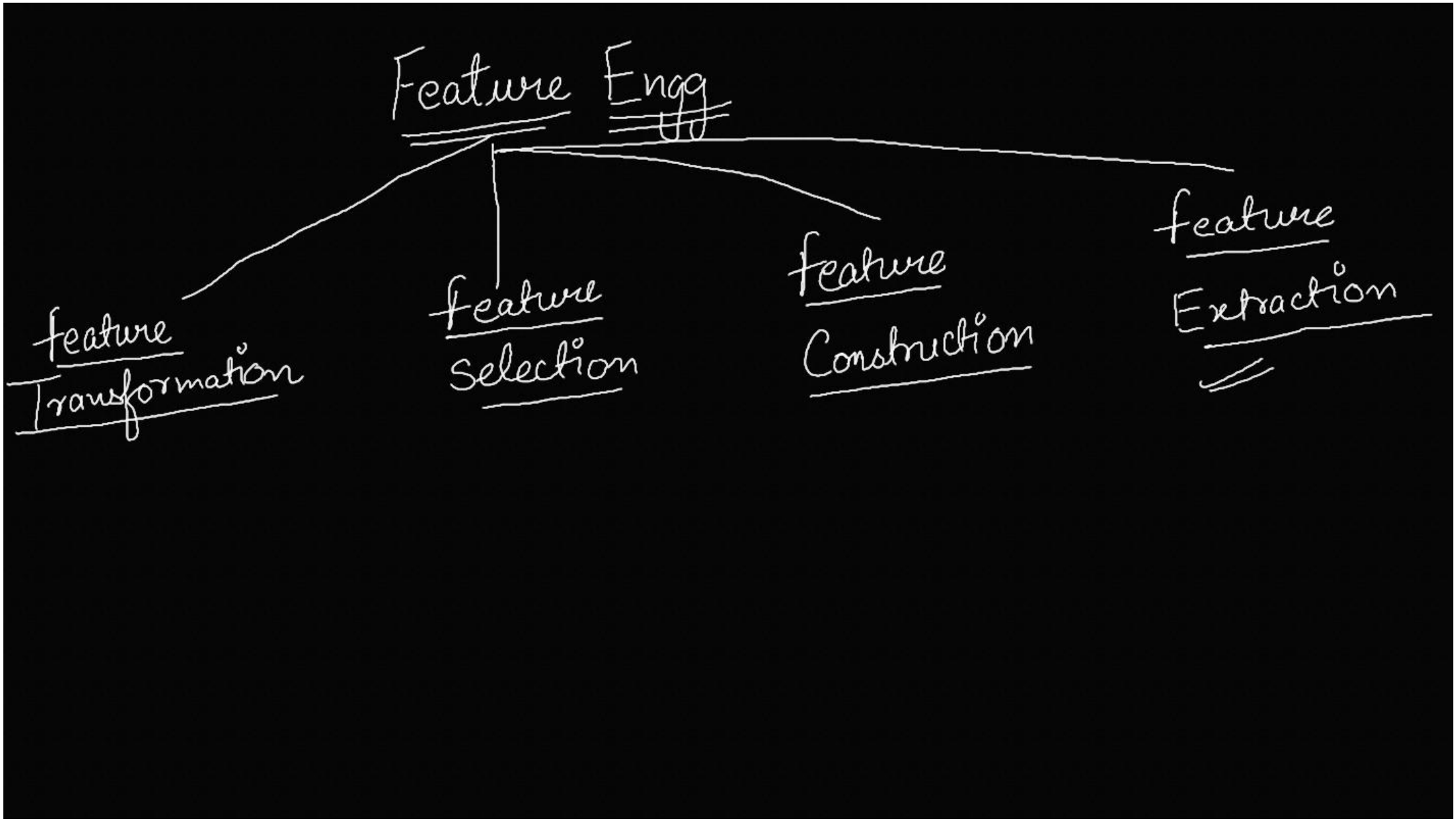
