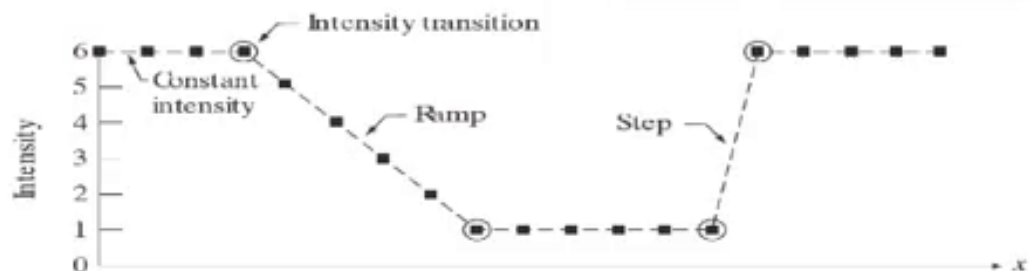$$\frac{\partial f}{\partial x} \cong f(x+1) - f(x)$$

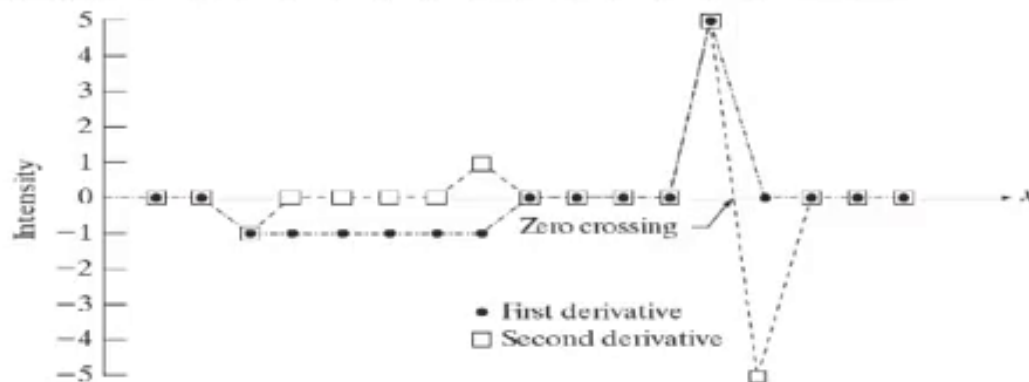| 0	0	0
0	-1	1
0	0	0

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

0	0	0
1	-2	1
0	0	0

Intensity transition
Constant intensity
Ramp
Step

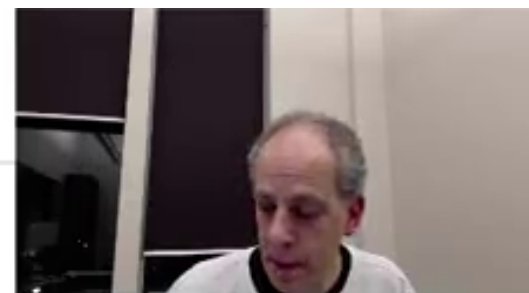| Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6	→ x
1st derivative		0	0	-1	-1	-1	-1	-1	0	0	0	0	0	5	0	0	0	0		
2nd derivative		0	0	-1	0	0	0	0	1	0	0	0	0	5	-5	0	0	0		

Zero crossing

- First derivative
- Second derivative

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \triangle f$$
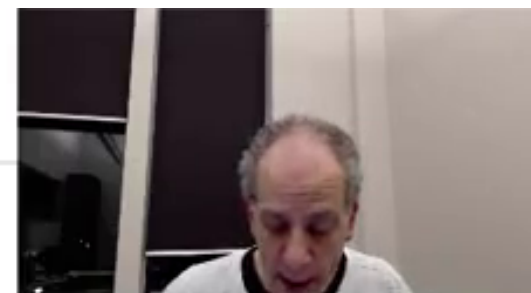
$$= f(x+1) + f(x-1) - 2f(x)$$

