Introduction to Probabilistic Graphical Models
Homework 3

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Instructions: Put all your files (code and report) in a zip file: *surname_name_hw3.zip* and submit it through moodle before October 26 2016, 13:59. Late submissions will not be accepted.

Question 1

Consider the following probability model:

$$p(x_1, x_2, x_3, x_4) = \frac{1}{Z}\phi_1(x_1, x_2)\phi_2(x_2, x_3)\phi_3(x_3, x_4)$$

1. Draw the associated undirected graphical model
2. Draw the associated factor graph
3. Describe an efficient algorithm to compute Z
4. Describe an efficient algorithm to compute the marginals $p(x_i)$.

Question 2

A distribution factorizes according to the following factorization

$$p(A, B, D, F, T, L, M, X) = p(F|T, L)p(M)p(T|A)p(B|M)p(X|F)p(L|M)p(D|F, B)p(A)$$

1. Draw the corresponding directed graphical model
2. Draw an equivalent factor graph and undirected graphical model
3. If all the variables have N states, compute the space to store the model specification.
4. Verify the following conditional independence statements using d-separation. State if they are true or false and explain why.
   - $A \perp\!\!\!\perp M|\emptyset$
   - $A \perp\!\!\!\perp M|X$
   - $T \perp\!\!\!\perp L|X$
   - $X \perp\!\!\!\perp L|F$
   - $X \perp\!\!\!\perp L|D$

Question 3

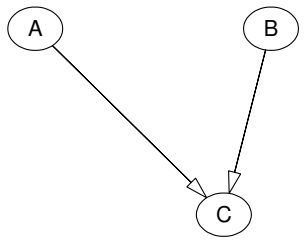
A Hierarchical Hidden Markov Model is given by the following specification

$$
\begin{aligned}
x_0 &\sim p(x_0) \\
z_0 &\sim p(z_0) \\
x_k &\sim p(x_k|x_{k1}) \\
y_k &\sim p(y_k|x_k) \\
z_k &\sim p(z_k|z_{k1}, y_k)
\end{aligned}
$$

1. Draw the corresponding directed graphical model
2. Draw an equivalent factor graph and undirected graphical model

Question 4

Consider the following graphical model:



Here, all variables are binary. $p(A = 1) = 0.9$, $p(B = 1) = 0.3$, $C = A \oplus B$ where $\oplus$ is the xor (exclusive or) operation.

1. Find the following quantities:
   - $p(C)$
   - $p(A, B|C)$
2. Write a program that will generate random probability tables $p(A)$, $p(B)$, and $p(C|A, B)$. Use the Beta distribution as the prior.
3. Using the randgen subroutine you developed in the previous assignment sheet, write a program that will generate random instances from the above model.