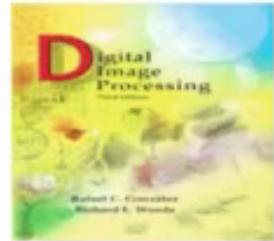


Why is Compression Needed?

$$1000 \times 1000 \times 2^4 \times 30 \times 60 \times 120 =$$

Very large number





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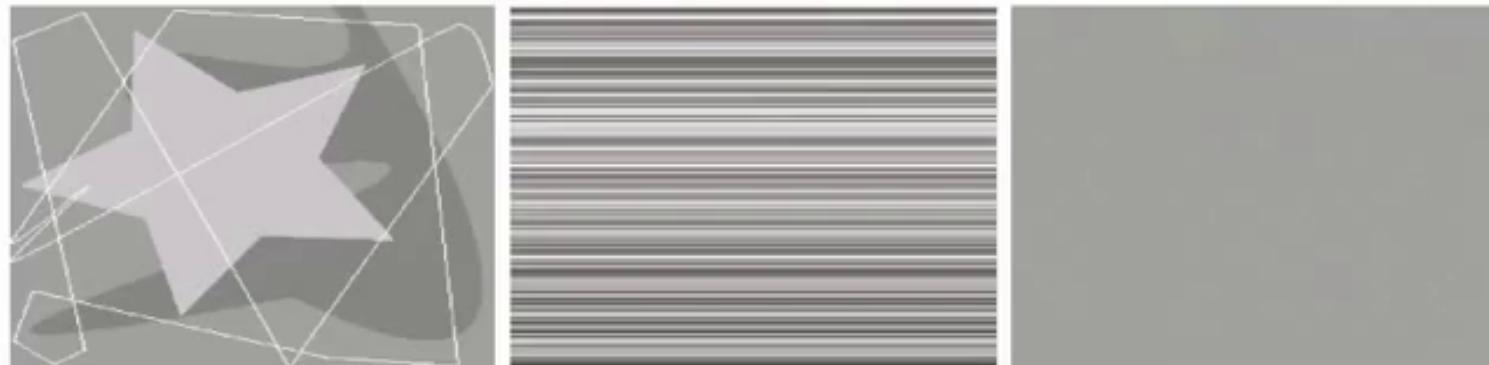
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Chapter 8

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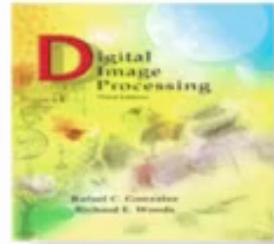


The reasons we can compress:



a b c

FIGURE 8.1 Computer generated $256 \times 256 \times 8$ bit images with (a) coding redundancy, (b) spatial redundancy, and (c) irrelevant information. (Each was designed to demonstrate one principal redundancy but may exhibit others as well.)



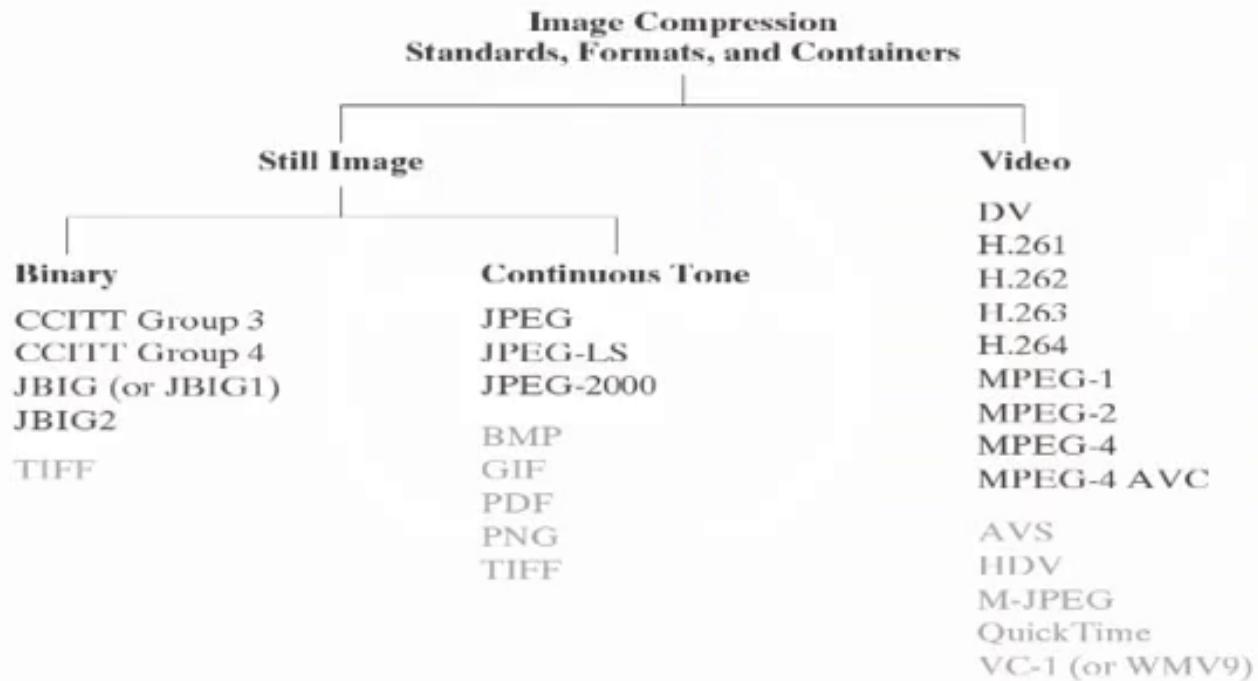
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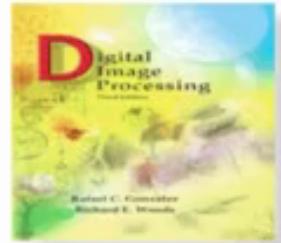
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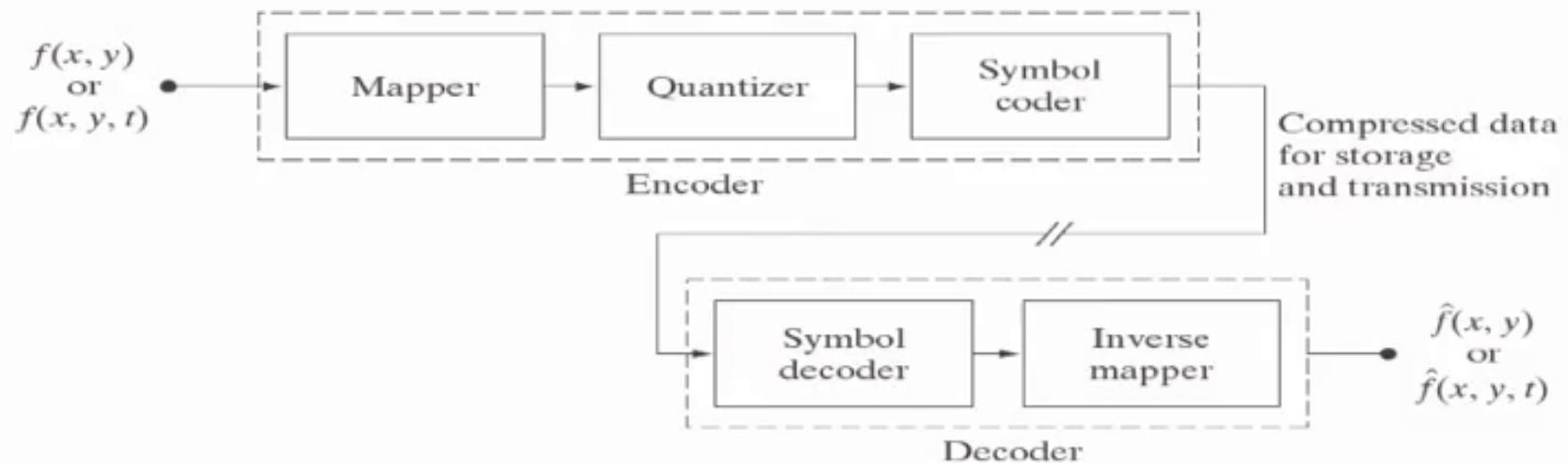
Digital Image Processing, 3rd ed.

