

Chapter 13 - Nonparametric Statistics

Note: Graphs are not to scale and are intended to convey a general idea.
Answers may vary due to rounding.

EXERCISE SET 13-1

1.

Non-parametric means hypotheses other than those using population parameters can be tested; distribution free means no assumptions about the population distributions have to be satisfied.

2.

When the assumptions for the parametric methods cannot be met, statisticians use non-parametric methods.

3.

Non-parametric methods have the following advantages:

- They can be used to test population parameters when the variable is not normally distributed.
- They can be used when data are nominal or ordinal in nature.
- They can be used to test hypotheses other than those involving population parameters.
- The computations are easier in some cases than the computations of the parametric counterparts.
- They are easier to understand.
- There are fewer assumptions that have to be met, and the assumptions are easier to verify.

4.

The disadvantages are as follows:

- They are less sensitive than their parametric counterparts.
- They tend to use less information than their parametric counterparts.

4. continued

c. They are less efficient than their para-metric counterparts.

5.

Distribution-free means the samples can be selected from populations that are not normally distributed.

6.

The efficiency of a nonparametric test means the percent rating of the nonparametric test compared to the corresponding parametric test to obtain the same results.

7.

DATA	25	36	36	39	63	68	74
RANK	1	2.5	2.5	4	5	6	7

8.

DATA	88	182	243	465	587
RANK	1	2	3	4	5

9.

DATA	2.1	6.2	11.4	12.7	18.6	20.7	22.5
RANK	1	2	3	4	5	6	7

10.

DATA	5.1	5.1	11.7	11.7	14.3
RANK	1.5	1.5	3.5	3.5	5
DATA	16.2	18.6	31.4	41.7	
RANK	6	7	8	9	

11.

DATA	12	22	22	38	44	50
RANK	1	2.5	2.5	4	5	6
DATA	54	56	56	62	73	88
RANK	7	8.5	8.5	10	11	12

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12.

DATA 16 32.7 46 58.6 62.8

RANK 1 2 3 4 5

DATA 83.4 83.4 88.3 148.3 321

RANK 6.5 6.5 8 9 10

EXERCISE SET 13-2

1.

The sign test uses only + or - signs.

2.

The median

3.

The smaller number of + or - signs

4.

The normal approximation

5.

+	+	+	+	-
+	-	-	-	-
+	+	+	+	+

H_0 : Median = 27

H_1 : Median \neq 27 (claim)

$\alpha = 0.05$ $n = 15$

C. V. = 3

Test value = 5

Since $5 > 3$, do not reject the null hypothesis. There is not enough evidence to support the claim that the median age is not 27 years.

6.

+	+	-	+	+
+	+	-	-	-
-	-	-	+	+
+	-	+	-	-

H_0 : Median = 3000 (claim)

H_1 : Median \neq 3000

6. continued

$\alpha = 0.05$ $n = 20$

C. V. = 5 Test value = 10

Since $10 > 5$, do not reject the null hypothesis. There is not enough evidence to reject the claim that the median is 3000. Yes, you could use 3000 as a guide.

7.

-	+	+	-	-
+	+	-	-	+
-	+	+		

H_0 : Median = \$35,642

H_1 : Median $>$ \$35,642 (claim)

$\alpha = 0.05$ $n = 13$

C. V. = 3 Test value = 6

Since $6 > 2$, do not reject the null hypothesis. There is not enough evidence to support the claim that the median is greater than \$35,642.

8.

-	-	+
-	-	+
+	+	-
-	+	+
-	-	-

H_0 : Median = \$1603

H_1 : Median $<$ \$1603 (claim)

$\alpha = 0.05$ $n = 15$

C. V. = 3 Test value = 6

Since $6 > 3$, do not reject the null hypothesis. There is not enough evidence to conclude that the median income is less than \$1603.

9.

H_0 : Median = 25 (claim)

H_1 : Median $>$ 25

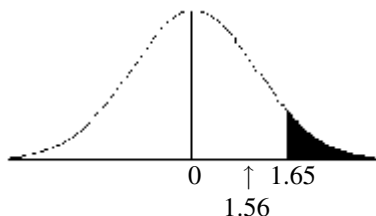
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9. continued

C. V. = + 1.65

$$z = \frac{(x - 0.5) - \left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}} = \frac{(31 - 0.5) - \frac{50}{2}}{\frac{\sqrt{50}}{2}}$$

$$z = \frac{5.5}{3.536} = 1.56$$



Do not reject the null hypothesis. There is not enough evidence to support the claim that more than 50% of the students favor the summer institute.

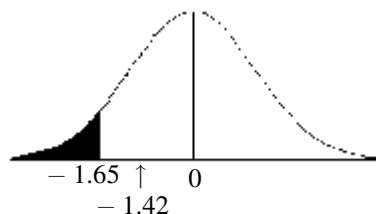
10.

H_0 : Median = 200

H_1 : Median < 200 (claim)

C. V. = - 1.65

$$z = \frac{(15 + 0.5) - \frac{40}{2}}{\frac{\sqrt{40}}{2}} = -1.42$$



Do not reject the null hypothesis. There is not enough evidence to support the claim that she sells less than 200 tickets per day.

11.

H_0 : Median = 150 (claim)

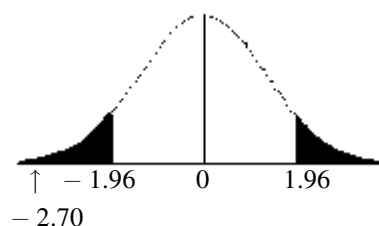
H_1 : Median \neq 150

C. V. = \pm 1.96

$$z = \frac{(x + 0.5) - \left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}} = \frac{(9 + 0.5) - \frac{35}{2}}{\frac{\sqrt{35}}{2}}$$

$$z = -2.70$$

11. continued



Reject. There is enough evidence to reject the claim that the median number of faculty members is 150.

12.

- - - + + -
- - - + +

H_0 : Median = 60 (claim)

H_1 : Median \neq 60

$$\alpha = 0.05n = 11$$

C. V. = 1 Test Value = 4

Since $4 > 1$, do not reject the null hypothesis. There is not enough evidence to reject the claim that the median is 60. Yes, considering the number of tornadoes, the median of 60 is relatively small.

13.

H_0 : Median = 49 (claim)

H_1 : Median \neq 49

$$z = \frac{(x + 0.5) - \left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}} = \frac{(36 + 0.5) - \frac{98}{2}}{\frac{\sqrt{98}}{2}}$$

$$z = -2.53$$

$$P\text{-value} = 0.0114$$

Reject. There is enough evidence to reject the claim that 50% of the students are against extending the school year.

14.

H_0 : Median = 52.7 (claim)

H_1 : Median \neq 52.7

$$z = \frac{(27 + 0.5) - \frac{75}{2}}{\frac{\sqrt{75}}{2}} = -2.31$$

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14. continued

P-value = 0.0209

Reject the null hypothesis. There is enough evidence to reject the claim that the median is 52.7. One reason that this information would be valuable is that the sponsors of the show could target viewers based on median age.

15.

A	B	C	D	E	F	G	H
+	+	-	+	+	-	+	+

H_0 : The number of sessions will not be reduced.

H_1 : The number of sessions will be reduced. (claim)

$\alpha = 0.05$ $n = 8$

C. V. = 1 Test value = 2

Since $2 > 1$, do not reject the null hypothesis. There is not enough evidence to support the claim that the number of sessions was reduced.

16.

1	2	3	4	5	6	7	8
-	+	-	-	-	-	+	+

H_0 : There is no difference in scores.

H_1 : There is a difference in scores. (claim)

$\alpha = 0.10$ $n = 8$

C. V. = 1 Test value = 3

Since $3 > 1$, do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in scores.

17.

A	B	C	D	E	F	G	H	I	J
-	+	+	+	+	+	-	-	+	+

17. continued

H_0 : The number of soft drinks will not change.

H_1 : The number of soft drinks will change. (claim)

$\alpha = 0.10$ $n = 10$

C. V. = 1 Test value = 3

Since $3 > 1$, do not reject the null hypothesis. There is not enough evidence to support the claim that the number of soft drinks was reduced.

18.

1	2	3	4	5	6
+	+	+	-	+	+
7	8	9	10	11	12
-	+	+	+	+	+

H_0 : The pill has no effect on the caloric intake of the person eating.

H_1 : The pill has an effect on the caloric intake of the person eating. (claim)

$\alpha = 0.02$ $n = 12$

C. V. = 1 Test value = 2

Since $2 > 1$, do not reject the null hypothesis. There is not enough evidence to support the claim that the pill has an effect on the caloric intake of a person.

19.

1	2	3	4	5	6	7	8	9	10
+	+	+	+	+	+	-	+	+	-

H_0 : The number of viewers is the same as last year. (claim)

H_1 : The number of viewers is not the same as last year.

$\alpha = 0.01$ $n = 10$

C. V. = 0 Test value = 2

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19. continued

Since $2 > 0$, do not reject the null hypothesis. There is not enough evidence to reject the claim that the number of viewers is the same as last year.

20.

1	2	3	4	5	6	7	8	9
+	+	-	0	-	+	+	+	+

H_0 : Increased maintenance does not reduce the number of defective parts a machine produces.

H_1 : Increased maintenance reduces the number of defective parts a machine produces. (claim)

$\alpha = 0.01$ $n = 8$

C. V. = 0 Test value = 2

Since $2 > 0$, do not reject the null hypothesis. There is not enough evidence to support the claim that increased maintenance reduces the number of defective parts manufactured by the machines.

