

Chapter 14: Integer, Nonlinear, and Advanced Optimization Methods



Statistics, Data Analysis, and
Decision Modeling, Fifth Edition
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Integer Optimization Models

- Integer linear optimization model (integer program): some or all decision variables are restricted to integer values
 - If only a subset of variables are integer, we have a mixed integer model



Nonlinear Optimization Models

- The objective function and/or constraint functions are nonlinear functions of the decision variables
 - Terms cannot be written as a constant times a variable



Example: Cutting Stock Problem

- Suppose that a company makes standard 110-inch-wide rolls of thin sheet metal, and slits them into smaller rolls to meet customer orders for widths of 12, 15, and 30 inches. The demands for these widths vary from week to week. Demands this week are 500 12" rolls, 715 15" rolls, and 630 30" rolls.
- Cutting patterns:

<i>Size of End Item</i>				
Pattern	12"	15"	30"	Scrap
1	0	7	0	5"
2	0	1	3	5"
3	1	0	3	8"
4	9	0	0	2"
5	2	1	2	11"
6	7	1	0	11"



IP Model

- Define X_i to be the number of 110” rolls to cut using cutting pattern i , for $i = 1, \dots, 6$.

$$\text{Min } 5X_1 + 5X_2 + 8X_3 + 2X_4 + 11X_5 + 11X_6$$

$$0X_1 + 0X_2 + 1X_3 + 9X_4 + 2X_5 + 7X_6 \geq 500 \text{ (12” rolls)}$$

$$7X_1 + 1X_2 + 0X_3 + 0X_4 + 1X_5 + 1X_6 \geq 715 \text{ (15” rolls)}$$

$$0X_1 + 3X_2 + 3X_3 + 0X_4 + 2X_5 + 0X_6 \geq 630 \text{ (30” rolls)}$$

$$X_i \geq 0 \text{ and integer}$$

Spreadsheet and *Solver* Models

	A	B	C	D	E
1	Cutting Stock Problem				
2					
3	Data				
4	Pattern	12-in rolls	15-in rolls	30-in rolls	Scrap
5	1	0	7	0	5
6	2	0	1	3	5
7	3	1	0	3	8
8	4	9	0	0	2
9	5	2	1	2	11
10	6	7	1	0	11
11	Demand	500	715	630	
12					
13	Model				
14		No. of rolls			
15	Pattern 1	72.14			
16	Pattern 2	210.00			
17	Pattern 3	0.00			
18	Pattern 4	55.56			
19	Pattern 5	0.00			
20	Pattern 6	0.00			
21					
22		12-in rolls	15-in rolls	30-in rolls	
23	Number produced	500	715	630	
24					
25		Total			
26	Scrap	1521.8254			
27					

Solver Parameters V9.5

Objective
 ...\$B\$26 (Min)

Variables
 ...\$B\$15:\$B\$20

Constraints
 ...\$B\$23:\$D\$23 >= \$B\$11:\$D\$11

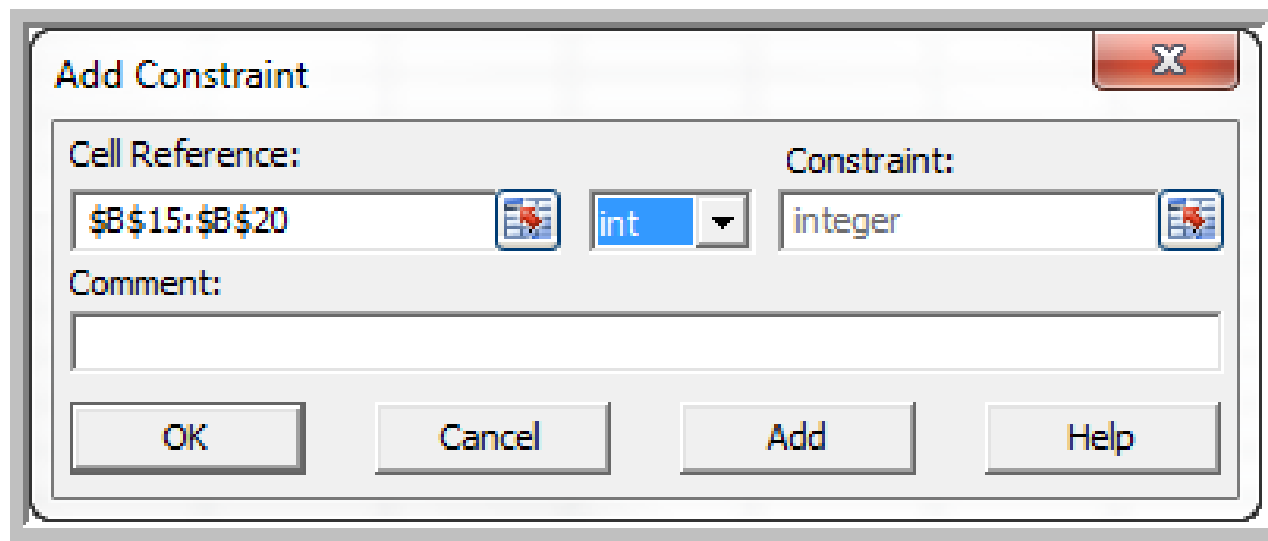
☒ Make Unconstrained Variables Non-Negative

Select a Solving Method: Standard LP/Quadratic

Solving Method
 Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.


Solve **Close**


Adding Integer Restrictions in *Solver*



The screenshot shows the 'Add Constraint' dialog box in Microsoft Excel's Solver. The dialog has a title bar with a close button (X). Inside, there are two main sections: 'Cell Reference:' and 'Constraint:'. The 'Cell Reference:' section contains a text box with '\$B\$15:\$B\$20' and a selection icon. The 'Constraint:' section contains a text box with 'integer' and a selection icon. Below these sections is a 'Comment:' text box. At the bottom, there are four buttons: 'OK', 'Cancel', 'Add', and 'Help'.

Add Constraint

Cell Reference: 

Constraint: 

Comment:

Optimal Integer Solution

	A	B	C	D	E
1	Cutting Stock Problem				
2					
3	Data				
4	Pattern	12-in rolls	15-in rolls	30-in rolls	Scrap
5	1	0	7	0	5
6	2	0	1	3	5
7	3	1	0	3	8
8	4	9	0	0	2
9	5	2	1	2	11
10	6	7	1	0	11
11	Demand	500	715	630	
12					
13	Model				
14		No. of rolls			
15	Pattern 1	73.00			
16	Pattern 2	210.00			
17	Pattern 3	0.00			
18	Pattern 4	56.00			
19	Pattern 5	0.00			
20	Pattern 6	0.00			
21					
22		12-in rolls	15-in rolls	30-in rolls	
23	Number produced	504	721	630	
24					
25		Total			
26	Scrap	1527			



IP Models With Binary Variables

- A **binary variable** x is simply a general integer variable that is restricted to being between 0 and 1:

$$0 \leq x \leq 1 \text{ and integer}$$

- We usually just write this as

$$x = 0 \text{ or } 1$$



Example: Project Selection

TABLE 14.1 Project Selection Data

	Project 1	Project 2	Project 3	Project 4	Project 5	Available Resources
Expected return (NPV)	\$180,000	\$220,000	\$150,000	\$140,000	\$200,000	
Cash requirements	\$55,000	\$83,000	\$24,000	\$49,000	\$61,000	\$150,000
Personnel requirements	5	3	2	5	3	12

