



COMPUTER VISION

Course ID: ME4201

Semester 232 – Class P01

ASSIGNMENT 1

Group 4

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List of Exercises

| | | |
|-------------|-------|---|
| Exercise 1. | | 2 |
| Exercise 2. | | 3 |
| Exercise 3. | | 7 |
| Exercise 4. | | 9 |

Exercise 1.

(Note: The websites in this exercise may take a while to load all the necessary information.)

The field of view (FOV) of a camera is a rectangle of shape $m \times n$. Therefore, the minimum FOV must be a rectangle inscribed by the arcs of the working space. So, the value of FOV is

$$\text{FOV} = (\text{Arc diameter} + \text{Rectangle length}) \times (\text{Arc diameter}) = 3000 \times 1000 \text{ mm}$$

Enter parameters such as FOV, smallest feature (1 mm), moving object... at the website Basler <http://tinyurl.com/2p8pv8m6>. From there, the website outputs 8 cameras according to the given requirements (visit <http://tinyurl.com/4x7xadjk> to see the results). The results are divided into 3 groups as shown below:

| | | | |
|---------------------------------------|---|---|--|
| Initial FOV (mm) | 3000 × 1000 | | |
| Model | Basler boost boA8100-16cc/16cm | Basler boost boA6500-36cc/36cm | Basler boost boA9344- 30cc/30cm/70cc/70cm |
| Sensor size (mm) | 26.2 × 17.4 | 21 × 15.8 | 29.9 × 22.4 |
| Sensor diagonal (mm) | 31.45 | 26.28 | 37.4 |
| Camera resolution (px) | 8192 × 5460 | 6580 × 4935 | 9344 × 7000 |
| Select FOV width (mm) | 3000 | | |
| New FOV height (mm) | $3000 \cdot \frac{5460}{8192} = 2000$ | $3000 \cdot \frac{4935}{6580} = 2250$ | $3000 \cdot \frac{7000}{9344} = 2248$ |
| Camera FOV (mm) | 3000 × 2000 | 3000 × 2250 | 3000 × 2248 |
| Select fixed focal length (mm) | 35 | | |
| Working distance (mm) | $\frac{35\sqrt{3000^2 + 2000^2}}{31.45} = 4013$ | $\frac{35\sqrt{3000^2 + 2250^2}}{26.28} = 4995$ | $\frac{35\sqrt{3000^2 + 2248^2}}{37.4} = 3509$ |

Table 1. Properties and calculation values of Basler cameras

The formulas used in the table above are:

- **Camera FOV** FOV height = FOV width $\cdot \frac{\text{Camera resolution height}}{\text{Camera resolution width}}$.
- **Working distance** Working distance = $\frac{\text{FOV} \times \text{Focal length}}{\text{Sensor size}}$.

Then, we access the Basler website <http://tinyurl.com/28prxsize>, choose appropriate camera and input the parameters Object width (3000 mm), Focal length (35 mm) to check the calculated value:

| Model | Formula's result | Website's result | Results link |
|-----------------------------|------------------|------------------|----------------------------|
| boA8100-16cc/16cm | 4013 mm | 4043 mm | Click here |
| boA6500-36cc/36cm | 4995 mm | 5035 mm | Click here |
| boA9344-30cc/30cm/70cc/70cm | 3509 mm | 3547 mm | Click here |

Table 2. Working distance results

Exercise 2.

The matrix of the picture is
$$\begin{bmatrix} 40 & 20 & 30 & 30 \\ 80 & 40 & 100 & 110 \\ 120 & 160 & 40 & 150 \\ 220 & 230 & 240 & 250 \end{bmatrix}.$$

a) The histogram equalization table of the picture matrix:

| Pixel | Number of pixels | Probability | Cumulative density function (CDF) | New pixel |
|-------|------------------|-------------|-----------------------------------|-----------|
| 20 | 1 | 0.0625 | 1 | 0 |
| 30 | 2 | 0.125 | 3 | 34 |
| 40 | 3 | 0.1875 | 6 | 85 |
| 80 | 1 | 0.0625 | 7 | 102 |
| 100 | 1 | 0.0625 | 8 | 119 |
| 110 | 1 | 0.0625 | 9 | 136 |
| 120 | 1 | 0.0625 | 10 | 153 |
| 150 | 1 | 0.0625 | 11 | 170 |
| 160 | 1 | 0.0625 | 12 | 187 |
| 220 | 1 | 0.0625 | 13 | 204 |
| 230 | 1 | 0.0625 | 14 | 221 |
| 240 | 1 | 0.0625 | 15 | 238 |
| 250 | 1 | 0.0625 | 16 | 255 |

