EM

If we want to find the maximal point of x for the function $f(x)$, we can introduce another parameter y to help

$$x_{max} \equiv \underset{x}{\operatorname{argmax}} f(x)$$

take $f(x)$ apart into

$$\begin{gathered} f(x) \equiv g(x, y) + \Delta(x, y) \\ y^k \equiv \underset{y}{\operatorname{argmax}} g(x^k, y) \equiv \underset{y}{\operatorname{argmin}} \Delta(x^k, y) \end{gathered}$$

where y^k is easy to find, $\min\limits_y \Delta(x^k, y) = 0$

step 1 [M]

fix $y \equiv y^k$

$$\begin{gathered} x^{k+1} \equiv \underset{x}{\operatorname{argmax}} g(x, y^k) \\ g(x^{k+1}, y^k) > g(x^k, y^k) = f(x^k) \end{gathered}$$

step 2 [E]

fix $x = x^{k+1}$

$$\begin{gathered} y^{k+1} \equiv \underset{y}{\operatorname{argmax}} g(x^{k+1}, y) \equiv \underset{y}{\operatorname{argmin}} \Delta(x^{k+1}, y) = 0 \\ \begin{aligned} f(x^{k+1}) &\equiv g(x^{k+1}, y) + \Delta(x^{k+1}, y) \\ &= g(x^{k+1}, y^k) + \Delta(x^{k+1}, y^k) \\ &= g(x^{k+1}, y^{k+1}) + \Delta(x^{k+1}, y^{k+1}) \\ &= g(x^{k+1}, y^{k+1}) \end{aligned} \\ f(x^{k+1}) = g(x^{k+1}, y^{k+1}) > g(x^{k+1}, y^k) \end{gathered}$$

In all

$$f(x^{k+1}) = g(x^{k+1}, y^{k+1}) > g(x^{k+1}, y^k) > f(x^k) = g(x^k, y^k)$$

So

$$\lim_{k\to\infty} f(x^k) = \lim_{k\to\infty} g(x^{k+1}, y^k) = \underset{x}{\mathrm{argmax}} f(x)$$

Comment:

By introducing another parameter y, and iterate step 1 [M] and step 2 [E], we eventually find the maximal point of x for the function $f(x)$