approximation of the distribution

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Abstract

This report considers approximation of the distribution function of $N(0,1)$ by the Monte Carlo methods, and forms a table that includes the true value for comparison.

Introduction

This report uses **bookdown** or **rmarkdown** to produce a report for considering approximation of the distribution function of $N(0,1)$ by the Monte Carlo methods, and do a Experiment with the approximation at $n \in \{10^2, 10^3, 10^4\}$ at $t \in \{0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72\}$ to form a table. And Draw box plots of the 100 approximation errors at each t using **ggplot2** [@R-ggplot2] for each n.

Math Equations

Consider approximation of the distribution function of $N(0,1)$,

$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy, (\#eq:cdf) \tag{1}$$

by the Monte Carlo methods:

$$\hat{\Phi}(t) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq t), \tag{2}$$

where X_i's are a random sample from $N(0,1)$, and $I(\cdot)$ is the indicator function.

Experimentation and Result

Experiment with the approximation at $n \in \{10^2, 10^3, 10^4\}$ at $t \in \{0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72\}$ to form a table.

Form a table

```
t=c(0.0,0.67, 0.84,1.28,1.65,2.32,2.58,3.09,3.72)

n1=100
w1=matrix(0,9,100)
x1=vector("numeric",9)
y1=c(rnorm(100,0,1))

for (i in 1:9)
  {for(j in 1:100)
    {w1[i,j]=sign(y1[j]<=t[i])}
  x1[i]=sum(w1[i,])/100}

n2=1000
w2=matrix(0,9,1000)
x2=vector("numeric",9)
y2=c(rnorm(1000,0,1))

for (i in 1:9) {
  for(j in 1:1000){
    w2[i,j]=sign(y2[j]<=t[i])
  }
  x2[i]=sum(w2[i,])/1000
}

n3=10000
w3=matrix(0,9,10000)
x3=vector("numeric",9)
y3=c(rnorm(100,0,1))

for (i in 1:9) {
  for(j in 1:10000){
    w3[i,j]=sign(y3[j] <= t[i])
  }
  x3[i]=sum(w1[i,])/10000
```

```
}

x4=pnorm(c(0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72),0,1)
x=data.frame(R.100=x1,R.1000=x2,R.10000=x3,True_value=x4)
colnames=c("n=100","n=1000","n=10000","True_value")
rownames=c("t=0","t=0.67","t=0.84","t=1.28","t=1.65","t=2.32","t=2.58","t=3.09","t=3.
dimnames(x)=list(rownames,colnames)
x
```

```
##        n=100 n=1000 n=10000 True_value
## t=0     0.44  0.521  0.0044  0.5000000
## t=0.67  0.73  0.758  0.0073  0.7485711
## t=0.84  0.78  0.812  0.0078  0.7995458
## t=1.28  0.89  0.919  0.0089  0.8997274
## t=1.65  0.95  0.960  0.0095  0.9505285
## t=2.32  0.98  0.993  0.0098  0.9898296
## t=2.58  0.98  0.996  0.0098  0.9950600
## t=3.09  1.00  0.999  0.0100  0.9989992
## t=3.72  1.00  1.000  0.0100  0.9999004
```