21.

3, 4, 6, 9, 12, 15, 15, 16, 18, 22, 25, 30

At $\alpha = 0.05$, the value from Table J with $n = 12$ is 2; hence, count in 3 numbers from each end to get $6 \leq MD \leq 22$.

22.

100, 101, 106, 115, 115, 141, 142, 143, 143, 145, 147, 147, 150, 152, 153, 155, 157, 160, 163, 164

MD = 146

$141 \leq MD \leq 153$

23.

4.2, 4.5, 4.7, 4.8, 5.1, 5.2, 5.6, 6.3, 7.1, 7.2, 7.8, 8.2, 9.3, 9.3, 9.5, 9.6

23. continued

At $\alpha = 0.02$, the value from Table J with $n = 16$ is 2; hence, count 3 numbers from each end to get $4.7 \leq MD \leq 9.3$

24.

1, 2, 3, 5, 6, 8, 10, 15, 15, 21, 24, 31, 33, 41, 42, 54, 56, 58, 65

MD = 21

$5 \leq MD \leq 54$

25.

12, 14, 14, 15, 16, 17, 18, 19, 19, 21, 23, 25, 27, 32, 33, 35, 39, 41, 42, 47

At $\alpha = 0.05$, the value from Table J with $n = 20$ is 5; hence, count in 6 numbers from each end to get $17 \leq MD \leq 33$.

EXERCISE SET 13-3

1.

The sample sizes n_1 and n_2 must be greater than or equal to 10.

2.

The t-test for independent samples

3.

H_0 : There is no difference in the speed skating times of the students at the two universities. (claim)

H_1 : There is a difference in the speed skating times of the students at the two universities.

C. V. = ± 1.96

1:35	1:38	1:39	1:40	1:42
------	------	------	------	------

UB	UB	UA	UA	UA
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1	2	3	4	5
---	---	---	---	---

1:48	1:48	1:51	1:58	2:00
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UB	UB	UA	UA	UB
----	----	----	----	----

6.5	6.5	8	9	10
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3. continued

2:01	2:03	2:05	2:06	2:10
UA	UA	UA	UB	UB
11	12	13	14	15

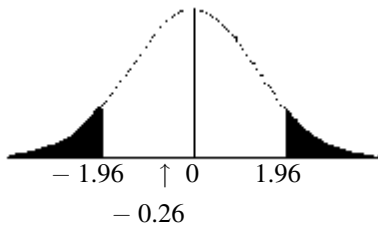
2:14	2:15	2:15	2:20	2:27
UB	UA	UB	UA	UB
16	17.5	17.5	19	20

$$R = 101.5$$

$$\mu_R = \frac{10(10 + 10 + 1)}{2} = 105$$

$$\sigma_R = \sqrt{\frac{10 \cdot 10(10 + 10 + 1)}{12}} = 13.23$$

$$Z = \frac{101.5 - 105}{13.23} = -0.26$$



Do not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the times.

4.

H_0 : There is no difference in the length of the sentences of the males and females.
(claim)

H_1 : There is a difference in the length of the sentences of the males and females.

$$C. V. = \pm 1.96$$

2	3	4	5	6	7	8	9	11
1	2	3	4	5	6	7	8	9
F	F	F	F	M	F	M	F	F

12	12	13	14	15	16	17	19	21
10.5	10.5	12	13	14	15	16	17	18
F	M	M	M	M	F	F	M	F

4. continued

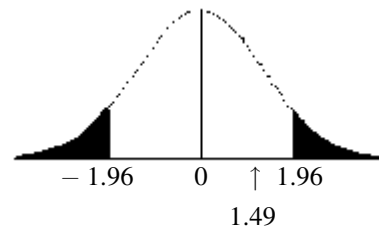
22	23	24	26	26	27	30	32
19	20	21	22.5	22.5	24	25	26
M	F	M	F	M	M	F	M

$$R = 191$$

$$\mu_R = \frac{12(12 + 14 + 1)}{2} = 162$$

$$\sigma_R = \sqrt{\frac{12 \cdot 14(12 + 14 + 1)}{12}} = 19.44$$

$$Z = \frac{191 - 162}{19.44} = 1.49$$



Do not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the sentences received by the men and women.

5.

H_0 : There is no difference in the number of credits transferred.

H_1 : There is a difference in the number of credits transferred. (claim)

$$C. V. = \pm 1.96 \quad \alpha = 0.05$$

35	37	42	45	46	48	58	58	59
1	2	3	4	5	6	7.5	7.5	9
C	S	C	S	S	C	S	S	C

60	61	62	63	63	64	64
10	11	12	13.5	13.5	15.5	15.5
C	C	C	C	S	C	S

65	66	68	71
17	18	19	20
C	S	S	S

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5. continued

R = 98 (for community college)

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

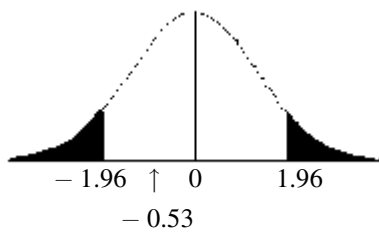
$$\mu_R = \frac{10(10 + 10 + 1)}{2} = \frac{10(21)}{2} = \frac{210}{2} = 105$$

$$\sigma_R = \sqrt{\frac{n_1 \cdot n_2 (n_1 + n_2 + 1)}{12}}$$

$$\sigma_R = \sqrt{\frac{10 \cdot 10 (10 + 10 + 1)}{12}} = \sqrt{\frac{(10)(10)(21)}{12}}$$

$$\sigma_R = \sqrt{175} = 13.23$$

$$Z = \frac{R - \mu_R}{\sigma_R} = \frac{98 - 105}{13.23} = -0.53$$



Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in the number of credits transferred.

6.

H₀: There is no difference in the lifetimes of the two brands of video games. (claim)

H₁: There is a difference in the lifetimes of the two brands of video games.

C. V. = ± 2.58

22	28	29	32	34	34	38	38
1	2	3	4	5.5	5.5	7.5	7.5
A	A	B	B	A	A	B	B

39	39	39	41	42	42	43
10	10	10	12	13.5	13.5	15.5
A	B	A	A	A	A	B

6. continued

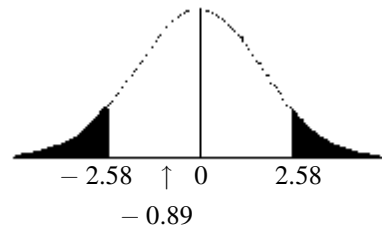
43	44	45	47	49	51	53
15.5	17	18	19	20	21	22
B	B	B	A	B	A	B

R = 113

$$\mu_R = \frac{11(11 + 11 + 1)}{2} = 126.5$$

$$\sigma_R = \sqrt{\frac{11(11)(23)}{12}} = 15.23$$

$$Z = \frac{113 - 126.5}{15.23} = -0.89$$



Do not reject the null hypothesis. There is not enough evidence to reject the claim that there is no difference in the lifetimes of the two brands of video games.

7.

H₀: There is no difference between the stopping distances of the two types of automobiles. (claim)

H₁: There is a difference between the stopping distances of the two types of automobiles.

C. V. = ± 1.65

186	187	188	188	190	192	193
1	2	3.5	3.5	5	6	7
M	M	M	M	M	M	C

194	195	196	198	200	203	204
8	9	10	11	12	13	14
M	M	C	C	C	M	C

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7. continued

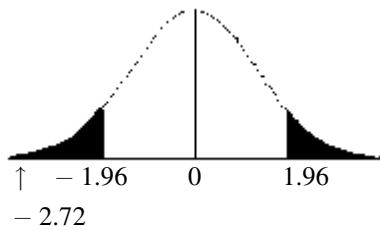
206	211	212	214	218	297
15	16	17	18	19	20
C	C	C	M	C	C

R = 69

$$\mu_R = \frac{10(10+10+1)}{2} = 105$$

$$\sigma_R = \sqrt{\frac{10 \cdot 10(10+10+1)}{12}} = 13.23$$

$$z = \frac{69-105}{13.23} = -2.72$$



Reject the null hypothesis. There is enough evidence to reject the claim that there is no difference in the stopping distances of the two types of automobiles.

8.

H_0 : There is no difference in the number of wins.

H_1 : There is a difference in the number of wins. (claim)

C. V. = ± 1.96 $\alpha = 0.05$

86	88	88	89	89	90	91	91	92
1	2.5	2.5	4.5	4.5	6	7.5	7.5	9
AL	NL	AL	NL	AL	NL	AL	NL	NL
95	95	95	96	96	97	100	100	
11	11	11	13.5	13.5	15	16.5	16.5	
AL	AL	NL	NL	NL	AL	AL	AL	
101	101	102	104	108	108			
18.5	18.5	20	21	22.5	22.5			
AL	AL	AL	AL	AL	AL			

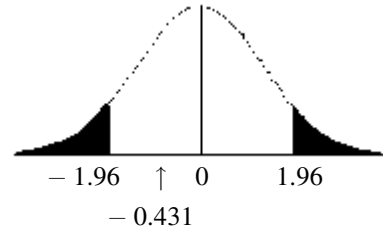
8. continued

R = 125

$$\mu_R = \frac{11(11+12+1)}{2} = 132$$

$$\sigma_R = \sqrt{\frac{11 \cdot 12(11+12+1)}{12}} = 16.248$$

$$Z = \frac{125-132}{16.248} = -0.431$$



Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in the number of wins.

