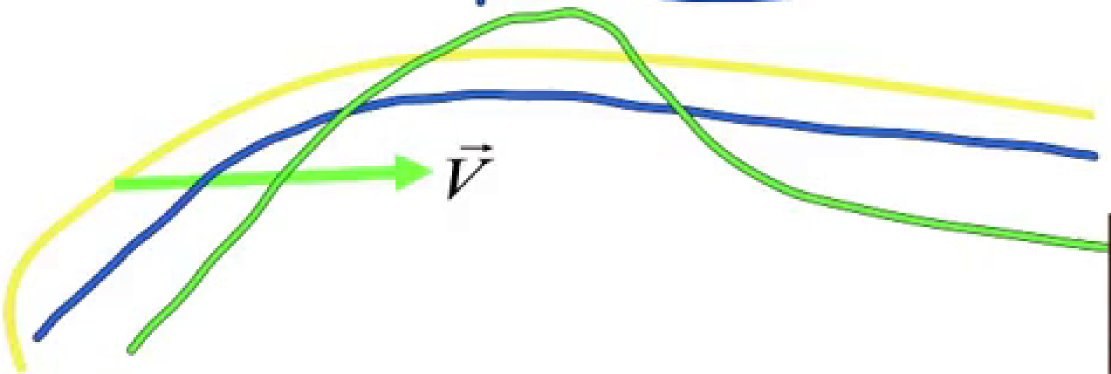
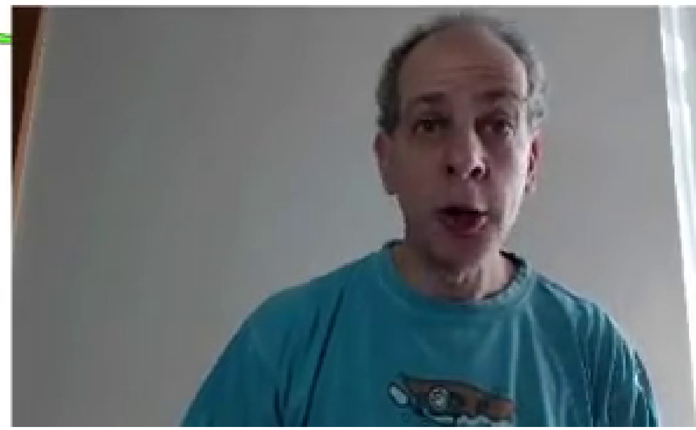
$$\frac{\partial C(p)}{\partial t} = \vec{V}(p,t)$$

$$\boxed{C_t = \vec{V}}$$

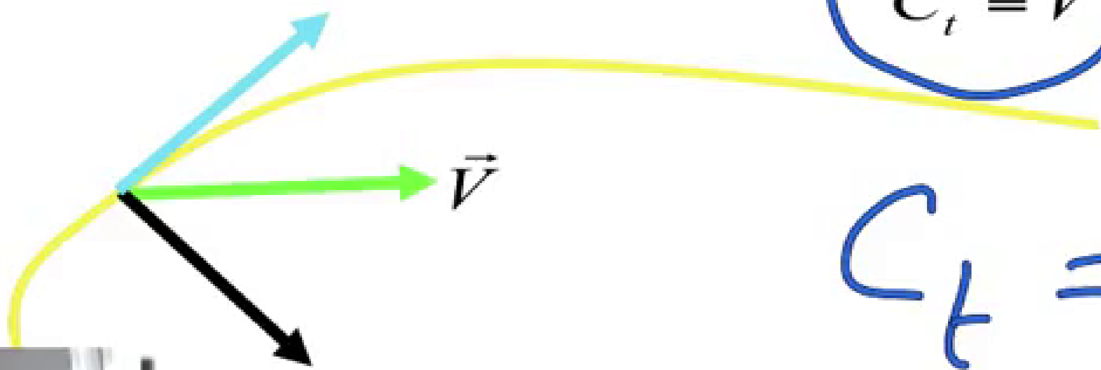




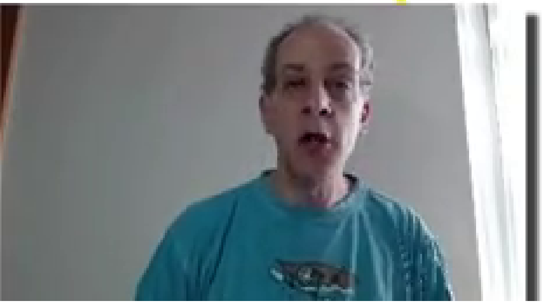
Important property

- Tangential components do not affect the geometry of an evolving curve

$$C_t = \vec{V} \Leftrightarrow C_t = \left\langle \vec{V}, \vec{n} \right\rangle \vec{n}$$