Repeat the experiment 100 times

```
n1=100
rp100_100<-function(){
  n1=100
  w1=matrix(0,9,100)
  x1=vector("numeric",9)
  y1=c(rnorm(100,0,1))

for (i in 1:9) {
  for(j in 1:100){
    w1[i,j]=sign(y1[j]<=t[i])
  }
  x1[i]=sum(w1[i,])/100
}
  return(x1)
}
m1=matrix(0,100,9)
for(i in 1:100)
    m1[i,]=rp100_100()
```

```
m1
```

```
##        [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##   [1,] 0.49 0.70 0.74 0.89 0.94 1.00 1.00 1.00 1.00
##   [2,] 0.51 0.78 0.82 0.92 0.96 0.99 0.99 0.99 1.00
##   [3,] 0.58 0.83 0.88 0.96 0.98 0.99 0.99 1.00 1.00
##   [4,] 0.51 0.75 0.80 0.93 0.98 0.99 0.99 1.00 1.00
##   [5,] 0.64 0.82 0.85 0.93 0.98 1.00 1.00 1.00 1.00
##   [6,] 0.54 0.74 0.86 0.91 0.96 1.00 1.00 1.00 1.00
##   [7,] 0.48 0.75 0.79 0.90 0.96 0.99 1.00 1.00 1.00
##   [8,] 0.47 0.75 0.79 0.85 0.91 0.99 0.99 1.00 1.00
##   [9,] 0.52 0.73 0.79 0.90 0.95 1.00 1.00 1.00 1.00
##  [10,] 0.50 0.74 0.76 0.90 0.92 0.98 0.99 1.00 1.00
##  [11,] 0.54 0.76 0.79 0.92 0.95 1.00 1.00 1.00 1.00
##  [12,] 0.52 0.72 0.73 0.85 0.90 0.98 0.99 1.00 1.00
##  [13,] 0.49 0.76 0.82 0.94 0.97 0.98 0.99 0.99 1.00
##  [14,] 0.48 0.69 0.78 0.86 0.95 1.00 1.00 1.00 1.00
##  [15,] 0.51 0.74 0.78 0.89 0.95 0.99 1.00 1.00 1.00
##  [16,] 0.50 0.75 0.78 0.89 0.96 0.98 0.98 1.00 1.00
##  [17,] 0.49 0.74 0.78 0.90 0.95 1.00 1.00 1.00 1.00
##  [18,] 0.57 0.72 0.76 0.92 0.97 0.98 0.99 0.99 0.99
##  [19,] 0.51 0.73 0.78 0.89 0.93 0.99 1.00 1.00 1.00
##  [20,] 0.57 0.76 0.83 0.90 0.97 0.98 0.99 1.00 1.00
##  [21,] 0.57 0.77 0.83 0.94 0.95 1.00 1.00 1.00 1.00
##  [22,] 0.42 0.71 0.78 0.95 0.99 1.00 1.00 1.00 1.00
##  [23,] 0.46 0.74 0.80 0.95 0.96 1.00 1.00 1.00 1.00
##  [24,] 0.54 0.73 0.79 0.88 0.96 0.98 0.99 1.00 1.00
##  [25,] 0.54 0.74 0.82 0.89 0.96 1.00 1.00 1.00 1.00
##  [26,] 0.46 0.69 0.73 0.83 0.89 0.98 1.00 1.00 1.00
##  [27,] 0.47 0.71 0.73 0.87 0.91 0.99 1.00 1.00 1.00
##  [28,] 0.50 0.74 0.79 0.88 0.92 0.99 1.00 1.00 1.00
##  [29,] 0.53 0.78 0.82 0.92 0.95 0.99 1.00 1.00 1.00
##  [30,] 0.42 0.67 0.70 0.88 0.92 0.99 0.99 1.00 1.00
##  [31,] 0.48 0.73 0.78 0.91 0.96 0.98 0.99 1.00 1.00
##  [32,] 0.54 0.72 0.78 0.92 0.97 0.99 0.99 0.99 1.00
##  [33,] 0.55 0.80 0.84 0.92 0.97 1.00 1.00 1.00 1.00
##  [34,] 0.50 0.74 0.79 0.94 0.96 0.98 0.98 0.98 1.00
##  [35,] 0.47 0.78 0.80 0.88 0.93 0.97 0.98 1.00 1.00
##  [36,] 0.43 0.68 0.73 0.85 0.90 0.98 0.99 1.00 1.00
##  [37,] 0.55 0.74 0.80 0.90 0.95 0.99 0.99 1.00 1.00
```

```
##  [38,] 0.58 0.79 0.84 0.90 0.97 0.99 1.00 1.00 1.00
##  [39,] 0.53 0.78 0.82 0.89 0.94 0.98 1.00 1.00 1.00
##  [40,] 0.45 0.72 0.80 0.92 0.97 0.99 0.99 1.00 1.00
##  [41,] 0.48 0.76 0.80 0.87 0.93 0.99 1.00 1.00 1.00
##  [42,] 0.50 0.74 0.78 0.88 0.89 0.99 1.00 1.00 1.00
##  [43,] 0.47 0.75 0.83 0.93 0.96 1.00 1.00 1.00 1.00
##  [44,] 0.44 0.71 0.80 0.88 0.97 0.98 0.99 1.00 1.00
##  [45,] 0.53 0.70 0.78 0.85 0.90 0.99 0.99 1.00 1.00
##  [46,] 0.48 0.67 0.75 0.87 0.92 0.99 1.00 1.00 1.00
##  [47,] 0.49 0.78 0.82 0.90 0.96 1.00 1.00 1.00 1.00
##  [48,] 0.43 0.77 0.83 0.92 0.97 0.98 0.98 1.00 1.00
##  [49,] 0.46 0.64 0.70 0.87 0.93 0.98 1.00 1.00 1.00
##  [50,] 0.47 0.72 0.79 0.89 0.94 0.98 0.99 1.00 1.00
##  [51,] 0.57 0.79 0.82 0.90 0.97 0.99 0.99 0.99 1.00
##  [52,] 0.52 0.75 0.81 0.91 0.94 0.99 0.99 0.99 1.00
##  [53,] 0.54 0.72 0.78 0.89 0.95 1.00 1.00 1.00 1.00
##  [54,] 0.48 0.76 0.80 0.91 0.96 0.99 1.00 1.00 1.00
##  [55,] 0.45 0.72 0.80 0.92 0.98 0.99 0.99 1.00 1.00
##  [56,] 0.52 0.71 0.74 0.85 0.96 0.99 1.00 1.00 1.00
##  [57,] 0.44 0.73 0.78 0.86 0.95 0.98 1.00 1.00 1.00
##  [58,] 0.60 0.80 0.83 0.91 0.94 0.99 0.99 1.00 1.00
##  [59,] 0.49 0.70 0.74 0.83 0.90 0.98 0.99 1.00 1.00
##  [60,] 0.50 0.78 0.79 0.91 0.99 1.00 1.00 1.00 1.00
##  [61,] 0.48 0.78 0.82 0.93 0.95 1.00 1.00 1.00 1.00
##  [62,] 0.60 0.83 0.87 0.94 0.96 0.99 0.99 1.00 1.00
##  [63,] 0.54 0.77 0.82 0.89 0.93 0.99 1.00 1.00 1.00
##  [64,] 0.48 0.67 0.75 0.87 0.96 0.99 1.00 1.00 1.00
##  [65,] 0.36 0.73 0.83 0.90 0.97 1.00 1.00 1.00 1.00
##  [66,] 0.55 0.76 0.84 0.91 0.96 0.98 0.99 1.00 1.00
##  [67,] 0.48 0.76 0.79 0.87 0.91 0.99 1.00 1.00 1.00
##  [68,] 0.55 0.75 0.77 0.88 0.95 0.99 0.99 1.00 1.00
##  [69,] 0.48 0.71 0.78 0.89 0.93 0.99 1.00 1.00 1.00
##  [70,] 0.46 0.74 0.79 0.86 0.89 0.97 0.99 1.00 1.00
##  [71,] 0.46 0.72 0.77 0.90 0.92 0.98 1.00 1.00 1.00
##  [72,] 0.56 0.83 0.87 0.91 0.93 0.97 0.98 1.00 1.00
##  [73,] 0.55 0.67 0.70 0.89 0.95 0.98 0.99 1.00 1.00
##  [74,] 0.50 0.72 0.75 0.89 0.95 1.00 1.00 1.00 1.00
##  [75,] 0.50 0.76 0.79 0.91 0.97 0.99 1.00 1.00 1.00
##  [76,] 0.47 0.71 0.77 0.86 0.93 1.00 1.00 1.00 1.00
##  [77,] 0.49 0.71 0.80 0.91 0.97 1.00 1.00 1.00 1.00
```

```
##  [78,] 0.40 0.76 0.84 0.91 0.94 0.99 1.00 1.00 1.00
##  [79,] 0.43 0.69 0.78 0.89 0.98 1.00 1.00 1.00 1.00
##  [80,] 0.41 0.66 0.74 0.88 0.95 1.00 1.00 1.00 1.00
##  [81,] 0.54 0.73 0.79 0.93 0.96 1.00 1.00 1.00 1.00
##  [82,] 0.49 0.75 0.80 0.90 0.94 1.00 1.00 1.00 1.00
##  [83,] 0.55 0.78 0.83 0.91 0.97 1.00 1.00 1.00 1.00
##  [84,] 0.45 0.74 0.78 0.90 0.95 0.97 0.98 1.00 1.00
##  [85,] 0.53 0.78 0.85 0.94 0.97 0.99 1.00 1.00 1.00
##  [86,] 0.53 0.71 0.77 0.90 0.95 1.00 1.00 1.00 1.00
##  [87,] 0.54 0.79 0.84 0.93 0.99 1.00 1.00 1.00 1.00
##  [88,] 0.52 0.74 0.80 0.89 0.93 1.00 1.00 1.00 1.00
##  [89,] 0.60 0.77 0.79 0.91 0.94 0.96 0.98 1.00 1.00
##  [90,] 0.48 0.75 0.82 0.91 0.95 1.00 1.00 1.00 1.00
##  [91,] 0.49 0.78 0.81 0.90 0.99 1.00 1.00 1.00 1.00
##  [92,] 0.50 0.76 0.84 0.92 0.96 1.00 1.00 1.00 1.00
##  [93,] 0.44 0.72 0.79 0.84 0.89 0.97 0.99 1.00 1.00
##  [94,] 0.50 0.78 0.83 0.91 0.93 0.99 1.00 1.00 1.00
##  [95,] 0.52 0.82 0.86 0.95 1.00 1.00 1.00 1.00 1.00
##  [96,] 0.46 0.67 0.71 0.88 0.94 1.00 1.00 1.00 1.00
##  [97,] 0.49 0.73 0.74 0.86 0.91 0.96 0.96 0.99 1.00
##  [98,] 0.47 0.73 0.77 0.91 0.99 1.00 1.00 1.00 1.00
##  [99,] 0.60 0.76 0.79 0.87 0.96 1.00 1.00 1.00 1.00
## [100,] 0.49 0.74 0.78 0.94 0.96 0.99 0.99 0.99 1.00
```

```
n2=1000
rp100_1000<-function(){
  n2=1000
  w2=matrix(0,9,1000)
  x2=vector("numeric",9)
  y2=c(rnorm(1000,0,1))

for (i in 1:9) {
  for(j in 1:1000){
    w2[i,j]=sign(y2[j]<=t[i])
  }
  x2[i]=sum(w2[i,])/1000
}
  return(x2)
}
m2=matrix(0,100,9)
```

```
for(i in 1:100)
    m2[i,]=rp100_100()
m2
```

```
##        [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##   [1,] 0.52 0.69 0.73 0.84 0.90 0.97 0.98 0.99 1.00
##   [2,] 0.50 0.72 0.77 0.89 0.95 0.98 0.98 0.99 1.00
##   [3,] 0.54 0.84 0.85 0.92 0.96 1.00 1.00 1.00 1.00
##   [4,] 0.54 0.73 0.78 0.88 0.93 0.98 0.99 1.00 1.00
##   [5,] 0.45 0.71 0.78 0.93 0.97 1.00 1.00 1.00 1.00
##   [6,] 0.45 0.69 0.76 0.92 0.96 1.00 1.00 1.00 1.00
##   [7,] 0.51 0.77 0.81 0.93 0.97 0.99 0.99 1.00 1.00
##   [8,] 0.49 0.76 0.78 0.89 0.95 0.99 1.00 1.00 1.00
##   [9,] 0.50 0.73 0.78 0.91 0.95 1.00 1.00 1.00 1.00
##  [10,] 0.50 0.76 0.82 0.89 0.95 1.00 1.00 1.00 1.00
##  [11,] 0.46 0.70 0.75 0.90 0.95 0.98 1.00 1.00 1.00
##  [12,] 0.58 0.77 0.81 0.95 0.97 0.99 1.00 1.00 1.00
##  [13,] 0.48 0.72 0.81 0.92 0.97 0.99 1.00 1.00 1.00
##  [14,] 0.52 0.79 0.86 0.92 0.98 0.99 0.99 0.99 1.00
##  [15,] 0.50 0.72 0.76 0.89 0.95 0.99 1.00 1.00 1.00
##  [16,] 0.48 0.76 0.79 0.89 0.95 0.99 0.99 0.99 0.99
##  [17,] 0.52 0.76 0.80 0.92 0.96 0.98 0.99 1.00 1.00
##  [18,] 0.45 0.76 0.77 0.89 0.95 0.99 0.99 1.00 1.00
##  [19,] 0.51 0.74 0.82 0.88 0.92 1.00 1.00 1.00 1.00
##  [20,] 0.44 0.72 0.80 0.95 0.97 1.00 1.00 1.00 1.00
##  [21,] 0.44 0.68 0.74 0.83 0.90 0.96 1.00 1.00 1.00
##  [22,] 0.42 0.76 0.84 0.92 0.98 1.00 1.00 1.00 1.00
##  [23,] 0.50 0.76 0.82 0.95 0.97 1.00 1.00 1.00 1.00
##  [24,] 0.47 0.75 0.79 0.92 0.98 1.00 1.00 1.00 1.00
##  [25,] 0.55 0.80 0.85 0.92 0.97 1.00 1.00 1.00 1.00
##  [26,] 0.41 0.70 0.76 0.86 0.92 1.00 1.00 1.00 1.00
##  [27,] 0.47 0.69 0.75 0.86 0.94 0.99 0.99 1.00 1.00
##  [28,] 0.56 0.80 0.83 0.92 0.94 0.98 0.99 1.00 1.00
##  [29,] 0.52 0.75 0.79 0.89 0.99 1.00 1.00 1.00 1.00
##  [30,] 0.48 0.78 0.81 0.93 0.96 0.99 0.99 1.00 1.00
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##  [32,] 0.47 0.73 0.80 0.89 0.95 1.00 1.00 1.00 1.00
##  [33,] 0.48 0.75 0.81 0.92 0.97 0.99 0.99 0.99 1.00
##  [34,] 0.43 0.71 0.81 0.89 0.96 1.00 1.00 1.00 1.00
```

```
##  [35,] 0.53 0.75 0.80 0.91 0.93 0.98 0.99 1.00 1.00
##  [36,] 0.55 0.78 0.83 0.92 0.95 0.98 0.98 1.00 1.00
##  [37,] 0.56 0.80 0.83 0.95 0.97 1.00 1.00 1.00 1.00
##  [38,] 0.50 0.77 0.82 0.94 0.95 0.97 0.99 0.99 1.00
##  [39,] 0.47 0.75 0.76 0.90 0.96 0.98 1.00 1.00 1.00
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##  [42,] 0.51 0.81 0.84 0.92 0.95 0.98 0.98 1.00 1.00
##  [43,] 0.53 0.78 0.79 0.90 0.97 0.98 0.99 1.00 1.00
##  [44,] 0.52 0.77 0.85 0.92 0.97 1.00 1.00 1.00 1.00
##  [45,] 0.47 0.69 0.74 0.87 0.89 0.97 0.98 1.00 1.00
##  [46,] 0.51 0.79 0.83 0.89 0.97 0.98 0.98 1.00 1.00
##  [47,] 0.45 0.76 0.79 0.90 0.97 1.00 1.00 1.00 1.00
##  [48,] 0.47 0.74 0.80 0.89 0.95 0.98 0.99 1.00 1.00
##  [49,] 0.54 0.73 0.80 0.90 0.95 0.97 0.99 1.00 1.00
##  [50,] 0.51 0.78 0.79 0.89 0.97 1.00 1.00 1.00 1.00
##  [51,] 0.60 0.79 0.83 0.91 0.97 0.98 0.99 1.00 1.00
##  [52,] 0.51 0.73 0.79 0.88 0.94 1.00 1.00 1.00 1.00
##  [53,] 0.44 0.70 0.77 0.90 0.95 0.99 0.99 1.00 1.00
##  [54,] 0.48 0.80 0.88 0.94 0.97 1.00 1.00 1.00 1.00
##  [55,] 0.55 0.68 0.75 0.90 0.92 0.98 0.98 1.00 1.00
##  [56,] 0.53 0.71 0.73 0.87 0.94 0.98 0.99 1.00 1.00
##  [57,] 0.45 0.69 0.79 0.92 0.96 0.99 1.00 1.00 1.00
##  [58,] 0.45 0.66 0.74 0.83 0.88 0.96 0.99 1.00 1.00
##  [59,] 0.54 0.78 0.84 0.91 0.98 0.99 1.00 1.00 1.00
##  [60,] 0.48 0.71 0.77 0.86 0.95 0.99 1.00 1.00 1.00
##  [61,] 0.51 0.79 0.82 0.92 0.96 0.98 0.98 1.00 1.00
##  [62,] 0.56 0.78 0.79 0.92 0.93 0.98 1.00 1.00 1.00
##  [63,] 0.57 0.81 0.85 0.92 0.95 0.99 1.00 1.00 1.00
##  [64,] 0.48 0.74 0.77 0.90 0.95 0.98 1.00 1.00 1.00
##  [65,] 0.45 0.75 0.77 0.89 0.94 0.99 1.00 1.00 1.00
##  [66,] 0.49 0.73 0.76 0.87 0.95 0.96 0.98 1.00 1.00
##  [67,] 0.55 0.80 0.84 0.94 0.97 1.00 1.00 1.00 1.00
##  [68,] 0.54 0.79 0.82 0.88 0.94 0.97 0.98 1.00 1.00
##  [69,] 0.59 0.76 0.78 0.86 0.96 0.99 0.99 0.99 1.00
##  [70,] 0.42 0.68 0.74 0.84 0.89 0.98 0.99 0.99 1.00
##  [71,] 0.54 0.83 0.85 0.90 0.96 0.98 1.00 1.00 1.00
##  [72,] 0.41 0.67 0.73 0.85 0.92 0.97 0.99 1.00 1.00
##  [73,] 0.43 0.73 0.77 0.85 0.91 0.99 1.00 1.00 1.00
##  [74,] 0.52 0.78 0.84 0.88 0.94 1.00 1.00 1.00 1.00
```

```
##  [75,] 0.54 0.76 0.85 0.97 0.98 0.99 0.99 0.99 1.00
##  [76,] 0.45 0.68 0.74 0.90 0.96 0.98 0.98 1.00 1.00
##  [77,] 0.52 0.78 0.80 0.89 0.92 0.98 0.98 1.00 1.00
##  [78,] 0.58 0.81 0.85 0.94 0.99 1.00 1.00 1.00 1.00
##  [79,] 0.49 0.71 0.78 0.89 0.94 1.00 1.00 1.00 1.00
##  [80,] 0.42 0.70 0.75 0.92 0.97 1.00 1.00 1.00 1.00
##  [81,] 0.55 0.72 0.76 0.91 0.95 1.00 1.00 1.00 1.00
##  [82,] 0.46 0.68 0.78 0.88 0.93 0.99 1.00 1.00 1.00
##  [83,] 0.53 0.75 0.82 0.88 0.91 0.99 0.99 1.00 1.00
##  [84,] 0.45 0.73 0.78 0.91 0.97 1.00 1.00 1.00 1.00
##  [85,] 0.56 0.78 0.82 0.90 0.95 1.00 1.00 1.00 1.00
##  [86,] 0.49 0.78 0.86 0.93 0.98 1.00 1.00 1.00 1.00
##  [87,] 0.61 0.79 0.86 0.95 0.96 1.00 1.00 1.00 1.00
##  [88,] 0.46 0.69 0.77 0.87 0.94 0.99 1.00 1.00 1.00
##  [89,] 0.49 0.69 0.78 0.90 0.98 1.00 1.00 1.00 1.00
##  [90,] 0.43 0.75 0.81 0.88 0.91 0.98 1.00 1.00 1.00
##  [91,] 0.46 0.73 0.78 0.87 0.92 0.99 1.00 1.00 1.00
##  [92,] 0.50 0.74 0.78 0.88 0.95 0.99 0.99 0.99 1.00
##  [93,] 0.53 0.76 0.82 0.92 0.99 1.00 1.00 1.00 1.00
##  [94,] 0.52 0.78 0.84 0.93 0.97 0.99 0.99 1.00 1.00
##  [95,] 0.36 0.61 0.67 0.87 0.94 0.98 0.99 0.99 1.00
##  [96,] 0.56 0.73 0.80 0.87 0.91 0.98 0.99 1.00 1.00
##  [97,] 0.52 0.74 0.79 0.90 0.95 0.98 0.99 1.00 1.00
##  [98,] 0.48 0.75 0.83 0.91 0.94 0.97 0.98 0.99 1.00
##  [99,] 0.49 0.70 0.78 0.91 0.96 0.97 0.99 1.00 1.00
## [100,] 0.50 0.76 0.82 0.92 0.97 1.00 1.00 1.00 1.00
```

```r
n3=10000
rp100_10000<-function(){
  n3=10000
  w3=matrix(0,9,10000)
  x3=vector("numeric",9)
  y3=c(rnorm(10000,0,1))

for (i in 1:9) {
  for(j in 1:10000){
    w3[i,j]=sign(y3[j]<=t[i])
  }
  x3[i]=sum(w3[i,])/10000
}
```

```
  return(x3)
}
m3=matrix(0,100,9)
for(i in 1:100)
    m3[i,]=rp100_100()
m3
```

```
##         [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9]
##   [1,] 0.44 0.79 0.84 0.96 0.98 1.00 1.00 1.00 1.00
##   [2,] 0.63 0.82 0.89 0.94 0.97 0.99 1.00 1.00 1.00
##   [3,] 0.59 0.83 0.86 0.93 0.97 0.99 0.99 1.00 1.00
##   [4,] 0.44 0.73 0.78 0.90 0.96 0.98 1.00 1.00 1.00
##   [5,] 0.49 0.78 0.83 0.91 0.95 0.98 0.98 1.00 1.00
##   [6,] 0.52 0.71 0.76 0.86 0.90 0.97 0.97 1.00 1.00
##   [7,] 0.56 0.76 0.81 0.87 0.95 0.98 0.99 1.00 1.00
##   [8,] 0.48 0.73 0.78 0.87 0.96 1.00 1.00 1.00 1.00
##   [9,] 0.58 0.80 0.85 0.97 0.98 1.00 1.00 1.00 1.00
##  [10,] 0.48 0.77 0.83 0.89 0.94 0.98 0.98 1.00 1.00
##  [11,] 0.49 0.74 0.80 0.90 0.94 0.98 0.98 1.00 1.00
##  [12,] 0.51 0.77 0.81 0.91 0.95 0.97 0.99 1.00 1.00
##  [13,] 0.55 0.80 0.83 0.86 0.91 0.99 0.99 0.99 0.99
##  [14,] 0.56 0.79 0.84 0.91 0.97 1.00 1.00 1.00 1.00
##  [15,] 0.48 0.71 0.79 0.91 0.95 1.00 1.00 1.00 1.00
##  [16,] 0.40 0.71 0.73 0.92 0.94 0.98 0.99 1.00 1.00
##  [17,] 0.50 0.78 0.84 0.93 0.98 1.00 1.00 1.00 1.00
##  [18,] 0.43 0.74 0.77 0.84 0.95 1.00 1.00 1.00 1.00
##  [19,] 0.46 0.72 0.76 0.89 0.94 0.97 0.99 0.99 1.00
##  [20,] 0.53 0.74 0.79 0.88 0.96 1.00 1.00 1.00 1.00
##  [21,] 0.59 0.77 0.82 0.92 0.97 0.99 0.99 0.99 1.00
##  [22,] 0.55 0.74 0.77 0.84 0.90 0.97 0.98 0.99 1.00
##  [23,] 0.47 0.74 0.77 0.92 0.96 1.00 1.00 1.00 1.00
##  [24,] 0.46 0.71 0.76 0.90 0.93 0.97 0.98 1.00 1.00
##  [25,] 0.53 0.83 0.86 0.94 0.97 0.99 1.00 1.00 1.00
##  [26,] 0.41 0.69 0.75 0.89 0.98 1.00 1.00 1.00 1.00
##  [27,] 0.43 0.74 0.79 0.93 0.96 0.99 1.00 1.00 1.00
##  [28,] 0.45 0.78 0.80 0.92 0.99 1.00 1.00 1.00 1.00
##  [29,] 0.48 0.76 0.81 0.87 0.96 0.99 0.99 1.00 1.00
##  [30,] 0.48 0.70 0.80 0.85 0.93 0.98 0.98 1.00 1.00
##  [31,] 0.59 0.74 0.81 0.94 0.98 1.00 1.00 1.00 1.00
```

```
##  [32,] 0.39 0.59 0.69 0.85 0.92 0.97 0.97 1.00 1.00
##  [33,] 0.55 0.76 0.81 0.89 0.93 0.97 0.97 1.00 1.00
##  [34,] 0.52 0.80 0.84 0.89 0.96 0.99 1.00 1.00 1.00
##  [35,] 0.52 0.75 0.82 0.86 0.94 0.99 1.00 1.00 1.00
##  [36,] 0.48 0.73 0.79 0.93 0.97 0.99 1.00 1.00 1.00
##  [37,] 0.55 0.80 0.83 0.92 0.94 0.98 0.99 1.00 1.00
##  [38,] 0.49 0.69 0.76 0.86 0.92 0.98 0.99 1.00 1.00
##  [39,] 0.50 0.66 0.73 0.85 0.96 0.99 0.99 0.99 1.00
##  [40,] 0.40 0.69 0.75 0.87 0.95 1.00 1.00 1.00 1.00
##  [41,] 0.47 0.71 0.76 0.92 0.96 1.00 1.00 1.00 1.00
##  [42,] 0.48 0.75 0.81 0.92 0.96 1.00 1.00 1.00 1.00
##  [43,] 0.48 0.74 0.74 0.89 0.94 0.98 0.99 1.00 1.00
##  [44,] 0.48 0.71 0.75 0.86 0.92 0.98 0.98 1.00 1.00
##  [45,] 0.59 0.82 0.86 0.95 0.97 1.00 1.00 1.00 1.00
##  [46,] 0.41 0.72 0.78 0.89 0.96 1.00 1.00 1.00 1.00
##  [47,] 0.42 0.66 0.69 0.82 0.93 0.97 0.99 1.00 1.00
##  [48,] 0.46 0.68 0.75 0.86 0.93 0.97 0.98 1.00 1.00
##  [49,] 0.47 0.63 0.68 0.86 0.94 0.99 0.99 1.00 1.00
##  [50,] 0.52 0.81 0.86 0.90 0.98 1.00 1.00 1.00 1.00
##  [51,] 0.44 0.69 0.72 0.85 0.96 0.98 0.99 1.00 1.00
##  [52,] 0.51 0.73 0.76 0.90 0.97 0.99 0.99 1.00 1.00
##  [53,] 0.55 0.72 0.79 0.90 0.93 0.98 0.98 1.00 1.00
##  [54,] 0.52 0.71 0.75 0.88 0.93 1.00 1.00 1.00 1.00
##  [55,] 0.52 0.72 0.77 0.90 0.95 0.99 1.00 1.00 1.00
##  [56,] 0.47 0.76 0.82 0.91 0.98 0.99 1.00 1.00 1.00
##  [57,] 0.54 0.80 0.82 0.92 0.94 1.00 1.00 1.00 1.00
##  [58,] 0.47 0.73 0.79 0.90 0.95 0.98 1.00 1.00 1.00
##  [59,] 0.45 0.72 0.79 0.89 0.94 0.99 0.99 1.00 1.00
##  [60,] 0.46 0.65 0.73 0.83 0.94 0.99 1.00 1.00 1.00
##  [61,] 0.52 0.76 0.82 0.93 0.98 1.00 1.00 1.00 1.00
##  [62,] 0.50 0.74 0.80 0.90 0.91 0.99 0.99 1.00 1.00
##  [63,] 0.54 0.76 0.80 0.85 0.93 0.99 0.99 0.99 1.00
##  [64,] 0.50 0.65 0.70 0.81 0.89 0.98 0.99 1.00 1.00
##  [65,] 0.45 0.76 0.83 0.88 0.93 0.99 0.99 1.00 1.00
##  [66,] 0.51 0.72 0.79 0.90 0.95 0.99 0.99 1.00 1.00
##  [67,] 0.52 0.76 0.86 0.92 0.98 0.99 1.00 1.00 1.00
##  [68,] 0.46 0.77 0.80 0.90 0.96 0.99 1.00 1.00 1.00
##  [69,] 0.51 0.74 0.84 0.90 0.96 0.99 1.00 1.00 1.00
##  [70,] 0.50 0.71 0.77 0.87 0.97 1.00 1.00 1.00 1.00
##  [71,] 0.54 0.75 0.80 0.92 0.97 0.99 0.99 0.99 0.99
```

```
##  [72,] 0.64 0.84 0.89 0.95 0.99 1.00 1.00 1.00 1.00
##  [73,] 0.49 0.70 0.75 0.85 0.92 0.99 1.00 1.00 1.00
##  [74,] 0.55 0.74 0.82 0.95 0.99 0.99 1.00 1.00 1.00
##  [75,] 0.48 0.73 0.76 0.87 0.93 0.97 0.98 1.00 1.00
##  [76,] 0.56 0.75 0.79 0.90 0.96 1.00 1.00 1.00 1.00
##  [77,] 0.42 0.68 0.70 0.87 0.95 0.97 0.98 1.00 1.00
##  [78,] 0.59 0.78 0.82 0.92 0.95 1.00 1.00 1.00 1.00
##  [79,] 0.54 0.82 0.88 0.98 0.99 1.00 1.00 1.00 1.00
##  [80,] 0.50 0.74 0.82 0.92 0.96 0.98 0.99 1.00 1.00
##  [81,] 0.40 0.70 0.77 0.87 0.93 0.98 1.00 1.00 1.00
##  [82,] 0.42 0.81 0.85 0.94 0.98 1.00 1.00 1.00 1.00
##  [83,] 0.56 0.75 0.79 0.92 0.94 0.99 0.99 1.00 1.00
##  [84,] 0.45 0.69 0.75 0.85 0.95 1.00 1.00 1.00 1.00
##  [85,] 0.51 0.71 0.79 0.90 0.97 1.00 1.00 1.00 1.00
##  [86,] 0.54 0.73 0.76 0.87 0.93 0.98 0.98 1.00 1.00
##  [87,] 0.47 0.69 0.77 0.83 0.92 0.99 0.99 1.00 1.00
##  [88,] 0.61 0.84 0.90 0.95 0.96 0.99 1.00 1.00 1.00
##  [89,] 0.55 0.76 0.79 0.89 0.92 0.97 0.98 1.00 1.00
##  [90,] 0.48 0.78 0.85 0.93 0.97 1.00 1.00 1.00 1.00
##  [91,] 0.51 0.77 0.79 0.91 0.95 1.00 1.00 1.00 1.00
##  [92,] 0.55 0.76 0.82 0.97 0.99 0.99 1.00 1.00 1.00
##  [93,] 0.52 0.81 0.84 0.91 0.97 0.99 1.00 1.00 1.00
##  [94,] 0.49 0.73 0.80 0.87 0.95 0.99 1.00 1.00 1.00
##  [95,] 0.56 0.75 0.80 0.88 0.92 0.98 0.98 0.99 0.99
##  [96,] 0.57 0.75 0.80 0.88 0.94 0.99 1.00 1.00 1.00
##  [97,] 0.49 0.74 0.79 0.90 0.96 0.98 0.99 1.00 1.00
##  [98,] 0.45 0.66 0.73 0.86 0.93 0.98 1.00 1.00 1.00
##  [99,] 0.43 0.65 0.75 0.83 0.97 1.00 1.00 1.00 1.00
## [100,] 0.49 0.67 0.74 0.85 0.95 1.00 1.00 1.00 1.00
```

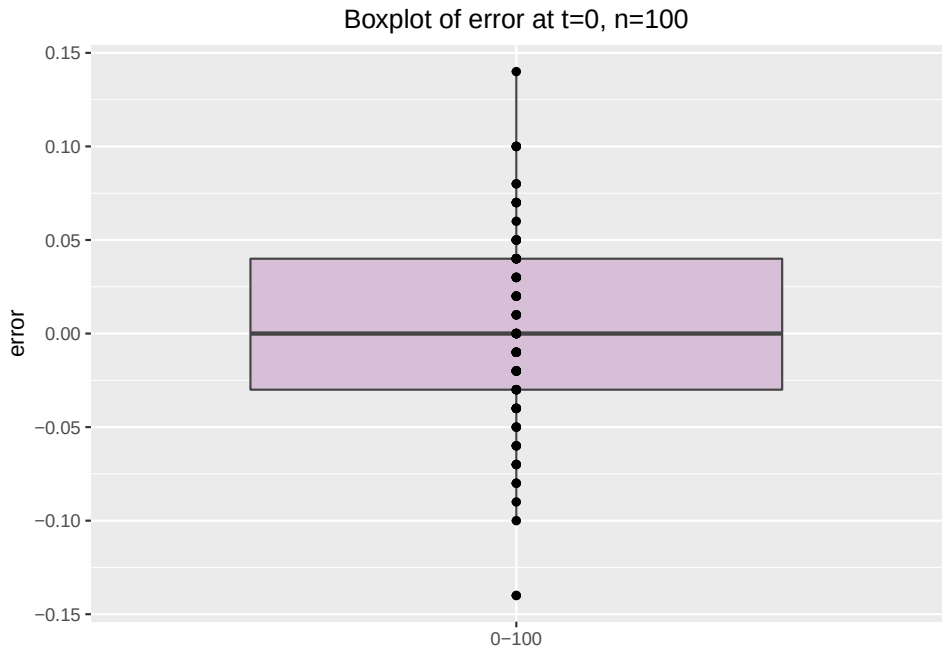
the 100 approximation errors

```
n=100
e100_1=m1[,1]-c(rep(pnorm(0,0,1),100))
e100_2=m1[,2]-c(rep(pnorm(0.67,0,1),100))
e100_3=m1[,3]-c(rep(pnorm(0.84,0,1),100))
e100_4=m1[,4]-c(rep(pnorm(1.28,0,1),100))
e100_5=m1[,5]-c(rep(pnorm(1.65,0,1),100))
e100_6=m1[,6]-c(rep(pnorm(2.32,0,1),100))
e100_7=m1[,7]-c(rep(pnorm(2.58,0,1),100))
e100_8=m1[,8]-c(rep(pnorm(3.09,0,1),100))
```

