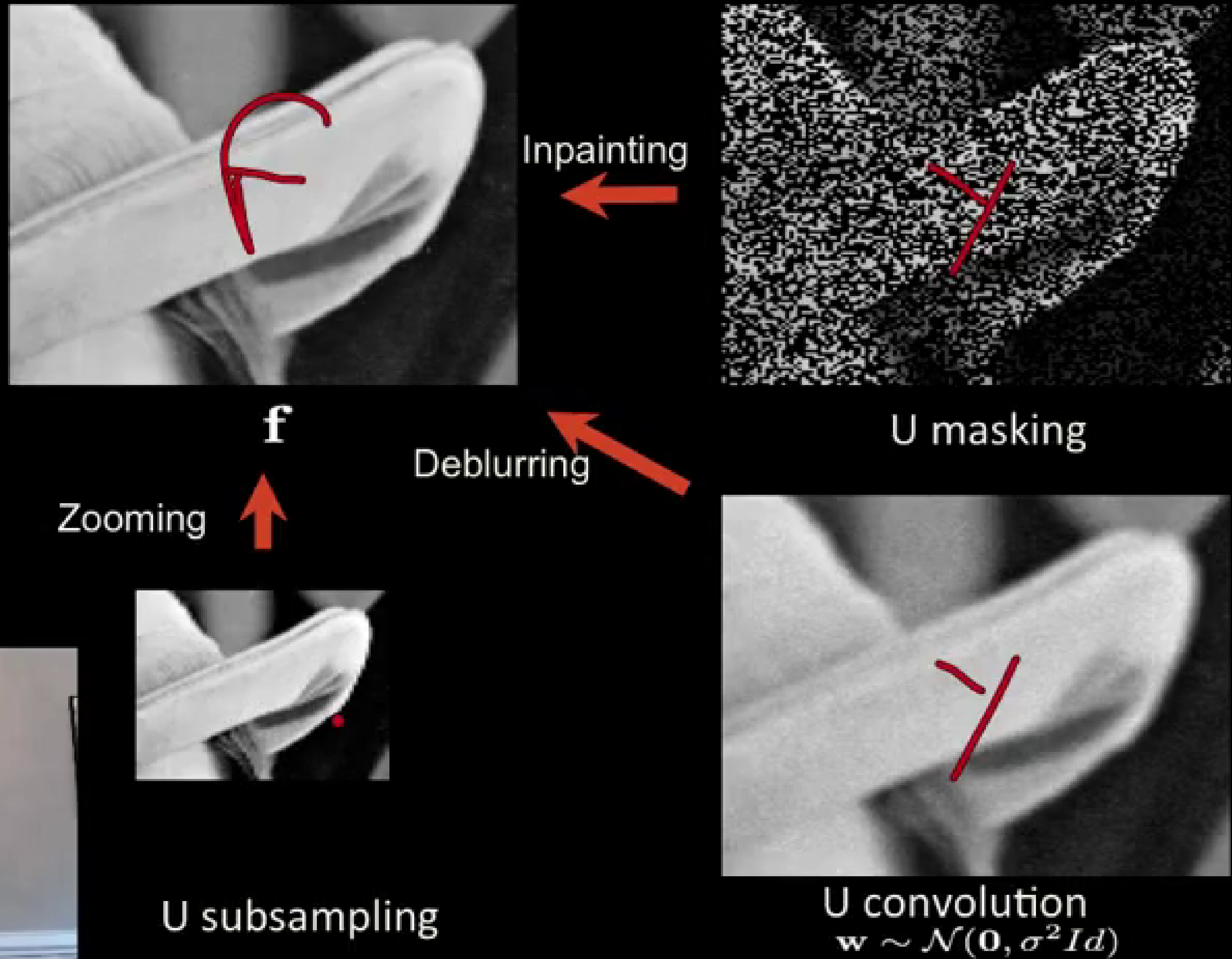
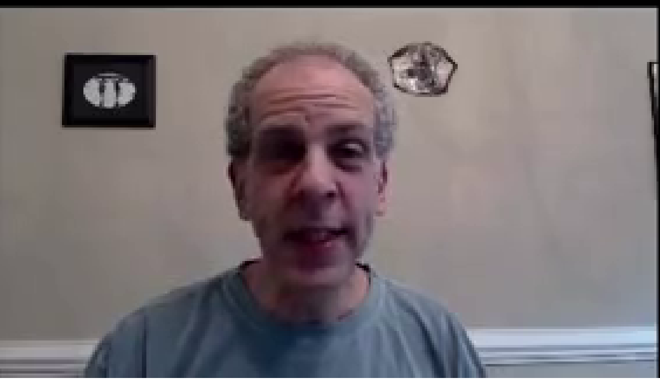
Inverse Problems

