Department of Electronic & Telecommunication Engineering

University of Moratuwa

BM 2101 – Analysis of Physiological Systems

ASSIGNMENT 02

BRANCHED CYLINDERS: DENDRITIC TREE APPROXIMATIONS

Name	**Index number**
Premathilaka H. D. M.	180497C

This report is submitted in partial fulfillment of the requirements for the module BM 2101 – Analysis of Physiological Systems.

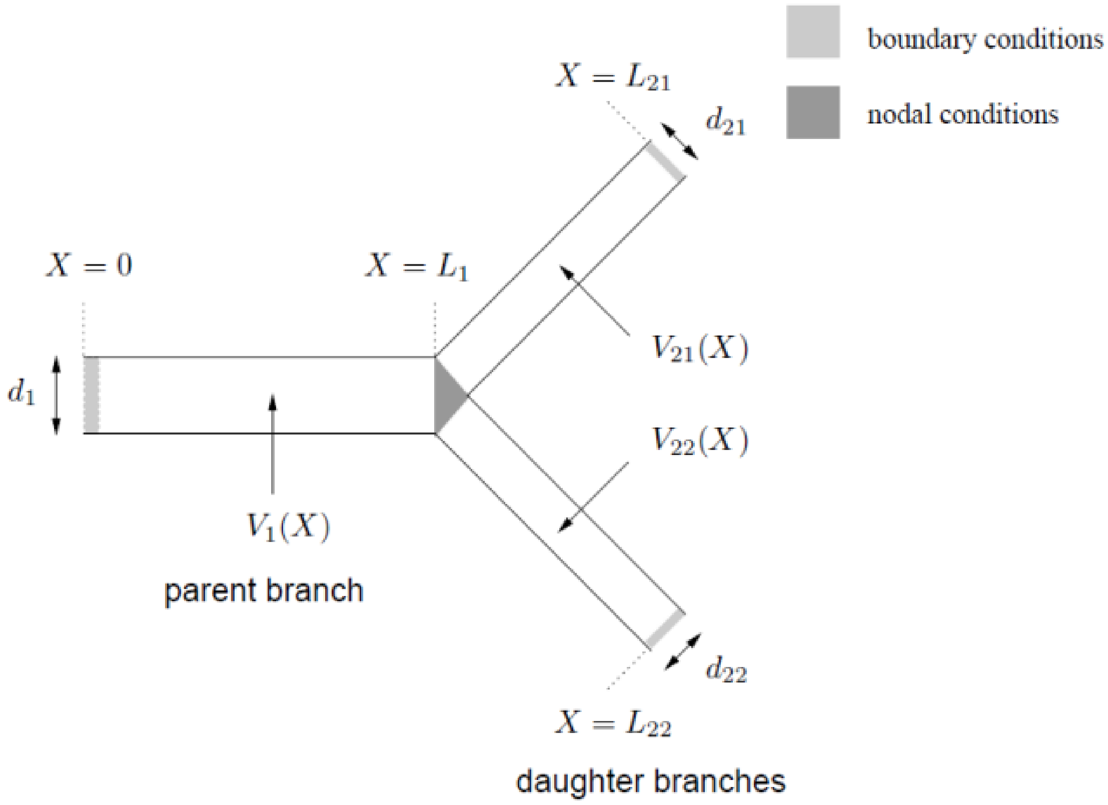
Date of submission: 05/02/2021

Question 01

By referring to the general solution (2) and the equations of constraints (3) - (6), convince yourself of the truth of equations (7).

Solution 01

- Consider the following first order branched cable.



- V_1, V_{21}, V_{22} are the membrane potentials of the respective branches
- d_1, d_{21}, d_{22} are the diameters of the parent and daughter branches
- $X = x/\lambda_C$ is the axial distance

- At the **steady state**, the membrane potential of each branch will satisfy,

$$\frac{d^2V}{dX^2} = V \qquad \text{-- (1)}$$

- Thus, a general solution for the membrane potential as a function of electrotonic distance will be,

$$V_1(X) = A_1 e^{-X} + B_1 e^{X} \qquad 0 \le X \le L_1 \qquad \text{----------------------------(2) [1]}$$

$$V_{21}(X) = A_{21} e^{-X} + B_{21} e^{X} \qquad L_1 \le X \le L_{21} \qquad \text{-----------------------------(2)[2]}$$

$$V_{22}(X) = A_{22} e^{-X} + B_{22} e^{X} \qquad L_1 \le X \le L_{22} \qquad \text{-----------------------------(2)[3]}$$

- Suppose an applied current I_{app} is injected at the inner conductor at $X = 0$. Then,

$$\left.\frac{dV_1}{dX}\right|_{X=0} = -(r_i\lambda_c)_1 I_{app} \qquad \text{---(3)}$$

- By differentiating equation (2)[1] and then evaluating at X = 0,

$$\left.\frac{dV_1}{dX}\right|_{X=0} = -A_1e^{-0} + B_1e^{0}$$

$$\left.\frac{dV_1}{dX}\right|_{X=0} = -A_1 + B_1$$

- By equating the above result with equation (3),

$$-(r_i\lambda_c)_1 I_{app} = -A_1 + B_1$$

$$A_1 - B_1 = (r_i\lambda_c)_1 I_{app} \qquad \text{---(A)}$$

- Given that the terminal ends of the daughter branches are held at rest (killed ends), we can say that,

$$V_{21}(L_{21}) = V_{22}(L_{22}) = 0 \qquad \text{---(4)}$$

- Now, we can substitute X = L_{21} in equation (2)[2] and X = L_{22} in equation (2)[3]. Thus we get,

$$V_{21}(L_{21}) = A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0 \qquad \text{---(B)}$$

$$V_{22}(L_{22}) = A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0 \qquad \text{---(C)}$$

