

An introduction to (s)imputation

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Missing data



Missing data

Reasons

- nonresponse, data loss
- Value is observed but deemed wrong and erased

Solutions

- Measure/observe again
- Ignore
- Take into account when estimating
- **Impute**



Imputation methodology

Model based

Estimate a value based on observed variables.

Donor imputation

Copy a value from a record that you did observe.

Proxy imputation

Copy or derive a value form other variables.

The simputation package

Provide

- a *uniform interface*,
- with *consistent behaviour*,
- across *commonly used methodologies*

To facilitate

- experimentation
- configuration for production

Assignment 1: Try the following code

Installation

```
install.packages("simputation", dependencies = TRUE)
```

Code to try

```
library(simputation)
data(retailers, package="validate")
ret <- retailers[3:6]
ret %>% impute_lm(other.rev ~ turnover) %>% head()
```



Assignment 1: Try the following code

```
library(simputation)
data(retailers,package="validate")
ret <- retailers[3:6]
ret %>% impute_lm(other.rev ~ turnover) %>% head()
```

##	staff	turnover	other.rev	total.rev
## 1	75	NA	NA	1130
## 2	9	1607	5427.113	1607
## 3	NA	6886	-33.000	6919
## 4	NA	3861	13.000	3874
## 5	NA	NA	37.000	5602
## 6	1	25	6341.683	25



Assignment 2: Try the following code

```
# note the 'rlm'!  
ret %>% impute_rlm(other.rev ~ turnover) %>% head()
```


Assignment 2: Try the following code

```
# note the 'rlm'!  
ret %>% impute_rlm(other.rev ~ turnover) %>% head()
```

```
##   staff turnover other.rev total.rev  
## 1     75        NA        NA     1130  
## 2      9     1607    17.25247     1607  
## 3    NA     6886   -33.00000     6919  
## 4    NA     3861    13.00000     3874  
## 5    NA        NA    37.00000     5602  
## 6      1      25    11.05605        25
```



The imputation package

An imputation procedure is specified by

1. The variable to impute
2. An imputation model
3. Predictor variables

The imputation interface

```
impute_<model>(data  
  , <imputed vars> ~ <predictor vars>  
  , [options])
```



Chaining methods

```
ret %>%  
  impute_rlm(other.rev ~ turnover) %>%  
  impute_rlm(other.rev ~ staff) %>% head()
```

##	staff	turnover	other.rev	total.rev
## 1	75	NA	64.88174	1130
## 2	9	1607	17.25247	1607
## 3	NA	6886	-33.00000	6919
## 4	NA	3861	13.00000	3874
## 5	NA	NA	37.00000	5602
## 6	1	25	11.05605	25



Assignment 3

Adapt this code so turnover is imputed, based on turnover and staff.

```
ret %>%  
  impute_rlm(other.rev ~ turnover) %>%  
  impute_rlm(other.rev ~ staff) %>% head()
```



(One) solution

```
ret %>%  
  impute_rlm(other.rev ~ turnover) %>%  
  impute_rlm(other.rev ~ staff) %>%  
  impute_rlm(turnover ~ staff + other.rev) %>% head()
```



Example: Multiple variables, same predictors

```
ret %>%  
  impute_rlm(other.rev + total.rev ~ turnover)  
  
ret %>%  
  impute_rlm( . - turnover ~ turnover)
```



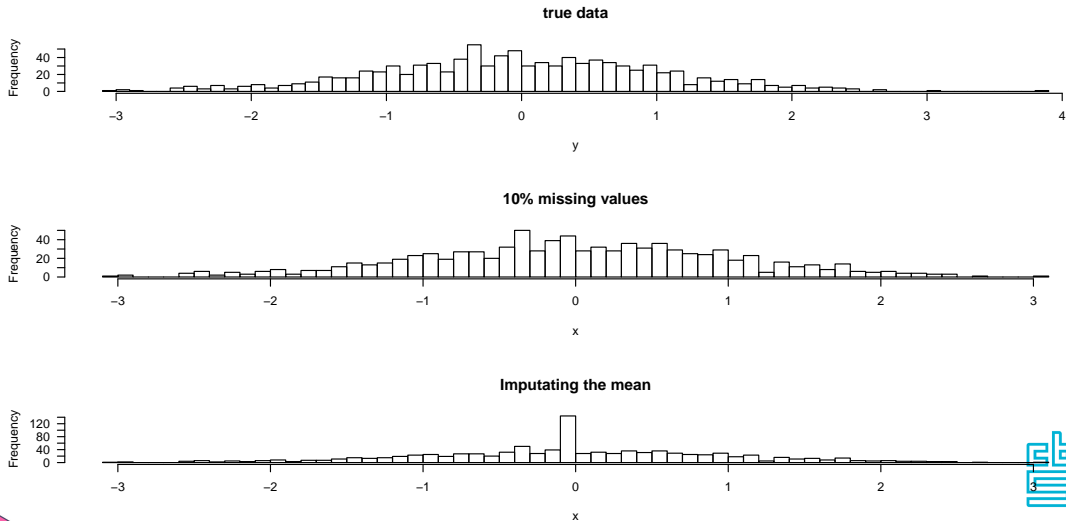
Example: grouping

```
retailers %>% impute_rlm(total.rev ~ turnover | size)

# or, using dplyr::group_by
retailers %>%
  group_by(size) %>%
  impute_rlm(total.rev ~ turnover)
```

