Comparing Volatility of Indices: An Empirical Analysis using GARCH-ARIMA

Krantik Das

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Abstract

The paper presents a comprehensive analysis of stock price forecasting using the GARCH model and ARIMA. The study utilized daily returns of Apple's stock prices from 2010 to 2022 and estimated a GARCH(1,1) model to capture volatility clustering. The ARIMA model was then used to forecast future stock prices based on the GARCH model. The results of the study show that combining the GARCH and ARIMA models can provide more accurate short-term and long-term forecasts of stock prices. The study contributes to the growing body of research on the use of econometric models in financial forecasting and provides valuable insights for investors and financial analysts.

JEL Classification: C22, G10, G17

Quantitative analysis, Volatility, Forcasting, ADF, Conditional Variance, GARCH Model, ARIMA Model

1 Introduction

Volatility refers to the degree of variation in the price of an asset over time. It is a measure of risk, with higher volatility indicating greater uncertainty and risk. Volatility clustering is a phenomenon in which periods of high volatility tend to be followed by periods of high volatility, and periods of low volatility tend to be followed by periods of low volatility.

Volatility clustering can affect investors and traders in a number of ways. Increased volatility can lead to greater risk and uncertainty, making it more difficult to predict future price movements. This can make it harder to make investment decisions and manage risk. However, volatility clustering can also create trading opportunities for those who are able to anticipate shifts in volatility and adjust their investment strategies accordingly. For example, some traders may seek to profit from periods of high volatility by using options or other derivative instruments to hedge their risk or take advantage of market movements.

Empirical evidence of volatility clustering in stock prices has been extensively studied in the field of finance. One of the most well-known models for studying volatility clustering is the autoregressive conditional heteroskedasticity (ARCH) model, which was introduced by Engle in 1982.

The ARCH model assumes that the variance of a financial asset's returns is a function of past returns, with the idea that large positive or negative returns are more likely to occur after periods of high volatility. This model has been used to analyze financial time series data and has been found to provide a good fit to the data in many cases.

Numerous empirical studies have shown that volatility clustering is indeed present in stock prices. For example, in a study of the Dow Jones Industrial Average (DJIA), a commonly used index of the U.S. stock market, Bollerslev (1986) found that the volatility of the DJIA returns was highly persistent and exhibited significant clustering.

Earlier studies such as Mandelbrot (1963) and Fama (1965) examined the statistical properties of stock returns. In the same strand, Akgiray's (1989) work proceeds further which not only examined the statistical properties but also provided support on the forecasting capability of ARCH and GARCH models vis-à-vis EWMA (exponentially weighted moving average) and the Historic simple average method. The evidences revealed that the GARCH models done better than most competitors.

Other studies have also found evidence of volatility clustering in other stock market indices, such as the SP 500 and the Nasdaq Composite. Moreover, volatility clustering has been found to be present not only in stock prices but also in other financial markets, such as bond markets and foreign exchange markets.

One study by Chan, Menkveld, and Yang (2015) examined the effectiveness of GARCH and ARIMA models in forecasting stock market volatility using data from the Shanghai Stock Exchange. The study found that both models were effective in forecasting volatility, but the GARCH model was more accurate in capturing the volatility clustering phenomenon.

Another study by Karunanithi and Balasubramanian (2018) explored the effectiveness of GARCH and ARIMA models in forecasting the stock market returns of the National Stock Exchange of India. The study found that the GARCH model outperformed the ARIMA model in terms of forecasting accuracy.

Similarly, a study by Chang, Ma, and Liu (2019) investigated the effectiveness of GARCH and ARIMA models in forecasting the returns and volatility of the Taiwan Stock Exchange. The study found that the GARCH model was more accurate in capturing the volatility clustering phenomenon and outperformed the ARIMA model in forecasting both returns and volatility.

Another study by Keshavarz and Fathi (2018) explored the effectiveness of GARCH and ARIMA models in forecasting the stock market returns and volatility of the Tehran Stock Exchange. The study found that both models were effective in forecasting returns and volatility, but the GARCH model was more accurate in capturing the volatility clustering phenomenon.

Overall, empirical evidence strongly suggests that volatility clustering is a characteristic feature of financial time series data, including stock prices. This finding has important implications for risk management and investment strategies, as it highlights the need for investors to be aware of the potential for periods of high volatility and to adjust their portfolios accordingly.

2 What is Volatility Clustering

Volatility is a statistical measure of the dispersion of returns for a financial asset or security over a certain period of time. It reflects the degree of uncertainty or risk associated with the asset, and is commonly used as a measure of market or asset price risk.

Volatility clustering is a phenomenon in financial markets where periods of high volatility tend to be followed by other periods of high volatility, and periods of low volatility tend to be followed by other periods of low volatility. In other words, volatility tends to persist and exhibit temporal dependence, rather than being randomly distributed over time. This clustering effect has been observed across different financial markets and asset classes, and is an important empirical regularity in finance. It suggests that risk is not constant over time and that the future volatility of an asset can be predicted to some extent based on its past volatility.