- The membrane potentials must be continuous at the nodes, provided that no currents are applied other than at the terminals. Therefore,

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1) \qquad \text{---(5)}$$

- Using the above condition, let's substitute X = L_1 to equations (2)[1], (2)[2] and (2)[3].
- Let's first consider,

$$V_1(L_1) = V_{21}(L_1)$$

$$A_1e^{-L_1} + B_1e^{L_1} = A_{21}e^{-L_1} + B_{21}e^{L_1}$$

- Then,

$$A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} = 0 \qquad \text{---(D)}$$

- Let's now consider,

$$V_{21}(L_1) = V_{22}(L_1)$$

$$A_{21}e^{-L_1} + B_{21}e^{L_1} = A_{22}e^{-L_1} + B_{22}e^{L_1}$$

- Then,

$$A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} = 0 \qquad \text{---------------------------------------(E)}$$

- Moreover, the current is conserved at the nodes. That is, the current flowing out of the parent branch must be equal to the sum of the currents flowing in to the daughter branches.
- The mathematical interpretation of the above concept is as follows.

$$\frac{-1}{(r_i\lambda_c)_1}\frac{dV_1}{dX}\bigg|_{X=L_1} = \frac{-1}{(r_i\lambda_c)_{21}}\frac{dV_{21}}{dX}\bigg|_{X=L_1} + \frac{-1}{(r_i\lambda_c)_{22}}\frac{dV_{22}}{dX}\bigg|_{X=L_1} \quad \text{------------------(6)}$$

- Before using the above equation, let's differentiate equations (2)[1], (2)[2] and (2)[3] with respect to X and evaluate at X = L_1.

$$\frac{dV_1}{dX}\bigg|_{X=L_1} = -A_1e^{-L_1} + B_1e^{L_1}$$

$$\frac{dV_{21}}{dX}\bigg|_{X=L_1} = -A_{21}e^{-L_1} + B_{21}e^{L_1}$$

$$\frac{dV_{22}}{dX}\bigg|_{X=L_1} = -A_{22}e^{-L_1} + B_{22}e^{L_1}$$

- By substituting the above results in equation (6),

$$\frac{-1}{(r_i\lambda_c)_1}[-A_1e^{-L_1} + B_1e^{L_1}] = \frac{-1}{(r_i\lambda_c)_{21}}[-A_{21}e^{-L_1} + B_{21}e^{L_1}] + \frac{-1}{(r_i\lambda_c)_{22}}[-A_{22}e^{-L_1} + B_{22}e^{L_1}]$$

- By simplifying the above equation,

$$\frac{1}{(r_i\lambda_c)_1}[-A_1e^{-L_1} + B_1e^{L_1}] = \frac{1}{(r_i\lambda_c)_{21}}[-A_{21}e^{-L_1} + B_{21}e^{L_1}] + \frac{1}{(r_i\lambda_c)_{22}}[-A_{22}e^{-L_1} + B_{22}e^{L_1}]$$

$$\frac{-A_1}{(r_i\lambda_c)_1}e^{-L_1} + \frac{B_1}{(r_i\lambda_c)_1}e^{L_1} = \frac{-A_{21}}{(r_i\lambda_c)_{21}}e^{-L_1} + \frac{B_{21}}{(r_i\lambda_c)_{21}}e^{L_1} + \frac{-A_{22}}{(r_i\lambda_c)_{22}}e^{-L_1} + \frac{B_{22}}{(r_i\lambda_c)_{22}}e^{L_1}$$

- Rearranging the above equation,

$$\frac{-A_1}{(r_i\lambda_c)_1}e^{-L_1} + \frac{B_1}{(r_i\lambda_c)_1}e^{L_1} + \frac{A_{21}}{(r_i\lambda_c)_{21}}e^{-L_1} - \frac{B_{21}}{(r_i\lambda_c)_{21}}e^{L_1} + \frac{A_{22}}{(r_i\lambda_c)_{22}}e^{-L_1} - \frac{B_{22}}{(r_i\lambda_c)_{22}}e^{L_1} = 0 \quad \text{-----(F)}$$

- Let's summarize the results obtained. (equations (A), (B), (C), (D), (E) and (F)). Label the system of equations as (7).

$$\left.\begin{array}{c}
A_1 - B_1 = (r_i\lambda_c)_1 I_{app} \\
A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0 \\
A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0 \\
A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} = 0 \\
A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} = 0 \\
\frac{-A_1}{(r_i\lambda_c)_1}e^{-L_1} + \frac{B_1}{(r_i\lambda_c)_1}e^{L_1} + \frac{A_{21}}{(r_i\lambda_c)_{21}}e^{-L_1} - \frac{B_{21}}{(r_i\lambda_c)_{21}}e^{L_1} + \frac{A_{22}}{(r_i\lambda_c)_{22}}e^{-L_1} - \frac{B_{22}}{(r_i\lambda_c)_{22}}e^{L_1} = 0
\end{array}\right\} \quad (7)$$

Question 02

By performing the matrix multiplication of equation (9), show that you obtain equations (7).

Solution 02

- Let's define the matrices A, X and b as follows.

