Homework 4 - Stat 488 Bayesian Analysis

Nicholas Marey

September 26, 2017

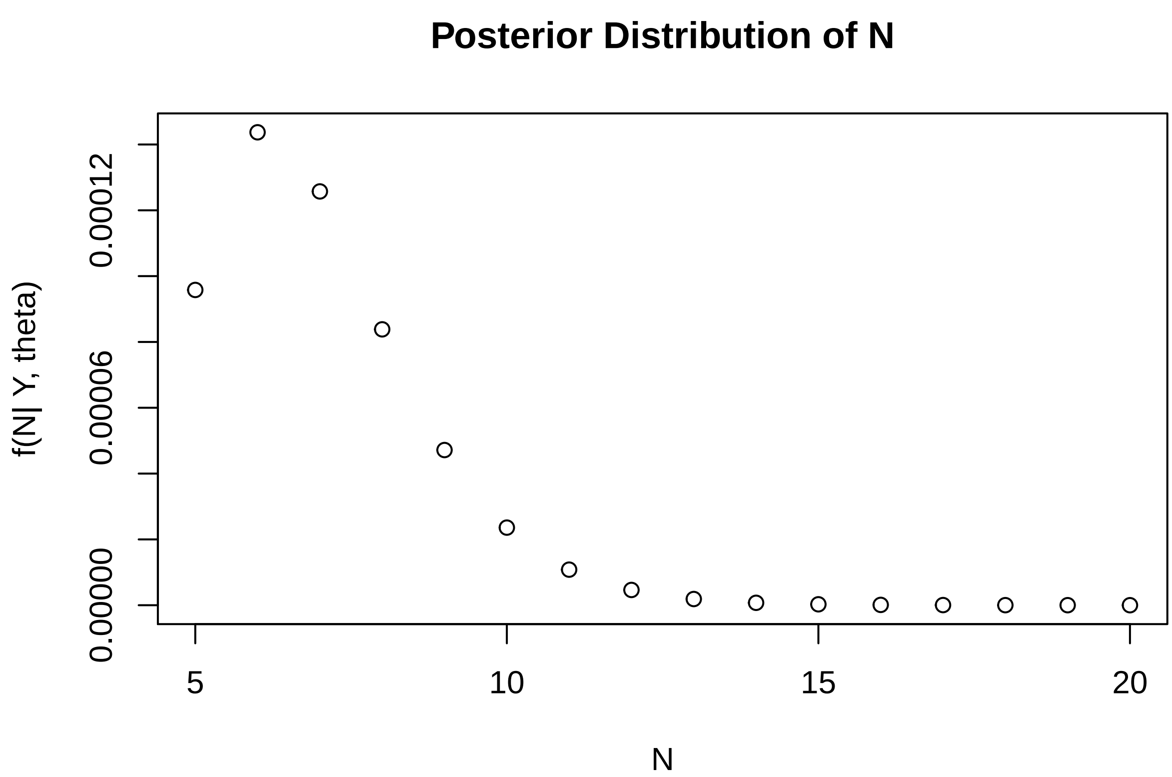
Question 1

Say that $Y \sim Binomial(N, \theta)$, where $N \in \{0, 1, 2, ...\}$ is the unknown parameter of interest and has prior $N \sim Poisson(1)$.

Part (a)

Given $Y = 5$ and $\theta = 0.5$, plot the posterior distribution of N.

