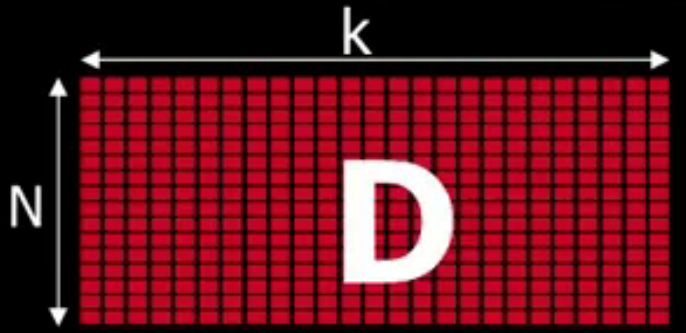
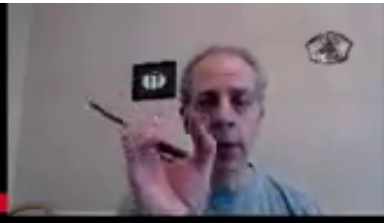
From Local to Global Treatment

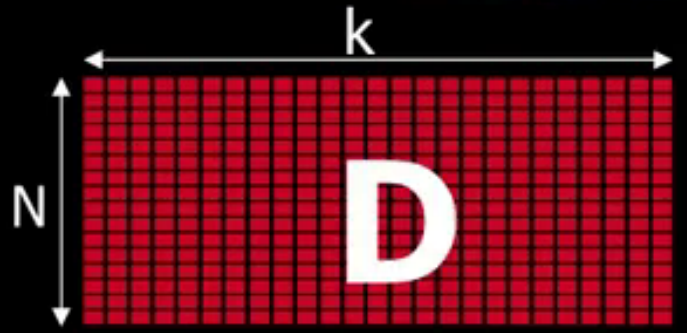


$$\hat{\underline{x}} = \underset{\underline{x},\{\underline{\alpha}_{ij}\}_{ij}}{\text{ArgMin}} \frac{1}{2}\left\|\underline{x} - \underline{y}\right\|_2^2 + \mu \sum_{ij} \left\|\mathbf{R}_{ij}\underline{x} - \mathbf{D}\underline{\alpha}_{ij}\right\|_2^2$$

$$\text{s.t.} \quad \left\|\underline{\alpha}_{ij}\right\|_0^0 \leq L$$

From Local to Global Treatment



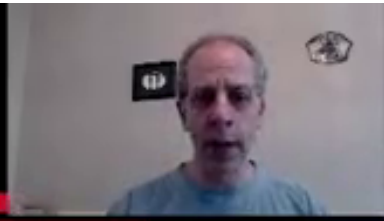


$$\hat{\underline{x}} = \underset{\underline{x},\{\underline{\alpha}_{ij}\}_{ij}}{\text{ArgMin}} \frac{1}{2}\left\|\underline{x}-\underline{y}\right\|_2^2 + \mu\sum_{ij}\left\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}_{ij}\right\|_2^2 \quad \text{s.t.} \quad \left\|\underline{\alpha}_{ij}\right\|_0^0 \le L$$

Extracts a patch in the ij location

Our prior

What Data to Train On?



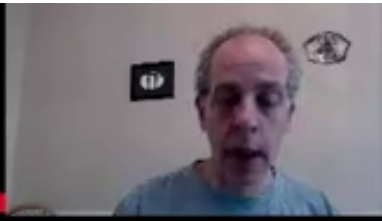
Option 1:

- Use a database of images

Option 2:

- Use the corrupted image itself !!

K-SVD Image Denoising



$$\hat{\underline{x}} = \underset{\underline{x},\{\underline{\alpha}_{ij}\}_{ij},\mathbf{D}}{\text{ArgMin}} \frac{1}{2}\left\|\underline{x}-\underline{y}\right\|_2^2 + \mu\sum_{ij}\left\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}_{ij}\right\|_2^2 \text{ s.t. } \left\|\underline{\alpha}_{ij}\right\|_0^0 \le L$$