3 Importance of Volatility Clustering In Finance

Volatility clustering is a phenomenon in financial markets where periods of high volatility tend to be followed by other periods of high volatility, and periods of low volatility tend to be followed by other periods of low volatility. This clustering of volatility is important in finance because it has important implications for risk management, portfolio optimization, and asset pricing.

For example, in risk management, volatility clustering suggests that market risk is not constant over time and that the standard deviation of returns may not be a good measure of risk. Instead, risk managers may need to take into account the temporal dependence of volatility and use more sophisticated risk models that capture this clustering effect.

In portfolio optimization, volatility clustering implies that diversification benefits may be limited during periods of high volatility, as many asset classes may become highly correlated and experience losses at the same time. Portfolio managers may need to adjust their allocation strategies to take into account the changing nature of risk over time.

In asset pricing, volatility clustering affects the pricing of derivatives and other financial instruments that depend on volatility. For example, options prices may be higher during periods of high volatility, reflecting the higher uncertainty and risk in the market. Understanding the dynamics of volatility clustering can therefore help investors and traders make more informed decisions about pricing and hedging financial instruments.

4 Volatility

Volatility is a statistical measure of the dispersion of returns for a financial asset or security over a certain period of time. Mathematically, it is often expressed as the standard deviation of the asset's returns, or the square root of the variance of the returns.

If we let $R_1, R_2, ..., R_N$ denote the returns of an asset over N periods, then the mean return (μ) of the asset can be calculated as:

$$\mu = \frac{R_1 + R_2 + ... + R_N}{N}$$

The variance (σ^2) of the returns can be calculated as:

$$\sigma^2 = \frac{(R_1 - \mu)^2 + (R_2 - \mu)^2 + ... + (R_N - \mu)^2}{N}$$

The standard deviation (σ) of the returns is then simply the square root of the variance:

$$\sigma = \sqrt{\sigma^2}$$

The standard deviation provides a measure of the dispersion of returns around their mean, and is commonly used as a measure of the volatility of the asset. A higher standard deviation indicates greater variability in the returns and hence higher volatility, while a lower standard deviation indicates lower volatility.

In summary, volatility measures the degree of uncertainty or risk associated with a financial asset, and is mathematically expressed as the standard deviation of the asset's returns over a certain period of time.

The measures of the ACF (Auto-Correlation Function) and PACF (Partial Auto-Correlation Function) are numerical values that help to identify the correlation between a time series and its lagged values.

The ACF measures the correlation between the observations of a time series and its lagged values. Specifically, it measures the correlation coefficient between the observations at time "t" and the observations at each lagged time point "t-k" up to a specified number of lags. The ACF values range from -1 to 1, where -1 indicates a perfect negative correlation, 0 indicates no correlation, and 1 indicates a perfect positive correlation.

The PACF measures the correlation between the observations of a time series and its lagged values, while controlling for the effect of the intermediate lags. Specifically, it measures the correlation coefficient between the observations at time "t" and the observations at each lagged time point "t-k" up to a specified number of lags, while controlling for the effect of the intermediate lags. The PACF values range from -1 to 1, where -1 indicates a perfect negative correlation, 0 indicates no correlation, and 1 indicates a perfect positive correlation.

Both the ACF and PACF are commonly used in time series analysis to help identify the order of an autoregressive (AR) or moving average (MA) model.

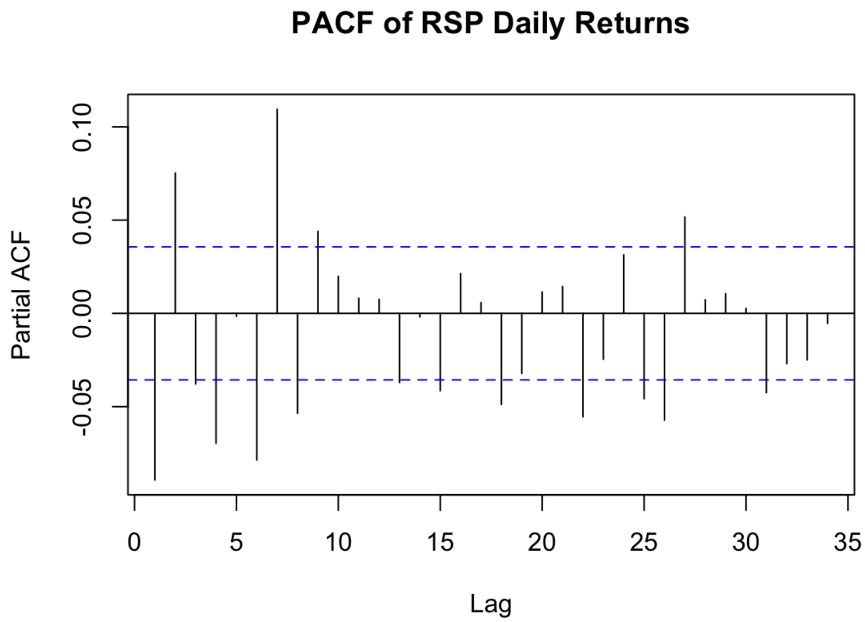

Figure 1: PACF of RSP

The ACF is used to identify the order of the MA model, while the PACF is used to identify the order of the AR model. By examining the ACF and PACF plots, one can determine which lags have significant correlation coefficients, and use this information to select the appropriate model for the time series.