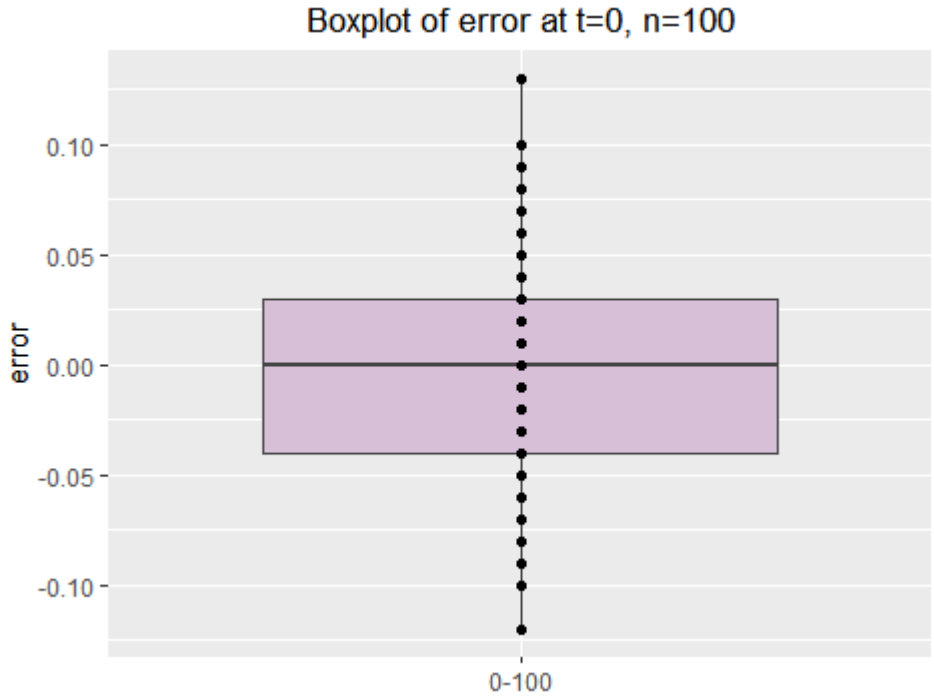
```
e100_9=m1[,9]-c(rep(pnorm(3.72,0,1),100))

n=1000
e1000_1=m2[,1]-c(rep(pnorm(0,0,1),100))
e1000_2=m2[,2]-c(rep(pnorm(0.67,0,1),100))
e1000_3=m2[,3]-c(rep(pnorm(0.84,0,1),100))
e1000_4=m2[,4]-c(rep(pnorm(1.28,0,1),100))
e1000_5=m2[,5]-c(rep(pnorm(1.65,0,1),100))
e1000_6=m2[,6]-c(rep(pnorm(2.32,0,1),100))
e1000_7=m2[,7]-c(rep(pnorm(2.58,0,1),100))
e1000_8=m2[,8]-c(rep(pnorm(3.09,0,1),100))
e1000_9=m2[,9]-c(rep(pnorm(3.72,0,1),100))

n=10000
e10000_1=m3[,1]-c(rep(pnorm(0,0,1),100))
e10000_2=m3[,2]-c(rep(pnorm(0.67,0,1),100))
e10000_3=m3[,3]-c(rep(pnorm(0.84,0,1),100))
e10000_4=m3[,4]-c(rep(pnorm(1.28,0,1),100))
e10000_5=m3[,5]-c(rep(pnorm(1.65,0,1),100))
e10000_6=m3[,6]-c(rep(pnorm(2.32,0,1),100))
e10000_7=m3[,7]-c(rep(pnorm(2.58,0,1),100))
e10000_8=m3[,8]-c(rep(pnorm(3.09,0,1),100))
e10000_9=m3[,9]-c(rep(pnorm(3.72,0,1),100))

A=cbind.data.frame(e100_1,e100_2,e100_3,e100_4,e100_5,e100_6,e100_7,e100_8,e100_9,e10
```

Box plots of the 100 approximation errors at each t using **ggplot2** [@R-ggplot2] for each n.

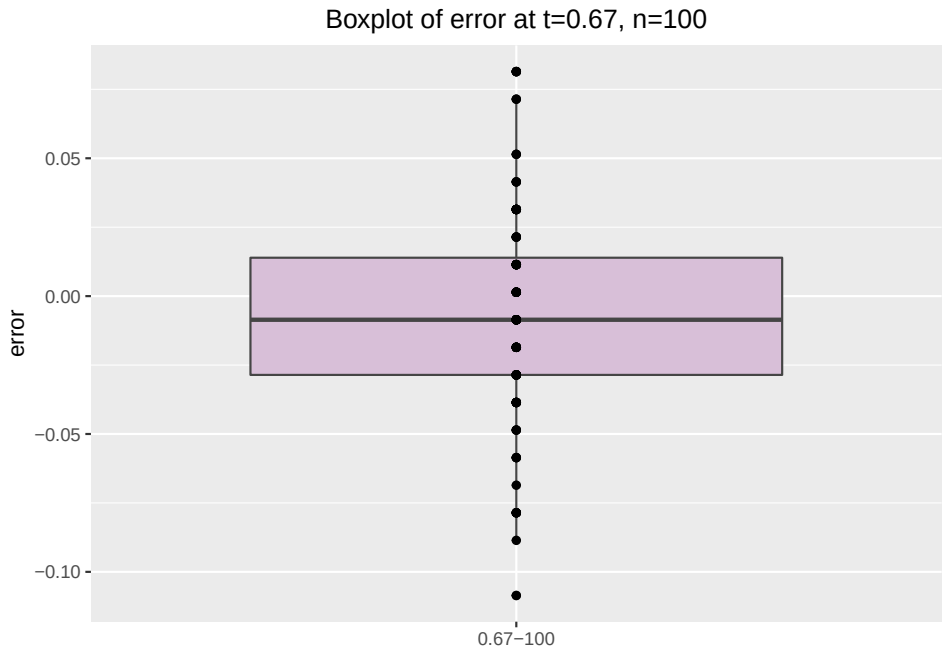
```
library(ggplot2)

plot1<-ggplot(data=A,aes(y=e100_1,x="0-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=0, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot1
```



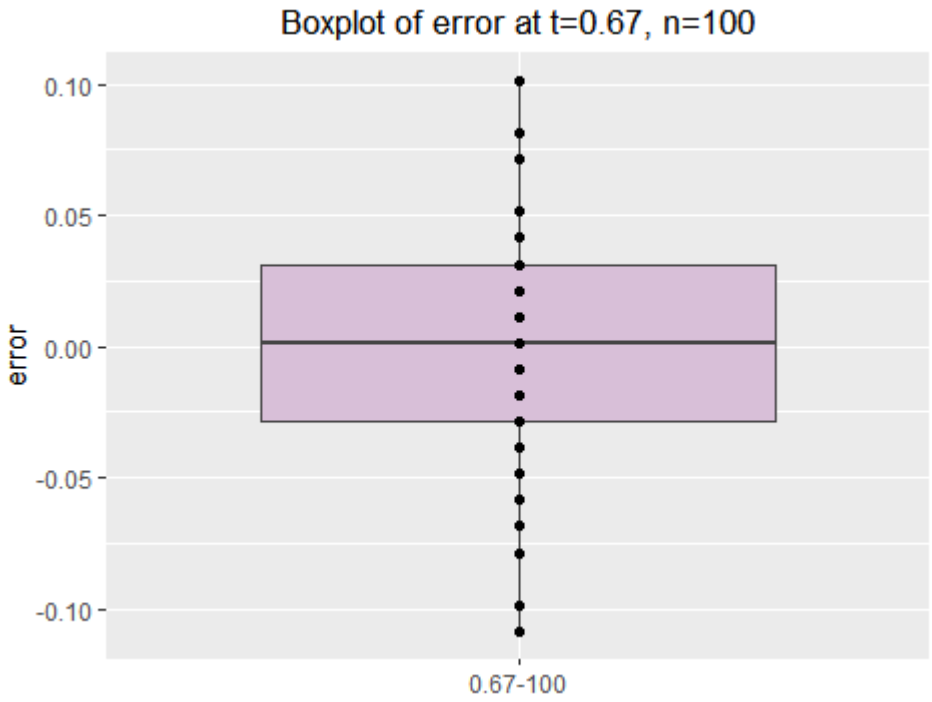

```
knitr::include_graphics("plot/Rplot1.png")
```



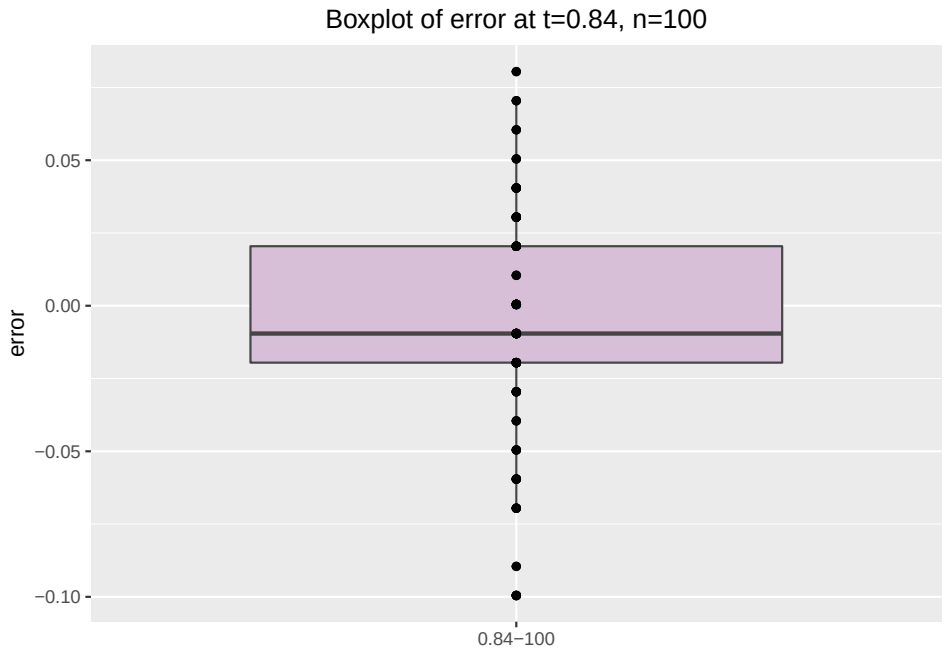

```
plot2<-ggplot(data=A,aes(y=e100_2,x="0.67-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=0.67, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot2
```



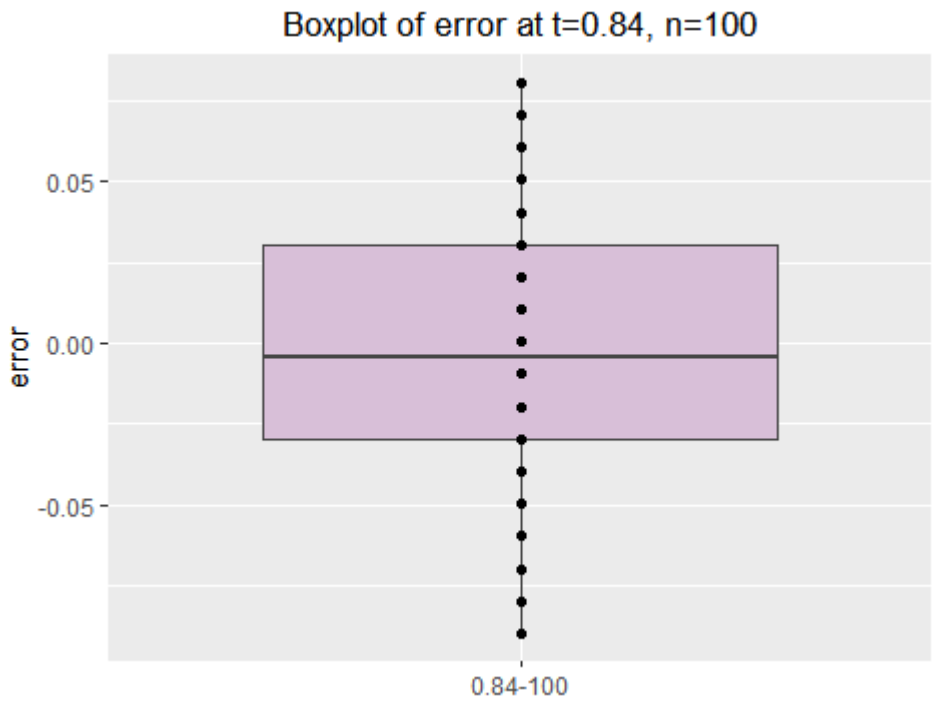

```
knitr::include_graphics("plot/Rplot2.png")
```



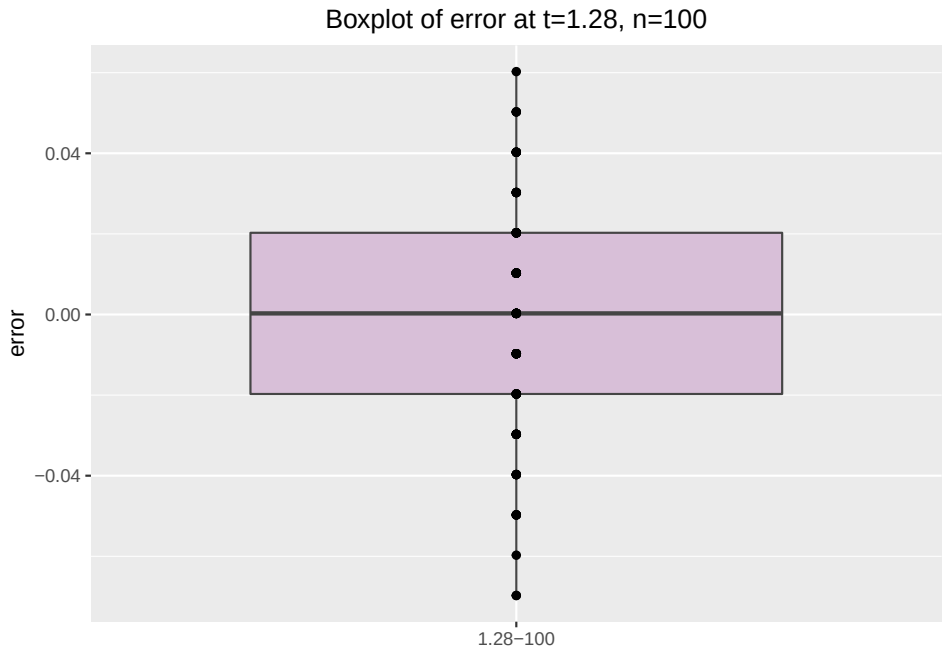

```
plot3<-ggplot(data=A,aes(y=e100_3,x="0.84-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=0.84, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot3
```



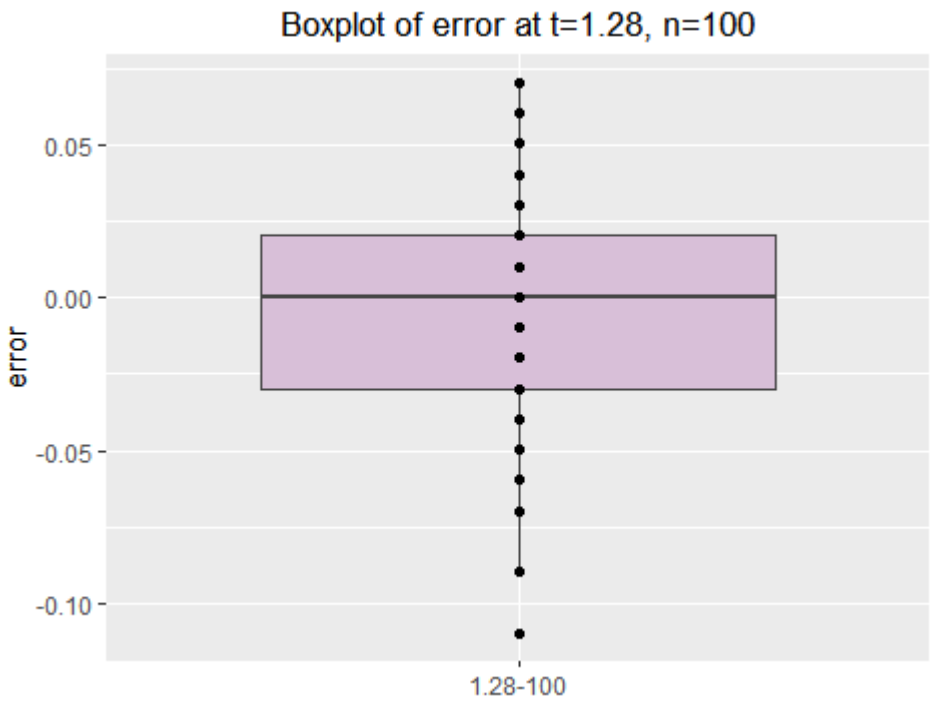

```
knitr::include_graphics("plot/Rplot3.png")
```



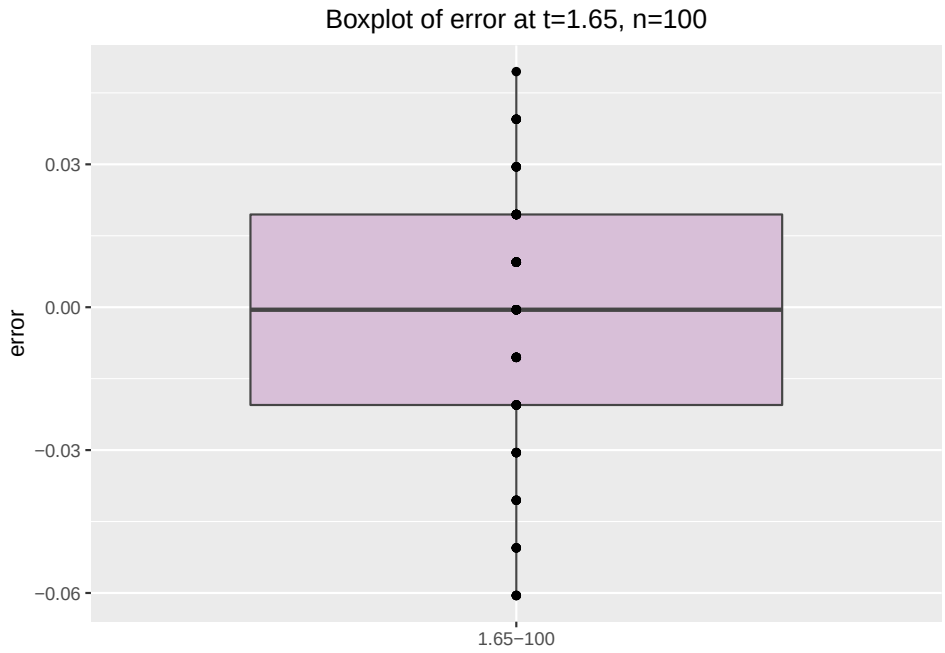

```
plot4<-ggplot(data=A,aes(y=e100_4,x="1.28-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=1.28, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot4
```