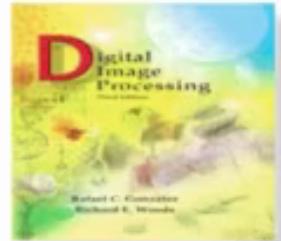
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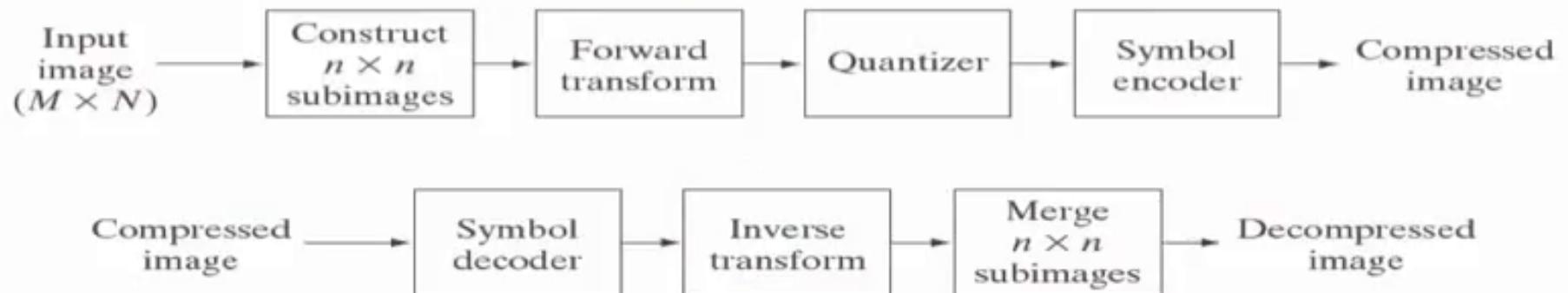
Gonzalez & Woods

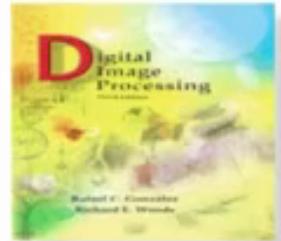
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JPEG





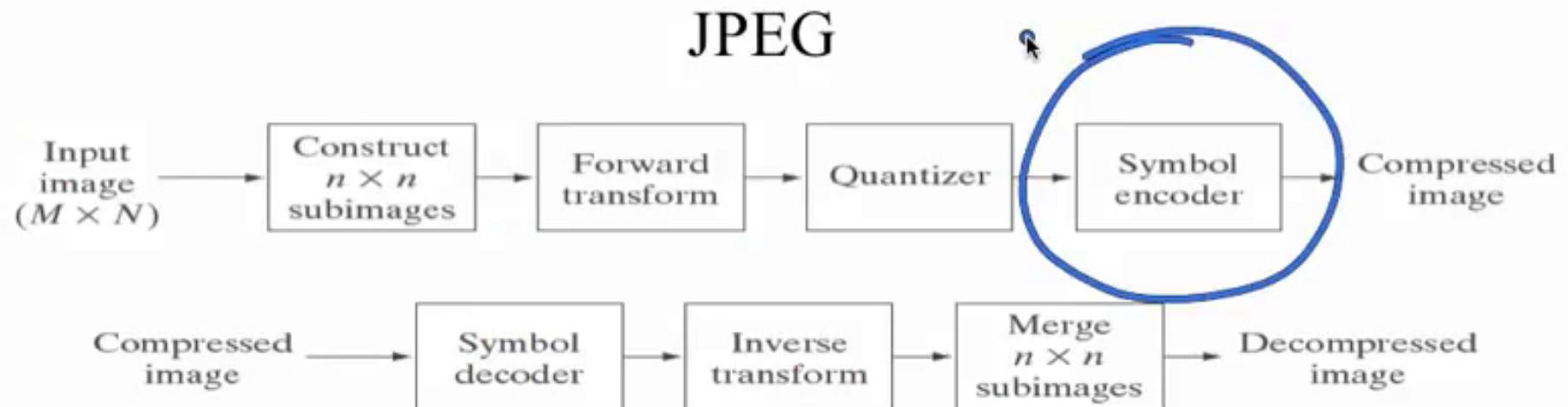
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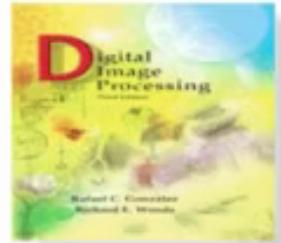
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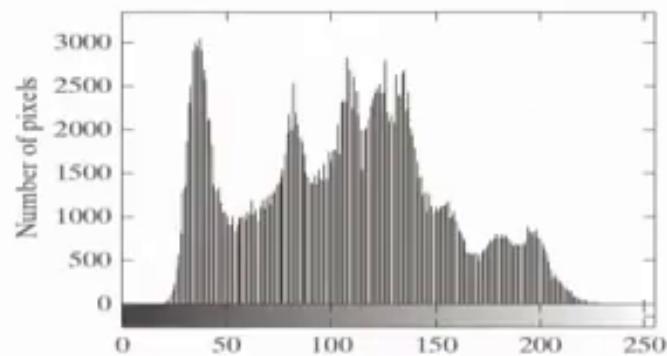
www.ImageProcessingPlace.com

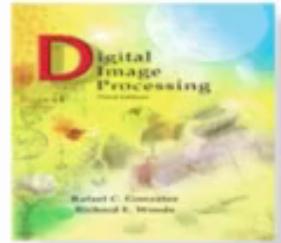
Chapter 8

Image Compression



Are all pixels equal?





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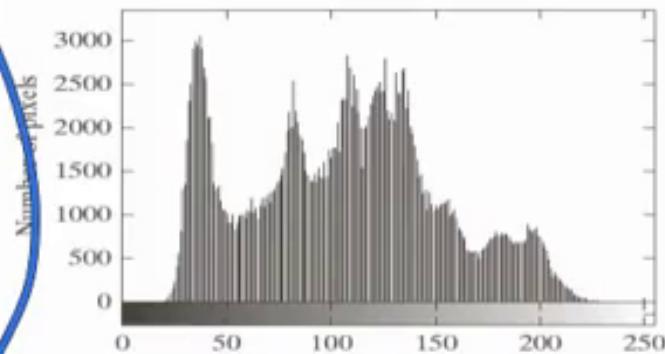
Image Compression



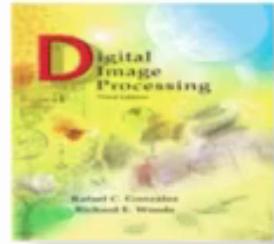
Are all pixels equal?



Lena



Histogram



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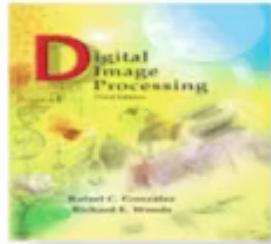
Chapter 8 Image Compression



Are all pixels/symbols equal?

r_k	$p_r(r_k)$	Code 1	$I_1(r_k)$	Code 2	$I_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$	0	—	8	—	0

•



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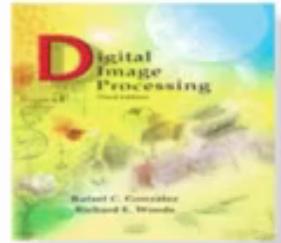


Are all pixels/symbols equal?

r_k	$p_r(r_k)$	Code 1	$I_I(r_k)$	Code 2	$I_2(r_k)$
$r_{87} = 87$	0.25	01010111	8	01	2
$r_{128} = 128$	0.47	10000000	8	1	1
$r_{186} = 186$	0.25	11000100	8	000	3
$r_{255} = 255$	0.03	11111111	8	001	3
r_k for $k \neq 87, 128, 186, 255$	0	—	8	—	0

$$0.25 \times 2 + 0.47 \times 1 + 0.25 \times 3 + 0.03 \times 3 = \\ 1.81$$





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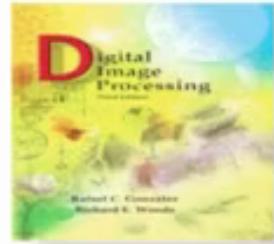
Chapter 8

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Huffman Coding

Original source		Source reduction			
Symbol	Probability	1	2	3	4
a_2	0.4	0.4	0.4	0.4	0.6
a_6	0.3	0.3	0.3	0.3	0.4
a_1	0.1	0.1	0.2	0.3	
a_4	0.1	0.1	0.1		
a_3	0.06	0.1			
a_5	0.04				



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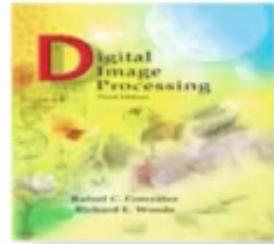
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Original source			Source reduction							
Symbol	Probability	Code	1		2		3		4	
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01	0.6	0
a_4	0.1	0100	0.1	0100	0.1	011	0.3	01	0.4	1
a_3	0.06	01010	0.1	0101	0.1	011	0.3	01	0.6	0
a_5	0.04	01011								



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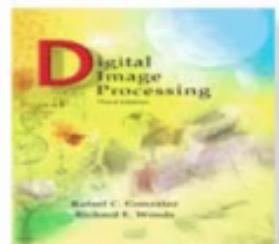
Chapter 8