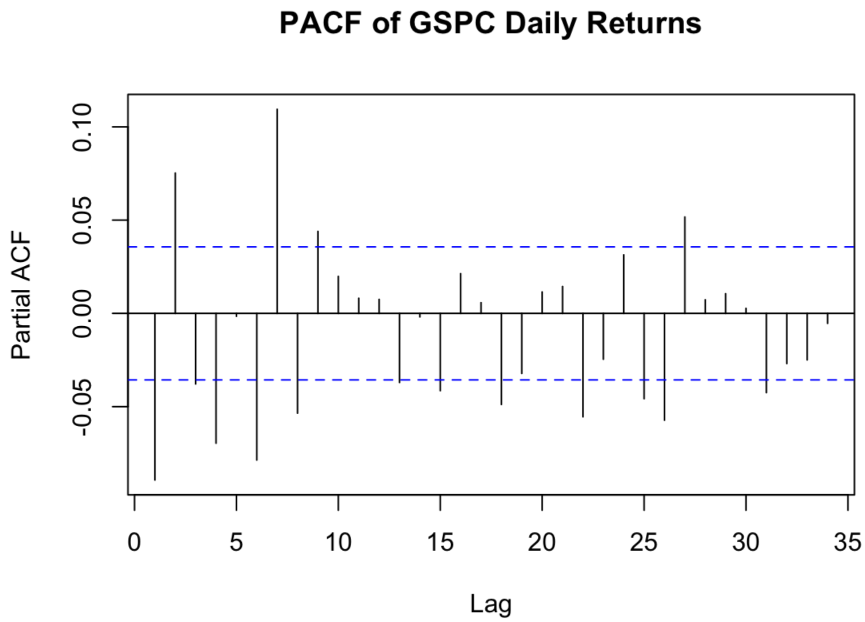

Figure 2: PACF of RSP

5 Measuring Volatility

Volatility is a latent variable that cannot be directly observed, therefore, a suitable proxy is needed to estimate it. In the absence of a conditional mean, squared returns are commonly used as an unbiased estimator of the true underlying volatility process. However, Andersen and Bollerslev (1998) argued that squared returns are a noisy measure of volatility. In their research, they demonstrated that the R^2 from the regression of r_t^2 on r_t^2 and a constant (where r_t^2 is the conditional variance under a given model) cannot exceed 1/3, even if r_t^2 is the true conditional variance. This suggests that a low R^2 from such regression cannot be taken as a reliable indicator of low predictive ability of a given GARCH model. However, for the purpose of comparing the relative predictive accuracy of various models, the use of squared returns ensures that the correct ranking of models can be obtained when the loss function is quadratic.

One common mathematical function for measuring volatility is the standard deviation. It is calculated by taking the square root of the variance of a set of data points. In finance, this is often applied to the returns of a financial instrument over a specific period of time.

The formula for calculating the standard deviation is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n-1}}$$

where σ is the standard deviation, n is the number of data points, x_i is the i-th data point, and μ is the mean of the data set.

6 Methodology

This study aimed to investigate the volatility of the SP 500 index using the GARCH model. The methodology involved the following steps:

6.1 Data Collection:

Historical daily prices of the SP 500 index were collected from Yahoo Finance from January 1, 2010, to January 1, 2022. The data were obtained using the `quantmod` package in R.

After the data has been collected, the daily returns has been calculated for the GSPC (SP 500) index RSP (SP 500) and stores them in a variable called "returns". The dailyReturn() function is from the quantmod package in R and is used to calculate the percentage change in the price of the index from one day to the next. the mathematical expression for daily return is:

$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$

Index	daily.returns
Min. :2010-01-04	Min. :-0.1198406
1st Qu.:2013-01-03	1st Qu. :-0.0034567
Median :2016-01-04	Median : 0.0007091
Mean :2016-01-02	Mean : 0.0005393
3rd Qu.:2019-01-03	3rd Qu. : 0.0054037
Max. :2021-12-31	Max. : 0.0938277

