Problem 11.24

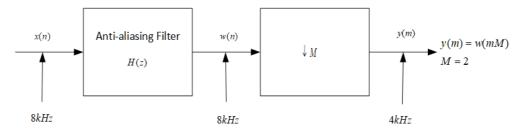
Generate a sinusoid with a 1000 Hz for 0.05 s using a sampling rate of 8 kHz,

- (a) Design a decimator to change the sampling rate to 4 kHz with specifications below:
 - Signal frequency range: 0-1800 Hz.
 - Hamming window required for FIR filter design
- (b) Write a MATLAB program to implement the down-sampling scheme,

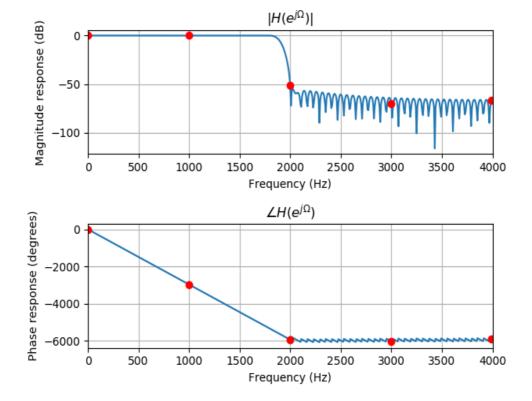
and plot the original signal and the down-sampled signal versus the sample number, respectively.

solution

(a) Design a decimator to change the sampling rate to 4 kHz



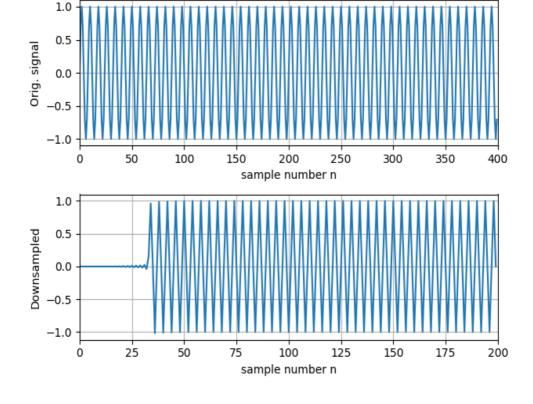
The anti-aliasing filter, filter length = 133, $f_c=1900~{
m Hz}$



```
h(n) = \
[-0.0043, 0.0019, 0.0047, -0.0012, -0.0051, 0.0004, 0.0053, 0.0004, -0.0054,
-0.0013, 0.0054, 0.0022, -0.0053, -0.0031, 0.005, 0.0041, -0.0045, -0.0049,
0.0039,\ 0.0058,\ -0.0031,\ -0.0065,\ 0.0022,\ 0.0072,\ -0.0012,\ -0.0077,\ 0.0,\ 0.0081,
0.0013, -0.0084, -0.0027, 0.0084, 0.0043, -0.0082, -0.0058, 0.0078, 0.0075,
-0.0071, -0.0092, 0.0062, 0.0109, -0.0049, -0.0126, 0.0032, 0.0143,
-0.0012, -0.0159, -0.0013, 0.0175, 0.0044, -0.0189, -0.0081, 0.0203, 0.0128,
-0.0215, -0.0188, 0.0225, 0.0269, -0.0234, -0.0388, 0.0241, 0.0588, -0.0246,
-0.1032, 0.0249, 0.3173, 0.475, 0.3173, 0.0249, -0.1032, -0.0246, 0.0588,
0.0241, -0.0388, -0.0234, 0.0269, 0.0225, -0.0188, -0.0215, 0.0128, 0.0203,
-0.0081, -0.0189, 0.0044, 0.0175, -0.0013, -0.0159, -0.0012, 0.0143, 0.0032,
-0.0126, -0.0049, 0.0109, 0.0062, -0.0092, -0.0071, 0.0075, 0.0078, -0.0058,
-0.0082, 0.0043, 0.0084, -0.0027, -0.0084, 0.0013, 0.0081, 0.0, -0.0077,
-0.0012, 0.0072, 0.0022, -0.0065, -0.0031, 0.0058, 0.0039, -0.0049, -0.0045,
0.0041, 0.005, -0.0031, -0.0053, 0.0022, 0.0054, -0.0013, -0.0054, 0.0004,
0.0053, 0.0004, -0.0051, -0.0012, 0.0047, 0.0019, -0.0043
h_w(n) = 
[-0.0003, 0.0002, 0.0004, -0.0001, -0.0004, 0.0, 0.0005, 0.0, -0.0006, -0.0002,
0.0007, \ 0.0003, \ -0.0008, \ -0.0005, \ 0.0009, \ 0.0008, \ -0.0009, \ -0.0011, \ 0.0009,
0.0015, -0.0009, -0.0019, 0.0007, 0.0024, -0.0004, -0.0029, 0.0, 0.0033, 0.0006,
-0.0038, -0.0013, 0.0042, 0.0022, -0.0044, -0.0033, 0.0046, 0.0045, -0.0045,
-0.006, 0.0041, 0.0075, -0.0035, -0.0092, 0.0024, 0.011, -0.0009, -0.0128,
-0.0011, 0.0147, 0.0037, -0.0165, -0.0072, 0.0183, 0.0117, -0.0199, -0.0176,
0.0214,\ 0.0258,\ -0.0226,\ -0.0378,\ 0.0236,\ 0.0581,\ -0.0244,\ -0.1027,\ 0.0248,
0.3172, 0.475, 0.3172, 0.0248, -0.1027, -0.0244, 0.0581, 0.0236, -0.0378,
-0.0226, 0.0258, 0.0214, -0.0176, -0.0199, 0.0117, 0.0183, -0.0072, -0.0165,
0.0037, 0.0147, -0.0011, -0.0128, -0.0009, 0.011, 0.0024, -0.0092, -0.0035,
0.0075, 0.0041, -0.006, -0.0045, 0.0045, 0.0046, -0.0033, -0.0044, 0.0022,
0.0042, -0.0013, -0.0038, 0.0006, 0.0033, 0.0, -0.0029, -0.0004, 0.0024, 0.0007,
-0.0019, -0.0009, 0.0015, 0.0009, -0.0011, -0.0009, 0.0008, 0.0009, -0.0005,
-0.0008, 0.0003, 0.0007, -0.0002, -0.0006, 0.0, 0.0005, 0.0, -0.0004, -0.0001,
0.0004, 0.0002, -0.0003]
```