$$C_t = \alpha \vec{t}$$

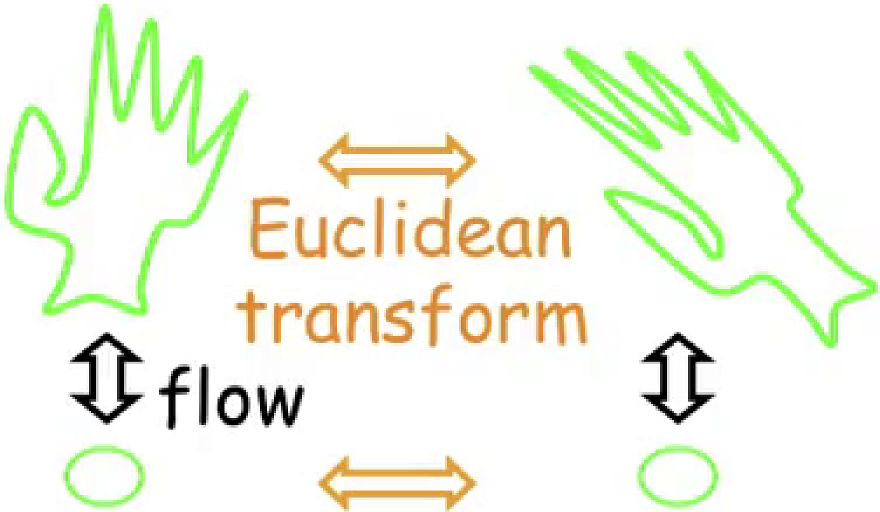


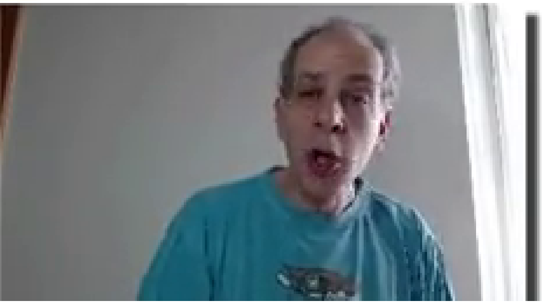
Curvature flow

- Euclidean geometric heat equation $C_t = \kappa\vec{n}$

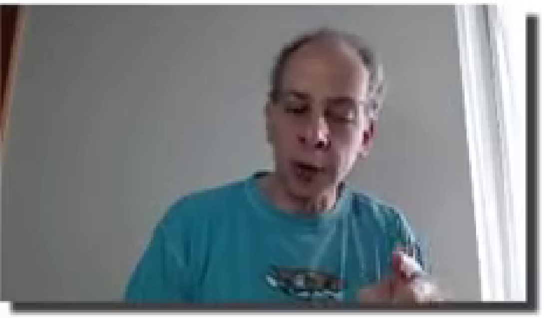
$$C_t = C_{ss} \quad \text{where } C_{ss} = \kappa\vec{n}$$

