

HTCS6702 - Cryptography

Introduction

Goals

- Understand real-world crypto via a rigorous approach
- When you encounter crypto in your career:
 - Understand the key terms
 - Understand the security guarantees provided
 - Know how to use crypto
 - Understand what goes on “under the hood”
- “Crypto mindset”

Non-goals

- Designing your own crypto schemes
- Implementing your own crypto for real-world use
- Course goal:
realize when to consult an expert!

Cryptography (historically)

“...the art of writing or solving codes...”

- Historically, cryptography focused exclusively on ensuring *private communication* between two parties sharing secret information in advance (using “codes” aka *private-key encryption*)

Modern cryptography

- Much broader scope!
 - Data integrity, authentication, protocols, ...
 - The *public-key setting*
 - Group communication
 - More-complicated trust models
 - Foundations (e.g., number theory, quantum-resistance) to systems (e.g., electronic voting, cryptocurrencies)



Modern cryptography

*Design, analysis, and implementation of **mathematical techniques** for securing information, systems, and distributed computations against adversarial attack*

Modern cryptography

- Cryptography is ubiquitous
 - Passwords, password hashing
 - Secure credit-card transactions over the internet
 - Encrypted WiFi
 - Disk encryption
 - Digitally signed software updates
 - Bitcoin
 - ...

Cryptography (historically)

“...the art of writing or solving codes...”

- Historically, cryptography was an *art*
 - Heuristic, unprincipled design and analysis
 - Schemes proposed, broken, repeat...

Modern cryptography

- Cryptography is now much more of a *science*
 - Rigorous analysis, firm foundations, deeper understanding, rich theory
- The “crypto mindset” has permeated other areas of computer security
 - Threat modeling
 - Proofs of security

Rough course outline

| | Secrecy | Integrity |
|---------------------|------------------------|------------------------------|
| Private-key setting | Private-key encryption | Message authentication codes |
| Public-key setting | Public-key encryption | Digital signatures |

- Building blocks
 - Pseudorandom (number) generators
 - Pseudorandom functions/block ciphers
 - Hash functions
 - Number theory

HTCS6702 - Cryptography

Classical Cryptography

Motivation



- Allows us to “ease into things...,” introduce notation
- Shows why unprincipled approaches are dangerous
- Illustrates why things are more difficult than they may appear

Classical cryptography

- Until the 1970s, exclusively concerned with ensuring *secrecy* of communication
- I.e., *encryption*