9.

H_0 : There is no difference in the number of hunting accidents in the two regions.

H_1 : There is a difference in the number of hunting accidents in the two regions. (claim)

C. V. = ± 1.96

2	3	5	5	6	7	8	8	9	10	11
1	2	3.5	3.5	5	6	7.5	7.5	9	10	12
E	E	E	E	E	E	E	W	W	W	W
11	11	13	13	14	15	17	17	21		
12	12	14.5	14.5	16	17	18.5	18.5	20		
W	E	E	W	E	W	W	W	W		

R = 71

$$\mu_R = \frac{n_1(n_1+n_2+1)}{2} = \frac{10(10+10+1)}{2} = 105$$

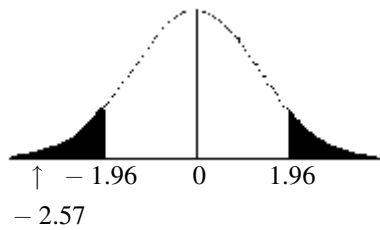
$$\sigma_R = \sqrt{\frac{n_1 \cdot n_2(n_1+n_2+1)}{12}}$$

$$\sigma_R = \sqrt{\frac{10 \cdot 10(10+10+1)}{12}} = 13.23$$

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9. continued

$$Z = \frac{R - \mu_R}{\sigma_R} = \frac{71 - 105}{13.23} = -2.57$$



Reject the null hypothesis. There is enough evidence to support the claim that there is a difference in the number of accidents in the two areas. The number of accidents may be related to the number of hunters in the two areas.

10.

H_0 : There is no difference in the size of enrollments.

H_1 : There is a difference in the size of enrollments. (claim)

$$C. V. = \pm 1.96$$

217	293	320	427	442	443	474
1	2	3	4	5	6	7.5
R	PC	PC	PC	PC	R	R

474	546	555	565	577	587	605
7.5	9	10	11	12	13	14.5
PC	PC	PC	R	R	PC	R

605	662	663	692	696	783	813
14.5	16	17	18	19	20	21
PC	PC	R	R	R	PC	R

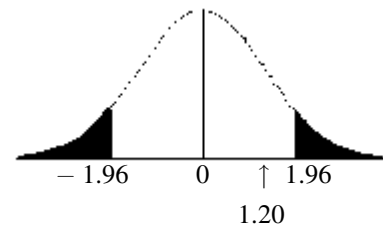
$$R = 127$$

$$\mu_R = \frac{10(10 + 11 + 1)}{2} = 110$$

$$\sigma_R = \sqrt{\frac{10 \cdot 11(10 + 11 + 1)}{12}} = 14.201$$

$$Z = \frac{127 - 110}{14.201} = 1.20$$

10. continued



Do not reject the null hypothesis. There is not enough evidence to conclude that there is a difference in enrollments.

11.

H_0 : There is no difference in job satisfaction.

H_1 : There is a difference in time until pain is relieved. (claim)

$$C. V. = \pm 1.65$$

51	57	57	59	62	63	65	68	69
1	2.5	2.5	4	5	6	7	8	9
M	U	M	M	M	M	M	U	U

71	73	76	78	79	80	83
10	11	12	13	14	15	16
M	U	U	U	U	M	M

84	85	86	87	92	94	95	96	98	99
17	18	19	20	21	22	23	24	25	26
U	U	M	U	M	U	M	M	U	U

$$R = 153.5$$

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} = \frac{13(13 + 13 + 1)}{2}$$

$$\mu_R = \frac{13(27)}{2} = 175.5$$

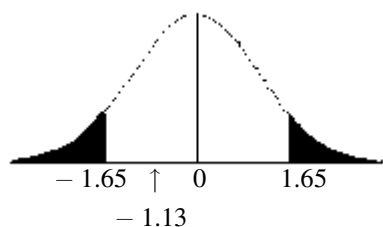
$$\sigma_R = \sqrt{\frac{n_1 \cdot n_2 (n_1 + n_2 + 1)}{12}}$$

$$\sigma_R = \sqrt{\frac{13 \cdot 13(13 + 13 + 1)}{12}} = 19.5$$

$$Z = \frac{R - \mu_R}{\sigma_R} = \frac{153.5 - 175.5}{19.5} = -1.13$$

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11. continued



Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in job satisfaction between the two groups.

12.

H_0 : There is no difference in the number of members of fraternities and sororities who participated in the blood drive. (claim)

H_1 : There is a difference in the number of members of fraternities and sororities who participated in the blood drive.

C. V. = ± 1.65

3	4	4	5	5	6	7	7	7
S	F	S	F	S	S	F	F	S
1	2.5	2.5	4.5	4.5	6	8.5	8.5	8.5

7	9	9	10	10	11	12	12
S	S	S	F	S	F	F	F
8.5	11	12	13.5	13.5	15	17	17

12	12	13	14	15	15
F	F	F	S	F	F
17	17	19	20	21.5	21.5

$R = 87.5$

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2}$$

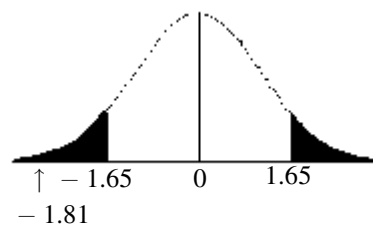
$$\mu_R = \frac{10(10 + 12 + 1)}{2} = 115$$

$$\sigma_R = \sqrt{\frac{n_1 \cdot n_2 (n_1 + n_2 + 1)}{12}}$$

$$\sigma_R = \sqrt{\frac{10 \cdot 12 (10 + 12 + 1)}{12}} = 15.17$$

$$Z = \frac{R - \mu_R}{\sigma_R} = \frac{87.5 - 115}{15.17} = -1.81$$

12. continued



Reject the null hypothesis. There is enough evidence to reject the claim that there is no difference in the number of fraternity and sorority members participating in student blood drives.

EXERCISE SET 13-4

1.

The t-test for dependent samples

2.

The Wilcoxon rank sum test is used when the random samples are independent while the Wilcoxon signed-rank test is used when the random samples are dependent.

3.

B	A	B - A	B - A	Rank	Signed Rank
106	112	-6	6	3	-3
85	84	1	1	1	1
117	105	12	12	6.5	6.5
163	167	-4	4	2	-2
154	142	12	12	6.5	6.5
106	113	-7	7	4	-4
152	143	9	9	5	5

Sum of the - ranks: $(-3) + (-2) + (-4) = -9$.

Sum of the + ranks:

$1 + 6.5 + 6.5 + 5 = 19$

$w_s = 9$

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4.

B	A	B - A	B - A	Rank	Signed Rank
25	29	-4	4	2.5	-2.5
38	45	-7	7	5	-5
62	51	11	11	7	7
49	45	4	4	2.5	2.5
63	71	-8	8	6	-6
29	32	-3	3	1	-1
74	74	0			
82	87	-5	5	4	-4

Sum of the - ranks: -18.5

Sum of the + ranks: 9.5

$$w_s = 9.5$$

5.

$$C. V. = 16 \quad w_s = 13$$

Since $13 \leq 16$, reject the null hypothesis.

6.

$$C. V. = 117 \quad w_s = 32$$

Since $32 \leq 117$, reject the null hypothesis.

7.

$$C. V. = 60 \quad w_s = 65$$

Since $65 > 60$, do not reject the null hypothesis.

8.

$$C. V. = 26 \quad w_s = 22$$

Since $22 \leq 26$, reject the null hypothesis.

9.

H_0 : The human dose is equal to the animal dose.

H_1 : The human dose is more than the animal dose. (claim)

9. continued

H	A	H - A	H - A	Rank	Signed Rank
0.67	0.13	0.54	0.54	7	+7
0.64	0.18	0.46	0.46	6	+6
1.20	0.42	0.78	0.78	8	+8
0.51	0.25	0.26	0.26	4	+4
0.87	0.57	0.30	0.30	5	+5
0.74	0.57	0.17	0.17	3	+3
0.50	0.49	0.01	0.01	1	+1
1.22	1.28	-0.06	0.06	2	-2

Sum of the - ranks: -2

Sum of the + ranks: +34

$$n = 8 \quad C. V. = 6$$

$$w_s = |-2| = 2$$

Since $2 < 6$, reject the null hypothesis.

There is enough evidence to support the claim that the human dose costs more than the animal dose. One reason is that some people might not be inclined to pay a lot of money for their pets' medication.

10.

H_0 : There is no difference in the assessed values.

H_1 : There is a difference in the assessed values. (claim)

$$n = 11 \quad \alpha = 0.05 \quad C. V. = 11$$

B	A	B - A	B - A	Rank	Signed Rank
184	161	23	23	7	+7
414	382	32	32	8	+8
22	22	0	0		
99	190	-91	91	9	-9
116	120	-4	4	2.5	-2.5
49	52	-3	3	1	-1
24	28	-4	4	2.5	-2.5
50	50	0	0		
282	297	-15	15	5.5	-5.5
25	40	-15	15	5.5	-5.5
141	148	-7	7	4	-4

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10. continued

Sum of the $-$ ranks:

$$(-9) + (-2.5) + (-1) + (-2.5) + (-5.5) + (-5.5) + (-4) = -30$$

Sum of the $+$ ranks: $7 + 8 = 15$

$$w_s = 15$$

Since $15 > 11$, do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in assessed values for the properties. Land values are probably not normally distributed.