(b) Write a MATLAB program to implement the down-sampling scheme,

and plot the original signal and the down-sampled signal versus the sample number, respectively.



Python script:

```
from fir_filter.choose_window_type import choose_window_type
from fir_filter.calc_window_len import calc_window_len
from fir_filter.calc_mag_angle import calc_mag_angle
from iir_filter.calc_mag_angle import plot_mag_angle_freq
from fir_filter.calc_freq_cutoff import calc_freq_cutoff
from fir_filter.fir_filter import print_approx, fir_filter
from fir_filter.window import window
from fir_filter.filter import filter
def plot_down_sample(list_origin, list_downsample, f_sample, M, interval,
path_fig="./test.png"):
    fig = plt.figure()
    plt.subplot(2, 1, 1)
    num_origin = ceil(interval * f_sample)
    plt.plot(list(range(num_origin)), list_origin[:num_origin])
    plt.xlim([0, interval * f_sample])
    plt.xlabel("sample number n")
    plt.ylabel("Orig. signal")
    plt.grid()
    plt.subplot(2, 1, 2)
    num_downsample = ceil(interval * f_sample/M)
    plt.plot(list(range(num_downsample)), list_downsample[:num_downsample])
    plt.xlim([0, interval * f_sample/M])
    plt.xlabel("sample number n")
    plt.ylabel("Downsampled")
    plt.grid()
    plt.tight_layout()
    fig.savefig(path_fig)
    plt.show()
```

```
# passband_ripple = 0.02
# stopband_attenuation = 60
# str_window_type = choose_window_type(passband_ripple, stopband_attenuation)
# print(str_window_type)
str_window_type = "Hamming"
f_s, M = 8000, 2
f_{pass}, f_{stop} = 1800, f_{s} / (2*M)
list_transient_band = [ [f_pass, f_stop] ]
filter_len = calc_window_len(str_window_type, list_transient_band, f_sample=f_s)
print(filter_len)
list_freq_cutoff = calc_freq_cutoff(list_transient_band)
print(list_freq_cutoff)
list_filter = fir_filter(list_freq_cutoff, f_s, filter_len,
str_filter_type="low_pass")
print_approx(list_filter)
# Hamming window function.
path_fig = "../p11_24_H(z).png"
list_filter_window = window(list_filter, str_window_type=str_window_type)
print_approx(list_filter_window)
list_mag, list_angle, list_omega = calc_mag_angle(list_filter_window)
plot_mag_angle_freq(list_mag, list_angle, list_omega, f_s, path_fig=path_fig)
# down sample
from math import sin, pi, ceil
import matplotlib.pyplot as plt
interval = 0.05
list_x = [sin(2*pi * 1000*ind / f_s) for ind in range(round(0.05*f_s))]
list_anti = filter(list_x, list_filter_window) # anti-aliasing filter
list_downsample = [elem for ind, elem in enumerate(list_anti) if ind % M == 0] #
down sample
plot_down_sample(list_x, list_downsample, f_s, M, interval=0.05,
path_fig="../p11_24_point.png")
```

