Third homework

zhangxiaorui0112

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Abstract

Use rmarkdown to produce a report, and this report is about the Monte Carlo methods to approximate the standard normal distribution function.

Monte Carlo methods

Consider approximation of the distribution function of $N(0,1)$,

$$\Phi(t) = \int_{\infty}^{t} \frac{1}{\sqrt{2}\pi} e^{-y^2/2} \mathrm{d}y, \tag{1,1}$$

by the Monte Carlo methods:

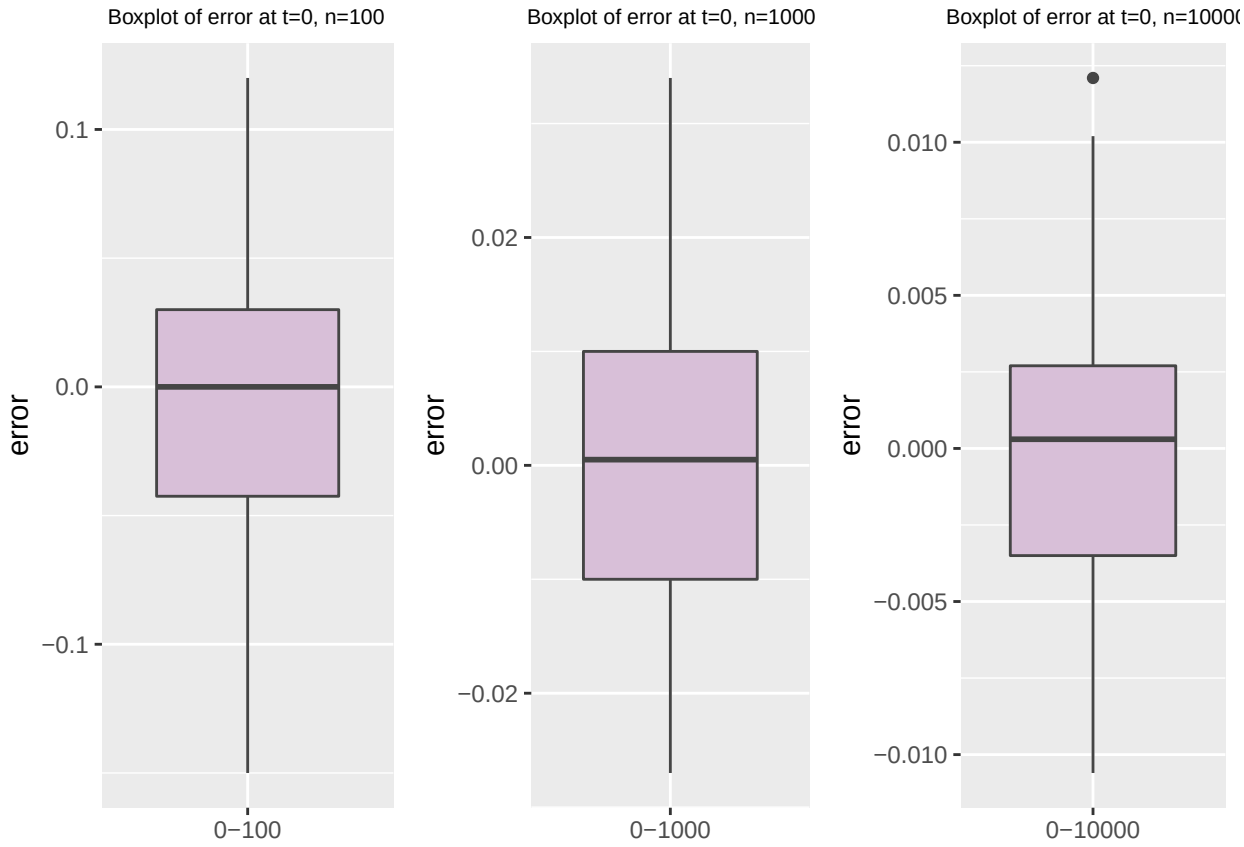
$$\hat{\Phi}(t) = \frac{1}{n} \sum_{i=1}^{n} I(X_i \leq t),$$

