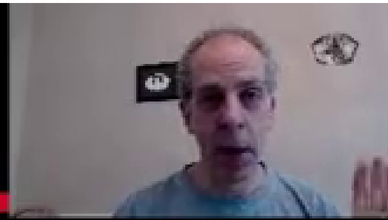
What Should **D** Be?



$$\underline{\hat{\alpha}} = \underset{\underline{\alpha}}{\arg\min} \|\underline{\alpha}\|_0^0 \ \text{ s.t. } \frac{1}{2}\|\mathbf{D}\underline{\alpha} - \underline{y}\|_2^2 \le \varepsilon^2 \Rightarrow \underline{\hat{x}} = \mathbf{D}\underline{\hat{\alpha}}$$

Our Assumption: Good-behaved Images have a sparse representation

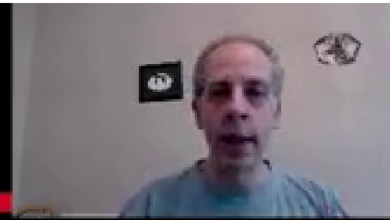
D should be chosen such that it sparsifies the representations

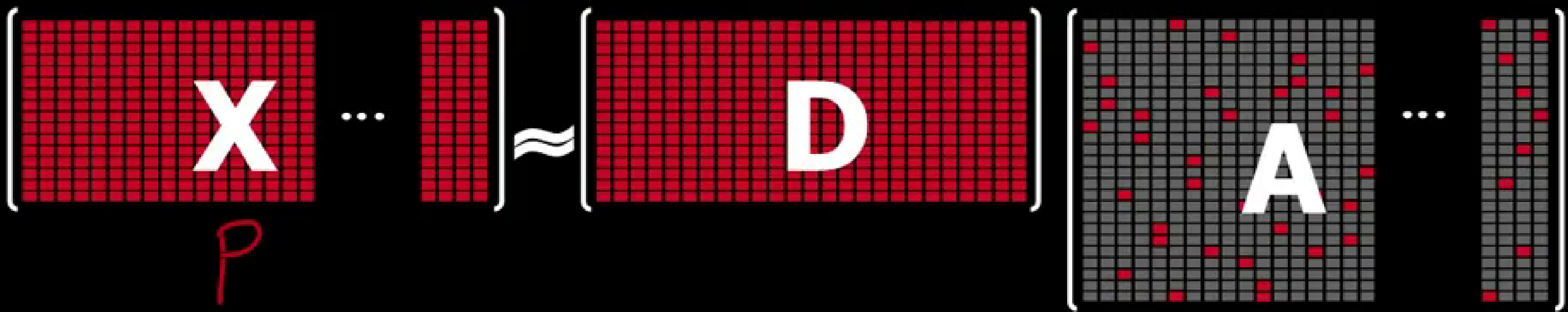
One approach to choose **D** is from a known set of transforms (Steerable wavelet, Curvelet, Contourlets, Bandlets, Shearlets …)