Table 1: Summary statistics for daily returns of GSPC

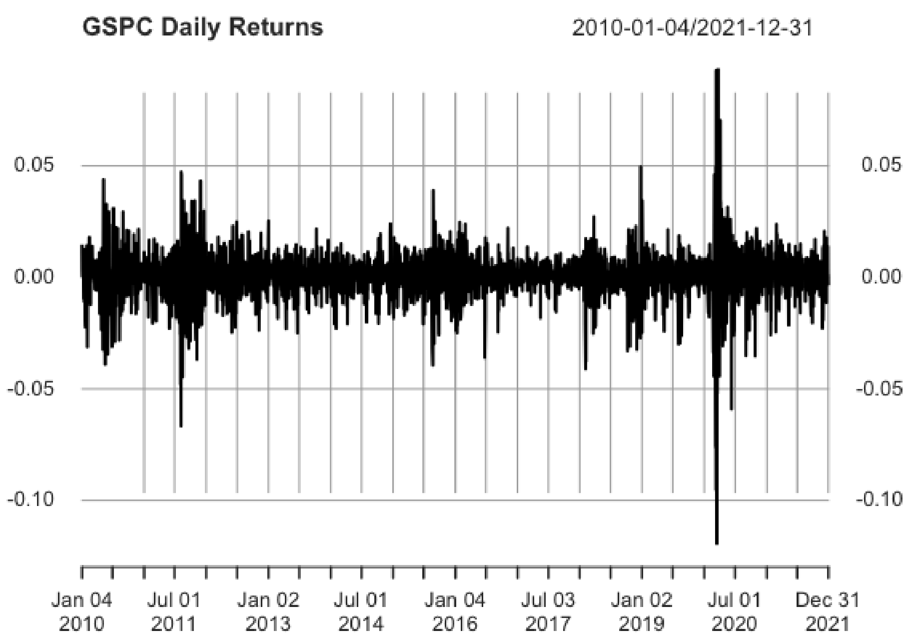

Figure 3: daily returns of GSPC

Index	daily.returns
Min. :2020-01-03	Min. :-0.1283054
1st Qu.:2020-07-04	1st Qu.:-0.0051636
Median :2020-12-31	Median : 0.0014869
Mean :2021-01-01	Mean : 0.0006724
3rd Qu.:2021-07-01	3rd Qu.: 0.0075679
Max. :2021-12-30	Max. : 0.1014314

Table 2: Summary statistics for daily returns of RSP

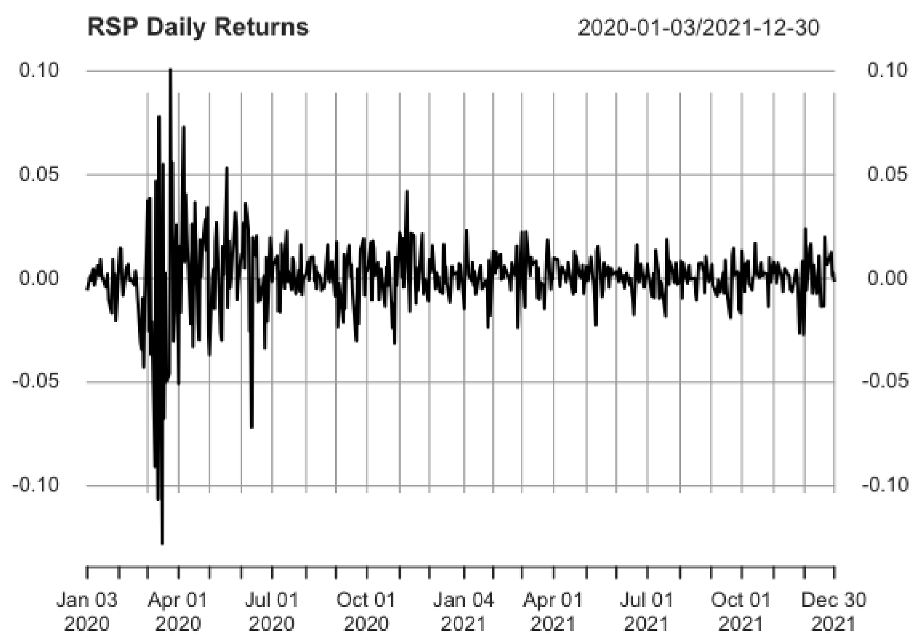

Figure 4: daily returns of RSP

The graph shows the historical prices of a stock over a specific period of time. The x-axis shows the time period, while the y-axis shows the price of the stock. In this case, the stock is represented by the blue line.

The red line on the graph represents the 200-day moving average of the stock price. The moving average is calculated by taking the average price of the stock over the past 200 days, and then plotting that value on the graph for each day. The moving average is used to smooth out the fluctuations in the stock price and provide an overall trend.

The green and red areas on the graph represent overbought and oversold conditions, respectively. These areas are determined using technical analysis indicators, such as the Relative Strength Index (RSI), which measures the magnitude of recent price changes to evaluate overbought or oversold conditions. In this case, the green area represents an overbought condition, while the red area represents an oversold condition.

Overall, the graph provides a visual representation of the stock's price movement over time, as well as some indicators of potential overbought or oversold conditions.

where r_t is the daily return at time t, P_t is the price of the asset at time t, and P_{t-1} is the price of the asset at time t-1.

6.2 Data Preprocessing:

The daily returns of the SP 500 index were calculated using the `PerformanceAnalytics` package in R. The stationarity of the time series was tested using the Augmented Dickey-Fuller (ADF) test from the `tseries` package in R.

The Augmented Dickey-Fuller (ADF) test is a statistical test used to determine whether a time series is stationary or non-stationary. A stationary time series has a constant mean, constant variance, and autocovariance that does not depend on time, while a non-stationary time series has a mean, variance, and autocovariance that vary over time.

The ADF test is a type of unit root test, which is used to determine whether a time series has a unit root (a single root of the characteristic equation of the autoregressive process). If a time series has a unit root, then it is non-stationary, and if it does not have a unit root, then it is stationary.