Maximize $\$180,000x_1 + \$220,000x_2 + \$150,000x_3 + \$140,000x_4 + \$200,000x_5$

$\$55,000x_1 + \$83,000x_2 + \$24,000x_3 + \$49,000x_4 + \$61,000x_5 \leq \$150,000$
(cash limitation)

$5x_1 + 3x_2 + 2x_3 + 5x_4 + 3x_5 \leq 12$ (personnel limitation)

Spreadsheet and Solver Models

	A	B	C	D	E	F	G
1	Project Selection Model						
2							
3	Data						
4		Project 1	Project 2	Project 3	Project 4	Project 5	Available
5	Expected Return (NPV)	\$ 180,000	\$ 220,000	\$ 150,000	\$ 140,000	\$ 200,000	Resources
6	Cash requirements	\$ 55,000	\$ 83,000	\$ 24,000	\$ 49,000	\$ 61,000	\$ 150,000
7	Personnel requirements	5	3	2	5	3	12
8							
9	Model						
10							
11	Project selection decisions	1	0	1	0	1	Total
12	Cash Used	\$ 55,000	\$ -	\$ 24,000	\$ -	\$ 61,000	\$ 140,000
13	Personnel Used	5	0	2	0	3	10
14	Return	\$ 180,000	\$ -	\$ 150,000	\$ -	\$ 200,000	\$ 530,000

Objective
 ...\$G\$14 (Max)

Variables
 ...Normal
 ...☒ \$B\$11:\$F\$11
 ...Recourse

Constraints
 ...Normal
 ...☒ \$G\$12:\$G\$13 <= \$G\$6:\$G\$7
 ...Chance
 ...Bound
 ...Conic
 ...Integers
 ...☒ \$B\$11:\$F\$11 = binary
 ...Uncertain Variables

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method: Standard LP/Quadratic



Site Location Model

- Response times from fire stations to districts

From/To	1	2	3	4	5	6	7
1	0	2	10	6	12	5	8
2	2	0	6	9	11	7	10
3	10	6	0	5	5	12	6
4	6	9	5	0	9	4	3
5	12	11	5	9	0	10	8
6	5	7	12	4	10	0	6
7	8	10	6	3	8	6	0

- Find best location to reach all districts within 8 minutes



IP Model

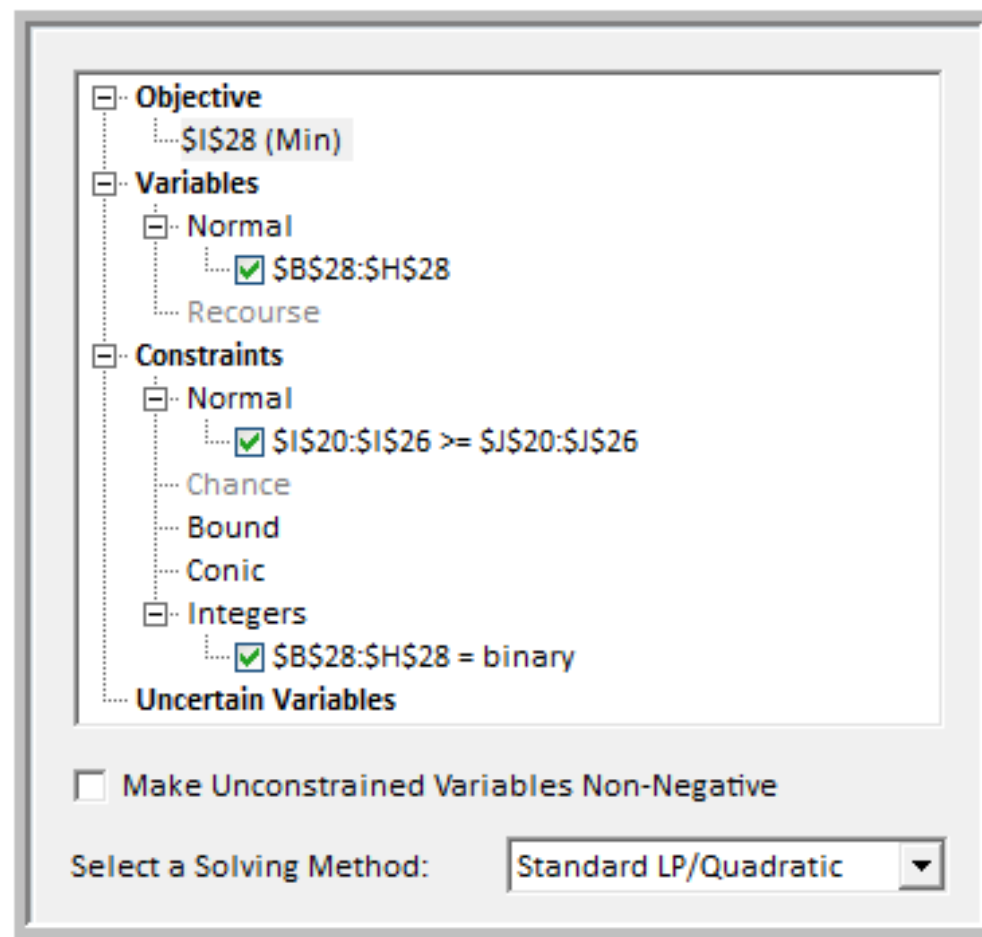
- $\text{Min } X1 + X2 + X3 + X4 + X5 + X6 + X7$
- Meet response time requirement for each district:
 - $X1 + X2 + X4 + X6 + X7 \geq 1$
 - $X1 + X2 + X3 + X6 \geq 1$
 - $X2 + X3 + X4 + X5 + X7 \geq 1$
 - $X1 + X3 + X4 + X6 + X7 \geq 1$
 - $X1 + X2 + X4 + X6 + X7 \geq 1$
 - $X1 + X3 + X4 + X5 + X6 + X7 \geq 1$

Spreadsheet Model

	A	B	C	D	E	F	G	H	I	J
1	Fire Station Location Model									
2										
3	Data									
4										
5	Response time		8							
6										
7	Response Times									
8	From/To	1	2	3	4	5	6	7		
9	1	0	2	10	6	12	5	8		
10	2	2	0	6	9	11	7	10		
11	3	10	6	0	5	5	12	6		
12	4	6	9	5	0	9	4	3		
13	5	12	11	5	9	0	10	8		
14	6	5	7	12	4	10	0	6		
15	7	8	10	6	3	8	6	0		
16										
17	Model									
18										
19	From/To	1	2	3	4	5	6	7	Covered?	Requirement
20	1	1	1	0	1	0	1	1	1	1
21	2	1	1	1	0	0	1	0	1	1
22	3	0	1	1	1	1	0	1	2	1
23	4	1	0	1	1	0	1	1	2	1
24	5	0	0	1	0	1	0	1	2	1
25	6	1	1	0	1	0	1	1	1	1
26	7	1	0	1	1	1	1	1	2	1
27									Total	
28	Location	0	0	1	0	0	0	1	2	



Solver Model



The Solver Model dialog box displays a tree view of the model structure:

- ☒ **Objective**
 - ☒ \$I\$28 (Min)
- ☒ **Variables**
 - ☒ Normal
 - ☒ \$B\$28:\$H\$28
 - ☐ Recourse
- ☒ **Constraints**
 - ☒ Normal
 - ☒ \$I\$20:\$I\$26 >= \$J\$20:\$J\$26
 - ☐ Chance
 - ☐ Bound
 - ☐ Conic
 - ☒ **Integers**
 - ☒ \$B\$28:\$H\$28 = binary
 - ☐ **Uncertain Variables**

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method: Standard LP/Quadratic ▼



Modeling Logical Conditions

TABLE 14.2 Modeling Logical Conditions Using Binary Variables

Logical Condition	Constraint Model Form
If A, then B	$B \geq A$ or $B - A \geq 0$
If not A, then B	$B \geq 1 - A$ or $A + B \geq 1$
If A, then not B	$B \leq 1 - A$ or $B + A \leq 1$
At most one of A and B	$A + B \leq 1$
If A, then B and C	$(B \geq A \text{ and } C \geq A)$ or $B + C \geq 2A$
If A and B, then C	$C \geq A + B - 1$ or $A + B - C \leq 1$



Supply Chain Facility Location

- $X_{ij} = 1$ if customer zone j is assigned to DC i , and 0 if not, and $Y_i = 1$ if DC i is chosen from among a set of k potential locations.
- C_{ij} = the total cost of satisfying the demand in customer zone j from DC i .

$$\begin{aligned} &\text{Min } \sum C_{ij}X_{ij} \\ &\sum X_{ij} = 1, \text{ for every } j \text{ (summed over } i\text{)} \\ &\sum Y_i = k \text{ (summed over } i\text{)} \\ &X_{ij} \leq Y_i, \text{ for every } i \text{ and } j \end{aligned}$$



Plant Location Model

TABLE 14.3 Plant Location Data

Plant	Distribution Center				Capacity
	Cleveland	Baltimore	Chicago	Phoenix	
Marietta	\$12.60	\$14.35	\$11.52	\$17.58	1,200
Minneapolis	\$9.75	\$16.26	\$8.11	\$17.92	800
Fayetteville	\$10.41	\$11.54	\$9.87	\$11.64	1,500
Chico	\$13.88	\$16.95	\$12.51	\$8.32	1,500
Demand	300	500	700	1,800	

Select a new plant from among Fayetteville and Chico

IP Model

$$\begin{aligned} \text{Minimize } & 12.60X_{11} + 14.35X_{12} + 11.52X_{13} + 17.58X_{14} + 9.75X_{21} + 16.26X_{22} \\ & + 8.11X_{23} + 17.92X_{24} + 10.41X_{31} + 11.54X_{32} + 9.87X_{33} + 11.64X_{34} + 13.88X_{41} \\ & + 16.95X_{42} + 12.51X_{43} + 8.32X_{44} \end{aligned}$$

$$X_{11} + X_{12} + X_{13} + X_{14} \leq 1200$$

$$X_{21} + X_{22} + X_{23} + X_{24} \leq 800$$

$$X_{31} + X_{32} + X_{33} + X_{34} \leq 1500Y_1$$

$$X_{41} + X_{42} + X_{43} + X_{44} \leq 1500Y_2$$

$$X_{11} + X_{21} + X_{31} + X_{41} = 300$$

$$X_{12} + X_{22} + X_{32} + X_{42} = 500$$

$$X_{13} + X_{23} + X_{33} + X_{43} = 700$$

$$X_{14} + X_{24} + X_{34} + X_{44} = 1800$$

$$Y_1 + Y_2 = 1$$

$$X_{ij} \geq 0, \text{ for all } i \text{ and } j$$

$$Y_1, Y_2 = 0, 1$$

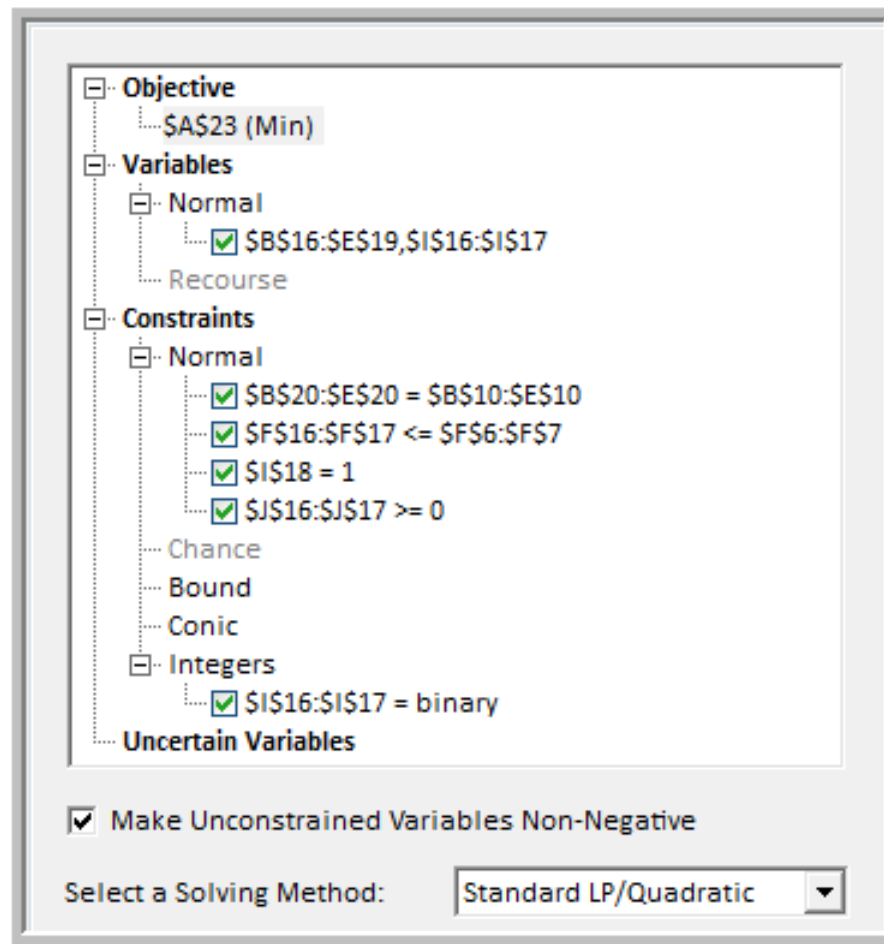
Ensures that exactly one DC is selected. Y_1 corresponds to Fayetteville; Y_2 corresponds to Chico

Spreadsheet Model

	A	B	C	D	E	F	G	H	I	J
1	Plant Location Model									
2										
3	Data									
4		Distribution Center								
5	Plant	Cleveland	Baltimore	Chicago	Phoenix	Capacity				
6	Marietta	\$ 12.60	\$ 14.35	\$ 11.52	\$ 17.58	1200				
7	Minneapolis	\$ 9.75	\$ 16.26	\$ 8.11	\$ 17.92	800				
8	Fayetteville	\$ 10.41	\$ 11.54	\$ 9.87	\$ 11.64	1500				
9	Chico	\$ 13.88	\$ 16.95	\$ 12.51	\$ 8.32	1500				
10	Demand	300	500	700	1800					
11										
12	Model									
13										
14	Amount Shipped	Distribution Center								
15	Plant	Cleveland	Baltimore	Chicago	Phoenix	Total shipped			New Plant Chosen	Surplus Capacity
16	Marietta	200	500	0	300	1000	Fayetteville	0	0	
17	Minneapolis	100	0	700	0	800	Chico	1	0	
18	Fayetteville	0	0	0	0	0	Total	1		
19	Chico	0	0	0	1500	1500				
20	Demand met	300	500	700	1800					
21										
22	Total cost									
23	\$	34,101								