Training

Learning from **Examples**

Measure of Quality for **D**

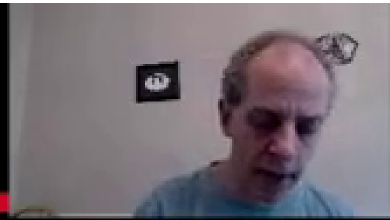


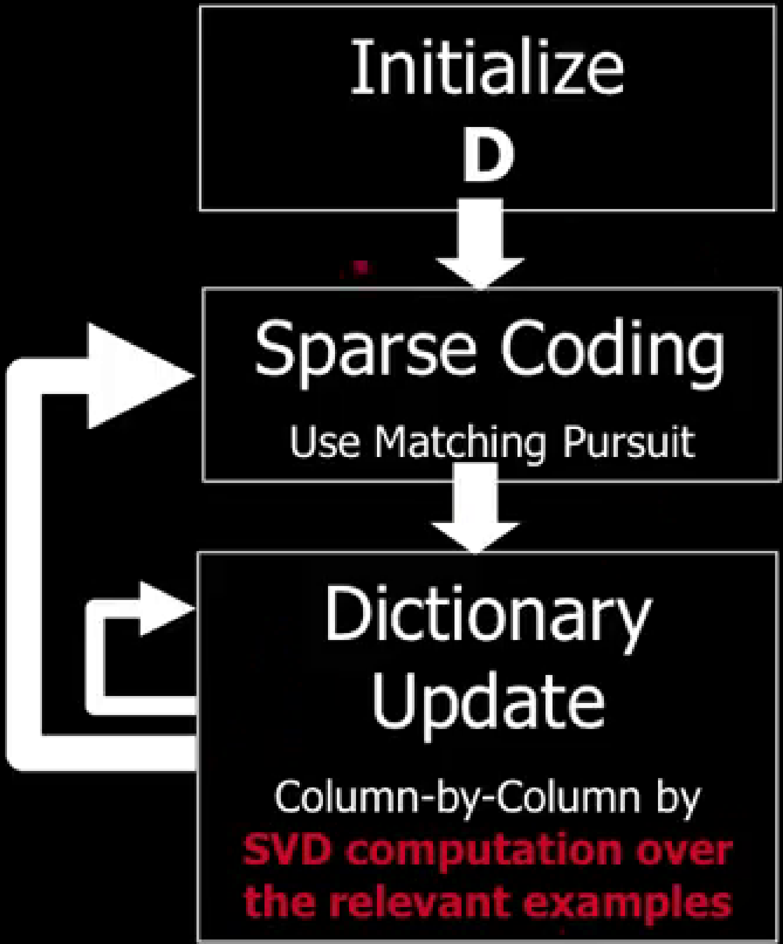


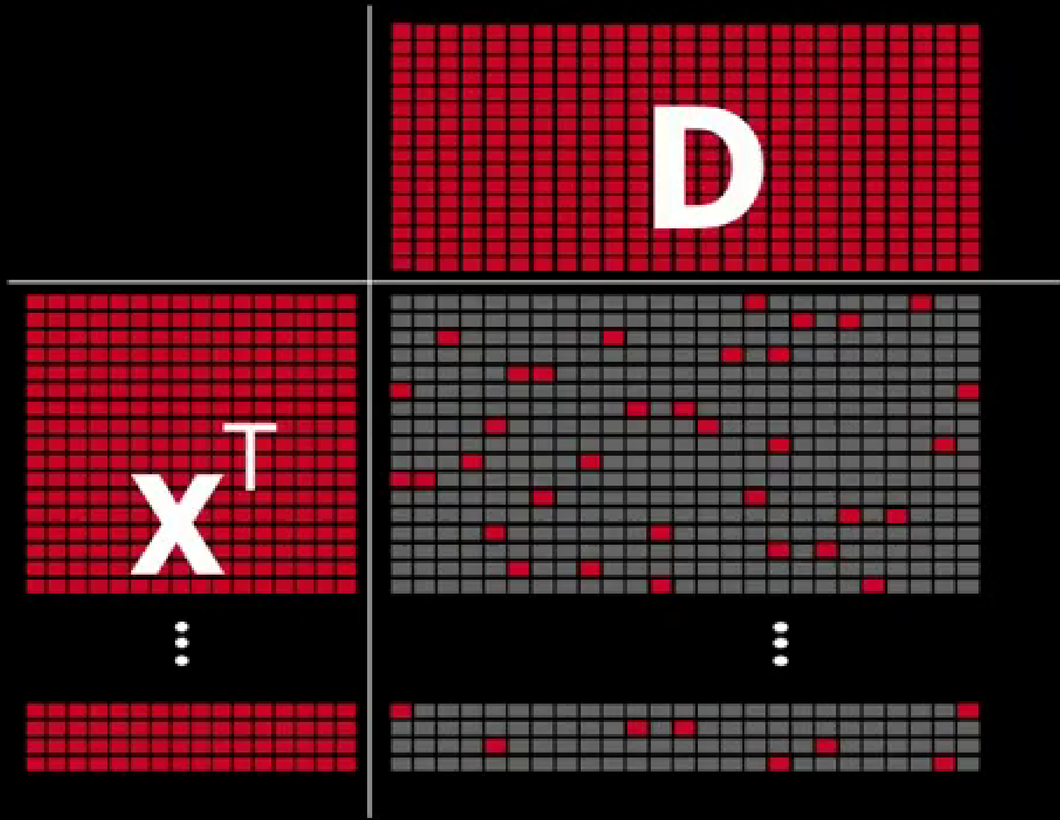
$$\underset{\mathbf{D},\mathbf{A}}{\mathrm{Min}} \sum_{j=1}^{P} \left\| \mathbf{D}\underline{\alpha}_j - \underline{x}_j \right\|_2^2 \quad \text{s.t. } \forall j, \left\| \underline{\alpha}_j \right\|_0^0 \le L$$