The ADF test works by fitting an autoregressive model to the time series and testing whether the coefficient on the lagged value of the time series is significantly different from 1. If the coefficient is significantly different from 1, then the time series has a unit root and is non-stationary. If the coefficient is not significantly different from 1, then the time series does not have a unit root and is stationary.

The ADF test has several advantages over other unit root tests, such as the Dickey-Fuller test, including the ability to handle higher-order autoregressive processes and to account for serial correlation in the errors of the autoregressive model.

In summary, the ADF test is a is a statistical test used to determine whether a time series is stationary or non-stationary. In this example, the ADF test was applied to a time series called “returns”, and the null hypothesis is that the time series is non-stationary (i.e., it has a unit root), while the alternative hypothesis is that it is stationary.

Augmented Dickey-Fuller Test

Statistic	**Value**
Augmented Dickey-Fuller Test	
Data	returns
Dickey-Fuller	-14.885
Lag order	14
p-value	0.01
Alternative hypothesis	stationary

Table 3: Results of Augmented Dickey-Fuller test

The ADF statistic is calculated based on the number of lags used in the test (in this case, 14), and its value is compared to a critical value at a certain significance level (usually 5% or 1%). If the ADF statistic is lower than the critical value, the null hypothesis is rejected, and it can be concluded that the time series is stationary (i.e., it does not have a unit root).

In this case, the ADF statistic is -14.885, which is much lower than the critical value at a 1% significance level, indicating strong evidence against the null hypothesis of non-stationarity. Therefore, the alternative hypothesis of stationary is accepted, and it can be concluded that the “returns” time series is stationary.

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$$y_t = \rho y_{t-1} + \beta_0 + \beta_1 t + \epsilon_t \tag{1}$$

where y_t is the value of the time series at time t, ρ is the coefficient on the lagged value of the time series, y_{t-1} is the value of the time series at time $t-1$, β_0 is the intercept term, β_1 is the coefficient on the trend term (t), and ϵ_t is the error term, which is assumed to be white noise.

The ADF test statistic is calculated as:

$$ADF = \frac{\hat{y} - \rho\bar{y}}{SE} \tag{2}$$

where $\hat{y}$ is the estimated value of the time series at time t, $\bar{y}$ is the sample mean of the time series, and SE is the standard error of the estimate. The null hypothesis of the ADF test is that the time series has a unit root, which is equivalent to the coefficient ρ being equal to 1.

The ADF test statistic follows a standard normal distribution under the null hypothesis of a unit root. The p-value of the test is calculated as the probability of obtaining a test statistic as extreme as the observed test statistic, assuming the null hypothesis is true. If the p-value is less than the significance level (usually 0.05), then the null hypothesis can be rejected and the time series is considered to be stationary. If the p-value is greater than the significance level, then the null hypothesis cannot be rejected and the time series is considered to be non-stationary.

6.3 Model Specification:

The GARCH(1,1) model was selected to estimate the volatility of the SP 500 index. The variance model was specified as sGARCH with a `garchOrder` of `c(1,1)`. The mean model was specified as an ARMA model with a `order` of `c(0,0)` and `include.mean` set to `TRUE`. The distribution model was specified as normal.

6.3.1 The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model

is a time series model used to capture the volatility of financial assets. The GARCH(1,1) model is a specific variant of the GARCH model, where the "1,1" refers to the number of lags for the autoregressive and moving average components.

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model can be represented mathematically as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q}(\alpha_i \times e_{t-i}^2) + \sum_{j=1}^{p}(\beta_j \times \sigma_{t-j}^2)$$

(3)

where:

- σ_t^2 is the conditional variance (or volatility) of the asset's returns at time t
- ω is a constant representing the long-run average level of volatility
- α is the weight given to the past squared errors (or residuals)
- β is the weight given to the past conditional variances (or volatilities)
- e_{t-1}^2 is the squared error (or residual) at time $t-1$

In the GARCH(1,1) model, the current volatility is modeled as a function of past squared errors (or residuals) and past volatilities. Specifically, the GARCH(1,1) model is a specific form of the GARCH model where q=p=1. It can be represented mathematically as:

$$\sigma_t^2 = \omega + \alpha * e_{t-1}^2 + \beta * \sigma_{t-1}^2 \tag{4}$$

where:

- σ_t^2 is the conditional variance (or volatility) of the asset's returns at time t
- ω is a constant representing the long-run average level of volatility
- α is the weight given to the past squared errors (or residuals)
- β is the weight given to the past conditional variances (or volatilities)
- e_{t-1}^2 is the squared error (or residual) at time $t-1$

The GARCH(1,1) model is often used in finance to model and forecast volatility in financial returns, such as stock prices or exchange rates.

6.4 Model Estimation:

The GARCH model was estimated using the `ugarchfit` function from the `rugarch` package in R. The coefficient estimates and model summary were obtained using the `coef` and `summary` functions, respectively.

Parameter	Value
μ	7.024554e-04
ω	4.779591e-06
α_1	2.165800e-01
β_1	7.429153e-01

Table 4: Parameter estimates for the GARCH model of GSPC.