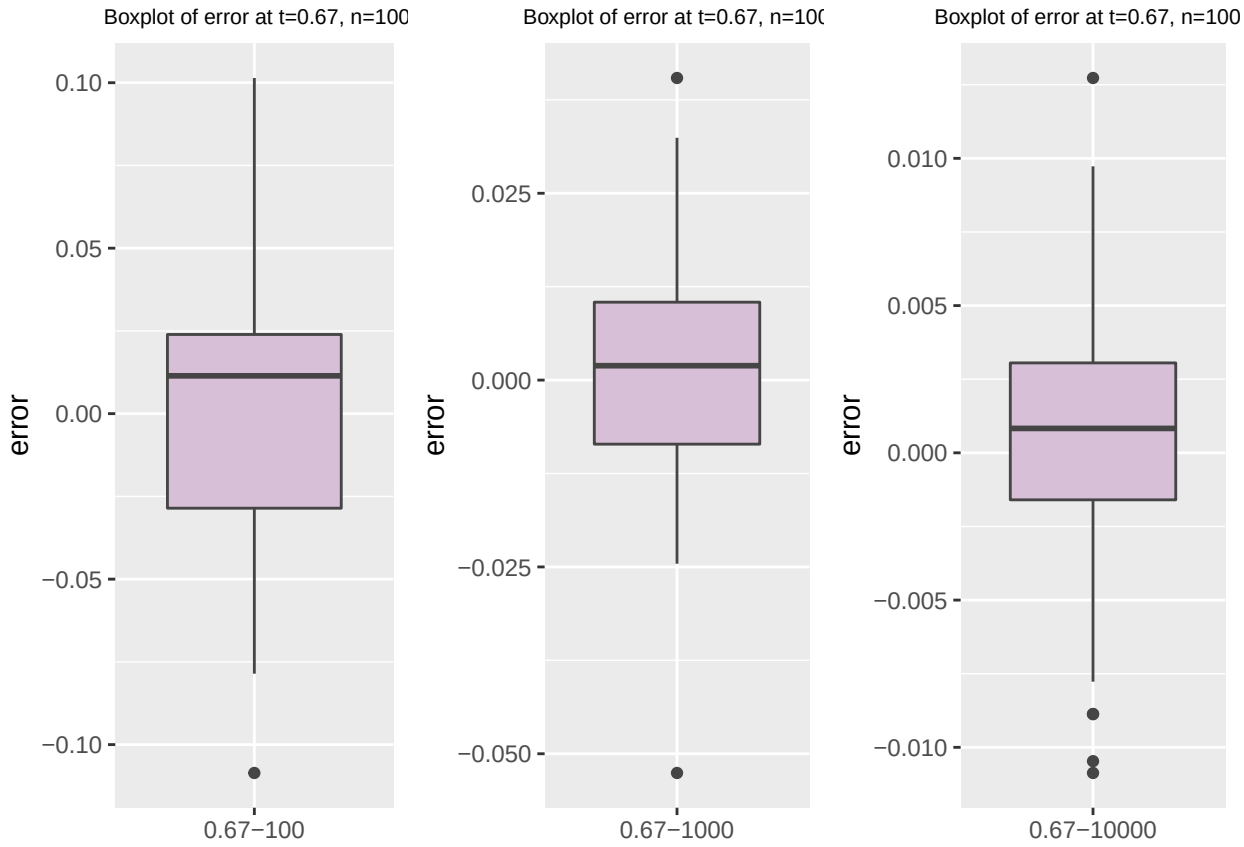
where X_is are a random sample from $N(0,1)$, and $I(\cdot)$ is the indicator function.

