Notes on QR decomposition in Weighted Least Squares/GLM

Let $X \in \mathbb{R}^{m\times n}, \quad Q \in \mathbb{R}^{m\times m}, \quad R \in \mathbb{R}^{m\times n}$ and $m \geq n$.

R is upper traingular
Q is upper triangular

The submatrices
$Q_1 = Q(1{:}m, 1{:}n)$
$R_1 = R(1{:}n, 1{:}n)$

$X = Q_1 R_1, \quad Q_1 \in \mathbb{R}^{m\times n}, R \in \mathbb{R}^{n\times n}$

The default regression formula is given by:

$$(XW^T X)\beta = X^T W y \tag{1}$$

solve for $\beta = (X^T W X)^{-1} X^T W y$ using an appropriate solver. However solve instead transform:

$$(W^{1/2}X)^T (W^{1/2}X)\beta = (W^{1/2}X)^T (W^{1/2}y) \tag{2}$$

Now redefine:
$X = W^{1/2}X; \quad y = W^{1/2}y$
so we can write (1) again as

$(X^TX)\beta = X^Ty$
so $X = Q_1R_1$
$(Q_1R_1)^T(Q_1R_1)\beta = (Q_1R_1)^Ty_1$
$R_1^TR_1\beta = R_1^TQ_1^Ty$
$(R_1^T)^{-1}R_1^TR_1R_1\beta = (R_1^T)^{-1}R_1^TQ_1^Ty$ $R_1\beta = Q_1^Ty$

R_1 is an upper triangular $n \times n$ matrix, β is a $n \times 1$
let $z = Q_1^Ty$, so the equation is $R_1\beta = z$
so solve conventionally $\beta = R_1^{-1}z$

So do:

1. QR decomposition of the matrix $W^{1/2}X$ (use `dgeqrf()` lapack function).

2. Calculation $z = Q_1^TW^{1/2}y$, where z is $n \times 1$, and Q_1 is $m \times n$, y is $m \times 1$.

3. Do a solve for $\beta = R_1^{-1}z$, where R is a $n \times n$ matrix solver for triangular matrix (`dtrtrs()` lapack function).

QR decomposition links:
netlib.org/lapack/lug/node40.html
netlib.org/lapack/lug/node46.html
milq.github.io/install-latex-ubuntu-debian/