $x = 1$, $z = 1$ then $O_1 = y^{'}$
- Realization of XNOR: In $O_1$, Set $z = 0$ then $O_1 = xy + x^{'}y^{'}$

- Realization of XOR: In $O_1$, Set $x = 1$ then $O_1 = y^{'}z + yz^{'}$
- Realization of NOR: In $O_1$, Set $x = 0$ then $O_1 = y^{'}z^{'}$

In 4-input 4-output rule 73 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wxyz + w^{'}xy^{'}z + wx^{'}y^{'} + wx^{'}z^{'} + wy^{'}z^{'}$$
$$O_2 = wxyz + wx^{'}yz^{'} + w^{'}xy^{'} + w^{'}xz^{'} + xy^{'}z^{'}$$
$$O_3 = wxyz + w^{'}xy^{'}z + w^{'}x^{'}y + w^{'}yz^{'} + x^{'}yz^{'}$$
$$O_4 = wxyz + wx^{'}yz^{'} + w^{'}x^{'}z + w^{'}y^{'}z + x^{'}y^{'}z$$

- Realization of AND: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = wx$
- Realization of NAND: In $O_1$, Set $w = 1$, $y = 0$ then $O_1 = x^{'} + z^{'}$

In 5-input 5-output rule 73 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = vwxyz{+}v^{'}wx^{'}y^{'}z{+}w^{'}xy^{'}{+}vw^{'}y^{'}{+}vw^{'}z^{'}{+}x^{'}yz^{'}{+}vx^{'}z^{'}$$
$$O_2 = vwxyz{+}vw^{'}xy^{'}z^{'}{+}v^{'}wx^{'}{+}v^{'}y^{'}z{+}v^{'}wy^{'}{+}x^{'}yz^{'}{+}wx^{'}z^{'}$$
$$O_3 = vwxyz{+}v^{'}wx^{'}yz^{'}{+}v^{'}y^{'}z{+}v^{'}xy^{'}{+}w^{'}xy^{'}{+}w^{'}xz^{'}{+}vw^{'}z^{'}$$
$$O_4 = vwxyz{+}v^{'}w^{'}xy^{'}z{+}v^{'}x^{'}y{+}v^{'}wx^{'}{+}w^{'}yz^{'}{+}vw^{'}z^{'}{+}x^{'}yz^{'}$$
$$O_5 = vwxyz{+}vw^{'}x^{'}yz^{'}{+}v^{'}x^{'}z{+}v^{'}wx^{'}{+}v^{'}y^{'}z{+}w^{'}y^{'}z{+}w^{'}xy^{'}$$

- Realization of OR: In $O_1$, Set $w = 0$, $y = 0$, $z = 1$ then $O_1 = x + v$

### 3.2.50 Rule 74

In 3-input 3-output rule 74 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'}z + yz^{'}$$
$$O_2 = xyz^{'} + x^{'}z$$
$$O_3 = x^{'}yz + xy^{'}$$

- Realization of NOT: In $O_1$, Set $x = 1$, $z = 1$ then $O_1 = y^{'}$
- Realization of FAN-OUT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x$
- Realization of AND: In $O_1$, Set $y = 0$ then $O_1 = xz$
- Realization of XOR: In $O_1$, Set $x = 1$ then $O_1 = y^{'}z + yz^{'}$

In 3-input 3-output rule 74 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = xy^{'} + xz^{'}$$
$$O_2 = x^{'}y + yz^{'}$$
$$O_3 = x^{'}z + y^{'}z$$

- Realization of NAND: In $O_1$, Set $x = 1$ then $O_1 = y^{'} + z^{'}$

In 4-input 4-output rule 74 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wx^{'}y^{'}z + w^{'}yz^{'}$$
$$O_2 = wxy^{'}z^{'} + w^{'}x^{'}z$$
$$O_3 = w^{'}xyz^{'} + wx^{'}y^{'}$$
$$O_4 = w^{'}x^{'}yz + xy^{'}z^{'}$$

- Realization of NOR: In $O_1$, Set $w = 1$, $z = 1$ then $O_1 = x^{'}y^{'}$

In 5-input 5-output rule 74 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = vwx^{'}y + vw^{'}y^{'}z + wxy^{'}z^{'} + v^{'}xy^{'} + v^{'}xz^{'}$$
$$O_2 = wxy^{'}z + v^{'}xyz^{'} + vwx^{'}z^{'} + v^{'}w^{'}y + w^{'}yz^{'}$$
$$O_3 = vxyz^{'} + v^{'}w^{'}yz + v^{'}wxy^{'} + v^{'}x^{'}z + w^{'}x^{'}z$$
$$O_4 = v^{'}wyz + vw^{'}x^{'}z + w^{'}xyz^{'} + vw^{'}y^{'} + vx^{'}y^{'}$$
$$O_5 = vw^{'}xz + v^{'}x^{'}yz + vwx^{'}y^{'} + wx^{'}z^{'} + wy^{'}z^{'}$$

- Realization of NOR: In $O_1$, Set $v = 1$, $x = 0$, $z = 1$ then $O_1 = wy + w^{'}y^{'}$

### 3.2.51 Rule 76

In 3-input 3-output rule 76 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'} + xz^{'}$$
$$O_2 = x^{'}y + yz^{'}$$
$$O_3 = x^{'}z + y^{'}z$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 1$, $z = 1$ then $O_1 = y^{'}$
- Realization of NAND gate: In $O_1$, set $x = 1$ then $O_1 = y^{'} + z^{'}$

### 3.2.52 Rule 77

In 3-input 3-output rule 77 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'} + xz^{'} + y^{'}z^{'}$$
$$O_2 = x^{'}y + yz^{'} + x^{'}z^{'}$$
$$O_3 = x^{'}z + y^{'}z + x^{'}y^{'}$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x$

- Realization of NOT gate: In $O_1$, set $x = 0$, $y = 0$ then $O_1 = z^{'}$
- Realization of NOR gate: In $O_1$, set $x = 0$ then $O_1 = y^{'}z^{'}$
- Realization of NAND gate: In $O_1$, set $x = 1$ then $O_1 = y^{'} + z^{'}$

In 4-input 4-output rule 77 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}yz^{'} + wy + wx^{'} + wz^{'}$$
$$O_2 = w^{'}y^{'}z + xz + w^{'}x + xy^{'}$$
$$O_3 = wx^{'}z^{'} + wy + x^{'}y + yz^{'}$$
$$O_4 = w^{'}xy^{'} + xz + w^{'}z + y^{'}z$$

- Realization of AND gate: In $O_1$, set $x = 1$, $z = 1$ then $O_1 = wy$
- Realization of OR gate: In $O_1$, set $x = 0$, $z = 0$ then $O_1 = y + w$

### 3.2.53 Rule 78

In 3-input 3-output rule 78 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'} + yz^{'}$$
$$O_2 = x^{'}z + yz^{'}$$
$$O_3 = x^{'}z + xy^{'}$$

