Short Exercises on Generative Adversarial Networks

Instructions. Each exercise is designed to take approximately 10–15 minutes. Show all necessary steps for calculations and provide concise explanations where requested.

Exercise 1: Generator Gradient Calculation

Consider the original generator loss

$$L_G = \mathbb{E}_{z \sim p_z}[\log(1 - D(G(z)))].$$

1. Assume that for a batch of samples, $D(G(z)) = 0.01$. Compute

$$\frac{d}{dD} \log(1 - D)$$

   evaluated at $D = 0.01$.

2. Now consider the non-saturating generator loss

$$L_G^{\text{NS}} = -\mathbb{E}_{z \sim p_z}[\log D(G(z))].$$

   Compute $\frac{d}{dD}(-\log D)$ at $D = 0.01$.

3. Explain why the non-saturating loss provides stronger gradients early in training.

Exercise 2: Wasserstein GAN Intuition

Let

$$p_{\text{data}} = \delta(x = 0), \quad p_g = \delta(x = \theta).$$

1. Compute the Wasserstein-1 (Earth Mover) distance between p_{data} and p_g.

2. Describe how this distance behaves as $\theta \to 0$.

3. Explain why this leads to meaningful gradients even when the supports do not overlap.