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Original source			Source reduction							
Symbol	Probability	Code	1	2	3	4	5	6	7	8
a_2	0.4	1	0.4	1	0.4	1	0.4	1	0.6	0
a_6	0.3	00	0.3	00	0.3	00	0.3	00	0.4	1
a_1	0.1	011	0.1	011	0.2	010	0.3	01		
a_4	0.1	0100	0.1	0100	0.1	011				
a_3	0.06	01010	0.1	0101						
a_5	0.04	01011								

$$\text{Entropy} = H = - \sum_{\text{symbols}} p(s) \log_2 p(s)$$



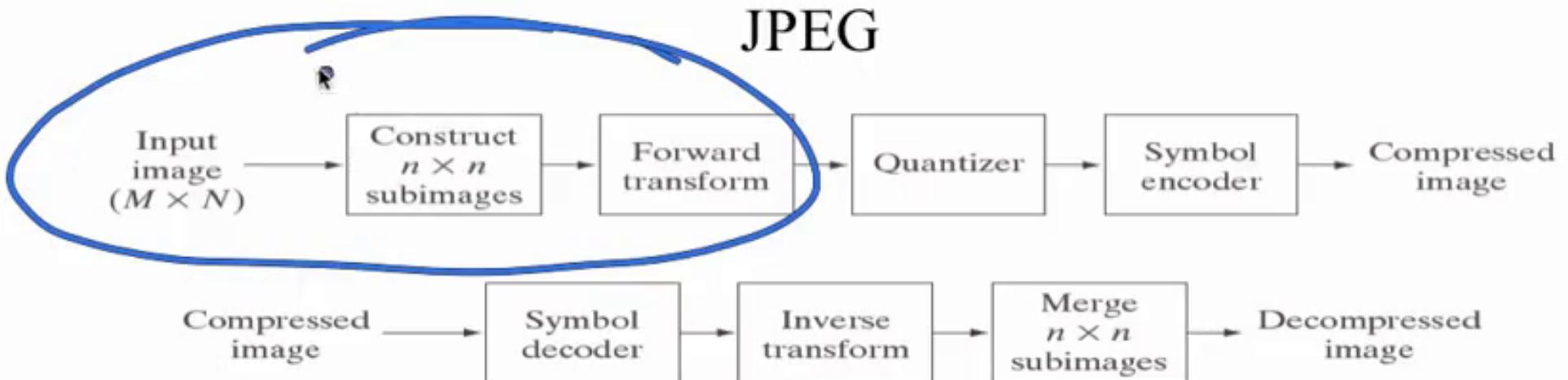
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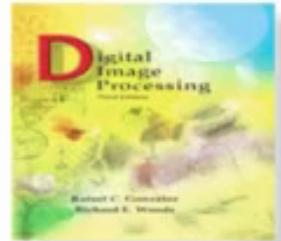
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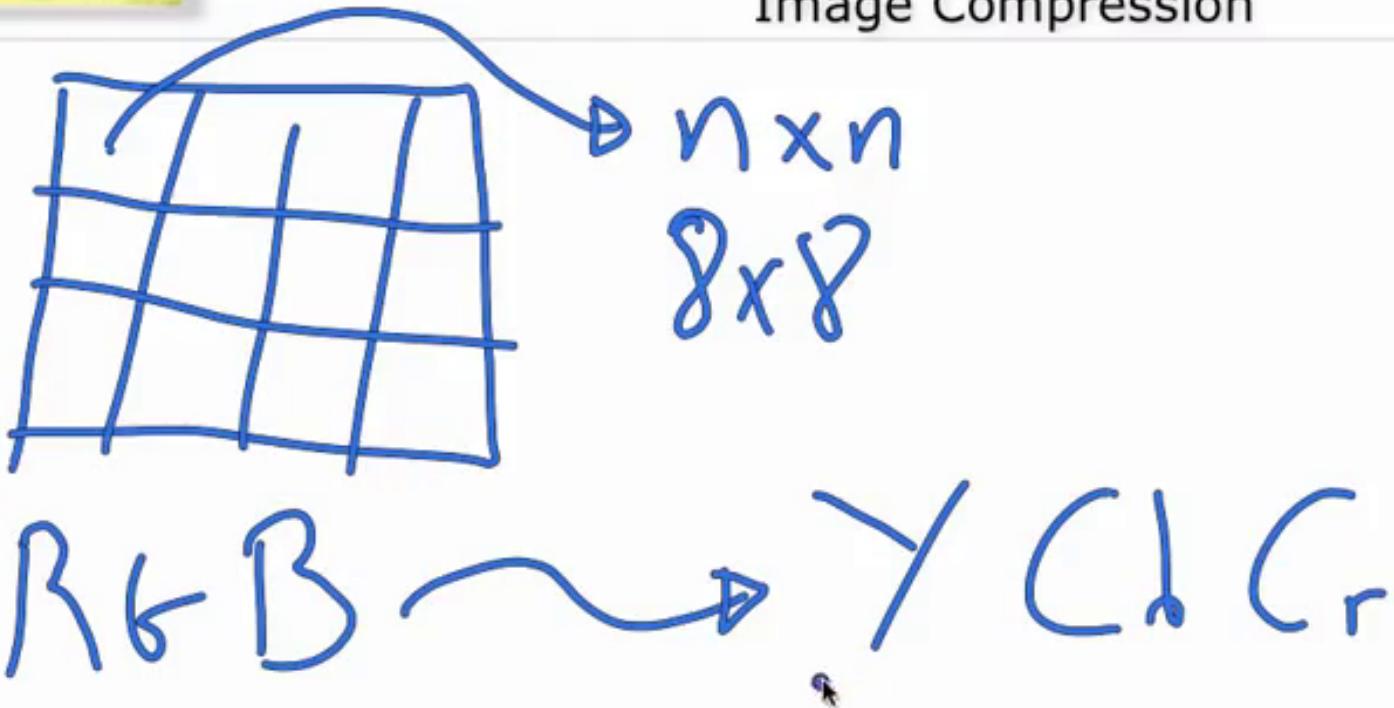
Digital Image Processing, 3rd ed.

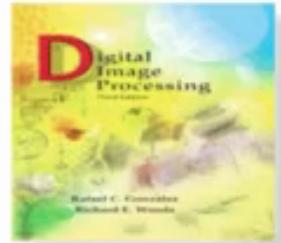
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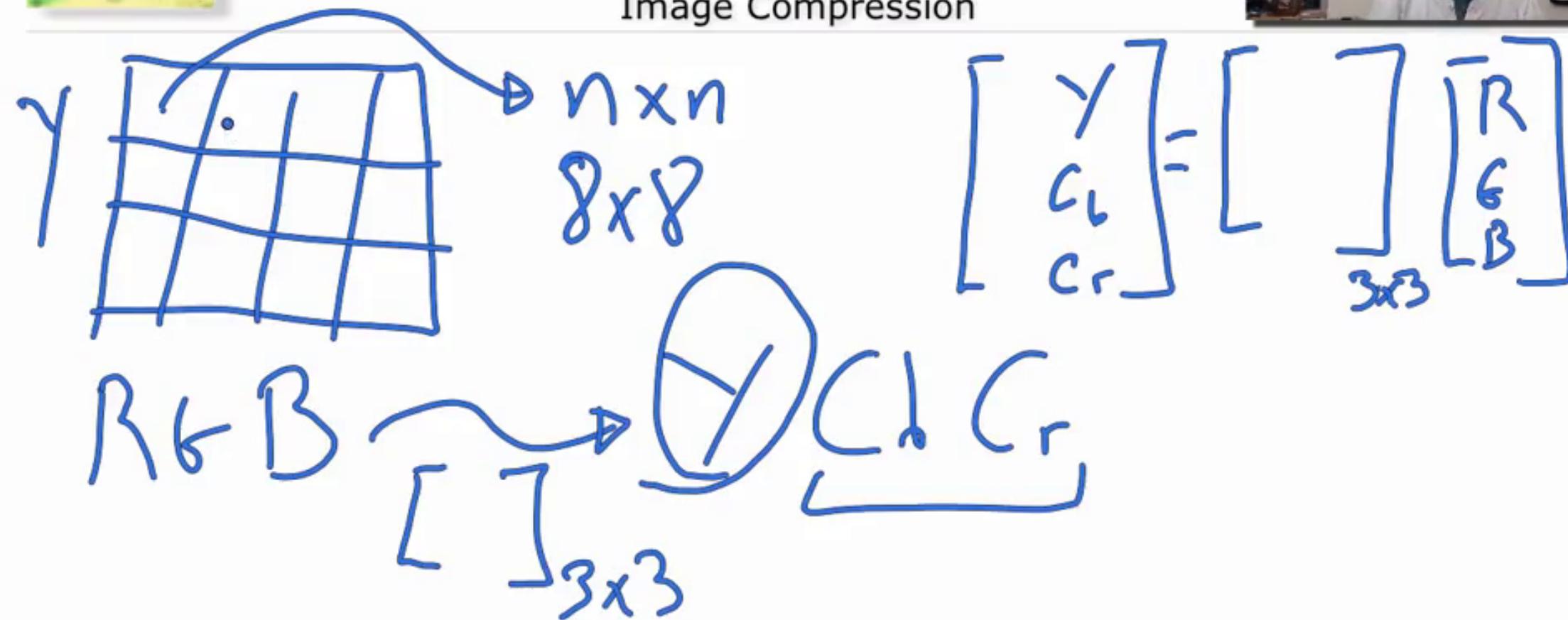
Digital Image Processing, 3rd ed.