11.

H_0 : The amount spent on lottery tickets does not change.

H_1 : The amount spent on lottery tickets is reduced. (claim)

$$n = 8 \quad \alpha = 0.05 \quad C. V. = 6$$

B	A	B - A	B - A	Rank	Signed Rank
86	72	14	14	7	+ 7
150	143	7	7	5	+ 5
161	123	38	38	8	+ 8
197	186	11	11	6	+ 6
98	102	- 4	4	3.5	- 3.5
56	53	3	3	1.5	+ 1.5
122	125	- 3	3	1.5	- 1.5
76	72	4	4	3.5	+ 3.5

Sum of the $-$ ranks: $- 5$

Sum of the $+$ ranks: $+ 31$

$$w_s = 5$$

Since $5 \leq 6$, reject the null hypothesis. There is enough evidence to support the claim that the workshop reduced the amount the participants spent on lottery tickets.

12.

H_0 : There is no difference in legal costs.

H_1 : There is a difference in legal costs. (claim)

$$n = 8 \quad \alpha = 0.05 \quad C. V. = 4$$

12. continued

1	2	1 - 2	1 - 2	Rank	Signed Rank
108	138	- 30	30	6	- 6
36	28	8	8	2.5	2.5
65	67	- 2	2	1	- 1
108	181	- 73	73	8	- 8
87	97	- 10	10	4	- 4
94	126	- 32	32	7	- 7
10	18	- 8	8	2.5	- 2.5
40	67	- 27	27	5	- 5

Sum of the $-$ ranks: $- 33.5$

Sum of the $+$ ranks: 2.5

$$w_s = 2.5$$

Since $2.5 \leq 4$, reject the null hypothesis. There is enough evidence to conclude that there is a difference in legal costs.

13.

H_0 : The prices of prescription drugs in the United States are equal to the prices in Canada.

H_1 : The drugs sold in Canada are cheaper. (claim)

$$n = 10 \quad \alpha = 0.05 \quad C. V. = 11$$

U. S.	C	US - C	US - C	Rank	Signed Rank
3.31	1.47	1.84	1.84	8	+ 8
2.27	1.07	1.20	1.20	4.5	+ 4.5
2.54	1.34	1.20	1.20	4.5	+ 4.5
3.13	1.34	1.79	1.79	7	+ 7
23.40	21.44	1.94	1.94	10	+ 10
3.16	1.47	1.69	1.69	6	+ 6
1.98	1.07	0.91	0.91	3	+ 3
5.27	3.39	1.88	1.88	9	+ 9
1.96	2.22	- 0.26	0.26	2	- 2
1.11	1.13	- 0.02	0.02	1	- 1

Sum of the $+$ ranks:

$$8 + 4.5 + 4.5 + 7 + 10 + 6 + 3 + 9 = 52$$

Sum of the $-$ ranks: $(- 2) + (- 1) = - 3$

$$w_s = |- 3| = 3$$

Since $3 < 11$, reject the null hypothesis. There is enough evidence to support the claim that the drugs are less expensive in Canada.

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14.

H_0 : There is no difference in the scores of the bowlers who use two different bowling balls.

H_1 : The bowling scores were higher when the bowlers used a new bowling ball.

(claim)

$$n = 8 \quad \alpha = 0.05 \quad C. V. = 4$$

Bowler	Day 1	Day 2	1 - 2	Rank	Signed Rank
A	141	158	17	1	- 1
B	176	144	32	4	+ 4
C	178	135	43	7	+ 7
D	174	153	21	2	+ 2
E	135	195	60	8	- 8
F	190	151	39	5	+ 5
G	182	151	31	3	+ 3
H	141	183	42	6	- 6

Sum of the + ranks: 21

Sum of the - ranks: - 15

$$w_s = |-15| = 15$$

Since $15 > 4$, do not reject the null hypothesis. There is not enough evidence to conclude that the bowling scores were higher when the bowlers used a new bowling ball.

EXERCISE SET 13-5

1.

H_0 : There is no difference in the results of the questionnaires among the three groups.

H_1 : There is a difference in the results of the questionnaires among the three groups.

(claim)

$$C. V. = 5.991$$

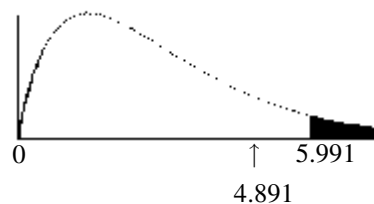
A	Rank	B	Rank	C	Rank
22	9	18	6.5	16	3
25	12.5	24	11	17	4.5
27	15.5	25	12.5	19	8
26	14	27	15.5	23	10
33	21	29	17	18	6.5

1. continued

A	Rank	B	Rank	C	Rank
35	22	31	19.5	31	19.5
30	18	17	4.5	15	1.5
36	23.5	15	1.5	36	23.5
$R_1 =$	135.5	$R_2 =$	88	$R_3 =$	76.5

$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} \right) - 3(N+1)$$

$$H = \frac{12}{24(24+1)} \left(\frac{135.5^2}{8} + \frac{88^2}{8} + \frac{76.5^2}{8} \right) - 3(24+1) = 4.891$$



Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in the results of the questionnaire.

2.

H_0 : There is no difference in mathematical literacy scores of the three groups.

H_1 : There is a difference in mathematical literacy scores of the three groups. (claim)

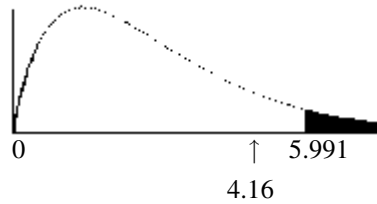
$$C. V. = 5.991$$

Western	Rank	Europe	Rank	Asia	Rank
527	11	520	9	523	10
406	3	510	7	547	12.5
474	5	513	8	547	12.5
381	1	548	14	391	2
411	4	496	6	549	15
$R_1 =$	24	$R_2 =$	44	$R_3 =$	52

$$H = \frac{12}{15(15+1)} \left(\frac{24^2}{5} + \frac{44^2}{5} + \frac{52^2}{5} \right) - 3(15+1) = 4.16$$

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2. continued



Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in mathematical literacy scores.

3.

H_0 : There is no difference in the scores on the questionnaire.

H_1 : There is a difference in the scores on the questionnaire. (claim)

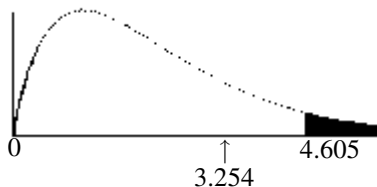
C. V. = 4.605

NM	Rank	M	Rank	D	Rank
37	14.5	40	19	38	16
39	17.5	36	13	35	12
32	8	32	8	21	2
31	5.5	33	10.5	19	1
37	14.5	39	17.5	31	5.5
32	8	33	10.5	24	3
		30	4		
$R_1 =$	68	$R_2 =$	82.5	$R_3 =$	39.5

$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} \right) - 3(N+1)$$

$$H = \frac{12}{19(19+1)} \left(\frac{68^2}{6} + \frac{82.5^2}{7} + \frac{39.5^2}{6} \right) - 3(19+1)$$

$$H = 3.254$$



3. continued

Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in the results of the questionnaire.

4.

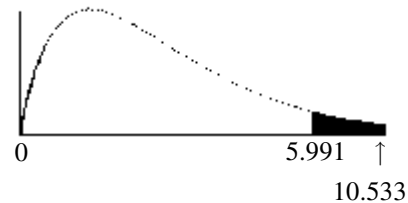
H_0 : There is no difference in the amounts of sodium in the different brands of microwave dinners.

H_1 : There is a difference in the amounts of sodium in the different brands of microwave dinners. (claim)

C. V. = 5.991

A	Rank	B	Rank	C	Rank
810	12	917	18	893	15.5
702	5	912	17	790	11
853	13	952	19	603	1
703	6	958	20	744	10
892	14	893	15.5	623	4
732	8			743	9
713	7			609	2
613	3				
$R_1 =$	68	$R_2 =$	89.5	$R_3 =$	52.5

$$H = \frac{12}{20(20+1)} \left(\frac{68^2}{8} + \frac{89.5^2}{5} + \frac{52.5^2}{7} \right) - 3(20+1) = 10.533$$



Reject the null hypothesis. There is enough evidence to support the claim that there is a difference in the amounts of sodium in the different brands of microwave dinners.

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5.

H_0 : There is no difference in the sugar content of the three different types of candy bars.

H_1 : There is a difference in the sugar content of the three different types of candy bars. (claim)

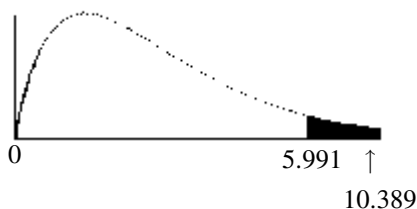
C. V. = 5.991

A	Rank	B	Rank	C	Rank
7.6	1	9.2	3	18.6	16
9.3	4	11.1	9	16.2	13
8.4	2	12.3	11	15.4	12
11.3	10	10.1	6	18	15
10.2	7.5	10.2	7.5	17.3	14
9.8	5				
$R_1 =$	29.5	$R_2 =$	36.5	$R_3 =$	70

$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} + \frac{R_4^2}{n_4} \right) - 3(N+1)$$

$$= \frac{12}{16(16+1)} \left(\frac{29.5^2}{6} + \frac{36.5^2}{5} + \frac{70^2}{2} \right) - 3(16+1)$$

$H = 10.389$



Reject the null hypothesis. There is enough evidence to support the claim that the sugar content of three of candy bars is different.