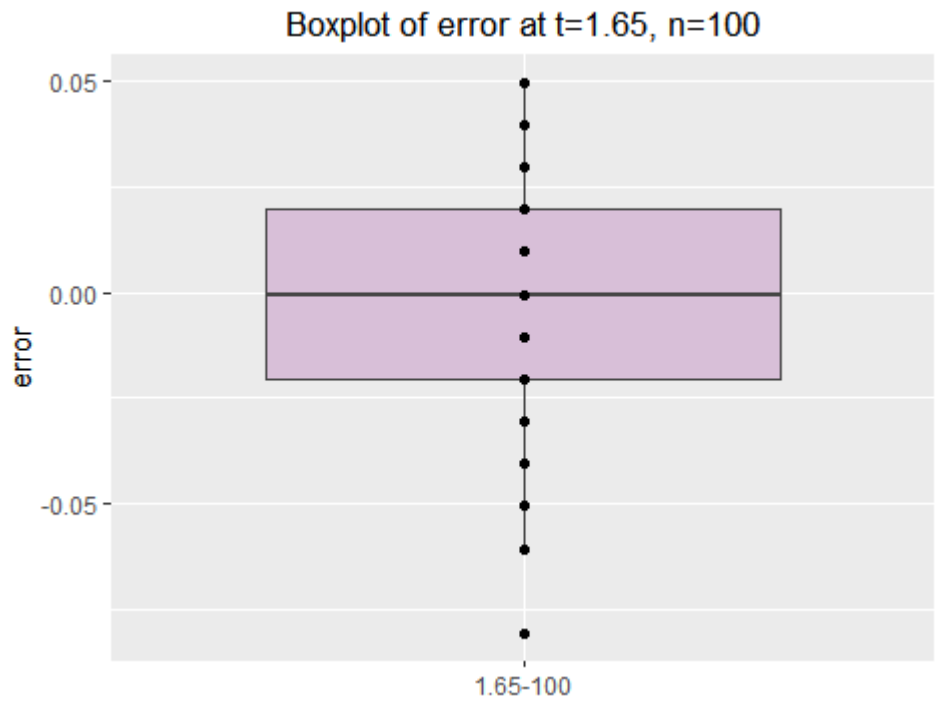

```
knitr::include_graphics("plot/Rplot4.png")
```



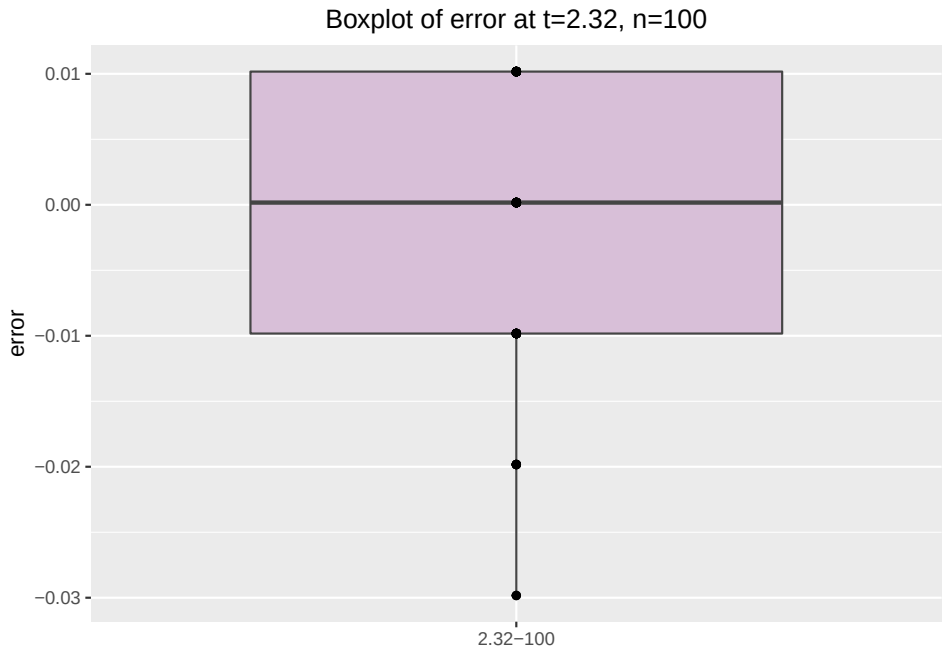

```
plot5<-ggplot(data=A,aes(y=e100_5,x="1.65-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=1.65, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot5
```



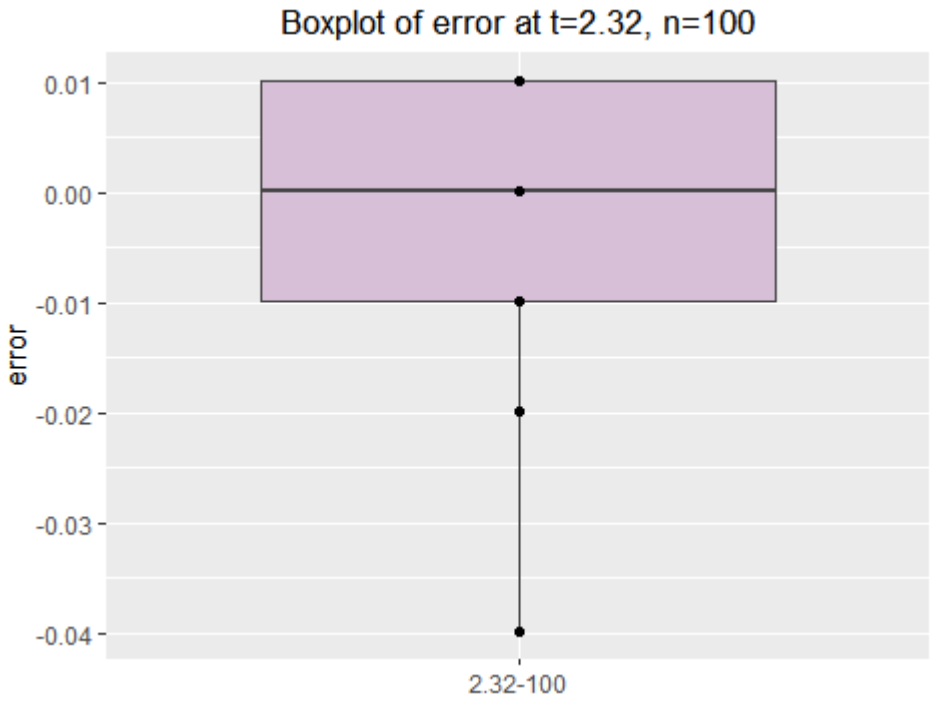

```
knitr::include_graphics("plot/Rplot5.png")
```



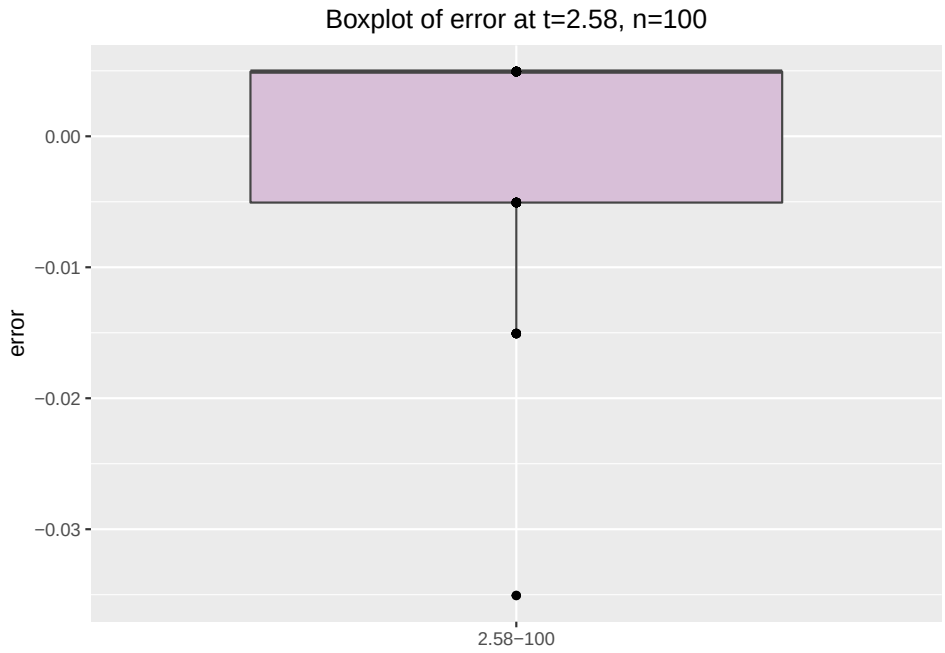

```
plot6<-ggplot(data=A,aes(y=e100_6,x="2.32-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=2.32, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot6
```



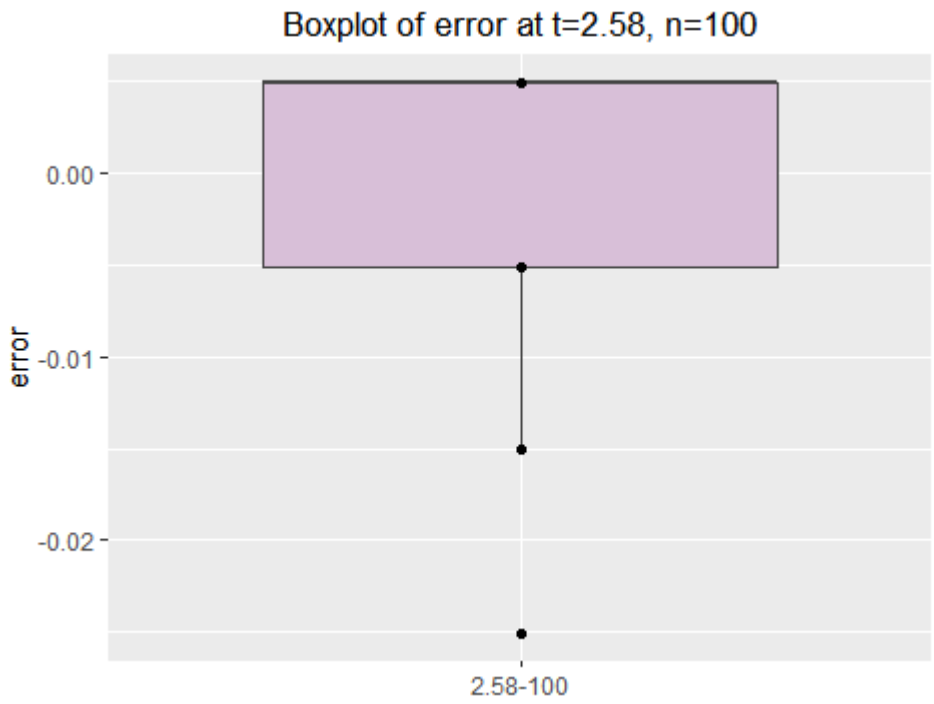

```
knitr::include_graphics("plot/Rplot6.png")
```



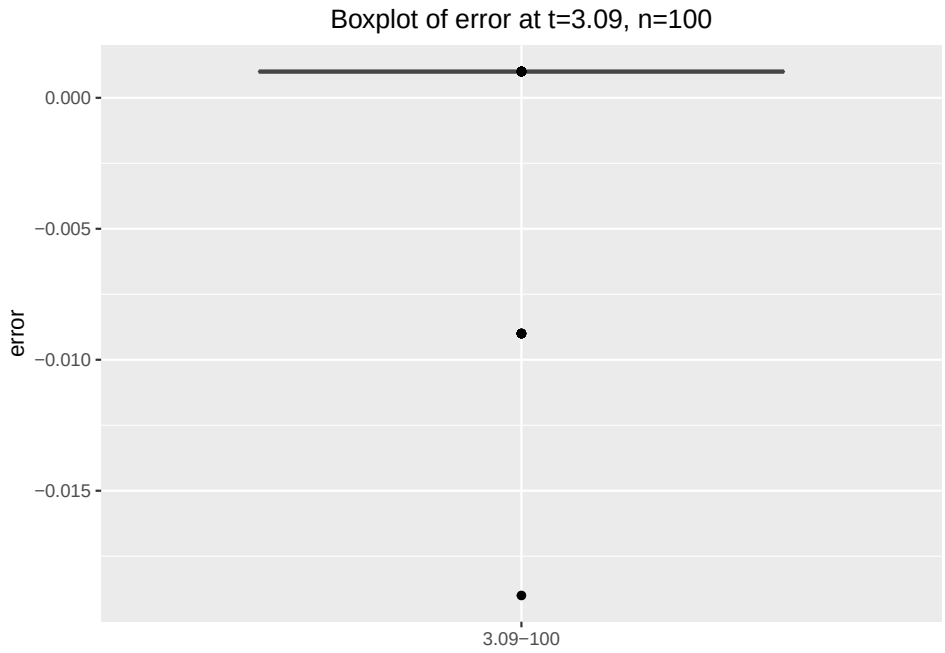

```
plot7<-ggplot(data=A,aes(y=e100_7,x="2.58-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=2.58, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot7
```



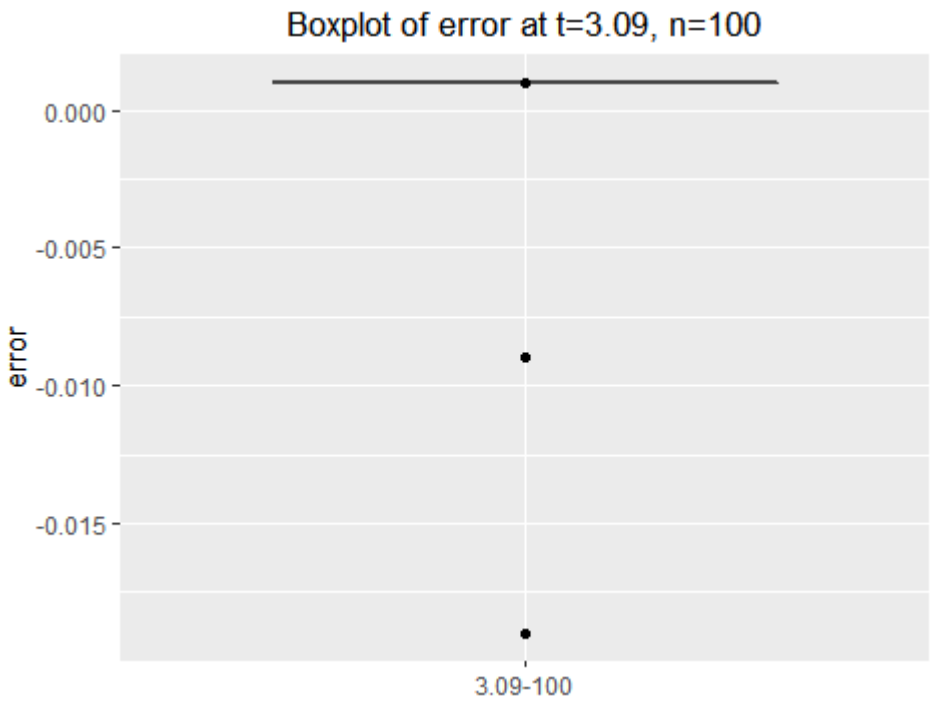

```
knitr::include_graphics("plot/Rplot7.png")
```



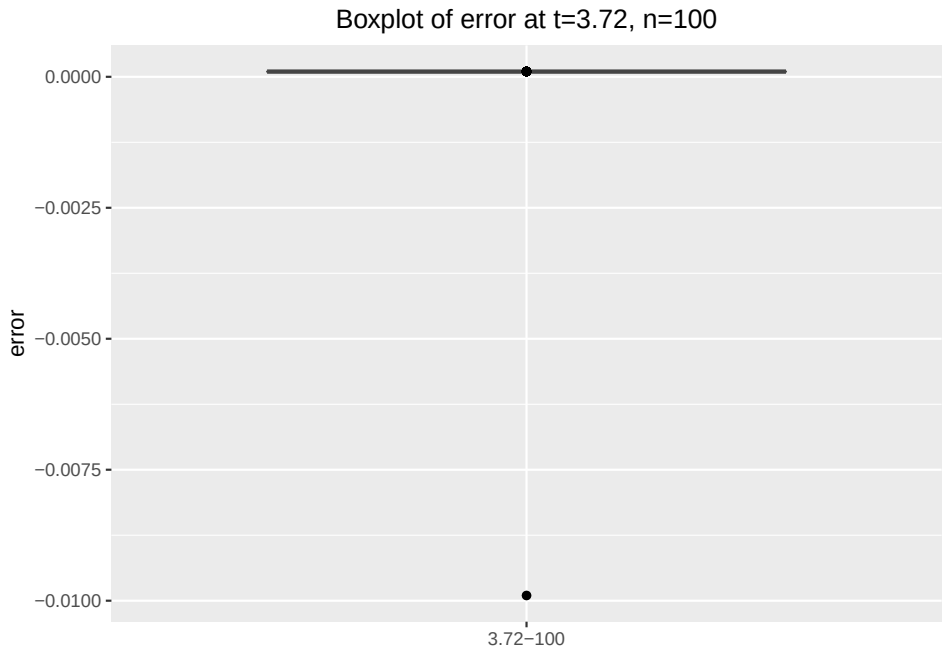

```
plot8<-ggplot(data=A,aes(y=e100_8,x="3.09-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=3.09, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot8
```



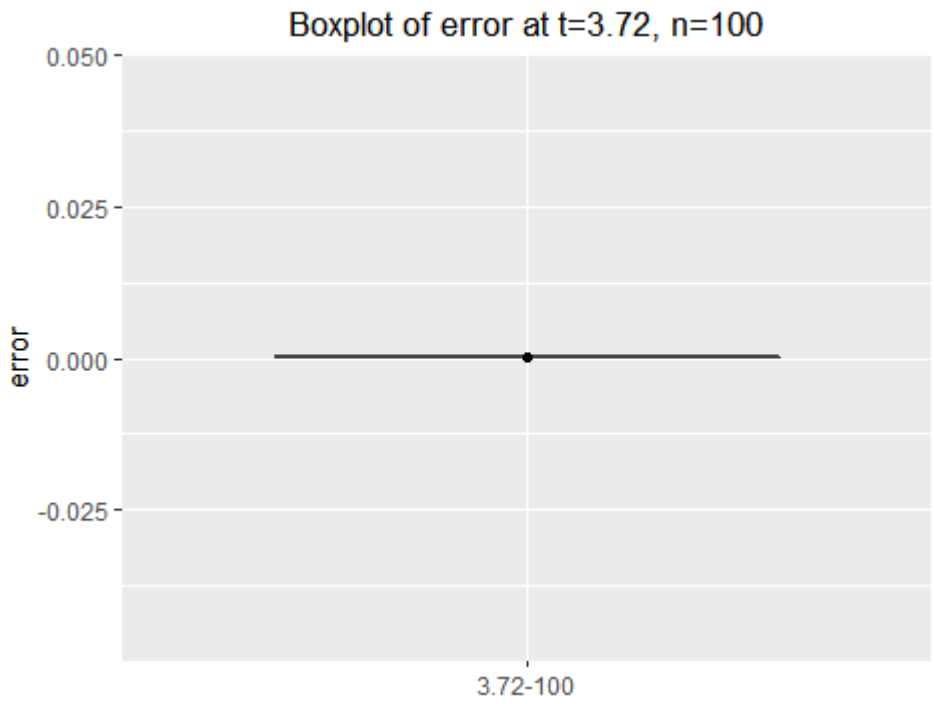

```
knitr::include_graphics("plot/Rplot8.png")
```