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ \frac{-e^{-L_1}}{(r_i\lambda_c)_1} & \frac{e^{L_1}}{(r_i\lambda_c)_1} & \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{21}} & \frac{e^{-L_1}}{(r_i\lambda_c)_{22}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{22}} \end{pmatrix}$$

$$x = \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} \quad \text{------------------------(8)} \qquad b = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \text{----------------------(10)}$$

- Let's verify whether the system of equations in (7) can be written in the form,

$$Ax = b \qquad \text{---(9)}$$

- Then,

$$\underbrace{\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ \frac{-e^{-L_1}}{(r_i\lambda_c)_1} & \frac{e^{L_1}}{(r_i\lambda_c)_1} & \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{21}} & \frac{e^{-L_1}}{(r_i\lambda_c)_{22}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{22}} \end{pmatrix}}_{\text{A}} \underbrace{\begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{\mathbf{b}}$$

- Now, let's consider the matrix multiplication A**x.**

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ \frac{-e^{-L_1}}{(r_i\lambda_c)_1} & \frac{e^{L_1}}{(r_i\lambda_c)_1} & \frac{e^{-L_1}}{(r_i\lambda_c)_{21}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{21}} & \frac{e^{-L_1}}{(r_i\lambda_c)_{22}} & \frac{-e^{L_1}}{(r_i\lambda_c)_{22}} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix}$$

- The above matrix multiplication will yield the following matrix.

$$\begin{pmatrix} A_1 - B_1 \\ A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} \\ A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} \\ A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} \\ A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} \\ \frac{-A_1}{(r_i\lambda_c)_1}e^{-L_1} + \frac{B_1}{(r_i\lambda_c)_1}e^{L_1} + \frac{A_{21}}{(r_i\lambda_c)_{21}}e^{-L_1} - \frac{B_{21}}{(r_i\lambda_c)_{21}}e^{L_1} + \frac{A_{22}}{(r_i\lambda_c)_{22}}e^{-L_1} - \frac{B_{22}}{(r_i\lambda_c)_{22}}e^{L_1} \end{pmatrix}_{6\times 1}$$

- By plugging the above result in equation (9),

$$\begin{pmatrix} A_1 - B_1 \\ A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} \\ A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} \\ A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} \\ A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} \\ \frac{-A_1}{(r_i\lambda_c)_1}e^{-L_1} + \frac{B_1}{(r_i\lambda_c)_1}e^{L_1} + \frac{A_{21}}{(r_i\lambda_c)_{21}}e^{-L_1} - \frac{B_{21}}{(r_i\lambda_c)_{21}}e^{L_1} + \frac{A_{22}}{(r_i\lambda_c)_{22}}e^{-L_1} - \frac{B_{22}}{(r_i\lambda_c)_{22}}e^{L_1} \end{pmatrix}_{6\times 1} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}_{6\times 1}$$

- Since the two matrices are equal, the corresponding entries in both matrices must be equal. Thus we get,

$$A_1 - B_1 = (r_i\lambda_c)_1 I_{app}$$

$$A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0$$

$$A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0$$

$$A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} = 0$$

$$A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} = 0$$

$$\frac{-A_1}{(r_i\lambda_c)_1}e^{-L_1} + \frac{B_1}{(r_i\lambda_c)_1}e^{L_1} + \frac{A_{21}}{(r_i\lambda_c)_{21}}e^{-L_1} - \frac{B_{21}}{(r_i\lambda_c)_{21}}e^{L_1} + \frac{A_{22}}{(r_i\lambda_c)_{22}}e^{-L_1} - \frac{B_{22}}{(r_i\lambda_c)_{22}}e^{L_1} = 0$$

- Therefore, the system of equations in (7) can be obtained by performing the matrix multiplication mentioned in (9), provided that the matrices A, x, b have the aforementioned values.

Question 03

By defining b, determine the values of the coefficients of equation (2) assuming the boundary conditions of sections 1 and 2. Make sure you define b as a column vector.

Solution 03

- In the previous question, we proved that the system of equations in (7) can be expressed in the form A**x** = **b**.
- Now, let's use the above matrix system to determine the values of A_1, B_1, A_{21}, B_{21}, A_{22} and B_{22}.

$$\boxed{Ax = b \implies A^{-1}.Ax = A^{-1}b \implies x = A^{-1}b}$$

- The MATLAB implementation of the above expression is 'x = A\b'.

```
%%
%Finding the solutions of the system Ax = b
x = A\b
```

```
x =
   0.000736975648372
   0.000016722595446
   0.001129071096534
  -0.000002798687438
   0.001129071096534
  -0.000002798687438
```

- Therefore,

$$\begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} 0.000737 \\ 0.000017 \\ 0.001129 \\ -0.000003 \\ 0.001129 \\ -0.000003 \end{pmatrix}$$

- $A_1 \approx 0.0007$
- $B_1 \approx 0$
- $A_{21} \approx 0.0011$
- $B_{21} \approx 0$
- $A_{22} \approx 0.0011$
- $B_{22} \approx 0$

Question 04

By using the coefficients found above and assuming that the coefficient array is stored in the variable x ordered according to equation (8), plot the steady state voltage profile in each branch by executing the following sequence of commands.

