



**DATA 8**  
Spring 2022

# Lecture 37

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Updating Probabilities

# Announcements

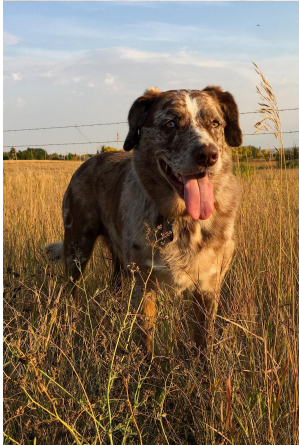
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- Homework 11 now **due tonight**
  - Project 3 Checkpoint **due tomorrow**
    - Entire project due next Friday (04/29)
  - Homework 12 due Thursday (04/28)
    - Turn in on Wednesday for a bonus point
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**Before Classifying**

# Dog or Wolf?

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# Start with a Representative Sample

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- Both the training and test sets must accurately represent the population on which you use your classifier
  - **Overfitting** happens when a classifier does very well on the training set, but can't do as well on the test set
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# Standardize if Necessary

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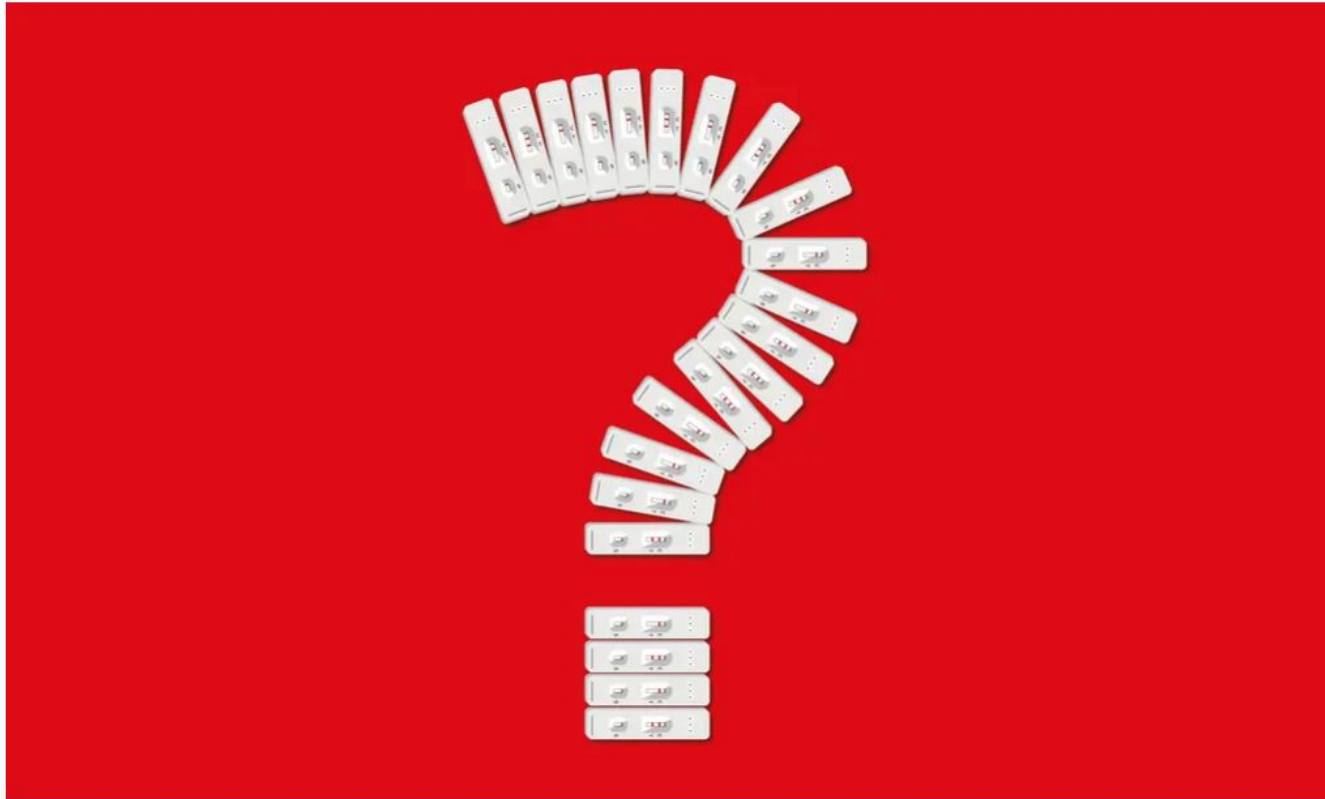
Chronic Kidney  
Disease data set

