Extreme Value Analysis of Stock Returns using ARMA-GJR-GARCH Model

Submitted by:
Srikrishna Das

Under the guidance of:
Neelesh S. Upadhye

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Department of Mathematics
Indian Institute of Technology Madras

1 Introduction

This report presents an extreme value analysis of stock returns for SBIN (State Bank of India) and INFY (Infosys) using the AR-GJR-GARCH model with Student's t-distribution. The analysis includes model fitting, diagnostic tests, and threshold selection for extreme value modeling.

2 Data Overview

Financial data for SBIN.NS and INFY.NS were downloaded using the yfinance library. Daily log returns were computed as follows:

2.1 Log Returns Sample

The first three observations of log returns are shown below:

Table 1: Sample Log Returns (in %)

Date	INFY.NS	SBIN.NS
1996-01-02	-0.4052	-3.2347
1996-01-03	0.6746	-2.7025
1996-01-04	-0.6624	-0.3464

These values represent the percentage change in log prices from one day to the next.

3 Visualization of Log Returns

3.1 SBIN Log Returns

Figure 1 shows the time series of log returns for State Bank of India (SBIN) over the entire period. The plot reveals periods of high volatility, particularly during market stress periods.

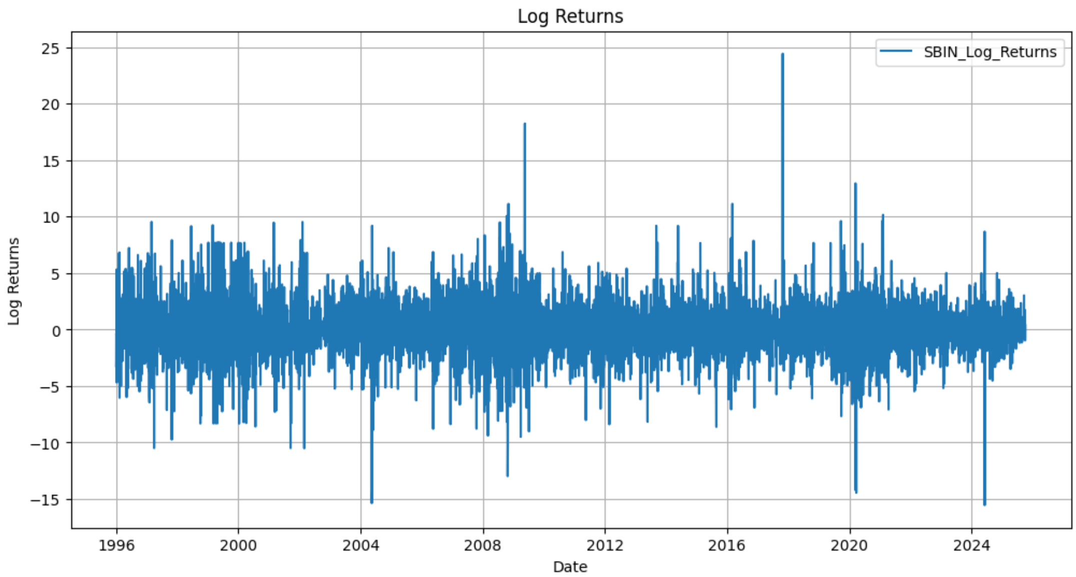

Figure 1: Log Returns of SBIN over time

3.2 INFY Log Returns

Figure 2 displays the log returns for Infosys (INFY). Similar to SBIN, the returns show volatility clustering, a common feature in financial time series.

Figure 2: Log Returns of INFY over time

4 Model Fitting and Diagnostics

An AR-GJR-GARCH(1,1) model with Student's t-distribution was fitted to both stocks. The data was split into 95% training and 5% testing sets.

4.1 SBIN.NS Model Results

Data Split:

- Train data shape: 7097 observations
- Test data shape: 374 observations

Model Selection Criteria:

- AIC: 30744.0794
- BIC: 30853.9357

Lower AIC and BIC values indicate a better fit of the model. These values will be used to compare different model specifications.

Diagnostic Tests:

Ljung-Box Test on Residuals (lag 20):

- LB statistic: 22.687
- p-value: 0.3044

The high p-value (¿0.05) indicates no significant autocorrelation in the residuals, suggesting the mean equation is adequate.

Ljung-Box Test on Squared Residuals (lag 20):

- LB statistic: 14.449
- p-value: 0.8070

The high p-value indicates no remaining ARCH effects, confirming the volatility model is well-specified.