Solution 04

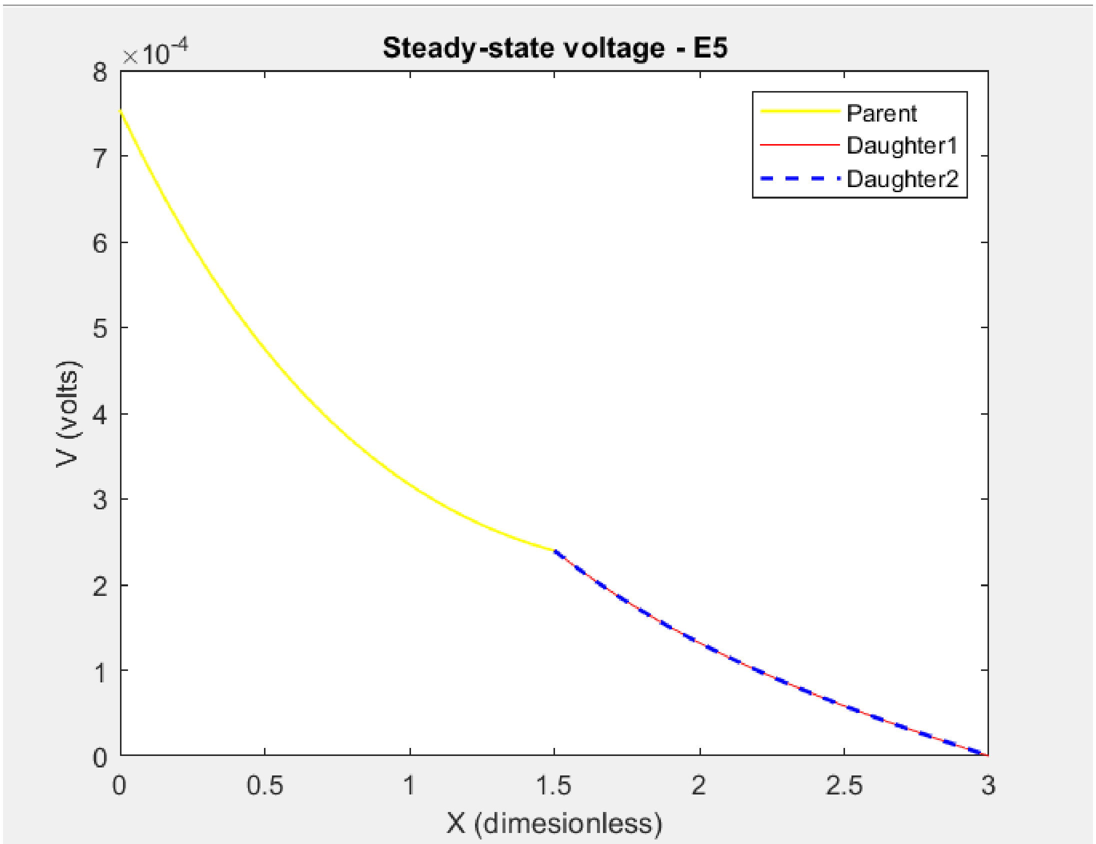
The values used in this section are from the calculations in question 03 (from the code in cable.m).

```
l1 = 1.5;                          % dimensionless
l21 = 3.0;                         % dimensionless
l22 = 3.0;                         % dimensionless

x = [0.000736975648371907;1.67225954462220e-05;0.00112907109653384;
    -2.79868743814432e-06;0.00112907109653384;-2.79868743814432e-06];

y1 = linspace(0,l1, 20);
y21 = linspace(l1,l21, 20);
y22 = linspace(l1,l22, 20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);

p = plot(y1, v1, 'y-', y21, v21, 'r-', y22, v22, 'b--');
p(1).LineWidth = 1;
p(2).LineWidth = 0.5;
p(3).LineWidth = 1.5;
xlabel('X (dimesionless)')
ylabel('V (volts)')
title('Steady-state voltage - E5')
legend('Parent','Daughter1', 'Daughter2')
```

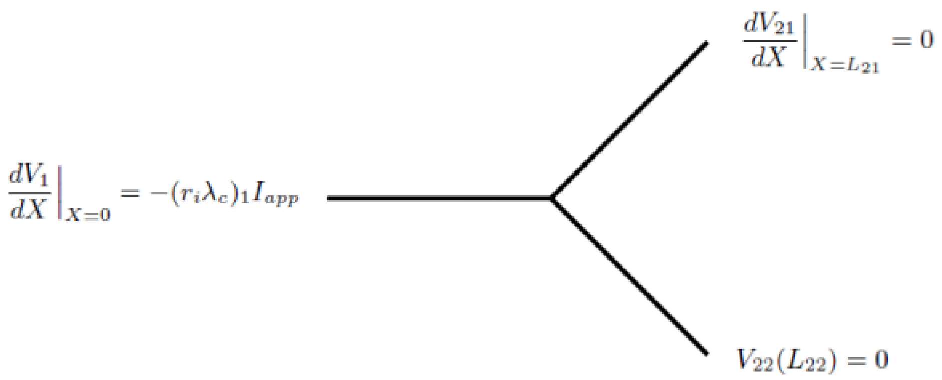


What do you note about the steady state voltage profile in the two daughter branches?

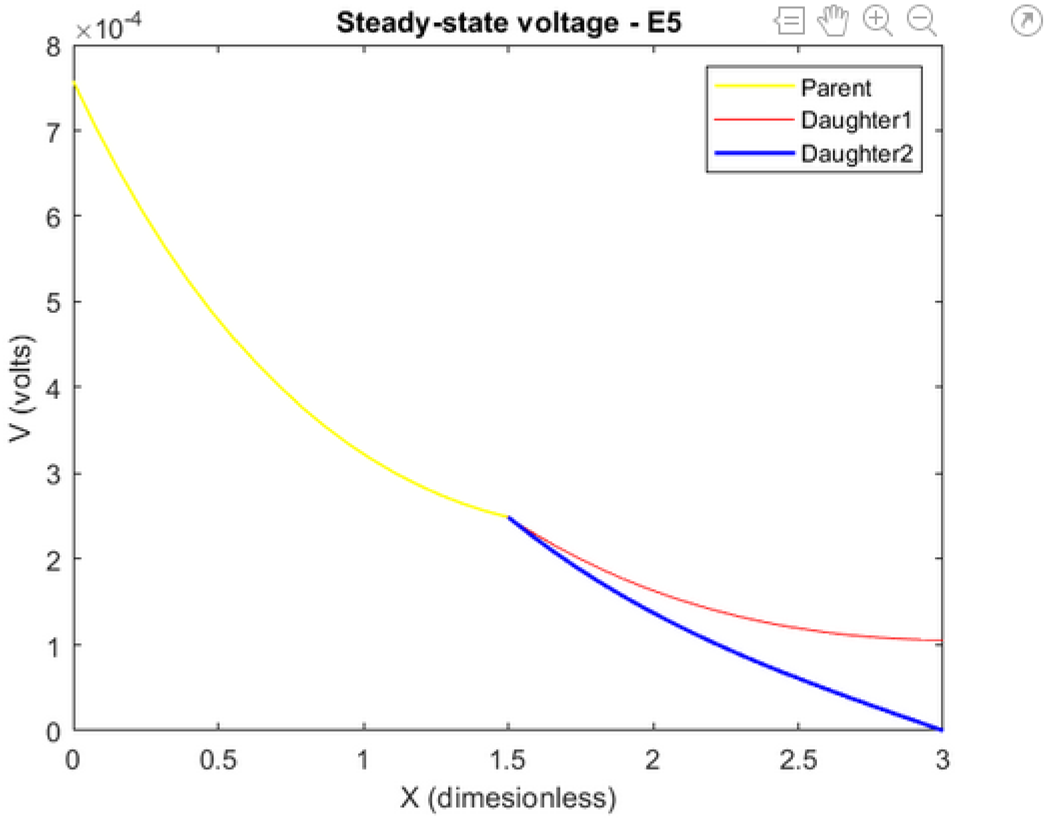
- The above figure indicates that the variation of the steady state voltage of the two daughter branches are the same, despite the differences in their diameters.
- This *might be* implying that the steady state voltage profiles of the daughter branches of a single branched cable are independent of their diameters.