Glucose	Hemoglobin	White Blood Cell Count	Class
117	11.2	6700	1
70	9.5	12100	1
380	10.8	4500	1
157	5.6	11000	1

- If the attributes are on very different numerical scales, distance can be affected
  - In such a situation, it is a good idea to convert all the variables to standard units
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# Updating Probabilities

# The obscure maths theorem that governs the reliability of Covid testing



[Source: Guardian](#)



# Updating Probabilities

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*Interpretation by Physicians of Clinical Laboratory Results (1978)*

"We asked 20 house officers, 20 fourth-year medical students and 20 attending physicians, selected in 67 consecutive hallway encounters at four Harvard Medical School teaching hospitals, the following question:

"If a test to detect a disease whose prevalence is  $1/1000$  has a false positive rate of 5%, what is the chance that a person found to have a positive result actually has the disease, assuming that you know nothing about the person's symptoms or signs?"

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# Updating Probabilities

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*Interpretation by Physicians of Clinical Laboratory Results (1978)*

"Eleven of 60 participants, or **18%**, **gave the correct answer**. These participants included four of 20 fourth-year students, three of 20 residents in internal medicine and four of 20 attending physicians. The most common answer, given by 27, was that **the chance that a person found to have a positive result actually has the disease was 95%**.

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# Conditional Probability

# Scenario 1

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- Scenario:
    - Class consists of second years (60%) and third years (40%)
    - 50% of the second years have declared their major
    - 80% of the third years have declared their major
  - **I pick one student at random.**
  - Which is more likely: Second year or Third year?
    - Second year, because they are 60% of the class
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# Scenario 2

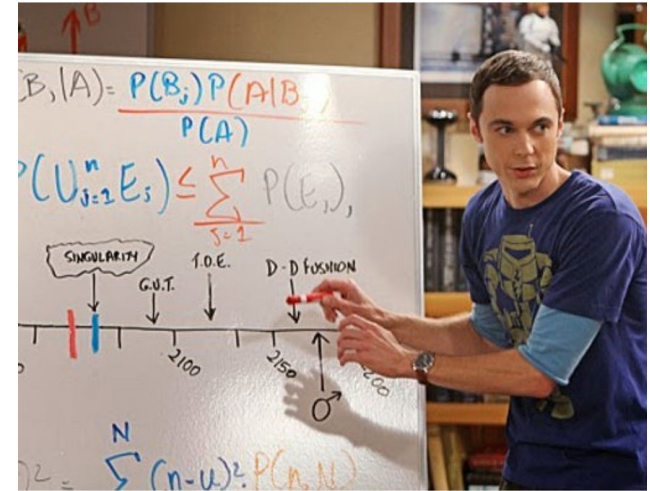
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- Slightly different scenario:
    - Class consists of second years (60%) and third years (40%)
    - 50% of the second years have declared their major
    - 80% of the third years have declared their major
  - **I pick one student at random...** (Demo)  
**That student has declared a major!**
  - Which is more likely: Second Year or Third Year?
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# Bayes' Rule