Problem 11.25

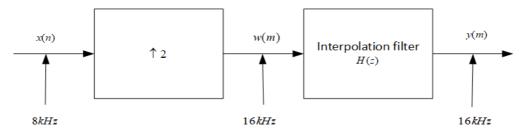
Generate a sinusoid with a 1000 Hz for 0.05 s using a sampling rate of 8 kHz,

- (a) Design an interpolator to change the sampling rate to 16 kHz with following specifications:
 - Signal frequency range: 0-3600 Hz
 - Hamming window required for FIR filter design
- (b) Write a MATLAB program to implement the up-sampling scheme,

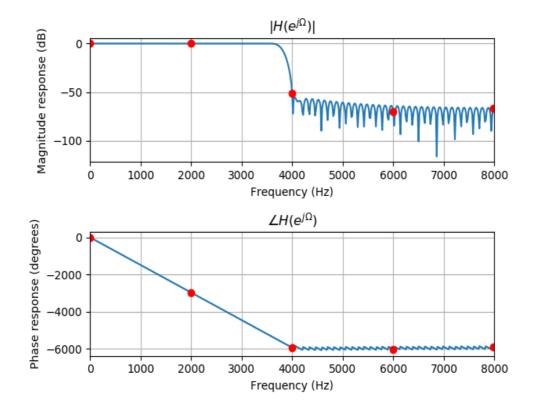
and plot the original signal and the up-sampled signal versus the sample number, respectively.