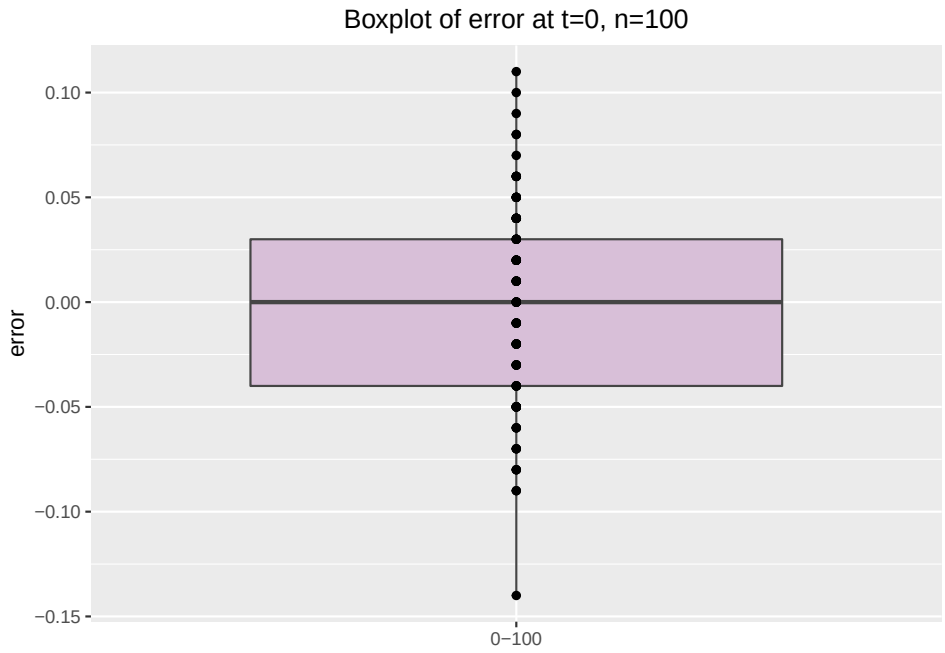

```
plot9<-ggplot(data=A,aes(y=e100_9,x="3.72-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=3.72, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot9
```



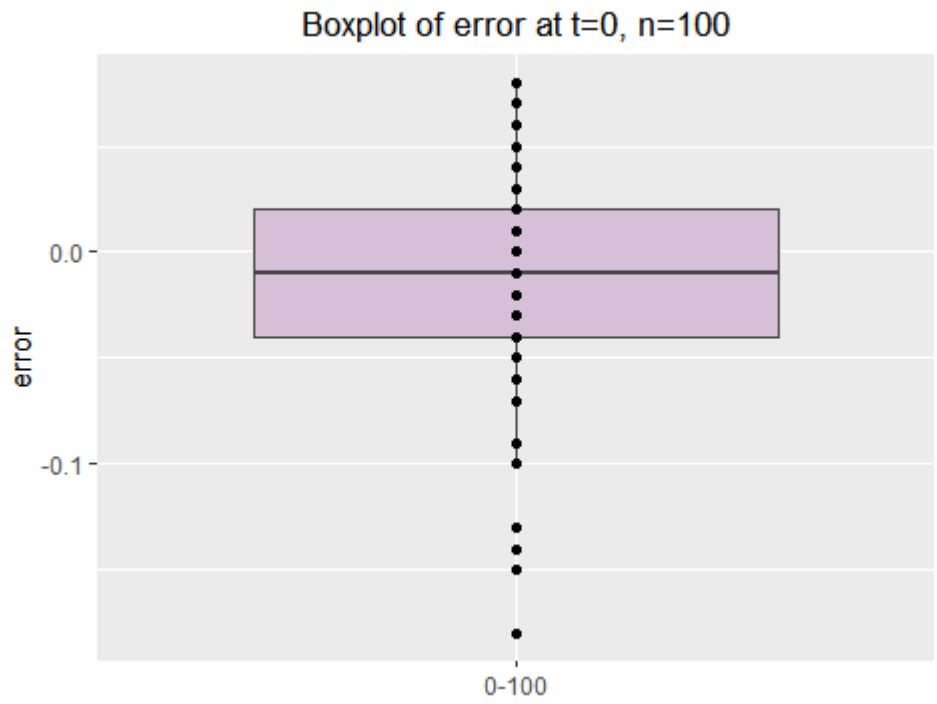

```
knitr::include_graphics("plot/Rplot9.png")
```



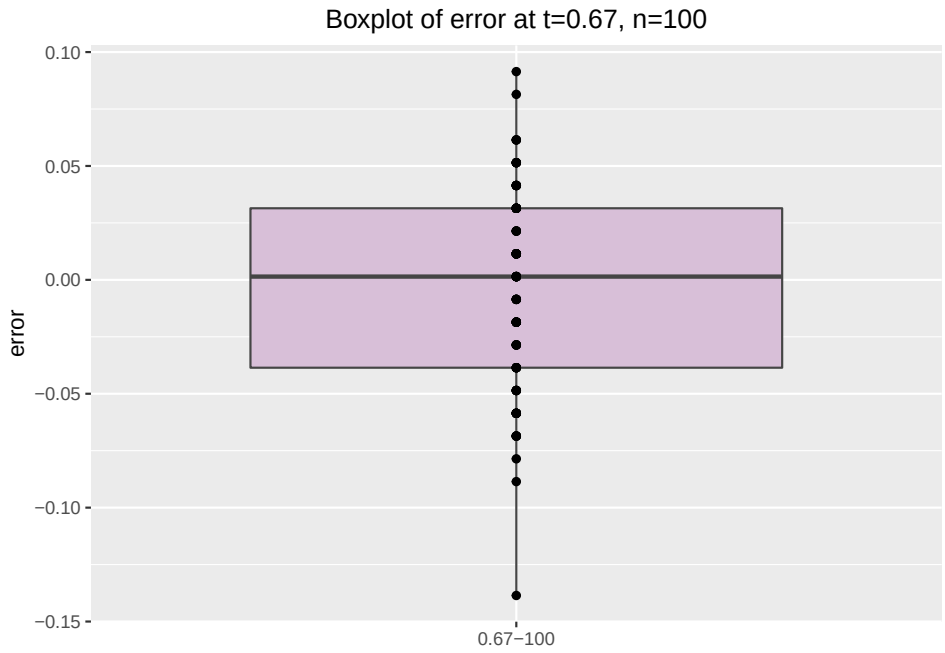

```
plot10<-ggplot(data=A,aes(y=e1000_1,x="0-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=0, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot10
```



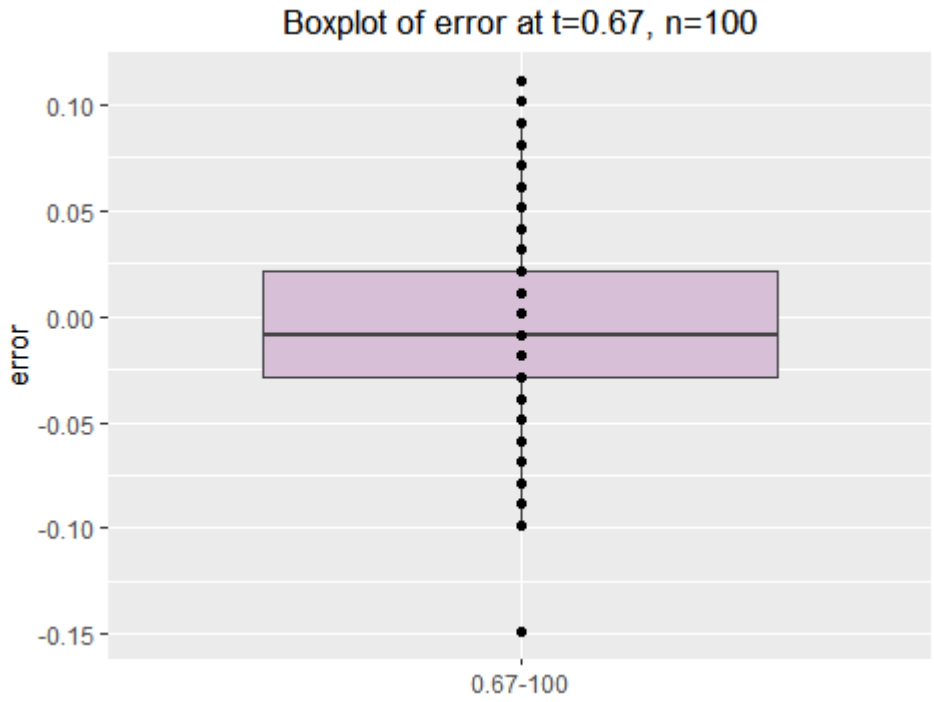

```
knitr::include_graphics("plot/Rplot10.png")
```



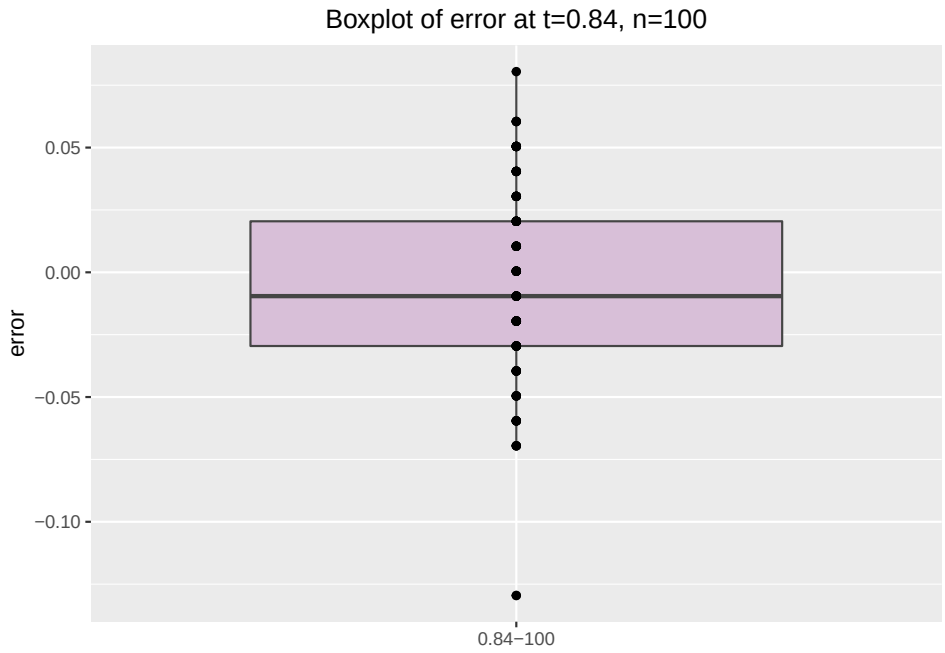

```
plot11<-ggplot(data=A,aes(y=e1000_2,x="0.67-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=0.67, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot11
```



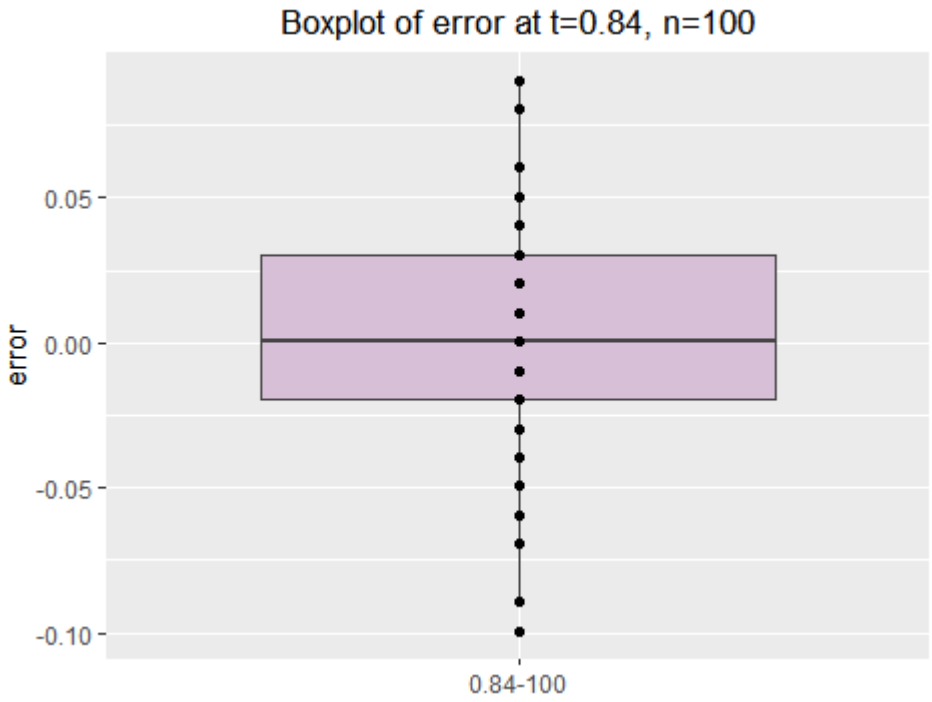

```
knitr::include_graphics("plot/Rplot11.png")
```



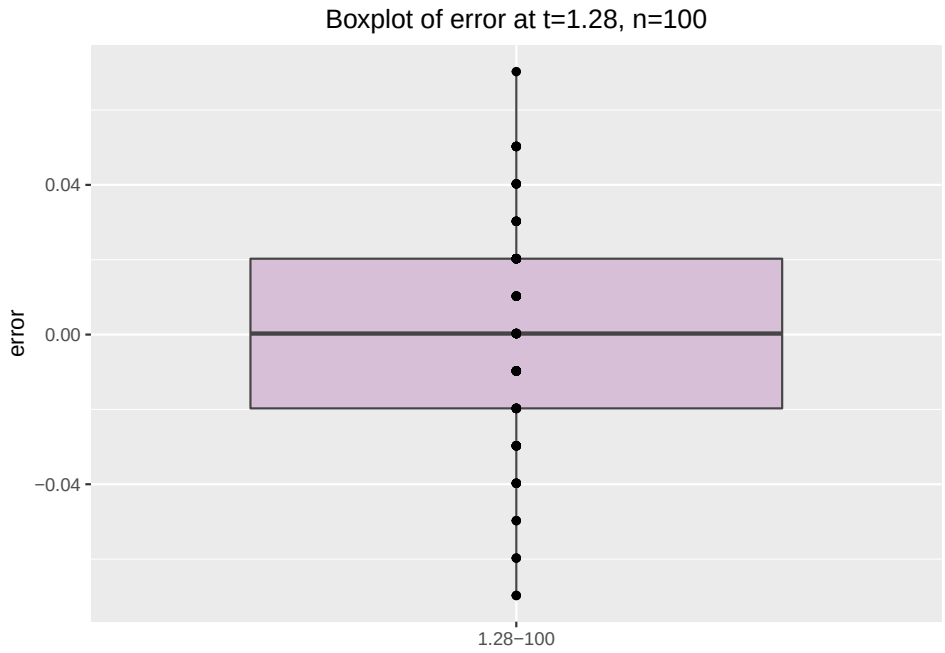

```
plot12<-ggplot(data=A,aes(y=e1000_3,x="0.84-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=0.84, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot12
```



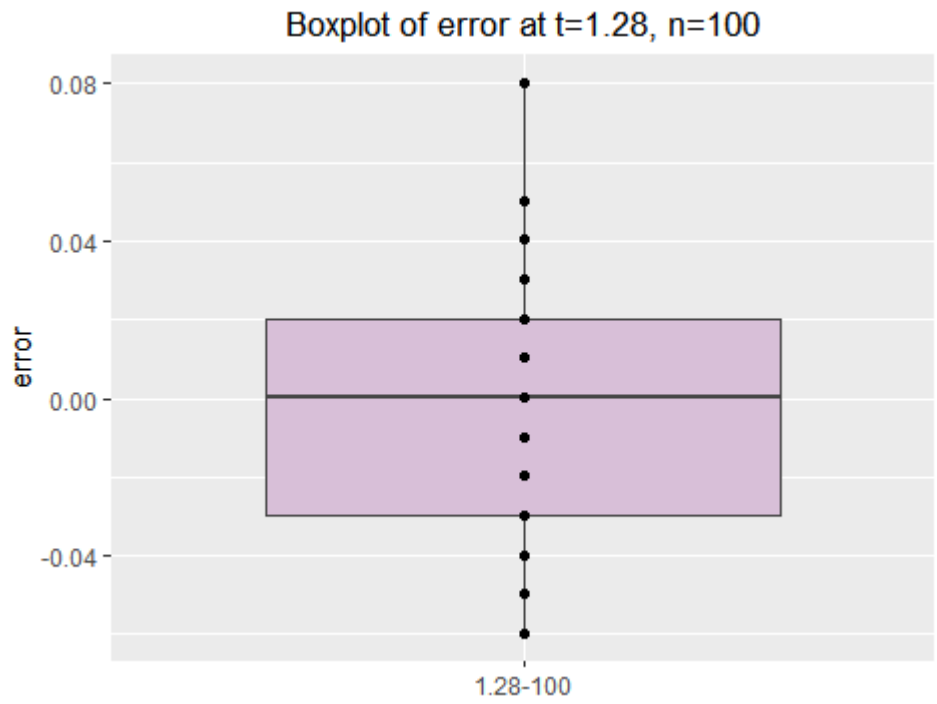

```
knitr::include_graphics("plot/Rplot12.png")
```



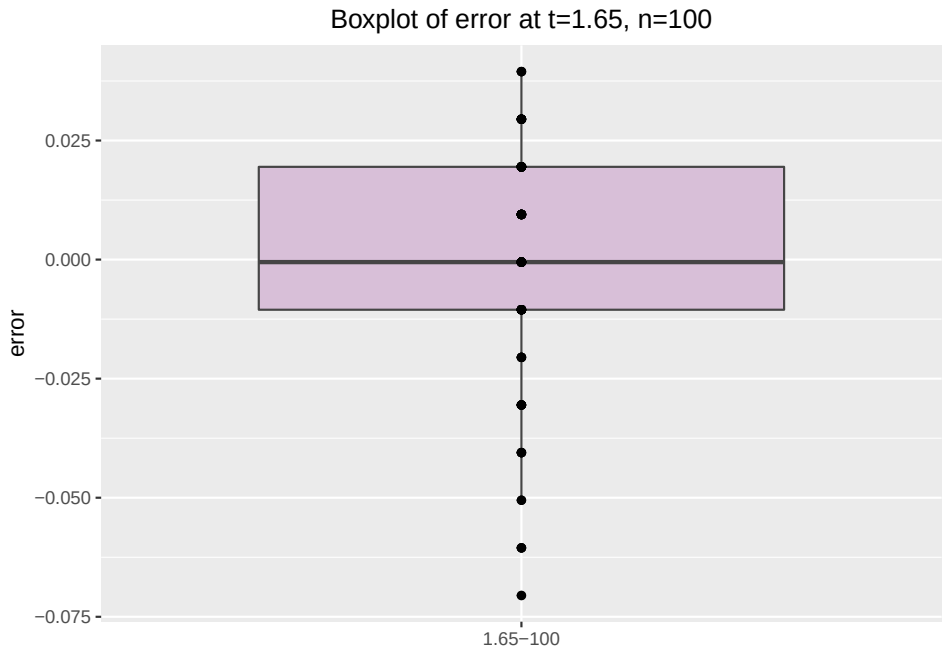

```
plot13<-ggplot(data=A,aes(y=e1000_4,x="1.28-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=1.28, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot13
```