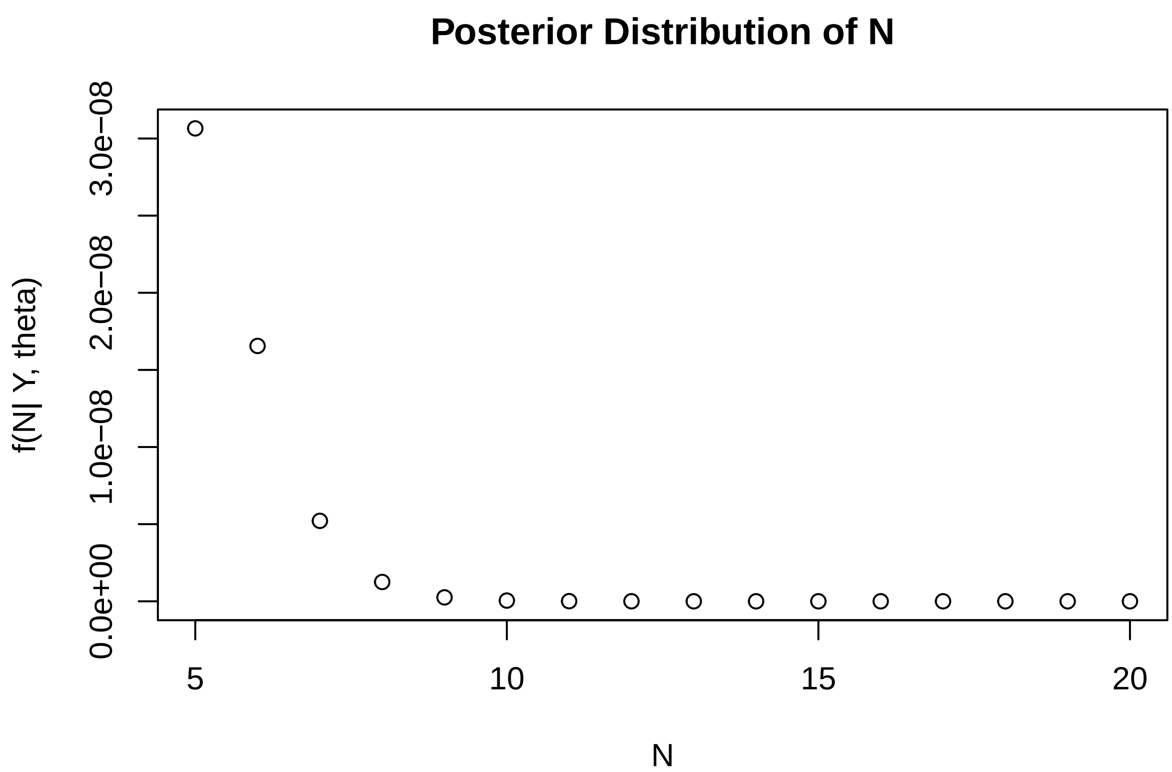
```
func <- function(N, Y, theta, lambda) {
    choose(N, Y) * theta^N * (1 - theta)^(N - Y) * ((lambda^Y * exp(-lambda))/factorial(Y))
}

n <- 5:20

plot(n, func(N = n, Y = 5, theta = 0.5, lambda = 1), ylab = "f(N| Y, theta)",
    xlab = "N", main = "Posterior Distribution of N")
```



Part (b)

Given $Y = 5$ and $\theta = 0.1$, plot the posterior distribution of N.

```
plot(n, func(N = n, Y = 5, theta = 0.1, lambda = 1), ylab = "f(N| Y, theta)",
    xlab = "N", main = "Posterior Distribution of N")
```



Part (c)

We know the number of complete passes thrown by the Bears quarterback and want to determine the distribution of the total number of passes attempted.

Question 2

A clinical trial was conducted to compare the effectiveness of three drugs. 100 patients were randomly assigned to each drug (300 total patients), and $Y_1 = 12$, $Y_2 = 18$, and $Y_3 = 10$ patients had successful outcomes in the three drug groups. Using uniform priors for the success probabilities of each drug:

Part (a)

Compute and plot the posterior distribution of the success probability for each drug.