solution

(a) Design an interpolator to change the sampling rate to 16 kHz



The interpolation filter, filter length = 133, $f_c=3800~{
m Hz}$

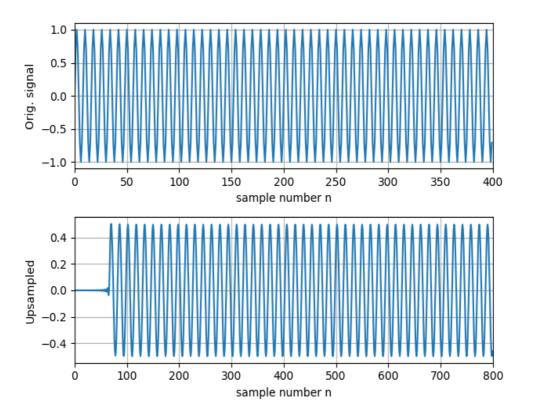


```
h(n) = 
[-0.0043, 0.0019, 0.0047, -0.0012, -0.0051, 0.0004, 0.0053, 0.0004, -0.0054,
-0.0013, 0.0054, 0.0022, -0.0053, -0.0031, 0.005, 0.0041, -0.0045, -0.0049,
0.0039,\ 0.0058,\ -0.0031,\ -0.0065,\ 0.0022,\ 0.0072,\ -0.0012,\ -0.0077,\ 0.0,\ 0.0081,
0.0013, -0.0084, -0.0027, 0.0084, 0.0043, -0.0082, -0.0058, 0.0078, 0.0075,
-0.0071, -0.0092, 0.0062, 0.0109, -0.0049, -0.0126, 0.0032, 0.0143,
-0.0012, -0.0159, -0.0013, 0.0175, 0.0044, -0.0189, -0.0081, 0.0203, 0.0128,
-0.0215, -0.0188, 0.0225, 0.0269, -0.0234, -0.0388, 0.0241, 0.0588, -0.0246,
-0.1032, 0.0249, 0.3173, 0.475, 0.3173, 0.0249, -0.1032, -0.0246, 0.0588,
0.0241, -0.0388, -0.0234, 0.0269, 0.0225, -0.0188, -0.0215, 0.0128, 0.0203,
-0.0081, -0.0189, 0.0044, 0.0175, -0.0013, -0.0159, -0.0012, 0.0143, 0.0032,
-0.0126, -0.0049, 0.0109, 0.0062, -0.0092, -0.0071, 0.0075, 0.0078, -0.0058,
-0.0082, 0.0043, 0.0084, -0.0027, -0.0084, 0.0013, 0.0081, 0.0, -0.0077,
-0.0012, 0.0072, 0.0022, -0.0065, -0.0031, 0.0058, 0.0039, -0.0049, -0.0045,
0.0041,\ 0.005,\ -0.0031,\ -0.0053,\ 0.0022,\ 0.0054,\ -0.0013,\ -0.0054,\ 0.0004,
0.0053, 0.0004, -0.0051, -0.0012, 0.0047, 0.0019, -0.0043
h_w(n) = 
[-0.0003, 0.0002, 0.0004, -0.0001, -0.0004, 0.0, 0.0005, 0.0, -0.0006, -0.0002,
0.0007, \ 0.0003, \ -0.0008, \ -0.0005, \ 0.0009, \ 0.0008, \ -0.0009, \ -0.0011, \ 0.0009,
0.0015, -0.0009, -0.0019, 0.0007, 0.0024, -0.0004, -0.0029, 0.0, 0.0033, 0.0006,
-0.0038, -0.0013, 0.0042, 0.0022, -0.0044, -0.0033, 0.0046, 0.0045, -0.0045,
-0.006, 0.0041, 0.0075, -0.0035, -0.0092, 0.0024, 0.011, -0.0009, -0.0128,
-0.0011, 0.0147, 0.0037, -0.0165, -0.0072, 0.0183, 0.0117, -0.0199, -0.0176,
0.0214,\ 0.0258,\ -0.0226,\ -0.0378,\ 0.0236,\ 0.0581,\ -0.0244,\ -0.1027,\ 0.0248,
0.3172, 0.475, 0.3172, 0.0248, -0.1027, -0.0244, 0.0581, 0.0236, -0.0378,
-0.0226, 0.0258, 0.0214, -0.0176, -0.0199, 0.0117, 0.0183, -0.0072, -0.0165,
0.0037, 0.0147, -0.0011, -0.0128, -0.0009, 0.011, 0.0024, -0.0092, -0.0035,
0.0075, 0.0041, -0.006, -0.0045, 0.0045, 0.0046, -0.0033, -0.0044, 0.0022,
0.0042, -0.0013, -0.0038, 0.0006, 0.0033, 0.0, -0.0029, -0.0004, 0.0024, 0.0007,
-0.0019, -0.0009, 0.0015, 0.0009, -0.0011, -0.0009, 0.0008, 0.0009, -0.0005,
-0.0008, 0.0003, 0.0007, -0.0002, -0.0006, 0.0, 0.0005, 0.0, -0.0004, -0.0001,
0.0004, 0.0002, -0.0003]
```

(b) Write a MATLAB program to implement the up-sampling scheme,

and plot the original signal and the up-sampled signal versus the sample number, respectively.