- Realization of FAN-OUT: In $O_1$, $y = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 1$, $z = 1$ then $O_1 = y^{'}$
- Realization of OR gate: In $O_1$, set $z = 0$ then $O_1 = x + y$
- Realization of NAND gate: In $O_1$, set $x = 1$ then $O_1 = y^{'} + z^{'}$

In 4-input 4-output rule 78 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}xz^{'} + wx^{'} + yz^{'}$$
$$O_2 = w^{'}x^{'}y + w^{'}z + xy^{'}$$
$$O_3 = x^{'}y^{'}z + wx^{'} + yz^{'}$$
$$O_4 = wy^{'}z^{'} + w^{'}z + xy^{'}$$

- Realization of XOR gate: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = w^{'}x + wx^{'}$

In 5-input 5-output rule 78 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = vx^{'}y + v^{'}wz^{'} + vw^{'} + xz^{'}$$
$$O_2 = v^{'}w^{'}x + wy^{'}z + v^{'}y + wx^{'}$$
$$O_3 = w^{'}x^{'}y + vxz^{'} + w^{'}z + xy^{'}$$
$$O_4 = v^{'}wy + x^{'}y^{'}z + vx^{'} + yz^{'}$$
$$O_5 = w^{'}xz + vy^{'}z^{'} + v^{'}z + wy^{'}$$

- Realization of AND gate: In $O_1$, set $x = 0$, $w = 1$, $z = 1$ then $O_1 = vy$
- Realization of NOR gate: In $O_1$, set $x = 0$, $y = 0$, $w = 1$ then $O_1 = v^{'}z^{'}$

### 3.2.54 Rule 90

In 3-input 3-output rule 90 block: After $1^{st}$ evolution get the following functions:

$$O_1 = y^{'}z + yz^{'}$$
$$O_2 = x^{'}z + xz^{'}$$
$$O_3 = x^{'}y + xy^{'}$$

- Realization of FANOUT: In $O_1$, Set $z = 0$ then $O_1 = y$
- Realization of NOT: In $O_1$, Set $z = 1$ then $O_1 = y^{'}$
- Realization of XOR: In $O_1$, $O_1 = y^{'}z + yz^{'}$

In 5-input 5-output rule 90 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = w^{'}xyz + wx^{'}yz + wxy^{'}z + wxyz^{'} + w^{'}x^{'}y^{'}z + w^{'}x^{'}yz^{'} + w^{'}xy^{'}z^{'} + wx^{'}y^{'}z^{'}$$
$$O_2 = v^{'}xyz + vx^{'}yz + vxy^{'}z + vxyz^{'} + v^{'}x^{'}y^{'}z + v^{'}x^{'}yz^{'} + v^{'}xy^{'}z^{'} + vx^{'}y^{'}z^{'}$$
$$O_3 = v^{'}wyz + vw^{'}yz + vwy^{'}z + vwyz^{'} + v^{'}w^{'}y^{'}z + v^{'}w^{'}yz^{'} + v^{'}wy^{'}z^{'} + vw^{'}y^{'}z^{'}$$
$$O_4 = v^{'}wxz + vw^{'}xz + vwx^{'}z + vwxz^{'} + v^{'}w^{'}x^{'}z + v^{'}w^{'}xz^{'} + v^{'}wx^{'}z^{'} + vw^{'}x^{'}z^{'}$$
$$O_5 = v^{'}wxy + vw^{'}xy + vwx^{'}y + vwxy^{'} + v^{'}w^{'}x^{'}y + v^{'}w^{'}x^{'}y + v^{'}wx^{'}y^{'} + vw^{'}x^{'}y^{'}$$

- Realization of XNOR: In $O_1$, Set $w = 0$, $z = 1$ then $O_1 = xy + x^{'}y^{'}$

### 3.2.55 Rule 104

In 3-input 3-output rule 104 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + xy^{'}z + xyz^{'}$$
$$O_2 = x^{'}yz + xy^{'}z + xyz^{'}$$
$$O_3 = x^{'}yz + xy^{'}z + xyz^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 0$, $y = 1$ then $O_1 = z$

- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of AND gate: In $O_1$, set $x = 0$ then $O_1 = yz$
- Realization of XOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y + xy^{'}$

In 5-input 5-output rule 104 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = v^{'}wx^{'}yz + v^{'}wxy^{'}z + vw^{'}xy + vx^{'}y^{'}z + vwx^{'}y^{'} + vxyz^{'}$$
$$O_2 = vw^{'}xy^{'}z + vw^{'}xyz^{'} + v^{'}wyz + wx^{'}yz + wxy^{'}z^{'} + vwy^{'}z^{'}$$
$$O_3 = v^{'}wx^{'}yz + vwx^{'}yz^{'} + vw^{'}xz + vxy^{'}z + v^{'}xyz^{'} + v^{'}wxz^{'}$$
$$O_4 = v^{'}wxy^{'}z + vw^{'}xy^{'}z + v^{'}w^{'}yz + v^{'}w^{'}xy + vwx^{'}y + vwyz^{'}$$
$$O_5 = vw^{'}xyz^{'} + vwx^{'}yz^{'} + v^{'}wxz + w^{'}x^{'}yz + vw^{'}x^{'}z + wxy^{'}z$$

- Realization of XNOR gate: In $O_1$, set $v = 1$, $w = 0$, $z = 1$ then $O_1 = xy + x^{'}y^{'}$

### 3.2.56 Rule 105

In 3-input 3-output rule 105 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + xy^{'}z + xyz^{'} + x^{'}y^{'}z^{'}$$
$$O_2 = x^{'}yz + xy^{'}z + xyz^{'} + x^{'}y^{'}z^{'}$$
$$O_3 = x^{'}yz + xy^{'}z + xyz^{'} + x^{'}y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of XOR: In $O_1$, Set $z = 1$ then $O_1 = x^{'}y + xy^{'}$
- Realization of XNOR: In $O_1$, Set $z = 0$ then $O_1 = xy + x^{'}y^{'}$

### 3.2.57 Rule 106

In 3-input 3-output rule 106 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'}z + x^{'}y + yz^{'}$$
$$O_2 = xyz^{'} + x^{'}z + y^{'}z$$
$$O_3 = x^{'}yz + xy^{'} + xz^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $x = 1$, $z = 1$ then $O_1 = y^{'}$
- Realization of XOR: In $O_1$, Set $z = 1$ then $O_1 = x^{'}y + xy^{'}$

- Realization of AND: In $O_1$, Set $y = 0$ then $O_1 = xz$
- Realization of NAND: In $O_1$, Set $y = 1$ then $O_1 = x^{'} + z^{'}$

In 3-input 3-output rule 106 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z$$
$$O_2 = xy^{'}z^{'}$$
$$O_3 = x^{'}yz^{'}$$

