

---

# CSC418 Assignment 1 Part A

---

Nicholas Dujay  
999194900

March 7, 2015

## 1 Question 1

Given that  $\vec{e} = (1, 2, 2)$   $\vec{g} = (1, 1, 3)$  and  $\vec{r} = (0, 1, 0)$ ,  $\vec{s}$ ,  $\vec{u}$  and  $\vec{v}$  are calculated like this:

$$\begin{aligned}\vec{s} &= -\frac{\vec{g}}{\|\vec{g}\|} = \left(-\frac{1}{\sqrt{11}}, -\frac{1}{\sqrt{11}}, -\frac{3}{\sqrt{11}}\right) \\ \vec{u} &= \frac{\vec{r} \times \vec{s}}{\|\vec{r} \times \vec{s}\|} = \left(-\frac{1}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}}\right) \\ \vec{v} &= \frac{\vec{s} \times \vec{u}}{\|\vec{s} \times \vec{u}\|} = \left(-\frac{1}{3\sqrt{2}}, -\frac{4}{3\sqrt{2}}, -\frac{1}{3\sqrt{2}}\right)\end{aligned}$$

From here, we can calculate the world to camera transformation matrix as follows:

$$M_{wc} = \left[ \begin{array}{ccc|c} -\frac{1}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \\ \frac{1}{3\sqrt{2}} & -\frac{4}{3\sqrt{2}} & -\frac{1}{3\sqrt{2}} & \frac{1}{3\sqrt{2}} \\ -\frac{1}{\sqrt{11}} & -\frac{1}{\sqrt{11}} & -\frac{3}{\sqrt{11}} & \frac{1}{\sqrt{11}} \\ 0 & 0 & 0 & 1 \end{array} \right]$$

Where the  $M_{wc}$  matrix has the following form:

$$M_{wc} = \left[ \begin{array}{ccc|c} u_x & u_y & u_z & \vec{u} \cdot \vec{e} \\ v_x & v_y & v_z & \vec{v} \cdot \vec{e} \\ s_x & s_y & s_z & \vec{s} \cdot \vec{e} \\ 0 & 0 & 0 & 1 \end{array} \right]$$

## 2 Question 2

First lets calculate  $m$  then project it to  $m'$ .

$$m = \frac{1}{2}(p + q) = \frac{1}{2} \left( \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} + \begin{bmatrix} q_x \\ q_y \\ q_z \\ 1 \end{bmatrix} \right) = \begin{bmatrix} \frac{1}{2}(p_x + q_x) \\ \frac{1}{2}(p_y + q_y) \\ \frac{1}{2}(p_z + q_z) \\ 1 \end{bmatrix}$$

$$m' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2}(p_x + q_x) \\ \frac{1}{2}(p_y + q_y) \\ \frac{1}{2}(p_z + q_z) \\ 1 \end{bmatrix} \cong \begin{bmatrix} -f \left( \frac{p_x + q_x}{p_z + q_z} \right) \\ -f \left( \frac{p_y + q_y}{p_z + q_z} \right) \\ -f \\ 1 \end{bmatrix}$$

Next lets calculate  $p'$  and  $q'$  then calculate the midpoint between them.

$$p' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} \cdot \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} \cong \begin{bmatrix} -f \frac{p_x}{p_z} \\ -f \frac{p_y}{p_z} \\ -f \\ 1 \end{bmatrix}$$

$q'$  is calculated similarly.

$$\frac{1}{2}(p' + q') = \frac{1}{2} \left( \begin{bmatrix} -f \frac{p_x}{p_z} \\ -f \frac{p_y}{p_z} \\ -f \\ 1 \end{bmatrix} + \begin{bmatrix} -f \frac{q_x}{q_z} \\ -f \frac{q_y}{q_z} \\ -f \\ 1 \end{bmatrix} \right) \cong \begin{bmatrix} -\frac{f}{2} \left( \frac{p_x}{p_z} + \frac{q_x}{q_z} \right) \\ -\frac{f}{2} \left( \frac{p_y}{p_z} + \frac{q_y}{q_z} \right) \\ -f \\ 1 \end{bmatrix}$$

In order for  $m' = \frac{1}{2}(p' + q')$ , we need  $-f \left( \frac{p_x + q_x}{p_z + q_z} \right) = -\frac{f}{2} \left( \frac{p_x}{p_z} + \frac{q_x}{q_z} \right)$  and  $-f \left( \frac{p_y + q_y}{p_z + q_z} \right) = -\frac{f}{2} \left( \frac{p_y}{p_z} + \frac{q_y}{q_z} \right)$ .

There's only one case where this is true, and it is when  $p_z = q_z$ .

