Introduction to Probabilistic Graphical Models
Homework 4

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Instructions: Put all your files (code and report) in a zip file: *surname_name_hw4.zip* and submit it through moodle before November 2 2016, 13:59. Late submissions will not be accepted.

Question 1

In numeric computations, we almost always work with the logarithm of the gamma function $\log(\Gamma(x))$, which is computed without explicit reference to $\Gamma(x)$ to avoid overflow. In matlab and python, this function is "gammaln". Using the gammaln function, write functions to evaluate the logarithms of the gamma, inverse gamma, and the beta densities.

Question 2

The exponential distribution is defined as

$$\mathcal{E}(x; \lambda) = \frac{1}{\lambda} \exp(-\frac{x}{\lambda}).$$

Verify that the exponential distribution is a special case of the Gamma distribution. Find the shape and scale parameters of the corresponding gamma distribution.

Question 3

Let

$$\begin{aligned} z &\sim \mathcal{G}(v; a, 1) \\ v &= bz \\ \lambda &= 1/v \end{aligned}$$

where $a, b > 0$ are known constants. Derive the marginal distributions $p(v)$ and $p(\lambda)$ and if possible express the result as known distributions.

Question 4

The Generalized gamma distribution is a three parameter family defined as (Stacey and Mihram 1965, Johnson and Kotz pp.393)

$$\mathcal{GG}(v; \alpha, \beta, c) = \frac{|c|}{\Gamma(\alpha)\beta^{c\alpha}} v^{c\alpha-1} \exp(-(v/\beta)^c)$$

Here, α is the shape, β is the scale and c is the power parameter.

1. Is the Generalized Gamma distribution an exponential family? If so, give the canonical parameters and the sufficient statistics.

2. Verify that the inverse Gamma distribution $\mathcal{IG}(v; a_i, b_i)$ and Gamma distribution $\mathcal{G}(v; a_g, b_g)$ are special cases. Give the corresponding settings of the power parameter.

3. Show that if $v \sim \mathcal{GG}(v; \alpha, \beta, c)$, $z = (v/\beta)^c$, then, z has the standard $\mathcal{G}(z; \alpha, 1)$ distribution. Using this fact and a function that samples from standard gamma, implement a function generates random samples from a generalized Gamma distribution.

Question 5

Consider the following model

$$
\begin{aligned}
s_1 &\sim p(s_1) = \mathcal{N}(s_1; 0, 1) \\
s_2 &\sim p(s_2) = \mathcal{N}(s_2; 0, 1) \\
x|s_1, s_2 &\sim p(x|s_1, s_2) = \mathcal{N}(x; s_1 + s_2, 1)
\end{aligned}
$$

1. Draw the directed, undirected graphical models and the factor graph

2. Find $p(s_1, s_2|x)$

3. Suppose we observe $x = 9$. Plot the posterior $p(s_1, s_2|x = 9)$.