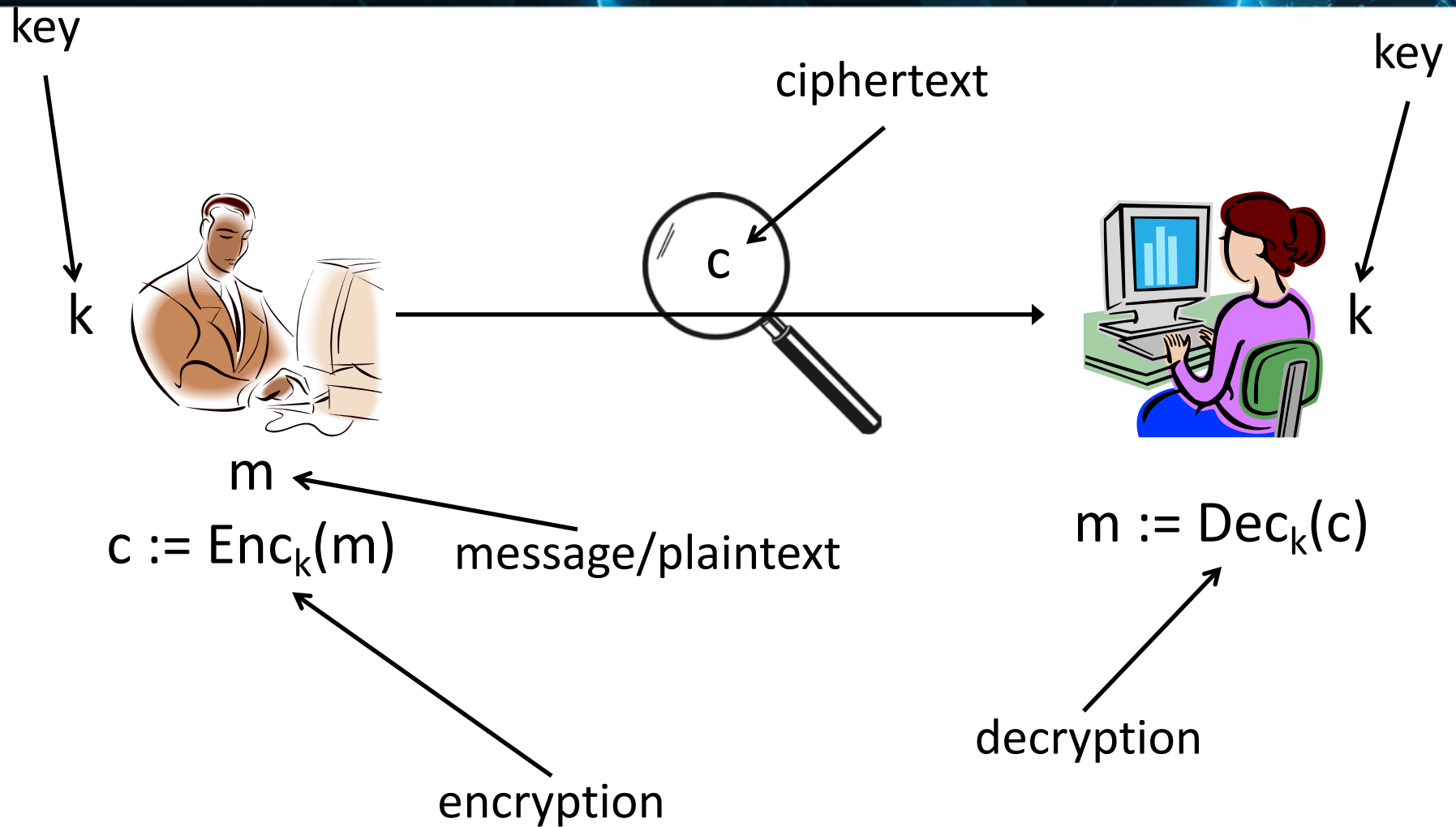
Classical cryptography

- Until the 1970s, relied exclusively on secret information (a *key*) shared in advance between the communicating parties