ARCH LM Test (lag 20):

- LM statistic: 14.6875
- p-value: 0.7940

This test also confirms no remaining conditional heteroskedasticity in the residuals.

4.2 INFY.NS Model Results

Data Split:

- Train data shape: 7097 observations
- Test data shape: 374 observations

Model Selection Criteria:

- AIC: 29512.4196
- BIC: 29622.2759

INFY has lower AIC and BIC values compared to SBIN, suggesting the model fits slightly better for INFY data.

Diagnostic Tests:

Ljung-Box Test on Residuals (lag 20):

- LB statistic: 26.932
- p-value: 0.1372

Ljung-Box Test on Squared Residuals (lag 20):

- LB statistic: 11.662
- p-value: 0.9272

ARCH LM Test (lag 20):

- LM statistic: 12.6437
- p-value: 0.8921

All diagnostic tests pass (p-values ¿ 0.05), indicating the model is well-specified for INFY as well.

5 Extreme Value Analysis

5.1 SBIN Extreme Value Analysis

After fitting the GARCH model, standardized residuals were extracted for extreme value analysis using the Generalized Pareto Distribution (GPD).

5.1.1 Mean Residual Life Plots

Figure 3 shows the mean residual life plots for both upper and lower extremes. These plots help identify appropriate thresholds for the GPD model. A linear pattern above a certain threshold suggests the GPD is appropriate.

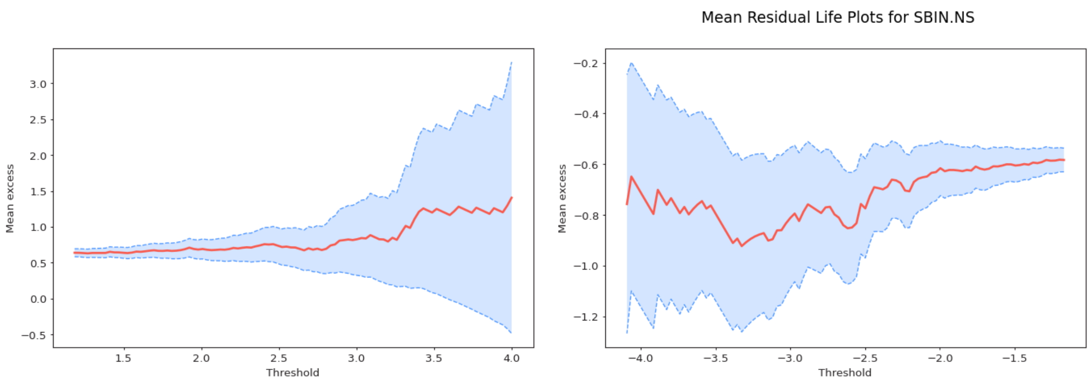

Figure 3: Mean Residual Life Plots for SBIN

5.1.2 Parameter Stability Plots

Figure 4 displays parameter stability plots showing how the GPD shape and scale parameters change with different threshold choices. Stable (flat) regions indicate good threshold candidates.

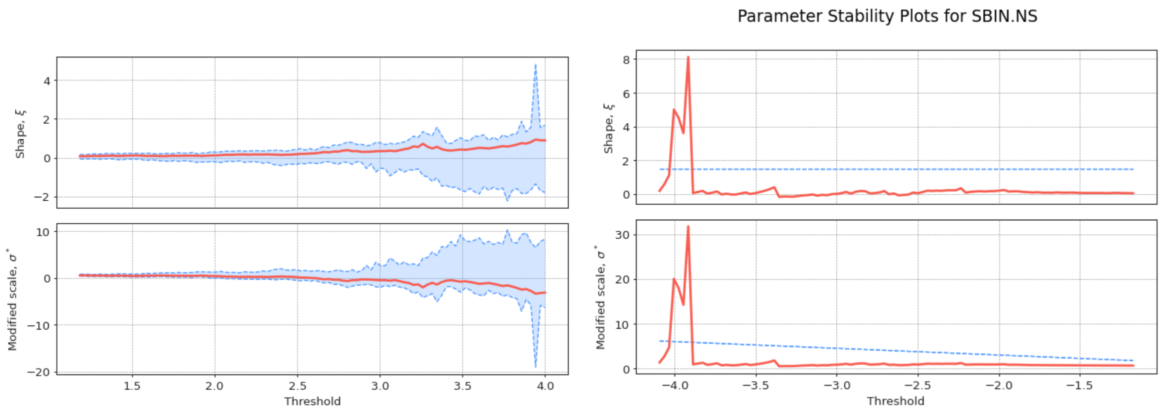

Figure 4: Parameter Stability Plots for SBIN

5.2 INFY Extreme Value Analysis

5.2.1 Mean Residual Life Plots

Figure 5 presents the mean residual life plots for INFY, used to guide threshold selection for modeling extreme values.