6.

H_0 : There is no difference in the number of job offers received by each group.

H_1 : There is a difference in the number of job offers received by each group. (claim)

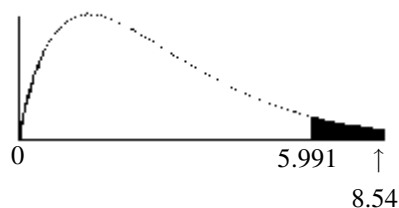
C. V. = 5.991

6. continued

A	Rank	B	Rank	C	Rank
6	8	2	3	10	13
8	11	1	2	12	14
7	10	0	1	9	12
5	6	3	4	13	15
6	8	6	8	4	5
$R_1 =$	43	$R_2 =$	18	$R_3 =$	59

$$H = \frac{12}{15(15+1)} \left(\frac{43^2}{5} + \frac{18^2}{5} + \frac{59^2}{5} \right) - 3(15+1)$$

$H = 8.54$



Reject the null hypothesis. There is enough evidence to support the claim that the number of job offers is different.

7.

H_0 : There is no difference in spending between regions.

H_1 : There is a difference in spending between regions.

C. V. = 5.991

E	Rank	M	Rank	W	Rank
6701	3	9854	15	7584	10
6708	4	8414	11	5474	1
9186	12	7279	7	6622	2
6786	5	7311	8	9673	14
9261	13	6947	6	7353	9
$R_1 =$	37	$R_2 =$	47	$R_3 =$	36

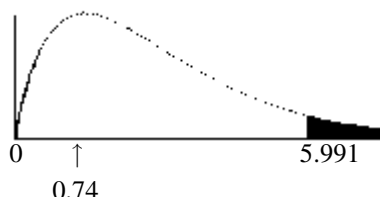
$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} + \frac{R_4^2}{n_4} \right) - 3(N+1)$$

$$H = \frac{12}{15(15+1)} \left(\frac{37^2}{5} + \frac{47^2}{5} + \frac{36^2}{5} \right) - 3(15+1)$$

$H = 0.74$

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7. continued



Do not reject the null hypothesis. There is not enough evidence to conclude that there is a difference in spending.

8.

H_0 : There is no difference in the prices of the three types of printer.

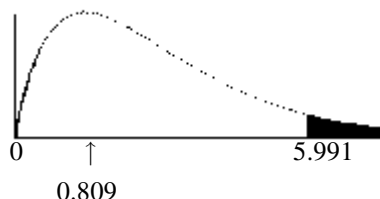
H_1 : There is a difference in the prices of the three types of printer. (claim)

C. V. = 5.991

Ink	Rank	Multi	Rank	Laser	Rank
149	6.5	98	2	192	9
199	12.5	119	5	159	8
249	16.5	149	6.5	198	10.5
239	15	249	16.5	198	10.5
99	3.5	99	3.5	229	14
79	1	199	12.5		
$R_1 =$	55	$R_2 =$	46	$R_3 =$	52

$$H = \frac{12}{17(17+1)} \left(\frac{55^2}{6} + \frac{46^2}{6} + \frac{52^2}{5} \right) - 3(17+1)$$

$H = 0.809$



Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in the prices of the printers. No, based on these samples one cannot conclude that one type of printer generally costs more than another type.

9.

H_0 : There is no difference in the number of crimes in the 5 precincts.

H_1 : There is a difference in the number of crimes in the 5 precincts. (claim)

C. V. = 13.277

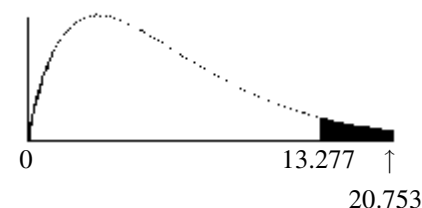
1	Rank	2	Rank	3	Rank
105	24	87	13	74	7.5
108	25	86	12	83	11
99	22	91	16	78	9
97	20	93	18	74	7.5
92	17	82	10	60	5
$R_1 =$	108	$R_2 =$	69	$R_3 =$	40

4	Rank	5	Rank
56	3	103	23
43	1	98	21
52	2	94	19
58	4	89	15
62	6	88	14
$R_4 =$	16	$R_5 =$	92

$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} + \frac{R_4^2}{n_4} + \frac{R_5^2}{n_5} \right) - 3(N+1)$$

$$H = \frac{12}{25(25+1)} \left(\frac{108^2}{5} + \frac{69^2}{5} + \frac{40^2}{5} + \frac{16^2}{5} + \frac{92^2}{5} \right) - 3(25+1)$$

$H = 20.753$



Reject the null hypothesis. There is enough evidence to support the claim that there is a difference in the number of crimes for the precincts.

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10.

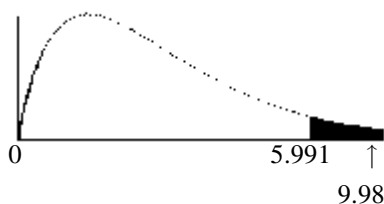
H_0 : There is no difference in caffeine content.

H_1 : There is a difference in caffeine content. (claim)

C. V. = 5.991

Tea	Rank	Coffee	Rank	Cola	Rank
70	10	120	13	35	3
40	4.5	80	11	48	8
30	2	160	15	55	9
25	1	90	12	43	7
40	4.5	140	14	42	6
$R_1 =$	22	$R_2 =$	65	$R_3 =$	33

$$H = \frac{12}{15(15+1)} \left(\frac{22^2}{5} + \frac{65^2}{5} + \frac{33^2}{5} \right) - 3(15+1) = 9.98$$



Reject the null hypothesis. There is enough evidence to conclude that there is a difference in caffeine content.

11.

H_0 : There is no difference in speeds.

H_1 : There is a difference in speeds. (claim)

C. V. = 5.991

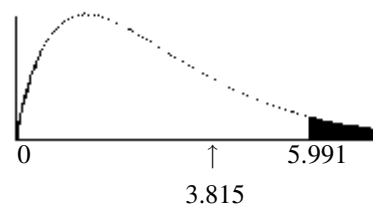
Pred.	Rank	Deer	Rank	Dom.	Rank
70	15	50	12.5	47.5	11
50	12.5	35	5.5	39.35	7
43	10	32	4	35	5.5
42	9	30	2.5	30	2.5
40	8	61	14	11	1
$R_1 =$	54.5	$R_2 =$	38.5	$R_3 =$	27

$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} \right) - 3(N+1)$$

11. continued

$$H = \frac{12}{15(15+1)} \left(\frac{54.5^2}{5} + \frac{38.5^2}{5} + \frac{27^2}{5} \right) - 3(15+1)$$

$$H = 3.815$$



Do not reject. There is not enough evidence to conclude there is a difference in speeds.

12.

H_0 : There is no difference in the prices of the supplements at the different stores.

H_1 : There is a difference in the prices of the supplements at the different stores. (claim)

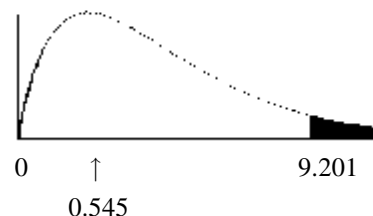
C. V. = 9.201

Groc.	Rank	Drug	Rank	Disc.	Rank
6.79	8	7.69	12.5	7.49	11
6.09	4	8.19	15	6.89	9
5.49	3	6.19	7	7.69	12.5
7.99	14	5.15	2	7.29	10
6.10	5	6.14	6	4.95	1
$R_1 =$	34	$R_2 =$	42.5	$R_3 =$	43.5

$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} \right) - 3(N+1)$$

$$H = \frac{12}{15(15+1)} \left(\frac{34^2}{5} + \frac{42.5^2}{5} + \frac{43.5^2}{5} \right) - 3(15+1)$$

$$H = 0.545$$



Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in the prices of the supplements at the different stores.

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EXERCISE SET 13-6

1.
0.392

2.
0.515

3.
0.783

4.
0.566

5.
 $H_0: \rho = 0$
 $H_1: \rho \neq 0$
C. V. = ± 0.786

Grade 4	R_1	Grade 8	R_2	$R_1 - R_2$	d^2
90	7	81	7	0	0
84	4	75	4	0	0
80	3	66	2	1	1
87	5	76	5	0	0
88	6	80	6	0	0
77	1	59	1	0	0
79	2	74	3	-1	1
					$\sum d^2 = 2$

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)} = 1 - \frac{6 \cdot 2}{7(7^2 - 1)}$$

$$r_s = 0.964$$

Reject the null hypothesis. There is a significant relationship between the mathematics achievement scores for the two grades.

6.
 $H_0: \rho = 0$
 $H_1: \rho \neq 0$
C. V. = ± 0.886

6. continued

Subway	R_1	Rail	R_2	$R_1 - R_2$	d^2
845	6	39	3	3	9
494	5	291	6	-1	1
425	4	142	5	-1	1
313	3	103	4	-1	1
108	2	33	1	1	1
41	1	38	2	-1	1
					$\sum d^2 = 14$

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

$$r_s = 1 - \frac{6 \cdot 14}{6(6^2 - 1)} = 0.6$$

Do not reject. There is no significant linear relationship between subway and commuter rail trips.

