

KNN

! Assume we have a training dataset D made up of $(x_i)_{i \in [1, n]}$ samples where $(n = |D|)$. The examples are described by a set of features F and any numeric features have been normalised to the range $[0, 1]$. Each training example is labelled with a class label $y_i \in Y$.

Objective: classify an unknown example q .
(q can be called a query)

For each $x_i \in D$ we can calculate the distance between q & x_i as follows.

$$d(q, x_i) = \sum_{f \in F} w_f \delta(q_f, x_{i,f})$$

This is a summation over all the features in F with w_f the weight for each feature

$$\delta(q_f, x_{i,f}) = \begin{cases} 0, & f \text{ discrete and } q_f = x_{i,f} \\ 1, & f \text{ discrete and } q_f \neq x_{i,f} \\ |q_f - x_{i,f}|, & f \text{ continuous} \end{cases}$$

The KNN are selected based on this distance metric.

Votes (weighted distance)

$$\text{Vote}(y_j) = \sum_{c=1}^K \frac{1}{d(q, x_c)^p} \cdot g(y_j, y_c)$$

$$g(a, b) = 1 \text{ if } a_{\text{label}} = b_{\text{label}} \text{ else } 0$$
