Introduction to Probabilistic Graphical Models
Homework 1

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Instructions: Put all your files (code and report) in a zip file: *surname_name_hw1.zip* and submit it through moodle before October 12 2016, 13:59 (let me know if you have any technical problems). Late submissions will not be accepted.

Question 1

In many practical applications, we often need to compute $s = \log \sum_{i=1}^{I} \exp(v_i)$, where each $v_i < 0$ and $|v_i|$ is very large. Derive (mathematically) and implement a *numerically stable* algorithm for computing 'log(sum(exp(v)))', where $v = \{v_i\}_{i=1}^{I}$ is a vector of numbers. Explain why it should work. Test your algorithm on log(sum(exp{-1234, -1235})).

Question 2

Implement a function that generates independent random samples from a specified distribution.

Y = RANDGEN(S, N, P) returns N samples from the discrete domain S, using positive weights P. P is a vector of probabilities (its entries sum up to 1).

For instance, if we want to generate 10 coin tosses with a fair coin, we can use: tosses = RANDGEN("HT", 10, [0.5, 0.5]), where H denotes heads and T denotes tails. If we would like to simulate an unfair coin, we can use tosses = RANDGEN("HT", 10, [0.2, 0.8]).

Explain the idea behind your algorithm.

Question 3

Suppose that we have three colored boxes r (red), b (blue), and g (green). Box r contains 3 apples, 4 oranges, and 3 limes, box b contains 1 apple, 1 orange, and 0 limes, and box g contains 3 apples, 3 oranges, and 4 limes.

1. If a box is chosen at random with probabilities $p(r) = 0.2$, $p(b) = 0.2$, $p(g) = 0.6$, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting an apple?
2. If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?

Question 4

Let $\lambda, \mathcal{D} \in \mathbb{R}$, $p(\lambda) = \mathcal{N}(\lambda; 0, \sigma_\lambda^2)$, and $p(\mathcal{D}|\lambda) = \mathcal{N}(\mathcal{D}; \lambda, \sigma_\mathcal{D}^2)$, where $\mathcal{N}(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-\frac{(x-\mu)^2}{2\sigma^2})$. Derive $p(\lambda|\mathcal{D})$. Plot $p(\lambda)$ and $p(\lambda|\mathcal{D} = 1)$ for $\sigma_\lambda^2 = 2, \sigma_\mathcal{D}^2 = 1$.