on-line.

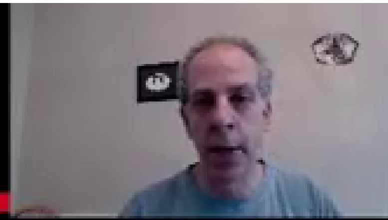
The K–SVD Algorithm – General







K–SVD: Sparse Coding Stage

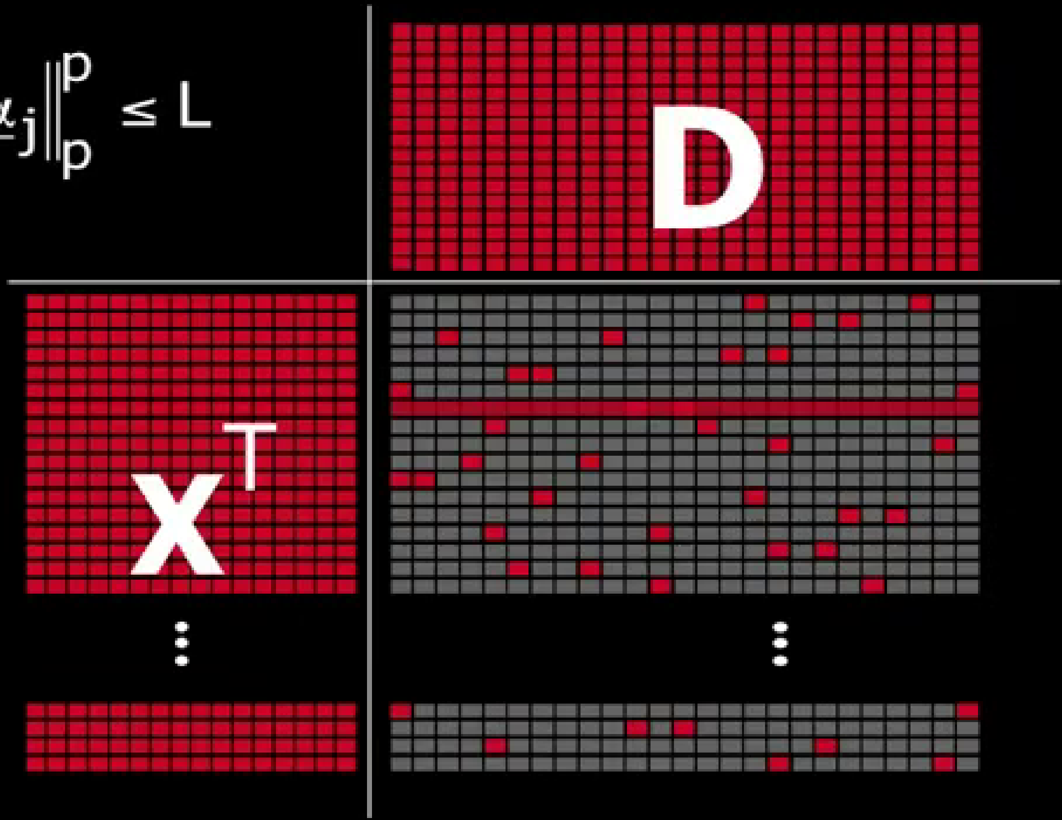


$$\underset{\mathbf{A}}{\text{Min}} \sum_{j=1}^{P} \left\| \mathbf{D}\underline{\alpha}_j - \underline{x}_j \right\|_2^2 \quad \text{s.t.} \quad \forall j, \left\| \underline{\alpha}_j \right\|_p^p \le L$$