$\underline{x}=\underline{y}$ and $\mathbf{D}$ known

Compute α_{ij} per patch

$$\underline{\alpha}_{ij} = \underset{\underline{\alpha}}{\text{Min}}\left\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}\right\|_2^2 \quad \text{s.t. } \left\|\underline{\alpha}\right\|_0^0 \le L$$

using the matching pursuit

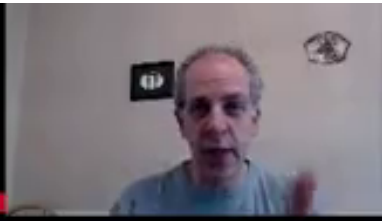
$\underline{x}$ and α_{ij} known

Compute $\mathbf{D}$ to minimize

$$\underset{\underline{\alpha}}{\text{Min}}\sum_{ij}\left\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}\right\|_2^2$$

using SVD, updating one column at a time

K-SVD Image Denoising



$$\hat{\underline{x}} = \underset{\underline{x},\{\underline{\alpha}_{ij}\}_{ij},\mathbf{D}}{\text{ArgMin}} \frac{1}{2}\left\|\underline{x}-\underline{y}\right\|_2^2 + \mu\sum_{ij}\left\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}_{ij}\right\|_2^2 \text{ s.t. } \left\|\underline{\alpha}_{ij}\right\|_0^0 \le L$$

$\underline{x}=\underline{y}$ and $\mathbf{D}$ known

Compute α_{ij} per patch

$$\underline{\alpha}_{ij} = \underset{\underline{\alpha}}{\text{Min}}\left\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}\right\|_2^2 \quad \text{s.t. } \left\|\underline{\alpha}\right\|_0^0 \le L$$

using the matching pursuit

$\underline{x}$ and α_{ij} known

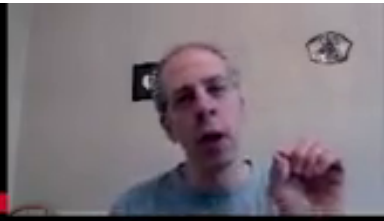
Compute $\mathbf{D}$ to minimize

$$\underset{\underline{\alpha}}{\text{Min}}\sum_{ij}\left\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}\right\|_2^2$$

g SVD, updating one column at a time

K-SVD

K-SVD Image Denoising



$$\hat{\underline{x}} = \underset{\underline{x},\{\underline{\alpha}_{ij}\}_{ij},\mathbf{D}}{\text{ArgMin}} \frac{1}{2}\|\underline{x}-\underline{y}\|_2^2 + \mu\sum_{ij}\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}_{ij}\|_2^2 \text{ s.t. } \|\underline{\alpha}_{ij}\|_0^0 \le L$$

$\underline{x}=\underline{y}$ and **D** known → Compute α_{ij} per patch

$$\underline{\alpha}_{ij} = \underset{\underline{\alpha}}{\text{Min}}\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}\|_2^2 \quad \text{s.t. } \|\underline{\alpha}\|_0^0 \le L$$

using the matching pursuit

$\underline{x}$ and α_{ij} known → Compute **D** to minimize

$$\underset{\underline{\alpha}}{\text{Min}}\sum_{ij}\|\mathbf{R}_{ij}\underline{x}-\mathbf{D}\underline{\alpha}\|_2^2$$

g SVD, updating one column at a time

D and α_{ij} known → Compute $\underline{x}$ by

$$\underline{x} = \left[I+\mu\sum_{ij}\mathbf{R}_{ij}^T\mathbf{R}_{ij}\right]^{-1}\left[\underline{y}+\mu\sum_{ij}\mathbf{R}_{ij}^T\mathbf{D}\underline{\alpha}_{ij}\right]$$