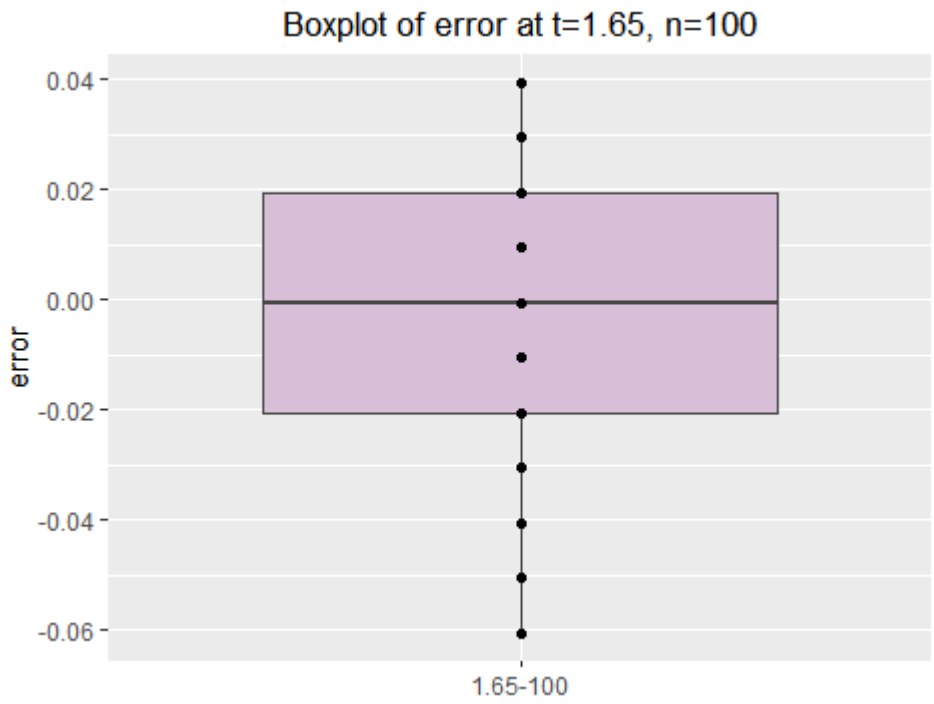

```
knitr::include_graphics("plot/Rplot13.png")
```



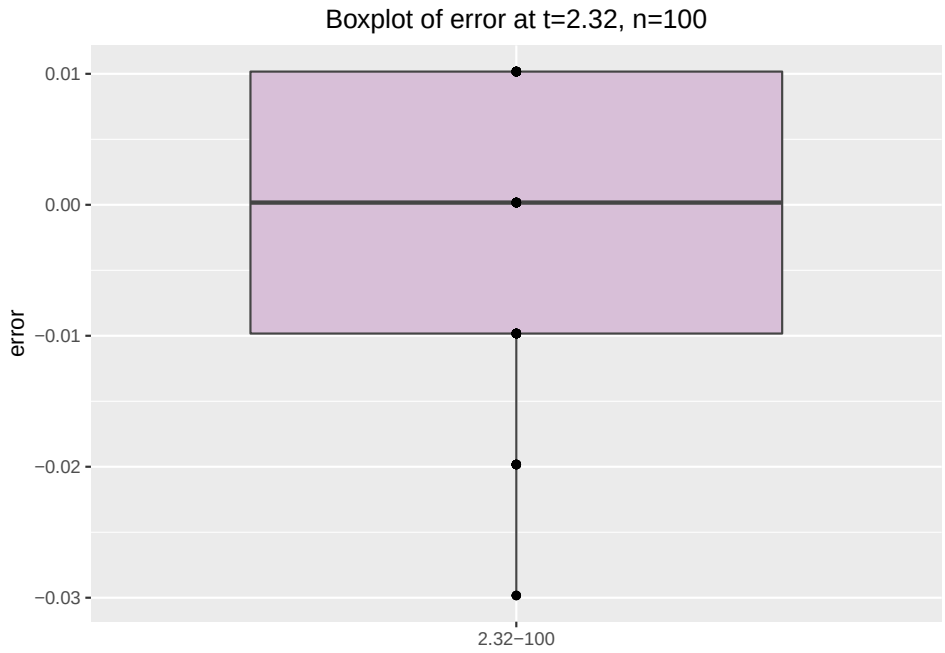

```
plot14<-ggplot(data=A,aes(y=e1000_5,x="1.65-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=1.65, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot14
```



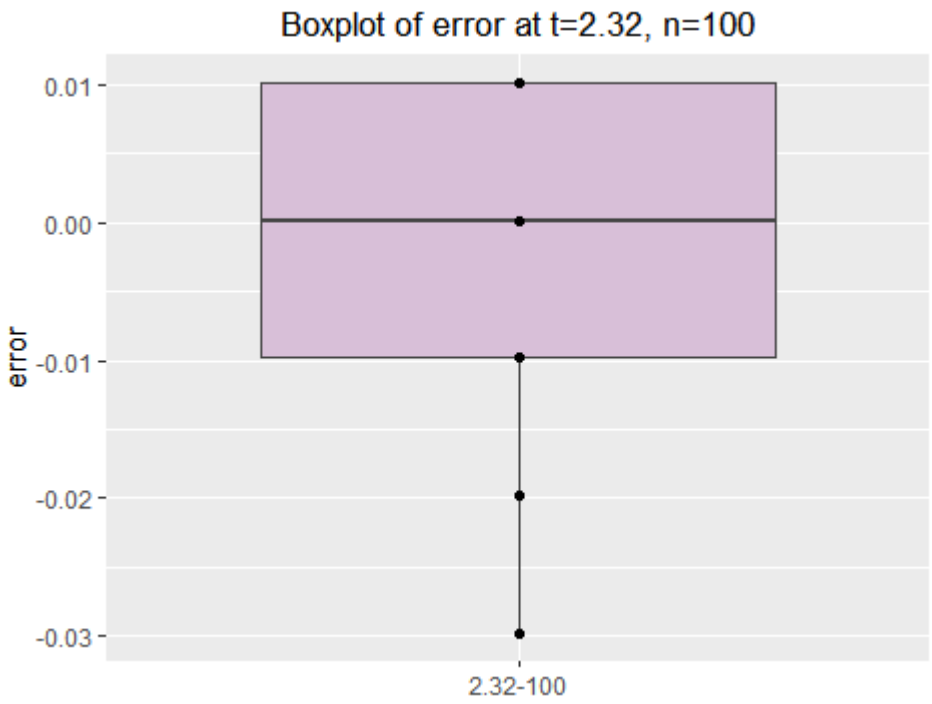

```
knitr::include_graphics("plot/Rplot14.png")
```



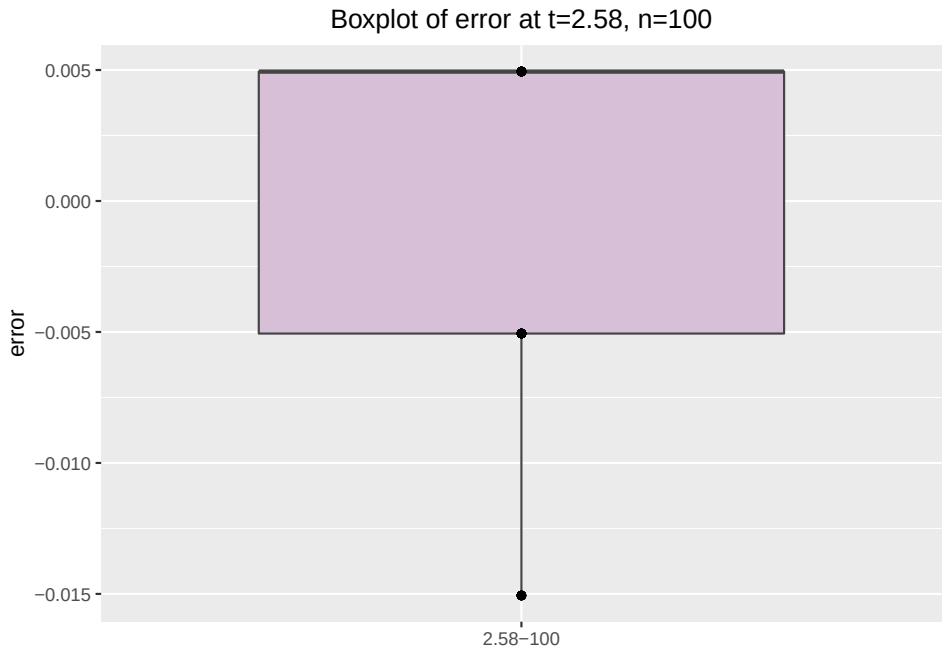

```
plot15<-ggplot(data=A,aes(y=e1000_6,x="2.32-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=2.32, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot15
```



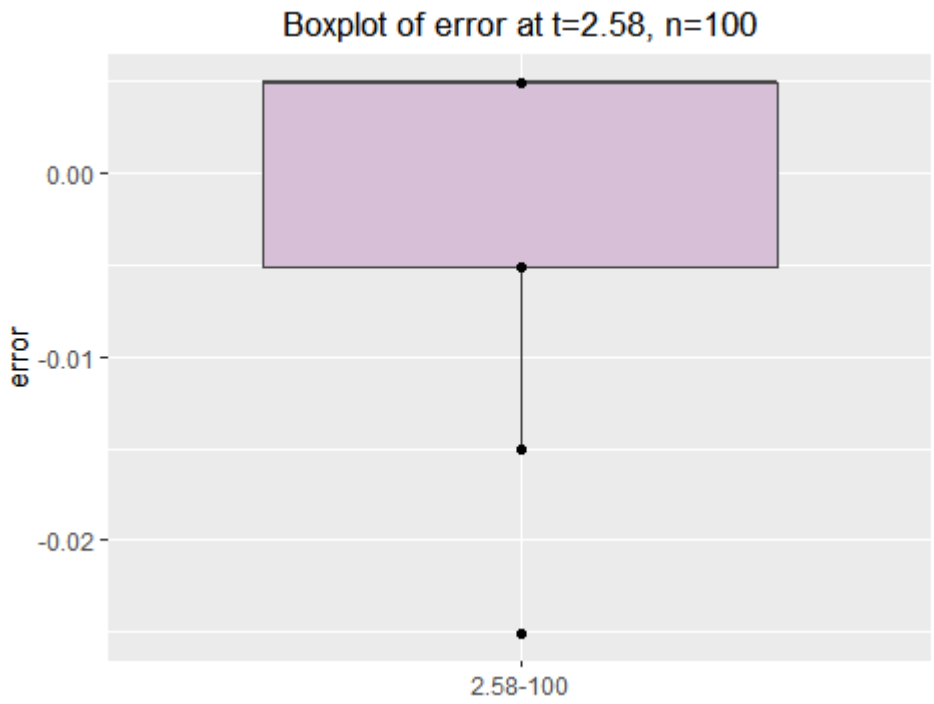

```
knitr::include_graphics("plot/Rplot15.png")
```



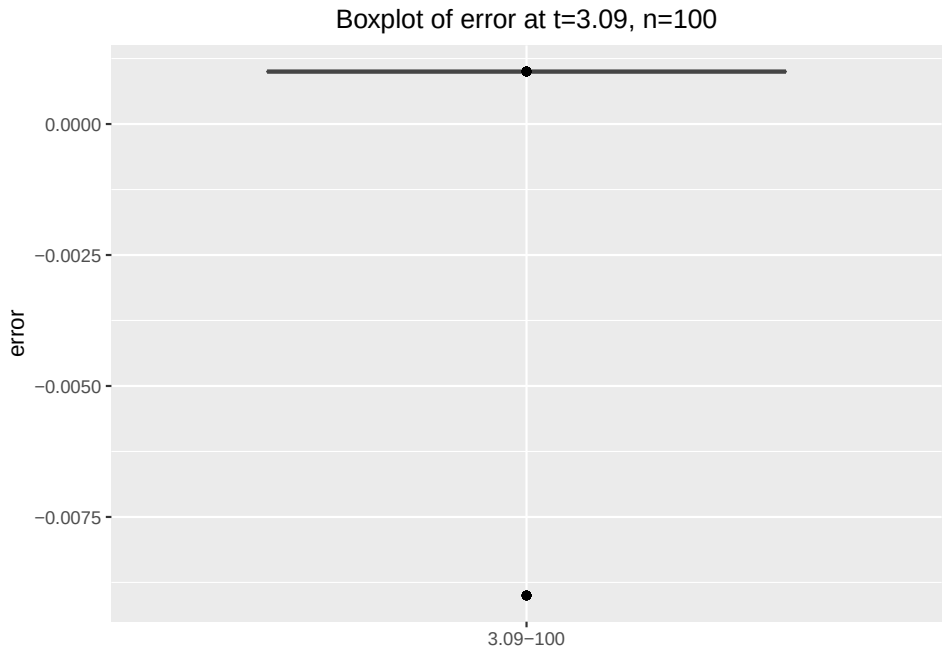

```
plot16<-ggplot(data=A,aes(y=e1000_7,x="2.58-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=2.58, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot16
```



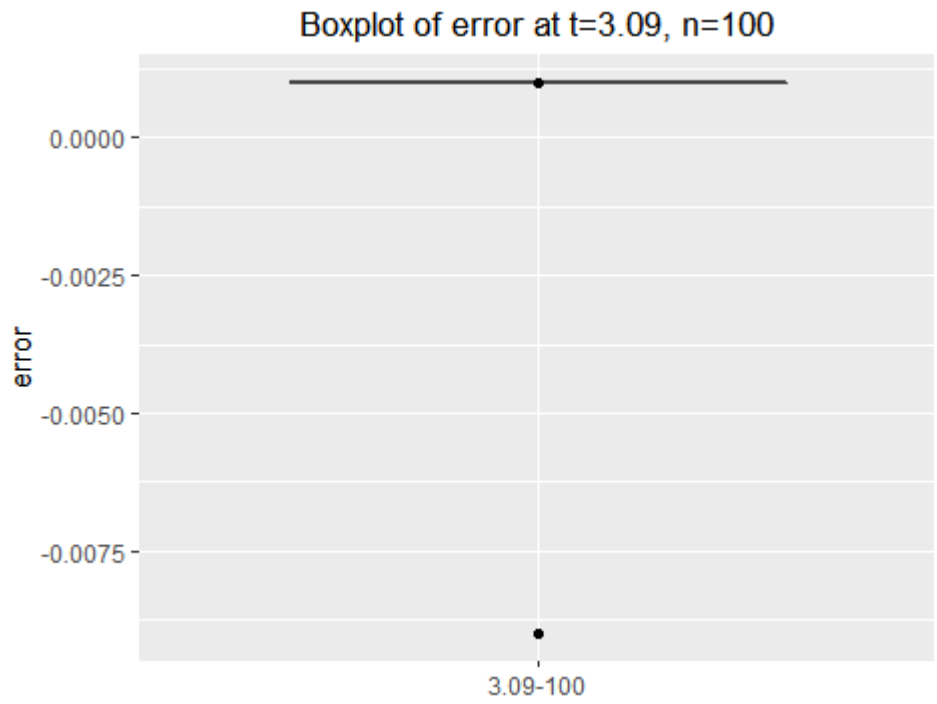

```
knitr::include_graphics("plot/Rplot16.png")
```



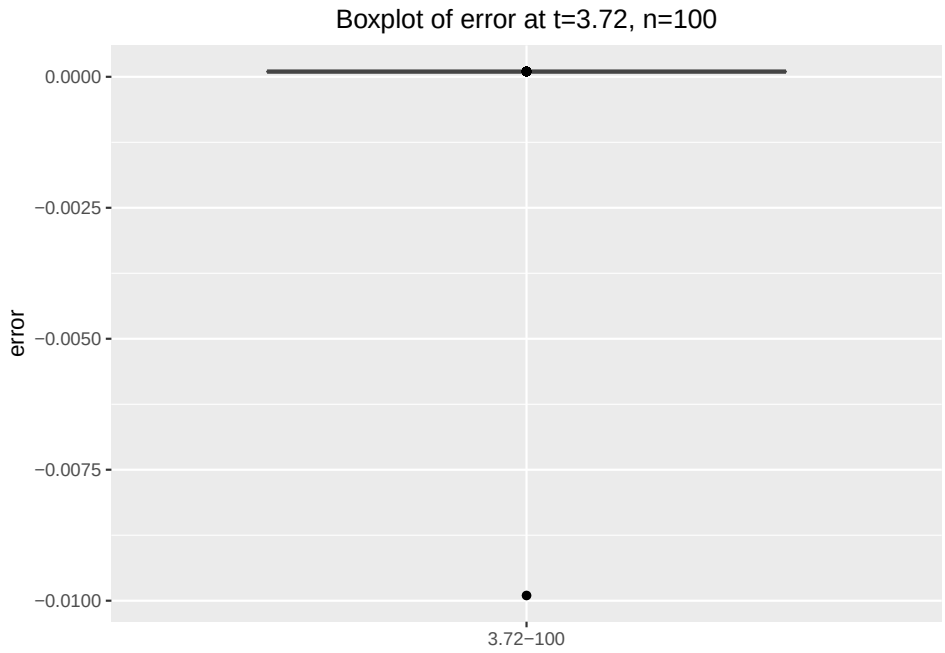

```
plot17<-ggplot(data=A,aes(y=e1000_8,x="3.09-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=3.09, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot17
```



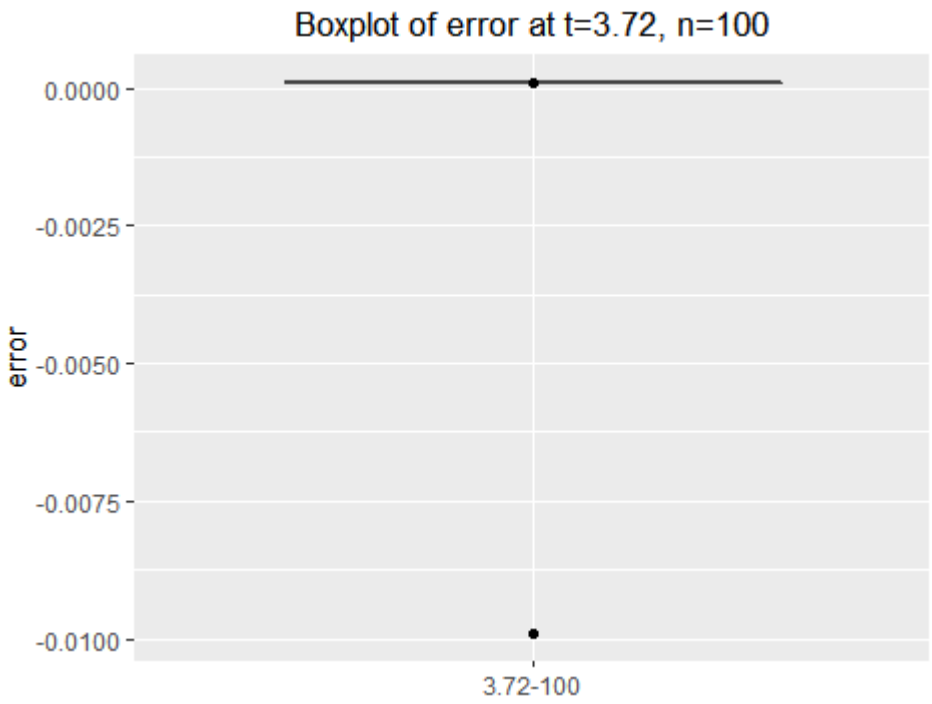

```
knitr::include_graphics("plot/Rplot17.png")
```