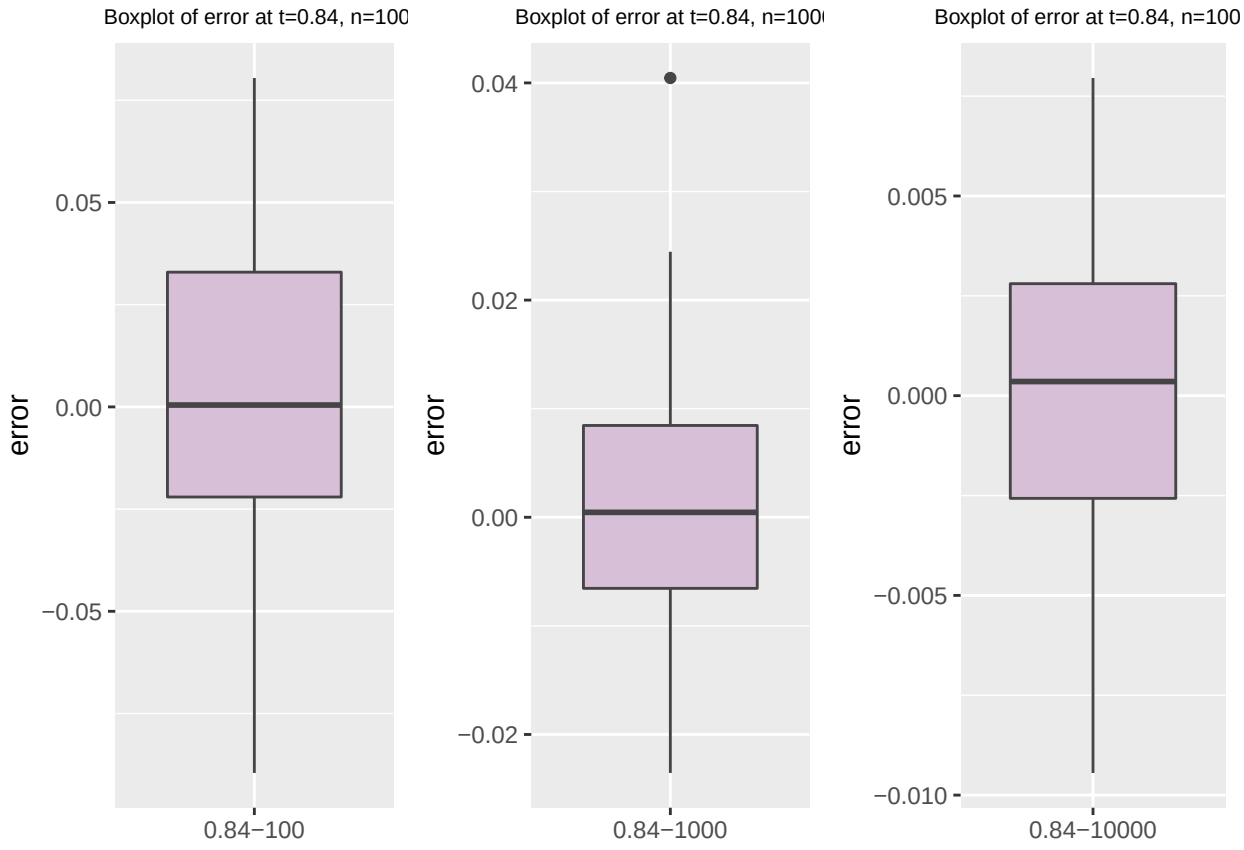
Results

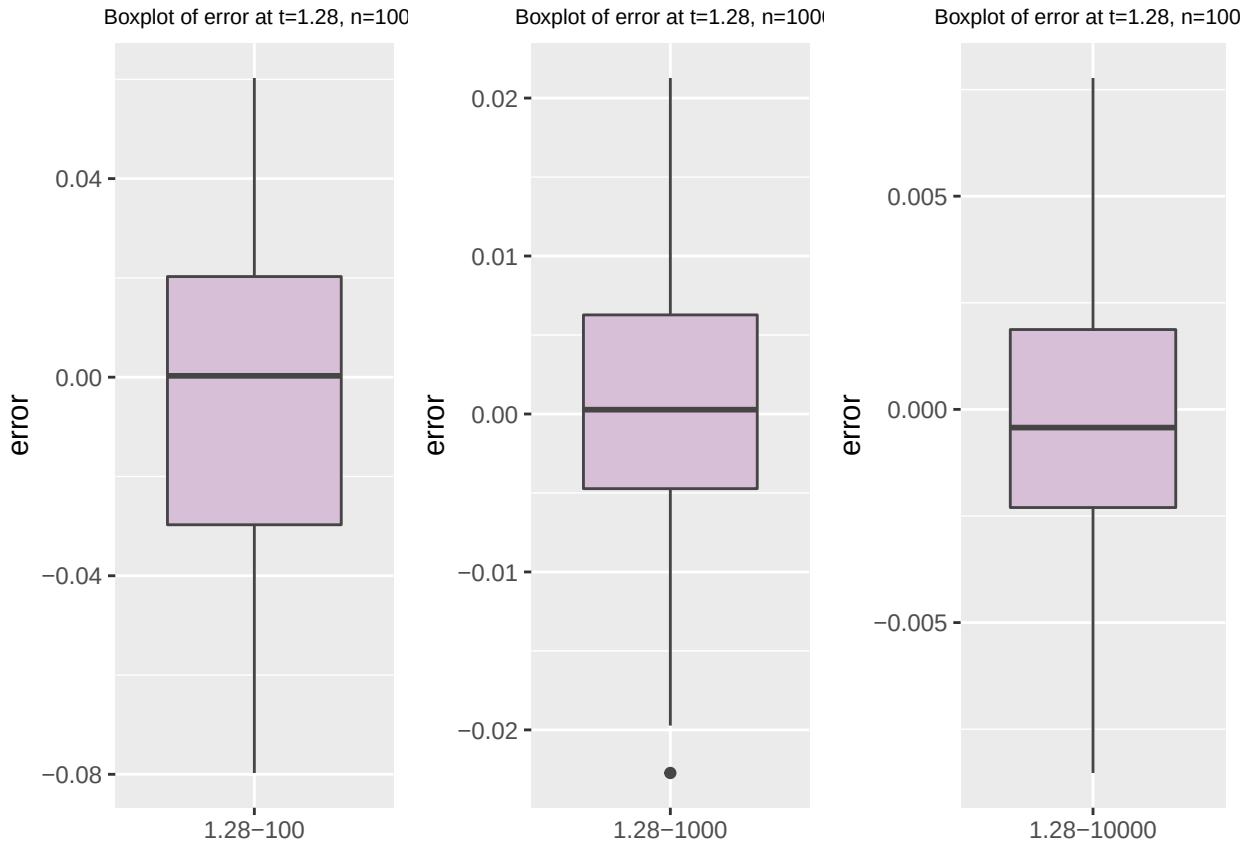
Experiment with the approximation at $n \in \{10^2, 10^3, 10^4\}$ at $t \in \{0.0, 0.67, 0.84, 1.28, 1.65, 2.32, 2.58, 3.09, 3.72\}$ to form a table.