<u>Section 04 – Solutions for different boundary conditions</u>

(a).



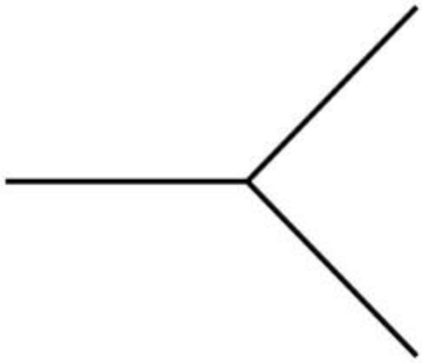

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
```



(b).

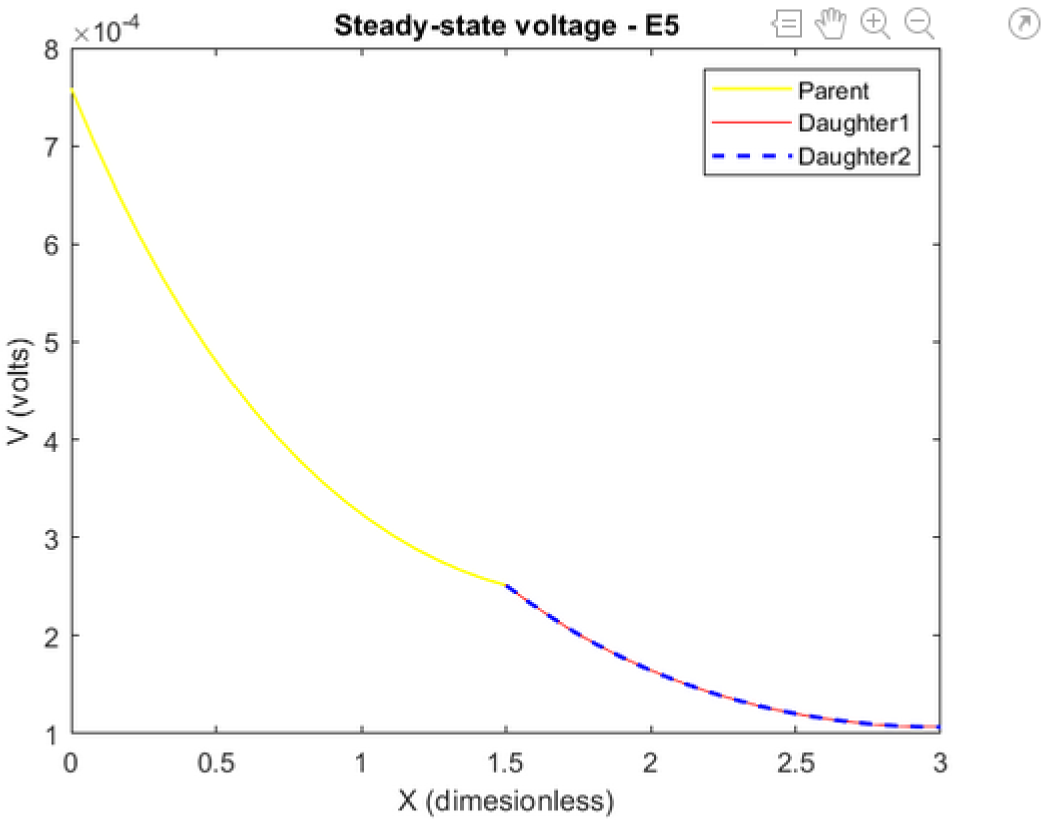
$$\frac{dV_{21}}{dX}\bigg|_{X=L_{21}} = 0$$

$$\frac{dV_1}{dX}\bigg|_{X=0} = -(r_i\lambda_c)_1 I_{app}$$



$$\frac{dV_{22}}{dX}\bigg|_{X=L_{22}} = 0$$

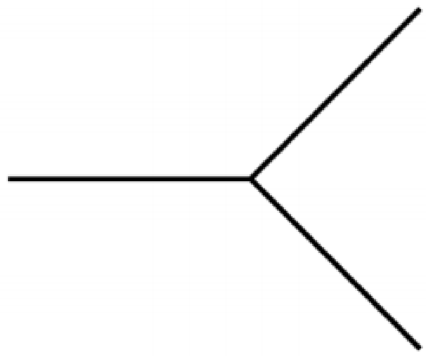
```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];
```



(c).

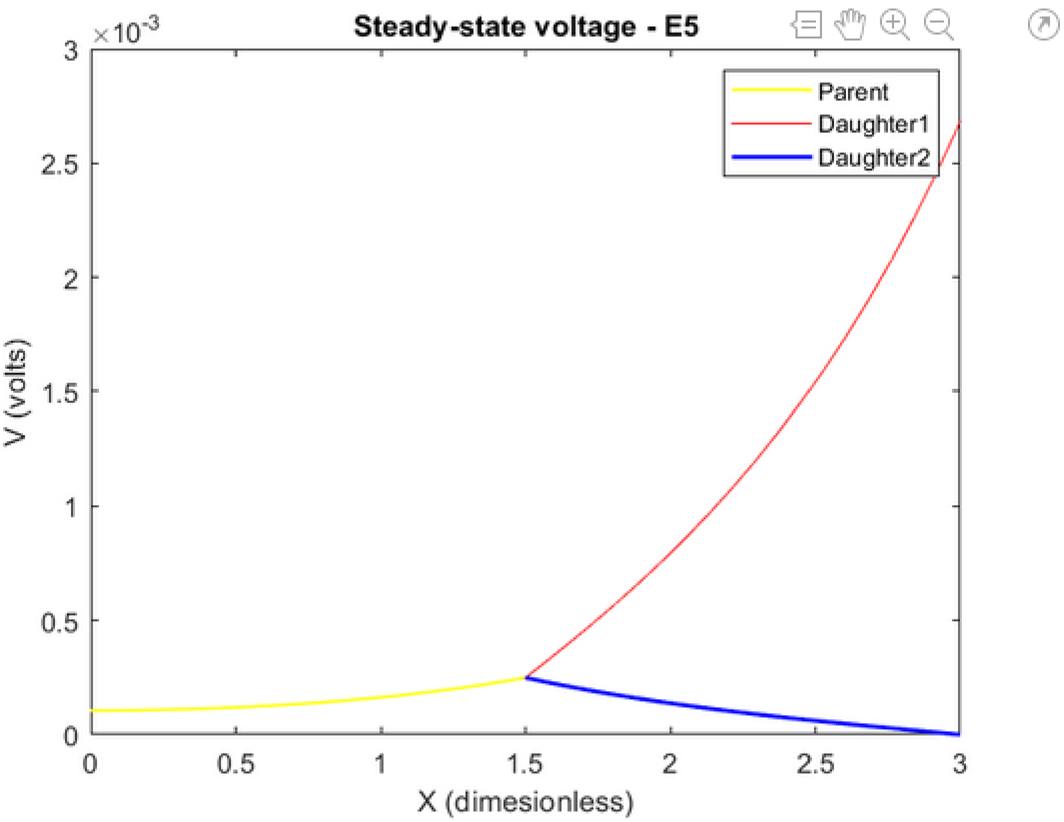