Solver Model



The Solver Model dialog box is shown with the following configuration:

- Objective:** \$A\$23 (Min)
- Variables:**
 - Normal: ☒ \$B\$16:\$E\$19,\$I\$16:\$I\$17
 - Recourse: ☐
- Constraints:**
 - Normal:
 - ☒ \$B\$20:\$E\$20 = \$B\$10:\$E\$10
 - ☒ \$F\$16:\$F\$17 <= \$F\$6:\$F\$7
 - ☒ \$I\$18 = 1
 - ☒ \$J\$16:\$J\$17 >= 0
 - Chance: ☐
 - Bound: ☐
 - Conic: ☐
 - Integers:
 - ☒ \$I\$16:\$I\$17 = binary
- Uncertain Variables:** ☐

☒ Make Unconstrained Variables Non-Negative

Select a Solving Method:



Modeling Fixed Costs

- Multiperiod Production Planning Model

- $Y_A = 1$ if production occurs during the Autumn, and 0 if not
- $Y_W = 1$ if production occurs during the Winter, and 0 if not
- $Y_S = 1$ if production occurs during the Spring, and 0 if not

Minimize $11P_A + 14P_W + 12.50P_S + 1.20I_A + 1.20I_W + 1.20I_S + 65(Y_A + Y_W + Y_S)$

$$P_A - I_A = 150$$

$$P_W + I_A - I_W = 400$$

$$P_S + I_W - I_S = 50$$

$$P_A \leq 600Y_A$$

$$P_W \leq 600Y_W$$

$$P_S \leq 600Y_S$$



Spreadsheet Implementation

	A	B	C	D
1	Kristin's Kreations Fixed Cost Model			
2				
3	Cost	Quarter 1	Quarter 2	Quarter 3
4	Production	\$ 11.00	\$ 14.00	\$ 12.50
5	Inventory	\$ 1.20	\$ 1.20	\$ 1.20
6	Demand	150	400	50
7	Fixed cost	\$ 65.00	\$ 65.00	\$ 65.00
8				
9		Quarter 1	Quarter 2	Quarter 3
10	Production	600	0	0
11	Inventory	450	50	0
12	Binary	1	0	0
13				
14	Binary constraints	600	0	0
15	Net production	150	400	50
16				
17		Cost		
18	Total	\$ 7,265.00		

	A	B	C	D
1	Kristin's Kreations			
2				
3	Cost	Quarter 1	Quarter 2	Quarter 3
4	Production	11	14	12.5
5	Inventory	1.2	1.2	1.2
6	Demand	150	400	50
7	Fixed cost	65	65	65
8				
9		Quarter 1	Quarter 2	Quarter 3
10	Production	600	0	0
11	Inventory	450	50	0
12	Binary	1	0	0
13				
14	Binary constraints	=600*B12	=600*C12	=600*D12
15	Net production	=B10-B11	=C10-C11+B11	=D10-D11+C11
16				
17		Cost		
18	Total	=SUMPRODUCT(B4:D5,B10:D11) + 65*(B12+C12+D12)		



Nonlinear Optimization

- Either objective function or constraint functions are not linear
- Models are unique in structure
- Solution techniques are different from linear and integer optimization



Hotel Pricing With Elastic Demand

- A 450-room hotel has the following history:

Room Type	Rate	Daily Avg. No. Sold	Revenue
Standard	\$85	250	\$21,250
Gold	\$98	100	\$9,800
Platinum	\$139	50	\$6,950
Total revenue			\$38,000

Room Type	Price Elasticity of Demand
Standard	-1.5
Gold	-2.0
Platinum	-1.0



Model Development

- Projected number of rooms of a given type sold =
(Historical Average Number of Rooms Sold) + (Elasticity)(New Price - Current Price)(Historical Average Number of Rooms Sold)/(Current Price)
- Define S = price of a standard room, G = price of a gold room, and P = price of a platinum room.

$$\begin{aligned}\text{Total Revenue} &= S(625 - 4.41176S) + G(300 - 2.04082G) + P(100 - 0.35971P) \\ &= 625S + 300G + 100P - 4.41176S^2 - 2.04082G^2 - 0.35971P^2\end{aligned}$$



Model

$$\text{Maximize } 625S + 300G + 100P - 4.41176S^2 - 2.04082G^2 - 0.35971P^2$$

$$70 \leq S \leq 90 \quad (\text{price range restrictions})$$

$$90 \leq G \leq 110$$

$$120 \leq P \leq 149$$

$$(625 - 4.41176S) + (300 - 2.04082G) + (100 - 0.35971P) \leq 450$$

$$\text{or } 1025 - 4.41176S - 2.04082G - 0.35971P \leq 450 \quad (\text{room limitation})$$

Spreadsheet and Solver Model

	A	B	C	D	E	F
1	Marquis Hotel					
2						
3	Data					
4		Current	Average		Total Room	
5	Room type	Rate	Daily Sold	Elasticity	Capacity	
6	Standard	\$ 85.00	250	-1.5	450	
7	Gold	\$ 98.00	100	-2		
8	Platinum	\$ 139.00	50	-1		
9						
10	Model				Projected	
11					Rooms	Projected
12	Room type	New Price	Price Range		Sold	Revenue
13	Standard	\$ 76.87	\$ 70.00	\$ 90.00	286	\$21,974.39
14	Gold	\$ 90.00	\$ 90.00	\$ 110.00	116	\$10,469.39
15	Platinum	\$ 145.04	\$ 120.00	\$ 149.00	48	\$ 6,936.87
16				Totals	450	\$39,380.65

Solver Parameters

Objective
 ...\$F\$16 (Max)

Variables
 ...Normal
 ...\$B\$13:\$B\$15
 ...Recourse

Constraints
 ...Normal
 ...\$E\$16 <= \$E\$6
 ...Chance
 ...Bound
 ...\$B\$13:\$B\$15 <= \$D\$13:\$D\$15
 ...\$B\$13:\$B\$15 >= \$C\$13:\$C\$15
 ...Conic
 ...Integers
 ...Uncertain Variables

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method: **Standard GRG Nonlinear**



Solving Nonlinear Models

- Select **Standard GRG Nonlinear** in Premium Solver as the solution procedure
- Sensitivity report is different for nonlinear models
 - Reduced gradient is analogous to reduced cost, but more difficult to interpret
 - Lagrange multipliers are similar to shadow prices, but give only approximate rates of change