which is a simple averaging of shifted patches

K-SVD

$$R_{ij}x = x_{ij} = D\hat{\alpha}$$

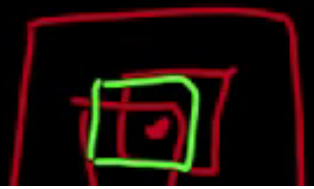
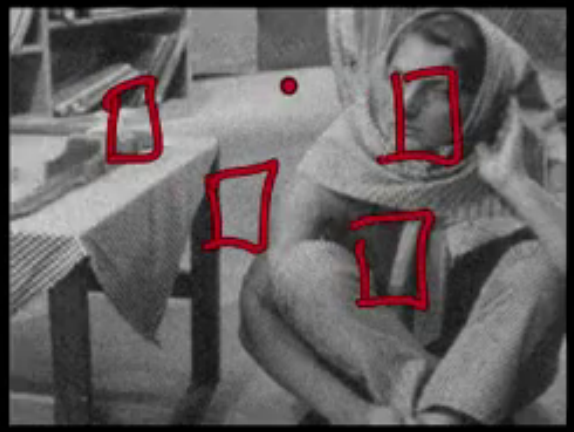
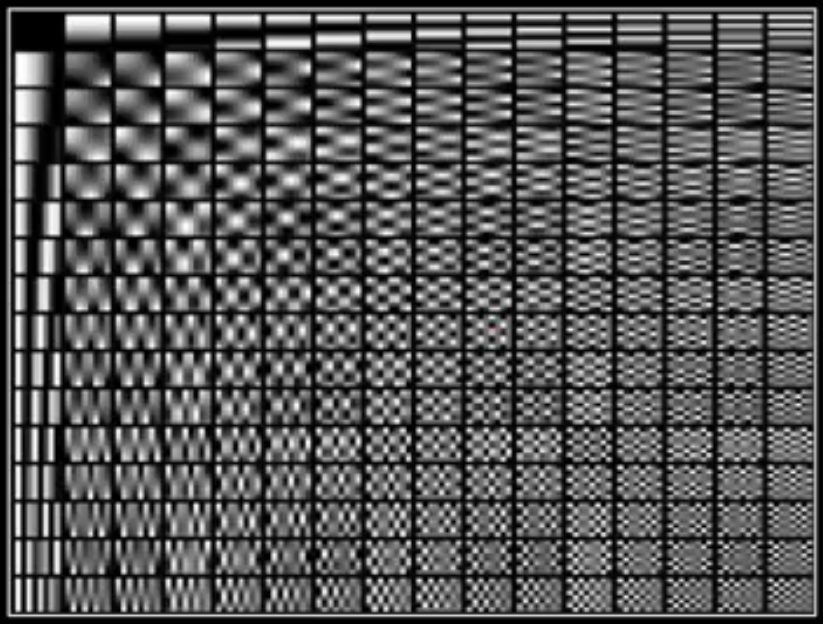


Image Denoising (Gray)

Source

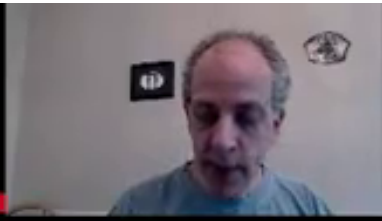


Noisy image
$\sigma = 20$



Initial dictionary (overcomplete DCT) 64×256

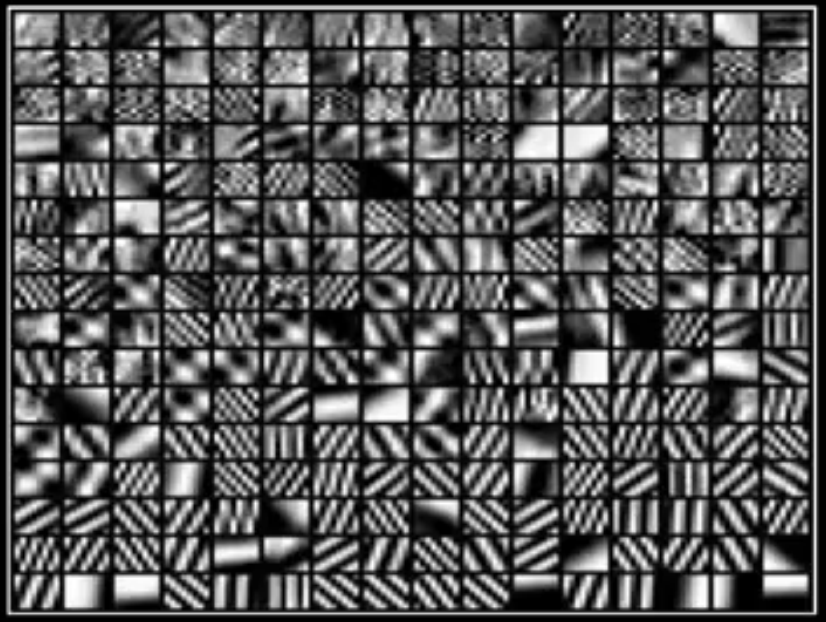
Image Denoising (Gray)