Table 3. Histogram equalization table

The new pixel values are calculated according to the following formula:

$$r(\text{Pixel}) = \frac{(\text{CDF} - \text{CDF}_{\min})(\text{Number of gray levels used} - 1)}{\text{Number of elements in the matrix} - \text{CDF}_{\min}}$$

From [Table 3](#), we have $\text{CDF}_{\min} = 1$. There are 16 elements in the matrix, and OpenCV libraries use 256 gray levels for histogram equalization. So, the formula becomes:

$$r(\text{Pixel}) = 17(\text{CDF} - 1)$$

Here are some values calculated according to the formula above:

$$\begin{aligned} r(40) &= 17(6 - 1) = 85 \\ r(100) &= 17(8 - 1) = 119 \\ r(220) &= 17(13 - 1) = 204 \\ r(240) &= 17(15 - 1) = 238 \end{aligned}$$

The image matrix after histogram equalization:

$$\begin{bmatrix} 85 & 0 & 34 & 34 \\ 102 & 85 & 119 & 136 \\ 153 & 187 & 85 & 170 \\ 204 & 221 & 238 & 255 \end{bmatrix}$$

b) The following code listing uses C++ and OpenCV 4.9.0 to perform histogram equalization in section a):

Code listing 1. Exercise 2.b)

```

1 #include "opencv2/opencv.hpp"
2
3 using namespace cv;
4 using namespace std;
5
6 void histogram(string const& name, Mat const& Image)
7 {
8     int bin = 255;
9     int histsize[] = { bin };
10    float range[] = { 0,255 };
11    const float* ranges[] = { range };
12    Mat hist;
13    int chanel[] = { 0 };
14    int hist_height = 256;
15    Mat hist_image = Mat::zeros(hist_height, bin, CV_8SC3);
16    calcHist(&Image, 1, chanel, Mat(), hist, 1, histsize, ranges,
17            true, false);
18    double max_val = 0;
19    minMaxLoc(hist, 0, &max_val);
20    for (int i = 0; i < bin; i++)
21    {
22        float binV = hist.at<float>(i);
23        int height = cvRound(binV * hist_height / max_val);
24        line(hist_image, Point(i, hist_height - height), Point(i,
25            hist_height), Scalar::all(255));
26    }
27    imshow(name, hist_image);
28 }
29
30 int main(int argv, char** argc)
31 {
32    float img[16] = { 40,20,30,30,
33                    80,40,100,110,
34                    120,160,40,150,
35                    220,230,240,250 };
36    Mat gray_img = Mat(4, 4, CV_32F, img);
37    Mat gray_img_his;
38    namedWindow("Old gray", WINDOW_FREERATIO);
39    namedWindow("New gray", WINDOW_FREERATIO);
40    gray_img.convertTo(gray_img, CV_8UC1);
41    equalizeHist(gray_img, gray_img_his);
42    cout << "Matrix = " << endl << " " << gray_img << endl << endl;
43    cout << "Matrix_histogram = " << endl << " " << gray_img_his <<
44        endl << endl;
45    imshow("Old gray", gray_img);
46    imshow("New gray", gray_img_his);
47    histogram("Old histogram", gray_img);
48    histogram("New histogram ", gray_img_his);
49    waitKey();
50 }

```

The results of the code above:

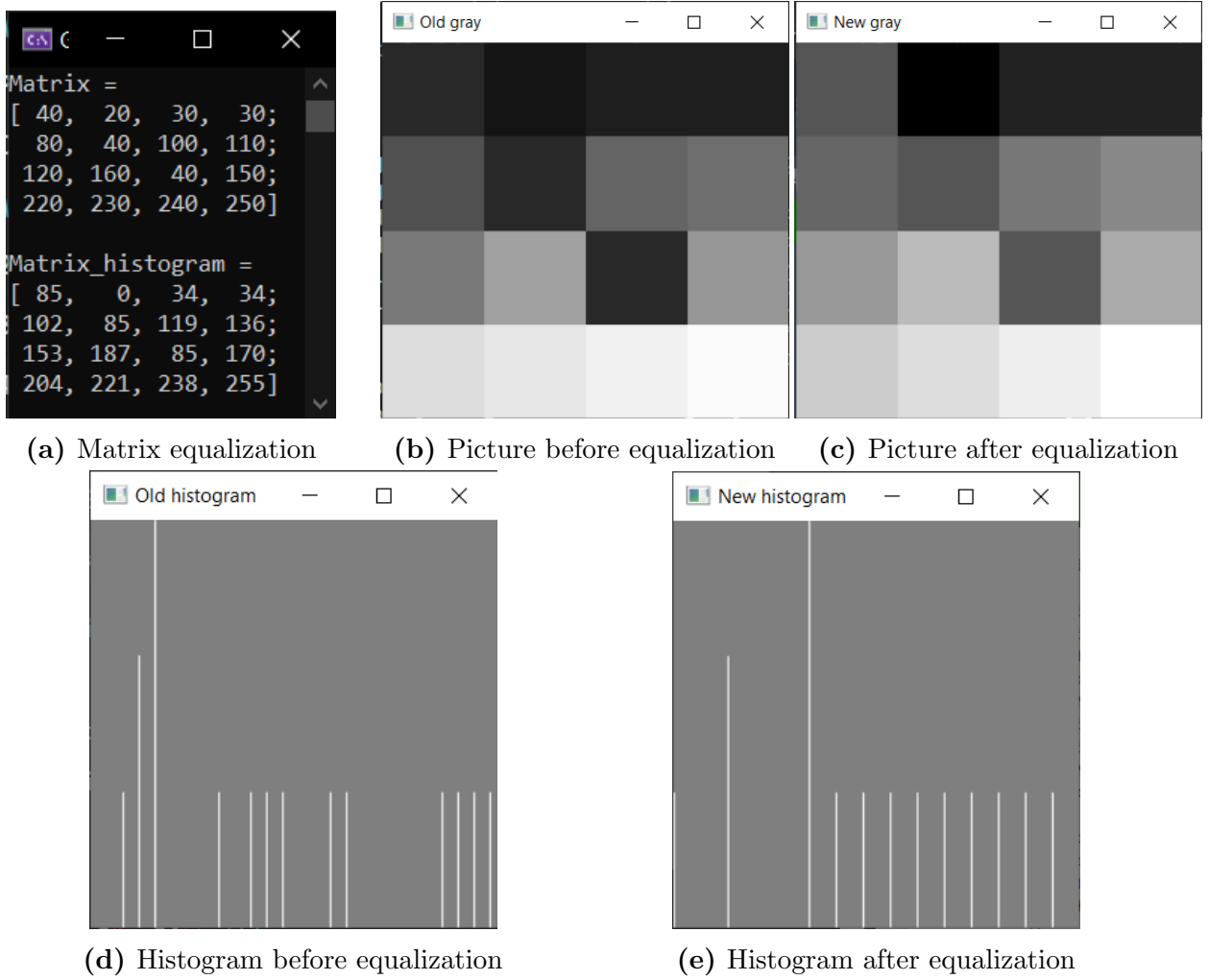


Figure 1. The picture before and after histogram equalization

c) We choose the value $T_0 = \frac{\max + \min}{2} = \frac{250 + 20}{2} = 135$ then we divide the elements of the matrix into 2 groups:

- Group of elements smaller than T_0 : 40, 20, 30, 30, 80, 40, 100, 110, 120, 40. The average value of this group is

$$m_1 = \frac{40 + 20 + 30 + 30 + 80 + 40 + 100 + 110 + 120 + 40}{10} = 61$$

- Group of elements greater than T_0 : 160, 150, 220, 230, 240, 250. The average value of this group is

$$m_2 = \frac{160 + 150 + 220 + 230 + 240 + 250}{6} = 208.33$$

The value $T_1 = \frac{m_1 + m_2}{2} = \frac{61 + 208.33}{2} = 134.67$. So $\Delta T = |T_1 - T_0| = |134.67 - 135| = 0.33$

We continue dividing the elements into 2 groups:

- Group of elements smaller than T_1 : 40, 20, 30, 30, 80, 40, 100, 110, 120, 40. The average value of this group is

$$m_1 = \frac{40 + 20 + 30 + 30 + 80 + 40 + 100 + 110 + 120 + 40}{10} = 61$$

- Group of elements greater than T_1 : 160, 150, 220, 230, 240, 250. The average value of this group is

$$m_2 = \frac{160 + 150 + 220 + 230 + 240 + 250}{6} = 208.33$$

The value $T_2 = \frac{m_1 + m_2}{2} = \frac{61 + 208.33}{2} = 134.67$. So $\Delta T = |T_2 - T_1| = 0$.

