Hence

$$\begin{split} \ln\left(p(\boldsymbol{X},\boldsymbol{Z}|\boldsymbol{\theta})\right) &= \sum_{n=1}^{N} a_n \ln p\left(\boldsymbol{x}_{\boldsymbol{n}}|\boldsymbol{z}_{\boldsymbol{n}},\boldsymbol{\Sigma},\boldsymbol{\mu}\right) + \sum_{n=1}^{N} a_n \ln p\left(\boldsymbol{z}_{\boldsymbol{n}}|\boldsymbol{\pi}\right) \\ &= \sum_{n=1}^{N} a_n \sum_{k=1}^{K} (\ln \pi_k) z_{kn} + \sum_{n=1}^{N} a_n \sum_{k=1}^{K} (\ln \phi_k(\boldsymbol{x}_{\boldsymbol{n}})) z_{kn} \\ &= \ln \boldsymbol{\pi}^T \cdot \boldsymbol{Z} \cdot \boldsymbol{a} + \\ &\sum_{n=1}^{N} a_n \sum_{k=1}^{K} \left[ -\frac{l}{2} \ln(2\pi) - \frac{1}{2} \ln |\boldsymbol{\Sigma}_k| - \frac{1}{2} (\boldsymbol{x}_{\boldsymbol{n}} - \mu_k)^T \boldsymbol{\Sigma}_k^{-1} (\boldsymbol{x}_{\boldsymbol{n}} - \mu_k) \right] z_{kn} \\ &= (\ln \boldsymbol{\pi}^T - \frac{1}{2} |\boldsymbol{\Sigma}|^T) \cdot \boldsymbol{Z} \cdot \boldsymbol{a} - \frac{l}{2} \ln(2\pi) ones(1, K) \cdot \boldsymbol{Z} \cdot \boldsymbol{a} \\ &- \frac{1}{2} \sum_{k=1}^{K} \left[ \operatorname{tr} \left( (\boldsymbol{X} - \mu_k \cdot \boldsymbol{1}^T)^T \boldsymbol{\Sigma}_k^{-1} (\boldsymbol{X} - \mu_k \cdot \boldsymbol{1}^T) \cdot \operatorname{diag}(\boldsymbol{a} \odot \boldsymbol{z}_k) \right) \right] \end{split}$$

Now find close form of Q

 $\sum_{m{Z}} p\left(m{Z}|m{X},m{ heta}^{
m old}
ight)$  is [probability distribution] random variable of Z [probability distribution] random variable

Here 
$$q^k = p\left(oldsymbol{Z}|oldsymbol{X},oldsymbol{ heta}^k
ight) = p\left(oldsymbol{Z}|oldsymbol{X},oldsymbol{ heta}^{
m old}
ight)$$

$$egin{aligned} oldsymbol{Q}\left(oldsymbol{ heta},oldsymbol{ heta}^{
m old}
ight) &\equiv \sum_{oldsymbol{Z}} p\left(oldsymbol{Z}|oldsymbol{X},oldsymbol{Q}|oldsymbol{ heta}(oldsymbol{x},oldsymbol{Z}|oldsymbol{ heta}) \ln p(oldsymbol{X},oldsymbol{Z}|oldsymbol{ heta}) + rac{1}{2} \left\{ \sum_{n=1}^{N} a_n \sum_{k=1}^{K} \left[ \sum_{oldsymbol{z}_n} p\left(oldsymbol{z}_n|oldsymbol{x}_n,oldsymbol{ heta}^{
m old}
ight) (oldsymbol{x}_n - \mu_k)^T \Sigma_k^{-1} (oldsymbol{x}_n - \mu_k) z_{kn} 
ight] 
ight\} \\ &= \left[ oldsymbol{\gamma} \cdot oldsymbol{a} \right]^T (\ln oldsymbol{\pi} - rac{1}{2} |oldsymbol{\Sigma}| - rac{l}{2} \ln(2\pi) \cdot 1.) \\ &- rac{1}{2} \sum_{k=1}^{K} \mathrm{tr} \Big( diag(oldsymbol{a} \odot oldsymbol{\gamma}_k) \cdot (X - \mu_k \cdot 1^T)^T \Sigma_k^{-1} (X - \mu_k \cdot 1^T) \Big) \end{aligned}$$