$$\mathbf{y} = \mathbf{U}\mathbf{f} + \mathbf{w}$$

$$\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma^2 Id)$$





Gaussian Mixture Models of Patches

$$\mathbf{y}_i = \mathbf{U}_i \mathbf{f}_i + \mathbf{w}_i$$

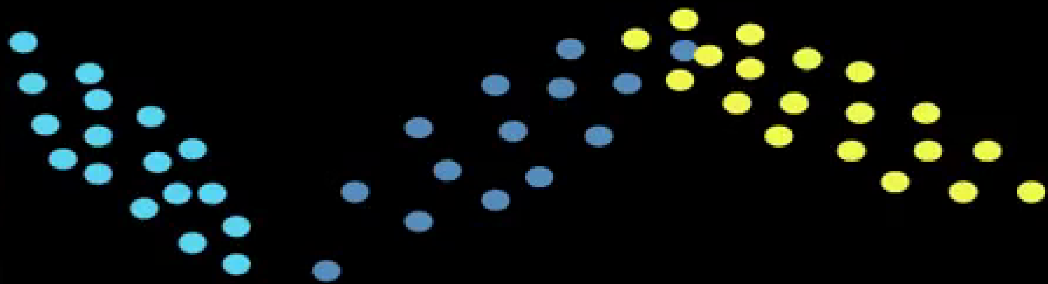
where

$$\mathbf{w}_i \sim \mathcal{N}(\mathbf{0}, \sigma^2 Id)$$

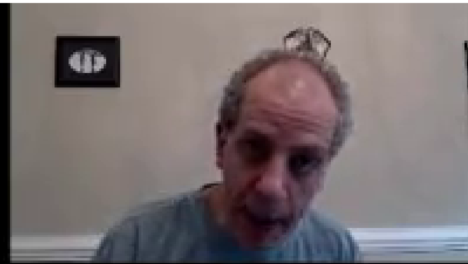
$\frac{8 \times 8}{64}$

- K Gaussian distributions or PCAs $\{\mathcal{N}(\mu_k, \Sigma_k)\}_{1 \leq k \leq K}$
- $\mathbf{f}_i \sim \mathcal{N}(\mu_k, \Sigma_k)$

$K = 10$



Gaussian Mixture Models of Patches



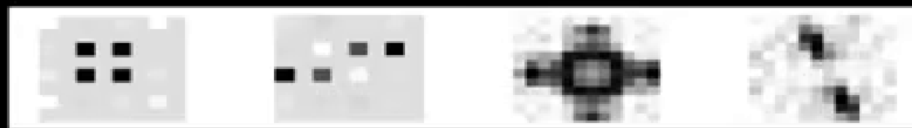
$$\mathbf{y}_i = \mathbf{U}_i \mathbf{f}_i + \mathbf{w}_i$$