If we instead use an orthographic projection, then  $m' = \frac{1}{2}(p' + q')$  for all cases:

$$m' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2}(p_x + q_x) \\ \frac{1}{2}(p_y + q_y) \\ \frac{1}{2}(p_z + q_z) \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(p_x + q_x) \\ \frac{1}{2}(p_y + q_y) \\ 1 \end{bmatrix}$$

$$p' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix} = \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

$$\frac{1}{2}(p' + q') = \frac{1}{2} \left( \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix} + \begin{bmatrix} q_x \\ q_y \\ 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} \frac{1}{2}(p_x + q_x) \\ \frac{1}{2}(p_y + q_y) \\ 1 \end{bmatrix} = m'$$

### 3 Question 3

The projected lines have the following form

$$l_i(u_i)' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f} & 0 \end{bmatrix} \cdot \begin{bmatrix} p_{ix} + u_i d_{ix} \\ p_{iy} + u_i d_{iy} \\ p_{iz} + u_i d_{iz} \\ 1 \end{bmatrix} = \begin{bmatrix} p_{ix} + u_i d_{ix} \\ p_{iy} + u_i d_{iy} \\ p_{iz} + u_i d_{iz} \\ \frac{1}{f} (p_{iz} + u_i d_{iz}) \end{bmatrix} \cong \begin{bmatrix} f \frac{p_{ix} + u_i d_{ix}}{p_{iz} + u_i d_{iz}} \\ f \frac{p_{iy} + u_i d_{iy}}{p_{iz} + u_i d_{iz}} \\ f \\ 1 \end{bmatrix}$$

The projection of each line will intersect in the canonical view space, so the final expression is independent of the original lines. However, since the original lines never intersect in camera space, the point of intersection is at  $u_i = \pm\infty$ . Therefore, we must take the limit of  $l'_i(u_i)$  as  $u_i \rightarrow \pm\infty$ .

$$\begin{aligned} \lim_{u_i \rightarrow \pm\infty} l_i(u_i) &= \lim_{u_i \rightarrow \pm\infty} \begin{bmatrix} f \frac{p_{ix} + u_i d_{ix}}{p_{iz} + u_i d_{iz}} \\ f \frac{p_{iy} + u_i d_{iy}}{p_{iz} + u_i d_{iz}} \\ f \\ 1 \end{bmatrix} = \begin{bmatrix} \lim_{u_i \rightarrow \pm\infty} f \frac{p_{ix} + u_i d_{ix}}{p_{iz} + u_i d_{iz}} \\ \lim_{u_i \rightarrow \pm\infty} f \frac{p_{iy} + u_i d_{iy}}{p_{iz} + u_i d_{iz}} \\ f \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} \lim_{u_i \rightarrow \pm\infty} f \frac{\frac{p_{ix} + u_i d_{ix}}{u_i} + d_{ix}}{\frac{p_{iz} + u_i d_{iz}}{u_i} + d_{iz}} \\ \lim_{u_i \rightarrow \pm\infty} f \frac{\frac{p_{iy} + u_i d_{iy}}{u_i} + d_{iy}}{\frac{p_{iz} + u_i d_{iz}}{u_i} + d_{iz}} \\ f \\ 1 \end{bmatrix} = \begin{bmatrix} \lim_{u_i \rightarrow \pm\infty} f \frac{\frac{p_{ix}}{u_i} + d_{ix}}{\frac{p_{iz}}{u_i} + d_{iz}} \\ \lim_{u_i \rightarrow \pm\infty} f \frac{\frac{p_{iy}}{u_i} + d_{iy}}{\frac{p_{iz}}{u_i} + d_{iz}} \\ f \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} f \frac{d_{ix}}{d_{iz}} \\ f \frac{d_{iy}}{d_{iz}} \\ f \\ 1 \end{bmatrix} \end{aligned}$$

## 4 Question 4

The canonical view transform using  $L = -1, R = 1, B = -1, T = 1, f = 1, F = 1001$  is this:

$$M_{cv} = \begin{bmatrix} \frac{2f}{L-R} & 0 & \frac{B+L}{L-R} & 0 \\ 0 & \frac{2f}{B-T} & \frac{B+T}{B-T} & 0 \\ 0 & 0 & \frac{f+F}{F-f} & \frac{2Ff}{F-f} \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{-1-1} & 0 & \frac{1+-1}{-1-1} & 0 \\ 0 & \frac{2}{-1-1} & \frac{-1+1}{-1-1} & 0 \\ 0 & 0 & \frac{1+1001}{1001-1} & \frac{2(1001)}{1001-1} \\ 0 & 0 & -\frac{1}{f} & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1.002 & 2.002 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Now transforming a point from camera space to canonical view space:

$$\begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix} = M_{cv} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} -x_c \\ -y_c \\ 1.002z_c + 2.002 \\ z_c \end{bmatrix} \cong \begin{bmatrix} -\frac{x_c}{z_c} \\ -\frac{y_c}{z_c} \\ 1.002 + \frac{2.002}{z_c} \\ 1 \end{bmatrix}$$

Since we are only interested in the pseudo depth value here, we now have a formula for the pseudo depth of each point.  $z_{c1} = -1, z_{c2} = -10, z_{c3} = -100, z_{c4} = -1000$

$$\begin{aligned} z_{v1} &= 1.002 + \frac{2.002}{z_{c1}} = -1 \\ z_{v2} &= 1.002 + \frac{2.002}{z_{c2}} = 0.8018 \\ z_{v3} &= 1.002 + \frac{2.002}{z_{c3}} = 0.98198 \\ z_{v4} &= 1.002 + \frac{2.002}{z_{c4}} = 0.999998 \end{aligned}$$

The relationship is not linear. Plot  $z_{ci}$  and  $z_{vi}$  on a graph and calculate slopes between each pair of points  $(-1, -1), (-10, 0.8018), (-100, 0.98198), (-1000, 0.999998)$  and the slopes between each point should be the same if the relationship is linear. However, the slopes are different between each point so therefore the relationship is not linear.

## 5 Question 5

## 6 Question 6