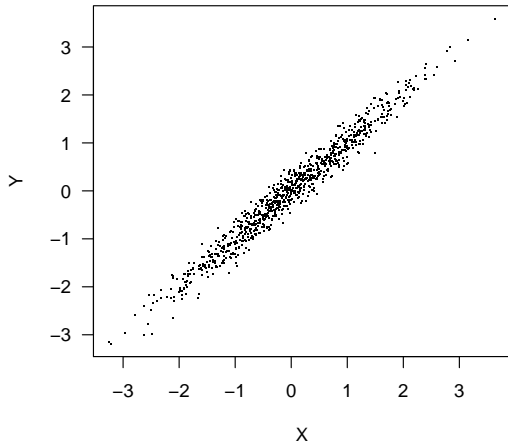


Imputation and univariate distribution

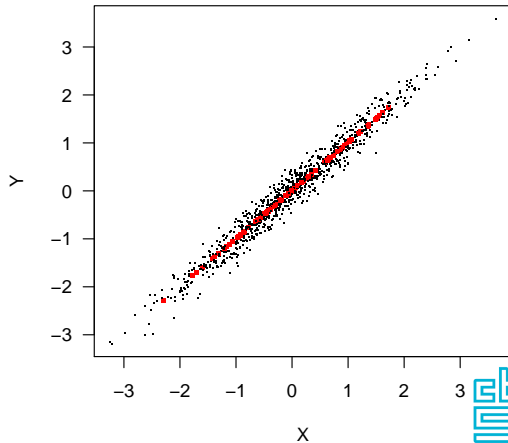


Imputation and bivariate distribution

10% missing in Y



Imputation with model $Y = a + bX$



Adding a random residual

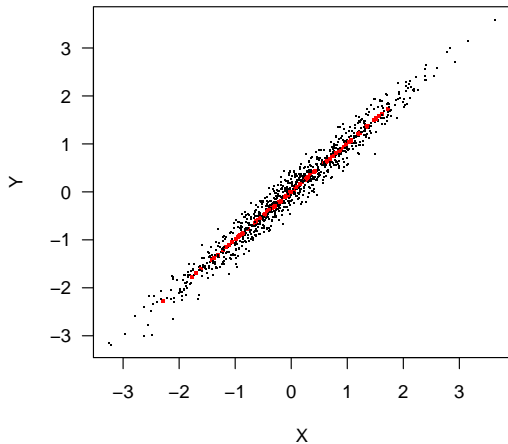
$$\hat{y}_i = \hat{f}(X_i) + \varepsilon_i$$

- \hat{y}_i estimated value for record i
 - $\hat{f}(X_i)$ model value
 - ε_i random perturbation
 - Either a residual from the model training
 - OR sampled from $N(0, \hat{\sigma})$
- + Better (multivariate) distribution
- Less reproducible

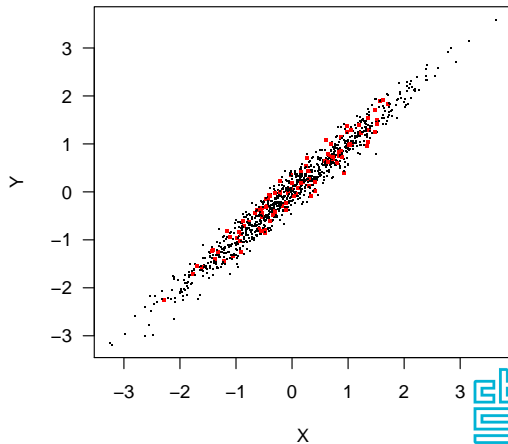


Adding a random residual

Imputation with model $Y = a + bX$



Imputation met $Y = a + bX + e$



Adding a residual with simulation

Try the following code

```
ret %>%  
  impute_rlm(other.rev ~ turnover  
    , add_residual = "normal") %>% head(3)
```

Options

- add_residual = "none": (default)
- add_residual = "normal": from $N(0, \hat{\sigma})$
- add_residual = "observed": from observed residuals

Compute the variance of other.rev after each option.



Five minutes for ten models.