# Purpose of Bayes' Rule

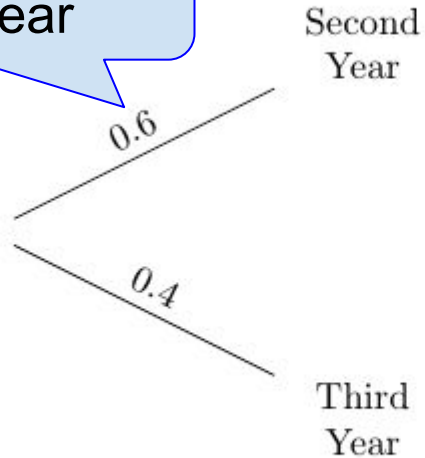
- Update your prediction based on new information
- In a multi-stage experiment, find the chance of an event at an earlier stage, given the result of a later stage



# Diagram and Terminology

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Prior probability of  
being a 2nd year

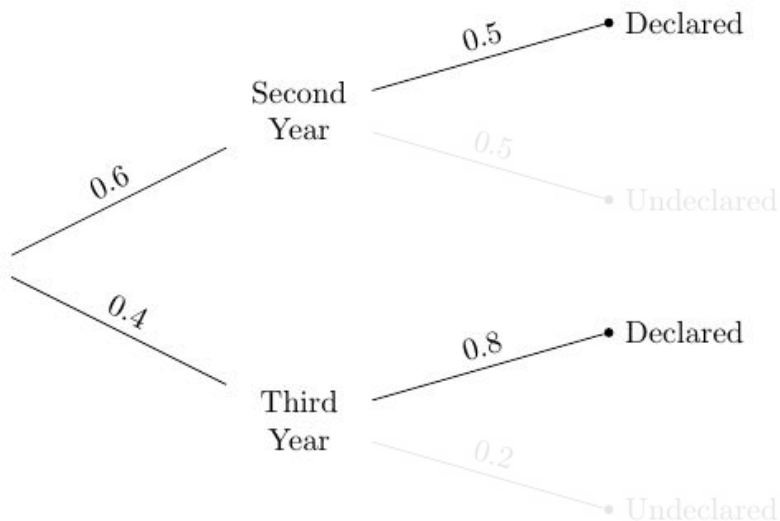


Likelihood of declared,  
given 2nd year



# Data & Calculation

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Pick a student at random.