Source

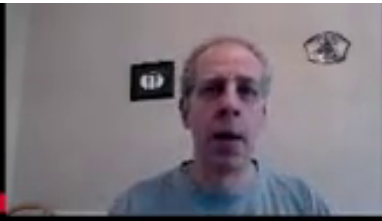
Result 30.829dB

Noisy image
$\sigma = 20$

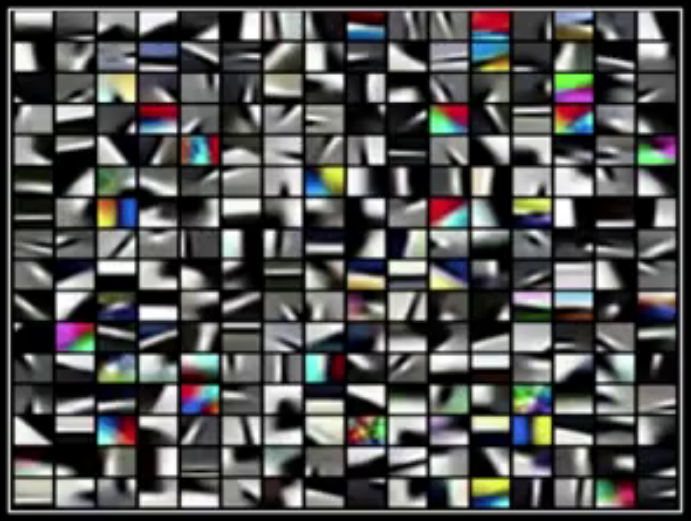


The obtained dictionary after
10 iterations

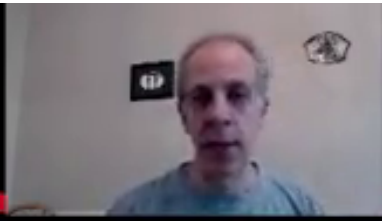
Denoising (Color)



- When turning to handle color images, the main difficulty is in defining the relation between the color layers – R, G, and B.
- The solution with the above algorithm is simple – consider 3D patches or 8-by-8 with the 3 color layers, and the dictionary will detect the proper relations.



Denoising (Color)

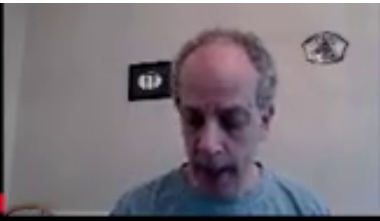


Original

Noisy (20.43dB)

Result (30.75dB)

Denoising (Color)

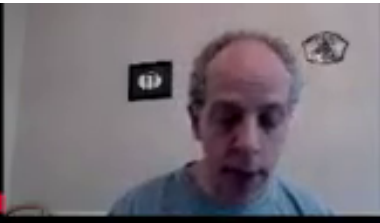


Original

Noisy (12.77dB)

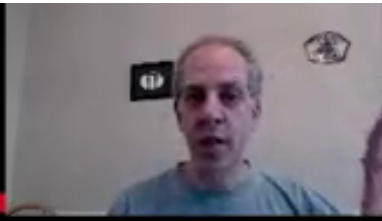
Result (29.87dB)