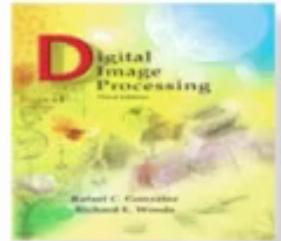
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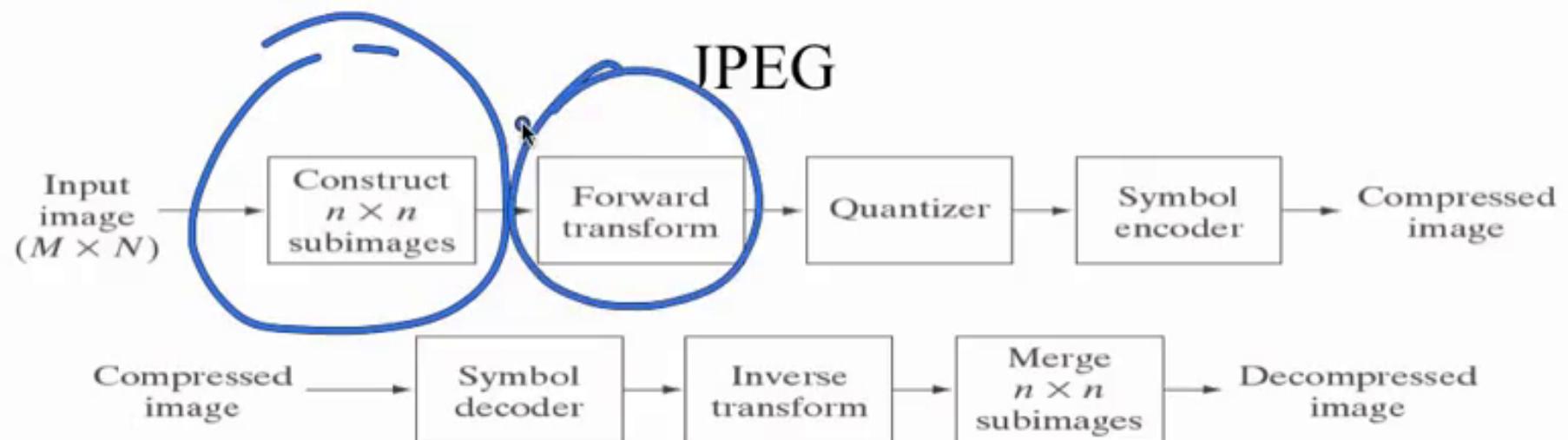


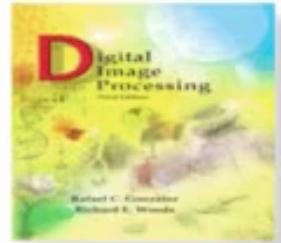
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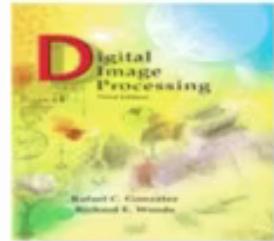
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MSE = Mean Square Error

$$MSE = \left[\frac{1}{\text{# of pixels}} \sum_{\text{pixels}} (\hat{f} - f)^2 \right]^{1/2}$$

$n \times n$ kahnen-Loeve
KLT



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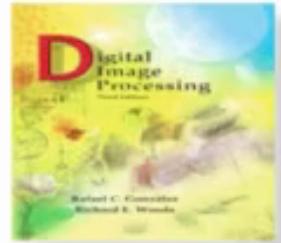
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$$T(u, v) = \sum_{x=0}^{n-1} \sum_{y=0}^{n-1} f(x, y) r(x, y, u, v)$$

$n \times n$

$$f(x, y) = \sum_{u=0}^{n-1} \sum_{v=0}^{n-1} T(u, v) s(x, y, u, v)$$



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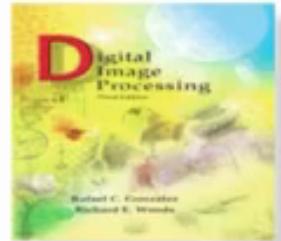


$$r(x, y, u, v) = s(x, y, u, v)$$

$$= \alpha(u)\alpha(v) \cos\left[\frac{(2x+1)u\pi}{2n}\right].$$

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{n}} & u=0 \\ \sqrt{\frac{2}{n}} & u \neq 0 \end{cases}$$
$$\cos\left[\frac{(2y+1)v\pi}{2n}\right]$$

DCT



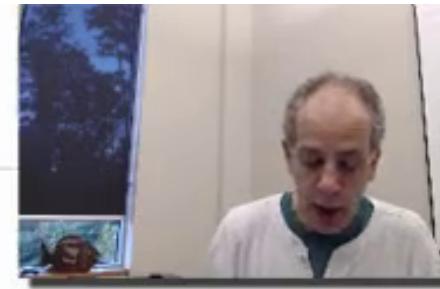
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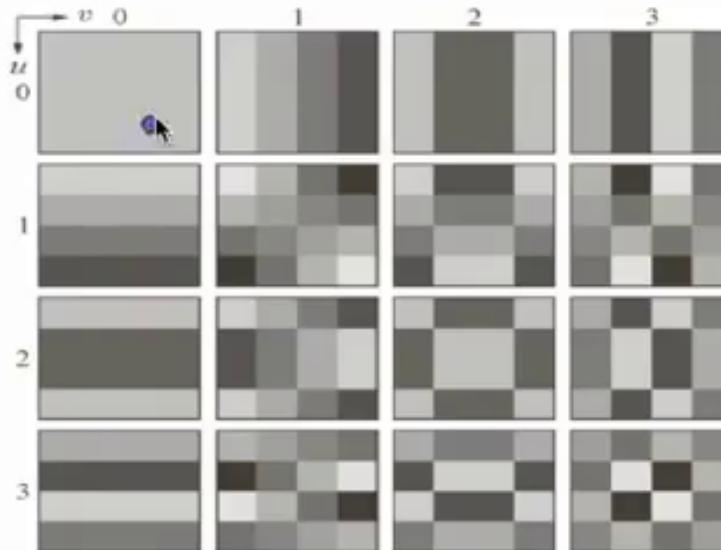
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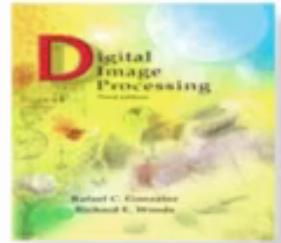
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Discrete Cosine Transform

$$n=4$$





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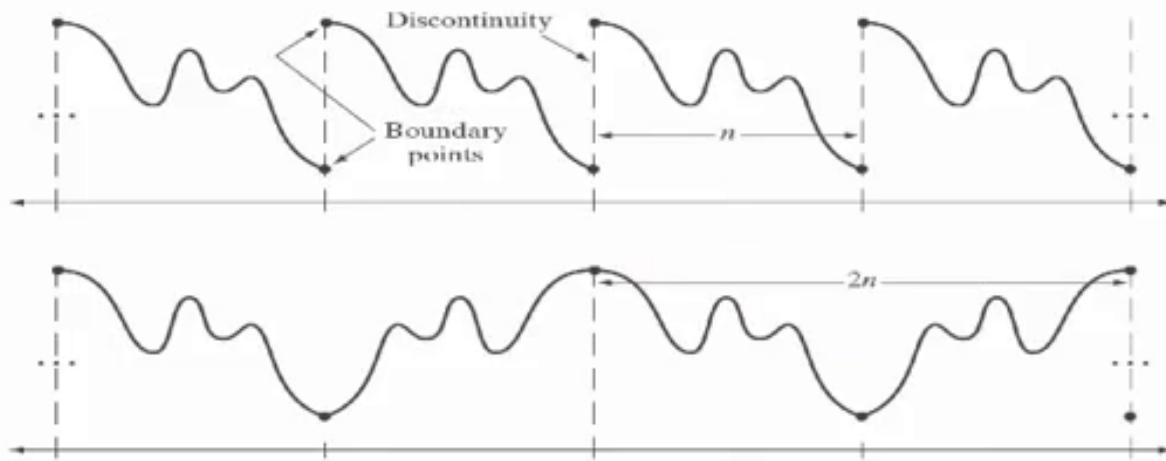
Gonzalez & Woods

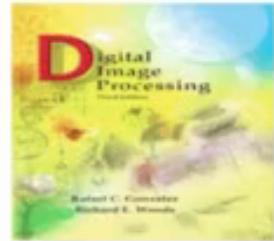
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Why DCT?