- Realization of NOR: In $O_1$, Set $z = 1$ then $O_1 = x^{'}y^{'}$

In 5-input 5-output rule 106 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wx^{'}yz + vx^{'}y^{'}z + w^{'}xy + v^{'}xy^{'} + xz^{'}$$
$$O_2 = vxy^{'}z + vwy^{'}z^{'} + x^{'}yz + w^{'}yz^{'} + v^{'}y$$
$$O_3 = vwyz^{'} + v^{'}wxz^{'} + vy^{'}z + v^{'}x^{'}z + w^{'}z$$
$$O_4 = v^{'}wxz + v^{'}w^{'}xy + vwz^{'} + vw^{'}y^{'} + vx^{'}$$
$$O_5 = vw^{'}xy + w^{'}x^{'}yz + v^{'}wx + wx^{'}z^{'} + wy^{'}$$

- Realization of XNOR: In $O_1$, Set $v = 1$, $z = 1$, $w = 0$ then $O_1 = xy + x^{'}y^{'}$

### 3.2.58 Rule 108

In 3-input 3-output rule 108 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}yz + xy^{'} + xz^{'}$$
$$O_2 = xy^{'}z + x^{'}y + yz^{'}$$
$$O_3 = xyz^{'} + x^{'}z + y^{'}z$$

- Realization of NOT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x^{'}$
- Realization of FAN-OUT: In $O_1$, Set $x = 0$, $z = 1$ then $O_1 = y$
- Realization of AND: In $O_1$, Set $x = 0$ then $O_1 = yz$
- Realization of XOR: In $O_1$, Set $z = 1$ then $O_1 = x^{'}y + xy^{'}$
- Realization of NAND: In $O_1$, Set $x = 1$ then $O_1 = y^{'} + z^{'}$

In 3-input 3-output rule 108 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xy^{'}z^{'}$$
$$O_2 = x^{'}yz^{'}$$
$$O_3 = x^{'}y^{'}z$$

- Realization of NOR: In $O_1$, Set $x = 1$ then $O_1 = y^{'}z^{'}$

### 3.2.59 Rule 128

In 3-input 3-output rule 128 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz$$
$$O_2 = xyz$$
$$O_3 = xyz$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of AND: In $O_1$, Set $z = 1$ then $O_1 = xy$

### 3.2.60 Rule 129

In 3-input 3-output rule 129 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz + x^{'}y^{'}z^{'}$$
$$O_2 = xyz + x^{'}y^{'}z^{'}$$
$$O_3 = xyz + x^{'}y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 0$, $z = 0$ then $O_1 = x^{'}$
- Realization of NOR: In $O_1$, Set $z = 0$ then $O_1 = x^{'}y^{'}$
- Realization of AND: In $O_1$, Set $z = 1$ then $O_1 = xy$

In 4-input 4-output rule 129 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xyz + w^{'}yz + w^{'}xz + w^{'}xy + wx^{'}y^{'} + wx^{'}z^{'} + wy^{'}z^{'} + x^{'}y^{'}z^{'}$$
$$O_2 = wyz + x^{'}yz + wx^{'}z + wx^{'}y + w^{'}xy^{'} + w^{'}xz^{'} + xy^{'}z^{'} + w^{'}y^{'}z^{'}$$
$$O_3 = wxz + xy^{'}z + wy^{'}z + wxy^{'} + w^{'}x^{'}y + w^{'}yz^{'} + x^{'}yz^{'} + w^{'}x^{'}z^{'}$$
$$O_4 = wxy + xyz^{'} + wyz^{'} + wxz^{'} + w^{'}x^{'}z + w^{'}y^{'}z + x^{'}y^{'}z + w^{'}x^{'}y^{'}$$

- Realization of XOR: In $O_1$, Set $x = 0$, $z = 1$ then $O_1 = w^{'}y + wy^{'}$
- Realization of XNOR: In $O_1$, Set $w = 1$, $z = 1$ then $O_1 = xy + x^{'}y^{'}$

In 5-input 5-output rule 129 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = wxyz + w^{'}x^{'}y^{'}z^{'} + v^{'}xz + vwz + vwy + vw^{'}y^{'} + vw^{'}z^{'} + vx^{'}z^{'}$$
$$O_2 = vxyz + v^{'}x^{'}y^{'}z^{'} + w^{'}xz + vwy + vwx + v^{'}wx^{'} + v^{'}wy^{'} + wx^{'}z^{'}$$
$$O_3 = vwyz + v^{'}w^{'}y^{'}z^{'} + wx^{'}z + wxy + vxy + v^{'}xy^{'} + w^{'}xy^{'} + w^{'}xz^{'}$$
$$O_4 = vwxz + v^{'}w^{'}x^{'}z^{'} + xy^{'}z + wyz + vxy + v^{'}x^{'}y + w^{'}yz^{'} + x^{'}yz^{'}$$
$$O_5 = vwxy + v^{'}w^{'}x^{'}y^{'} + v^{'}x^{'}y + v^{'}y^{'}z + w^{'}y^{'}z + wyz + vyz + vxz$$

- Realization of OR: In $O_1$, Set $y = 0$, $v = 0$, $z = 1$ then $O_1 = x + w$
- Realization of NAND: In $O_1$, Set $v = 1$, $y = 0$, $z = 0$ then $O_1 = w^{'} + x^{'}$

### 3.2.61 Rule 130

In 3-input 3-output rule 130 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz + x^{'}y^{'}z$$
$$O_2 = xyz + xy^{'}z^{'}$$
$$O_3 = xyz + x^{'}yz^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 1$, $z = 1$ then $O_1 = y$
- Realization of XNOR: In $O_1$, set $y = 1$ then $O_1 = xz + x^{'}z^{'}$
- Realization of NOT: In $O_1$, set $x = 0$, $y = 1$ then $O_1 = z^{'}$

In 5-input 5-output rule 130 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = vwxy + v^{'}w^{'}x$$
$$O_2 = wxyz + w^{'}x^{'}y$$
$$O_3 = vxyz + x^{'}y^{'}z$$
$$O_4 = vwyz + vy^{'}z^{'}$$
$$O_5 = vwxz + v^{'}wz^{'}$$

- Realization of AND: In $O_5$, set $v = 1$, $w = 1$ then $O_5 = xy$
- Realization of NOR: In $O_5$, set $x = 1$, $y = 0$ then $O_5 = v^{'}w^{'}$

### 3.2.62 Rule 131

In 3-input 3-output rule 131 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz + x^{'}z^{'}$$
$$O_2 = xyz + x^{'}y^{'}$$
$$O_3 = xyz + y^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 0$, $z = 0$ then $O_1 = x^{'}$
- Realization of AND: In $O_1$, Set $z = 1$ then $O_1 = xy$
- Realization of NOR: In $O_1$, Set $y = 0$ then $O_1 = x^{'}z^{'}$