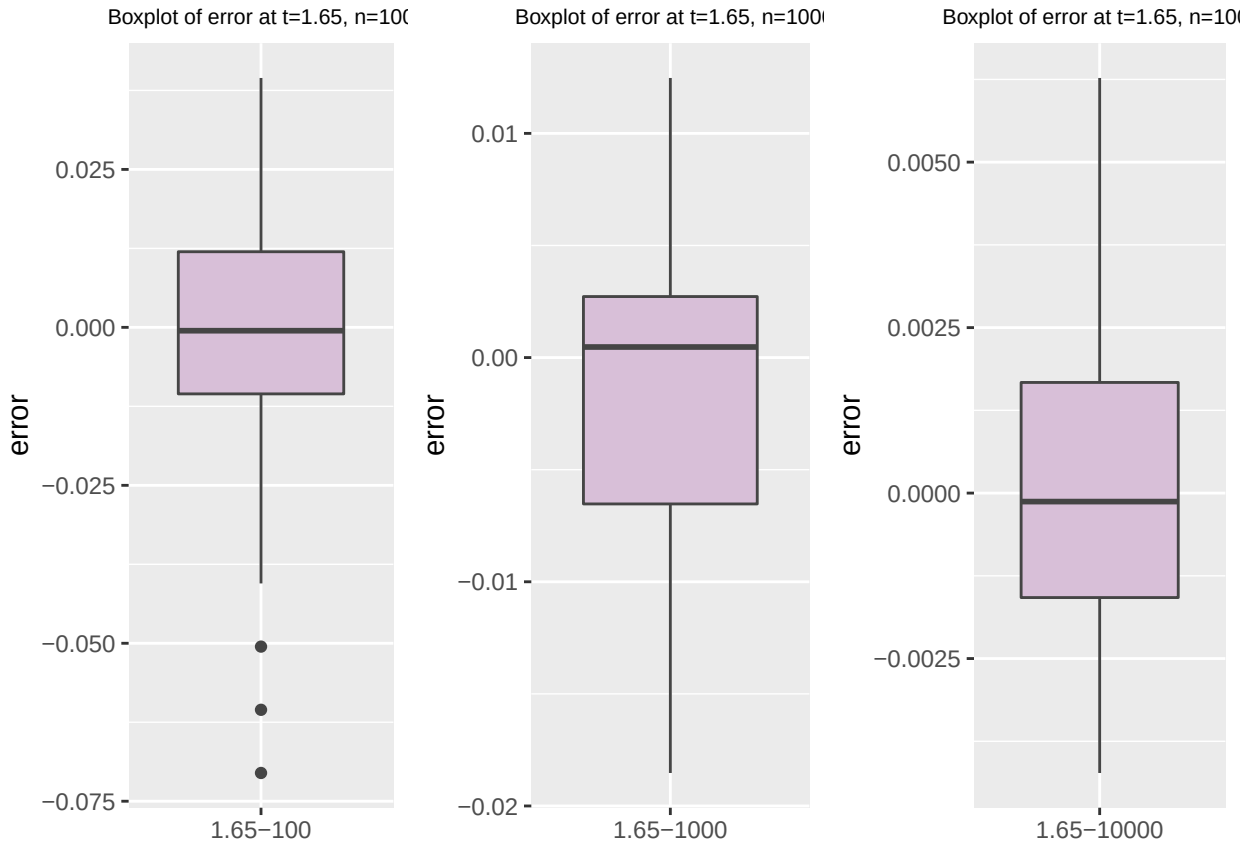
```
##       100  1000 10000 true_value
## 0    0.44 0.498 0.498      0.500
## 0.67 0.79 0.767 0.750      0.749
## 0.84 0.83 0.770 0.800      0.800
## 1.28 0.88 0.906 0.899      0.900
## 1.65 0.88 0.946 0.951      0.951
## 2.32 0.99 0.992 0.990      0.990
## 2.58 1.00 0.995 0.995      0.995
## 3.09 1.00 0.999 0.998      0.999
## 3.72 1.00 1.000 1.000      1.000
```