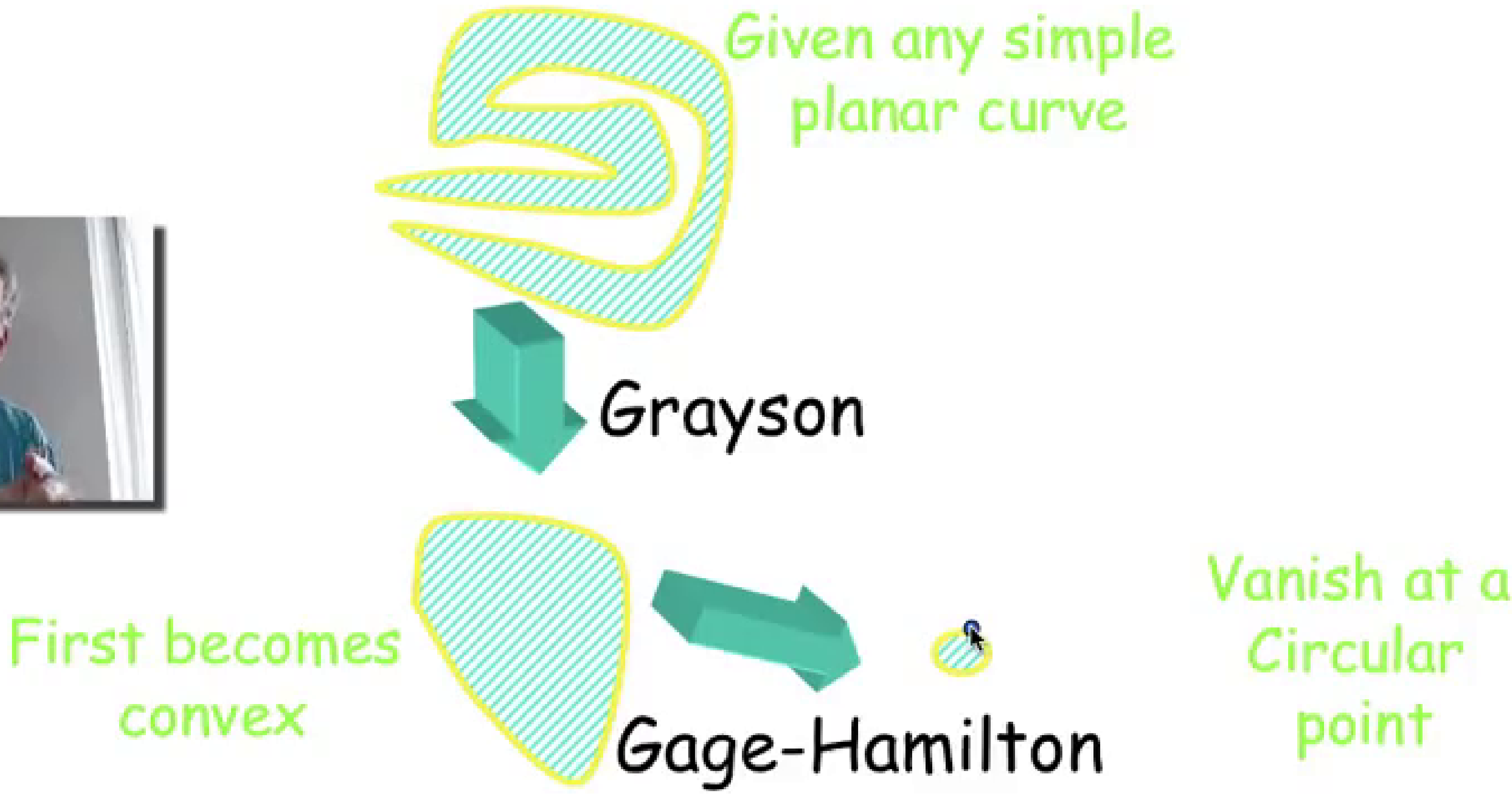
$$\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial s^2}$$