where

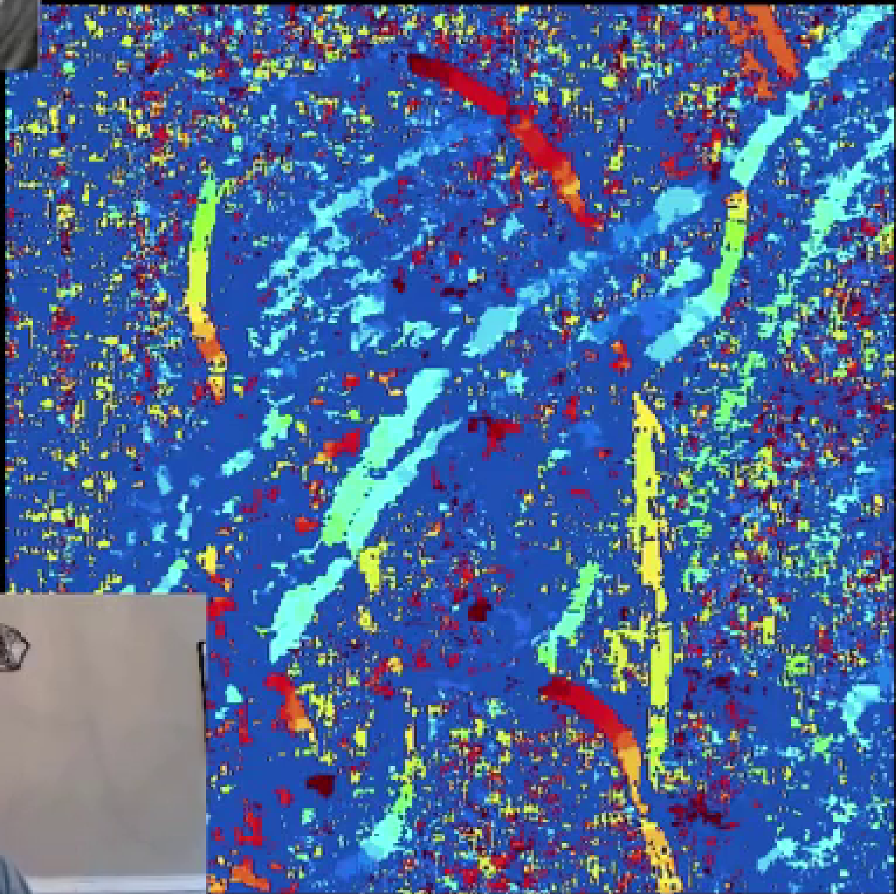
$$\mathbf{w}_i \sim \mathcal{N}(\mathbf{0}, \sigma^2 Id)$$

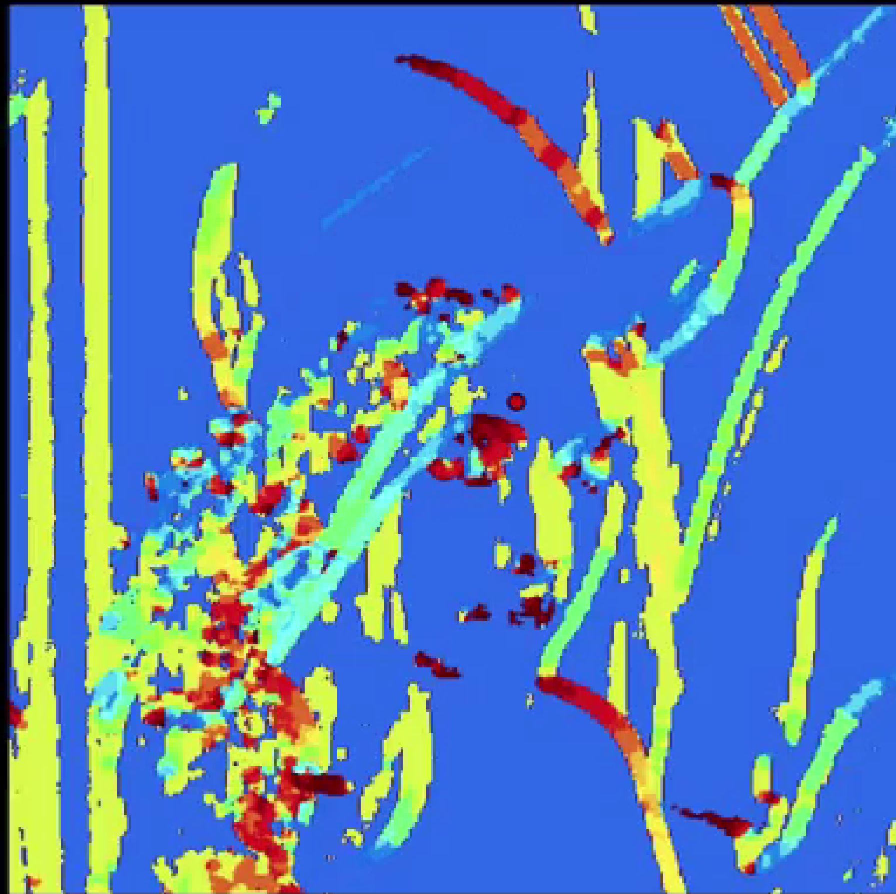
- Estimate $\{(\mu_k, \Sigma_k)\}_{1 \leq k \leq K}$ from $\{\mathbf{y}_i\}_{1 \leq i \leq I}$
- Identify the Gaussian k_i that generates $\mathbf{f}_i \ \forall i$
- Estimate $\tilde{\mathbf{f}}_i$ from $\mathcal{N}(\mu_{k_i}, \Sigma_{k_i}) \ \forall i$