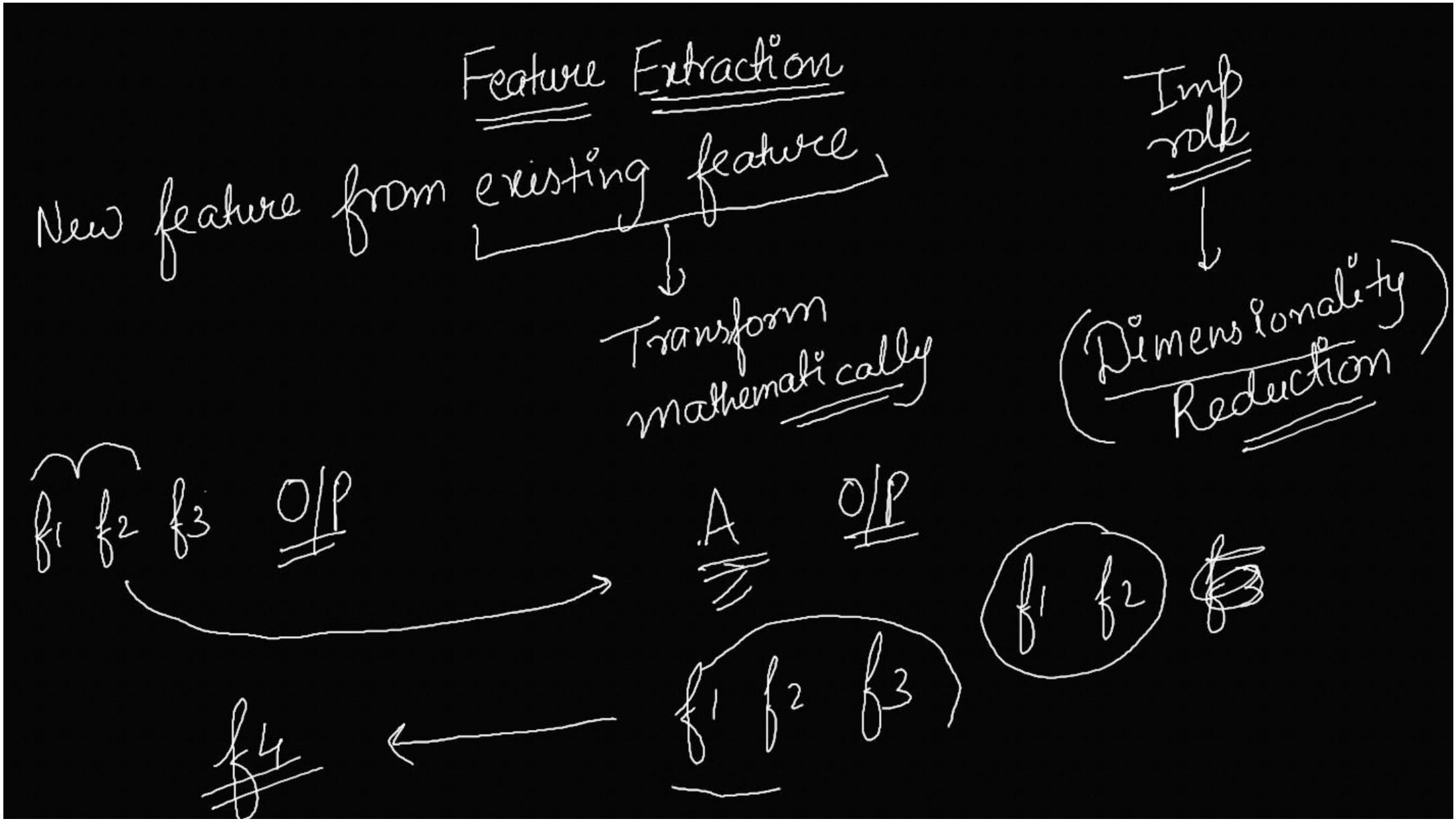