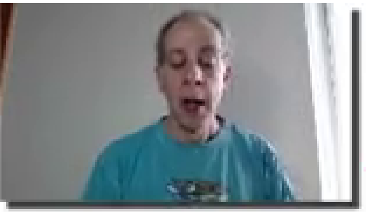




Curvature flow $C_t = \kappa \vec{n}$





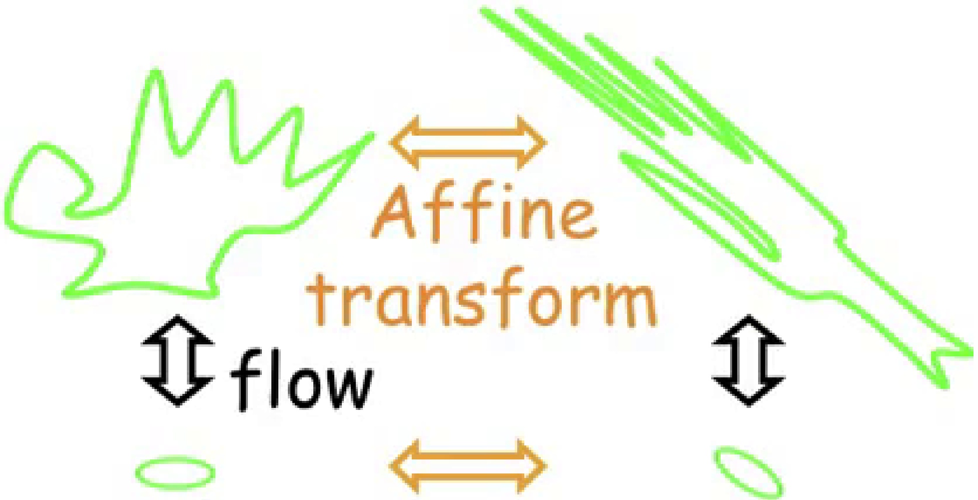


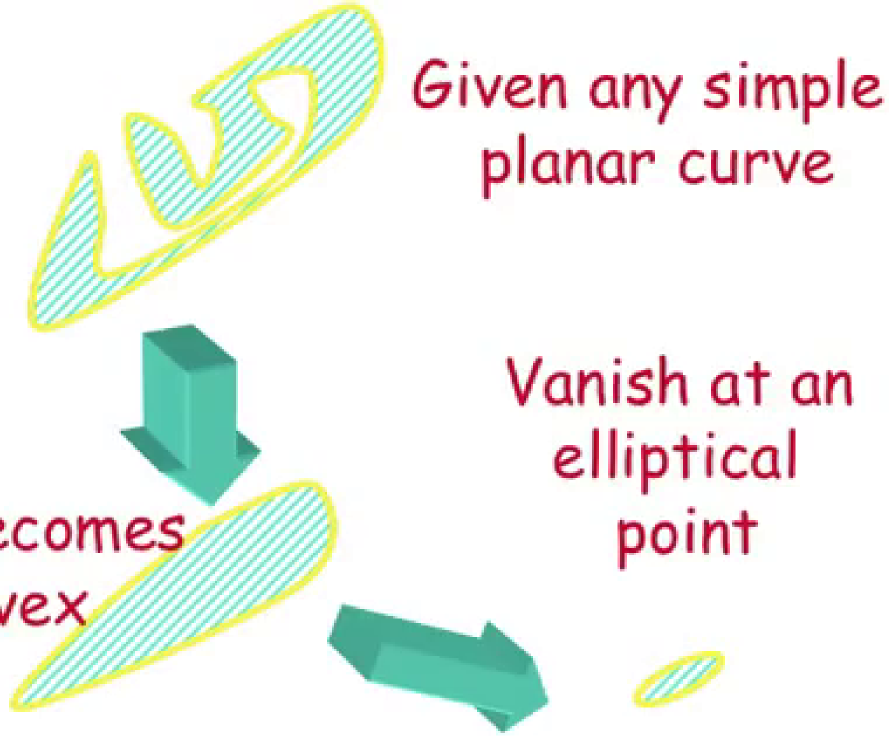
Affine heat equation

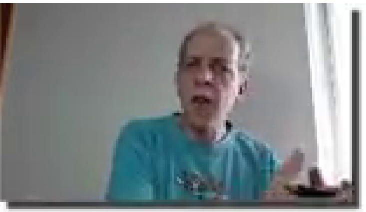
$C_t = C_{vv}$

- Special (equi-)affine heat flow $C_t = \kappa^{1/3}\vec{n}$

$$C_t = \langle C_{vv}, \vec{n} \rangle \vec{n} \quad \text{where } \langle C_{vv}, \vec{n} \rangle = \kappa^{1/3}$$