Up-sampled signal = $\frac{1}{L}$ **original signal**, (L=2)



```
from fir_filter.choose_window_type import choose_window_type
from fir_filter.calc_window_len import calc_window_len
from fir_filter.calc_mag_angle import calc_mag_angle
from iir_filter.calc_mag_angle import plot_mag_angle_freq
from fir_filter.calc_freq_cutoff import calc_freq_cutoff
from fir_filter.fir_filter import print_approx, fir_filter
from fir_filter.window import window
from fir_filter.filter import filter
def plot_up_sample(list_origin, list_upample, f_sample, L, interval,
path_fig="./test.png"):
    fig = plt.figure()
    plt.subplot(2, 1, 1)
    num_origin = ceil(interval * f_sample)
    plt.plot(list(range(num_origin)), list_origin[:num_origin])
    plt.xlim([0, interval * f_sample])
    plt.xlabel("sample number n")
    plt.ylabel("Orig. signal")
    plt.grid()
    plt.subplot(2, 1, 2)
    num_upsample = ceil(interval * f_sample * L)
    plt.plot(list(range(num_upsample)), list_upsample[:num_upsample])
    plt.xlim([0, interval * f_sample * L])
    plt.xlabel("sample number n")
    plt.ylabel("Upsampled")
    plt.grid()
    plt.tight_layout()
    fig.savefig(path_fig)
    plt.show()
# passband_ripple = 0.02
# stopband_attenuation = 60
# str_window_type = choose_window_type(passband_ripple, stopband_attenuation)
```

```
# print(str_window_type)
str_window_type = "Hamming"
f_s, L = 8000, 2
f_sL = f_s * L
f_{pass}, f_{stop} = 3600, f_{s} / 2
list_transient_band = [ [f_pass, f_stop] ]
filter_len = calc_window_len(str_window_type, list_transient_band,
f_sample=f_sL)
print(filter_len)
list_freq_cutoff = calc_freq_cutoff(list_transient_band)
print(list_freq_cutoff)
list_filter = fir_filter(list_freq_cutoff, f_sL, filter_len,
str_filter_type="low_pass")
print_approx(list_filter)
# Hamming window function.
path_fig = "../p11_25_H(z).png"
list_filter_window = window(list_filter, str_window_type=str_window_type)
print_approx(list_filter_window)
list_mag, list_angle, list_omega = calc_mag_angle(list_filter_window)
plot_mag_angle_freq(list_mag, list_angle, list_omega, f_sL, path_fig=path_fig)
# down sample
from math import sin, pi, ceil
import matplotlib.pyplot as plt
import itertools
interval = 0.05
list_x = [sin(2*pi * 1000*ind / f_s) for ind in range(round(0.05*f_s))]
list\_zeros = [[elem] + [0] * (L-1) for elem in <math>list\_x]
list_zeros = list(itertools.chain.from_iterable(list_zeros))
list_upsample = filter(list_zeros, list_filter_window) # anti-aliasing filter
plot_up_sample(list_x, list_upsample, f_s, L, interval=0.05,
path_fig="../p11_25_point.png")
```

