University of Toronto

CSC418 Assignment 1 Part A

Nicholas Dujay
999194900

March 7, 2015

1 Question 1

Given that $\vec{e} = (1, 2, 2)$ $\vec{g} = (1, 1, 3)$ and $\vec{r} = (0, 1, 0)$, $\vec{s}$ $\vec{u}$ and $\vec{v}$ are calculated like this:

$$\vec{s} = -\frac{\vec{g}}{\|\vec{g}\|} = \left(-\frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}}\right)$$
$$\vec{u} = \frac{\vec{r} \times \vec{s}}{\|\vec{r} \times \vec{s}\|} = \left(-\frac{1}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\right)$$
$$\vec{v} = \frac{\vec{s} \times \vec{u}}{\|\vec{s} \times \vec{u}\|} = \left(-\frac{1}{3\sqrt{2}}, -\frac{4}{3\sqrt{2}}, -\frac{1}{3\sqrt{2}}\right)$$

From here, we can calculate the world to camera transformation matrix as follows:

$$M_{wc} = \left[\begin{array}{ccc|c} -\frac{1}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{3\sqrt{2}} & -\frac{4}{3\sqrt{2}} & -\frac{1}{3\sqrt{2}} & \frac{11}{3\sqrt{2}} \\ -\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & -\frac{3}{\sqrt{11}} & \frac{9}{\sqrt{11}} \\ 0 & 0 & 0 & 1 \end{array}\right]$$

Where the M_{wc} matrix has the following form:

$$M_{wc} = \left[\begin{array}{ccc|c} u_x & u_y & u_z & \vec{u} \cdot \vec{e} \\ v_x & v_y & v_z & \vec{v} \cdot \vec{e} \\ s_x & s_y & s_z & \vec{s} \cdot \vec{e} \\ 0 & 0 & 0 & 1 \end{array}\right]$$

2 Question 2

First lets calculate m then project it to m'.

$$m = \frac{1}{2}(p+q) = \frac{1}{2}\left(\begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} + \begin{bmatrix} q_x \\ q_y \\ q_z \\ 1 \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ \frac{1}{2}(p_z+q_z) \\ 1 \end{bmatrix}$$

$$m' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ \frac{1}{2}(p_z+q_z) \\ 1 \end{bmatrix} \cong \begin{bmatrix} -f\left(\frac{p_x+q_x}{p_z+q_z}\right) \\ -f\left(\frac{p_y+q_y}{p_z+q_z}\right) \\ -f \\ 1 \end{bmatrix}$$

Next lets calculate p' and q' then calculate the midpoint between them.

$$p' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} \cdot \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \cong \begin{bmatrix} -f\frac{p_x}{p_z} \\ -f\frac{p_y}{p_z} \\ -f \\ 1 \end{bmatrix}$$

q' is calculated similarly.

$$\frac{1}{2}(p'+q') = \frac{1}{2}\left(\begin{bmatrix} -f\frac{p_x}{p_z} \\ -f\frac{p_y}{p_z} \\ -f \\ 1 \end{bmatrix} + \begin{bmatrix} -f\frac{q_x}{q_z} \\ -f\frac{q_y}{q_z} \\ -f \\ 1 \end{bmatrix}\right) \cong \begin{bmatrix} -\frac{f}{2}\left(\frac{p_x}{p_z}+\frac{q_x}{q_z}\right) \\ -\frac{f}{2}\left(\frac{p_y}{p_z}+\frac{q_y}{q_z}\right) \\ -f \\ 1 \end{bmatrix}$$

In order for $m' = \frac{1}{2}(p'+q')$, we need $-f\left(\frac{p_x+q_x}{p_z+q_z}\right) = -\frac{f}{2}\left(\frac{p_x}{p_z}+\frac{q_x}{q_z}\right)$ and $-f\left(\frac{p_y+q_y}{p_z+q_z}\right) = -\frac{f}{2}\left(\frac{p_y}{p_z}+\frac{q_y}{q_z}\right)$. There's only one case where this is true, and it is when $p_z = q_z$.
If we instead use an orthographic projection, then $m' = \frac{1}{2}(p'+q')$ for all cases:

$$m' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ \frac{1}{2}(p_z+q_z) \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ 1 \end{bmatrix}$$

$$p' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

$$\begin{aligned} \frac{1}{2}(p'+q') &= \frac{1}{2}\left(\begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} + \begin{bmatrix} q_x \\ q_y \\ 1 \end{bmatrix}\right) \\ &= \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ 1 \end{bmatrix} = m' \end{aligned}$$

3 Question 3

4 Question 4

5 Question 5

6 Question 6