Likelihood

$$Y_1, Y_2, Y_3|\theta \sim Binomial(n, \theta)$$

$$f(y_1, y_2, y_3|\theta) = \binom{n}{y_i}(\theta)^{y_i}(1-\theta)^{n-y_i}$$

Prior Distribution

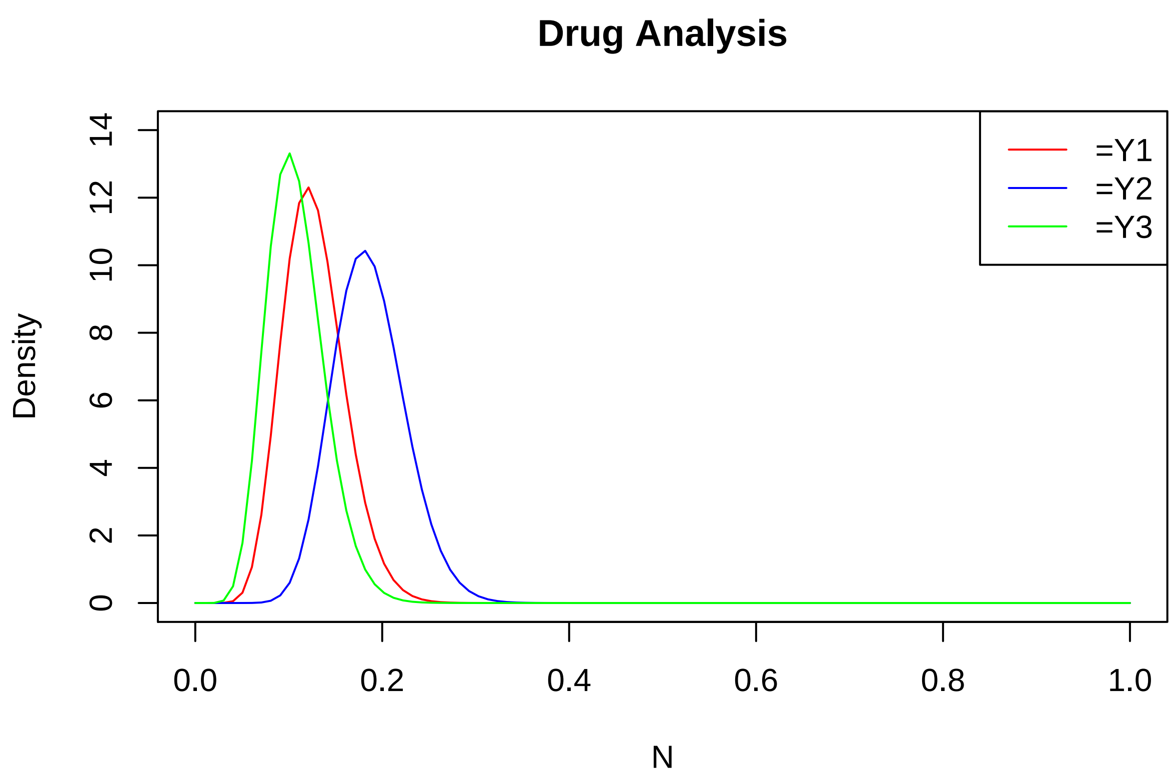
$$\theta \sim Beta(a, b)$$

$$f(\theta) = \frac{(a+b)}{(a)(b)}\theta^{a-1}(1-\theta)^{b-1}$$

Posterior Distribution

$$\theta|y \sim Beta(y+a, n-y+b)$$

```
plot(seq(0, 1, length = 100), dbeta(seq(0, 1, length = 100), 12 + 1, 100 - 12 +
    1), type = "l", col = "red", ylim = c(0, 14), ylab = "Density", xlab = "N",
    main = "Drug Analysis")
lines(seq(0, 1, length = 100), dbeta(seq(0, 1, length = 100), 18 + 1, 100 -
    18 + 1), col = "blue")
lines(seq(0, 1, length = 100), dbeta(seq(0, 1, length = 100), 10 + 1, 100 -
    10 + 1), col = "green")
legend("topright", legend = c("=Y1", "=Y2", "=Y3"), lty = c(1, 1, 1), col = c("red",
    "blue", "green"))
```



Part (b)

Compute the posterior probability that drug 2 is the best drug.

```r
drug1 <- rbeta(1e+06, 12 + 1, 100 - 12 + 1)
drug2 <- rbeta(1e+06, 18 + 1, 100 - 18 + 1)
drug3 <- rbeta(1e+06, 10 + 1, 100 - 10 + 1)

mean(drug2 > drug1 | drug2 > drug3)
```

```
## [1] 0.97832
```

The posterior probability that drug 2 is the best drug is 0.98725