$$f(y+1) + f(y-1) - 2f(y)$$

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1

0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

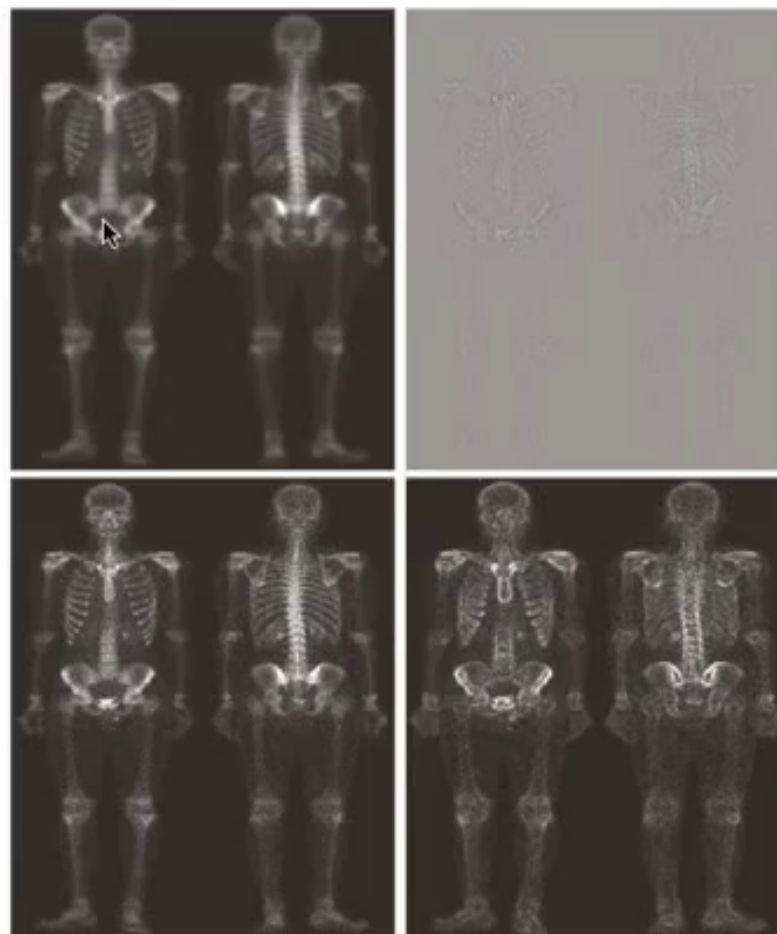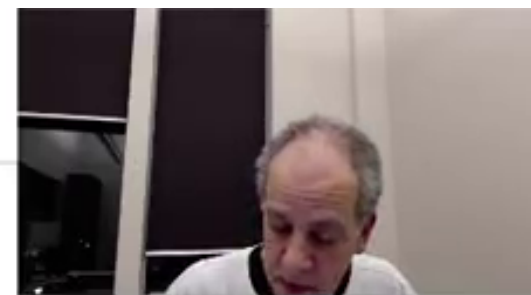| −1	−1	−1
−1	8	−1
−1	−1	−1

a
b c
d e
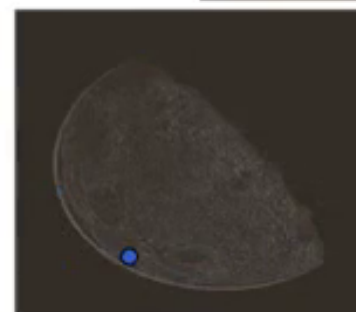
FIGURE 3.38
(a) Blurred image of the North Pole of the moon.
(b) Laplacian without scaling.
(c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

a b
c d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).
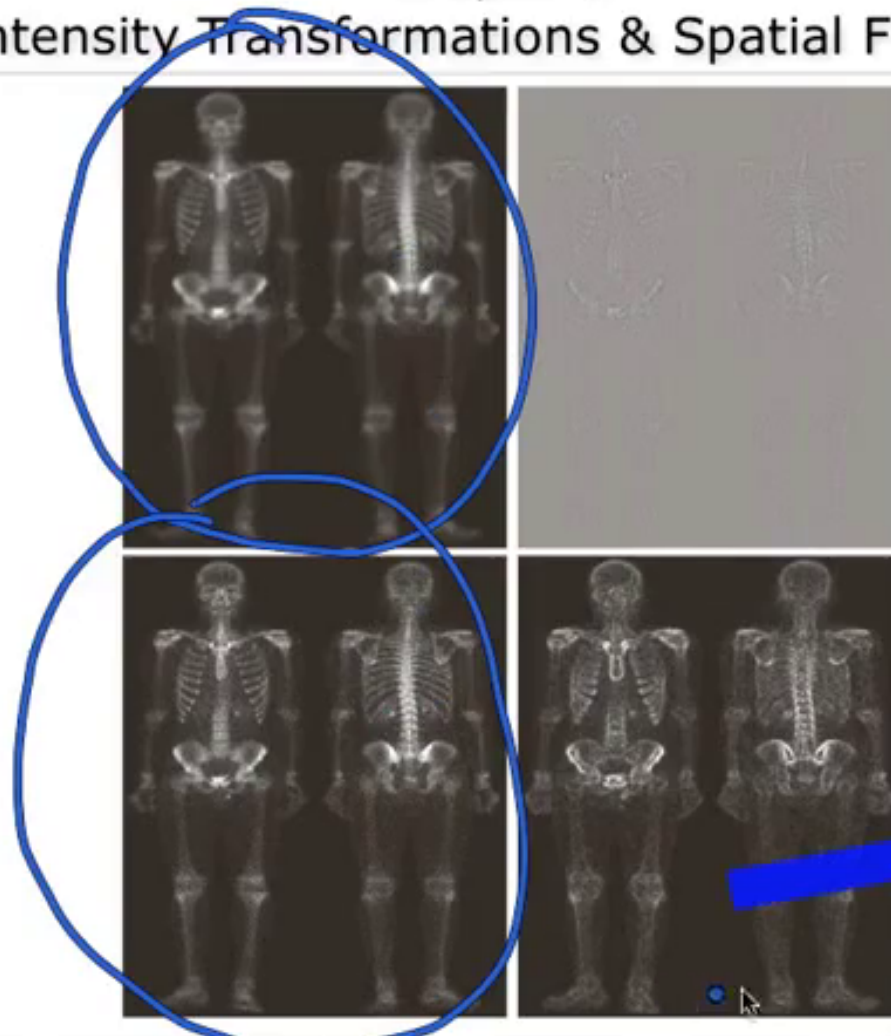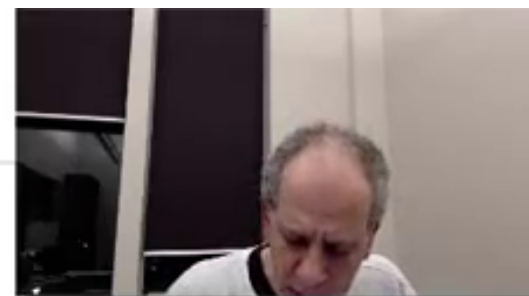(d) Sobel gradient of (a).