Solver Answer Report

	A	B	C	D	E	F	G
11							
12		Target Cell (Max)					
13		Cell	Name	Original Value	Final Value		
14		\$F\$16 Totals Revenue		0	39380.65104		
15							
16							
17		Adjustable Cells					
18		Cell	Name	Original Value	Final Value		
19		\$B\$13 Standard New Price	\$	-	\$ 76.87		
20		\$B\$14 Gold New Price	\$	-	\$ 90.00		
21		\$B\$15 Platinum New Price	\$	-	\$ 145.04		
22							
23		Constraints					
24		Cell	Name	Cell Value	Formula	Status	Slack
25		\$E\$16 Totals Sold		450.00000004	\$E\$16<=\$E\$6	Binding	0
26		\$B\$13 Standard New Price	\$	76.87	\$B\$13>=\$C\$13	Not Binding	6.87476046
27		\$B\$14 Gold New Price	\$	90.00	\$B\$14>=\$C\$14	Binding	0
28		\$B\$15 Platinum New Price	\$	145.04	\$B\$15>=\$C\$15	Not Binding	25.0414271
29		\$B\$13 Standard New Price	\$	76.87	\$B\$13<=\$D\$13	Not Binding	13.1252395
30		\$B\$14 Gold New Price	\$	90.00	\$B\$14<=\$D\$14	Not Binding	20
31		\$B\$15 Platinum New Price	\$	145.04	\$B\$15<=\$D\$15	Not Binding	3.95857289



Solver Sensitivity Report

	A	B	C	D	E
4					
5	Target Cell (Max)				
6		Cell	Name	Final Value	
7		\$F\$16	Totals Revenue	39380.65104	
8					
9	Adjustable Cells				
10				Final Value	Reduced Gradient
11		Cell	Name	Final Value	Reduced Gradient
12		\$B\$13	Standard New Price	\$ 76.87	\$ -
13		\$B\$14	Gold New Price	\$ 90.00	\$ (42.69)
14		\$B\$15	Platinum New Price	\$ 145.04	\$ -
15					
16	Constraints				
17				Final Value	Lagrange Multiplier
18		Cell	Name	Final Value	Lagrange Multiplier
19		\$E\$16	Totals Sold	450.0000004	12.08293216



Markowitz Portfolio Model

- Select stocks to minimize portfolio variance

$$\sum_{i=1}^k s_i^2 x_i^2 + \sum_{i=1}^k \sum_{j>i} 2s_{ij} x_i x_j$$

and ensure a specified expected return

- s_i^2 = the sample variance in the return of stock i
- s_{ij} = the sample covariance between stocks i and j



Example

Variance-Covariance Matrix

	Stock 1	Stock 2	Stock 3
Stock 1	.025	.015	-.002
Stock 2		.030	.005
Stock 3			.004
Exp. return	10%	12%	7%

Minimize Variance =

$$.025x_1^2 + .030x_2^2 + .004x_3^2 + 0.03x_1x_2 - 0.004x_1x_3 + 0.010x_2x_3$$

$$x_1 + x_2 + x_3 = 1$$

$$10x_1 + 12x_2 + 7x_3 \geq 10 \text{ (required return)}$$

$$x_1, x_2, x_3 \geq 0$$

Spreadsheet Model

	A	B	C	D	E	F	G
1	Markowitz Model						
2							
3	Data						
4		Expected			Variance-Covariance Matrix		
5		Return			Stock 1	Stock 2	Stock 3
6	Stock 1	10%		Stock 1	0.025	0.015	-0.002
7	Stock 2	12%		Stock 2		0.03	0.005
8	Stock 3	7%		Stock 3			0.004
9	Target Return	10%					
10							
11	Model						
12					Variance Calculations		
13		Allocation			Squared Terms	Cross-Products	
14	Stock 1	0.25			0.001579256	0.003387	
15	Stock 2	0.45			0.006053361	-0.000301067	
16	Stock 3	0.30			0.000358718	0.001345191	
17	Total	1					
18							
19							
20		Return	Variance				
21	Portfolio	10.0%	0.012				



Sensitivity Report

	A	B	C	D	E
4					
5		Objective Cell (Min)			
6		Cell	Name	Final Value	
7		\$C\$21	Portfolio Variance	0.01242246	
8					
9		Decision Variable Cells			
10				Final Value	Reduced Gradient
11		Cell	Name	Final Value	Reduced Gradient
12		\$B\$14	Stock 1 Allocation	0.25	0.00
13		\$B\$15	Stock 2 Allocation	0.45	0.00
14		\$B\$16	Stock 3 Allocation	0.30	0.00
15					
16		Constraints			
17				Final Value	Lagrange Multiplier
18		Cell	Name	Final Value	Lagrange Multiplier
19		\$B\$17	Total Allocation	1	-0.038363637
20		\$B\$21	Portfolio Return	10.0%	63.2%

Risk versus Return Profile





Evolutionary Solver for Nonsmooth Optimization

- Used for difficult nonlinear problems and models using Excel functions such as IF, ABS, MIN, and MAX
- *Evolutionary Solver* uses heuristics—intelligent rules for systematically searching among solutions—that remember the best solutions they find, then modifying or combining them in attempting to find better solutions.

Fixed Cost Model

The objective function in cell B16 is =SUMPRODUCT(B4:D5,B10:D11) + IF(B10>0,B7,0)+IF(C10>0,C7,0)+IF(D10>0,D7,0).

	A	B	C	D
1	Kristin's Kreations Evolutionary Solver Model			
2				
3	Cost	Quarter 1	Quarter 2	Quarter 3
4	Production	\$ 11.00	\$ 14.00	\$ 12.50
5	Inventory	\$ 1.20	\$ 1.20	\$ 1.20
6	Demand	150	400	50
7	Fixed cost	\$ 65.00	\$ 65.00	\$ 65.00
8				
9		Quarter 1	Quarter 2	Quarter 3
10	Production	550	0	50
11	Inventory	400	0	0
12				
13	Net production	150	400	50
14				
15		Cost		
16	Total	\$7,285.00		

The screenshot shows the Excel Solver Parameters dialog box. The 'Objective' is set to '\$B\$16 (Min)'. The 'Variables' section is expanded, showing 'Normal' constraints for '\$B\$10:\$D\$11' with a checkbox checked. The 'Constraints' section is also expanded, showing 'Normal' constraints for '\$B\$13:\$D\$13 = \$B\$6:\$D\$6' with a checkbox checked. The 'Bound' section is expanded, showing two constraints for '\$B\$10:\$D\$11': '\$B\$10:\$D\$11 <= 600' and '\$B\$10:\$D\$11 >= 0', both with checkboxes checked. The 'Conic' and 'Integers' sections are collapsed. The 'Uncertain Variables' section is also collapsed. At the bottom, the 'Make Unconstrained Variables Non-Negative' checkbox is checked. The 'Select a Solving Method' dropdown is set to 'Standard Evolutionary'.



Location Model

- Locate a tool bin to minimize the weighted distance between the location coordinates and each production cell

Cell	X-coordinate	Y-coordinate	Demand
Fabrication	1	4	12
Paint	1	2	24
Subassembly 1	2.5	2	13
Subassembly 2	3	5	7
Assembly	4	4	17

Minimize $12(|X - 1| + |Y - 4|) + 24(|X - 1| + |Y - 2|) + 13(|X - 2.5| + |Y - 2|) + 7(|X - 3| + |Y - 5|) + 17(|X - 4| + |Y - 4|)$

Spreadsheet and *Solver* Models

	A	B	C	D
1	Edwards Manufacturing			
2				
3	Data			
4				
5	Cell	x-coordinate	y-coordinate	Demand
6	Fabrication	1	4	12
7	Paint	1	2	24
8	Subassembly 1	2.5	2	13
9	Subassembly 2	3	5	7
10	Assembly	4	4	17
11	Maximum	4	5	
12				
13	Model			
14	Tool bin location	2.499997179	2.489551412	
15				
16	Cell	Weighted Distance		
17	Fabrication	36.1253492		
18	Paint	47.7491662		
19	Subassembly 1	6.364205031		
20	Subassembly 2	21.07315986		
21	Assembly	51.17767394		
22	Total	162.4895542		