- Realization of XNOR: In $O_1$, Set $y = 1$ then $O_1 = xz + x^{'}z^{'}$

In 3-input 3-output rule 131 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xy + x^{'}y^{'} + z$$
$$O_2 = yz + y^{'}z^{'} + x$$
$$O_3 = xz + x^{'}z^{'} + y$$

- Realization of OR: In $O_1$, Set $y = 1$ then $O_1 = x + z$

In 4-input 4-output rule 131 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wxy + x^{'}y^{'}z + w^{'}x^{'}y^{'} + wyz^{'} + w^{'}z$$
$$O_2 = wyz + wy^{'}z^{'} + x^{'}y^{'}z^{'} + w^{'}xz + wx^{'}$$
$$O_3 = wxz + w^{'}xz^{'} + w^{'}y^{'}z^{'} + wx^{'}y + xy^{'}$$
$$O_4 = wxy + w^{'}x^{'}y + w^{'}x^{'}z^{'} + xy^{'}z + yz^{'}$$

- Realization of NAND: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x^{'} + w^{'}$
- Realization of XNOR: In $O_1$, Set $y = 1$, $x = 0$ then $O_1 = wz^{'} + w^{'}z$

### 3.2.63 Rule 132

In 3-input 3-output rule 132 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz + xy^{'}z^{'}$$
$$O_2 = xyz + x^{'}yz^{'}$$
$$O_3 = xyz + x^{'}y^{'}z$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 1$, $z = 0$ then $O_1 = y^{'}$
- Realization of AND gate: In $O_1$, set $z = 1$ then $O_1 = xy$
- Realization of XNOR gate: In $O_1$, set $x = 1$ then $O_1 = yz + y^{'}z^{'}$

### 3.2.64 Rule 133

In 3-input 3-output rule 133 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz + y^{'}z^{'}$$
$$O_2 = xyz + x^{'}z^{'}$$
$$O_3 = xyz + x^{'}y^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 0$, $x = 0$ then $O_1 = z^{'}$
- Realization of AND: In $O_1$, Set $z = 1$ then $O_1 = xy$
- Realization of NOR: In $O_1$, Set $x = 0$ then $O_1 = y^{'}z^{'}$
- Realization of XNOR: In $O_1$, Set $x = 1$ then $O_1 = yz + y^{'}z^{'}$

In 3-input 3-output rule 133 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = yz + y^{'}z^{'} + x$$
$$O_2 = xz + x^{'}z^{'} + y$$
$$O_3 = xy + x^{'}y^{'} + z$$

- Realization of OR: In $O_1$, Set $z = 1$ then $O_1 = x + y$

In 4-input 4-output rule 133 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wxz + w^{'}y + wy^{'} + x^{'}z^{'}$$
$$O_2 = wxy + x^{'}z + xz^{'} + w^{'}y^{'}$$
$$O_3 = wxz + w^{'}y + wy^{'} + x^{'}z^{'}$$
$$O_4 = wxy + x^{'}z + xz^{'} + w^{'}y^{'}$$

- Realization of XOR: In $O_1$, Set $x = 0$, $z = 1$ then $O_1 = w^{'}y + wy^{'}$
- Realization of NAND: In $O_1$, Set $y = 1$, $z = 0$ then $O_1 = x^{'} + w^{'}$

### 3.2.65 Rule 134

In 3-input 3-output rule 134 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz + x^{'}yz^{'} + xy^{'}z^{'}$$
$$O_2 = xyz + x^{'}y^{'}z + x^{'}yz^{'}$$
$$O_3 = xyz + x^{'}y^{'}z + xy^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 1$, $z = 0$ then $O_1 = x^{'}$
- Realization of XNOR: In $O_1$, Set $y = 1$ then $O_1 = xz + x^{'}z^{'}$
- Realization of XOR: In $O_1$, Set $z = 0$ then $O_1 = x^{'}y + xy^{'}$
- Realization of AND: In $O_1$, Set $z = 1$ then $O_1 = xy$

In 4-input 4-output rule 134 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wxz + w^{'}x^{'}y + yz^{'}$$
$$O_2 = wxy + x^{'}y^{'}z + w^{'}z$$
$$O_3 = xyz + wy^{'}z^{'} + wx^{'}$$
$$O_1 = wyz + w^{'}xz^{'} + xy^{'}$$

- Realization of NAND: In $O_1$, Set $w = 0$, $y = 1$ then $O_1 = x^{'} + z^{'}$
- Realization of NOR: In $O_1$, Set $y = 1$, $z = 0$ then $O_1 = w^{'}x^{'}$

### 3.2.66 Rule 136

In 3-input 3-output rule 136 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy$$
$$O_2 = yz$$
$$O_3 = xz$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$ then $O_1 = x$
- Realization of AND: In $O_1$, Set $z = 1$ or $z = 0$ then $O_1 = xy$

### 3.2.67 Rule 137

In 3-input 3-output rule 137 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z^{'} + xy$$
$$O_2 = x^{'}y^{'}z^{'} + yz$$
$$O_3 = x^{'}y^{'}z^{'} + xz$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 0$, $z = 0$ then $O_1 = x^{'}$
- Realization of XNOR: In $O_1$, Set $z = 0$ then $O_1 = xy + x^{'}y^{'}$
- Realization of AND: In $O_1$, Set $z = 1$ then $O_1 = xy$

In 3-input 3-output rule 137 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xyz + x^{'}y^{'} + x^{'}z^{'} + y^{'}z^{'}$$
$$O_1 = xyz + x^{'}y^{'} + x^{'}z^{'} + y^{'}z^{'}$$
$$O_1 = xyz + x^{'}y^{'} + x^{'}z^{'} + y^{'}z^{'}$$

- Realization of NAND: In $O_1$, Set $z = 0$ then $O_1 = x^{'} + y^{'}$

In 4-input 4-output rule 137 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}x^{'}yz + w^{'}xyz^{'} + wxy + wx^{'}z^{'} + x^{'}y^{'}z^{'}$$
$$O_2 = wx^{'}y^{'}z + wx^{'}yz^{'} + xyz + w^{'}xy^{'} + w^{'}y^{'}z^{'}$$
$$O_1 = w^{'}xy^{'}y + wxy^{'}z^{'} + wyz + x^{'}yz^{'} + w^{'}x^{'}z^{'}$$
$$O_1 = w^{'}xyz^{'} + wx^{'}yz^{'} + wxz + w^{'}y^{'}z + w^{'}x^{'}y^{'}$$

- Realization of XOR: In $O_1$, Set $x = 0$, $y = 1$ then $O_1 = w^{'}z + wz^{'}$

In 5-input 5-output rule 137 block: After $3^{rd}$ evolution get the following functions:

$O_1 = v^{'}w^{'}xy^{'} + vw^{'}x^{'}y^{'}z + v^{'}w^{'}xyz^{'} + v^{'}wxy^{'}z^{'} + v^{'}w^{'}xy^{'}z^{'} + v^{'}w^{'}x^{'}y^{'}z^{'} + vxyz + v^{'}x^{'}yz + v^{'}wx^{'}z + vwy^{'}z + wx^{'}yz^{'} + vx^{'}yz^{'} + vwx^{'}z^{'}$

$O_2 = v^{'}w^{'}x^{'}yz + v^{'}w^{'}xyz^{'} + v^{'}wx^{'}yz^{'} + vw^{'}x^{'}yz^{'} + vwx^{'}y^{'}z^{'} + v^{'}w^{'}x^{'}y^{'}z^{'} + vwyz + v^{'}wxz + v^{'}wy^{'}z + v^{'}wxy^{'} + w^{'}xy^{'}z + vw^{'}y^{'}z + vw^{'}xy^{'}$

$O_3 = v^{'}w^{'}x^{'}yz + v^{'}w^{'}xy^{'}z + v^{'}wx^{'}y^{'}z + vw^{'}x^{'}y^{'}z + v^{'}wxy^{'}z^{'} + v^{'}w^{'}x^{'}y^{'}z^{'} + vwxz + vw^{'}xy + w^{'}xyz^{'} + vw^{'}xz^{'} + wx^{'}yz^{'} + vx^{'}yz^{'} + vwx^{'}z^{'}$

$O_4 = vw^{'}x^{'}y^{'}z + v^{'}w^{'}xyz^{'} + vw^{'}x^{'}yz^{'} + vw^{'}xy^{'}z^{'} + v^{'}w^{'}x^{'}y^{'}z^{'} + vwxy + wx^{'}yz + v^{'}x^{'}yz + v^{'}wx^{'}y + v^{'}xy^{'}z + v^{'}wy^{'}z + v^{'}wxz^{'}$

$O_5 = v^{'}w^{'}x^{'}yz + v^{'}wx^{'}y^{'}z + v^{'}wx^{'}yz^{'} + v^{'}wxy^{'}z^{'} + vwx^{'}y^{'}z^{'} + v^{'}w^{'}x^{'}y^{'}z^{'} + wxyz + vxy^{'}z + w^{'}xy^{'}z + vw^{'}y^{'}z + w^{'}xyz^{'} + vw^{'}yz^{'} + vw^{'}xz^{'}$

- Realization of NOR: In $O_1$, Set $x = 1$, $y = 0$, $z = 1$ then $O_1 = v^{'}w^{'}$

### 3.2.68 Rule 138

In 3-input 3-output rule 138 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy + yz^{'}$$
$$O_2 = yz + x^{'}z$$
$$O_3 = xz + xy^{'}$$

- Realization of FAN-OUT: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 0$, $y = 1$ then $O_1 = z^{'}$
- Realization of AND gate: In $O_1$, set $z = 1$ then $O_1 = xy$

### 3.2.69 Rule 140

In 3-input 3-output rule 138 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy + xz^{'}$$
$$O_2 = yz + x^{'}y$$
$$O_3 = xz + y^{'}z$$

- Realization of FAN-OUT: In $O_1$, set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 1$, $y = 0$ then $O_1 = z^{'}$
- Realization of AND gate: In $O_1$, set $z = 1$ then $O_1 = xy$

### 3.2.70 Rule 142

In 3-input 3-output rule 142 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy + yz^{'} + xz^{'}$$
$$O_2 = yz + x^{'}z + x^{'}y$$
$$O_3 = xz + xy^{'} + y^{'}z$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $x = 0$, $y = 1$ then $O_1 = z^{'}$
- Realization of OR: In $O_1$, Set $z = 0$ then $O_1 = x + y$
- Realization of AND: In $O_1$, Set $z = 1$ then $O_1 = xy$

In 4-input 4-output rule 142 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}xz^{'} + xy + w^{'}y + yz^{'}$$
$$O_2 = w^{'}x^{'}y + yz + w^{'}z + x^{'}z$$
$$O_3 = x^{'}y^{'}z + wz + wx^{'} + wy^{'}$$
$$O_4 = wy^{'}z^{'} + wx + xy^{'} + xz^{'}$$

- Realization of NAND: In $O_1$, Set $x = 0$, $y = 1$ then $O_1 = w^{'} + z^{'}$
- Realization of NOR: In $O_1$, Set $x = 1$, $y = 0$ then $O_1 = w^{'}z^{'}$

### 3.2.71 Rule 146

In 3-input 3-output rule 146 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz + x^{'}y^{'}z + x^{'}yz^{'}$$
$$O_2 = xyz + x^{'}y^{'}z + xy^{'}z^{'}$$
$$O_3 = xyz + x^{'}yz^{'} + xy^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set y=1,z=1 then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of XNOR: In $O_1$, Set $z = 1$ then $O_1 = xy + x^{'}y^{'}$
- Realization of AND: In $O_1$, Set $x = 1$ then $O_1 = yz$
- Realization of XOR: In $O_1$, Set $x = 0$ then $O_1 = y^{'}z + yz^{'}$

In 5-input 5-output rule 146 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = vw^{'}x^{'}y^{'}z + vwx^{'}y^{'}z^{'} + vxyz + vwxy + w^{'}x^{'}yz^{'} + w^{'}xy^{'}z^{'}$$
$$O_2 = v^{'}wxy^{'}z^{'} + vwx^{'}y^{'}z^{'} + wxyz + vwyz + v^{'}x^{'}y^{'}z + v^{'}x^{'}yz^{'}$$
$$O_3 = v^{'}w^{'}xyz^{'} + v^{'}wxy^{'}z^{'} + vxyz + vwxz + v^{'}w^{'}y^{'}z + vw^{'}y^{'}z^{'}$$
$$O_4 = v^{'}w^{'}x^{'}yz + v^{'}w^{'}xyz^{'} + vwyz + vwxy + v^{'}wx^{'}z^{'} + vw^{'}x^{'}z^{'}$$
$$O_5 = v^{'}w^{'}x^{'}yz + vw^{'}x^{'}y^{'}z + wxyz + vwxz + v^{'}w^{'}xy^{'} + v^{'}wx^{'}y^{'}$$

- Realization of NOR: In $O_1$, Set $v = 1$, $y = 0$, $z = 1$ then $O_1 = w^{'}x^{'}$

In 5-input 5-output rule 146 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = vwyz + vwxz + v^{'}w^{'}x^{'}z + v^{'}w^{'}y^{'}z + v^{'}wx^{'}z^{'} + v^{'}wy^{'}z^{'}$$
$$O_2 = vwxz + vwxy + v^{'}w^{'}xy^{'} + vw^{'}x^{'}y^{'} + v^{'}w^{'}xz^{'} + vw^{'}x^{'}z^{'}$$
$$O_3 = wxyz + vwxy + v^{'}w^{'}x^{'}y + v^{'}wx^{'}y^{'} + w^{'}x^{'}yz^{'} + wx^{'}y^{'}z^{'}$$
$$O_4 = wxyz + vxyz + v^{'}x^{'}y^{'}z + w^{'}x^{'}y^{'}z + v^{'}xy^{'}z^{'} + w^{'}xy^{'}z^{'}$$
$$O_1 = vxyz + vwyz + v^{'}w^{'}yz^{'} + v^{'}x^{'}yz^{'} + vw^{'}y^{'}z^{'} + vx^{'}y^{'}z^{'}$$