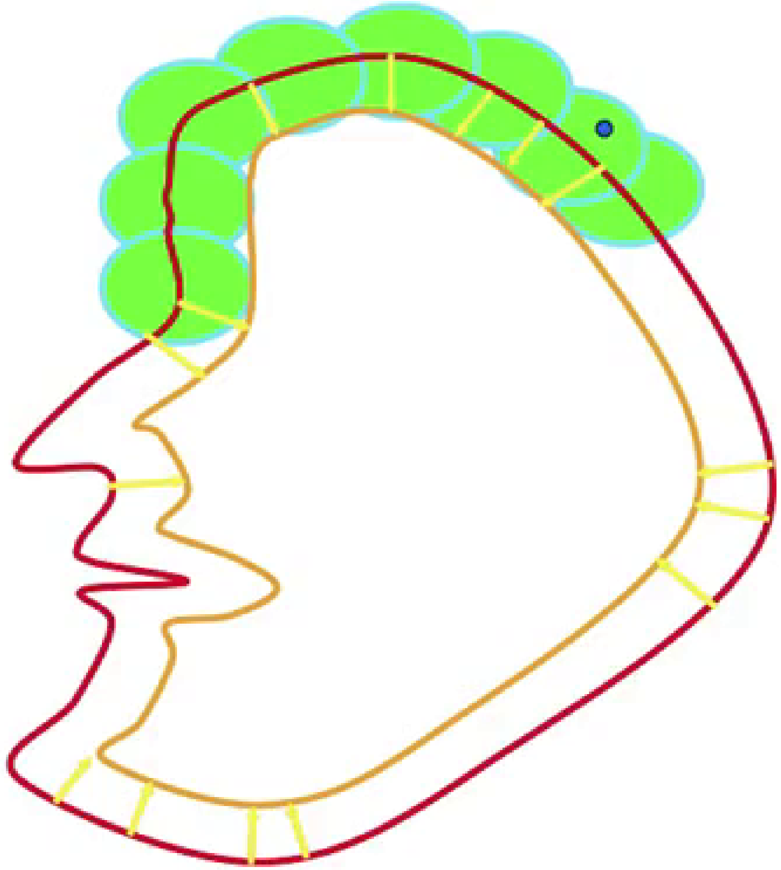


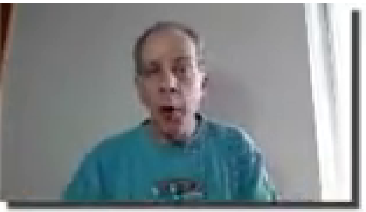


Constant flow

$$C_t = \vec{n}$$

- Offset curves
- Equal-height contours of the distance transform
- Envelope of all disks of equal radius centered along the curve (Huygens principle)