$$\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}} = (r_i\lambda_c)_{21} I_{app}$$

$$\left.\frac{dV_1}{dX}\right|_{X=0} = 0$$



$$V_{22}(L_{22}) = 0$$

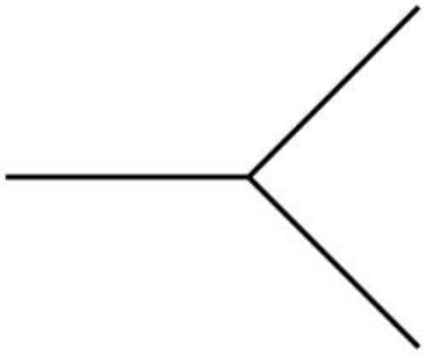
```
A(2,:) = [ 0 0 -exp(-l21) exp(l21) 0 0];
b(1) = 0;
b(2) = rl21*iapp;
```



(d).

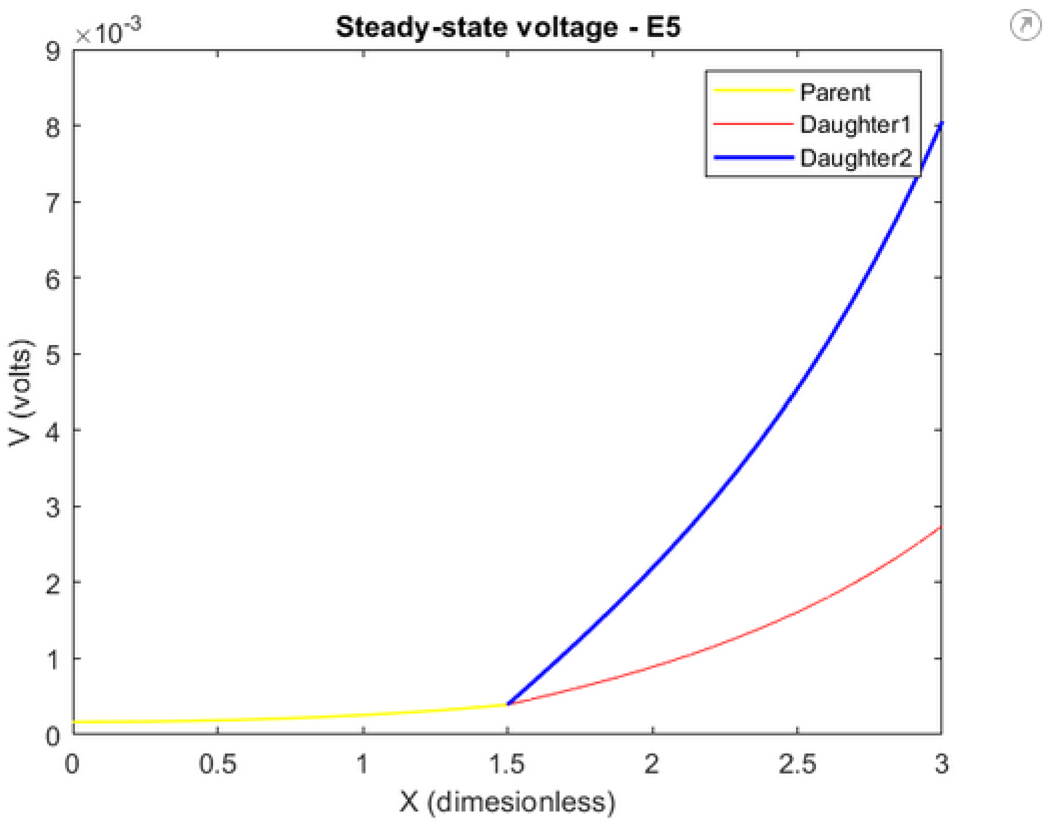
$$\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}} = (r_i\lambda_c)_{21}I_{app}$$

$$\left.\frac{dV_1}{dX}\right|_{X=0} = 0$$



$$\left.\frac{dV_{22}}{dX}\right|_{X=L_{22}} = (r_i\lambda_c)_{22}I_{app}$$