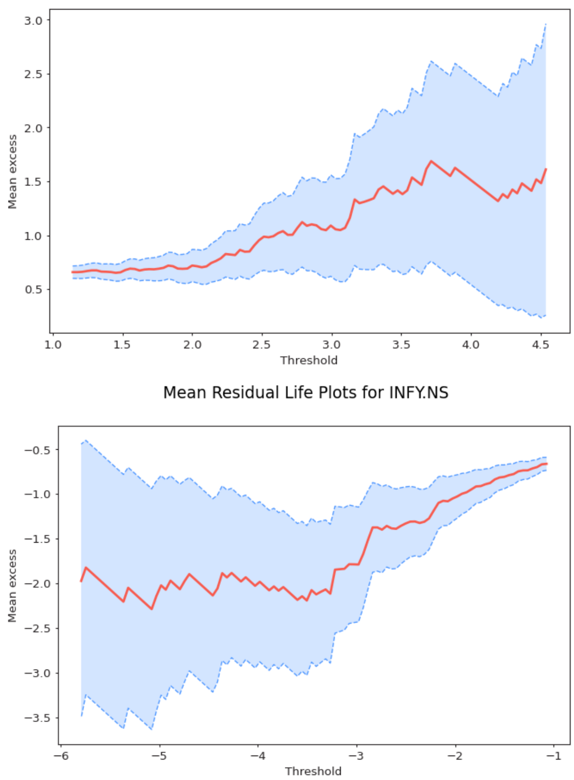

Figure 5: Mean Residual Life Plots for INFY

5.2.2 Parameter Stability Plots

Figure 6 shows parameter stability for INFY extremes.

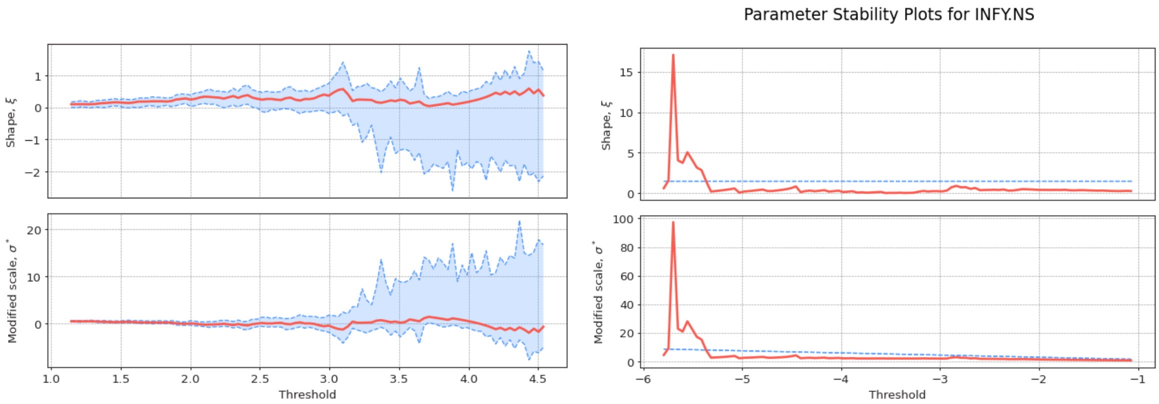

Figure 6: Parameter Stability Plots for INFY

6 Threshold Selection for SBIN

6.1 Upper Threshold Selection

The optimal upper threshold was selected by maximizing the p-value of the Kolmogorov-Smirnov (KS) test for GPD fit.

Selected Upper Threshold:

- Quantile: 0.970
- Threshold value: 1.9050
- Number of exceedances: 213
- Shape parameter: 0.1058
- Scale parameter: 0.6206
- KS p-value: 0.5947
- Anderson-Darling statistic: 0.6661

The high p-value (0.5951) indicates the GPD fits the upper tail well.

6.2 Lower Threshold Selection

Selected Lower Threshold:

- Quantile: 0.025
- Threshold value: -1.9664
- Number of exceedances: 178
- Shape parameter: 0.1533

- Scale parameter: 0.5307
- KS p-value: 0.7756
- Anderson-Darling statistic: 0.4125

The very high p-value (0.7753) confirms an excellent fit of the GPD to the lower tail.