7.
 $H_0: \rho = 0$
 $H_1: \rho \neq 0$
C. V. = ± 0.700

Releases	R_1	Receipts	R_2	$R_1 - R_2$	d^2
361	9	3844	9	0	0
270	7	1967	8	-1	1
306	8	1371	7	1	1
22	5	1064	6	-1	1
35	6	667	5	1	1
10	2	241	4	-2	4
8	1	188	3	-2	4
12	3	154	2	1	1
21	4	125	1	3	9
					$\sum d^2 = 22$

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

$$r_s = 1 - \frac{6 \cdot 22}{9(9^2 - 1)} = 0.817$$

Reject. There is a significant relationship between the number of new releases and gross receipts.

Chapter 13 - Nonparametric Statistics

8.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.786$$

Hospitals	R ₁	Nursing Homes	R ₂	R ₁ - R ₂	d ²
107	2	230	2	0	0
61	1	134	1	0	0
202	7	704	7	0	0
133	5	376	4	1	1
145	6	431	5	1	1
117	4	538	6	-2	4
108	3	373	3	0	0
					$\sum d^2 = 6$

$$r_s = 1 - \frac{6 \cdot 6}{7(7^2 - 1)} = 0.893$$

Reject the null hypothesis. There is a significant relationship between the number of hospitals and the number of nursing homes in a state.

9.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.738$$

Calories	R ₁	Cholesterol	R ₂	R ₁ - R ₂	d ²
580	7.5	205	3	4.5	20.25
580	7.5	225	6	1.5	2.25
270	1	285	8	-7	49
470	6	270	7	-1	1
420	4	185	1.5	2.5	6.25
415	3	215	4	-1	1
330	2	185	1.5	0.5	0.25
430	5	220	5	0	0
					$\sum d^2 = 80$

$$r_s = 1 - \frac{6 \cdot 80}{8(8^2 - 1)} = 1 - \frac{480}{504} = 0.048$$

Do not reject the null hypothesis. There is not enough evidence to say that a significant relationship exists between calories and cholesterol amounts in fast-food sandwiches.

10.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.886$$

Consumer group	R ₁	Testing lab	R ₂	R ₁ - R ₂	d ²
6	1	9	2	1	1
15	4	13	4	0	0
20	6	18	5	-1	1
9	2	2	1	-1	1
17	5	19	6	1	1
12	3	11	3	0	0
					$\sum d^2 = 4$

$$r_s = 1 - \frac{6 \cdot 4}{6(6^2 - 1)} = 0.8857$$

Very close! Do not reject the null hypothesis. There is no significant relationship between the ratings of the consumer group and the testing lab.

11.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.786$$

Instructor	R ₁	Student	R ₂	R ₁ - R ₂	d ²
1	1	2	2	-1	1
4	4	6	6	-2	4
6	6	7	7	-1	1
7	7	5	5	2	4
5	5	4	4	1	1
2	2	3	3	-1	1
3	3	1	1	2	4
					$\sum d^2 = 16$

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)} = 1 - \frac{6 \cdot 16}{7(7^2 - 1)} = 0.714$$

Do not reject the null hypothesis. There is not enough evidence to say that there is a relationship in the rankings of the textbook between the instructors and the students.

Chapter 13 - Nonparametric Statistics

12.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.886$$

Theft	R ₁	Burglary	R ₂	R ₁ - R ₂	d ²
220.5	3	913.6	5	-2	4
499.4	6	909.2	4	2	4
285.6	4	803.6	3	1	1
159.2	2	520.9	2	0	0
104.3	1	477.8	1	0	0
444	5	993.7	6	-1	1
					$\sum d^2 = 10$

$$r_s = 1 - \frac{6 \cdot 10}{6(6^2 - 1)} = 0.714$$

Do not reject the null hypothesis. There is not enough evidence to conclude a significant relationship between the number of vehicle thefts and burglaries.

13.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.900$$

Students	R ₁	Cost	R ₂	R ₁ - R ₂	d ²
10	3	7200	2	1	1
6	1	9393	5	-4	16
17	5	7385	3	2	4
8	2	4500	1	1	1
11	4	8203	4	0	0
					$\sum d^2 = 22$

$$r_s = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

$$r_s = 1 - \frac{6 \cdot 22}{5(5^2 - 1)} = -0.10$$

Do not reject the null hypothesis. There is no significant relationship between the number of cyber school students and the cost per pupil. In this case, the cost per pupil is different in each district.

14.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.643$$

Humans	R ₁	Animals	R ₂	R ₁ - R ₂	d ²
0.67	4	0.13	1	3	9
0.64	3	0.18	2	1	1
1.20	7	0.42	4	3	9
0.51	2	0.25	3	-1	1
0.87	6	0.57	6	0	0
0.74	5	0.58	7	-2	4
0.50	1	0.49	5	-4	16
1.22	8	1.28	8	0	0
					$\sum d^2 = 40$

$$r_s = 1 - \frac{6 \cdot 40}{8(8^2 - 1)} = 0.524$$

Do not reject the null hypothesis. There is no significant relationship between the costs of the drugs.

15.

H₀: The occurrence of cavities is random.

H₁: The null hypothesis is not true.

The median of the data set is two. Using A = above and B = below, the runs (going across) are shown:

B AA B AAA B A BB AAAA B A B
A B A B A B AAA B A BB

There are 21 runs. The expected number of runs is between 10 and 22. Therefore, the null hypothesis should not be rejected. The number of cavities occurs at random.

16.

H₀: The numbers occur at random. (claim)

H₁: The null hypothesis is not true.

EE O EE OO EE OO EE OO EEEEE O
EEE O E O

Chapter 13 - Nonparametric Statistics

16. continued

There are 14 runs, and since this value is between 8 and 20, the decision is do not reject the null hypothesis. There is not enough evidence to reject the claim of randomness.

17.

H_0 : The type of admissions occur at random. (claim)

H_1 : The null hypothesis is not true.

SS F S F S FF S F SS FFF S FF SSS FFFF S
FFF S F SS F SS FF SS

There are 23 runs and this value is between 13 and 27. Hence, do not reject the null hypothesis. The admissions occur at random.

18.

H_0 : The integers generated by a calculator occur at random. (claim)

H_1 : The null hypothesis is not true.

111111 2 1111 22 1 2 1 22 1 2 11 2 11

There are 13 runs and since this is between 7 and 17, the null hypothesis is not rejected. The integers are random.

19.

H_0 : The ups and downs in the stock market occur at random. (claim)

H_1 : The ups and downs in the stock market do not occur at random.

UUUU DD UUU D UUUUU DDD U D

There are 8 runs and since this is between 5 and 15, the null hypothesis is not rejected. The ups and downs in the stock market occur at random.

20.

H_0 : The gender of the shoppers occurs at random. (claim)

H_1 : The null hypothesis is not true.

20. continued

F MM FF M F MM FF MMM FFFF
M

There are 10 runs. The expected number of runs is between 6 and 16, hence the null hypothesis is not rejected. There is not enough evidence to reject the hypothesis that the gender of the shoppers in line is random.

21.

H_0 : The number of absences of employees occurs at random. (claim)

H_1 : The null hypothesis is not true.

The median of the data is 12. Using A = above and B = below, the runs are shown as follows:

A B AAAAAAA BBBB BBBB AAAAA
BBBB

There are 6 runs. The expected number of runs is between 9 and 21, hence the null hypothesis is rejected since 6 is not between 9 and 21. The number of absences do not occur at random.

22.

H_0 : The days customers are able to ski occur at random. (claim)

H_1 : The null hypothesis is not true.

SSSSSS NNNNNNNNN SSS NN
SSSSSSSS

There are 5 runs. The expected number of runs is between 9 and 20, hence the null hypothesis should be rejected. There is enough evidence to reject the claim that the days customers are able to ski occur at random.

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23.

H_0 : The number of on-demand movie rentals occurs at random. (claim)

H_1 : The null hypothesis is not true.

The median of the data is 6.5. Using A = above and B = below, the runs are shown as follows:

BB AAA BB A B A BB AAAA BBB A

There are 10 runs. The expected number of runs is between 6 and 16, hence the null hypothesis is not rejected since 10 is between 6 and 16. The number of movie rentals occur at random.

24.

Answers will vary.

25.

H_0 : The gender of the patients at a medical center occurs at random. (claim)

H_1 : The null hypothesis is not true.

C. V. = ± 1.96

$n_1 = 28$ (men) and $n_2 = 22$ (women)

FF MMMMM FFF MMMMMM F MM
FFFF MMM F M F MMMMMM F MM F
M FF M FFFFFFF M

There are $G = 20$ runs. The test statistic is:

$$z = \frac{G - \mu_G}{\sigma_G}$$

$$\mu_G = \frac{2(28)(22)}{28+22} + 1 = 25.64$$

$$\sigma_G = \sqrt{\frac{2(28)(22)[2(28)(22) - 28 - 22]}{(28+22)^2(28+22-1)}} = 3.448$$

$$z = \frac{20 - 25.64}{3.448} = -1.636 \text{ or } -1.64$$

Do not reject the null hypothesis. There is not enough evidence to reject the claim that the sequence is random.

26.

H_0 : The gender of the speeders occurs at random. (claim)

H_1 : The null hypothesis is not true.