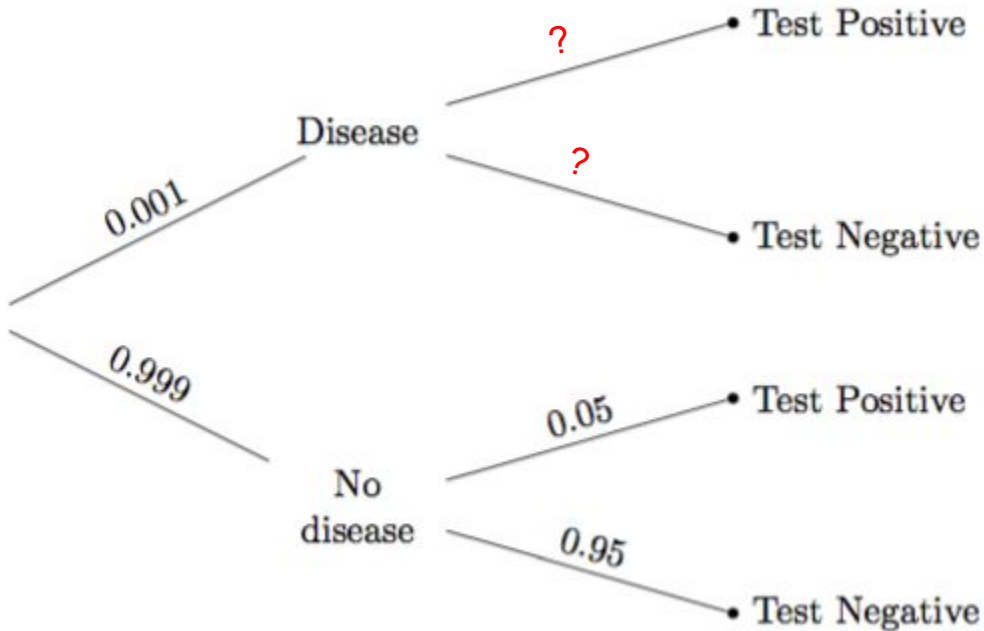
**Posterior probability:**

$P(\text{Second Year} \mid \text{Declared})$

$$\begin{aligned} & \frac{0.6 \times 0.5}{(0.6 \times 0.5) + (0.4 \times 0.8)} \\ &= 0.4839\dots \end{aligned}$$

# Example: Doctors & Clinical Tests

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Problem did not give the *true positive* rate.

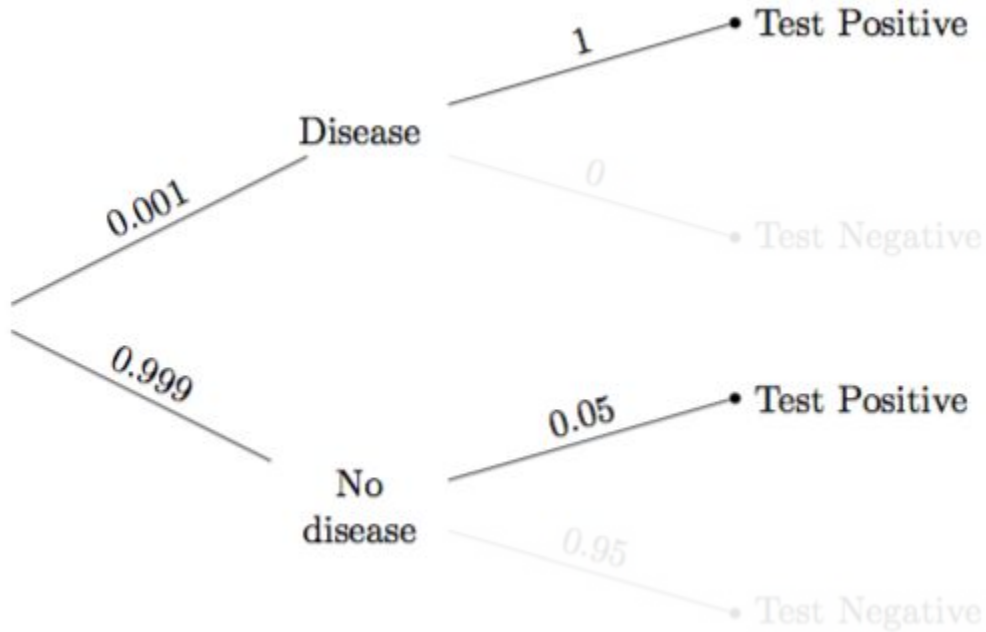
That's the chance the test says "positive" if the person has the disease.

It was assumed to be 100%.

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# Data and Calculation

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$P(\text{Disease} \mid \text{Test} +)$

=

$$0.001 * 1$$

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$$(0.001 * 1) + (0.999 * 0.05)$$

$$= 0.0196270...$$

(Demo)

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# Subjective Probabilities

# Subjective Probabilities

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A probability of an outcome is...