Problem 11.26

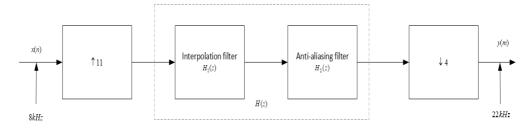
Generate a sinusoid with a frequency of 500 Hz for 0.1 s using a sampling rate of 8 kHz,

- (a) Design an interpolation and decimation processing algorithm to change the sampling rate to $22\ \text{kHz}$
 - Signal frequency range: 0-3400 Hz.
 - Hamming window required for FIR filter design
- (b) Write a MATLAB program to implement the scheme,

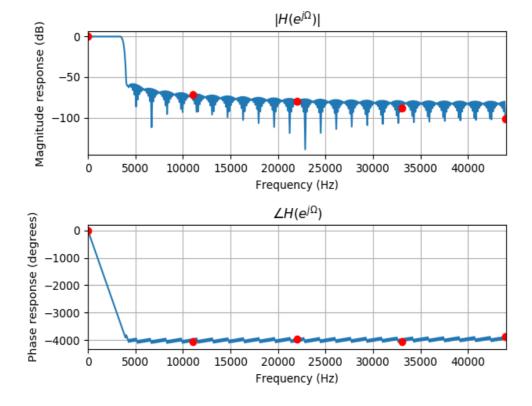
and plot the original signal and the sampled signal at the rate of 22 kHz versus the sample number, respectively.

solution

(a) Design an interpolation and decimation processing algorithm to change the sampling rate to $22\ \text{kHz}$