1. Impute a proxy

$$\hat{y} = \mathbf{x} \text{ or } \mathbf{y} = f(\mathbf{x}),$$

where \mathbf{x} is another (proxy) variable (e.g. VAT value for turnover), and f a user-defined (optional) transformation.

```
# imputation  
impute_proxy()
```



2. Linear model

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}},$$

where

$$\hat{\boldsymbol{\beta}} = \arg \min_{\boldsymbol{\beta}} \sum_i \epsilon_i^2$$

```
# imputation:  
impute_lm()
```



3. Regularized linear model (elasticnet)

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}},$$

where

$$\hat{\boldsymbol{\beta}} = \arg \min_{\boldsymbol{\beta}} \frac{1}{2} \sum_i \epsilon_i^2 + \lambda \left[\frac{1-\alpha}{2} \|\boldsymbol{\beta}^*\|^2 + \alpha \|\boldsymbol{\beta}^*\|_1 \right]$$

- $\alpha = 0$ (Lasso) \cdots $\alpha = 1$ (Ridge)
- $\boldsymbol{\beta}^*$: $\boldsymbol{\beta}$ w/o intercept.

imputation:

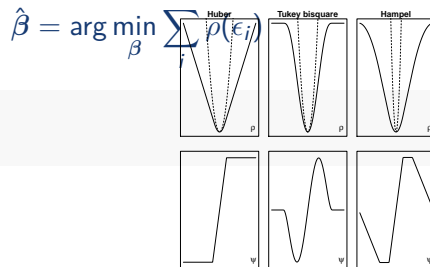
```
impute_en()
```



4. M -estimator

$$\hat{\mathbf{y}} = \mathbf{X}\hat{\boldsymbol{\beta}},$$

where



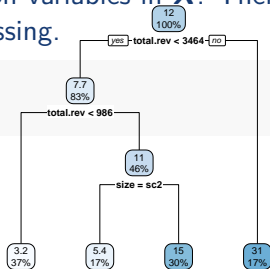
```
# simulation:  
impute_rlm()
```

5. Classification and regression tree (CART)

$$\hat{y} = T(\mathbf{X}),$$

where T represents a set of binary questions on variables in \mathbf{X} . There are spare questions for when one of the predictors is missing.

```
# imputation:  
impute_cart()
```



6. Random forest

$$\hat{\mathbf{y}} = \frac{1}{|\text{Forest}|} \sum_{i \in \text{Forest}} T_i(\mathbf{x}),$$

where each T_i is a simple decision tree without spare questions. For categorical \mathbf{y} , the majority vote is chosen.

```
# simulation  
impute_rf()
```



7. Expectation-Maximization

Dataset $\mathbf{X} = \mathbf{X}_{obs} \cup \mathbf{X}_{mis}$. Assume $\mathbf{X} \sim P(\theta)$.

1. Choose a $\hat{\theta}$.
2. Repeat until convergence:
 - a. $Q(\theta|\hat{\theta}) = \ell(\theta|\mathbf{X}_{obs}) + E_{mis}[\ell(\mathbf{X}_{mis}|\theta, \mathbf{X}_{obs})|\hat{\theta}]$
 - b. $\hat{\theta} = \arg \max_{\theta} Q(\theta|\hat{\theta})$
3. $\hat{\mathbf{X}}_{mis} = \arg \max_{\mathbf{X}_{mis}} P(\mathbf{X}_{mis}|\hat{\theta})$

```
# imputation (multivariate normal):  
impute_em()
```



8. missForest

Dataset $\mathbf{X} = \mathbf{X}_{obs} \cup \mathbf{X}_{mis}$.

1. Trivial imputation of \mathbf{X}_{mis} (median for numeric variables, mode for categorical variables)
2. Repeat until convergence:
 - a. Train random forest models on the completed data
 - b. Re-impute based on these models.

```
# simulation:  
impute_mf()
```



9.a Random hot deck

1. Split the data records into groups (optional)
2. Impute missing values by copying a value from a random record in the same group

imputation

```
impute_rhd(data, imputed_variables ~ grouping_variables)
```



9.b Sequential hot-deck

1. Sort the dataset
2. For each row in the sorted dataset, impute missing values from the last observed.

```
# imputation  
impute_shd(data, imputed_variables ~ sorting_variables)
```



9.c k -nearest neighbours

For each record with one or more missings:

1. Find the k nearest neighbours (Gower's distance) with observed values
2. Sample value(s) from the k records.

```
# imputation  
impute_knn(data, imputed_variables ~ distance_variables)
```



10. Predictive mean matching

1. For each variable X_i with missing values, estimate a model \hat{f}_i .
2. Estimate all values, observed or not.
3. For each missing value, impute the observed value, of which the prediction is closest to the prediction of the missing value.

```
# imputation: (currently buggy!)  
impute_pmm()
```