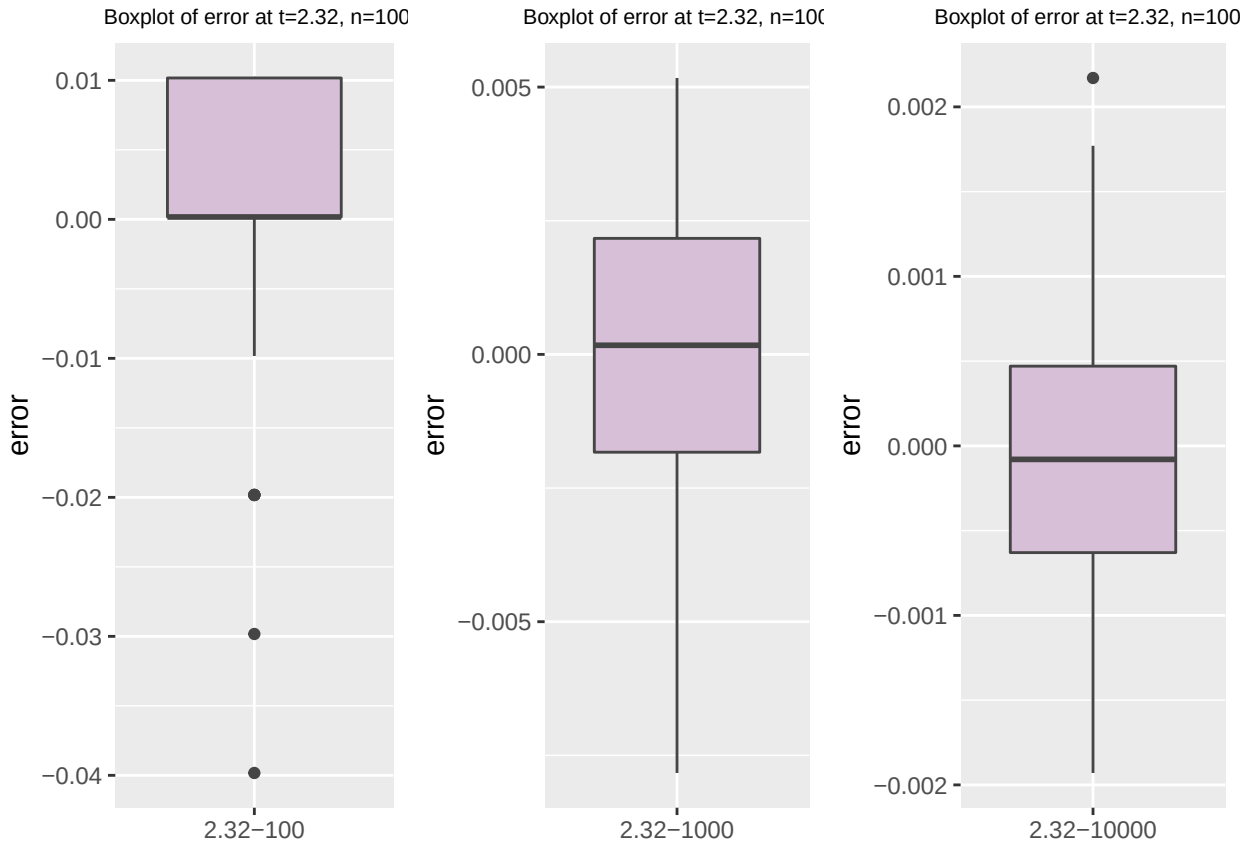

Boxplot of error at t=0, n=100
Boxplot of error at t=0, n=1000
Boxplot of error at t=0, n=10000
error
0.1
0.0
−0.1
0.02
0.00
−0.02
0.010
0.005
0.000
−0.005
−0.010
0−100
0−1000
0−10000