Objective
 ...\$B\$22 (Min)

Variables
 ...Normal
 ...☒ \$B\$14:\$C\$14
 ...Recourse

Constraints
 ...Normal
 ...Chance
 ...Bound
 ...☒ \$B\$14:\$C\$14 <= 5
 ...☒ \$B\$14:\$C\$14 >= 0
 ...Conic
 ...Integers
 ...Uncertain Variables

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method: Standard Evolutionary



Job Sequencing

- A custom manufacturing company has ten jobs waiting to be processed. Each job i has an estimated processing time (P_i) and a due date (D_i) that was requested by the customer, as shown in the table below:

Job	1	2	3	4	5	6	7	8	9	10
Time	8	7	6	4	10	8	10	5	9	5
Due date	20	27	39	28	23	40	25	35	29	30



Performance Measures

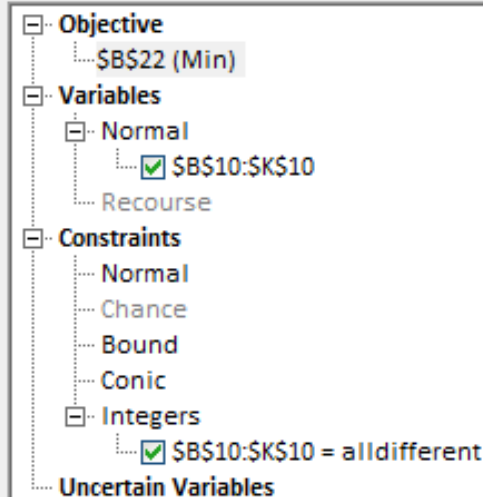
- *Lateness* (L_i) is the difference between the completion time and the due date (either positive or negative).
- *Tardiness* (T_i) is the amount of time by which the completion time exceeds the due date; thus tardiness is zero if a job is completed early.



Spreadsheet Model

	A	B	C	D	E	F	G	H	I	J	K
1	Job Sequencing										
2											
3	Data										
4	Job	1	2	3	4	5	6	7	8	9	10
5	Time	8	7	6	4	10	8	10	5	9	5
6	Due date	26	27	39	28	23	40	25	35	29	30
7											
8	Model										
9	Sequence	1	2	3	4	5	6	7	8	9	10
10	Job Assigned	5	7	1	2	4	9	10	8	3	6
11	Processing time	10	10	8	7	4	9	5	5	6	8
12	Completion time	10	20	28	35	39	48	53	58	64	72
13	Due Date	23	25	26	27	28	29	30	35	39	40
14	Lateness	-13	-5	2	8	11	19	23	23	25	32
15	Tardiness	0	0	2	8	11	19	23	23	25	32
16											
17	Average Completion Time	42.7									
18	Maximum Number Tardy	8									
19	Total Lateness	125									
20	Average Lateness	12.5									
21	Variance of Lateness	188.85									
22	Total Tardiness	143									
23	Average Tardiness	14.3									
24	Variance of Tardiness	121.21									

Solver Model with Alldifferent Constraint



The Solver Parameters dialog box shows the following structure:

- Objective: \$B\$22 (Min)
- Variables:
 - Normal: ☒ \$B\$10:\$K\$10
 - Recourse
- Constraints:
 - Normal
 - Chance
 - Bound
 - Conic
 - Integers: ☒ \$B\$10:\$K\$10 = alldifferent
 - Uncertain Variables

☐ Make Unconstrained Variables Non-Negative

Select a Solving Method: Standard Evolutionary

Add Constraint

Cell Reference:

\$B\$12:\$K\$12

dif

Constraint:

alldifferent

Comment:

OK

Cancel

Add

Help

Minimum Total Tardiness Solution Using *Evolutionary Solver*

	A	B	C	D	E	F	G	H	I	J	K
1	Job Sequencing										
2											
3	Data										
4	Job	1	2	3	4	5	6	7	8	9	10
5	Time	8	7	6	4	10	8	10	5	9	5
6	Due date	26	27	39	28	23	40	25	35	29	30
7											
8	Model										
9	Sequence	1	2	3	4	5	6	7	8	9	10
10	Job Assigned	2	5	1	4	10	8	3	6	9	7
11	Processing time	7	10	8	4	5	5	6	8	9	10
12	Completion time	7	17	25	29	34	39	45	53	62	72
13	Due Date	27	23	26	28	30	35	39	40	29	25
14	Lateness	-20	-6	-1	1	4	4	6	13	33	47
15	Tardiness	0	0	0	1	4	4	6	13	33	47
16											
17	Average Completion Time	38.3									
18	Maximum Number Tardy	7									
19	Total Lateness	81									
20	Average Lateness	8.1									
21	Variance of Lateness	331.69									
22	Total Tardiness	108									
23	Average Tardiness	10.8									
24	Variance of Tardiness	236.96									