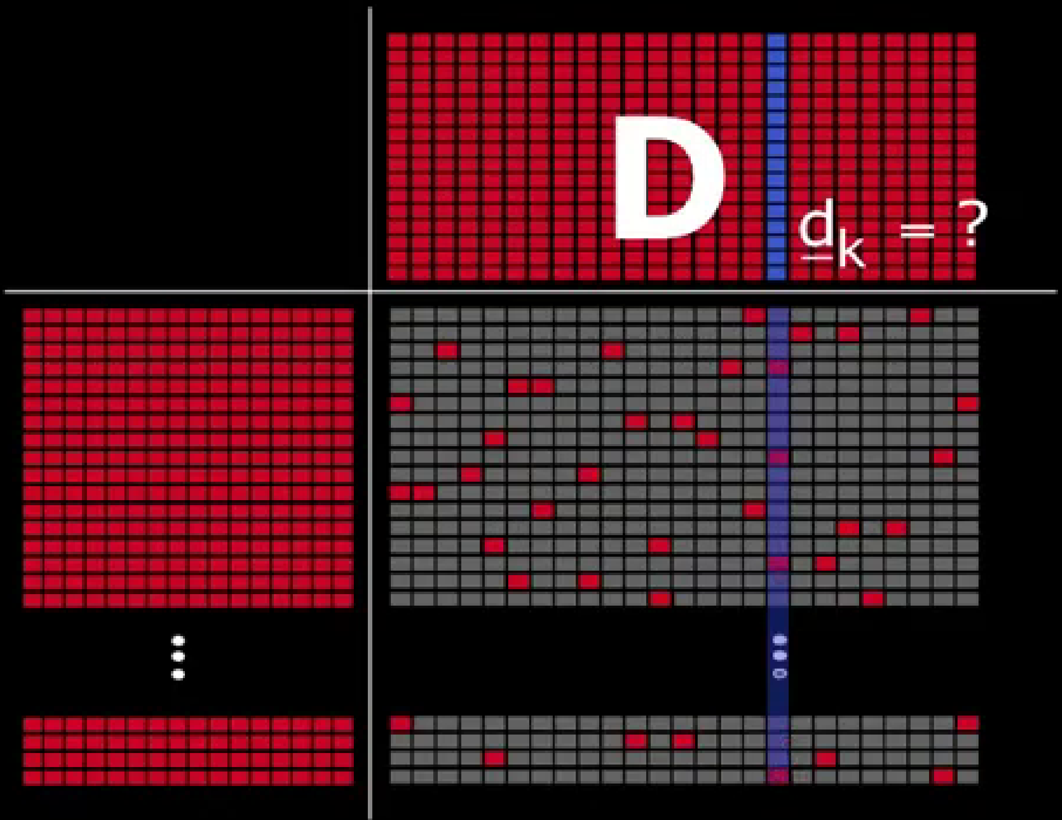
D is known!
For the jth item
we solve

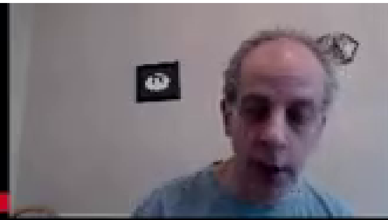
$$\underset{\underline{\alpha}}{\text{Min}} \left\| \mathbf{D}\underline{\alpha} - \underline{x}_j \right\|_2^2 \quad \text{s.t.} \quad \left\| \underline{\alpha} \right\|_p^p \le L$$