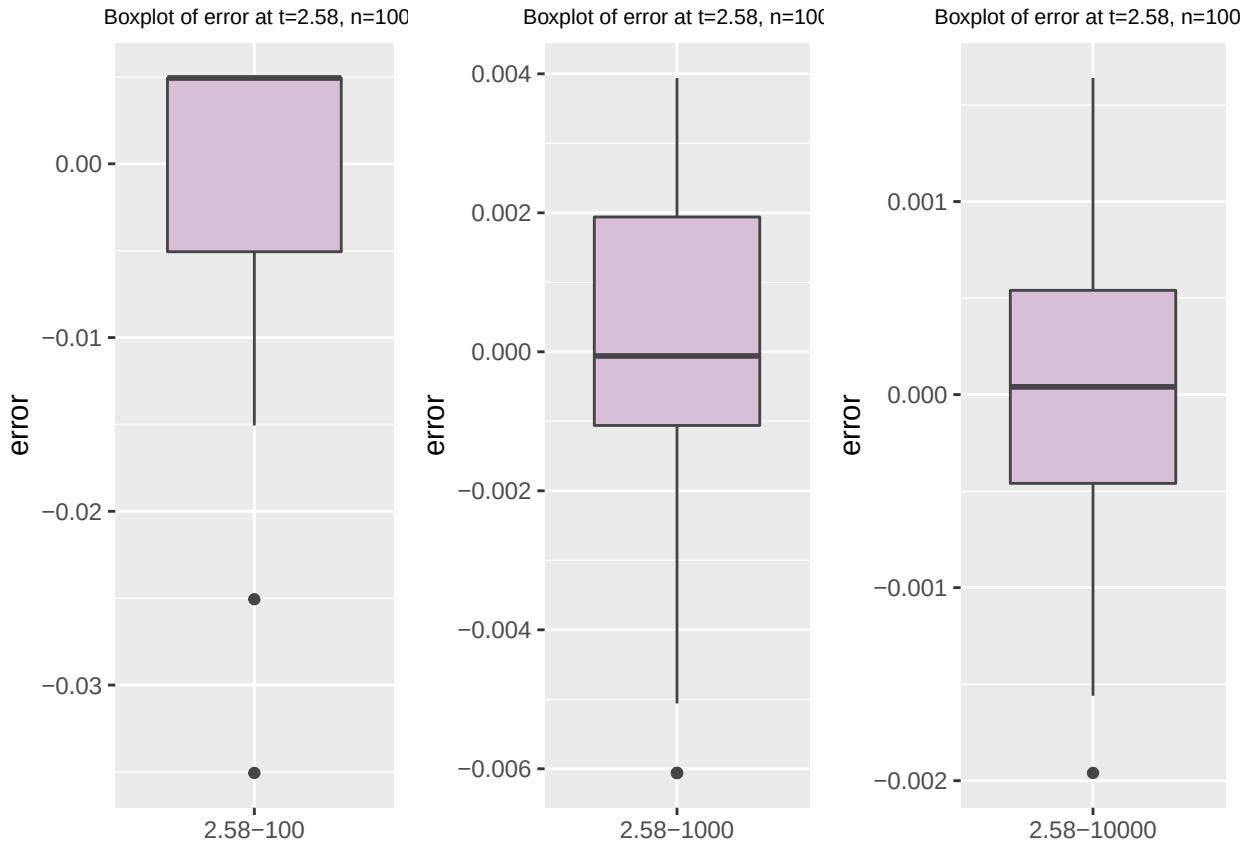

Boxplot of error at t=0.67, n=100
error
0.10
0.05
0.00
−0.05
−0.10
0.67−100
Boxplot of error at t=0.67, n=10(
error
0.025
0.000
−0.025
−0.050
0.67−1000
Boxplot of error at t=0.67, n=100
error
0.010
0.005
0.000
−0.005
−0.010
0.67−10000