C. V. = ± 1.96

$n_1 = 30$ (males) and $n_2 = 20$ (females)

MMM FF M F M F MM F MMM F MM
FFF MM F MM F MMMMM F M FFF
MMM FF M FFF MMMM

There are $G = 25$ runs. The test statistic is:

$$z = \frac{G - \mu_G}{\sigma_G}$$

$$\mu_G = \frac{2(30)(20)}{30+20} + 1 = 25$$

$$\sigma_G = \sqrt{\frac{2(30)(20)[2(30)(20) - 30 - 20]}{(30+20)^2(30+20-1)}} = 11.265$$

$$z = \frac{25 - 25}{11.265} = 0$$

Do not reject the null hypothesis. There is not enough evidence to reject the claim that the sequence occurs at random

27.

H_0 : The patients who were treated for an accident or illness occur at random. (claim)

H_1 : The null hypothesis is not true.

C. V. = ± 1.96

$n_1 = 24$ (illness) and $n_2 = 36$ (accident)

I A I AAAAA I A II AA II AAA I A I
AAA II AA II AA I A I A I AA II AAA
II A I AA II AAA

There are $G = 34$ runs. The test statistic is:

$$z = \frac{G - \mu_G}{\sigma_G}$$

$$\mu_G = \frac{2(24)(36)}{24+36} + 1 = 29.8$$

$$\sigma_G = \sqrt{\frac{2(24)(36)[2(24)(36) - 24 - 36]}{(24+36)^2(24+36-1)}} = 3.684$$

Chapter 13 - Nonparametric Statistics

27. continued

$$z = \frac{34 - 29.8}{3.684} = 1.14$$

Do not reject the null hypothesis. There is not enough evidence to reject the claim that the sequence occurs at random.

28.

H_0 : The answers “true” and “false” occur at random. (claim)

H_1 : The answers do not occur at random.

FF TTT FFF T F TT FFFF TT FF TT
FFF T F TTT

There are 14 runs and since this is between 10 and 22, the null hypothesis is not rejected. Hence, the answers occur at random.

29.

$$r = \frac{\pm 1.96}{\sqrt{50-1}} = \pm 0.28$$

30.

$$r = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 2.58}{\sqrt{30-1}} = \pm 0.479$$

31.

$$r = \frac{\pm 2.33}{\sqrt{35-1}} = \pm 0.400$$

32.

$$r = \frac{\pm z}{\sqrt{n-1}} = \frac{\pm 1.65}{\sqrt{60-1}} = \pm 0.215$$

33.

$$r = \frac{\pm 2.58}{\sqrt{40-1}} = \pm 0.413$$

REVIEW EXERCISES - CHAPTER 13

1.

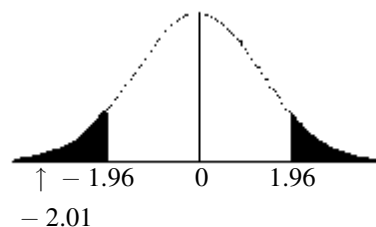
H_0 : median = \$9.00 (claim)

H_1 : median \neq \$9.00

1. continued

$$C. V. = \pm 1.96 \quad \alpha = 0.05$$

$$Z = \frac{(9 + 0.5) - \left(\frac{30}{2}\right)}{\frac{\sqrt{30}}{2}} = -2.01$$



Reject. There is enough evidence to reject the claim that the median price is \$9.00.

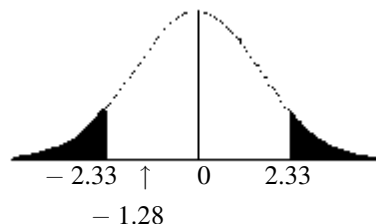
2.

H_0 : median = 420 minutes (claim)

H_1 : median \neq 420 minutes

$$C. V. = \pm 1.96$$

$$Z = \frac{(11 + 0.5) - \left(\frac{30}{2}\right)}{\frac{\sqrt{30}}{2}} = -1.28$$



Do not reject the null hypothesis. There is not enough evidence to reject the claim that the median is equal to 420 minutes.

3.

H_0 : There is no difference in prices.

H_1 : There is a difference in prices. (claim)

$$R^+ = 7, R^- = 1$$

$$C. V. = 0$$

$$\text{Test value} = 1$$

Do not reject the null hypothesis. There is not enough evidence to conclude a difference in prices. Comments: Examine what affects the result of this test.

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4.

H_0 : There is no difference in the record high temperatures of the two cities.

H_1 : There is a difference in the record high temperatures of the two cities.

(claim) $\alpha = 0.05$

47	47	50	51	51	52	52	57
W	W	W	W	W	D	D	D
1.5	1.5	3	4.5	4.5	6.5	6.5	8

60	66	66	69	71	80	80	86
D	D	W	W	D	W	D	W
9	10.5	10.5	12	13	14.5	14.5	17

86	86	88	89	89	91	93	94
W	D	D	D	W	W	D	D
17	17	19	20.5	20.5	22	23	24

$R = 128.5$ for Whitehorse

$$\mu_R = \frac{12(12 + 12 + 1)}{2} = 150$$

$$\sigma_R = \sqrt{\frac{12 \cdot 12(12 + 12 + 1)}{12}} = 17.32$$

$$Z = \frac{R - \mu_R}{\sigma_R} = \frac{128.5 - 150}{17.32} = -1.24$$

P-value = 0.2150

Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in the record high temperatures.

5.

H_0 : There is no difference in the amount of hours worked.

H_1 : There is a difference in the amount of hours worked.

C. V. = ± 1.645 at $\alpha = 0.10$

12	15	17	18	18	19	19	20	20
1	2	3	4.5	4.5	6.5	6.5	8.5	8.5
A	A	A	A	L	A	L	A	L

5. continued

21	22	22	24	24	25	25	26
10	11.5	11.5	13.5	13.5	15.5	15.5	17
L	A	L	A	L	L	A	L

28	30	31	35
18	19	20	21
L	A	L	L

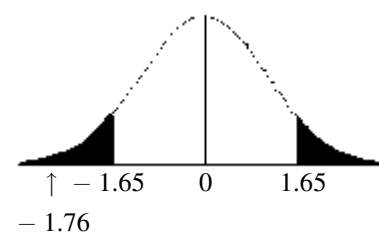
$R = 85$

$$\mu_R = \frac{n_1(n_1 + n_2 + 1)}{2} = \frac{10(10 + 11 + 1)}{2} = 110$$

$$\sigma_R = \sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}$$

$$\sigma_R = \sqrt{\frac{10 \cdot 11(10 + 11 + 1)}{12}} = 14.201$$

$$Z = \frac{R - \mu_R}{\sigma_R} = \frac{85 - 110}{14.201} = -1.76$$



Reject the null hypothesis. There is enough evidence to conclude a difference in the hours worked.

At $\alpha = 0.05$, C. V. = ± 1.96 . The decision would be to not reject the null hypothesis.

6.

H_0 : The additive did not improve the gas mileage.

H_1 : The additive did improve the gas mileage. (claim)

Chapter 13 - Nonparametric Statistics

6. continued

B	A	B - A	B - A	Rank	Signed Rank
13.6	22.6	-9.0	9.0	9	-9
18.2	21.9	-3.7	3.7	4	-4
16.1	25.3	-9.2	9.2	10	-10
15.3	28.6	-13.3	13.3	12	-12
19.2	15.2	4.0	4.0	5	5
18.8	16.3	2.5	2.5	3	3
18.3	23.7	-5.4	5.4	7	-7
19.5	20.8	-1.3	1.3	1.5	-1.5
18.2	25.3	-7.1	7.1	8	-8
16.7	27.2	-10.5	10.5	11	-11
21.3	17.2	4.1	4.1	6	6
17.2	18.5	-1.3	1.3	1.5	-1.5

Sum of + ranks = 14

Sum of - ranks = 53

C. V. = 14 $\alpha = 0.05$ $n = 11$

$w_s = 14$

Reject the null hypothesis. There is enough evidence to support the claim that the additive improved the gas mileage.

7.

H_0 : There is no difference in the amount spent.

H_1 : There is a difference in the amount spent. (claim)

B	A	B - A	B - A	Rank	Signed Rank
7	6	1	1	1	1
5.5	10	-4.5	4.5	7	-7
4.5	7	-2.5	2.5	6	-6
10	12	-2	2	4	-4
6.75	8.5	-1.75	1.75	2	-2
5	7	-2	2	4	-4
6	8	-2	2	4	-4

Sum of the + ranks: 1

Sum of the - ranks: -27

$w_s = 1$ C. V. = 2 $\alpha = 0.05$ $n = 7$

Reject the null hypothesis. There is enough evidence to conclude that there is a difference in amount spent at the 0.05 level of significance.

8.

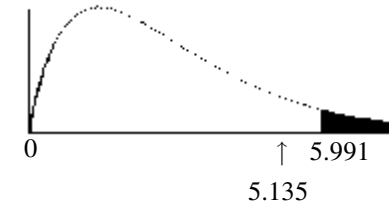
H_0 : There is no difference in the breaking strengths of the cables.

H_1 : There is a difference in the breaking strengths of the cables. (claim)

C. V. = 5.991 d. f. = 2 $\alpha = 0.05$

Manufacturer A	Rank	Manufacturer B	Rank	Manufacturer C	Rank
602	15	416	3.5	372	1
587	14	404	2	431	5
433	6	483	7	552	12.5
551	11	504	8	508	9
552	12.5	516	10	416	3.5
$R_1 =$	58.5	$R_2 =$	30.5	$R_3 =$	31

$$H = \frac{12}{15(16)} \left(\frac{58.5^2}{5} + \frac{30.5^2}{5} + \frac{31^2}{5} \right) - 3(15 + 1) = 5.135$$



Do not reject the null hypothesis. There is not enough evidence to support the claim that there is a difference in the breaking strengths of the cables.

