University of Toronto

CSC418 Assignment 1 Part A

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1 Question 1

Given that $\vec{e} = (1, 2, 2)$ $\vec{g} = (1, 1, 3)$ and $\vec{r} = (0, 1, 0)$, $\vec{s}$ $\vec{u}$ and $\vec{v}$ are calculated like this:

$$\begin{aligned}
\vec{s} &= -\frac{\vec{g}}{\|\vec{g}\|} = \left(-\frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}}\right) \\
\vec{u} &= \frac{\vec{r} \times \vec{s}}{\|\vec{r} \times \vec{s}\|} = \left(-\frac{1}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\right) \\
\vec{v} &= \frac{\vec{s} \times \vec{u}}{\|\vec{s} \times \vec{u}\|} = \left(-\frac{1}{3\sqrt{2}}, -\frac{4}{3\sqrt{2}}, -\frac{1}{3\sqrt{2}}\right)
\end{aligned}$$

From here, we can calculate the world to camera transformation matrix as follows:

$$M_{wc} = \left[\begin{array}{ccc|c}
-\frac{1}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\
\frac{1}{3\sqrt{2}} & -\frac{4}{3\sqrt{2}} & -\frac{1}{3\sqrt{2}} & \frac{11}{3\sqrt{2}} \\
-\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & -\frac{3}{\sqrt{11}} & \frac{9}{\sqrt{11}} \\
0 & 0 & 0 & 1
\end{array}\right]$$

Where the M_{wc} matrix has the following form:

$$M_{wc} = \left[\begin{array}{ccc|c}
u_x & u_y & u_z & \vec{u} \cdot \vec{e} \\
v_x & v_y & v_z & \vec{v} \cdot \vec{e} \\
s_x & s_y & s_z & \vec{s} \cdot \vec{e} \\
0 & 0 & 0 & 1
\end{array}\right]$$

2 Question 2

First lets calculate m then project it to m'.

$$m = \frac{1}{2}(p+q) = \frac{1}{2}\left(\begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} + \begin{bmatrix} q_x \\ q_y \\ q_z \\ 1 \end{bmatrix}\right) = \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ \frac{1}{2}(p_z+q_z) \\ 1 \end{bmatrix}$$

$$m' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ \frac{1}{2}(p_z+q_z) \\ 1 \end{bmatrix} \cong \begin{bmatrix} -f\left(\frac{p_x+q_x}{p_z+q_z}\right) \\ -f\left(\frac{p_y+q_y}{p_z+q_z}\right) \\ -f \\ 1 \end{bmatrix}$$

Next lets calculate p' and q' then calculate the midpoint between them.

$$p' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} \cdot \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \cong \begin{bmatrix} -f\frac{p_x}{p_z} \\ -f\frac{p_y}{p_z} \\ -f \\ 1 \end{bmatrix}$$

q' is calculated similarly.

$$\frac{1}{2}(p'+q') = \frac{1}{2}\left(\begin{bmatrix} -f\frac{p_x}{p_z} \\ -f\frac{p_y}{p_z} \\ -f \\ 1 \end{bmatrix} + \begin{bmatrix} -f\frac{q_x}{q_z} \\ -f\frac{q_y}{q_z} \\ -f \\ 1 \end{bmatrix}\right) \cong \begin{bmatrix} -\frac{f}{2}\left(\frac{p_x}{p_z}+\frac{q_x}{q_z}\right) \\ -\frac{f}{2}\left(\frac{p_y}{p_z}+\frac{q_y}{q_z}\right) \\ -f \\ 1 \end{bmatrix}$$

In order for $m' = \frac{1}{2}(p'+q')$, we need $-f\left(\frac{p_x+q_x}{p_z+q_z}\right) = -\frac{f}{2}\left(\frac{p_x}{p_z}+\frac{q_x}{q_z}\right)$ and $-f\left(\frac{p_y+q_y}{p_z+q_z}\right) = -\frac{f}{2}\left(\frac{p_y}{p_z}+\frac{q_y}{q_z}\right)$. There's only one case where this is true, and it is when $p_z = q_z$.
If we instead use an orthographic projection, then $m' = \frac{1}{2}(p'+q')$ for all cases:

$$m' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ \frac{1}{2}(p_z+q_z) \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ 1 \end{bmatrix}$$

$$p' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

$$\begin{aligned} \frac{1}{2}(p'+q') &= \frac{1}{2}\left(\begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} + \begin{bmatrix} q_x \\ q_y \\ 1 \end{bmatrix}\right) \\ &= \begin{bmatrix} \frac{1}{2}(p_x+q_x) \\ \frac{1}{2}(p_y+q_y) \\ 1 \end{bmatrix} = m' \end{aligned}$$

3 Question 3

4 Question 4

The canonical view transform using $L = -1, R = 1, B = -1, T = 1, f = 1, F = 1001$ is this:

$$M_{cv} = \begin{bmatrix} \frac{2f}{L-R} & 0 & \frac{R+L}{L-R} & 0 \\ 0 & \frac{2f}{B-T} & \frac{B+T}{B-T} & 0 \\ 0 & 0 & \frac{f+F}{F-f} & \frac{2Ff}{F-f} \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{-1-1} & 0 & \frac{1+-1}{-1-1} & 0 \\ 0 & \frac{2}{-1-1} & \frac{-1+1}{-1-1} & 0 \\ 0 & 0 & \frac{1+1001}{1001-1} & \frac{2(1001)}{1001-1} \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1.002 & 2.002 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Now transforming a point from camera space to canonical view space:

$$\begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} = M_{cv} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} -x_c \\ -y_c \\ 1.002z_c + 2.002 \\ z_c \end{bmatrix} \cong \begin{bmatrix} -\frac{x_c}{z_c} \\ -\frac{y_c}{z_c} \\ 1.002 + \frac{2.002}{z_c} \\ 1 \end{bmatrix}$$

Since we are only interested in the pseudo depth value here, we now have a formula for the pseudo depth of each point. $z_{c1} = -1, z_{c2} = -10, z_{c3} = -100, z_{c4} = -1000$

$$z_{v1} = 1.002 + \frac{2.002}{z_{c1}} = -1$$
$$z_{v2} = 1.002 + \frac{2.002}{z_{c2}} = 0.8018$$
$$z_{v3} = 1.002 + \frac{2.002}{z_{c3}} = 0.98198$$
$$z_{v4} = 1.002 + \frac{2.002}{z_{c4}} = 0.999998$$

The relationship is not linear. Plot z_{ci} and z_{vi} on a graph and calculate slopes between each pair of points $(-1, -1), (-10, 0.8018), (-100, 0.98198), (-1000, 0.999998)$ and the slopes between each point should be the same if the relationship is linear. However, the slopes are different between each point so therefore the relationship is not linear.

5 Question 5

6 Question 6