Solved by
A Pursuit Algorithm

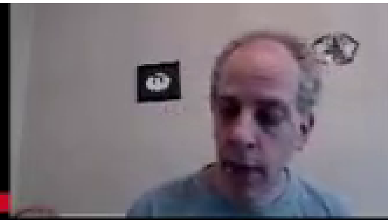


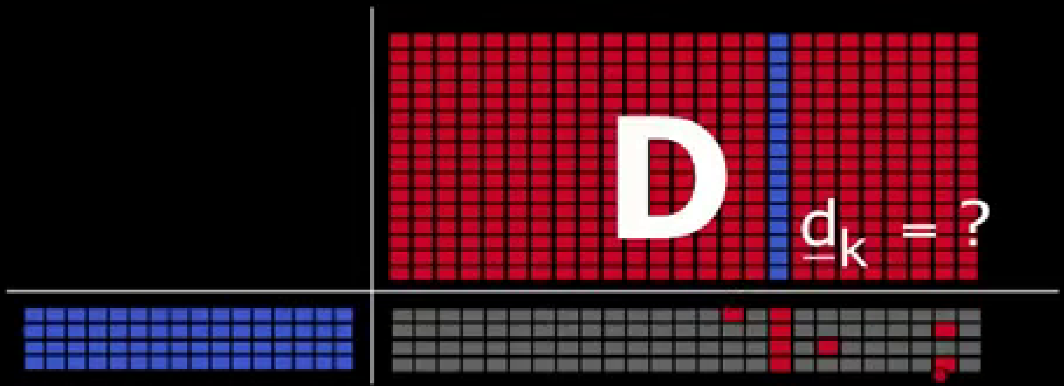
K–SVD: Dictionary Update Stage





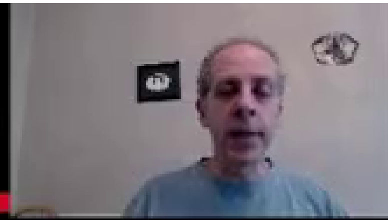
K–SVD: Dictionary Update Stage

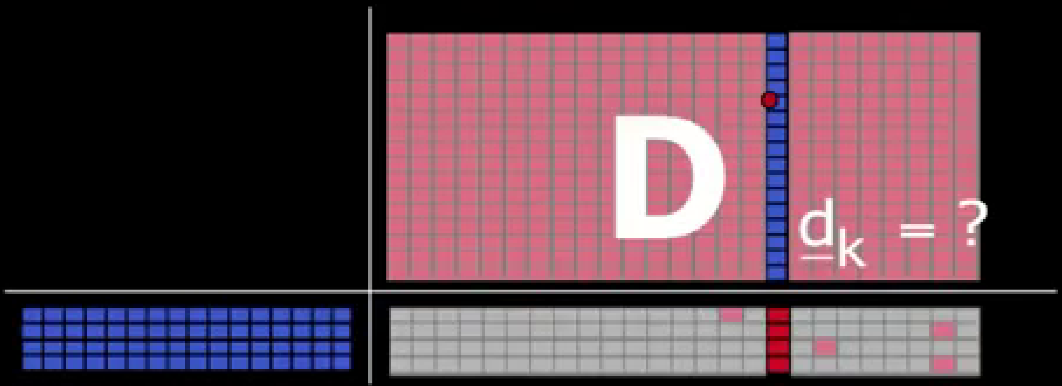




We refer only to the examples that use the column $\underline{d}_k$

K–SVD: Dictionary Update Stage

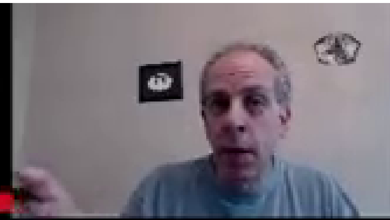


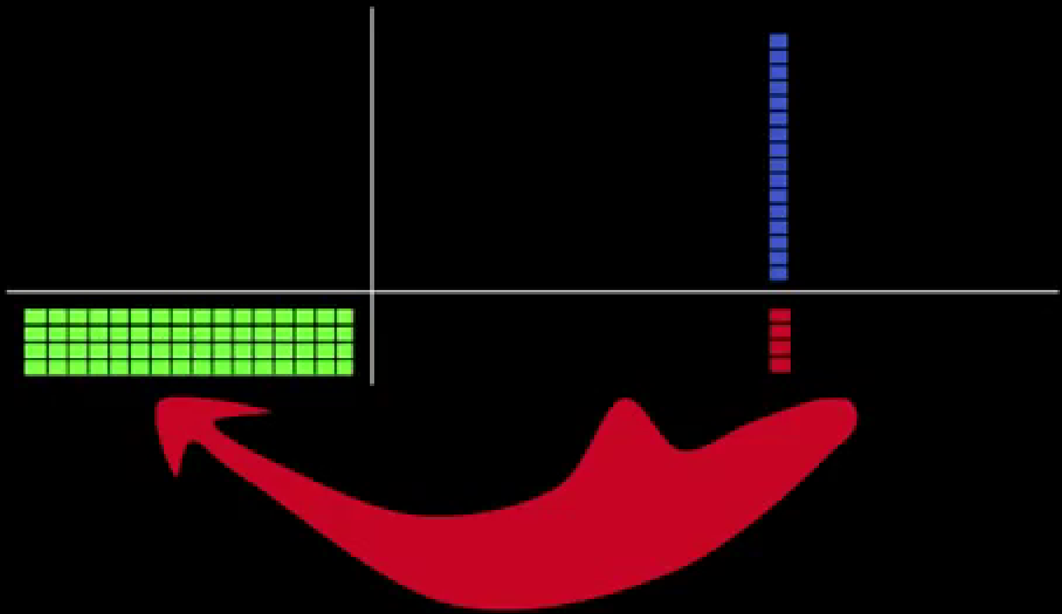


We refer only to the examples that use the column $\underline{d}_k$

Fixing all **A** and **D** apart from the k^{th} column, and seek both $\underline{d}_k$ and the k^{th} column in **A** to better fit the **residual**!

K–SVD: Dictionary Update Stage





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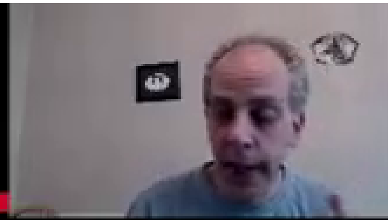
Fixing all **A** and **D** apart from the k^{th} column, and seek both $\underline{d}_k$ and the k^{th} column in **A** to better fit the **residual**!

K–SVD: Dictionary Update Stage



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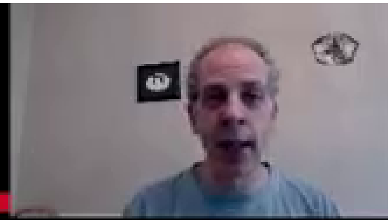