Therefore, the Otsu thresholding value is $T = 134.67$ and the result matrix is shown below:

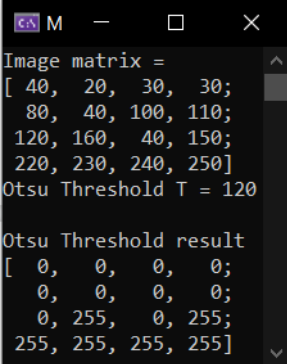
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 255 & 0 & 255 \\ 255 & 255 & 255 & 255 \end{bmatrix}$$

The following code listing uses C++ and OpenCV 4.9.0 to check the result above:

Code listing 2. Exercise 2.c)

```
1 #include <opencv2/opencv.hpp>
2
3 using namespace cv;
4 using namespace std;
5
6 int main() {
7     Mat source = (Mat_<uint8_t>(4, 4) << 40, 20, 30, 30,
8         80, 40, 100, 110,
9         120, 160, 40, 150,
10        220, 230, 240, 250);
11     Mat dst;
12     double thresh = 0, maxValue = 255;
13     long double thres = threshold(source, dst, thresh, maxValue,
14     THRESH_OTSU);
15
16     cout << "Image matrix = " << endl << " " << source << endl;
17     cout << "Otsu Threshold T = " << thres << endl;
18     cout << "Otsu Threshold result" << endl << " " << dst << endl;
19
20     waitKey(0);
21     return 0;
22 }
```

The result from the code above:



```
Image matrix =
[ 40, 20, 30, 30;
 80, 40, 100, 110;
120, 160, 40, 150;
220, 230, 240, 250]
Otsu Threshold T = 120

Otsu Threshold result
[ 0, 0, 0, 0;
 0, 0, 0, 0;
 0, 255, 0, 255;
255, 255, 255, 255]
```

Figure 2. Otsu Thresholding result from OpenCV

Exercise 3.

The matrix of the picture is $\begin{bmatrix} 1 & 2 & 3 & 4 & 10 & 11 \\ 4 & 10 & 11 & 16 & 12 & 18 \\ 16 & 12 & 18 & 22 & 24 & 25 \\ 23 & 24 & 25 & 25 & 4 & 7 \end{bmatrix}$ and the template is $\begin{bmatrix} 4 & 10 & 11 \\ 16 & 12 & 18 \\ 23 & 24 & 25 \end{bmatrix}$.

Denote the element in the x -th row and y -th column of the image matrix is $I(x, y)$ and the element in the x' -th row and y' -th column of the template matrix is $T(x', y')$. The Normalized Cross-Correlation (NCC) coefficients are determined according to the following formula:

$$R(x, y) = \frac{\sum_{x', y'} [T(x', y') \cdot I(x + x' - 1, y + y' - 1)]}{\sqrt{\sum_{x', y'} T^2(x', y') \cdot \sum_{x', y'} I^2(x + x' - 1, y + y' - 1)}}$$

Substitute the $\sum_{x', y'} T^2(x', y') = 4^2 + 10^2 + 11^2 + 16^2 + 12^2 + 18^2 + 23^2 + 24^2 + 25^2 = 2691$ into the $R(x, y)$ formula, we obtain:

$$R(x, y) = \frac{\sum_{x', y'} [T(x', y') \cdot I(x + x' - 1, y + y' - 1)]}{\sqrt{2691 \sum_{x', y'} I^2(x + x' - 1, y + y' - 1)}}$$

Here are some values obtained from the formula above:

$$\begin{aligned} R(1, 1) &= \frac{1 \cdot 4 + 2 \cdot 10 + 3 \cdot 11 + 4 \cdot 16 + 10 \cdot 12 + 11 \cdot 18 + 16 \cdot 23 + 12 \cdot 24 + 18 \cdot 25}{\sqrt{2691(1^2 + 2^2 + 3^2 + 4^2 + 10^2 + 11^2 + 16^2 + 12^2 + 18^2)}} = 0.953826374 \\ R(1, 2) &= \frac{2 \cdot 4 + 3 \cdot 10 + 4 \cdot 11 + 10 \cdot 16 + 11 \cdot 12 + 16 \cdot 18 + 12 \cdot 23 + 18 \cdot 24 + 22 \cdot 25}{\sqrt{2691(2^2 + 3^2 + 4^2 + 10^2 + 11^2 + 16^2 + 12^2 + 18^2 + 22^2)}} = 0.969316528 \\ R(1, 3) &= \frac{3 \cdot 4 + 4 \cdot 10 + 10 \cdot 11 + 11 \cdot 16 + 16 \cdot 12 + 12 \cdot 18 + 18 \cdot 23 + 22 \cdot 24 + 24 \cdot 25}{\sqrt{2691(3^2 + 4^2 + 10^2 + 11^2 + 16^2 + 12^2 + 18^2 + 22^2 + 24^2)}} = 0.978928905 \\ R(2, 1) &= \frac{4 \cdot 4 + 10 \cdot 10 + 11 \cdot 11 + 16 \cdot 16 + 12 \cdot 12 + 18 \cdot 18 + 23 \cdot 23 + 24 \cdot 24 + 25 \cdot 25}{\sqrt{2691(4^2 + 10^2 + 11^2 + 16^2 + 12^2 + 18^2 + 23^2 + 24^2 + 25^2)}} = 1 \end{aligned}$$