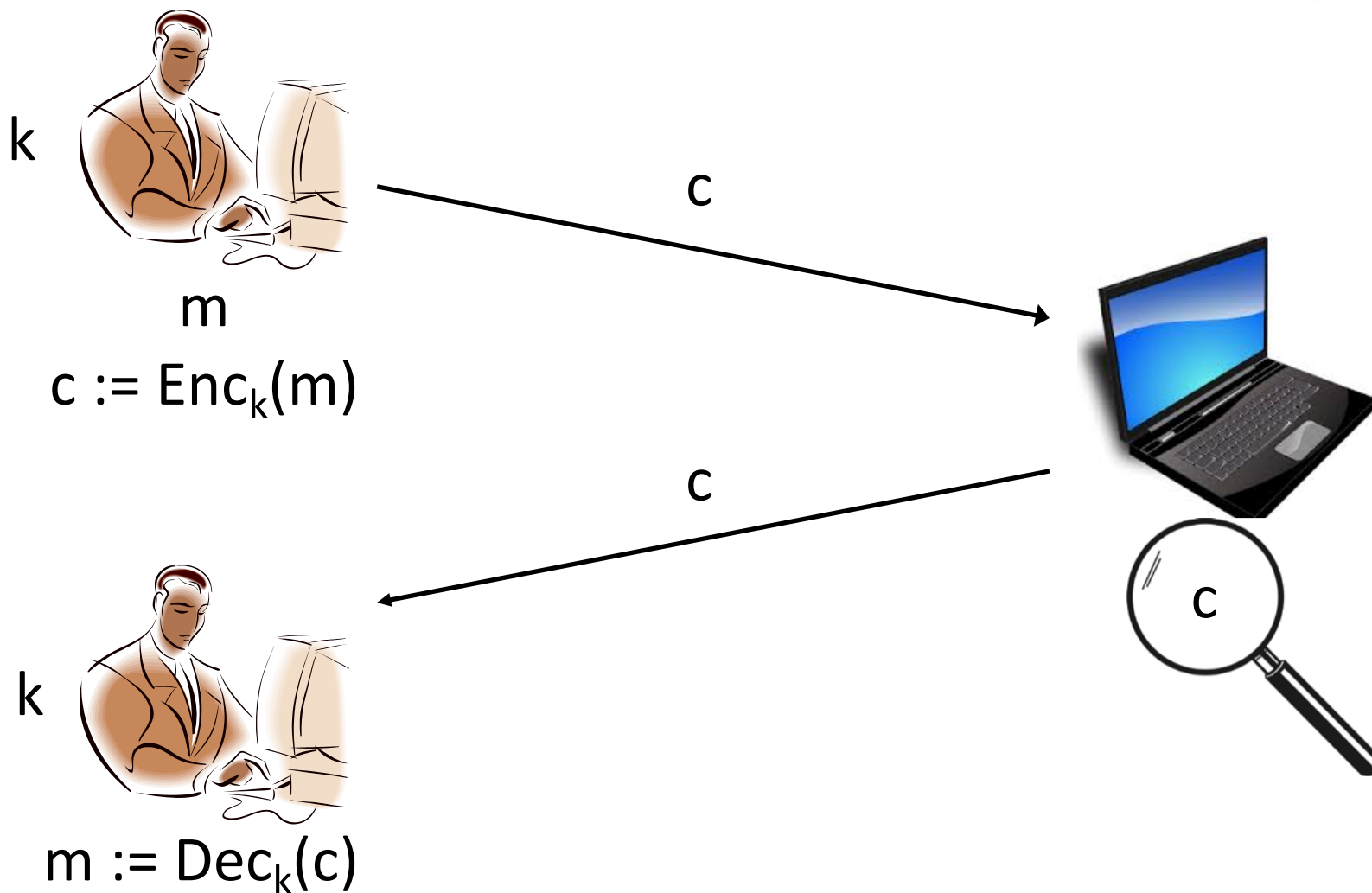
Private-key cryptography

- aka secret-key / shared-key / symmetric-key cryptography

Private-key encryption



Private-key encryption



Private-key encryption

- A *private-key encryption scheme* is defined by a message space \mathcal{M} and algorithms (Gen, Enc, Dec):
 - Gen (key-generation algorithm): outputs $k \in \mathcal{K}$
 - Enc (encryption algorithm): takes key k and message $m \in \mathcal{M}$ as input; outputs ciphertext c
 $c \leftarrow \text{Enc}_k(m)$
 - Dec (decryption algorithm): takes key k and ciphertext c as input; outputs m or “error”
 $m := \text{Dec}_k(c)$

For all $m \in \mathcal{M}$ and k output by Gen,
 $\text{Dec}_k(\text{Enc}_k(m)) = m$

Kerckhoffs's principle

- *The encryption scheme* is not secret
 - The attacker knows the encryption scheme
 - The only secret is the *key*
 - The key must be chosen at random; kept secret
- Some arguments in favor of this principle
 - Easier to keep *key* secret than *algorithm*
 - Easier to change *key* than to change *algorithm*
 - Standardization
 - Ease of deployment
 - Public validation

The shift cipher

- Consider encrypting English text
- Associate 'a' with 0; 'b' with 1; ...; 'z' with 25
- $k \in \mathcal{K} = \{0, \dots, 25\}$
- To encrypt using key k , shift every letter of the plaintext by k positions (with wraparound)
- Decryption just does the reverse

```
helloworld  
jgnnqyqtnf
```

Modular arithmetic

- $x = y \bmod N$ if and only if N divides $x - y$
- $[x \bmod N]$ = the remainder when x is divided by N
 - i.e., the unique value $y \in \{0, \dots, N-1\}$ such that $x = y \bmod N$
- $25 = 35 \bmod 10$
- $25 \neq [35 \bmod 10]$
- $5 = [35 \bmod 10]$

The shift cipher, formally

- $\mathcal{M} = \{\text{strings over lowercase English alphabet}\}$
- Gen: choose uniform $k \in \{0, \dots, 25\}$
- $\text{Enc}_k(m_1 \dots m_t)$: output $c_1 \dots c_t$, where
$$c_i := [m_i + k \bmod 26]$$
- $\text{Dec}_k(c_1 \dots c_t)$: output $m_1 \dots m_t$, where
$$m_i := [c_i - k \bmod 26]$$
- Can verify that correctness holds...

Is the shift cipher secure?

- No -- only 26 possible keys!
 - Given a ciphertext, try decrypting with every possible key
 - Only one possibility will “make sense”
 - (What assumptions are we making here?)
- Example of a “brute-force” or “exhaustive-search” attack

Example

- Ciphertext `uryybjbeyq`
- Try every possible key...
 - `tqxxaiadxp`
 - `spwwzhzcwo`
 - ...
 - `helloworld`