Hence

$$\begin{aligned}
\ln\Big(p(\boldsymbol{X},\boldsymbol{Z}|\boldsymbol{\theta})\Big) &= \sum_{n=1}^{N} a_n \ln p\left(\boldsymbol{x_n}|\boldsymbol{z_n},\boldsymbol{\Sigma},\boldsymbol{\mu}\right) + \sum_{n=1}^{N} a_n \ln p\left(\boldsymbol{z_n}|\boldsymbol{\pi}\right) \\
&= \sum_{n=1}^{N} a_n \sum_{k=1}^{K} (\ln \pi_k) z_{kn} + \sum_{n=1}^{N} a_n \sum_{k=1}^{K} (\ln \phi_k(\boldsymbol{x_n})) z_{kn} \\
&= \ln \boldsymbol{\pi}^T \cdot \boldsymbol{Z} \cdot \boldsymbol{a} + \\
&\sum_{n=1}^{N} a_n \sum_{k=1}^{K} \Big[-\frac{l}{2}\ln(2\pi) - \frac{1}{2}\ln|\Sigma_k| - \frac{1}{2}(\boldsymbol{x_n}-\mu_k)^T \Sigma_k^{-1}(\boldsymbol{x_n}-\mu_k)\Big] z_{kn} \\
&= (\ln \boldsymbol{\pi}^T - \frac{1}{2}|\boldsymbol{\Sigma}|^T) \cdot \boldsymbol{Z} \cdot \boldsymbol{a} - \frac{l}{2}\ln(2\pi) ones(1,K) \cdot \boldsymbol{Z} \cdot \boldsymbol{a} \\
&- \frac{1}{2}\sum_{k=1}^{K} \Big[\mathrm{tr}\Big((X-\mu_k\cdot 1^T)^T \Sigma_k^{-1}(X-\mu_k\cdot 1^T)\cdot \mathrm{diag}(\boldsymbol{a}\odot \boldsymbol{z_k})\Big)\Big]
\end{aligned}$$

Now find close form of Q

$\sum_{\boldsymbol{Z}} p\left(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta}^{\text{ old }}\right)$ is [probability distribution] random variable **of** Z [probability distribution] random variable

Here $q^k = p\left(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta}^k\right) = p\left(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta}^{\text{ old }}\right)$

$$\begin{aligned}
\boldsymbol{Q}\left(\boldsymbol{\theta},\boldsymbol{\theta}^{\text{ old }}\right) &\equiv \sum_{\boldsymbol{Z}} p\left(\boldsymbol{Z}|\boldsymbol{X},\boldsymbol{\theta}^{\text{ old }}\right) \ln p(\boldsymbol{X},\boldsymbol{Z}|\boldsymbol{\theta}) \\
&= \left\{\left[\sum_{n=1}^{N} a_n \sum_{z_n} p\left(\boldsymbol{z_n}|\boldsymbol{x_n},\boldsymbol{\theta}^{\text{ old }}\right) \boldsymbol{z_n}^T\right](\ln \boldsymbol{\pi} - \frac{1}{2}|\boldsymbol{\Sigma}| - \frac{l}{2}\ln(2\pi)\cdot 1.)\right\} \\
&- \frac{1}{2}\left\{\sum_{n=1}^{N} a_n \sum_{k=1}^{K}\left[\sum_{z_n} p\left(\boldsymbol{z_n}|\boldsymbol{x_n},\boldsymbol{\theta}^{\text{ old }}\right)(\boldsymbol{x_n}-\mu_k)^T \Sigma_k^{-1}(\boldsymbol{x_n}-\mu_k) z_{kn}\right]\right\} \\
&= \left[\boldsymbol{\gamma}\cdot \boldsymbol{a}\right]^T (\ln \boldsymbol{\pi} - \frac{1}{2}|\boldsymbol{\Sigma}| - \frac{l}{2}\ln(2\pi)\cdot 1.) \\
&- \frac{1}{2}\sum_{k=1}^{K} \mathrm{tr}\Big(diag(\boldsymbol{a}\odot \boldsymbol{\gamma}_k)\cdot (X-\mu_k\cdot 1^T)^T \Sigma_k^{-1}(X-\mu_k\cdot 1^T)\Big)
\end{aligned}$$