Feature Engg
feature Transformation
feature Selection
feature Construction
feature Extraction


Feature Extraction
New feature from existing feature
Transform mathematically
Imp role
(Dimensionality Reduction)
f1 f2 f3 O/P
A
O/P
f1 f2
f1 f2 f3
f4

Dimension ⇒ no of features

f_1 f_2 f_3 f_4 f_5 "Curse of Dimensionality"

dim = 3

dim = 5

No of features ⇒ 1000+

Curse of Dimensionality

① Longer training time ⟶ resource higher use
↳ ⟶ money ↑

② Overfitting ↑

90% → Training ⟶ 98%
10% → Testing ⟶ 60%
overfitting

98%
90%

③ High chances of irrelevant features
↳ Model accuracy ↓

④ O/P = 1
I/P = 999
Unable to predict relationship b/w I/P & O/P features

Model is unable to find pattern in data.

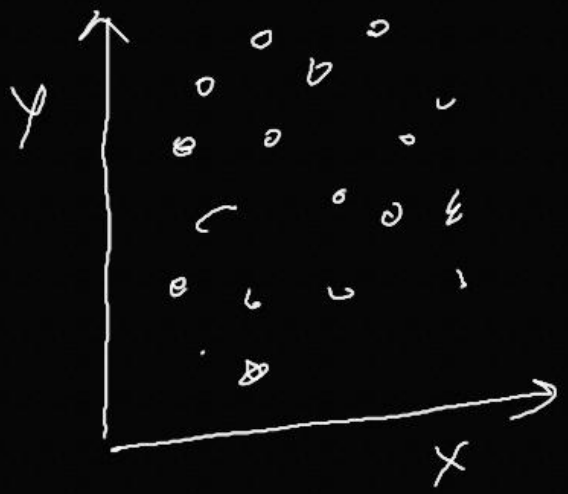
Root cause

Suppose => 100 points.