Here

$$p(oldsymbol{z_n}|oldsymbol{x_n},oldsymbol{ heta}^{ ext{old}}) = rac{p(oldsymbol{x_n}|oldsymbol{z_n},oldsymbol{ heta}^{ ext{old}})p(oldsymbol{z_n}|oldsymbol{ heta}^{ ext{old}})}{\sum\limits_{oldsymbol{z_n}}p(oldsymbol{x_n}|oldsymbol{z_n},oldsymbol{ heta}^{ ext{old}})p(oldsymbol{z_n}|oldsymbol{ heta}^{ ext{old}})}$$

So, have  $1.^T \boldsymbol{\gamma} = ones(1, N)$ 

$$egin{aligned} p(oldsymbol{z_n} = k | oldsymbol{x_n}, oldsymbol{ heta}^{ ext{old}}) = rac{oldsymbol{\phi}_k^{ ext{old}}(oldsymbol{x_n}) \pi_k^{ ext{old}}}{\sum\limits_{k=1}^K oldsymbol{\phi}_k^{ ext{old}}(oldsymbol{x_n}) \pi_k^{ ext{old}}} \equiv \gamma_{kn} \ oldsymbol{\gamma} = oldsymbol{\phi}^{ ext{old}}(X) \odot (\pi_k^{ ext{old}} \cdot 1.^T) \odot (rac{1.}{1.\cdot (oldsymbol{\pi}^{ ext{old}})^T oldsymbol{\phi}^{ ext{old}}(X)}) \end{aligned}$$

maximize Q

## To sum up

$$egin{aligned} oldsymbol{\pi} &= rac{ig[oldsymbol{\gamma} \cdot oldsymbol{a}ig]}{1.^Tig[oldsymbol{\gamma} \cdot oldsymbol{a}ig]} = rac{ig[oldsymbol{\gamma} \cdot oldsymbol{a}ig]}{1.^Toldsymbol{a}} \ \mu_k &= rac{X(oldsymbol{a}\odotoldsymbol{\gamma_k})}{1.^T(oldsymbol{a}\odotoldsymbol{\gamma_k})} \ \Sigma_k &= rac{(X - \mu_k \cdot 1^T)diag(oldsymbol{a}\odotoldsymbol{\gamma_k}) \cdot (X - \mu_k \cdot 1^T)^T}{1.^T(oldsymbol{a}\odotoldsymbol{\gamma_k})} \end{aligned}$$

where, have  $1.^T oldsymbol{\gamma} = ones(1,N)$ 

$$egin{aligned} p(oldsymbol{z_n} = k | oldsymbol{x_n}, oldsymbol{ heta}^{ ext{old}}) = rac{oldsymbol{\phi}_k^{ ext{old}}(oldsymbol{x_n}) \pi_k^{ ext{old}}}{\sum\limits_{k=1}^K oldsymbol{\phi}_k^{ ext{old}}(oldsymbol{x_n}) \pi_k^{ ext{old}}} \equiv \gamma_{kn} \ oldsymbol{\gamma} = oldsymbol{\phi}^{ ext{old}}(X) \odot (\pi_k^{ ext{old}} \cdot 1.^T) \odot (rac{1.}{1.\cdot (oldsymbol{\pi}^{ ext{old}})^T oldsymbol{\phi}^{ ext{old}}(X)}) \end{aligned}$$

here  $\gamma_k$  is (N x 1) matrix

$$oldsymbol{\gamma} = egin{bmatrix} dots \ oldsymbol{\gamma}_k^T \ dots \ \end{bmatrix}$$

Actually, Hinton calculate the  $\pi$ 

$$\pi_k = \operatorname{sigmoid}\left(\lambda\left(eta_a - (eta_u + \ln|\Sigma_k|)(oldsymbol{a^T}oldsymbol{\gamma_k})
ight)
ight)$$