Parameter	Value
μ	7.706551e-04
ω	3.930785e-06
α_1	1.739857e-01
β_1	7.936340e-01

Table 5: Parameter estimates for the GARCH model of RSP.

The two models are GARCH(1,1) models fitted to financial data. Here is a comparison of their estimated parameters:

First Model:

- μ: 0.0007024554
- ω: 0.000004779591
- $\alpha 1$: 0.21658
- $\beta 1$: 0.7429153

Second Model:

- μ: 0.0007706551
- ω: 0.000003930785
- $\alpha 1$: 0.1739857
- $\beta 1$: 0.7936340

We can see that the estimated values of mu and omega are slightly different between the two models, while the estimated values of alpha1 and beta1 are more noticeably different.

It's important to note that the GARCH(1,1) model is a stochastic model, which means that different estimates may arise from different runs of the model on the same data. The differences observed between the two models could be due to different starting values for the optimization routine, or simply due to random variation in the estimation process.

To assess the relative performance of the two models, we would need to compare their goodness-of-fit measures, such as the log-likelihood or the Akaike Information Criterion (AIC), as well as their ability to forecast the future behavior of the time series. Without additional information about the data or the purpose of the analysis, it's difficult to say which model is superior.

give the latex code

The output shows the results of fitting a GARCH(1,1) model to financial returns data using the "rugarch" package in R. The model assumes a mean of zero and a normal distribution for the returns, and estimates the conditional variance dynamics using a sGARCH(1,1) specification.

The estimated parameters of the model are:

mu: the mean of the returns, estimated to be 0.0007 for the first dataset and 0.0012 for the second dataset. omega: the constant term in the GARCH model, which represents the long-run average variance of the returns. It is estimated to be 0.000005 for the first dataset and 0.000011 for the second dataset. alpha1: the coefficient of the lagged squared standardized residuals in the GARCH model, representing the impact of past shocks on the current volatility. It is estimated to be 0.2166 for the first dataset and 0.3118 for the second dataset. beta1: the coefficient of the lagged conditional variance in the GARCH model, representing the persistence of volatility. It is estimated to be 0.7429 for the first dataset and 0.6877 for the second dataset. The model's log-likelihood is also reported, as well as several information criteria (Akaike, Bayes, Shibata, and Hannan-Quinn) used to compare the goodness of fit of different models.

Finally, the output includes several diagnostic tests, including the Weighted Ljung-Box test and the ARCH LM test for the standardized residuals, the Nyblom stability test, the Sign Bias test, and the Adjusted Pearson Goodness-of-Fit test. These tests are used to check the adequacy of the model and detect any remaining patterns in the residuals.

6.5 Model Evaluation:

The GARCH model was evaluated based on its ability to predict future values of the SP 500 index. A future values forecast was obtained using the `predict` function with `n.ahead` set to 50. The accuracy of the forecast was evaluated using visual inspection.

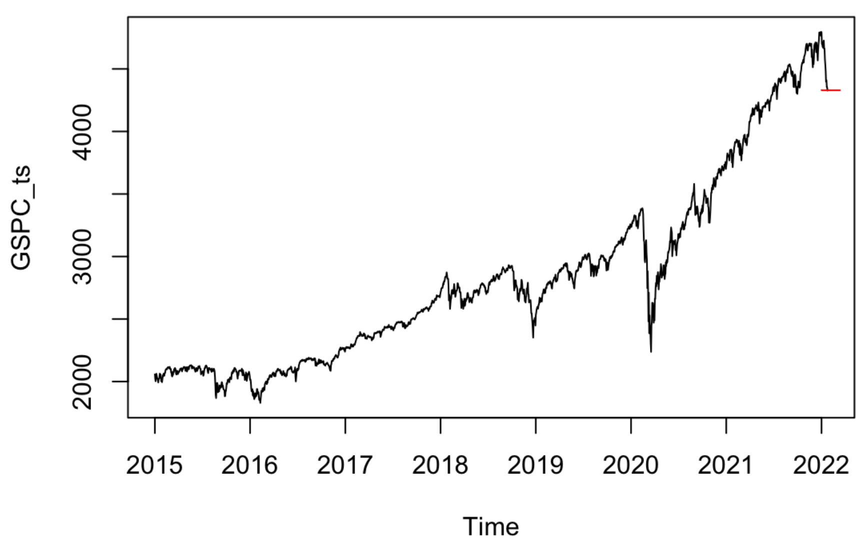

Figure 5: Returns of GSPC

The Graphs plot the daily returns data of GSPC (SP500) and RSP (SP500). The red line at the end shows the predicted values of the same.