Inpainting



Original 80% missing Result

Inpainting

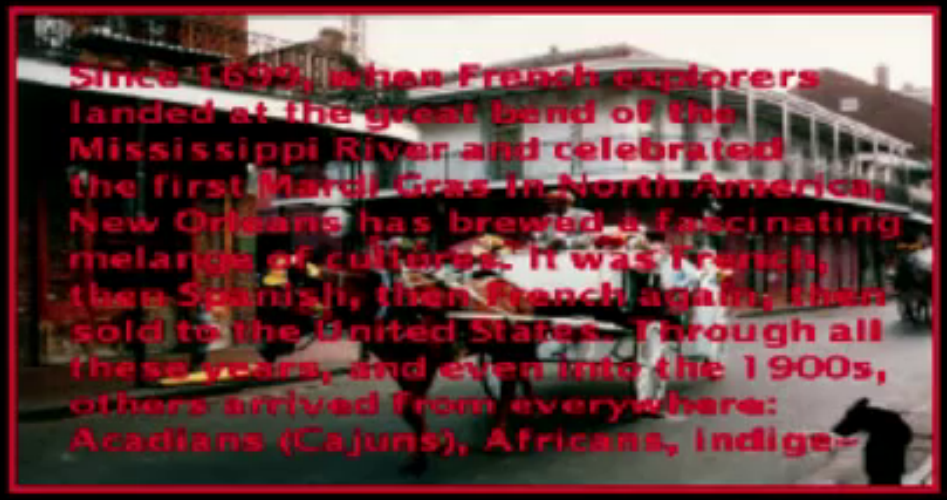


Original

80% missing

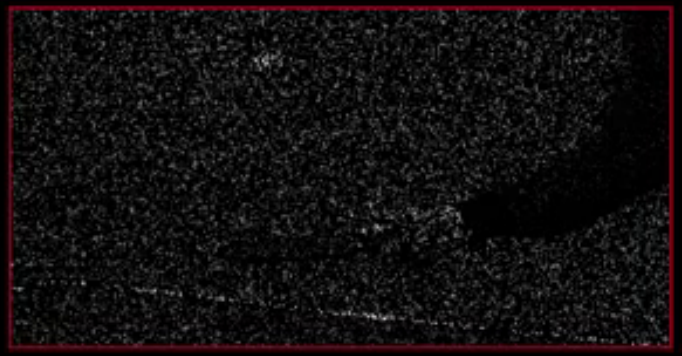
Result

Inpainting

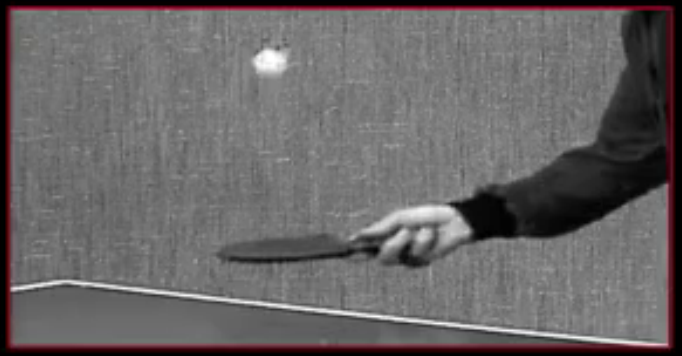

Since 1699, when French explorers
landed at the great bend of the
Mississippi River and celebrated
the first Mardi Gras in North America,
New Orleans has brewed a fascinating
melange of cultures. It was French,
then Spanish, then French again, then
sold to the United States. Through all
these years, and even into the 1900s,
others arrived from everywhere:
Acadians (Cajuns), Africans, Indige-

Video Inpainting

Original

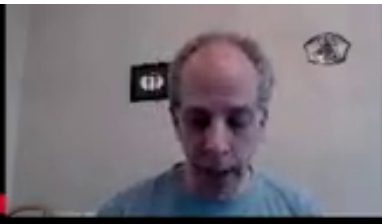


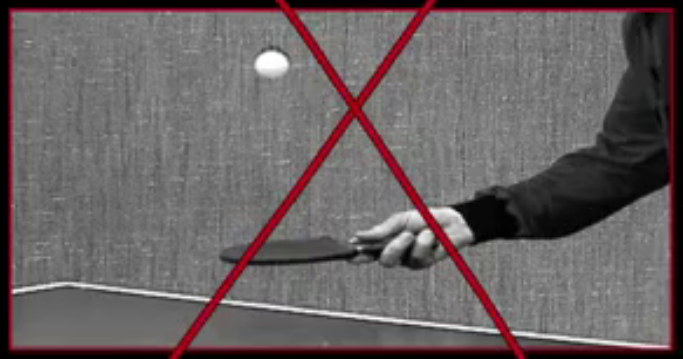
80% missing



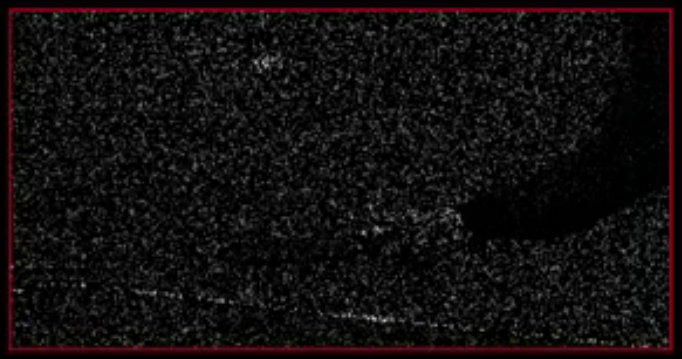
Result

Video Inpainting

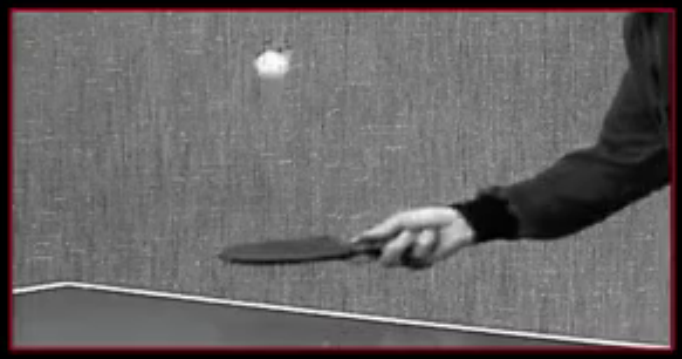




Original



80% missing



Result

Demosaicing