```
b(3) = rl22*iapp;
```

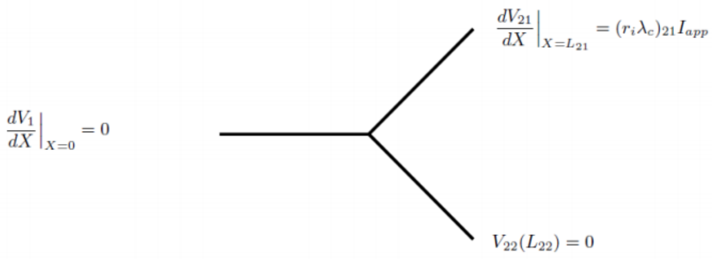


Question 05

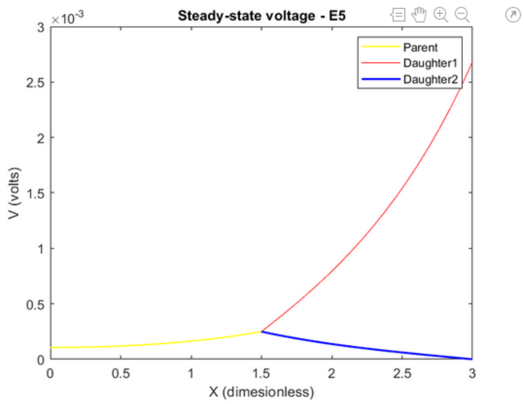
What is the meaning of the positive right hand sides of $\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}}$ and $\left.\frac{dV_{22}}{dX}\right|_{X=L_{22}}$ in 2(c) and 2(d)?

Solution 05

Let’s first consider the figure 2(c) and the corresponding voltage variations.

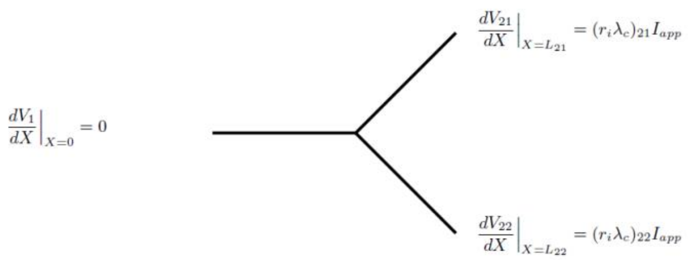


- $\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}} = (r_i\lambda_c)_{21}I_{app} > 0$ implies that a current is injected from the terminal end of daughter branch (22).
- The X values are measured from the parent end and in this case we are injecting a current from the terminal end of a daughter branch. Therefore, the steady state voltage will show a decrease in the direction of decreasing X values. Thus, the gradient at X = L_{22} is positive.

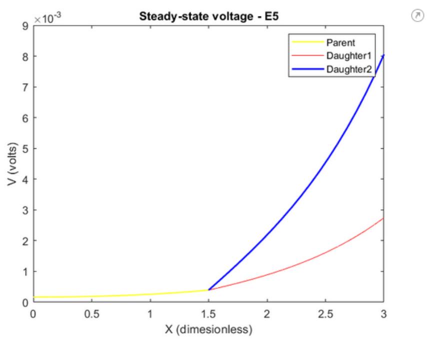


- The voltage at the terminal end of the daughter branch (21) is zero (i.e. $V_{22}(L_{22}) = 0$) which indicates that this terminal end is held at rest.
- On the other hand, the derivative of the steady state voltage at the parent end is zero which denotes that its voltage is a constant. From the graph shown, we can conclude that the steady state voltage at the parent end is a <u>non-zero</u> constant.

Now let’s consider the figure 2(d) and the corresponding voltage variations.



- $\left.\frac{dV_{21}}{dX}\right|_{X=L_{21}} = (r_i\lambda_c)_{21}I_{app} > 0$ and $\left.\frac{dV_{22}}{dX}\right|_{X=L_{22}} = (r_i\lambda_c)_{22}I_{app} > 0$. Therefore, in this case, an applied current is injected at both terminal ends of the daughter branches. (At X = L_{21} and X = L_{22}).
- As in the figure 2(c), the derivative of the steady state voltage at the parent end is zero which denotes that its voltage is a constant. From the graph shown, we can conclude that the steady state voltage at the parent end is a <u>non-zero</u> constant.



Question 06