- Realization of OR: In $O_1$, Set $v = 1$, $w = 1$, $z = 1$ then $O_1 = x + y$
- Realization of NAND: In $O_1$, Set $v = 0$, $w = 1$, $z = 0$ then $O_1 = x^{'} + y^{'}$

### 3.2.72 Rule 150

In 3-input 3-output rule 150 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xyz + x^{'}y^{'}z + x^{'}yz^{'} + xy^{'}z^{'}$$
$$O_1 = xyz + x^{'}y^{'}z + x^{'}yz^{'} + xy^{'}z^{'}$$
$$O_1 = xyz + x^{'}y^{'}z + x^{'}yz^{'} + xy^{'}z^{'}$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of NOT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of XNOR: In $O_1$, Set $z = 1$ then $O_1 = xy + x^{'}y^{'}$
- Realization of AND: In $O_1$, Set $x = 1$ then $O_1 = yz$
- Realization of XOR: In $O_1$, Set $z = 0$ then $O_1 = x^{'}y + xy^{'}$

### 3.2.73 Rule 152

In 3-input 3-output rule 152 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z + xy$$
$$O_2 = xy^{'}z^{'} + yz$$
$$O_3 = x^{'}yz^{'} + xz$$

- Realization of FAN-OUT: In $O_1$, set $x = 1$, $z = 0$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $x = 0$, $z = 1$ then $O_1 = y^{'}$
- Realization of AND gate: In $O_1$, set $z = 0$ then $O_1 = xy$
- Realization of XNOR gate: In $O_1$, set $z = 1$ then $O_1 = x^{'}y^{'} + xy$

In 4-input 4-output rule 152 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}x^{'}yz^{'} + wyz + wxy$$
$$O_2 = w^{'}x^{'}y^{'}z + xyz + wxz$$
$$O_3 = wx^{'}y^{'}z^{'} + wyz + wxy$$
$$O_4 = w^{'}xy^{'}z^{'} + xyz + wyz$$

- Realization of OR gate: In $O_1$, set $w = 1$, $y = 1$ then $o1 = x + z$

### 3.2.74 Rule 156

In 3-input 3-output rule 156 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z + xy + xz^{'}$$
$$O_2 = xy^{'}z^{'} + yz + x^{'}y$$
$$O_3 = x^{'}yz^{'} + xz + y^{'}z$$

- Realization of NOT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 1$ then $O_1 = x$
- Realization of XOR: In $O_1$, Set $y = 0$ then $O_1 = x^{'}z + xz^{'}$
- Realization of XNOR: In $O_1$, Set $z = 1$ then $O_1 = x^{'}y^{'} + xy$

In 4-input 4-output rule 156 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wy + x^{'}y + wz^{'}$$
$$O_2 = xz + w^{'}x + y^{'}z$$
$$O_3 = wy + x^{'}y + wz^{'}$$
$$O_4 = xz + w^{'}x + y^{'}z$$

- Realization of FANOUT: In $O_1$, Set $x = 1$, $z = 1$ then $O_1 = wy$
- Realization of OR: In $O_1$, Set $x = 0$, $z = 0$ then $O_1 = w + y$

In 5-input 5-output rule 156 block: After $1^{st}$ evolution get the following functions:

$$O_1 = v^{'}w^{'}xz + v^{'}w^{'}x^{'}y + vwx^{'}y^{'} + w^{'}xy^{'}z^{'} + vxy + vz^{'}$$
$$O_2 = vw^{'}x^{'}y + w^{'}x^{'}y^{'}z + wxy^{'}z^{'} + w^{'}x^{'}yz^{'} + wyz + v^{'}w$$
$$O_3 = vwx^{'}y^{'} + v^{'}x^{'}y^{'}z + v^{'}xyz^{'} + vx^{'}y^{'}z^{'} + vxz + w^{'}x$$
$$O_4 = v^{'}w^{'}yz + v^{'}wy^{'}z^{'} + vxy^{'}z^{'} + vw^{'}y^{'}z^{'} + vwy + x^{'}y$$
$$O_5 = vw^{'}x^{'}z + v^{'}w^{'}xz^{'} + v^{'}wyz^{'} + v^{'}wx^{'}z^{'} + wxz + y^{'}z$$

- Realization of NOR: In $O_1$, Set $x = 1$, $z = 1$, $y = 0$ then $O_1 = v^{'}w^{'}$
- Realization of NAND: In $O_1$, Set $v = 1$, $w = 1$, $y = 0$ then $O_1 = x^{'} + z^{'}$

### 3.2.75 Rule 160

In 3-input 3-output rule 160 block: After $1^{st}$ evolution get the following functions:

$$O_1 = yz$$
$$O_2 = xz$$
$$O_3 = xy$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$ then $O_1 = z$
- Realization of AND: In $O_1 = yz$

### 3.2.76 Rule 161

In 3-input 3-output rule 161 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x^{'}y^{'}z^{'} + yz$$
$$O_2 = x^{'}y^{'}z^{'} + xz$$
$$O_3 = x^{'}y^{'}z^{'} + xy$$

- Realization of FAN-OUT: In $O_1$, Set $x = 1$, $z = 1$ then $O_1 = y$
- Realization of NOT: In $O_1$, Set $y = 0$, $z = 0$ then $O_1 = x^{'}$
- Realization of XNOR: In $O_1$, Set $x = 0$ then $O_1 = yz + y^{'}z^{'}$
- Realization of AND: In $O_1$, Set $x = 1$ then $O_1 = yz$
- Realization of NOR: In $O_1$, Set $z = 0$ then $O_1 = x^{'}y^{'}$

In 3-input 3-output rule 161 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xyz + x^{'}y^{'} + x^{'}z^{'} + y^{'}z^{'}$$
$$O_2 = xyz + x^{'}y^{'} + x^{'}z^{'} + y^{'}z^{'}$$
$$O_3 = xyz + x^{'}y^{'} + x^{'}z^{'} + y^{'}z^{'}$$

- Realization of NAND: In $O_1$, Set $z = 0$ then $O_1 = x^{'} + y^{'}$

In 4-input 4-output rule 161 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}yz + xyz^{'} + x^{'}y^{'}z^{'} + wy + wx^{'} + wz^{'}$$
$$O_2 = w^{'}yz + wy^{'}z + w^{'}y^{'}z^{'} + xz + w^{'}x + xy^{'}$$
$$O_3 = wx^{'}z + wxz^{'} + w^{'}x^{'}z^{'} + wy + x^{'}y + yz^{'}$$
$$O_4 = w^{'}xy + wxy^{'} + w^{'}x^{'}y^{'} + xz + w^{'}z + yz^{'}$$