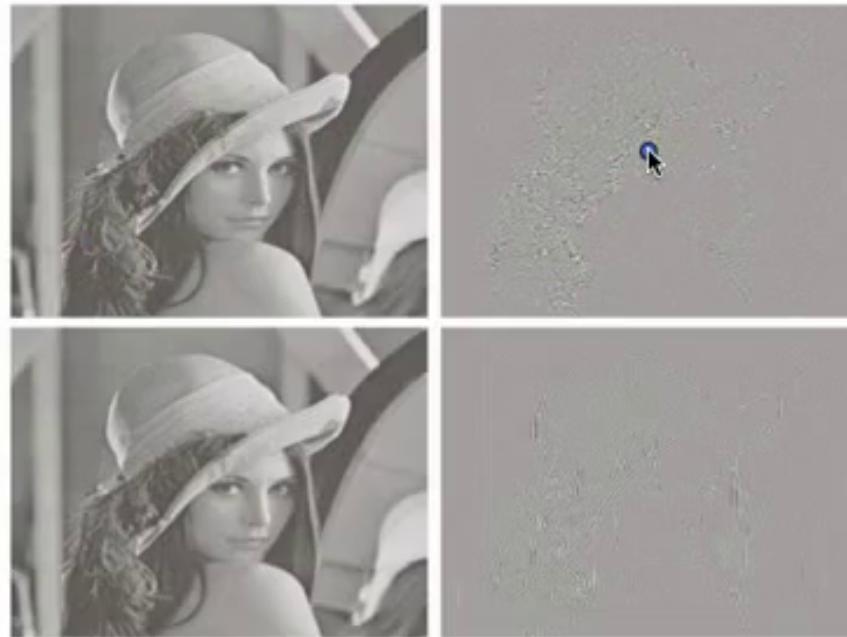
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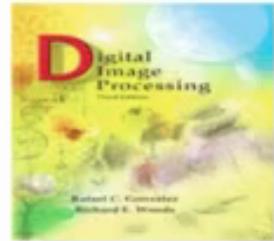
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Image Compression



a b
c d

FIGURE 8.28
Approximations
of Fig. 8.9(a) using
12.5% of the
 8×8 DCT
coefficients:
(a)–(b) threshold
coding results;
(c)–(d) zonal
coding results. The
difference images
are scaled by 4.



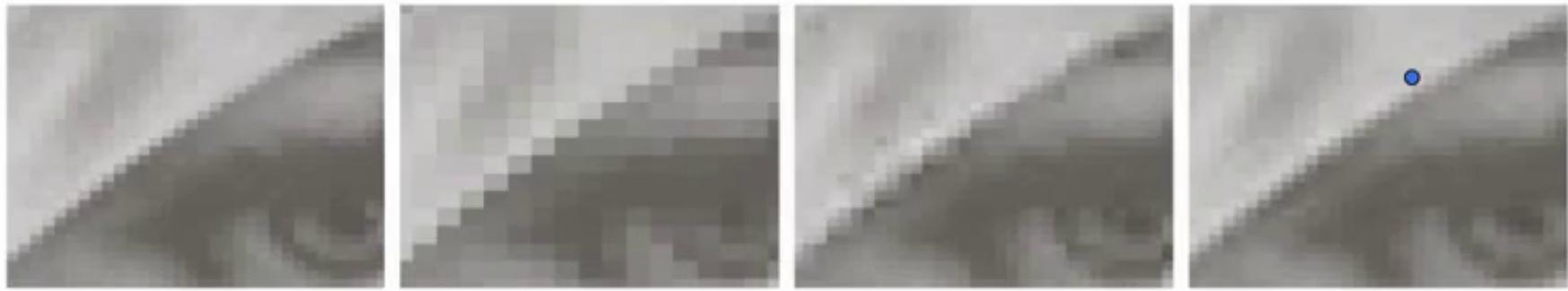
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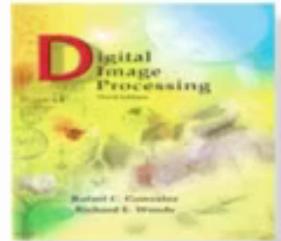
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a b c d

FIGURE 8.27 Approximations of Fig. 8.27(a) using 25% of the DCT coefficients and (b) 2×2 subimages, (c) 4×4 subimages, and (d) 8×8 subimages. The original image in (a) is a zoomed section of Fig. 8.9(a).



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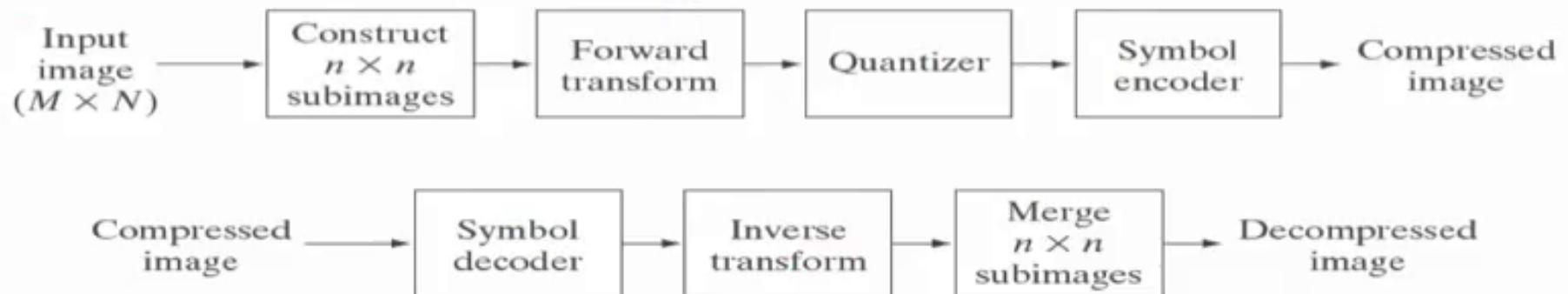
www.ImageProcessingPlace.com

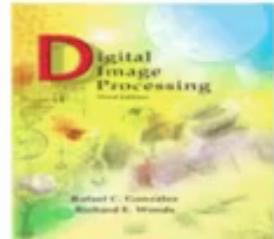
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JPEG





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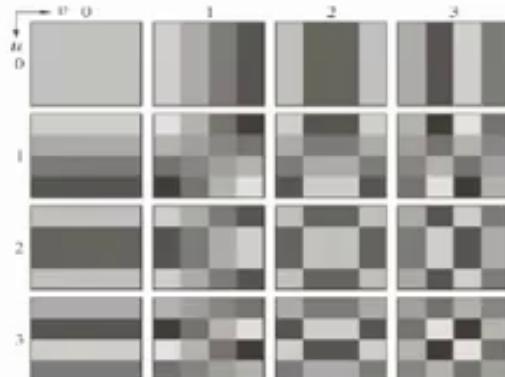


1	1	1	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

8	7	6	4	3	2	1	0
7	6	5	4	3	2	1	0
6	5	4	3	3	1	1	0
4	4	3	3	2	1	0	0
3	3	3	2	1	1	0	0
2	2	1	1	1	0	0	0
1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0

a b
c d

FIGURE 8.29
A typical
(a) zonal mask,
(b) zonal bit
allocation,
(c) threshold
mask, and
(d) thresholded
coefficient
ordering
sequence. Shading
highlights the
coefficients that
are retained.



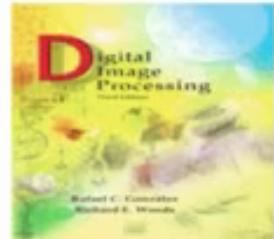
1	1	0	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	0	0	0	0	0	0
1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

0	1	5	6	14	15	27	28
2	4	7	13	16	26	29	42
3	8	12	17	25	30	41	43
9	11	18	24	31	40	44	53
10	19	23	32	39	45	52	54
20	22	33	38	46	51	55	60
21	34	37	47	50	56	59	61
35	36	48	49	57	58	62	63

$$\hat{T}(u, v) = \sum_x \sum_y F(x, y) r(x, y, u, v)$$

$$\hat{f}(x, y) = \sum_u \sum_v \hat{T}(u, v) r(x, y, u, v)$$

$$\hat{T}(0, 0) \quad \hat{T}(0, 1) \quad \hat{T}(1, 0)$$



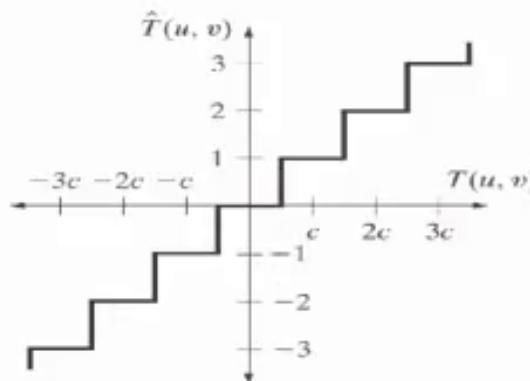
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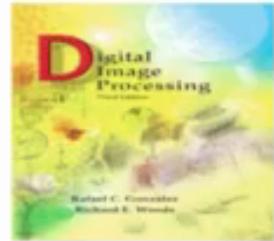
16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

a b

FIGURE 8.30
(a) A threshold coding quantization curve [see Eq. (8.2-29)]. (b) A typical normalization matrix.

Students: Good place to take a break if you need it.