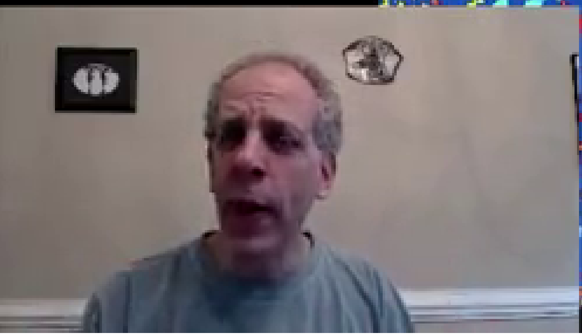
Efficiently solved via MAP-EM



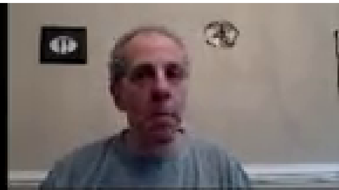
MAP - EM





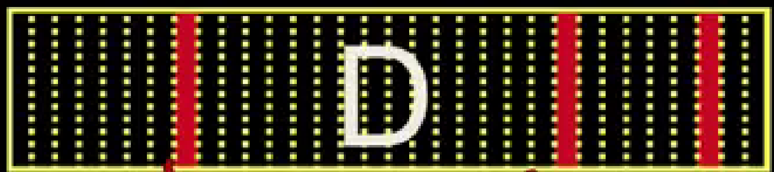


Structured and Collaborative Sparsity



Sparse estimate **V.S.** Piecewise linear estimate

K

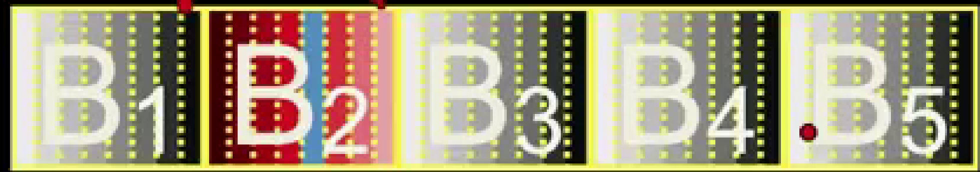


L

- Full degree of freedom in atom selection

$\binom{K}{L} \sim 10^{14}$

GMM ~ PCA



K ~ 10

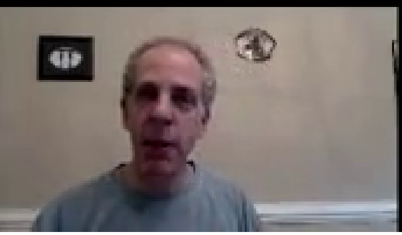
- Linear *collaborative* filtering in each basis.
- Nonlinear basis selection, degree of freedom $K \sim 10$.

Experiments: Inpainting

Zoom (original)

20% available 6.69 dB

PLE 30.07 dB



Experiments: Zooming