9.

H_0 : There is no difference in beach temperatures.

H_1 : There is a difference in beach temperatures.

C. V. = 7.815

CI. Pac.	Rank	W. Gulf	Rank	E. Gulf	Rank
67	4	86	21	87	25
68	5	86	21	87	25
66	3	84	13	86	21
69	6	85	16.5	86	21
63	2	79	8	85	16.5
62	1	85	16.5	84	13
			.5	85	16.5
$R_1 =$	21	$R_2 =$	96	$R_3 =$	138

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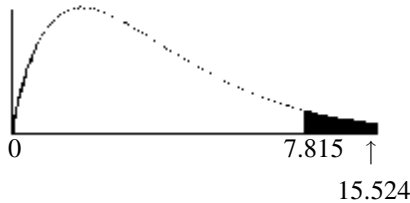
9. continued

S. Atl.	Rank
76	7
81	10
82	11
84	13
80	9
86	21
87	25
$R_4 =$	96

$$H = \frac{12}{N(N+1)} \left(\frac{R_1^2}{n_1} + \frac{R_2^2}{n_2} + \frac{R_3^2}{n_3} \right) - 3(N+1)$$

$$H = \frac{12}{26(26+1)} \left(\frac{21^2}{6} + \frac{96^2}{6} + \frac{138^2}{7} + \frac{96^2}{7} \right) - 3(26+1)$$

$$H = 15.524$$



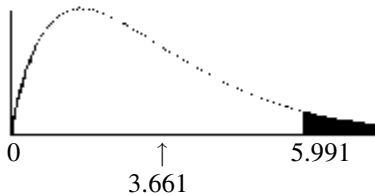
Reject the null hypothesis. There is enough evidence to conclude a difference in beach temperatures.

Without the Southern Pacific:

$$C. V. = 5.991 \text{ at } \alpha = 0.05$$

$$H = \frac{12}{20(20+1)} \left(\frac{60^2}{6} + \frac{96^2}{7} + \frac{54^2}{7} \right) - 3(20+1)$$

$$H = 3.661$$



Do not reject the null hypothesis. There is not enough evidence to conclude a difference in the temperatures. The conclusion is not the same without the Southern Pacific temperatures.

10.

Homework	R_1	Exam	R_2	$R_1 - R_2$	d^2
63	5	85	5	0	0
55	3	71	2	1	1
58	4	75	4	0	0
87	7	98	8	-1	1
89	8	93	7	1	1
52	2	63	1	1	1
46	1	72	3	-2	4
75	6	89	6	0	0
105	9	100	9	0	0
					$\sum d^2 = 8$

$$r_s = 1 - \frac{6.8}{9(9^2-1)} = 0.933$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.700$$

Reject. There is a significant relationship between the rankings.

11.

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.786$$

Pages	R_1	Sources	R_2	$R_1 - R_2$	d^2
15	1	10	1	0	0
25	4	18	4.5	-0.5	0.25
23	3	18	4.5	-1.5	2.25
30	6	15	3	3	9
18	2	13	2	0	0
28	5	23	7	-2	4
35	7	20	6	1	1
					$\sum d^2 = 16.5$

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)} = 1 - \frac{6(16.5)}{7(48)} = 0.7054$$

Do not reject the null hypothesis. There is not a significant relationship between the number of pages and the number of references.

Chapter 13 - Nonparametric Statistics

12.

H_0 : The scoring leaders of the NBA occur at random. (claim)

H_1 : The scoring leaders of the NBA do not occur at random.

W E WWW EE EE EEEEE W
EEEEEE W EEEEE

The number of runs is 10. This value is between 7 and 17. Do not reject. The number of scoring leaders occurs at random.

13.

H_0 : The grades of students who finish the exam occur at random. (claim)

H_1 : The null hypothesis is not true.

The median grade is 73. Using A = above and B = below, the runs are:

AAAA B AAAA BBBB AAAAA BB
A BBBB BBB

Since there are 8 runs and this does not fall between 9 and 21, the null hypothesis is rejected. The grades do not occur at random.

14.

H_0 : The choice of the book(fiction or nonfiction) occurs at random. (claim)

H_1 : The null hypothesis is not true.

$n_1 = 29$ (fiction) and $n_2 = 19$ (nonfiction)

FFFFFF NN F NNNN FFFFFFFF NNN F
NNNNN FFF NNN FFFFFFFF NN FF

There are $G = 13$ runs. The test statistic is:

$$z = \frac{G - \mu_G}{\sigma_G}$$

$$\mu_G = \frac{2(29)(19)}{29+19} + 1 = 23.958$$

$$\sigma_G = \sqrt{\frac{2(29)(19)[2(29)(19) - 29 - 19]}{(29+19)^2(29+19-1)}} = 3.275$$

14. continued

$$z = \frac{13 - 23.958}{3.275} = -3.35$$

Reject the null hypothesis. There is enough evidence to reject the claim that the selection occurs at random.

CHAPTER 13 QUIZ

1. False

2. False, they are less sensitive.

3. True

4. True

5. a

6. c

7. d

8. b

9. Non-parametric

10. Nominal, ordinal

11. Sign

12. Sensitive

13.

H_0 : Median = \$230,500

H_1 : Median \neq \$230,500 (claim)

There are three — signs.

Do not reject since 3 is greater than the critical value of 2. There is not enough evidence to support the claim that the median is not \$230,500.

Chapter 13 - Nonparametric Statistics

14.

H_0 : Median = 1200 (claim)

H_1 : Median \neq 1200

There are ten $-$ signs. Do not reject since 10 is greater than the critical value 6. There is not enough evidence to reject the claim that the median is 1,200.

15.

H_0 : There will be no change in the weight of the turkeys after the special diet.

H_1 : The turkeys will weigh more after the special diet. (claim)

There is one $+$ sign. Reject the null hypothesis since the critical value is 0. There is enough evidence to support the claim that the turkeys gained weight on the special diet.

16.

H_0 : There is not difference in the amounts of money received by the teams.

H_1 : There is a difference in the amounts of money received by the teams. (claim)

C. V. = ± 1.96

$z = -0.79$

Do not reject the null hypothesis. There is not enough evidence to support the claim that the distributions are different.

17.

H_0 : The distributions are the same.

H_1 : The distributions are different. (claim)

C. V. = ± 1.65

$z = -0.144$

Do not reject the null hypothesis. There is not enough evidence to support the claim that the distributions are different.

18.

H_0 : There is no difference in the GPAs before and after the workshop.

H_1 : There is a difference in GPAs before and after the workshop. (claim)

18. continued

C. V. = 2 Test statistic = 0

Reject the null hypothesis. There is enough evidence to support the claim that there is a difference in the GPAs of the students.

19.

H_0 : There is no difference in the amounts of sodium in the three sandwiches.

H_1 : There is a difference in the amounts of sodium in the sandwiches. (claim)

$H = 11.795$

C. V. = 5.991

Reject the null hypothesis. There is enough evidence to support the claim that there is a difference in the amounts of sodium in the three sandwiches.

20.

H_0 : There is no difference in reaction times.

H_1 : There is a difference in reaction times. (claim) $H = 6.9$

$0.025 < P\text{-value} < 0.05$ (0.032)

Reject the null hypothesis. There is enough evidence to support the claim that there is a difference in the reaction times of the monkeys.

21.

H_0 : $\rho = 0$

H_1 : $\rho \neq 0$

C. V. = ± 0.600

$r_s = 0.633$

Reject the null hypothesis. There is a significant relationship between the drug prices.

22.

H_0 : $\rho = 0$

H_1 : $\rho \neq 0$

$r_s = 0.943$

C. V. = ± 0.829

Chapter 13 - Nonparametric Statistics

22. continued

Reject the null hypothesis. There is a significant relationship between the amount of money spent on Head Start and the number of students enrolled in the program.

23.

H_0 : The gender of babies occurs at random.

H_1 : The null hypothesis is false.

$\alpha = 0.05$ C. V. = 8, 19

There are 10 runs, which is between 8 and 19. Do not reject the null hypothesis. There is not enough evidence to reject the null hypothesis that the gender occurs at random.

24.

H_0 : There is no difference in the rpm of the motors before and after the reconditioning.

H_1 : There is a difference in the rpm of the motors before and after reconditioning. (claim)

$\alpha = 0.05$ $n = 10$ C. V. = 8

Test statistic = 0

Reject the null hypothesis. There is enough evidence to support the claim that there is a difference in the rpm of the motors before and after the reconditioning.

25.

H_0 : The numbers occur at random

H_1 : The null hypothesis is false.

$\alpha = 0.05$ C. V. = 9, 21

The median number is 538.

There are 20 runs and since this is between 9 and 21, the null hypothesis is not rejected. There is not enough evidence to reject the null hypothesis that the numbers occur at random.

26.

H_0 : The showing of the type of movie (black and white or color) occurs at random. (claim)

H_1 : The showing of the type of movie does not occur at random.

C. V. = ± 1.96

$z = -5.54$

Reject the null hypothesis. There is enough evidence to reject the claim that the showing of the type of movie occurs at random.