The combined filter, filter length = 485, $f_c=3700~{
m Hz}$



```
[0.0012, 0.001, 0.0007, 0.0004, 0.0001, -0.0003, -0.0006, -0.0009, -0.0012,
-0.0013, -0.0014, -0.0013, -0.0012, -0.001, -0.0007, -0.0004, -0.0, 0.0003,
0.0007, \ 0.001, \ 0.0012, \ 0.0014, \ 0.0014, \ 0.0014, \ 0.0013, \ 0.001, \ 0.0007, \ 0.0004,
-0.0, -0.0004, -0.0008, -0.0011, -0.0013, -0.0015, -0.0015, -0.0015, -0.0013,
-0.0011, -0.0007, -0.0003, 0.0001, 0.0005, 0.0009, 0.0012, 0.0014, 0.0016,
0.0016, 0.0015, 0.0014, 0.0011, 0.0007, 0.0003, -0.0001, -0.0006, -0.001,
-0.0013, -0.0015, -0.0017, -0.0017, -0.0016, -0.0014, -0.0011, -0.0007, -0.0003,
0.0002, 0.0006, 0.0011, 0.0014, 0.0017, 0.0018, 0.0018, 0.0017, 0.0015, 0.0012,
0.0007, 0.0003, -0.0002, -0.0007, -0.0012, -0.0016, -0.0018, -0.002, -0.002,
-0.0018, -0.0016, -0.0012, -0.0007, -0.0002, 0.0003, 0.0009,
0.0013, 0.0017, 0.002, 0.0021, 0.0021, 0.002, 0.0017, 0.0013, 0.0007, 0.0002,
-0.0004, -0.001, -0.0015, -0.0019, -0.0022, -0.0023, -0.0023, -0.0021, -0.0018,
-0.0013, -0.0007, -0.0001, 0.0005, 0.0011, 0.0017, 0.0021, 0.0024, 0.0025,
0.0025, 0.0023, 0.0019, 0.0014, 0.0007, 0.0001, -0.0006, -0.0013, -0.0019,
-0.0024, -0.0027, -0.0028, -0.0027, -0.0025, -0.002, -0.0015, -0.0007, 0.0,
0.0008, 0.0015, 0.0022, 0.0027, 0.003, 0.0032, 0.0031, 0.0027, 0.0022, 0.0016,
0.0008, -0.0001, -0.001, -0.0018, -0.0025, -0.0031, -0.0035, -0.0036, -0.0034,
-0.0031, -0.0025, -0.0017, -0.0008, 0.0002, 0.0013, 0.0022, 0.003, 0.0036,
0.004, 0.0041, 0.0039, 0.0035, 0.0028, 0.0018, 0.0008, -0.0004, -0.0016,
-0.0027, -0.0036, -0.0043, -0.0048, -0.0049, -0.0046, -0.0041, -0.0032, -0.0021,
-0.0008, 0.0007, 0.0021, 0.0034, 0.0045, 0.0053, 0.0058, 0.006, 0.0056, 0.0049,
0.0038, 0.0024, 0.0008, -0.001, -0.0028, -0.0044, -0.0059, -0.0069, -0.0075,
-0.0077, -0.0072, -0.0063, -0.0048, -0.0029, -0.0008, 0.0016, 0.004, 0.0063,
0.0082,\ 0.0097,\ 0.0106,\ 0.0108,\ 0.0102,\ 0.0089,\ 0.0068,\ 0.004,\ 0.0008,\ -0.0028,
-0.0066, -0.0102, -0.0134, -0.016, -0.0177, -0.0183, -0.0176, -0.0155, -0.012,
-0.0071, -0.0008, 0.0068, 0.0153, 0.0245, 0.0341, 0.0437, 0.053, 0.0617, 0.0693,
0.0756, 0.0802, 0.0831, 0.0841, 0.0831, 0.0802, 0.0756, 0.0693, 0.0617, 0.053,
0.0437, \ 0.0341, \ 0.0245, \ 0.0153, \ 0.0068, \ -0.0008, \ -0.0071, \ -0.012, \ -0.0155,
-0.0176, -0.0183, -0.0177, -0.016, -0.0134, -0.0102, -0.0066, -0.0028, 0.0008,
0.004, 0.0068, 0.0089, 0.0102, 0.0108, 0.0106, 0.0097, 0.0082, 0.0063, 0.004,
0.0016, -0.0008, -0.0029, -0.0048, -0.0063, -0.0072, -0.0077, -0.0075, -0.0069,
-0.0059, -0.0044, -0.0028, -0.001, 0.0008, 0.0024, 0.0038, 0.0049, 0.0056,
0.006, 0.0058, 0.0053, 0.0045, 0.0034, 0.0021, 0.0007, -0.0008, -0.0021,
-0.0032, -0.0041, -0.0046, -0.0049, -0.0048, -0.0043, -0.0036, -0.0027, -0.0016,
-0.0004, 0.0008, 0.0018,
0.0028, 0.0035, 0.0039, 0.0041, 0.004, 0.0036, 0.003, 0.0022, 0.0013, 0.0002,
-0.0008, -0.0017, -0.0025, -0.0031, -0.0034, -0.0036, -0.0035, -0.0031, -0.0025,
-0.0018, -0.001, -0.0001, 0.0008, 0.0016, 0.0022, 0.0027, 0.0031, 0.0032, 0.003,
0.0027, \ 0.0022, \ 0.0015, \ 0.0008, \ 0.0, \ -0.0007, \ -0.0015, \ -0.002, \ -0.0025, \ -0.0027,
-0.0028, -0.0027, -0.0024, -0.0019, -0.0013, -0.0006, 0.0001, 0.0007, 0.0014,
0.0019, \ 0.0023, \ 0.0025, \ 0.0025, \ 0.0024, \ 0.0021, \ 0.0017, \ 0.0011, \ 0.0005, \ -0.0001,
-0.0007, -0.0013, -0.0018, -0.0021, -0.0023, -0.0023, -0.0022, -0.0019, -0.0015,
-0.001, -0.0004, 0.0002, 0.0007, 0.0013, 0.0017, 0.002, 0.0021, 0.0021, 0.002,
0.0017, \ 0.0013, \ 0.0009, \ 0.0003, \ -0.0002, \ -0.0007, \ -0.0012, \ -0.0016, \ -0.0018,
-0.002, -0.002, -0.0018, -0.0016, -0.0012, -0.0007, -0.0002, 0.0003, 0.0007,
0.0012, 0.0015, 0.0017, 0.0018, 0.0018, 0.0017, 0.0014, 0.0011, 0.0006, 0.0002,
-0.0003, -0.0007, -0.0011, -0.0014, -0.0016, -0.0017, -0.0017, -0.0015, -0.0013,
-0.001, -0.0006, -0.0001, 0.0003, 0.0007, 0.0011, 0.0014, 0.0015, 0.0016,
0.0016, \ 0.0014, \ 0.0012, \ 0.0009, \ 0.0005, \ 0.0001, \ -0.0003, \ -0.0007, \ -0.0011,
-0.0013, -0.0015, -0.0015, -0.0015, -0.0013, -0.0011, -0.0008, -0.0004, -0.0,
0.0004, 0.0007, 0.001, 0.0013, 0.0014, 0.0014, 0.0014, 0.0012, 0.001, 0.0007,
0.0003, -0.0, -0.0004, -0.0007, -0.001, -0.0012, -0.0013, -0.0014, -0.0013,
-0.0012, -0.0009, -0.0006, -0.0003, 0.0001, 0.0004, 0.0007, 0.001, 0.0012]
```