Crystal Ball Results

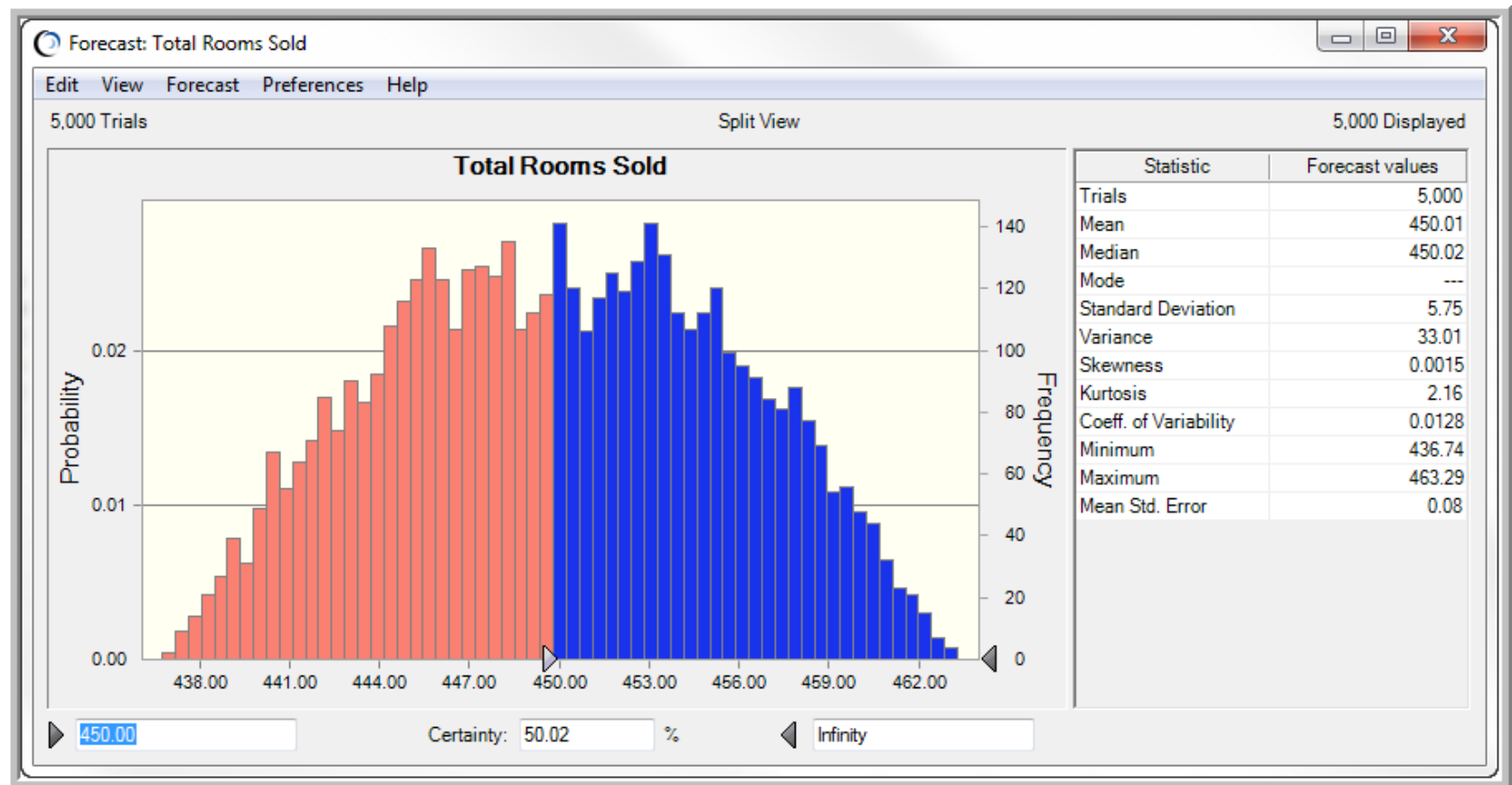




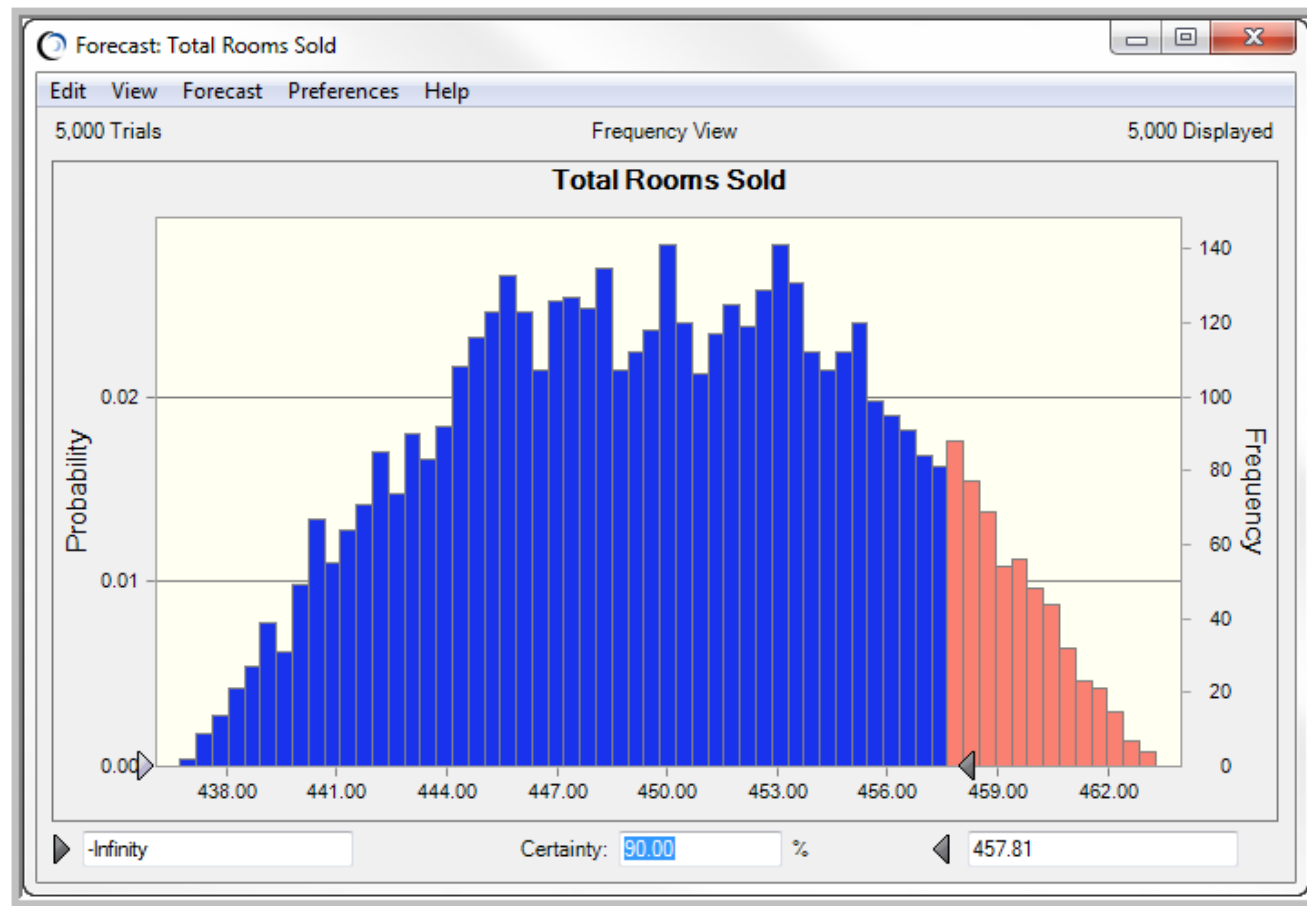
Risk Analysis and Optimization

- *Crystal Ball* may be used to conduct post-optimality risk analysis to understand the impact of uncertainty of optimization model parameters.
- Example – Hotel Pricing Model
 - Assume price-demand elasticities may vary by plus or minus 25%