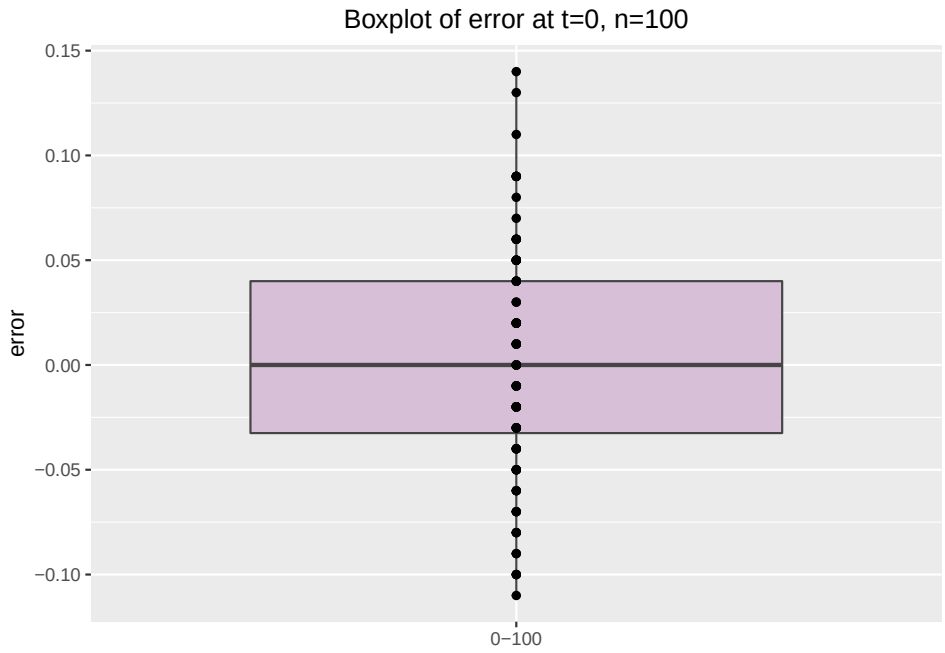

```
plot18<-ggplot(data=A,aes(y=e1000_9,x="3.72-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=3.72, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot18
```



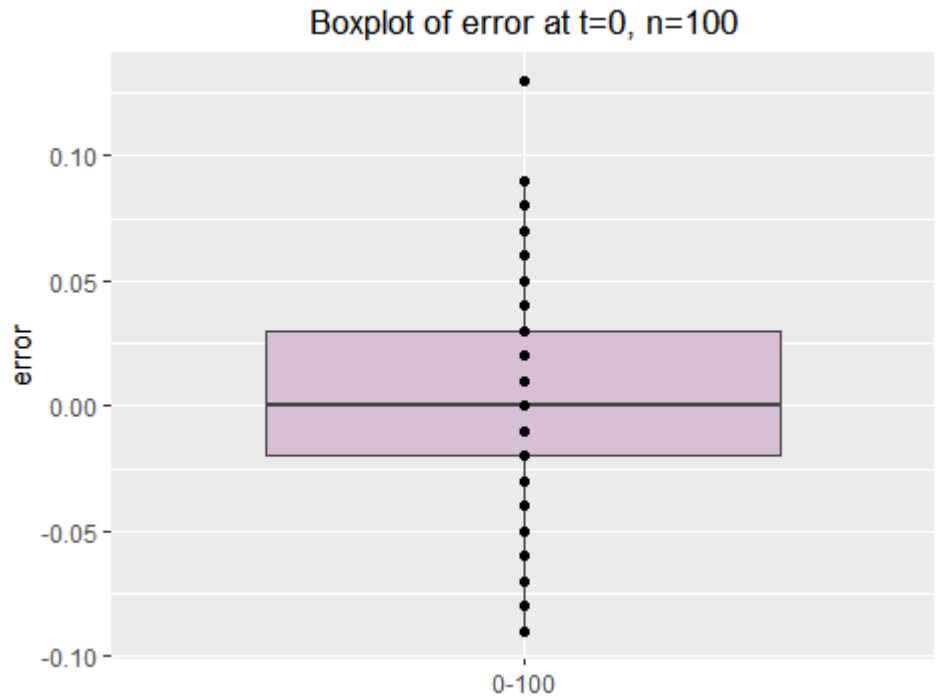

```
knitr::include_graphics("plot/Rplot18.png")
```



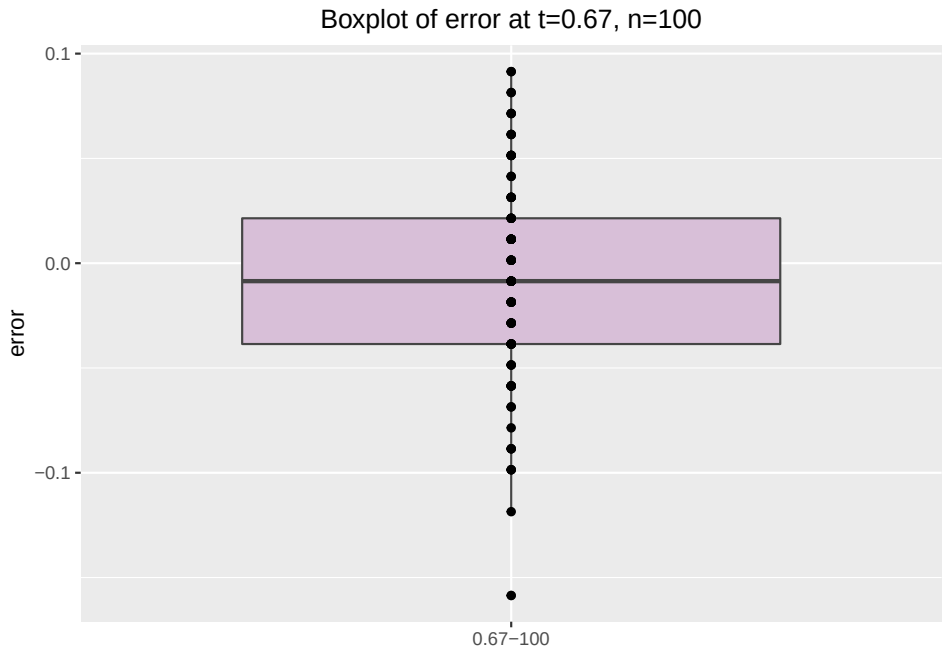

```
plot19<-ggplot(data=A,aes(y=e10000_1,x="0-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=0, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot19
```



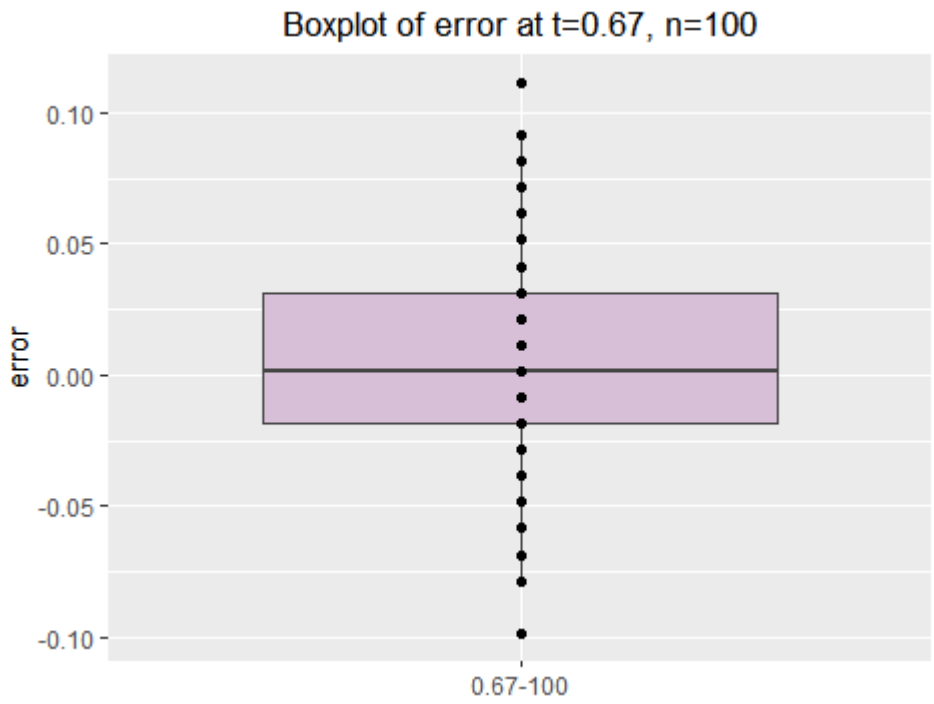

```
knitr::include_graphics("plot/Rplot19.png")
```



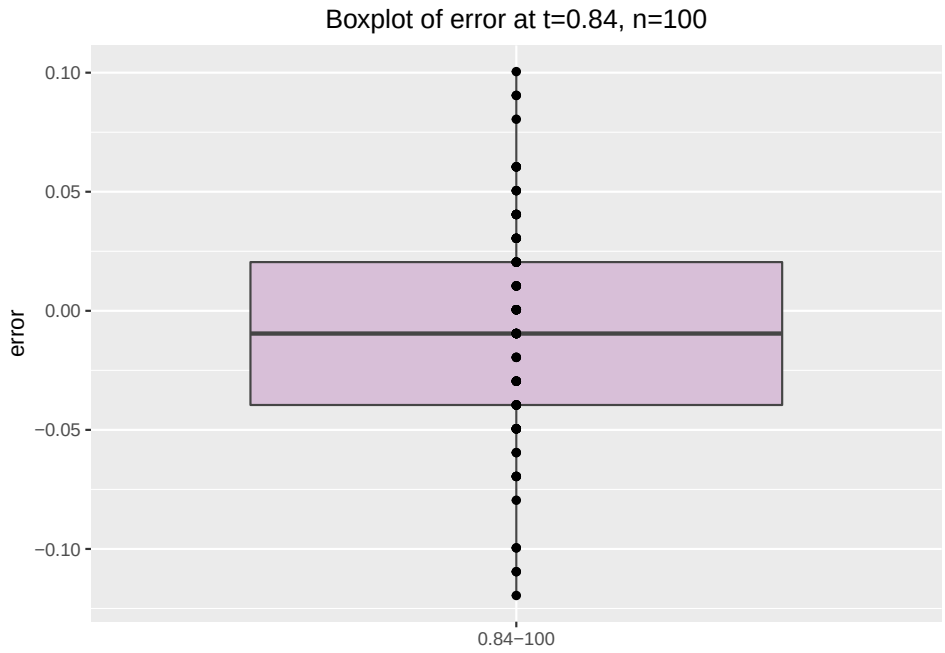

```
plot20<-ggplot(data=A,aes(y=e10000_2,x="0.67-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=0.67, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot20
```



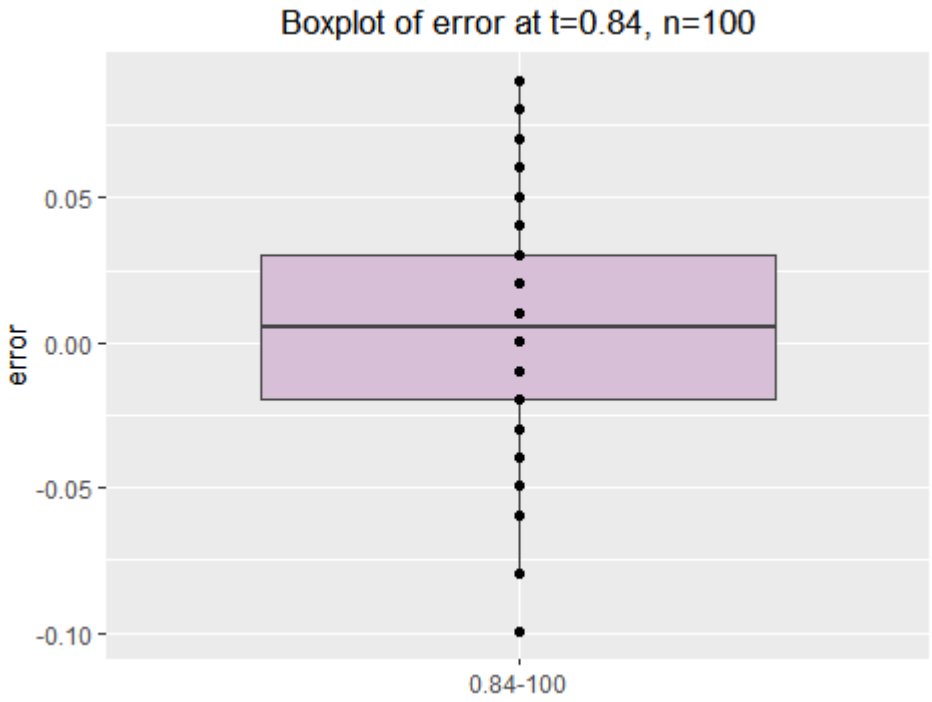

```
knitr::include_graphics("plot/Rplot20.png")
```



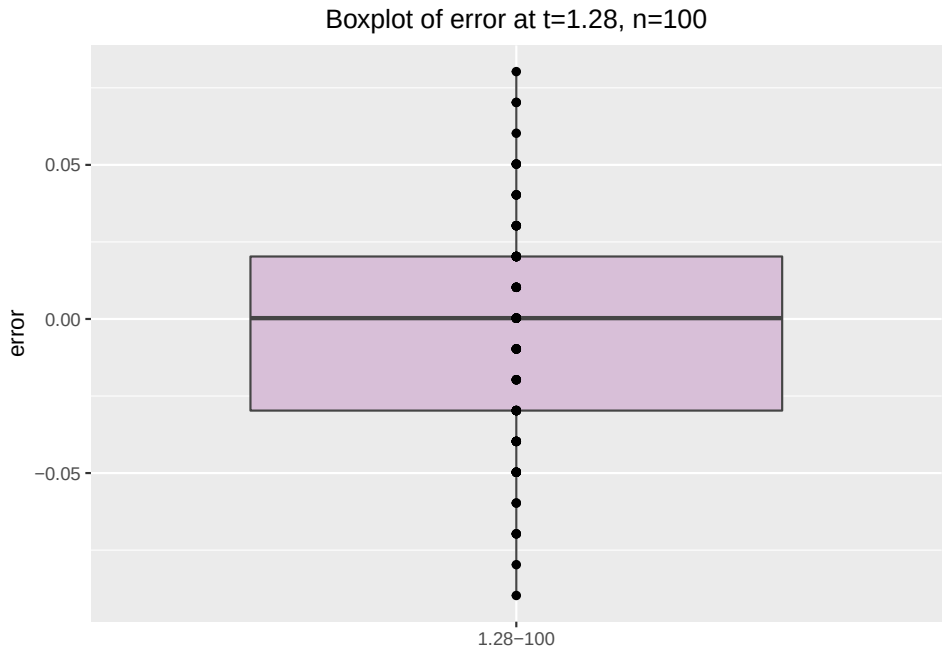

```
plot21<-ggplot(data=A,aes(y=e10000_3,x="0.84-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=0.84, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot21
```



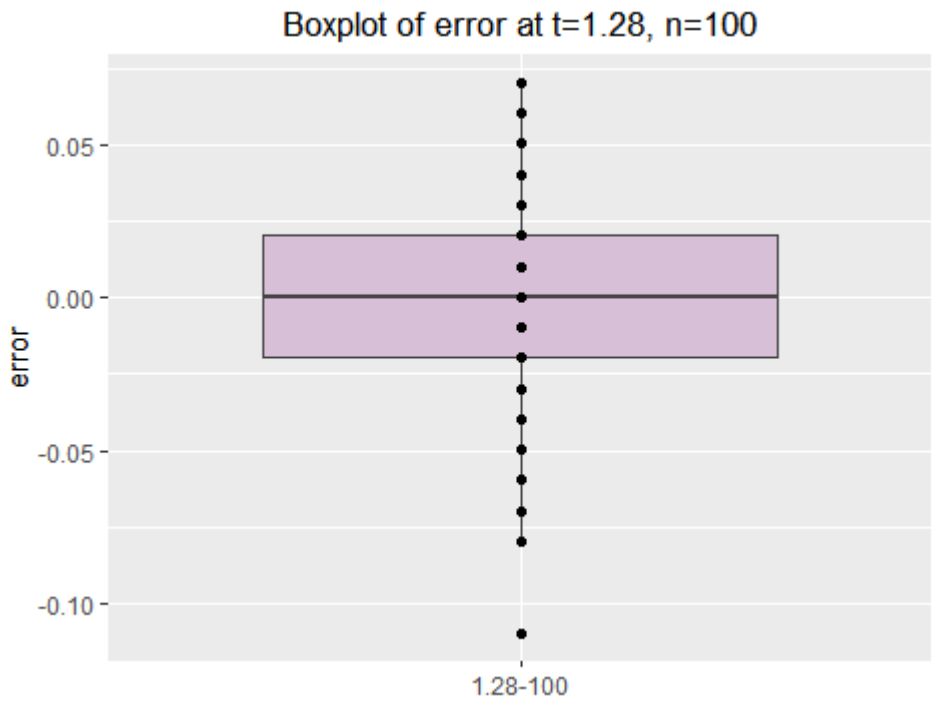

```
knitr::include_graphics("plot/Rplot21.png")
```



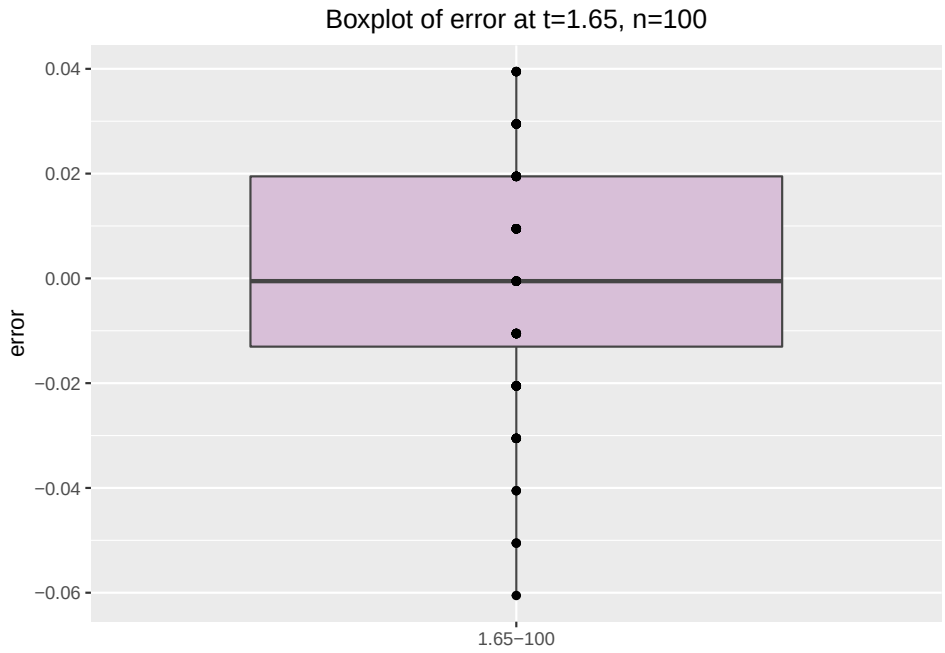

```
plot22<-ggplot(data=A,aes(y=e10000_4,x="1.28-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=1.28, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot22
```