- Realization of XOR: In $O_1$, Set $w = 0$, $y = 1$ then $O_1 = x^{'}z + xz^{'}$

### 3.2.77 Rule 162

In 3-input 3-output rule 162 block: After $1^{st}$ evolution get the following functions:

$$O_1 = yz + x^{'}y$$
$$O_2 = xz + y^{'}z$$
$$O_3 = xz + xz^{'}$$

- Realization of FAN-OUT: In $O_1$, set $x = 1$, $z = 1$ then $O_1 = y$

- Realization of NOT gate: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x^{'}$
- Realization of AND gate: In $O_1$, set $x = 0$ then $O_1 = yz$

In 5-input 5-output rule 162 block: After $1^{st}$ evolution get the following functions:

$$O_1 = vxy + vxz^{'} + w^{'}x$$
$$O_2 = wyz + v^{'}wy + x^{'}y$$
$$O_3 = vxz + w^{'}xz + y^{'}z$$
$$O_4 = vwy + vx^{'}y + vz^{'}$$
$$O_5 = wxz + wy^{'}z + v^{'}w$$

- Realization of NAND gate: In $O_1$, set $v = 1$, $x = 1$, $y = 0$ then $O_1 = z^{'} + w^{'}$

### 3.2.78 Rule 164

In 3-input 3-output rule 164 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy^{'}z^{'} + yz$$
$$O_2 = x^{'}yz^{'} + xz$$
$$O_3 = x^{'}y^{'}z + xy$$

- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $x = 1$, $z = 0$ then $O_1 = y^{'}$
- Realization of XNOR gate: In $O_1$, set $x = 1$ then $O_1 = yz + y^{'}z^{'}$

### 3.2.79 Rule 168

In 3-input 3-output rule 168 block: After $1^{st}$ evolution get the following functions:

$$O_1 = yz + xy$$
$$O_2 = xz + yz$$
$$O_3 = xy + xz$$

- Realization of FAN-OUT: In $O_1$, Set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of AND: In $O_1$, Set $z = 0$ then $O_1 = xy$
- Realization of OR: In $O_1$, Set $y = 1$ then $O_1 = z + x$

### 3.2.80 Rule 170

In 3-input 3-output rule 170 block: After $1^{st}$ evolution get the following functions:

$$O_1 = y$$
$$O_2 = z$$
$$O_3 = x$$

- Realization of FAN-OUT: $O_1 = y$

### 3.2.81 Rule 172

In 3-input 3-output rule 172 block: After $1^{st}$ evolution get the following functions:

$$O_1 = yz + xz^{'}$$
$$O_2 = xz + x^{'}y$$
$$O_3 = xy + y^{'}z$$

- Realization of AND gate: In $O_1$, set $x = 0$ then $O_1 = yz$
- Realization of FAN-OUT: In $O_1$, set $z = 1$, $x = 0$ then $O_1 = y$
- Realization of NOT gate: In $O_1$, set $x = 1$, $y = 0$ then $O_1 = z^{'}$

In 4-input 4-output rule 172 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = w^{'}xz + wy + wz^{'}$$
$$O_2 = wx^{'}y + xz + w^{'}x$$
$$O_3 = xy^{'}z + wy + x^{'}y$$
$$O_4 = wyz^{'} + xz + y^{'}z$$

- Realization of OR gate: In $O_1$, set $w = 1$, $z = 0$ then $O_1 = y + w$
- Realization of XOR gate: In $O_1$, set $x = 1$, $y = 0$ then $O_1 = w^{'}z + wz^{'}$

In 4-input 4-output rule 172 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = wx^{'}y + wy^{'}z^{'} + xz$$
$$O_2 = xy^{'}z + w^{'}xz^{'} + wy$$
$$O_3 = w^{'}x^{'}y + wyz^{'} + xz$$
$$O_4 = w^{'}xz + x^{'}y^{'}z + wy$$

- Realization of NOR gate: In $O_1$, set $y = 0$, $z = 0$ then $O_1 = y^{'}z^{'}$

In 5-input 5-output rule 172 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = vwx^{'}z^{'} + wyz + wxz + vxz + vy^{'}z^{'}$$
$$O_2 = v^{'}wxy^{'} + vxz + vxy + vwy + v^{'}wz^{'}$$
$$O_3 = w^{'}xyz^{'} + wyz + wxz + vwy + v^{'}w^{'}x$$
$$O_4 = v^{'}x^{'}yz + wxz + vxz + vxy + w^{'}x^{'}y$$
$$O_5 = vw^{'}y^{'}z + wyz + vxy + vwy + x^{'}y^{'}z$$

- Realization of NAND gate: In $O_1$, set $w = 1$, $v = 1$, $z = 0$ then $O_1 = x^{'} + y^{'}$

In 5-input 5-output rule 172 block: After $3^{rd}$ evolution get the following functions:

$$O_1 = wyz + vw^{'}x + vwz^{'} + vy^{'}z^{'}$$
$$O_2 = vxz + v^{'}wx + wx^{'}y + v^{'}wz^{'}$$
$$O_3 = vwy + w^{'}xy + v^{'}w^{'}x + xy^{'}z$$
$$O_4 = wxz + x^{'}yz + w^{'}x^{'}y + vyz^{'}$$
$$O_5 = vxy + v^{'}wz + vy^{'}z + x^{'}y^{'}z$$

- Realization of XNOR gate: In $O_1$, set $x = 0$, $w = 0$, $v = 1$ then $O_1 = y^{'}z^{'} + yz$

### 3.2.82 Rule 178

In 3-input 3-output rule 178 block: After $1^{st}$ evolution get the following functions:

$$O_1 = yz + x^{'}z + x^{'}y$$
$$O_2 = xz + y^{'}z + xy^{'}$$
$$O_3 = xy + yz^{'} + xz^{'}$$

- Realization of NOT: In $O_1$, Set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of FAN-OUT: In $O_1$, Set $x = 1$, $z = 1$ then $O_1 = y$
- Realization of NOR: In $O_1$, Set $x = 0$ then $O_1 = z + y$
- Realization of AND: In $O_1$, Set $x = 1$ then $O_1 = yz$

In 4-input 4-output rule 178 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = x^{'}yz^{'} + wy + wx^{'} + wz^{'}$$
$$O_2 = w^{'}y^{'}z + xz + w^{'}x + xy^{'}$$
$$O_3 = wx^{'}z^{'} + wy + x^{'}y + yz^{'}$$
$$O_1 = w^{'}xy^{'} + xz + w^{'}z + y^{'}z$$

