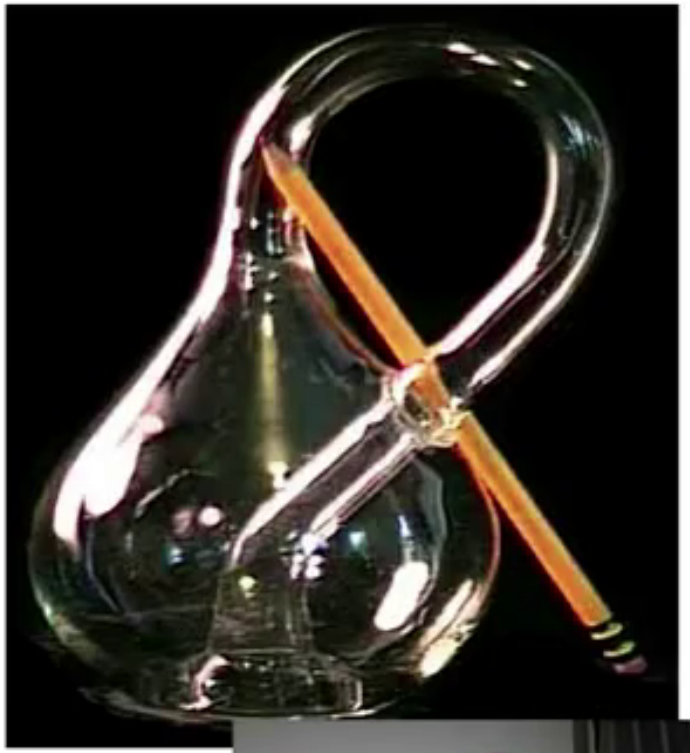
Surfaces

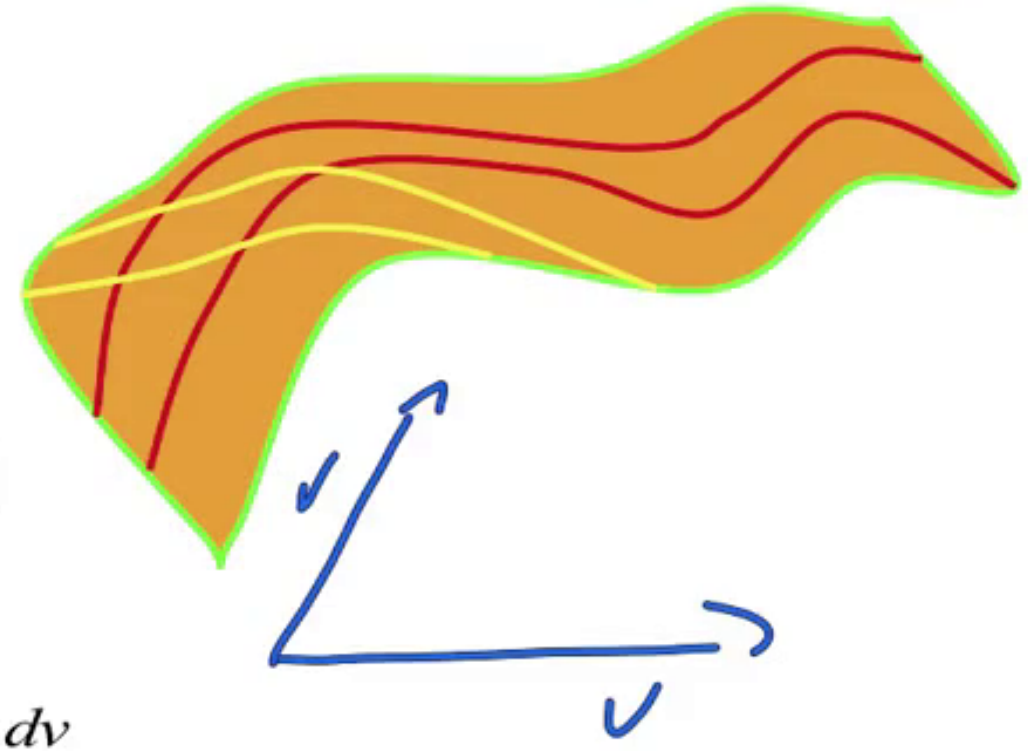
- Topology (Klein Bottle)