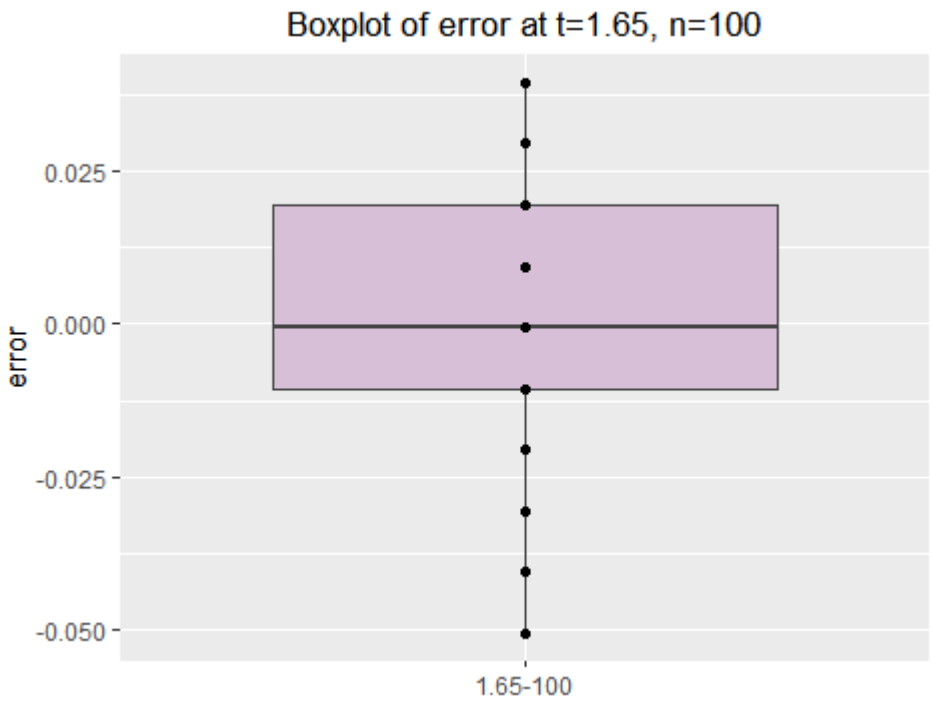

```
knitr::include_graphics("plot/Rplot22.png")
```



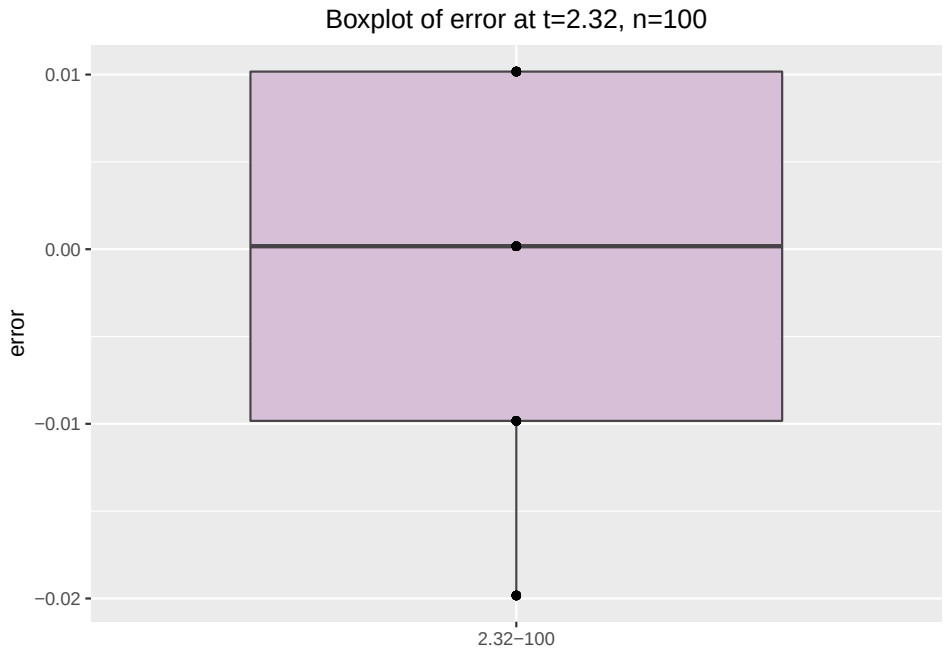

```
plot23<-ggplot(data=A,aes(y=e10000_5,x="1.65-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=1.65, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot23
```



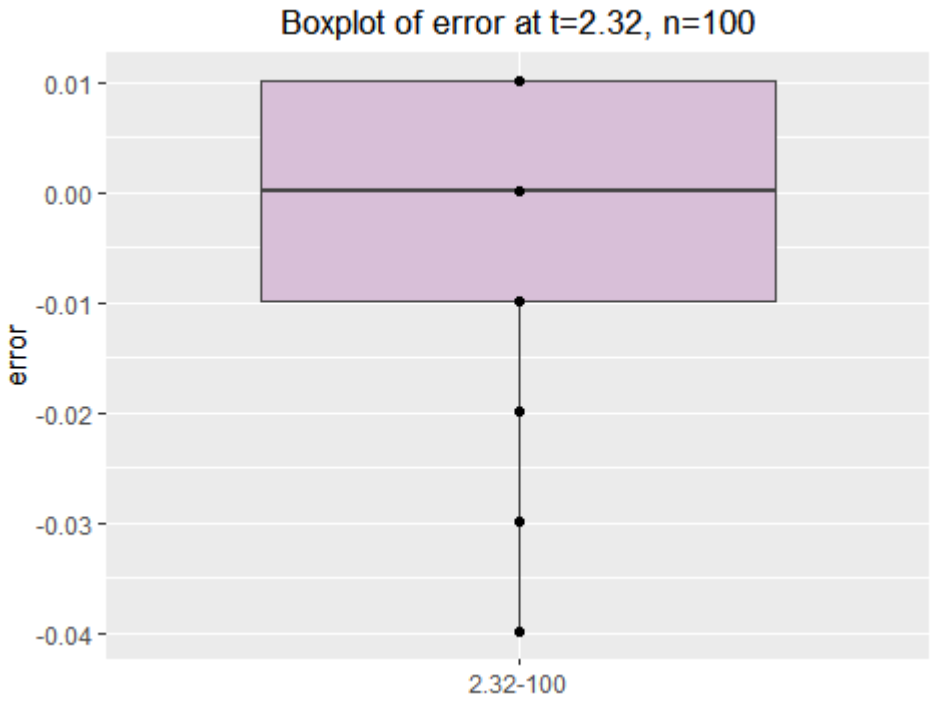

```
knitr::include_graphics("plot/Rplot23.png")
```



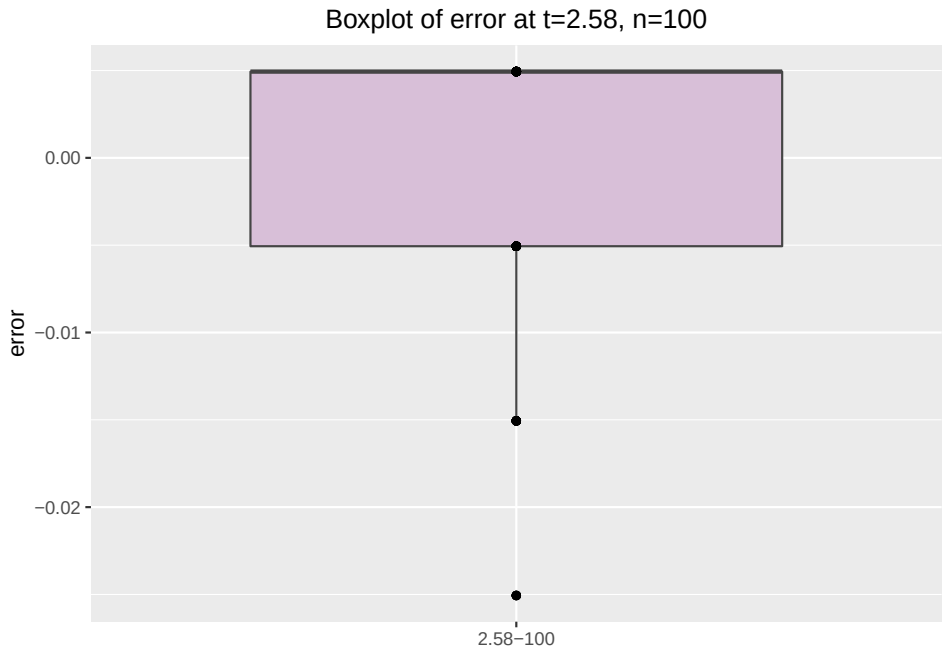

```
plot24<-ggplot(data=A,aes(y=e10000_6,x="2.32-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=2.32, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot24
```



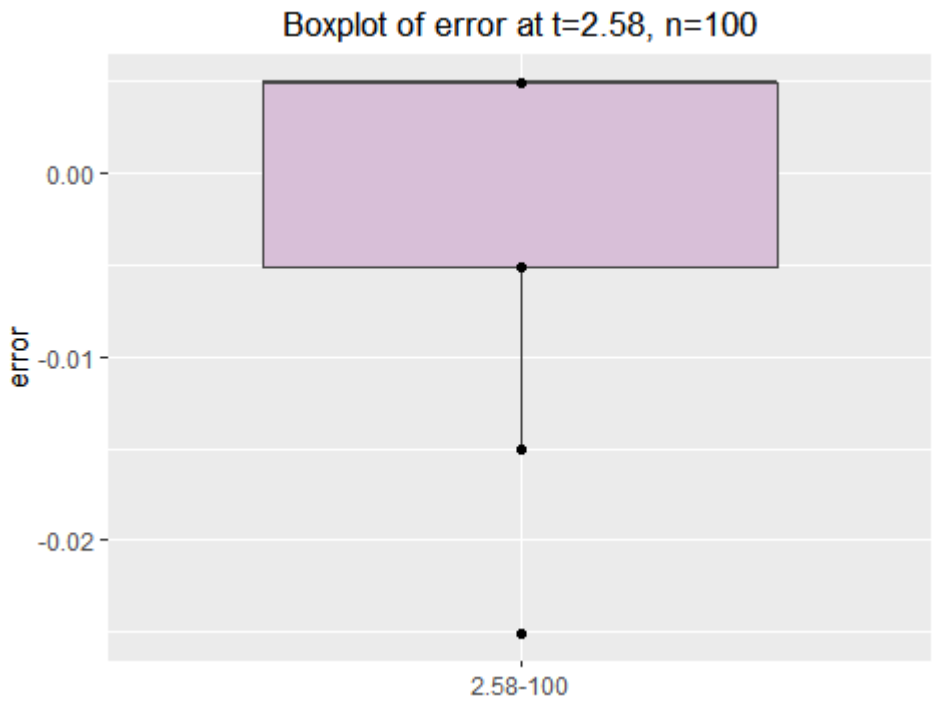

```
knitr::include_graphics("plot/Rplot24.png")
```



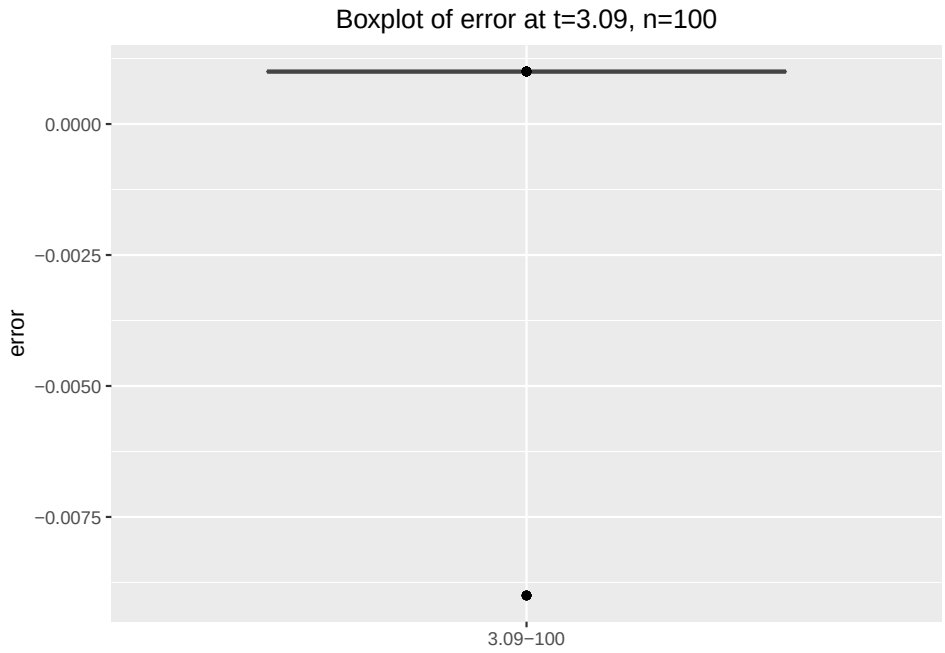

```
plot25<-ggplot(data=A,aes(y=e10000_7,x="2.58-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=2.58, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot25
```



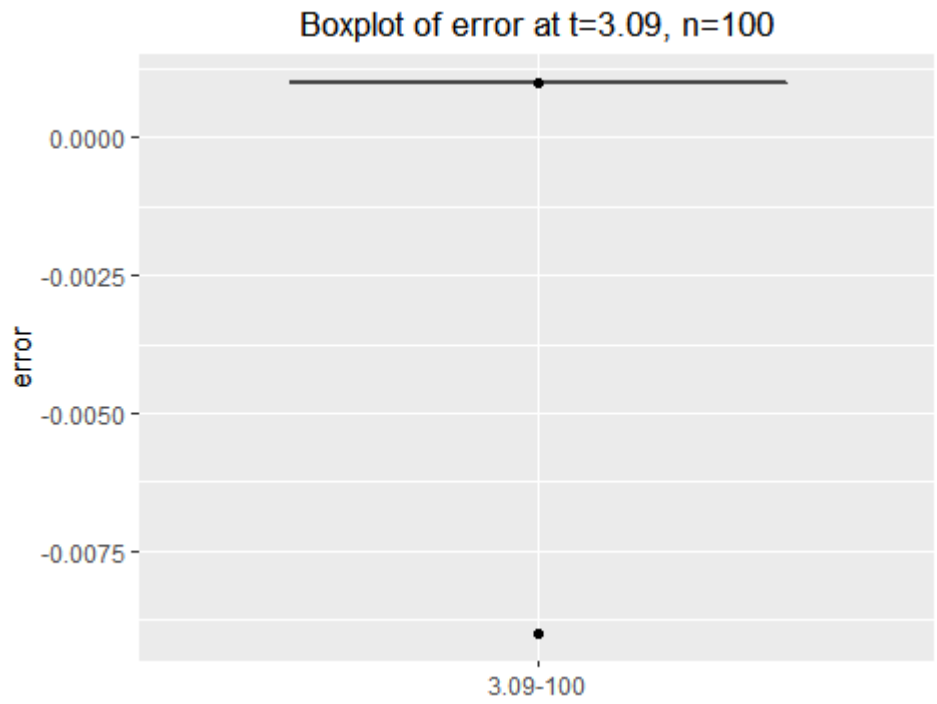

```
knitr::include_graphics("plot/Rplot25.png")
```



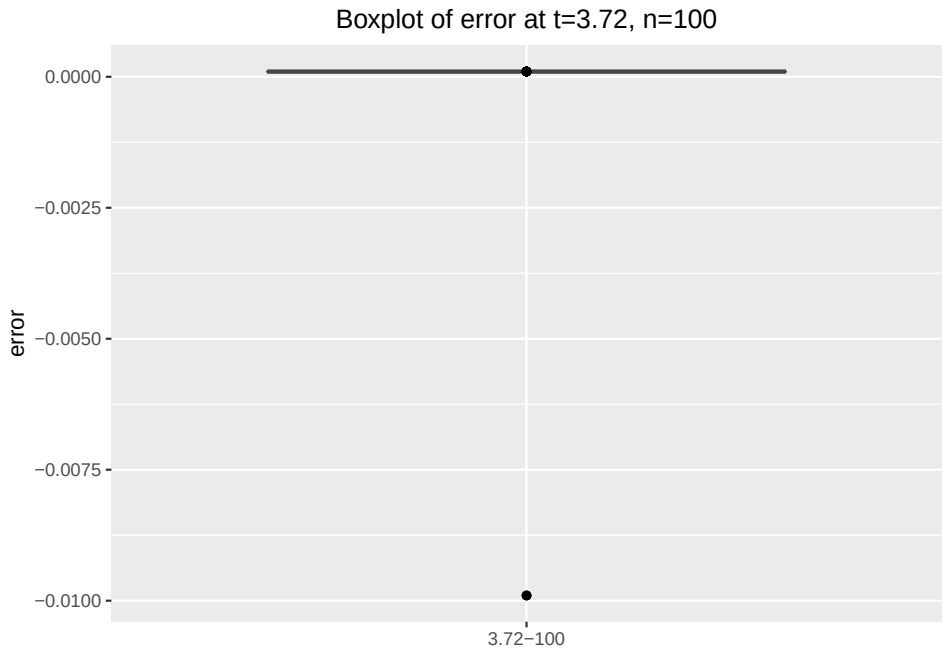

```
plot26<-ggplot(data=A,aes(y=e10000_8,x="3.09-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=3.09, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot26
```



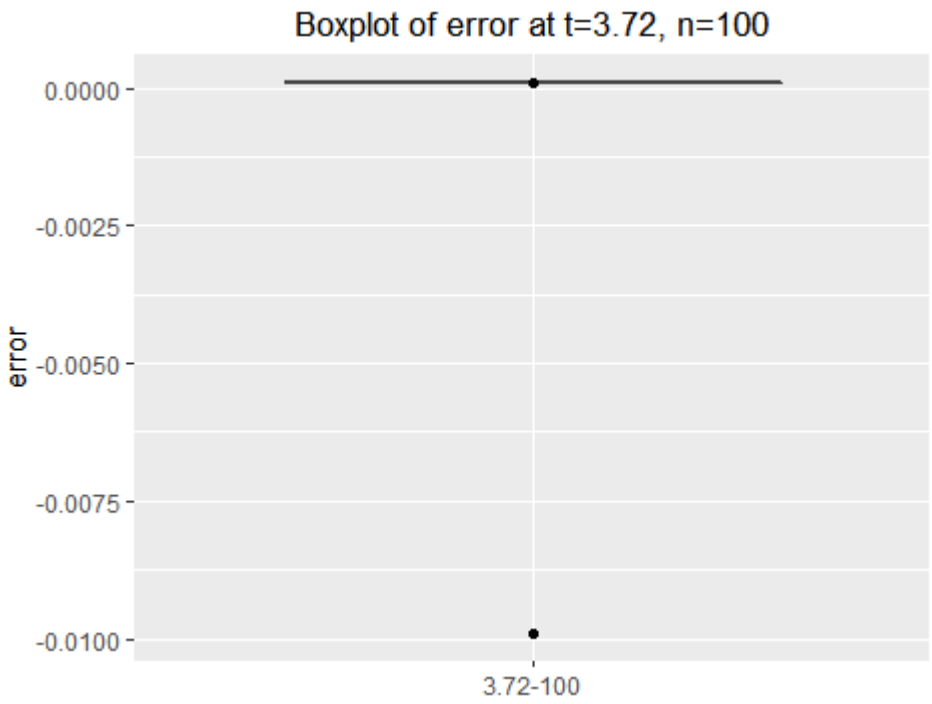

```
knitr::include_graphics("plot/Rplot26.png")
```




```
plot27<-ggplot(data=A,aes(y=e10000_9,x="3.72-100"))+geom_boxplot(
    fill="thistle",colour="gray27")+geom_point()+
    labs(title="Boxplot of error at t=3.72, n=100",y="error",
         x=NULL)+theme(plot.title=element_text(size=13,hjust=0.5))
plot27
```




```
knitr::include_graphics("plot/Rplot27.png")
```



Summary and Discussion

In this report, I formed a table that shows the difference between approximate value and true value, and draw the box plots of the 100 approximation errors which provides more intutive distinction.