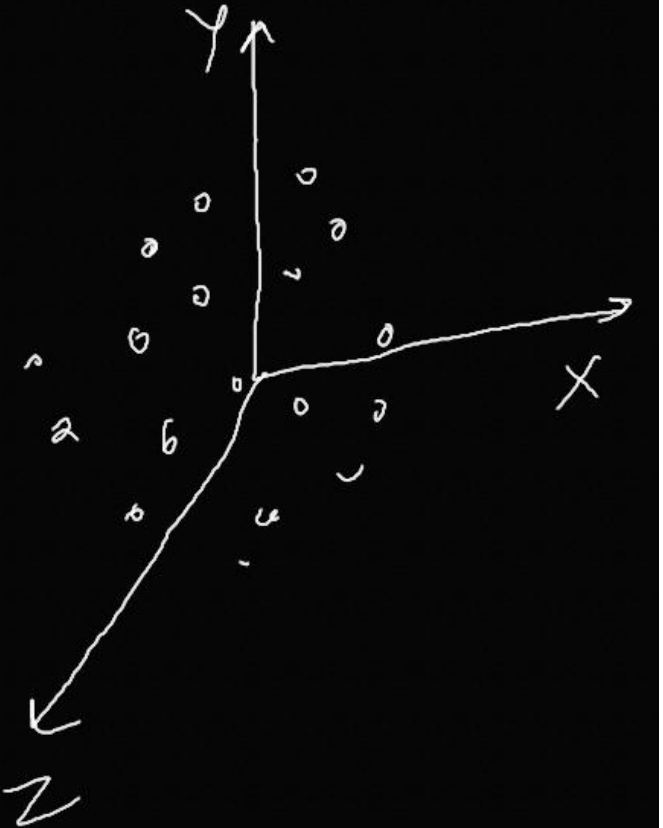
1-D

Point distance ↑

2-D



3-D



Points → chairs

1-D ⇒ Classroom → 100 chairs ⇒

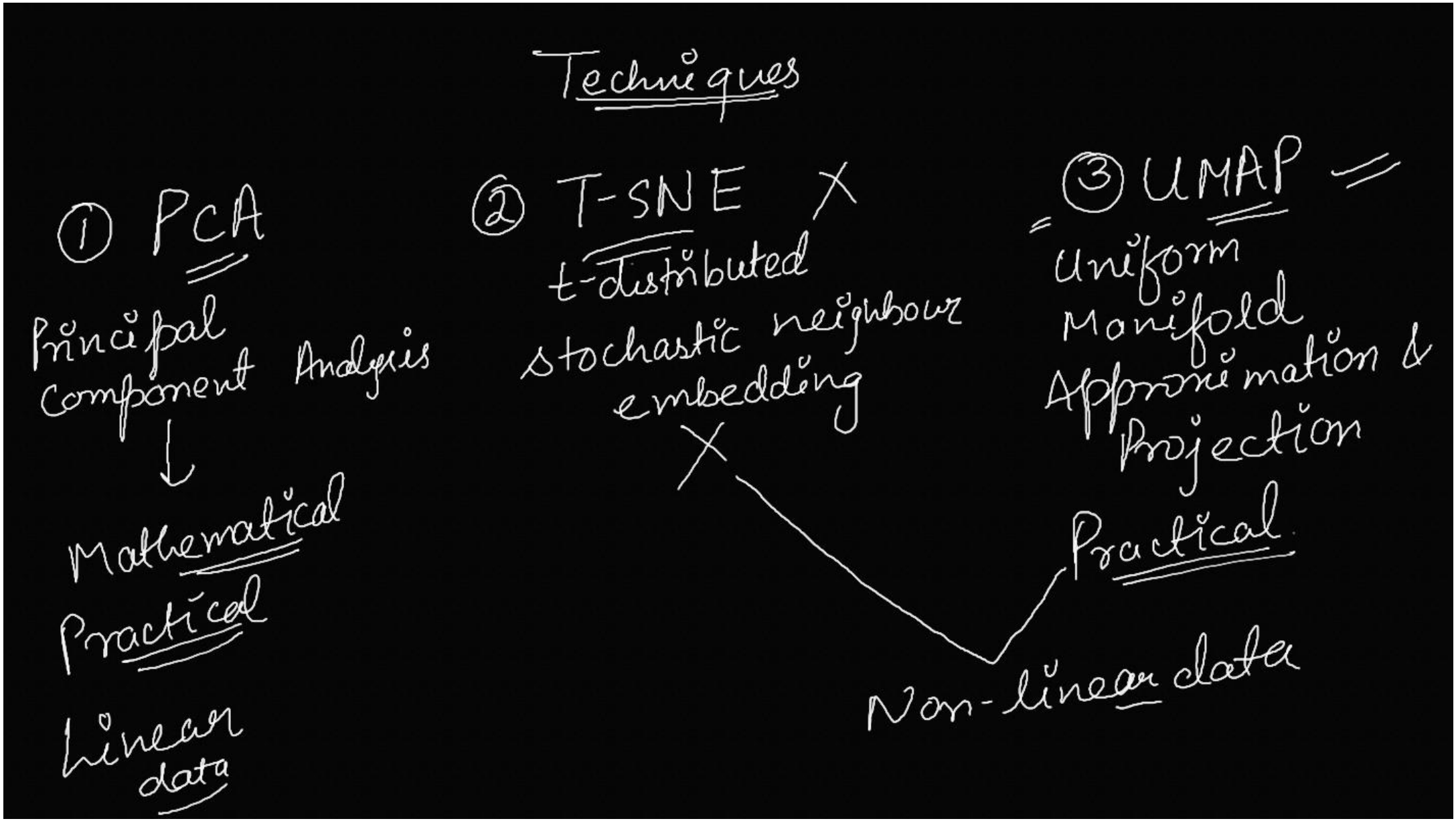
2-D ⇒ football field → 1000 chairs

3-D ⇒ Building ⇒ 1 ~~room~~ floor = 1 chair

100 ~~rooms~~ floors

Point ↑

gap


Techniques
① PCA
Principal component Analysis
Mathematical
Practical
Linear data
② T-SNE
t-distributed stochastic neighbour embedding
③ UMAP
Uniform Manifold Approximation & Projection
Practical
Non-linear data

Data

PCA

Sample	x_1	x_2
1	2	3
2	3	4
3	4	5

① Calculate mean of each feature

$\bar{x}_1 = 3$, $\bar{x}_2 = 4$

② Get standardized value.