The NCC coefficients matrix is shown below:

$$\begin{bmatrix} R(1, 1) & R(1, 2) & R(1, 3) & R(1, 4) \\ R(2, 1) & R(2, 2) & R(2, 3) & R(2, 4) \end{bmatrix} = \begin{bmatrix} 0.953826374 & 0.969316528 & 0.978928905 & 0.999848171 \\ 1 & 0.982228851 & 0.888346493 & 0.802686196 \end{bmatrix}$$

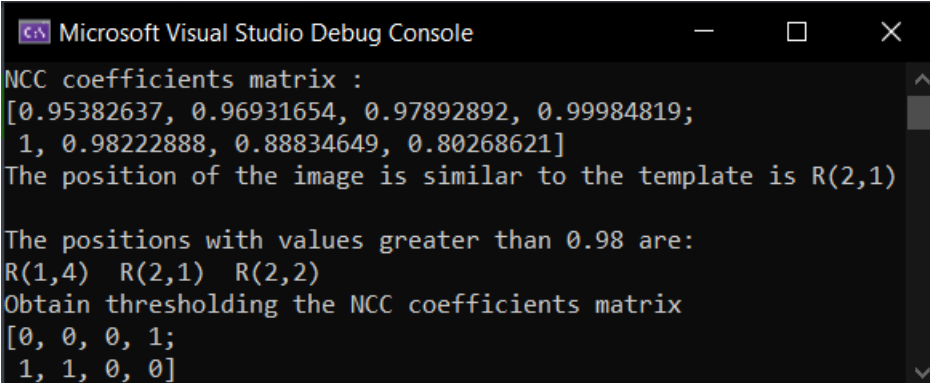
The position of the image is similar to the template is $R(2, 1)$. Applying binary thresholding to the NCC coefficients matrix with the threshold value $T = 0.98$ and the maximum value 1, we obtain the thresholded matrix below:

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

The following code listing is a C++ code using OpenCV 4.9.0 to check the calculation above:

Code listing 3. Exercise 3

```
1 #include <opencv2/opencv.hpp>
2
3 using namespace cv;
4 using namespace std;
5
6 int main()
7 {
8     Mat img = (Mat_<float>(4, 6) << 1, 2, 3, 4, 10, 11,
9         4, 10, 11, 16, 12, 18,
10        16, 12, 18, 22, 24, 25,
11        23, 24, 25, 25, 4, 7);
12     Mat temp = (Mat_<float>(3, 3) << 4, 10, 11,
13        16, 12, 18,
14        23, 24, 25);
15
16     Mat ncc;
17     matchTemplate(img, temp, ncc, TM_CCORR_NORMED);
18     cout << "NCC coefficients matrix :" << endl << ncc << endl;
19     double minVal, maxVal;
20     Point minLoc, maxLoc;
21     minMaxLoc(ncc, &minVal, &maxVal, &minLoc, &maxLoc);
22     cout << "The position of the image is similar to the template is R
23 (" << maxLoc.y + 1 << "," << maxLoc.x + 1 << ")" << endl;
24     Mat bin;
25     threshold(ncc, bin, 0.98, 1, THRESH_BINARY);
26     vector<Point> locs;
27     findNonZero(bin, locs);
28     cout << "The positions with values greater than 0.98 are:" << endl
29 ;
30     for (auto loc : locs) {
31         cout << "R(" << loc.y + 1 << "," << loc.x + 1 << ")\t";
32     }
33     cout << "\nObtain thresholding the NCC coefficients matrix" <<
34 endl << " " << bin << endl;
35     waitKey();
36 }
```



```
Microsoft Visual Studio Debug Console
NCC coefficients matrix :
[0.95382637, 0.96931654, 0.97892892, 0.99984819;
 1, 0.98222888, 0.88834649, 0.80268621]
The position of the image is similar to the template is R(2,1)

The positions with values greater than 0.98 are:
R(1,4) R(2,1) R(2,2)
Obtain thresholding the NCC coefficients matrix
[0, 0, 0, 1;
 1, 1, 0, 0]
```

Figure 3. Template matching with NCC coefficients

Exercise 4.