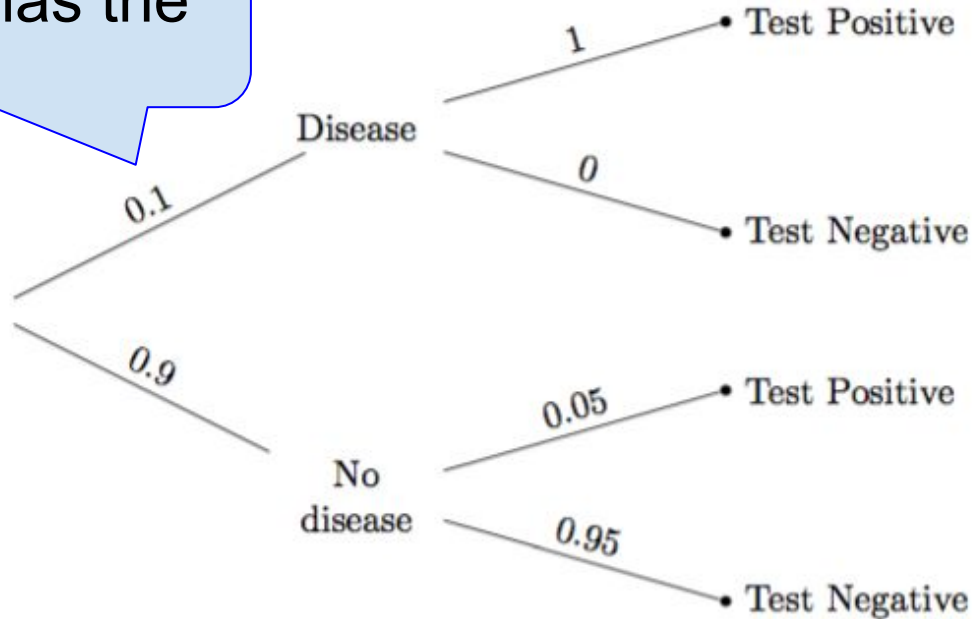
- The frequency with which it will occur in repeated trials, *or*
- The subjective degree of belief that it will (or has) occurred

Why use subjective priors?

- In order to quantify a belief that is relevant to a decision
  - If the subject of your prediction was not selected randomly from the population
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# A Subjective Opinion

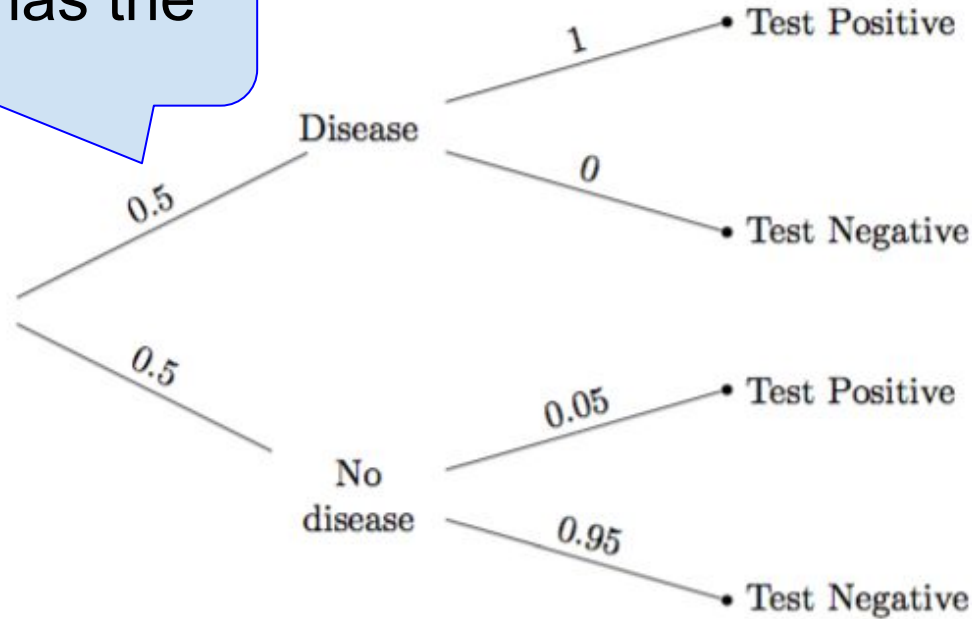
prior probability that  
the person has the  
disease



(Demo)

# A Different Subjective Opinion

prior probability that  
the person has the  
disease



(Demo)