

$$f_{xy}(x, y) = \begin{cases} e^{-x-y} & 0 < x < \infty \\ & 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

$$P(x < y) : \int_0^{\infty} \int_0^y e^{-(x-y)} dx dy$$

$$= \int_0^{\infty} \left[\frac{e^{-x-y}}{-1} \right]_0^y dy$$

$$= \int_0^{\infty} \left(\frac{e^{-2y}}{-1} - \frac{e^{-y}}{-1} \right) dy$$

$$= \int_0^{\infty} (-e^{-2y} + e^{-y}) dy$$

$$= \left[\frac{e^{-2y}}{2} - e^{-y} \right]_0^{\infty}$$

Upper limit Vanishes the exponentials to zero

$$\Rightarrow \text{applying lower} = -\frac{1}{2} + 1$$

$$\boxed{P(x < y) = 0.5}$$