The matrix of the picture is
$$\begin{bmatrix} 9 & 10 & 11 & 10 & 9 & 10 & 11 \\ 10 & 9 & 100 & 100 & 100 & 10 & 11 \\ 10 & 100 & 10 & 11 & 10 & 100 & 11 \\ 10 & 9 & 100 & 100 & 100 & 13 & 11 \\ 10 & 10 & 10 & 10 & 10 & 13 & 11 \end{bmatrix}$$
. Adding duplicate border to the image matrix
$$\begin{bmatrix} 9 & 9 & 10 & 11 & 10 & 9 & 10 & 11 & 11 \\ 9 & 9 & 10 & 11 & 10 & 9 & 10 & 11 & 11 \\ 10 & 10 & 9 & 100 & 100 & 100 & 10 & 11 & 11 \\ 10 & 10 & 100 & 10 & 11 & 10 & 100 & 11 & 11 \\ 10 & 10 & 9 & 100 & 100 & 100 & 13 & 11 & 11 \\ 10 & 10 & 10 & 10 & 10 & 10 & 13 & 11 & 11 \\ 10 & 10 & 10 & 10 & 10 & 10 & 13 & 11 & 11 \end{bmatrix}.$$

a) The Gaussian Filter function is $G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$. Substituting $\sigma = 0.3$ into $G_\sigma(x, y)$, then we calculate the kernel matrix:

$$\begin{bmatrix} G_{0.3}(-1, -1) & G_{0.3}(0, -1) & G_{0.3}(1, -1) \\ G_{0.3}(-1, 0) & G_{0.3}(0, 0) & G_{0.3}(1, 0) \\ G_{0.3}(-1, 1) & G_{0.3}(0, 1) & G_{0.3}(1, 1) \end{bmatrix} = \begin{bmatrix} 0.000026 & 0.006836 & 0.000026 \\ 0.006836 & 1.768388 & 0.006836 \\ 0.000026 & 0.006836 & 0.000026 \end{bmatrix}$$

The sum of all the elements of the matrix is 1.79584. Therefore, we obtain the filter kernel matrix:

$$\frac{1}{1.79584} \begin{bmatrix} 0.000026 & 0.006836 & 0.000026 \\ 0.006836 & 1.768388 & 0.006836 \\ 0.000026 & 0.006836 & 0.000026 \end{bmatrix}$$

The matrix after filtering with Gaussian kernel is

$$\begin{bmatrix} 9.007643 & 9.997518 & 11.332445 & 10.345263 & 9.355418 & 10.001339 & 10.996164 \\ 9.993696 & 9.700515 & 98.968198 & 99.313280 & 98.964392 & 10.689050 & 10.997488 \\ 10.342585 & 98.619280 & 11.034269 & 11.675240 & 11.034343 & 98.642151 & 11.338822 \\ 9.997518 & 9.700515 & 98.964392 & 99.313280 & 98.979663 & 13.654627 & 11.008953 \\ 9.999985 & 9.997518 & 10.343924 & 10.345263 & 10.355448 & 12.982143 & 11.007673 \end{bmatrix}$$

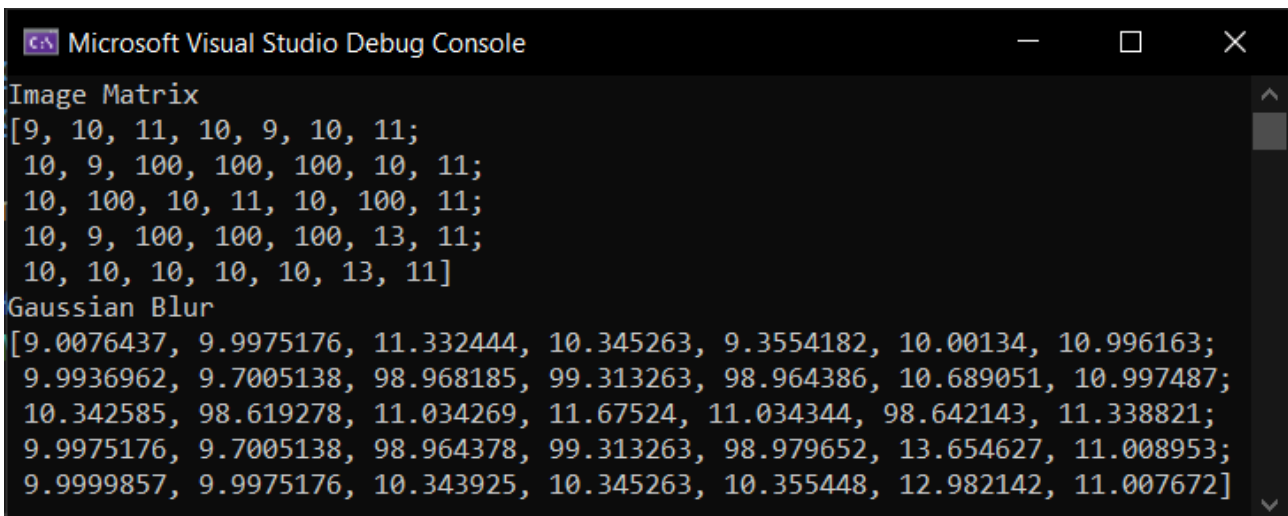
The elements of the matrix after filtering are calculated using the convolution formula. Here are some of the calculated element values:

$$\begin{aligned} a(1, 1) &= \frac{9 \cdot 0.000026 + 9 \cdot 0.006836 + 10 \cdot 0.000026 + 9 \cdot 0.006836 + 9 \cdot 1.768388}{1.79584} \\ &\quad + \frac{10 \cdot 0.006836 + 10 \cdot 0.000026 + 10 \cdot 0.006836 + 9 \cdot 0.000026}{1.79584} = 9.007643 \\ a(1, 2) &= \frac{9 \cdot 0.000026 + 10 \cdot 0.006836 + 11 \cdot 0.000026 + 9 \cdot 0.006836 + 10 \cdot 1.768388}{1.79584} \\ &\quad + \frac{11 \cdot 0.006836 + 10 \cdot 0.000026 + 9 \cdot 0.006836 + 100 \cdot 0.000026}{1.79584} = 9.997518 \\ a(2, 2) &= \frac{9 \cdot 0.000026 + 10 \cdot 0.006836 + 11 \cdot 0.000026 + 10 \cdot 0.006836 + 9 \cdot 1.768388}{1.79584} \\ &\quad + \frac{100 \cdot 0.006836 + 10 \cdot 0.000026 + 100 \cdot 0.006836 + 10 \cdot 0.000026}{1.79584} = 9.700515 \\ a(3, 3) &= \frac{9 \cdot 0.000026 + 100 \cdot 0.006836 + 100 \cdot 0.000026 + 100 \cdot 0.006836 + 10 \cdot 1.768388}{1.79584} \\ &\quad + \frac{11 \cdot 0.006836 + 9 \cdot 0.000026 + 100 \cdot 0.006836 + 100 \cdot 0.000026}{1.79584} = 11.034269 \end{aligned}$$

The following code listing is a C++ code using OpenCV 4.9.0 to check the calculation above:

Code listing 4. Exercise 4.a)

```
1 #include <opencv2/opencv.hpp>
2
3 using namespace cv;
4 using namespace std;
5
6 int main()
7 {
8     float sigma = 0.3;
9     Mat src = (Mat_<float>(5, 7) << 9, 10, 11, 10, 9, 10, 11,
10         10, 9, 100, 100, 100, 10, 11,
11         10, 100, 10, 11, 10, 100, 11,
12         10, 9, 100, 100, 100, 13, 11,
13         10, 10, 10, 10, 10, 13, 11);
14     Mat dst;
15     GaussianBlur(src, dst, Size(3, 3), sigma, sigma,
16     BORDER_REPLICATE);
17     cout << "Image Matrix" << endl << "" << src << endl;
18     cout << "Gaussian Blur" << endl << "" << dst << endl;
19     waitKey();
20 }
```



```
Microsoft Visual Studio Debug Console

Image Matrix
[9, 10, 11, 10, 9, 10, 11;
 10, 9, 100, 100, 100, 10, 11;
 10, 100, 10, 11, 10, 100, 11;
 10, 9, 100, 100, 100, 13, 11;
 10, 10, 10, 10, 10, 13, 11]
Gaussian Blur
[9.0076437, 9.9975176, 11.332444, 10.345263, 9.3554182, 10.00134, 10.996163;
 9.9936962, 9.7005138, 98.968185, 99.313263, 98.964386, 10.689051, 10.997487;
 10.342585, 98.619278, 11.034269, 11.67524, 11.034344, 98.642143, 11.338821;
 9.9975176, 9.7005138, 98.964378, 99.313263, 98.979652, 13.654627, 11.008953;
 9.9999857, 9.9975176, 10.343925, 10.345263, 10.355448, 12.982142, 11.007672]
```

Figure 4. Gaussian Blur filtering result from OpenCV

b) The Sobel operator uses two different kernel matrices:

- Taking the derivative with respect to x : $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$
- Taking the derivative with respect to y : $M_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

Using the Sobel M_x operator, we obtain the image after filtering:

$$\begin{bmatrix} 2 & 96 & 91 & -6 & -90 & -83 & 4 \\ 89 & 182 & 93 & -2 & -91 & -175 & -86 \\ 178 & 180 & 4 & 0 & 1 & -176 & -179 \\ 88 & 180 & 93 & 0 & -82 & -176 & -95 \\ -1 & 90 & 91 & 0 & -78 & -86 & -8 \end{bmatrix}$$