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UNIVERSITY

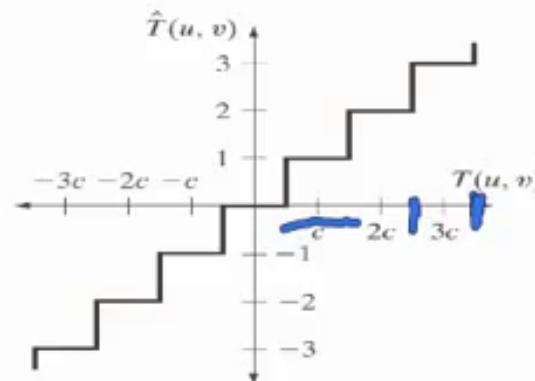


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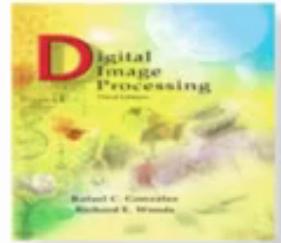


16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

a b

FIGURE 8.30
(a) A threshold coding quantization curve [see Eq. (8.2-29)]. (b) A typical normalization matrix.

$$\left[\frac{T(0,0)}{16} \right] \times 16 \simeq \hat{T}(0,0)$$
$$0..15 \simeq 0$$
$$16..31 \simeq 1$$



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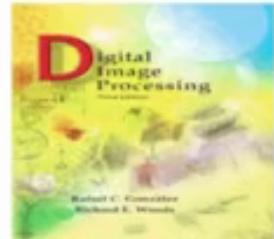
Chapter 8

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52	55	61	66	70	61	64	73
63	59	66	90	109	85	69	72
62	59	68	113	144	104	66	73
63	58	71	122	154	106	70	69
67	61	68	104	126	88	68	70
79	65	60	70	77	63	58	75
85	71	64	59	55	61	65	83
87	79	69	68	65	76	78	94

EXAMPLE 8.17:
JPEG baseline
coding and
decoding.



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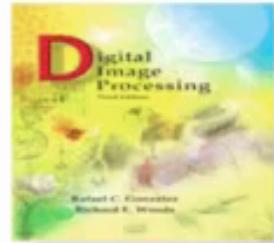
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FIGURE 8.31 Approximations of Fig. 8.9(a) using the DCT and normalization array of Fig. 8.30(b): (a) Z , (b) $2Z$, (c) $4Z$, (d) $8Z$, (e) $16Z$, and (f) $32Z$.



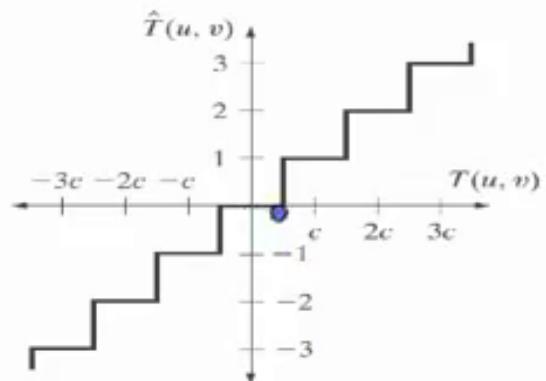
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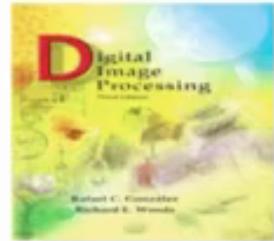
Image Compression



16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

a b

FIGURE 8.30
(a) A threshold coding quantization curve [see Eq. (8.2-29)]. (b) A typical normalization matrix.



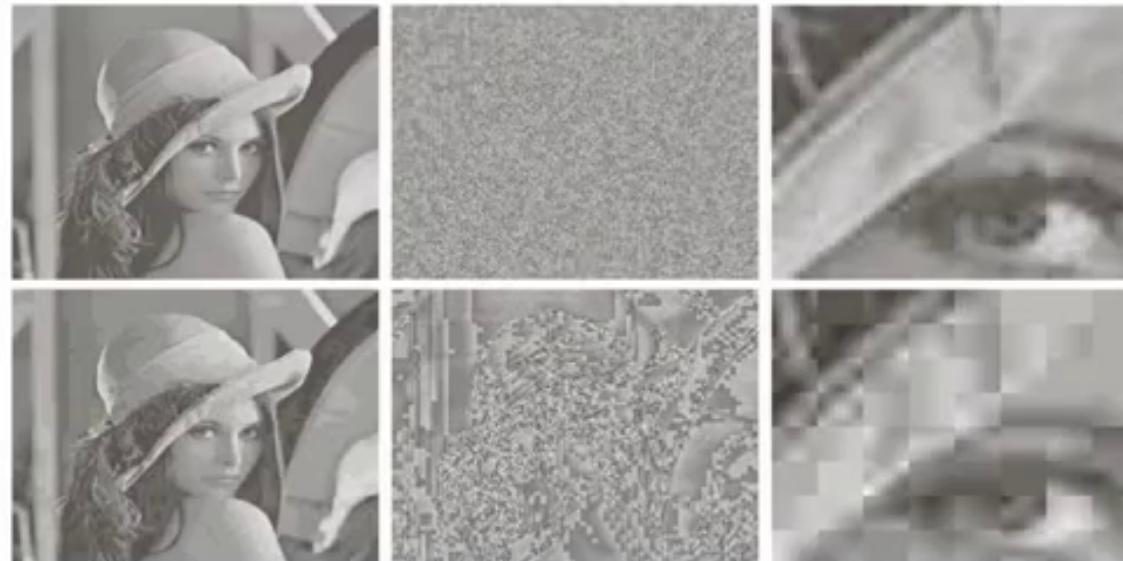
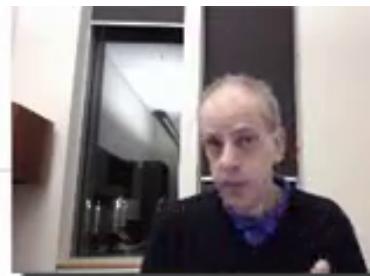
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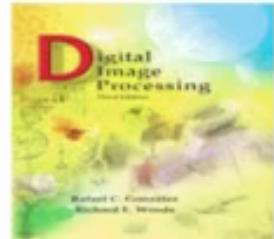
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a b c
d e f

FIGURE 8.32 Two JPEG approximations of Fig. 8.9(a). Each row contains a result after compression and reconstruction, the scaled difference between the result and the original image, and a zoomed portion of the reconstructed image.

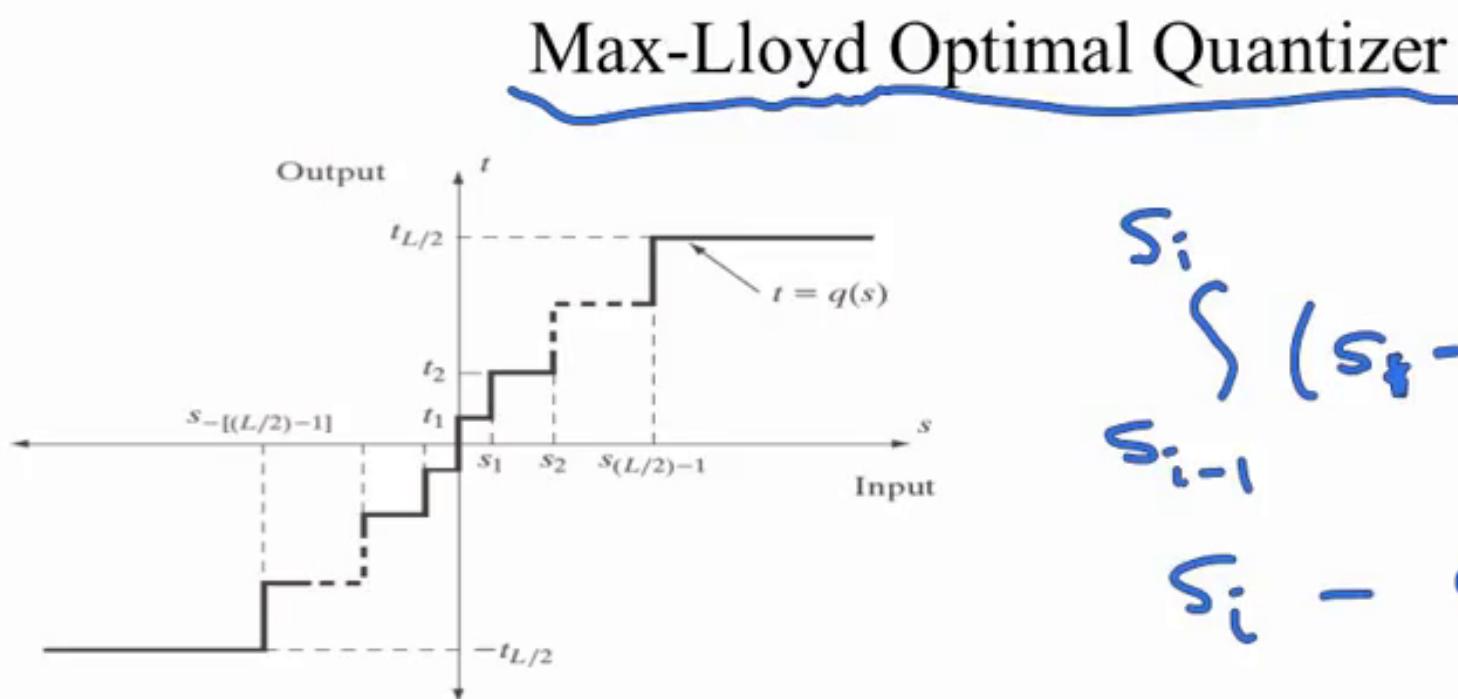


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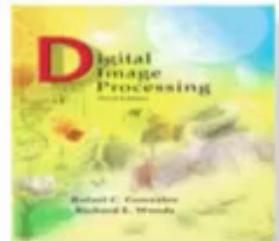
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Error $(s_i - t_i)^2$

$$s_i \sum_{s_{i-1}}^s (s_i - t_i) p(s) ds$$
$$s_i - \text{average } t_i, t_{i+1}$$