Surface

$$S(u,v) = \{x(u,v), y(u,v), z(u,v)\}$$

- Normal $\vec{N} = \dfrac{S_u \times S_v}{|S_u \times S_v|}$
- Area element $dA = |S_u \times S_v|$
- Total area $A = \iint |S_u \times S_v| \, du\, dv$



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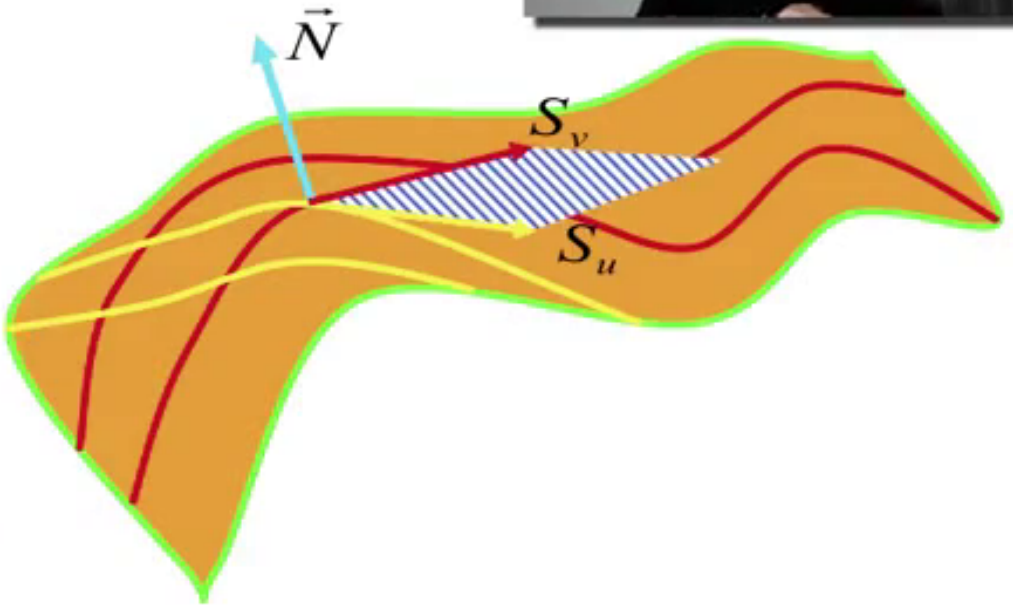
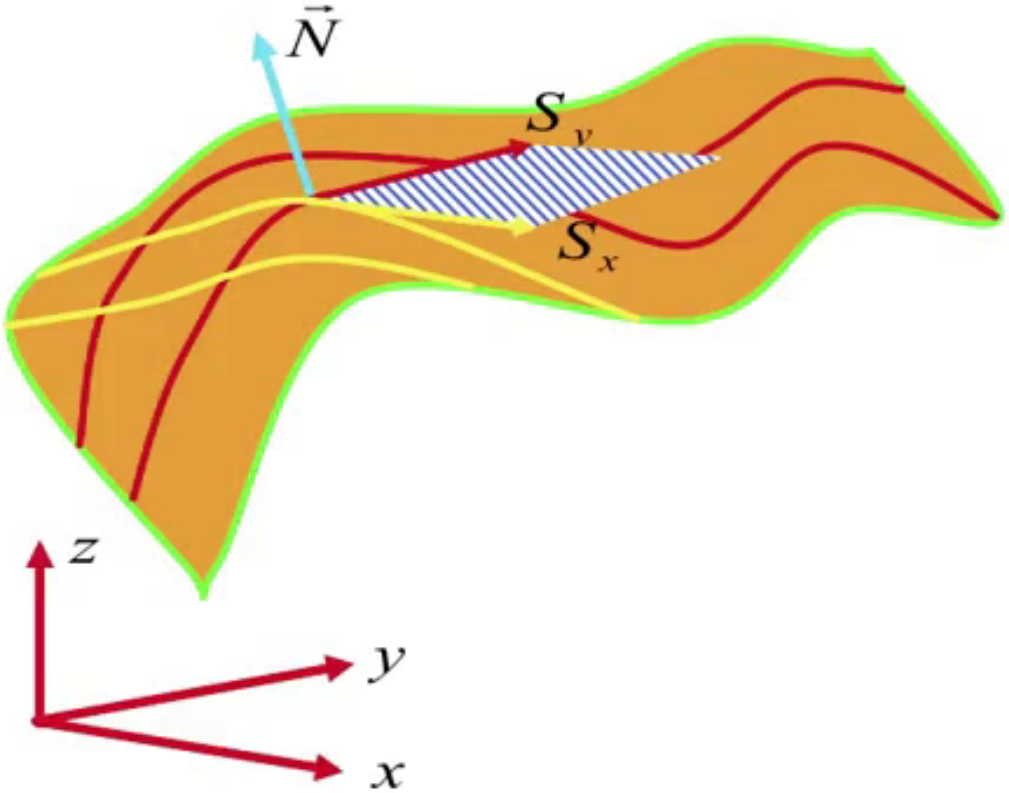