Here are some values that were calculated through the Sobel M_x operator:

$$a(1,1) = 9 \cdot (-1) + 9 \cdot 0 + 10 \cdot 1 + 9 \cdot (-2) + 9 \cdot 0 + 10 \cdot 2 + 10 \cdot (-1) + 10 \cdot 0 + 9 \cdot 1 = 2$$

$$a(2,2) = 9 \cdot (-1) + 10 \cdot 0 + 11 \cdot 1 + 10 \cdot (-2) + 9 \cdot 0 + 100 \cdot 2 + 10 \cdot (-1) + 100 \cdot 0 + 10 \cdot 1 = 182$$

Using the Sobel M_y operator, we obtain the image after filtering:

$$\begin{bmatrix} 2 & 88 & 267 & 360 & 272 & 91 & 0 \\ 93 & 180 & 89 & 2 & 93 & 181 & 90 \\ 0 & 0 & 0 & 0 & 3 & 6 & 3 \\ -90 & -180 & -91 & -2 & -88 & -174 & -87 \\ 1 & -88 & -269 & -360 & -270 & -90 & 0 \end{bmatrix}$$

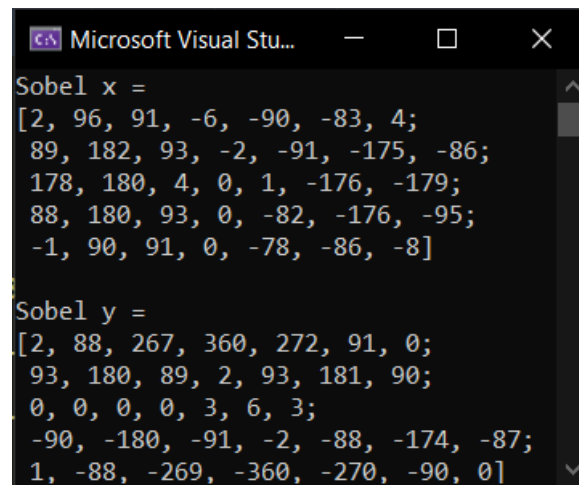
Here are some values that were calculated through the Sobel M_y operator:

$$a(1,1) = 9 \cdot (-1) + 9 \cdot (-2) + 10 \cdot (-1) + 9 \cdot 0 + 9 \cdot 0 + 10 \cdot 0 + 10 \cdot 1 + 10 \cdot 2 + 9 \cdot 1 = 2$$

$$a(2,2) = 9 \cdot (-1) + 10 \cdot (-2) + 11 \cdot (-1) + 10 \cdot 0 + 9 \cdot 0 + 100 \cdot 0 + 10 \cdot 1 + 100 \cdot 2 + 10 \cdot 1 = 180$$

Code listing 5. Exercise 4.b)

```
1 #include <opencv2/opencv.hpp>
2
3 using namespace cv;
4 using namespace std;
5
6 int main()
7 {
8     Mat src = (Mat_<double>(5, 7) << 9, 10, 11, 10, 9, 10, 11,
9         10, 9, 100, 100, 100, 10, 11,
10        10, 100, 10, 11, 10, 100, 11,
11        10, 9, 100, 100, 100, 13, 11,
12        10, 10, 10, 10, 10, 13, 11);
13
14     Mat sobelx, sobely;
15     Sobel(src, sobelx, CV_64F, 1, 0, 3, 1, 0, BORDER_REPLICATE);
16     Sobel(src, sobely, CV_64F, 0, 1, 3, 1, 0, BORDER_REPLICATE);
17     cout << "Sobel x = " << endl << " " << sobelx << endl << endl;
18     cout << "Sobel y = " << endl << " " << sobely << endl << endl;
19     waitKey();
20 }
```



```
Microsoft Visual Stu...  
Sobel x =  
[2, 96, 91, -6, -90, -83, 4;  
89, 182, 93, -2, -91, -175, -86;  
178, 180, 4, 0, 1, -176, -179;  
88, 180, 93, 0, -82, -176, -95;  
-1, 90, 91, 0, -78, -86, -8]  
Sobel y =  
[2, 88, 267, 360, 272, 91, 0;  
93, 180, 89, 2, 93, 181, 90;  
0, 0, 0, 0, 3, 6, 3;  
-90, -180, -91, -2, -88, -174, -87;  
1, -88, -269, -360, -270, -90, 0]
```

Figure 5. Sobel filtering result from OpenCV