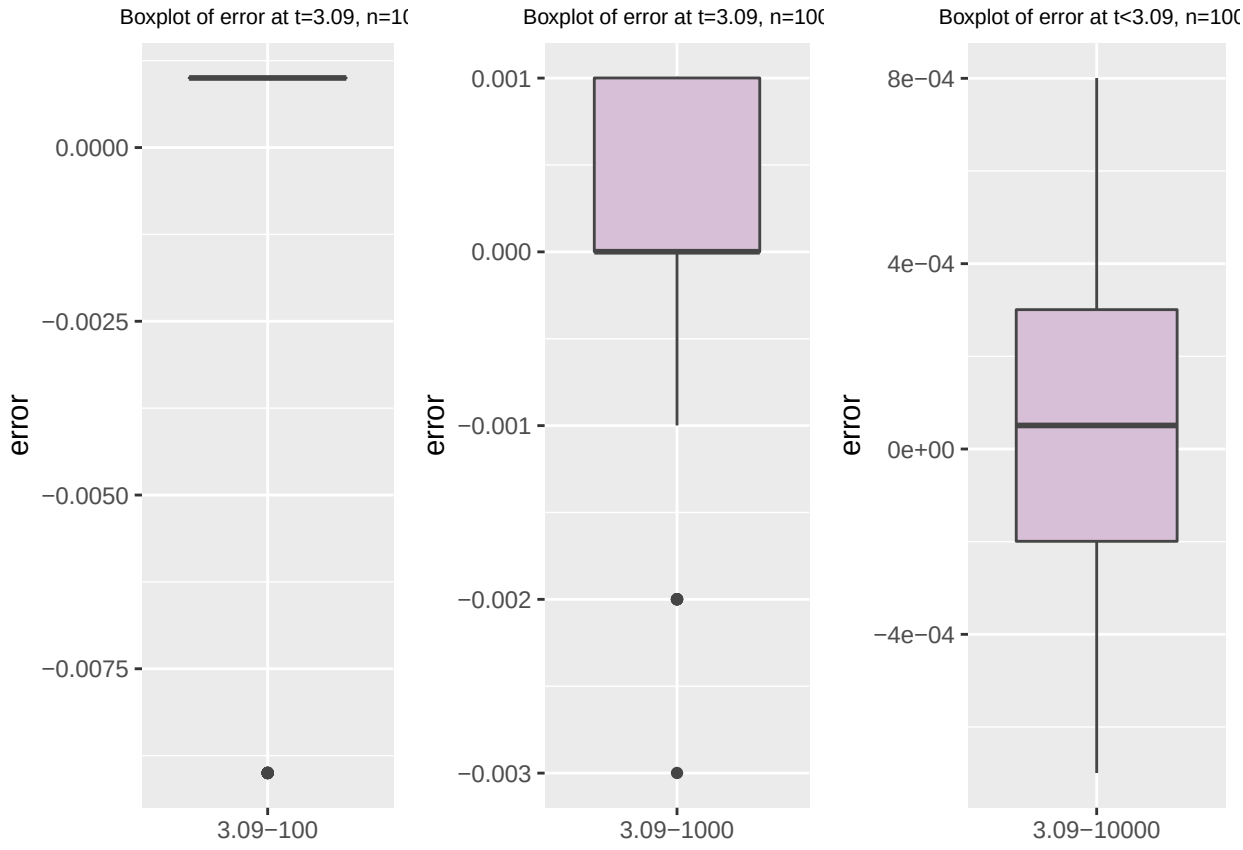

Boxplot of error at t=0.84, n=100
error
0.05
0.00
−0.05
0.84−100
Boxplot of error at t=0.84, n=1000
error
0.04
0.02
0.00
−0.02
0.84−1000
Boxplot of error at t=0.84, n=10000
error
0.005
0.000
−0.005
−0.010
0.84−10000