7 Smooth CDF Construction for SBIN

A smooth cumulative distribution function (CDF) was constructed by combining:

- GPD for the lower tail (below threshold -1.9664)
- Student's t-distribution for the body (between thresholds)
- GPD for the upper tail (above threshold 1.9050)
- Logistic splice function for smooth transitions

Figure 7 shows the resulting smooth CDF across the entire range of standardized residuals.

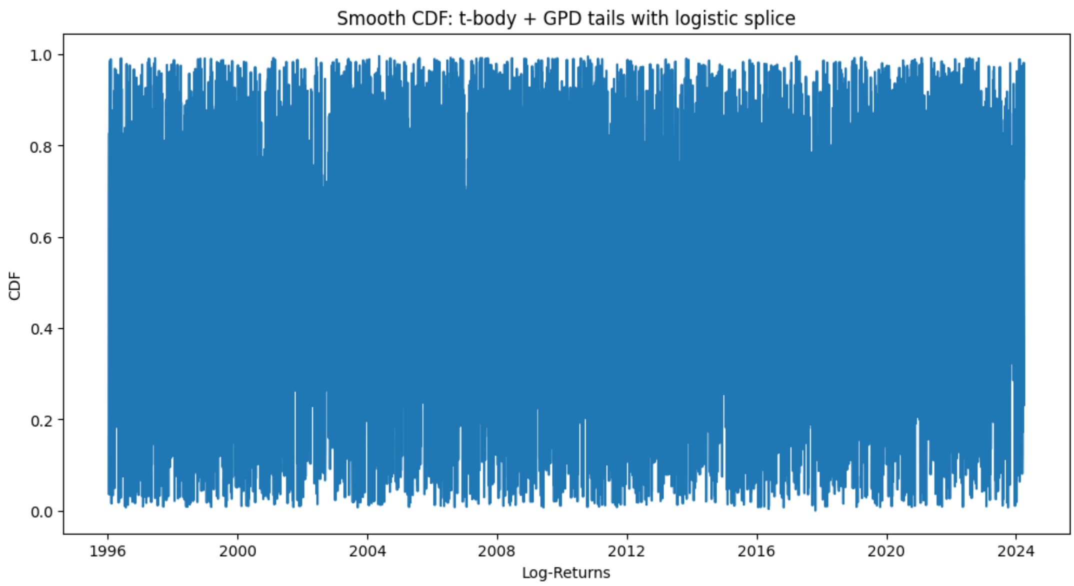

Figure 7: Smooth CDF: t-body + GPD tails with logistic splice

8 Probability Integral Transform (PIT) Validation for SBIN

8.1 PIT Histogram

The Probability Integral Transform was applied to validate the fitted distribution. If the model is correctly specified, the PIT values should be uniformly distributed on [0,1].

Figure 8 shows the histogram of PIT values.