Fixing all **A** and **D** apart from the k^{th} column, and seek both $\underline{d}_k$ and the k^{th} column in **A** to better fit the **residual**!

We should solve:

$$\min_{\underline{d}_k,\,\alpha_k} \left\| \alpha_k \underline{d}_k^T - \mathbf{E} \right\|_F^2$$

K–SVD: Dictionary Update Stage



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Fixing all **A** and **D** apart from the k^{th} column, and seek both $\underline{d}_k$ and the k^{th} column in **A** to better fit the **residual**!

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$$\min_{\underline{d}_k,\alpha_k} \left\| \alpha_k \underline{d}_k^T - \mathbf{E} \right\|_F^2$$

SVD

To Summarize So Far ...

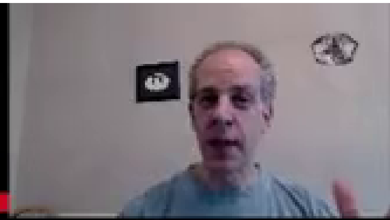

Image denoising (and many other problems in image processing) requires a model for the desired image

What do we do?

We proposed a model for signals/ images based on sparse and redundant representations

Problems?

We have seen approximation methods that find the sparsest solution, and theoretical results that guarantee their success. We also saw a way to learn **D**

What next?

Will it all work in applications?

D → subset X