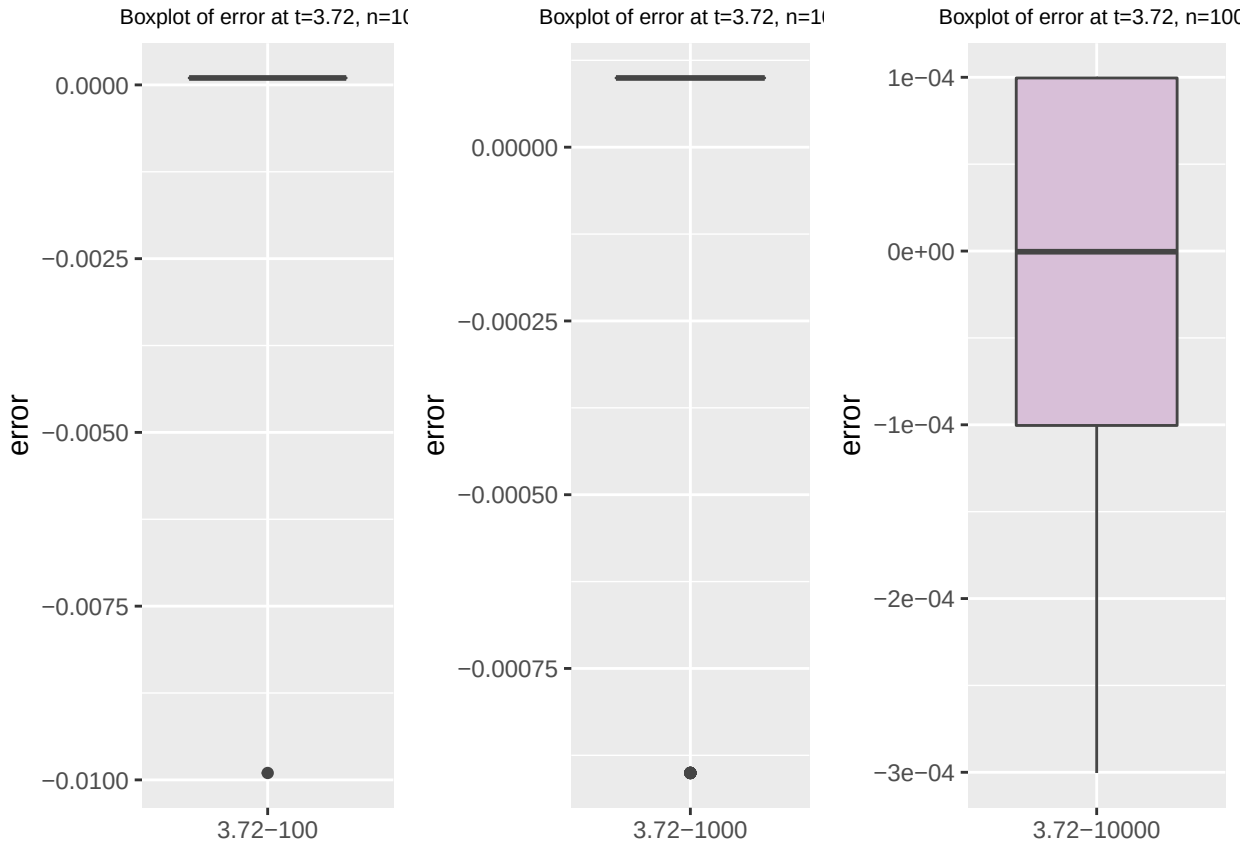

Boxplot of error at t=1.28, n=100
error
0.04
0.00
−0.04
−0.08
1.28−100
Boxplot of error at t=1.28, n=1000
error
0.02
0.01
0.00
−0.01
−0.02
1.28−1000
Boxplot of error at t=1.28, n=10000
error
0.005
0.000
−0.005
1.28−10000


Boxplot of error at t=1.65, n=10(
error
0.025
0.000
−0.025
−0.050
−0.075
1.65−100
Boxplot of error at t=1.65, n=100(
error
0.01
0.00
−0.01
−0.02
1.65−1000
Boxplot of error at t=1.65, n=10(
error
0.0050
0.0025
0.0000
−0.0025
1.65−10000


Boxplot of error at t=2.32, n=100
error
0.01
0.00
−0.01
−0.02
−0.03
−0.04
2.32−100
Boxplot of error at t=2.32, n=10(
error
0.005
0.000
−0.005
2.32−1000
Boxplot of error at t=2.32, n=100
error
0.002
0.001
0.000
−0.001
−0.002
2.32−10000


Boxplot of error at t=2.58, n=100
error
0.00
−0.01
−0.02
−0.03
2.58−100
Boxplot of error at t=2.58, n=10(
error
0.004
0.002
0.000
−0.002
−0.004
−0.006
2.58−1000
Boxplot of error at t=2.58, n=100
error
0.001
0.000
−0.001
−0.002
2.58−10000


Boxplot of error at t=3.09, n=1(
Boxplot of error at t=3.09, n=10(
Boxplot of error at t<3.09, n=10(
error
0.0000
−0.0025
−0.0050
−0.0075
3.09−100
0.001
0.000
−0.001
−0.002
−0.003
3.09−1000
8e−04
4e−04
0e+00
−4e−04
3.09−10000


Boxplot of error at t=3.72, n=1(
Boxplot of error at t=3.72, n=1(
Boxplot of error at t=3.72, n=10(
error
0.0000
−0.0025
−0.0050
−0.0075
−0.0100
3.72−100
0.00000
−0.00025
−0.00050
−0.00075
3.72−1000
1e−04
0e+00
−1e−04
−2e−04
−3e−04
3.72−10000