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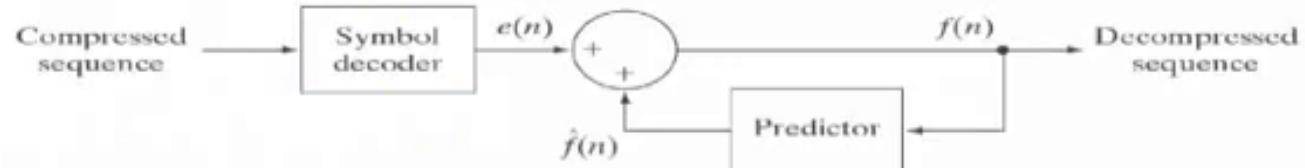
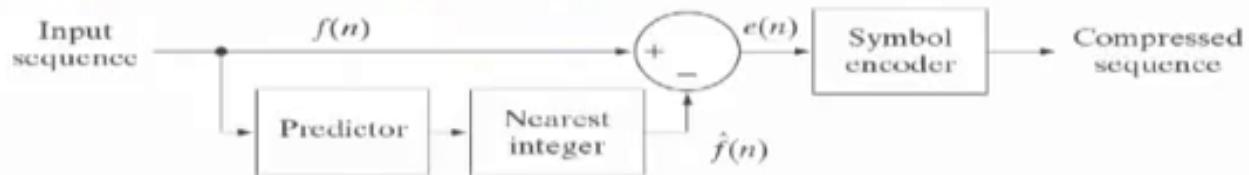
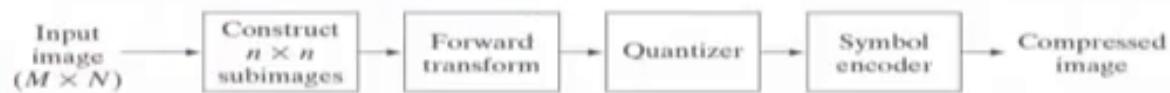
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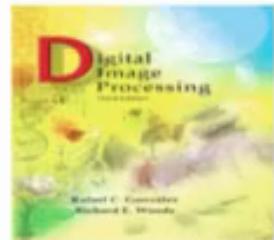
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Predictive lossless compression





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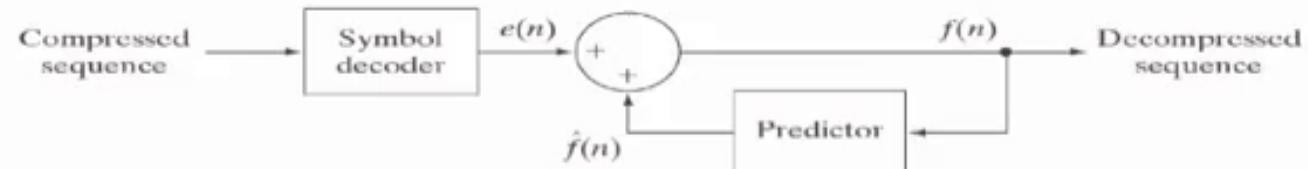
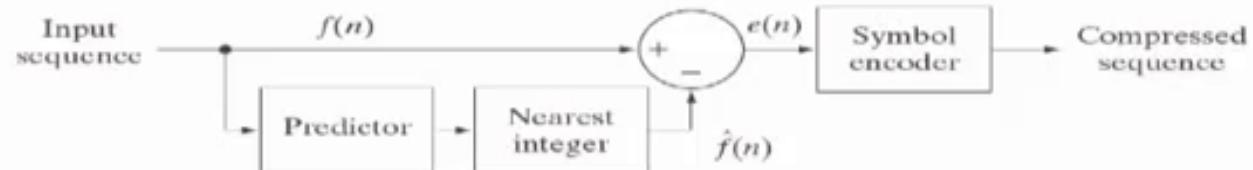
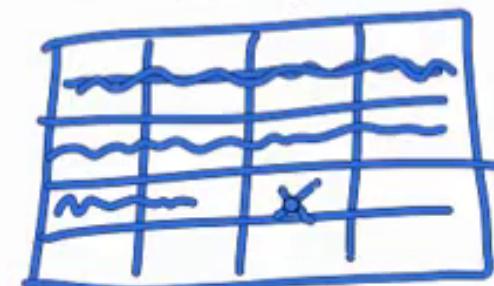
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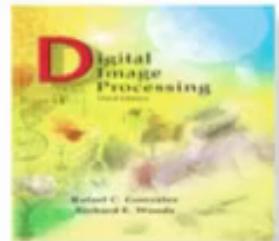
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Predictive lossless compression





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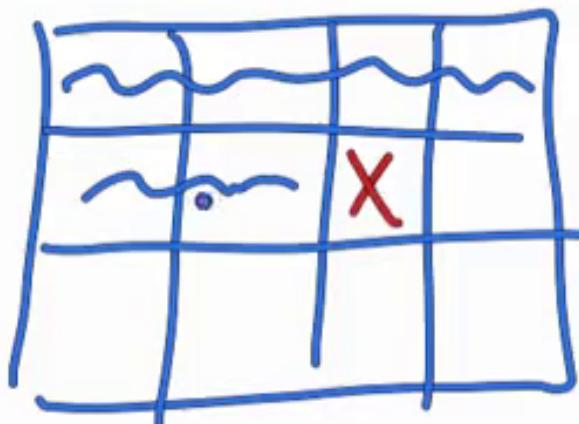
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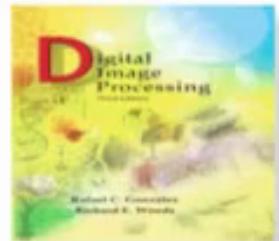
Image Compression



Examples of predictors and JPEG-LS



$$e = f(x-1, y) - f(x, y)$$



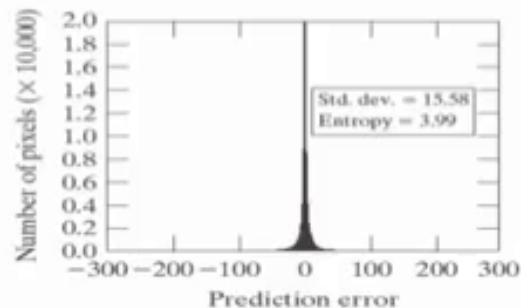
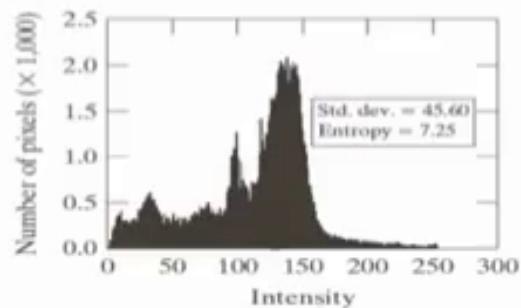
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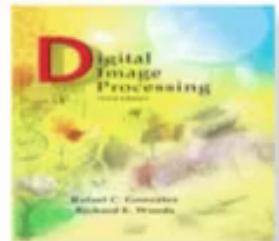
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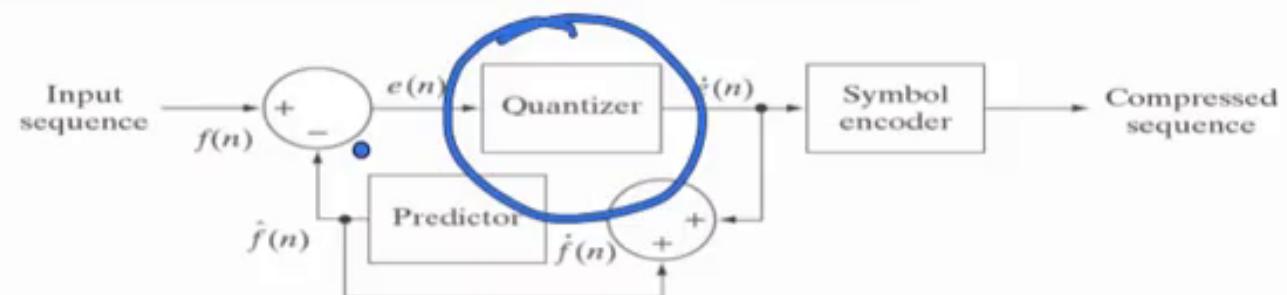
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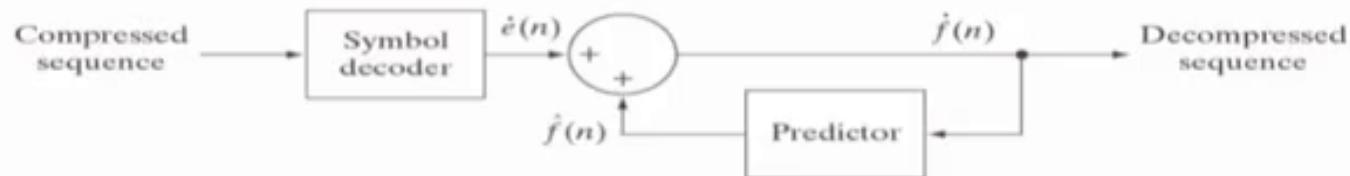
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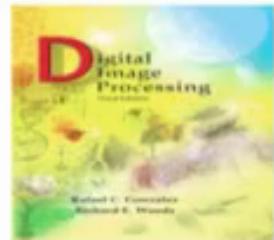
Image Compression



a
b

FIGURE 8.41
A lossless
predictive
coding model:
(a) encoder;
(b) decoder.