Sample	x_1	x_2
1	-1	-1
2	0	0
3	1	1

③ Calculate Co-Variance matrix $n \rightarrow$ no of rows $= 3$

$$Var(x_1) = \frac{1}{n-1}\left((-1)^2 + (0)^2 + (1)^2\right) = \frac{1}{2} \cdot 2 = 1$$

$$Var(x_2) = \frac{1}{2}(\quad " \quad) = 1 =$$

$$\text{Co-var}(x_1, x_2) = \frac{1}{n-1} \Sigma (x_1 \cdot x_2)$$

$$= \frac{1}{2}\left[\left((-1)(-1) + 0 + (1)(1)\right)\right] = 1$$

Co-variance matrix $= \begin{bmatrix} Cov(x_1, x_1) & Cov(x_1, x_2) \\ Cov(x_2, x_1) & Cov(x_2, x_2) \end{bmatrix}$

$= \begin{bmatrix} | & | \\ | & | \end{bmatrix}$ → Co-variance matrix

④ Calculate eigen value

$$\det(Cov - \lambda I) = 0$$

det → determinant
Cov → Co-var matrix
λ → eigen value
I → Identity matrix

(a) Identity matrix → diagonal = 1
others = 0

2×2

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3×3

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Determinant

2 × 2

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$\Rightarrow ad - bc$

$$\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$$

$\Rightarrow 4 - 6$

$\Rightarrow -2 \rightarrow$ det

3 × 3 $\Rightarrow$

$$\begin{bmatrix} + & - & + \\ 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$\Rightarrow 1(45 - 48) - 2(36 - 42) + 3(32 - 35)$

$\Rightarrow 0 \rightarrow$ det

$$\det(\underbrace{Cov - \lambda I}) = 0$$

$$Cov - \lambda I = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\underline{\underline{A}} = \begin{bmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix}$$

$$\det(A) = (1-\lambda)(1-\lambda) - 1$$

$$= (1-\lambda)^2 - 1$$

$$\therefore (a-b)^2 = a^2 + b^2 - 2ab$$

$$\det(A) = 1 + \lambda^2 - 2\lambda - 1$$

$$= \lambda^2 - 2\lambda$$

$$\lambda^2 - 2\lambda = 0$$

$$\lambda(\lambda - 2) = 0$$

$\lambda = 0$ $\qquad$ $\lambda - 2 = 0$

$\lambda = 2$

Eigen values

$$\lambda_1 = 0$$

$$\lambda_2 = 2$$

⑤ find eigen vector

$v \rightarrow$ eigen vector

Case 1: $\lambda_1 = 0$

$$(Cov - \lambda I) \times v = 0$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - 0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \times v = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$v_1 + v_2 = 0$ > same

$v_1 + v_2 = 0$

$v_1 + v_2 = 0$

$$\boxed{v_1 = -v_2}$$

$$\boxed{v_1 = -v_2}$$

$v_1 = -1$

$v_2 = 1$

$$V_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \Longrightarrow \text{vector 1}$$

Case 2: $\lambda_2 = 2$

$$(Cov - \lambda I) \cdot v = 0$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0$$

$$\Rightarrow -1 \cdot v_1 + 1 \cdot v_2 = 0$$

$$-v_1 + v_2 = 0$$

$$\boxed{v_1 = v_2}$$

$v_1 = 1$

$v_2 = 1$

$$V_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

↓

vector 2

(6) Normalization of vectors

$v_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ —normalize→ $v_1 = \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ Optional

$v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ⟶ $v_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

Normalization

① Magnitude

$\|v\| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$ ⟶ Divide each component

① form feature vector

$$V_1 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \lambda = 0$$

$$V_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \rightarrow \lambda = 2$$

$\lambda = 2$ ← larger eigen value

$$\text{feature vector} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

⑧ Transform

New data = feature vector × standardized data

Sample 1 :- $(x_1 = -1 \;,\; x_2 = +1)$

f.v = $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

new data = $-1 \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times (-1)$

$= -\frac{\sqrt{2}}{}$

Sample 2 :- 0

Sample 3 :- $-\sqrt{2}$

New data

Sample	PC1
1	$-\sqrt{2}$
2	0
3	$\sqrt{2}$