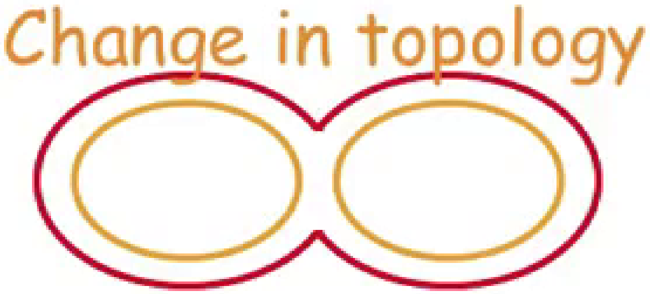


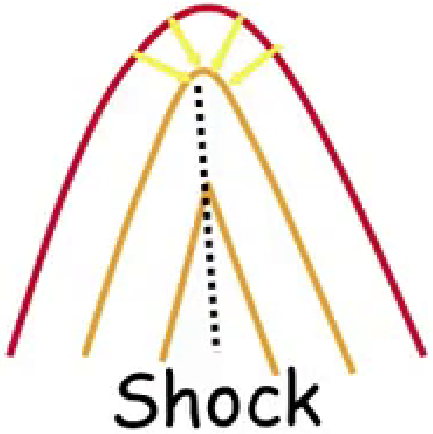


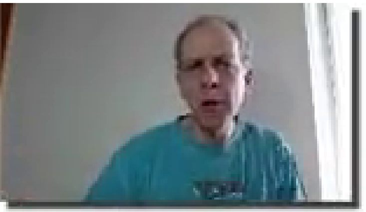
Constant flow

$$C_t = \vec{n}$$

- Offset curves





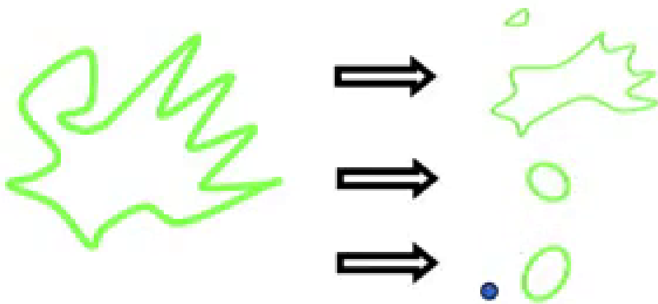


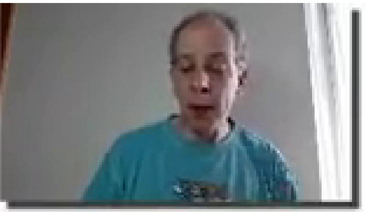
So far we defined

Constant flow $C_t = \vec{n}$

Curvature flow $C_t = \kappa \vec{n}$

Equi-affine flow $C_t = \kappa^{1/3} \vec{n}$





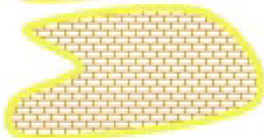
$$C_t = V\vec{n}$$

$$\frac{\partial}{\partial t}L = \frac{\partial}{\partial t}\oint \left\langle C_p, C_p \right\rangle^{1/2} dp = 2\oint \left\langle \frac{\partial}{\partial t}C_p, C_p \right\rangle dp = \ldots = -\int_0^L \kappa V ds$$

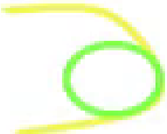
$$\frac{\partial}{\partial t}A = \frac{1}{2}\frac{\partial}{\partial t}\oint \left(C, C_p\right) dp = \oint \left(\frac{\partial}{\partial t}C, C_p\right) dp + \oint \left(C, \frac{\partial}{\partial t}C_p\right) dp = \ldots = -\int_0^L V ds$$

$$\frac{\partial}{\partial t}\kappa = \frac{\partial}{\partial t}\left(\frac{\left(C_p, C_{pp}\right)}{\left\langle C_p, C_p \right\rangle^{3/2}}\right) = \ldots = V_{ss} + \kappa^2 V$$

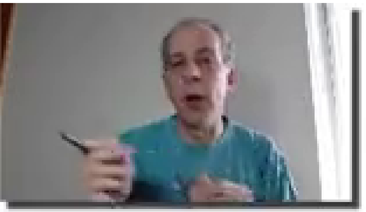
Length $\quad L_t = -\int_0^L \kappa V ds$



Area $\quad A_t = \int_0^L V ds$



Curvature $\quad \kappa_t = V_{ss} + \kappa^2 V$



Constant flow $(V=1)$

Length $L_t = -\int_0^L \kappa V ds = -\int_0^L \kappa ds = -2\pi$

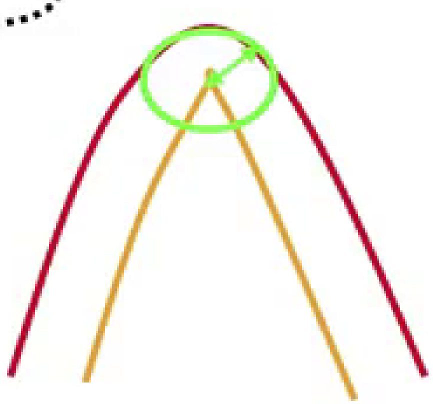
Area $A_t = -\int_0^L V ds = -\int_0^L ds = -L$

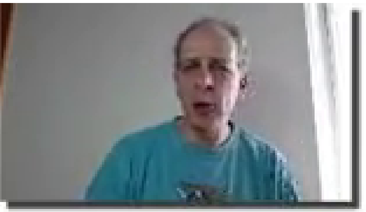
Curvature $\kappa_t = V_{ss} + \kappa^2 V = \kappa^2$

The curve vanishes at $t = \frac{L(0)}{2\pi}$

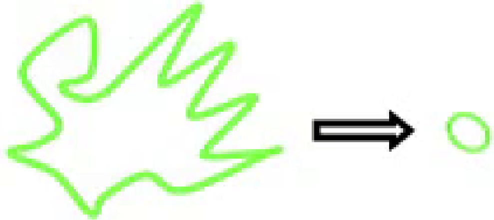
Riccati eq. $\kappa(p,t) = \frac{\kappa(p,0)}{1 - t\kappa(p,0)}$

Singularity (`shock') at $t = \rho(p,0)$





Curvature flow ($V = \kappa$)