h(n) =

```
[0.0001, 0.0001, 0.0001, 0.0, 0.0, -0.0, -0.0001, -0.0001, -0.0001, -0.0001,
-0.0001, -0.0001, -0.0001, -0.0001, -0.0001, -0.0, -0.0, 0.0, 0.0001, 0.0001,
0.0001, 0.0001, 0.0001, 0.0001, 0.0001, 0.0001, 0.0001, 0.0, -0.0, -0.0,
-0.0001, -0.0001, -0.0002, -0.0002, -0.0002, -0.0002, -0.0002, -0.0001, -0.0001,
-0.0, 0.0, 0.0001, 0.0001, 0.0002, 0.0002, 0.0002, 0.0003, 0.0003, 0.0002,
0.0002, 0.0001, 0.0001, -0.0, -0.0001, -0.0002, -0.0002, -0.0003, -0.0003,
-0.0004, -0.0003, -0.0003, -0.0002, -0.0002,
-0.0001, 0.0, 0.0002, 0.0003, 0.0003, 0.0004, 0.0005, 0.0005, 0.0005, 0.0004,
0.0003, 0.0002, 0.0001, -0.0001, -0.0002, -0.0004, -0.0005, -0.0006, -0.0006,
-0.0006, -0.0006, -0.0005, -0.0004, -0.0003, -0.0001, 0.0001, 0.0003, 0.0005,
0.0006, 0.0007, 0.0008, 0.0008, 0.0008, 0.0007, 0.0005, 0.0003, 0.0001, -0.0002,
-0.0004, -0.0006, -0.0008, -0.001, -0.001, -0.001, -0.001, -0.0008, -0.0006,
-0.0004, -0.0001, 0.0003, 0.0006, 0.0008, 0.0011, 0.0012, 0.0013, 0.0013,
0.0012, 0.001, 0.0007, 0.0004, 0.0, -0.0004, -0.0007, -0.0011, -0.0014, -0.0016,
-0.0017, -0.0016, -0.0015, -0.0012, -0.0009, -0.0005, 0.0, 0.0005, 0.001,
0.0014, 0.0017, 0.002, 0.0021, 0.002, 0.0018, 0.0015, 0.0011, 0.0005, -0.0001,
-0.0007, -0.0013, -0.0018, -0.0022, -0.0025, -0.0026, -0.0025, -0.0023, -0.0018,
-0.0013, -0.0006, 0.0002, 0.001, 0.0017, 0.0023, 0.0028, 0.0031, 0.0033, 0.0031,
0.0028, 0.0022, 0.0015, 0.0006, -0.0003, -0.0013, -0.0022, -0.003, -0.0036,
-0.004, -0.0041, -0.0039, -0.0035, -0.0027, -0.0018, -0.0007, 0.0006,
0.0018, 0.003, 0.004, 0.0047, 0.0052, 0.0053, 0.0051, 0.0044, 0.0035, 0.0022,
0.0007, -0.0009, -0.0026, -0.0041, -0.0054, -0.0064, -0.007, -0.0072, -0.0068,
-0.0059, -0.0046, -0.0028, -0.0007, 0.0015, 0.0038, 0.006, 0.0079, 0.0093,
0.0102, 0.0104, 0.0099, 0.0086, 0.0066, 0.0039, 0.0007, -0.0028, -0.0064, -0.01,
-0.0132, -0.0157, -0.0174, -0.0181, -0.0174, -0.0154, -0.0119, -0.007, -0.0008,
0.0067, 0.0152, 0.0244, 0.034, 0.0436, 0.053, 0.0616, 0.0692, 0.0755, 0.0802,
0.0831, 0.0841, 0.0831, 0.0802, 0.0755, 0.0692, 0.0616, 0.053, 0.0436, 0.034,
0.0244,\ 0.0152,\ 0.0067,\ -0.0008,\ -0.007,\ -0.0119,\ -0.0154,\ -0.0174,\ -0.0181,
-0.0174, -0.0157, -0.0132, -0.01, -0.0064, -0.0028, 0.0007, 0.0039, 0.0066,
0.0086, 0.0099, 0.0104, 0.0102, 0.0093, 0.0079, 0.006, 0.0038, 0.0015, -0.0007,
-0.0028, -0.0046, -0.0059, -0.0068, -0.0072, -0.007, -0.0064, -0.0054, -0.0041,
-0.0026, -0.0009, 0.0007, 0.0022, 0.0035, 0.0044, 0.0051, 0.0053, 0.0052,
0.0047, 0.004, 0.003, 0.0018, 0.0006, -0.0007, -0.0018, -0.0027, -0.0035,
-0.0039, -0.0041, -0.004, -0.0036, -0.003, -0.0022, -0.0013, -0.0003, 0.0006,
0.0015, 0.0022, 0.0028, 0.0031, 0.0033, 0.0031, 0.0028, 0.0023, 0.0017, 0.001,
0.0002, -0.0006, -0.0013, -0.0018, -0.0023, -0.0025, -0.0026, -0.0025, -0.0022,
-0.0018, -0.0013, -0.0007, -0.0001, 0.0005, 0.0011, 0.0015, 0.0018, 0.002,
0.0021, 0.002, 0.0017, 0.0014, 0.001, 0.0005, 0.0, -0.0005, -0.0009, -0.0012,
-0.0015, -0.0016, -0.0017, -0.0016, -0.0014, -0.0011, -0.0007, -0.0004, 0.0,
0.0004, \ 0.0007, \ 0.001, \ 0.0012, \ 0.0013, \ 0.0013, \ 0.0012, \ 0.0011, \ 0.0008, \ 0.0006,
0.0003, -0.0001, -0.0004, -0.0006, -0.0008, -0.001, -0.001, -0.001, -0.001,
-0.0008, -0.0006, -0.0004, -0.0002, 0.0001, 0.0003, 0.0005, 0.0007, 0.0008,
0.0008, 0.0008, 0.0007, 0.0006, 0.0005, 0.0003, 0.0001,
-0.0001, -0.0003, -0.0004, -0.0005, -0.0006, -0.0006, -0.0006, -0.0006, -0.0005,
-0.0004, -0.0002, -0.0001, 0.0001, 0.0002, 0.0003, 0.0004, 0.0005, 0.0005,
0.0005, 0.0004, 0.0003, 0.0003, 0.0002, 0.0, -0.0001, -0.0002, -0.0002, -0.0003,
-0.0003, -0.0004, -0.0003, -0.0003, -0.0002, -0.0002, -0.0001, -0.0, 0.0001,
0.0001, 0.0002, 0.0002, 0.0003, 0.0003, 0.0002, 0.0002, 0.0002,
0.0001, \ 0.0001, \ 0.0, \ -0.0, \ -0.0001, \ -0.0002, \ -0.0002, \ -0.0002, \ -0.0002,
-0.0002, -0.0001, -0.0001, -0.0, -0.0, 0.0, 0.0001, 0.0001, 0.0001, 0.0001,
0.0001, 0.0001, 0.0001, 0.0001, 0.0001, 0.0, -0.0, -0.0, -0.0, -0.0001, -0.0001,
-0.0001, -0.0001, -0.0001, -0.0001, -0.0001, -0.0001, -0.0001, -0.0, 0.0, 0.0,
0.0001, 0.0001, 0.0001]
```

and plot the original signal and the sampled signal at the rate of 22 kHz versus the sample number, respectively.

Up-sampled signal = $\frac{1}{L}$ original signal, (L=11)