a b
c d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).
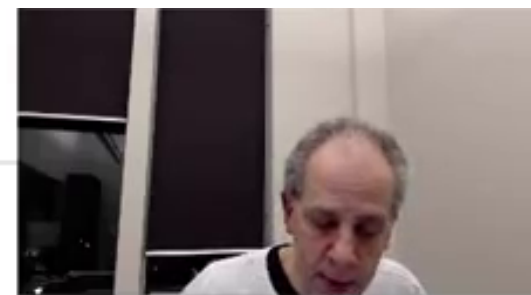(d) Sobel gradient of (a).

Sobel Edges

Unsharp

$\frac{1}{9}$

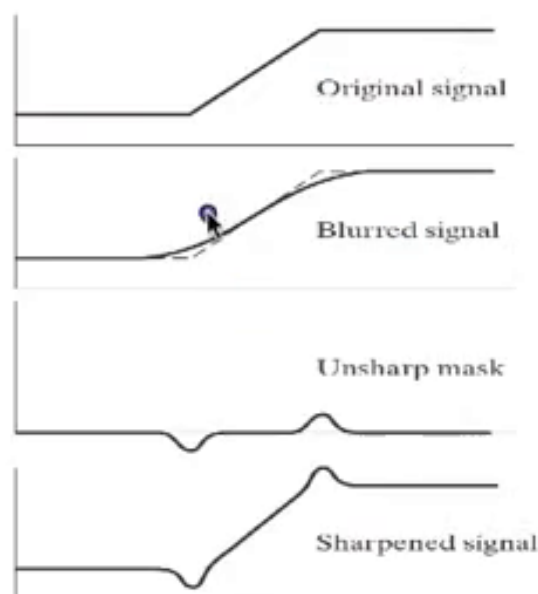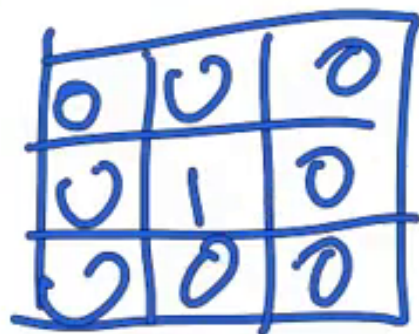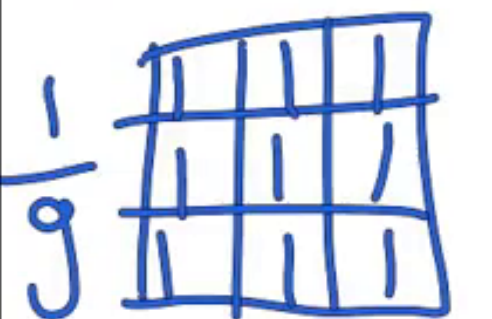| 0	0	0
0	1	0
0	0	0

a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

Original signal

Blurred signal

Unsharp mask

Sharpened signal