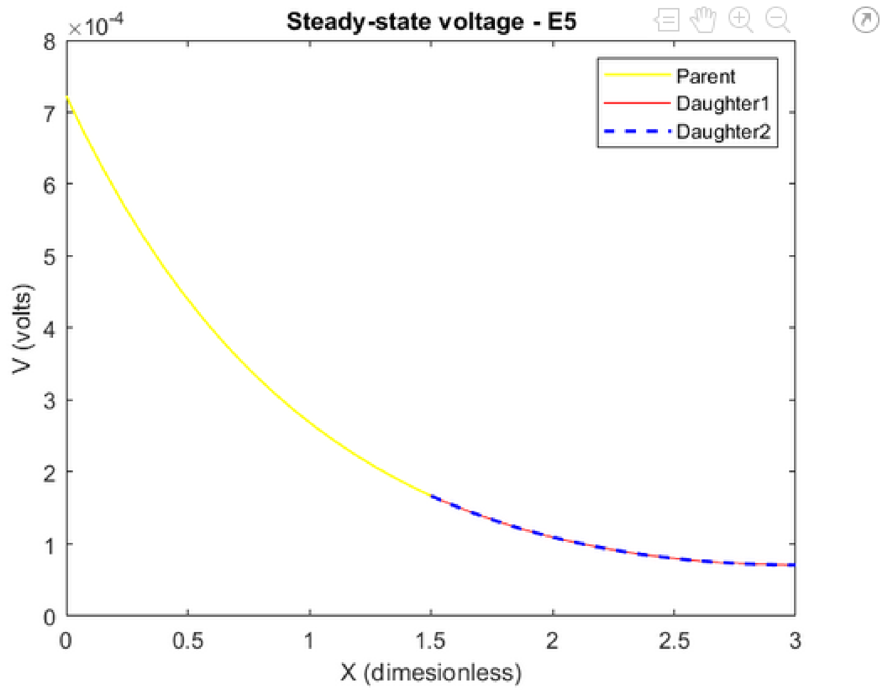
Recalculate the coefficients of equation (2) and replot the steady state voltage profile for the boundary conditions of figure 2(b) and 2(d) for $d_{21} = d_{22} = 47.2470 \times 10^{-4}$ cm. What do you notice?

Solution 06

i). Figure 2(b)

- By executing the code; x = A\b,

$$\begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} 0.0007216 \\ 0.0000013 \\ 0.0007132 \\ 0.0000018 \\ 0.0007132 \\ 0.0000018 \end{pmatrix}$$

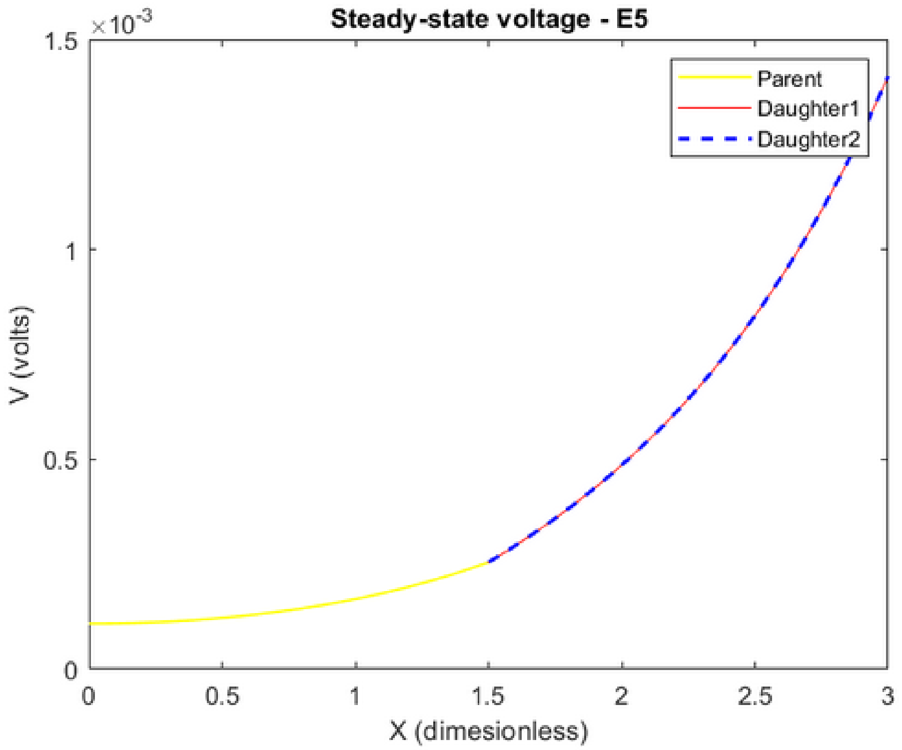


- We can see that the above curve replicates a smooth continuous function. In other words, there is a smooth transition between the steady state voltage of the parent branch and the daughter branches, at $X = L_1$ when the diameters of the daughter branches are equal.
- Moreover, the variations of the steady state voltages of the daughter branches still trace the same curve.
- As the applied current is injected from the parent end while the terminal ends of the daughter branches are maintained at a constant value, we can observe a gradual decrement in the steady state voltage as the X value increases.

ii). Figure 2(d)

- By executing the code; x = A\b,

$$\begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} 0.0000541 \\ 0.0000541 \\ -0.0002847 \\ 0.0000710 \\ -0.0002847 \\ 0.0000710 \end{pmatrix}$$



- The above curve replicates a nearly smooth continuous function. In other words, there is smooth transition between the steady state voltage of the parent branch and the daughter branches, at X = L_1 when the diameters of the daughter branches are equal.
- Moreover, the variations of the steady state voltages of the daughter branches still trace the same curve. Note that in this case, $A_1 = B_1$, $A_{21} = B_{21}$, $A_{22} = B_{22}$.
- As the applied current is injected from the terminal ends of the daughter branches while SS potential gradient at the parent end is zero, we can observe a gradual increment in the steady state voltage as the X value increases.