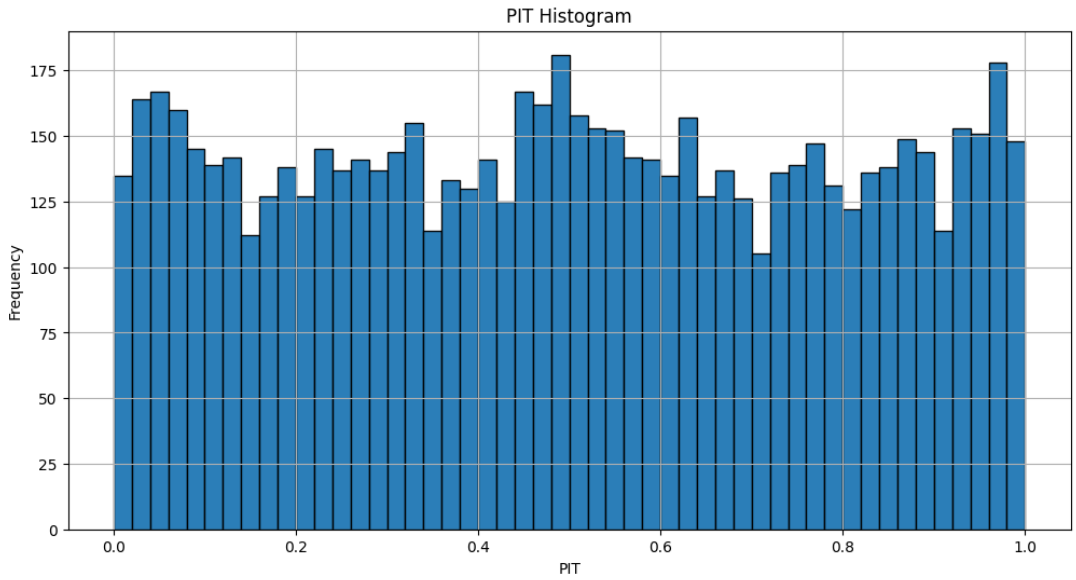

Figure 8: PIT Histogram

8.2 PIT Test Results for SBIN

PIT Validation Results: KS Statistic = 0.0116, p-value = 0.2974

8.2.1 Global PIT Optimization for SBIN.NS

Systematic search across threshold combinations to maximize KS p-value for global PIT:

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- **Optimal Combination:** (i=9, j=9)
- **KS Statistic:** 0.0116
- **p-value:** 0.2974
- **Upper Threshold:** Quantile = 0.970, Threshold = 1.9050, Exceedances = 213
- **Lower Threshold:** Quantile = 0.025, Threshold = -1.9664, Exceedances = 178

The optimized p-value (0.2976) is greater than 0.05, indicating that the smooth CDF with optimized thresholds provides a good fit to the data. This validates the combined GPD-t-GPD model for the standardized residuals.

9 Threshold Selection for INFY

9.1 Upper Threshold Selection

Selected Upper Threshold:

- Quantile: 0.935
- Threshold value: 1.4235

- Number of exceedances: 461
- Shape parameter: 0.1432
- Scale parameter: 0.5641
- KS p-value: 0.6600
- Anderson-Darling statistic: 0.8893

The high p-value indicates good GPD fit for the upper tail of INFY returns.

9.2 Lower Threshold Selection

Selected Lower Threshold:

- Quantile: 0.040
- Threshold value: -1.5829
- Number of exceedances: 284
- Shape parameter: 0.3542
- Scale parameter: 0.5462
- KS p-value: 0.9623
- Anderson-Darling statistic: 0.2552

The very high p-value (0.9623) confirms excellent fit for the lower tail of INFY returns.

10 Smooth CDF Construction for INFY

A smooth cumulative distribution function (CDF) was constructed by combining:

- GPD for the lower tail (below threshold -1.5829)
- Student's t-distribution for the body (between thresholds)
- GPD for the upper tail (above threshold 1.4235)
- Logistic splice function for smooth transitions

Figure 9 shows the resulting smooth CDF across the entire range of standardized residuals.

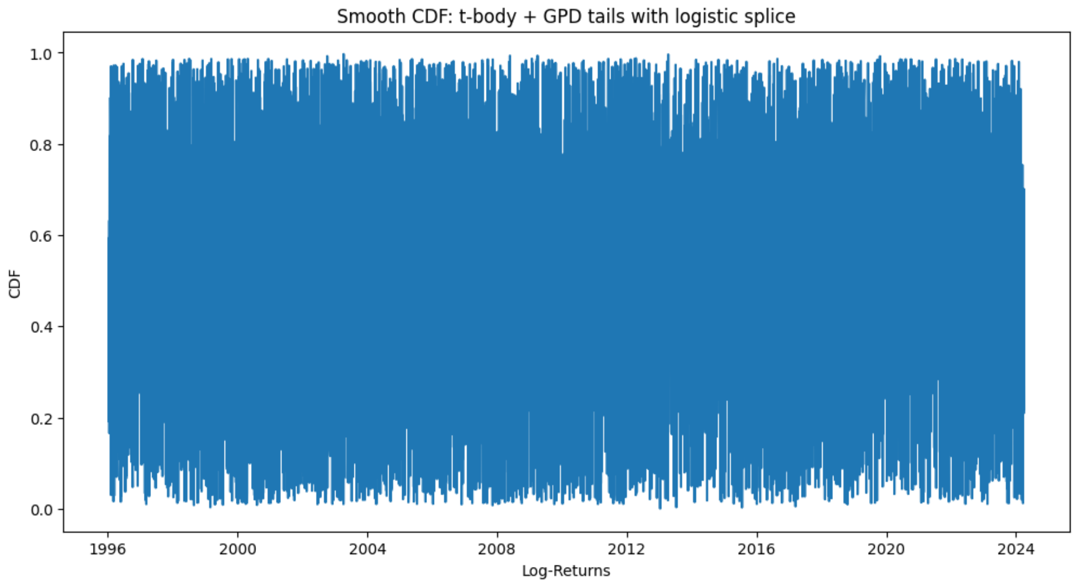

Figure 9: Smooth CDF: t-body + GPD tails with logistic splice (INFY)

11 Probability Integral Transform (PIT) Validation for INFY

11.1 PIT Histogram

The Probability Integral Transform was applied to validate the fitted distribution. If the model is correctly specified, the PIT values should be uniformly distributed on [0,1].