Here

$$p(\boldsymbol{z_n}|\boldsymbol{x_n},\boldsymbol{\theta}^{\text{old}}) = \frac{p(\boldsymbol{x_n}|\boldsymbol{z_n},\boldsymbol{\theta}^{\text{old}})p(\boldsymbol{z_n}|\boldsymbol{\theta}^{\text{old}})}{\sum_{z_n} p(\boldsymbol{x_n}|\boldsymbol{z_n},\boldsymbol{\theta}^{\text{old}})p(\boldsymbol{z_n}|\boldsymbol{\theta}^{\text{old}})}$$

So, have $1.^T\boldsymbol{\gamma} = ones(1,N)$

$$p(\boldsymbol{z_n}=k|\boldsymbol{x_n},\boldsymbol{\theta}^{\text{old}}) = \frac{\phi_k^{\text{old}}(\boldsymbol{x_n})\pi_k^{\text{old}}}{\sum_{k=1}^{K}\phi_k^{\text{old}}(\boldsymbol{x_n})\pi_k^{\text{old}}} \equiv \gamma_{kn}$$

$$\boldsymbol{\gamma} = \boldsymbol{\phi}^{\text{old}}(X)\odot(\pi_k^{\text{old}}\cdot 1.^T)\odot(\frac{1.}{1.\cdot(\boldsymbol{\pi}^{\text{old}})^T\boldsymbol{\phi}^{\text{old}}(X)})$$

maximize Q

To sum up

$$\begin{aligned}
\boldsymbol{\pi} &= \frac{[\boldsymbol{\gamma} \cdot \boldsymbol{a}]}{1.^T[\boldsymbol{\gamma} \cdot \boldsymbol{a}]} = \frac{[\boldsymbol{\gamma} \cdot \boldsymbol{a}]}{1.^T \boldsymbol{a}} \\
\mu_k &= \frac{X(\boldsymbol{a} \odot \boldsymbol{\gamma_k})}{1.^T(\boldsymbol{a} \odot \boldsymbol{\gamma_k})} \\
\Sigma_k &= \frac{(X - \mu_k \cdot 1^T) diag(\boldsymbol{a} \odot \boldsymbol{\gamma_k}) \cdot (X - \mu_k \cdot 1^T)^T}{1.^T(\boldsymbol{a} \odot \boldsymbol{\gamma_k})}
\end{aligned}$$

where, have $1.^T \boldsymbol{\gamma} = ones(1, N)$

$$p(\boldsymbol{z_n} = k | \boldsymbol{x_n}, \boldsymbol{\theta}^{\text{old}}) = \frac{\phi_k^{\text{old}}(\boldsymbol{x_n}) \pi_k^{\text{old}}}{\sum_{k=1}^{K} \phi_k^{\text{old}}(\boldsymbol{x_n}) \pi_k^{\text{old}}} \equiv \gamma_{kn}$$

$$\boldsymbol{\gamma} = \boldsymbol{\phi}^{\text{old}}(X) \odot (\pi_k^{\text{old}} \cdot 1.^T) \odot (\frac{1.}{1. \cdot (\boldsymbol{\pi}^{\text{old}})^T \boldsymbol{\phi}^{\text{old}}(X)})$$

here $\boldsymbol{\gamma_k}$ is (N x 1) matrix

$$\boldsymbol{\gamma} = \begin{bmatrix} \vdots \\ \boldsymbol{\gamma}_k^T \\ \vdots \end{bmatrix}$$

Actually, Hinton calculate the $\boldsymbol{\pi}$

$$\pi_k = \text{sigmoid}\left(\lambda\left(\beta_a - (\beta_u + \ln|\Sigma_k|)\left(\boldsymbol{a^T \gamma_k}\right)\right)\right)$$