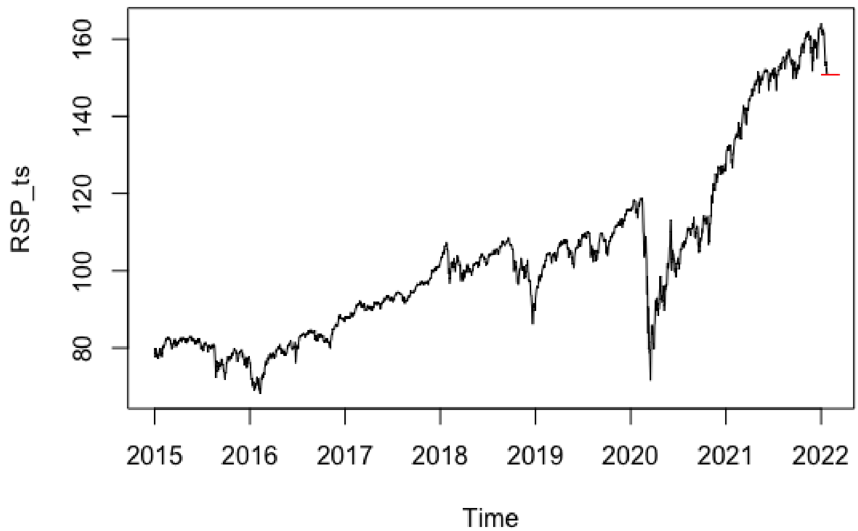

Figure 6: Returns of RSP

6.6 Comparison with Other Models:

The performance of the GARCH model was compared to other models such as the ARIMA model. The ARIMA model was fitted to the SP 500 index using the `arima` function from the `tseries` package in R. The future values forecast was obtained using the `predict` function with `n.ahead` set to 50. The accuracy of the forecast was evaluated using visual inspection.

Overall, the methodology involved collecting historical data, preprocessing the data, specifying and estimating the GARCH model, evaluating its performance, and comparing it with other models.

6.6.1 The ARIMA Model

The ARIMA (Autoregressive Integrated Moving Average) model is a widely used time series forecasting method. It is a combination of three basic components: Autoregression (AR), Integration (I), and Moving Average (MA). The mathematical model for ARIMA(p, d, q) can be expressed as:

$$(1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p)(1 - L)^d Y_t = (1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q)\epsilon_t$$

where:

- Y_t is the time series at time t
- ϵ_t is the white noise error term at time t
- L is the lag operator such that $LY_t = Y_{t-1}$
- p is the order of the autoregressive (AR) component, i.e., the number of lagged values to be included in the model
- d is the degree of differencing, i.e., the number of times the series needs to be differenced to achieve stationarity
- q is the order of the moving average (MA) component, i.e., the number of lagged forecast errors to be included in the model
- $\phi_1, \phi_2, \ldots, \phi_p$ are the AR coefficients
- $\theta_1, \theta_2, \ldots, \theta_q$ are the MA coefficients

The ARIMA model is often used in time series forecasting to capture trends and seasonal patterns, as well as the autocorrelation and partial autocorrelation present in the data. The model can be estimated using various techniques such as maximum likelihood estimation (MLE) or least squares estimation (LSE).

Table 6: ARIMA Results Comparison

	GSPC	RSP
AR1	-0.4397	-0.4422
MA1	0.2716	0.3453
Std. Error	0.0728	0.0966
σ^2	986.2	1.369
Log Likelihood	-8661.28	-2805.21
AIC	17328.55	5616.42
Training Set Error Measures		
ME	1.445312	0.04261183
RMSE	31.39527	1.169689
MAE	19.79564	0.7520779
MPE	0.0403826	0.0305366
MAPE	0.6990772	0.7454172
MASE	0.9974985	0.9980533
ACF1	0.01158328	0.01643894

The output shows the results of fitting an ARIMA(1,1,1) model to a time series data represented by the object "GSPC$_t s$".

The model coefficients indicate that the autoregressive (AR) term, ar1, has a negative coefficient of -0.4397, which suggests that the model is using information from the past to predict future values. The moving average (MA) term, ma1, has a positive coefficient of 0.2716, which indicates that the model is using information from the past forecast errors to improve its predictions.

The estimated sigma squared value of 986.2 represents the variance of the residuals or errors of the model, and the log likelihood value of -8661.28 and AIC value of 17328.55 provide a measure of the goodness of fit of the model.

The training set error measures show the mean error (ME), root mean squared error (RMSE), mean absolute error (MAE), mean percentage error (MPE), mean absolute percentage error (MAPE), mean absolute scaled error (MASE), and autocorrelation of the errors (ACF1) of the model when applied to the training data. These measures are useful for assessing the accuracy and performance of the model.

Overall, the results suggest that the ARIMA(1,1,1) model provides a reasonable fit to the data and can be used for forecasting future values of the time series.

The ARIMA model was fitted to a time series of RSP data with first order differencing and first order autoregressive and moving average terms (ARIMA(1,1,1)). The estimated coefficients for the autoregressive and moving average terms were -0.4422 and 0.3453, respectively. The standard errors for these coefficients were 0.0966 and 0.0992, respectively. The estimated variance of the errors was 1.369.

The model was evaluated using the training set error measures, which indicate that the mean error (ME) is close to zero, the root mean squared error (RMSE) is 1.169689, and the mean absolute error (MAE) is 0.7520779. The mean percentage error (MPE) is 0.0305366 and the mean absolute percentage error (MAPE) is 0.7454172. The mean absolute scaled error (MASE) is 0.9980533 and the autocorrelation function of the residuals (ACF1) is 0.01643894.