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Temporal prediction (MPEG)

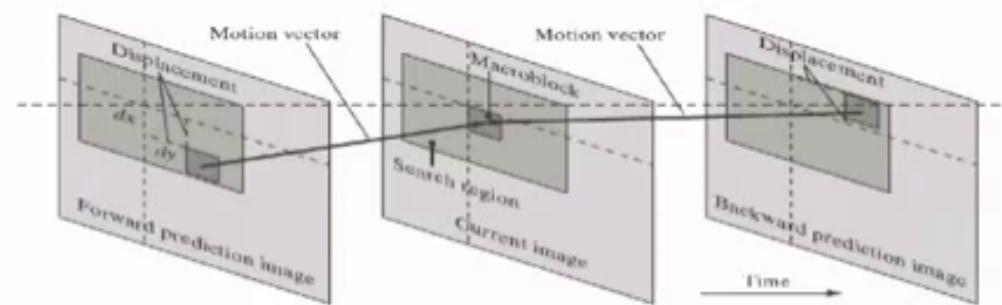
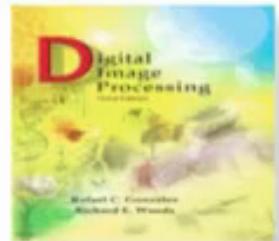


FIGURE 8.36
Macroblock motion specification.

Students: Good place to take a break if you need it.

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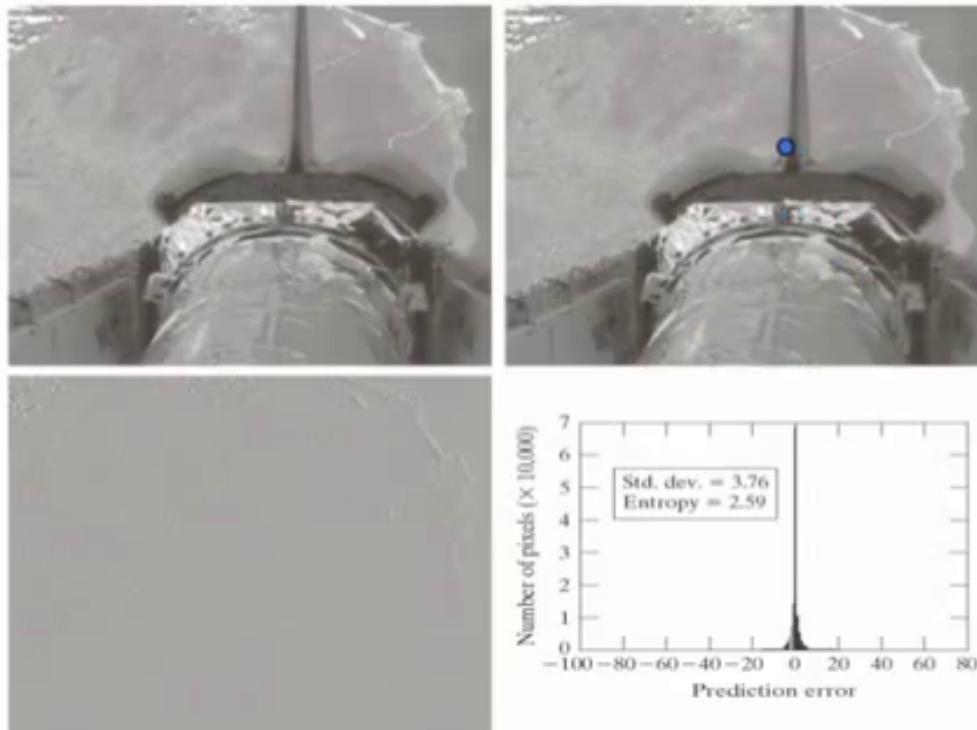
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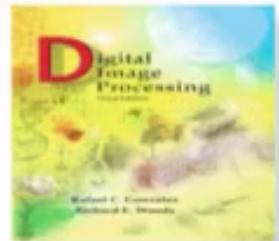
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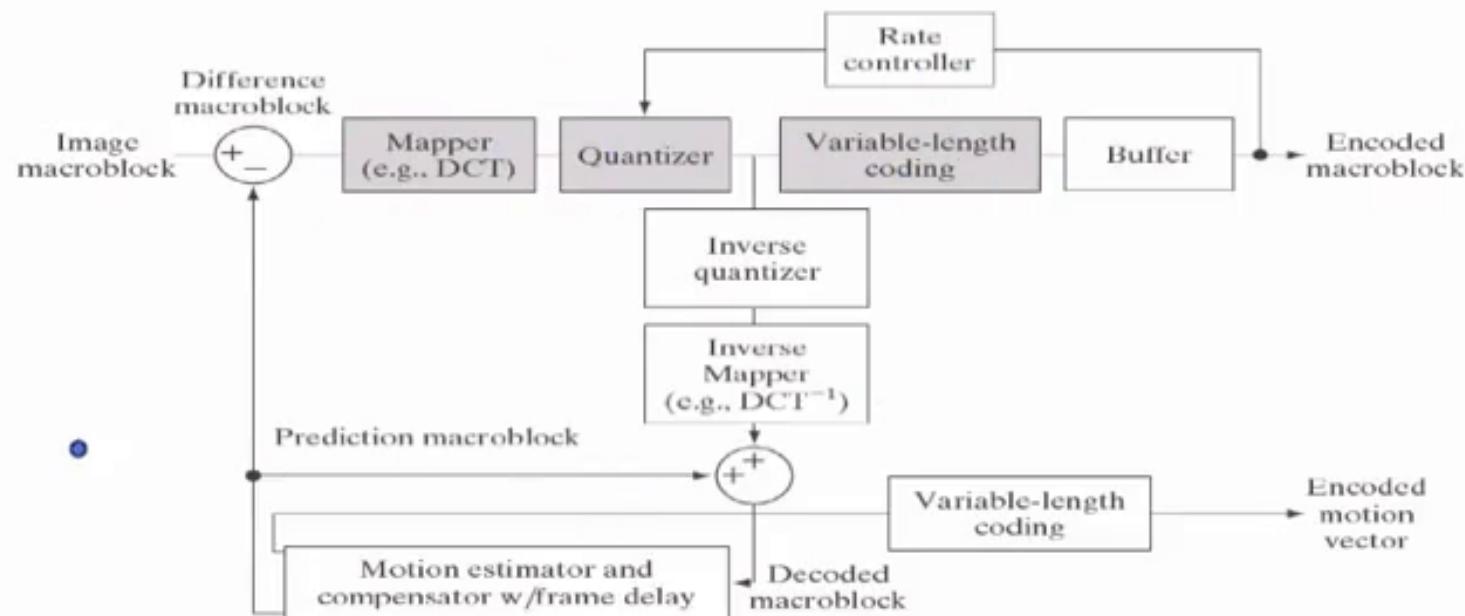
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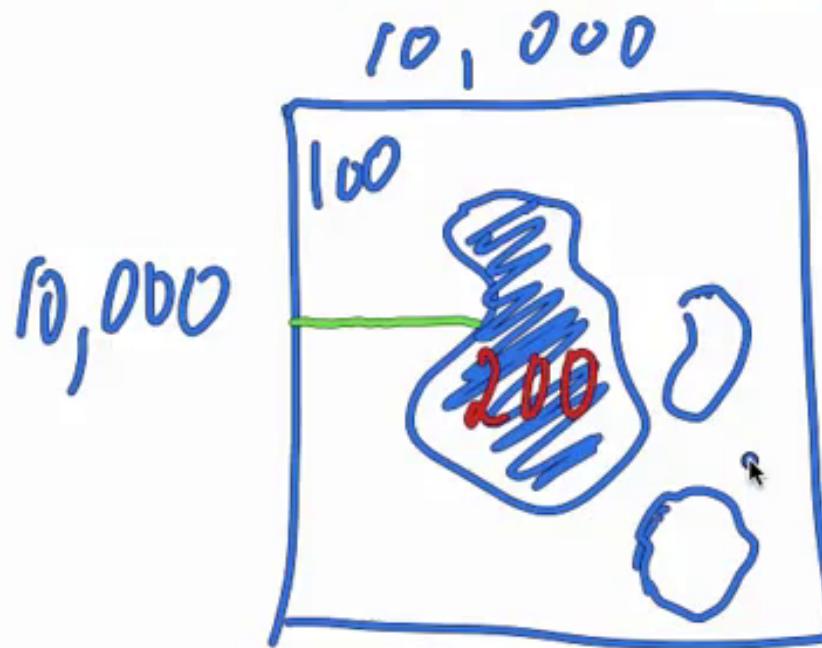
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Bonus: Run-length Coding



3000, 1000