Example: Surface as graph of function

- A surface, $S : \mathbf{R}^2 \rightarrow \mathbf{R}^3$

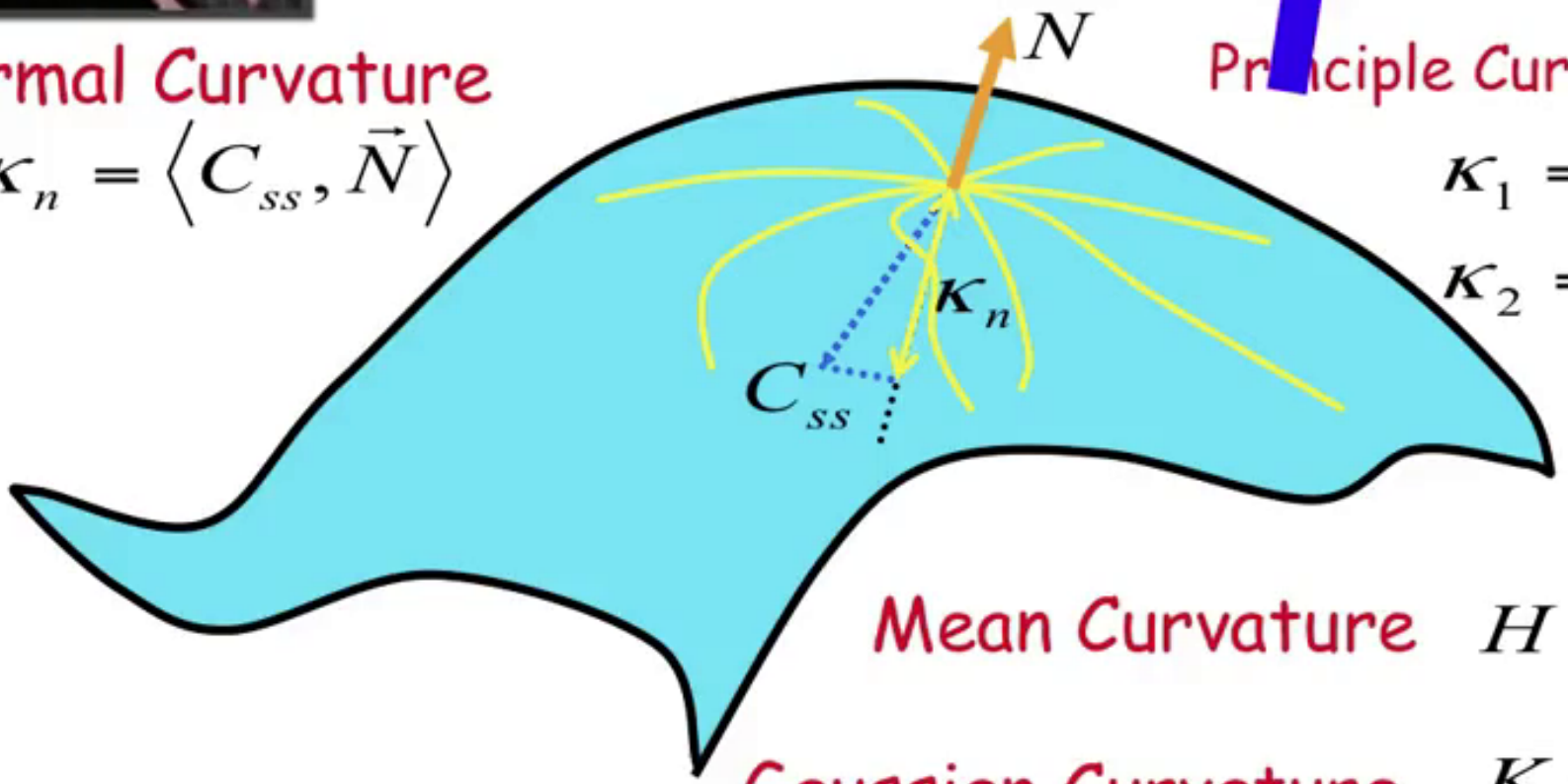
$$S(u,v) = \{x = u, y = v, z(u,v)\}$$



Principal

Normal Curvature

$$\kappa_n = \langle C_{ss}, \vec{N} \rangle$$



Principle Curvatures

$$\kappa_1 = \max_\theta(\kappa)$$

$$\kappa_2 = \min_\theta(\kappa)$$

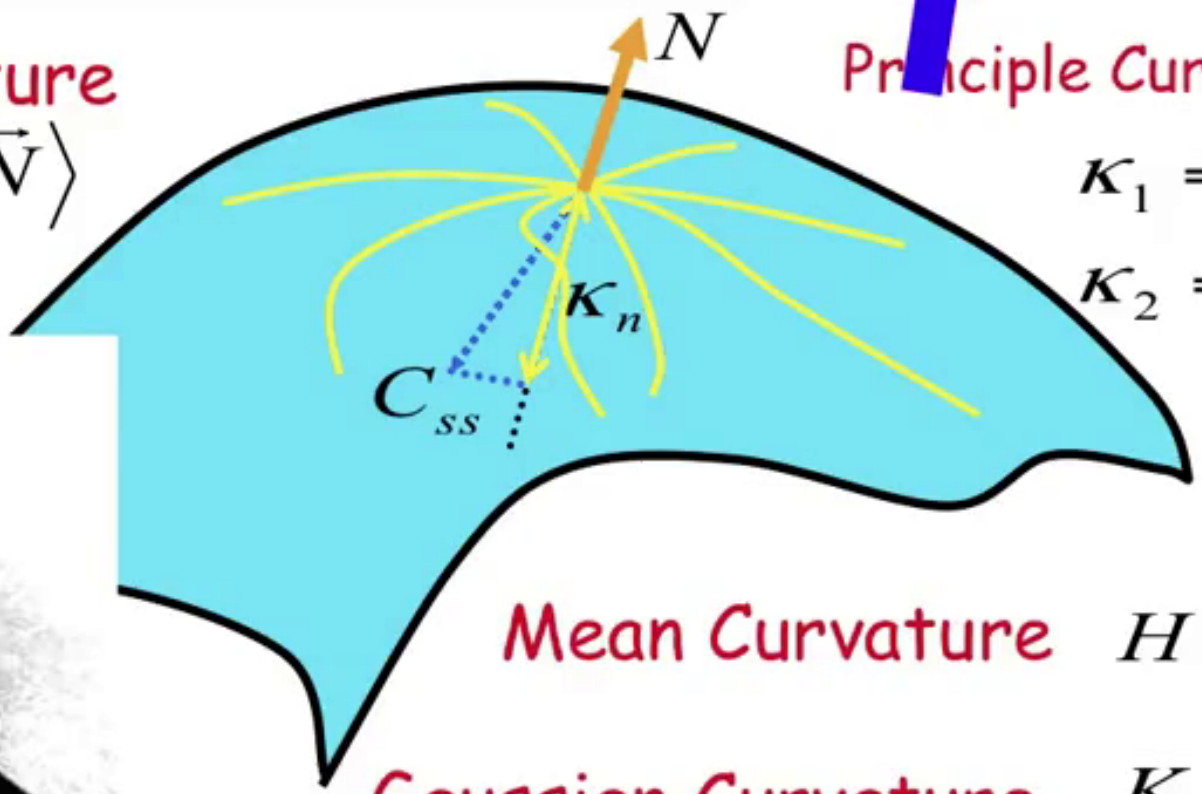
Mean Curvature $H = \frac{\kappa_1 + \kappa_2}{2}$

Gaussian Curvature $K = \kappa_1 \kappa_2$

Principal

Normal Curvature

$$\kappa_n = \left\langle C_{ss}, \vec{N} \right\rangle$$



Principle Curvatures

$$\kappa_1 = \max_\theta(\kappa)$$

$$\kappa_2 = \min_\theta(\kappa)$$

Gauss

Mean Curvature $H = \dfrac{\kappa_1 + \kappa_2}{2}$

Gaussian Curvature $K = \kappa_1 \kappa_2$