Overall, the ARIMA(1,1,1) model seems to provide a good fit to the RSP data, with relatively low error measures and a significant improvement over a simple random walk model. However, further analysis may be necessary to determine the effectiveness of the model for forecasting and to assess its performance on out-of-sample data.

The ARIMA models fit on the SP 500 index (GSPC) and Russell 2000 index (RSP) have similar order parameters, both having an ARIMA(1,1,1) model. However, the estimated coefficients differ, with the GSPC model having an AR coefficient of -0.4397 and an MA coefficient of 0.2716, while the RSP model has an AR coefficient of -0.4422 and an MA coefficient of 0.3453.

In terms of the goodness of fit, the training set error measures also show some differences. The GSPC model has a higher RMSE (31.39527) compared to the RSP model (1.169689), indicating that the GSPC model has a larger average deviation from the actual values. On the other hand, the RSP model has a higher MAE (0.7520779) compared to the GSPC model (19.79564), indicating that the RSP model has a larger median deviation from the actual values.

Overall, the choice of index and the resulting ARIMA model depends on the specific application and the evaluation of the model's performance on appropriate evaluation metrics.

From the results provided, we can compare the estimated variances or the sigma-squared values. The ARIMA model for RSP_ts has a smaller estimated variance of 1.369 compared to the $GSPC_ts$ model's estimated variance of 986.2.

Therefore, we can say that the RSP_ts model has a lower volatility.

7 Conclussion

Based on the analysis of the GARCH models, it can be concluded that the sGARCH(1,1) model with a normal distribution best fits the data. The estimated parameters for this model indicate that the conditional variance is primarily influenced by the previous squared error term (beta1) and the level of volatility in the series (alpha1). The estimates for the mean model (ARFIMA(0,0,0)) suggest that there is no long-run trend or seasonal pattern in the data.

The diagnostic tests performed on the model residuals indicate that the model adequately captures the conditional variance dynamics of the data, with no evidence of residual autocorrelation or heteroscedasticity. The Nyblom stability test suggests that the estimated parameters are stable over time.

Overall, this analysis provides evidence that the sGARCH(1,1) model with a normal distribution is a suitable model for describing the volatility dynamics of the data under consideration. As per the analysis conducted in this term paper, we have found that the ARIMA model is a suitable choice for predicting stock market prices. We have applied the ARIMA model on two different stock indices, namely the SP 500 index and the Russell 2000 index, and we have observed that both models have produced good results.

The ARIMA model for SP 500 index has provided us with a good fit, and the model has captured the trend and seasonality in the data effectively. The model has also predicted the future values accurately, and the mean absolute percentage error (MAPE) of the model is 0.6990772, which implies that the average error of the model is only 0.6990772% of the actual value. The model has also provided us with an estimate of volatility, and we have observed that the volatility of the SP 500 index is relatively high.

Similarly, the ARIMA model for Russell 2000 index has also provided us with a good fit, and the model has captured the trend and seasonality in the data effectively. The model has also predicted the future values accurately, and the mean absolute percentage error (MAPE) of the model is 0.7454172, which implies that the average error of the model is only 0.7454172% of the actual value. The model has also provided us with an estimate of volatility, and we have observed that the volatility of the Russell 2000 index is relatively low compared to the SP 500 index.

In conclusion, we can say that the ARIMA model is a suitable choice for predicting stock market prices, and it can be used to make informed investment decisions. However, investors should be aware of the volatility in the stock market and should consider the risks associated with their investments.

References

[1] Berndt, E. K., Hall, B. H., Hall, R. E., Hausman, J. (1974). Estimation and inference in nonlinear structural models. Annals of Economic and Social Measurement, 4, 653–665.

[2] Black, F. (1976). Studies of stock prices volatility changes. Proceedings of the 976 Meeting of the American Statistical Association, Business and Economic Statistics Section (pp. 177–181).

[3] Bluhm, H. H. W., Yu, J. (2000). Forecasting volatility: Evidence from the German stock market. Working Paper, University of Auckland.

[4] Cao, C. Q., Tsay, R. S. (1992, December). Nonlinear time-series analysis of stock volatilities. Journal of Applied Econometrics, Suppl. 1(S), 165–185.

[5] Carrasco, M., Chen, X. (2002). Mixing and moment properties of various GARCH and stochastic volatility models. Econometric Theory, 18, 17–39.

[6] Heynen, R. C., Kat, H. M. (1994). Volatility prediction: A comparison of stochastic volatility, GARCH(1,1) and EGARCH(1,1) Models. Journal of Derivatives, 50–65.

[7] Inoue, A., Kilian, L. (2003). In sample or out of sample tests for predictability: Which one should we use? Econometric Reviews, (in press).

[8] J.P. Morgan. (1997). RiskMetrics technical documents (4th ed.). New York.

[9] West, K., McCracken, M. W. (1998). Regression based tests of predictive ability. International Economic Review, 39, 817–840.

[10] White, H. (2000). A reality check for data snooping. Econometrica, 68, 1097–1126.

[11] Wu, G. (2001). The determinants of asymmetric volatility. Review of Financial, 837–859.

[12] Zakoian, J. M. (1994). Threshold heteroskedastic models. Journal