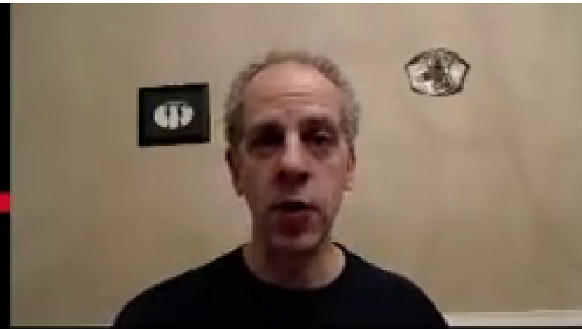
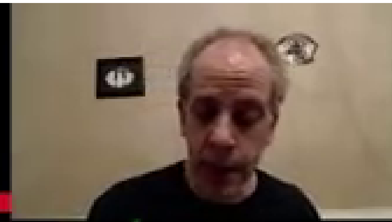
# Noise Removal?

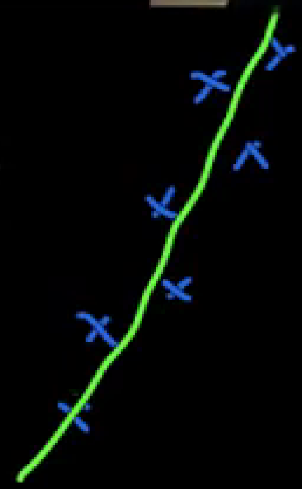


Our story begins with image denoising …

# Denoising By Energy Minimization



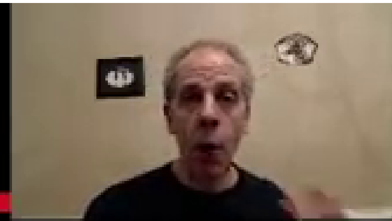
$$f(\underline{x}) = \frac{1}{2}\left\|\underline{x} - \underline{y}\right\|_2^2 \quad + \quad G(\underline{x})$$



$\underline{y}$ : Given measurements

$\underline{x}$ : Unknown to be recovered

# Denoising By Energy Minimization



$$f(\underline{x}) = \underbrace{\frac{1}{2}\left\|\underline{x} - \underline{y}\right\|_2^2}_{\text{Relation to measurements}} + \underbrace{G(\underline{x})}_{\text{Prior or regularization}}$$

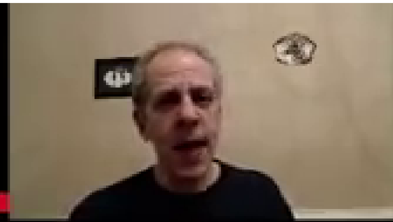
$\underline{y}$ : Given measurements

$\underline{x}$ : Unknown to be recovered

- This is in-fact a Bayesian point of view, adopting the Maximum-A-posteriori Probability (MAP) estimation.
- Clearly, the wisdom in such an approach is within the choice of the prior – **modeling the images** of interest.

Thomas Bayes
1702 - 1761

# The Evolution of G($\underline{x}$)



During the past several decades we have made all sort of guesses about the prior G($\underline{x}$) for images:

$$G(\underline{x}) = \lambda\|\underline{x}\|_2^2$$

**Energy**

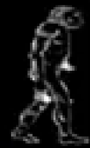
$$G(\underline{x}) = \lambda\|\mathbf{L}\underline{x}\|_2^2$$

**Smoothness**

$$G(\underline{x}) = \lambda\|\mathbf{L}\underline{x}\|_{\mathbf{W}}^2$$

**Adapt+ Smooth**

$$G(\underline{x}) = \lambda\rho\{\mathbf{L}\underline{x}\}$$



**Robust Statistics**

$$G(\underline{x}) = \lambda\||\nabla\underline{x}|\|_1$$

**Total-Variation**

$$G(\underline{x}) = \lambda\|\mathbf{W}\underline{x}\|_1$$

**Wavelet Sparsity**

$$G(\underline{x}) = \lambda\|\underline{\alpha}\|_0^0 \quad \text{for} \quad \underline{x} = \mathbf{D}\underline{\alpha}$$

**Sparse & Redundant**

- Hidden Markov Models,
- Compression algorithms as priors,
- …