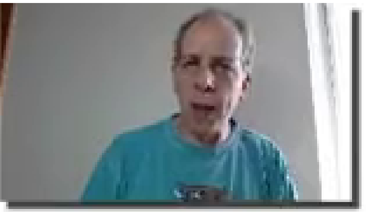


Length $\quad L_t = -\int_0^L \kappa V ds = -\int_0^L \kappa^2 ds$

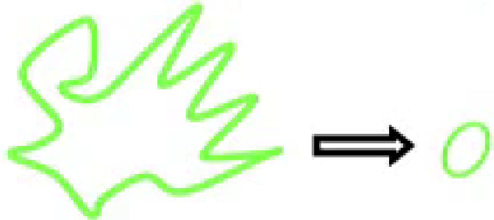
Area $\quad A_t = -\int_0^L V ds = -\int_0^L \kappa ds = -2\pi$

Curvature $\quad \kappa_t = V_{ss} + \kappa^2 V = \kappa_{ss} + \kappa^3$

The curve vanishes at $\quad t = \frac{A(0)}{2\pi}$



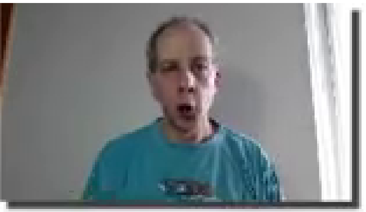
Equi-Affine flow ($V = \kappa^{1/3}$)



Length $$L_t = -\int_0^L \kappa V ds = -\int_0^L \kappa^{4/3} ds$$

Area $$A_t = -\int_0^L V ds = -\int_0^L \kappa^{1/3} ds$$

Curvature $$\kappa_t = V_{ss} + \kappa^2 V = \tfrac{1}{3}\kappa^{-2/3}\kappa_{ss} - \tfrac{2}{9}\kappa^{-5/3}{\kappa_s}^2 + \kappa^{7/3}$$

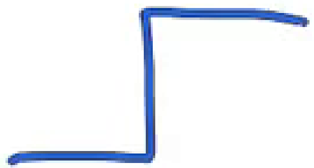


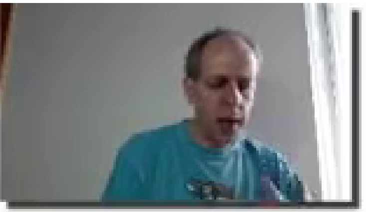
Geodesic active contours

$$C_t = \overbrace{\left(g(x,y)\kappa - \langle \nabla g(x,y), \vec{n}\rangle\right)}^{\checkmark}\vec{n}$$

$$C_t = g\,\kappa\,\vec{n}$$

$$g \sim \frac{1}{\underline{\nabla I}}$$



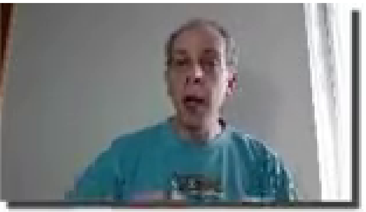


Geodesic active contours

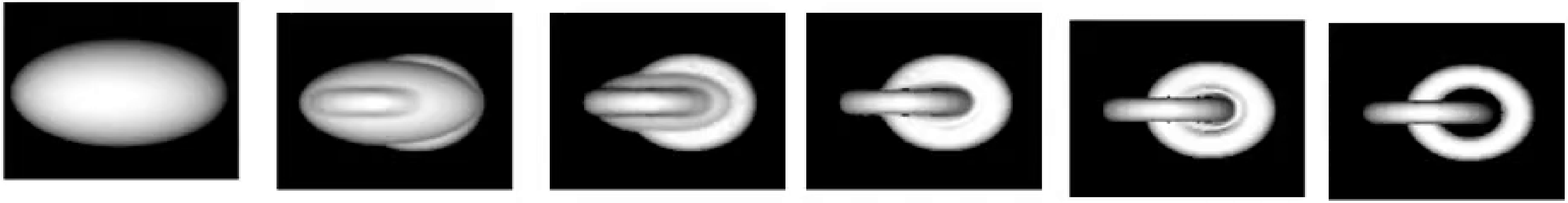
$$C_t = \overbrace{\left(g(x,y)\kappa - \langle \nabla g(x,y), \vec{n} \rangle\right)}^{\checkmark}\vec{n}$$

$$C_t = g\,\kappa\vec{n} + g\vec{n}$$

$$g \sim \frac{1}{\nabla I}$$



Surface evolution…



$$\frac{\partial S}{\partial t} = g \, k \vec{N}$$