- Realization of NOR: In $O_1$, Set $y = 1$, $w = 0$ then $O_1 = x^{'}z^{'}$
- Realization of NAND: In $O_1$, Set $w = 1$, $y = 0$ then $O_1 = x^{'} + z^{'}$

### 3.2.83 Rule 184

In 3-input 3-output rule 184 block: After $1^{st}$ evolution get the following functions:

$$O_1 = xy + x^{'}z$$
$$O_2 = yz + xy^{'}$$
$$O_3 = xz + yz^{'}$$

- Realization of FAN-OUT: In $O_1$,set $y = 1$, $z = 0$ then $O_1 = x$
- Realization of NOT gate: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x^{'}$
- Realization of AND gate: In $O_1$, set $z = 0$ then $O_1 = xy$
- Realization of OR gate: In $O_1$, set $z = 1$ then $O_1 = x + y$

In 5-input 5-output rule 184 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = wxz + vwx + vw^{'}z + v^{'}yz^{'} + w^{'}yz^{'}$$
$$O_2 = wxy + vxy + v^{'}w^{'}z + vwx^{'} + v^{'}x^{'}z$$
$$O_3 = xyz + wyz + vw^{'}x^{'} + wxy^{'} + vw^{'}y^{'}$$
$$O_4 = vyz + vxz + wx^{'}y^{'} + xyz^{'} + wx^{'}z^{'}$$
$$O_5 = vwz + vwy + v^{'}yz + v^{'}xy^{'} + xy^{'}z^{'}$$

- Realization of NAND gate: In $O_1$, set $v = 1$, $w = 0$, $z = 0$ then $O_1 = x^{'} + y^{'}$
- Realization of NOR gate: In $O_1$, set $v = 0$, $w = 1$, $z = 1$ then $O_1 = x^{'}y^{'}$

In 5-input 5-output rule 184 block: After $4^{th}$ evolution get the following functions:

$$O_1 = v^{'}w^{'}xy + vwy^{'}z^{'} + vyz + vxz + v^{'}wz + v^{'}wx^{'}$$
$$O_2 = w^{'}x^{'}yz + v^{'}wxz^{'} + vwz + vwy + vw^{'}x + w^{'}xy^{'}$$
$$O_3 = v^{'}w^{'}xy + vx^{'}y^{'}z + wxz + vwx + wx^{'}y + x^{'}yz^{'}$$
$$O_4 = w^{'}x^{'}yz + vwy^{'}z^{'} + wxy + vxy + xy^{'}z + v^{'}y^{'}z$$
$$O_5 = vx^{'}y^{'}z + v^{'}wxz^{'} + xyz + wyz + vyz^{'} + vw^{'}z^{'}$$

- Realization of XOR gate: In $O_1$, set $v = 0$, $w = 0$, $z = 1$ then $O_1 = x^{'}y + xy^{'}$
- Realization of XNOR gate: In $O_1$, set $v = 1$, $w = 0$, $z = 1$ then $O_1 = xy + x^{'}y^{'}$

### 3.2.84 Rule 200

In 3-input 3-output rule 200 block: After $1^{st}$ evolution get the following functions:

$$O1 = xz + xy$$
$$O2 = yz + xy$$
$$O3 = yz + xz$$

- Realization of AND: In $O_1$, set $z = 0$ then $O_1 = xy$
- Realization of OR: In $O_1$, set $x = 1$ then $O_1 = z + y$
- Realization of FAN-OUT: In $O_1$, set $y = 0$, $z = 1$ then $O_1 = x$

### 3.2.85 Rule 204

In 3-input 3-output rule 204 block: After $1^{st}$ evolution get the following functions:

$$O_1 = x$$
$$O_2 = y$$
$$O_3 = z$$

- Realization of FAN-OUT: $O_1 = x$

### 3.2.86 Rule 232

In 3-input 3-output rule 232 block: After $1^{st}$ evolution get the following functions:

$$O_1 = yz + xz + xy$$
$$O_2 = yz + xz + xy$$
$$O_3 = yz + xy + xz$$

- Realization of AND: In $O_1$, set $z = 0$ then $O_1 = xy$
- Realization of FAN-OUT: In $O_1$, set $y = 1$, $z = 0$ then $O_1 = x$

In 4-input 4-output rule 232 block: After $2^{nd}$ evolution get the following functions:

$$O_1 = xyz + wz + wy + wx$$
$$O_2 = wyz + xz + xy + wx$$
$$O_3 = wxz + yz + xy + wy$$
$$O_4 = wxy + yz + xz + wz$$

- Realization of OR: In $O_1$, set $w = 1$, $z = 0$ then $O_1 = x + y$

## 3.3 Combinational Logic Design Using Cellular Automata

In this section we design combinational logic circuit like multiplexer, half ADDER using cellular automata. To realize multiplexer and half ADDER logic, like earlier we set some input bit and then evolve the CA. After some evolution we get the desire output.

For example, 3-input 3-output 46 block of Figure 3.4 work as a multiplexer. If we consider input X and Z of this block as a data input of a 2-input multiplexer and input Y as select line, then output $O_1$ of the block works as desired output of the multiplexer.

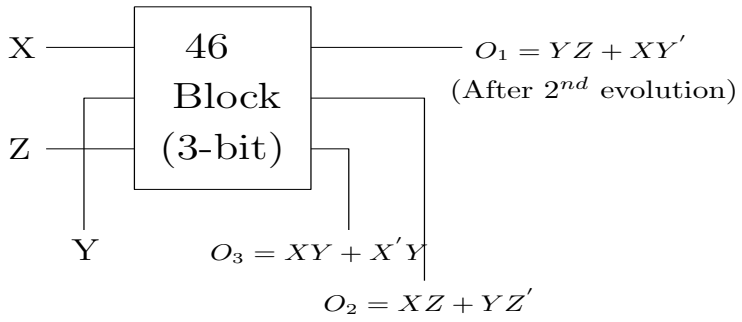


**Figure 3.4:** Multiplexer using 46 block

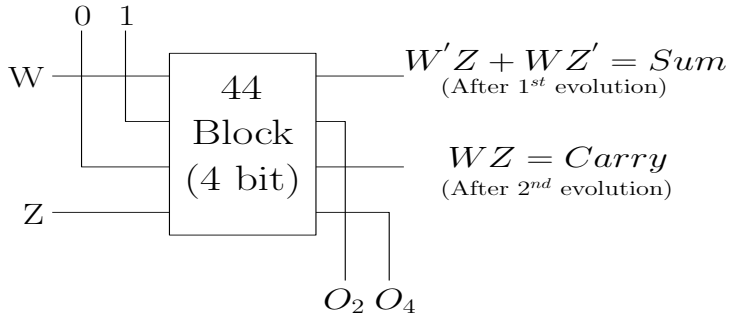


**Figure 3.5:** Implementation of Half Adder using 44 block

# Bibliography