Belaș Darius Vlad
grupa 142

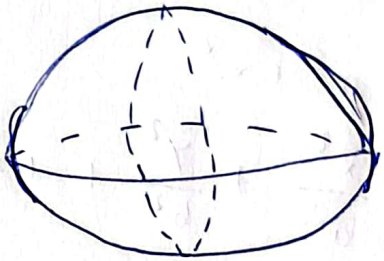
Cuadrice - tabel, determinare, invariantii, clasa

Def: **Cuadrice** → multimea solutiilor unor ecuatii de forma

$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz +$
$+ 2a_{10}x + 2a_{30}z + 2a_{20}y + a_{00} = 0$, cu $a_{11}, a_{22}, a_{33}, a_{12}, a_{13}, a_{23}$ minim unul nenul.

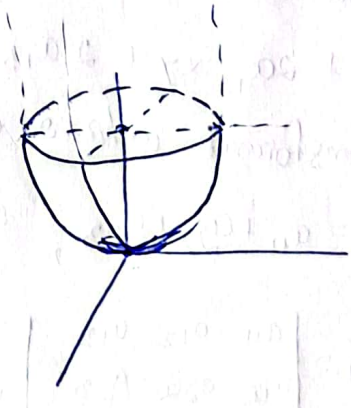
Ecuatiile reduse ($a, b, c \in (0, \infty)$)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **elipsoid** →



$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ **hiperboloid cu pânză**

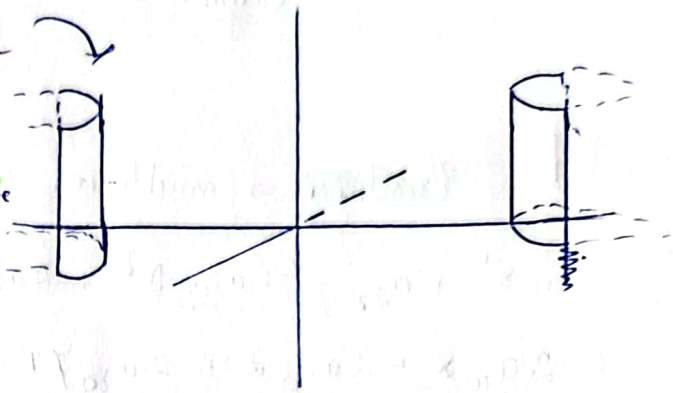
$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ **h cu ip**



$\frac{x^2}{a^2} + \frac{y^2}{b^2} - z = 0$ **paraboloid eliptic**

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **cilindru eliptic**

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cilindrul hiperbolic



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ 2 plane concurente

$x^2 - a^2 = 0$ 2 plane paralele

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 0$ punct

$\begin{cases} a^2 + x^2 \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} + 1 = 0 \end{cases}$ multimi vide

Invariatii

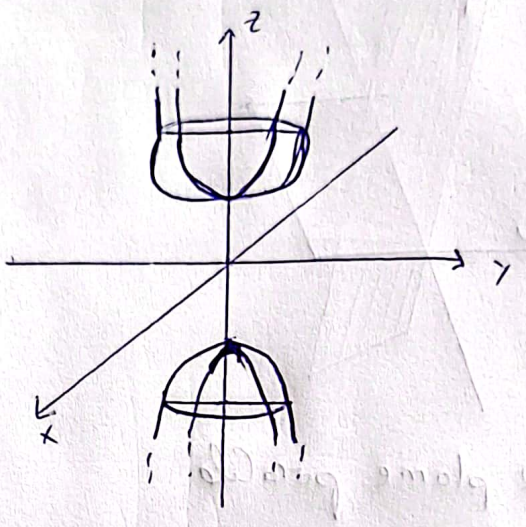
La schimbarea de reper, ecuația unei cuadrice $a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_{10}x + 2a_{20}y + 2a_{30}z + a_{00} = 0$ se transformă într-o ecuație de aceeași formă, iar parametrii

$I = a_{11} + a_{22} + a_{33}$, $\alpha = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{13} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{23} & a_{33} \end{vmatrix}$,

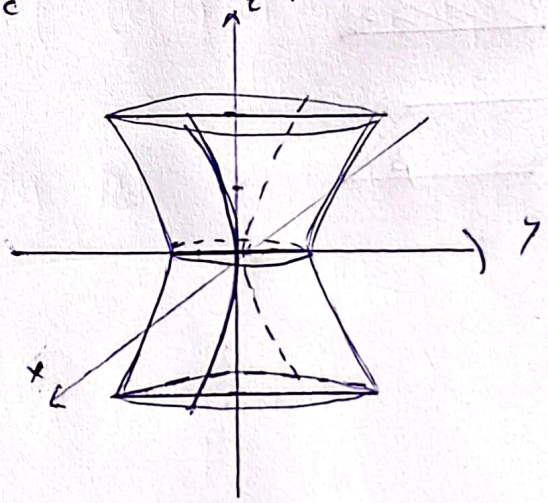
$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$

în $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{10} \\ a_{12} & a_{22} & a_{23} & a_{20} \\ a_{13} & a_{23} & a_{33} & a_{30} \\ a_{10} & a_{20} & a_{30} & a_{00} \end{vmatrix}$ unde în ecuația valoarea

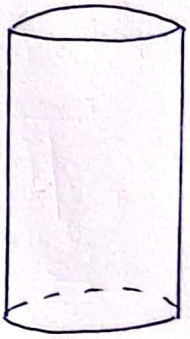
$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} + 1 = 0$ hiperboloid cu o pânză



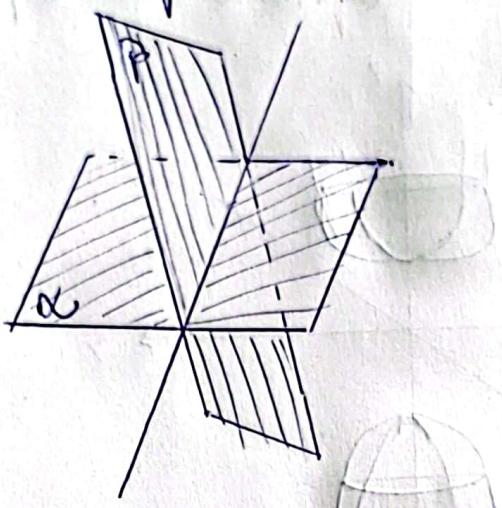
$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} - 1 = 0$ hiperboloid cu 2 pânze



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cilindru eliptic



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$ 2 plane concurente



$x^2 - a^2 = 0$ 2 plane paralele