Figure 10 shows the histogram of PIT values.

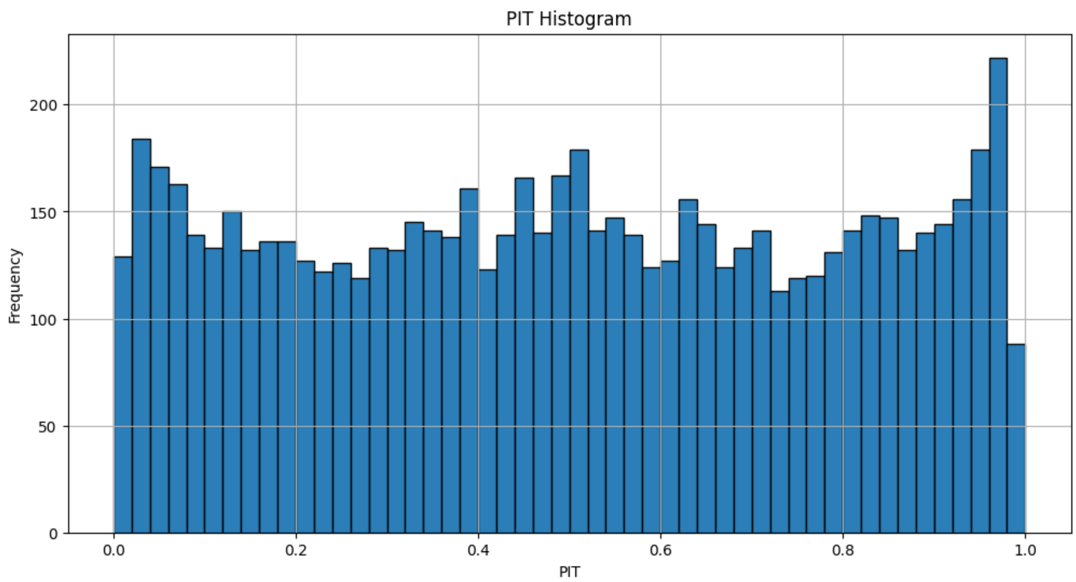

Figure 10: PIT Histogram for INFY

11.2 PIT Test Results

PIT Validation Results: KS Statistic = 0.0125, p-value = 0.2192

11.2.1 Global PIT Optimization for INFY

Systematic search across threshold combinations to maximize KS p-value for global PIT:

[itemsep=0.3em]

- **Optimal Combination:** (i=2, j=1)
- **KS Statistic:** 0.0125
- **p-value:** 0.2197
- **Upper Threshold:** Quantile = 0.935, Threshold = 1.4235, Exceedances = 461
- **Lower Threshold:** Quantile = 0.040, Threshold = -1.5829, Exceedances = 284

The optimized p-value (0.2197) is greater than 0.05, indicating that the smooth CDF with optimized thresholds provides a good fit to the data. This validates the combined GPD-t-GPD model for the standardized residuals of INFY.

12 Summary Tables

12.1 Summary Table for SBIN.NS

Metric	Value
Model	AR-GJR-GARCH-StudentsT + Composite (t-body + GPD tails)
LB (ε) p-value	0.3044
LB (ε^2) p-value	0.8070
ARCH-LM p-value	0.7940
Upper Threshold	1.9050
Upper Exceedances	213
GPD Upper ξ	0.1058
GPD Upper β	0.6207
Lower Threshold	-1.9664
Lower Exceedances	178
GPD Lower ξ	0.1533
GPD Lower β	0.5308
PIT KS p-value	0.2976

12.2 Summary Table for INFY.NS

Metric	Value
Model	AR-GJR-GARCH-StudentsT + Composite (t-body + GPD tails)
LB (ε) p-value	0.1372
LB (ε^2) p-value	0.9272
ARCH-LM p-value	0.8921
Upper Threshold	1.4235
Upper Exceedances	461
GPD Upper ξ	0.1432
GPD Upper β	0.5640
Lower Threshold	-1.5829
Lower Exceedances	284
GPD Lower ξ	0.3543
GPD Lower β	0.5462
PIT KS p-value	0.2197