Crystal Ball Results



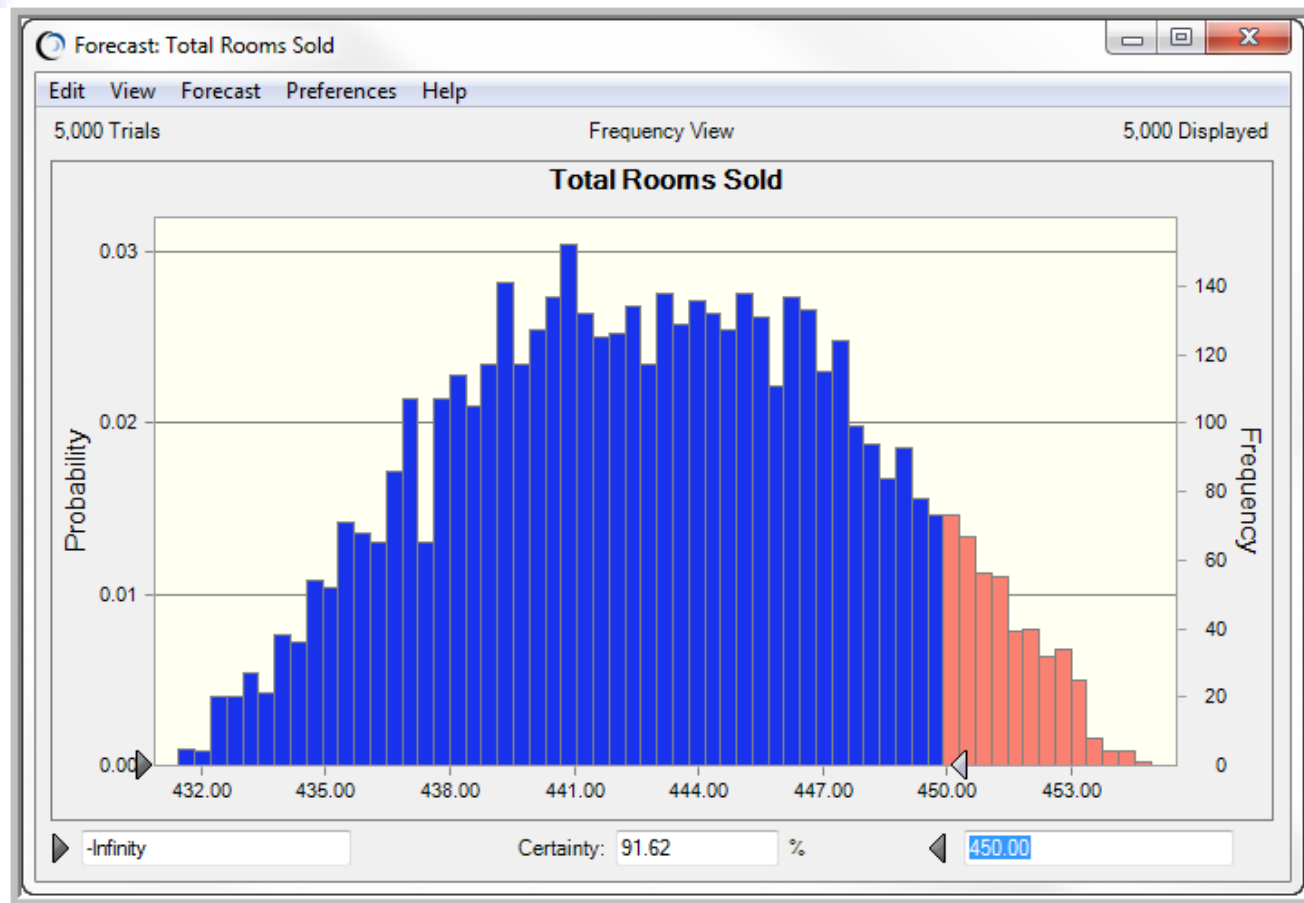
Forecast Chart for a 10% Risk of Exceeding Capacity



Solver Solution for 443 Room Capacity

	A	B	C	D	E	F
1	Marquis Hotel					
2						
3	Data					
4		Current	Average		Total Room	
5	Room type	Rate	Daily Sold	Elasticity	Capacity	
6	Standard	\$ 85.00	250	-1.5	443	
7	Gold	\$ 98.00	100	-2		
8	Platinum	\$ 139.00	50	-1		
9						
10	Model				Projected	
11					Rooms	Projected
12	Room type	New Price	Price Range		Sold	Revenue
13	Standard	\$ 78.34	\$ 70.00	\$ 90.00	279	\$21,886.69
14	Gold	\$ 90.00	\$ 90.00	\$ 110.00	116	\$10,469.39
15	Platinum	\$ 146.51	\$ 120.00	\$ 149.00	47	\$ 6,929.72
16				Totals	443	\$39,285.80

Crystal Ball Confirmation Run





OptQuest: Combining Optimization and Simulation

- *OptQuest* searches for optimal solutions within *Crystal Ball* simulation model spreadsheets.
- *OptQuest* is also designed to find solutions that satisfy a wide variety of constraints or a set of goals that you may define.



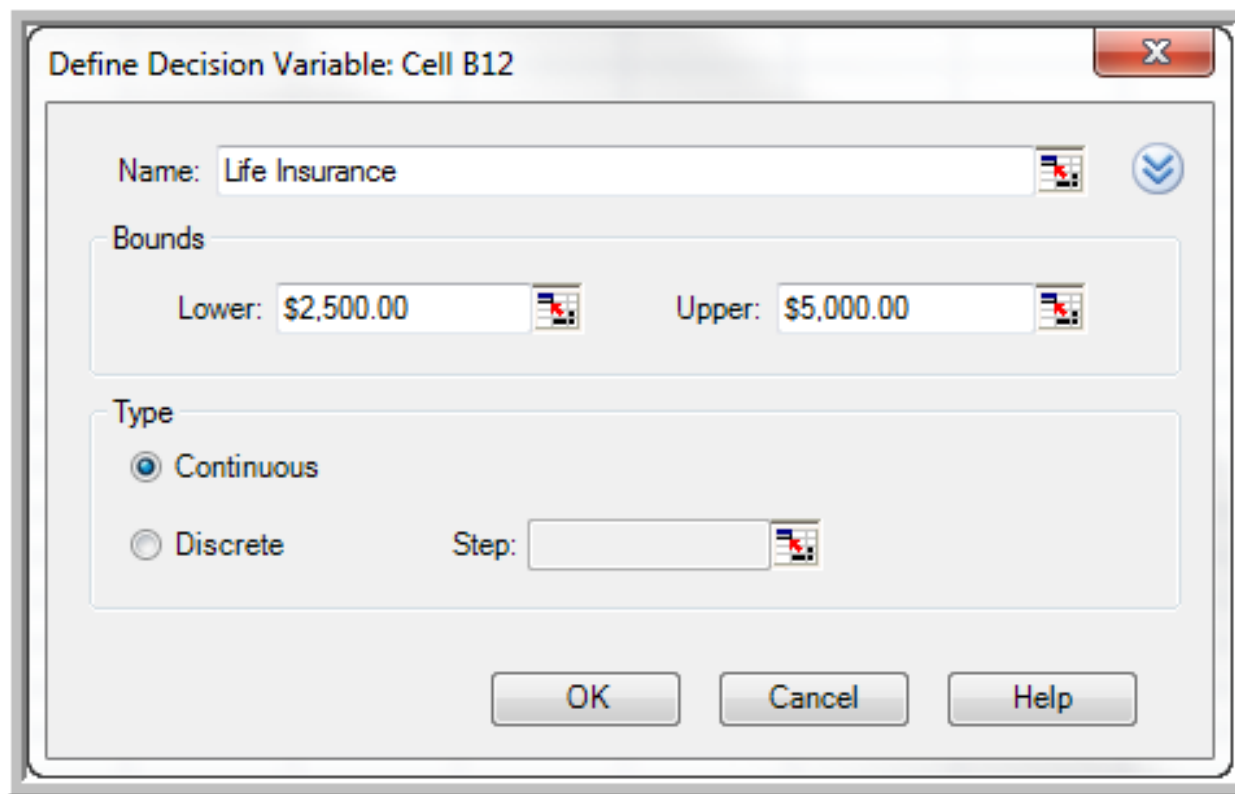
Using *OptQuest*

1. Create a *Crystal Ball* model of the decision problem.
2. Define the decision variables within *Crystal Ball*.
3. Run *OptQuest* from the *Crystal Ball* Tools group.
4. Specify objectives, decision variables, constraints, and other options as appropriate.
5. Solve the optimization problem.

Portfolio Allocation Model

	A	B	C	D	E
1	Portfolio Allocation Model				
2		Annual			Risk factor
3	Investment	return	Minimum	Maximum	per dollar
4	Life Insurance	5.0%	\$ 2,500.00	\$ 5,000.00	-0.5
5	Bond mutual funds	7.0%	\$ 30,000.00	none	1.8
6	Stock mutual funds	11.0%	\$ 15,000.00	none	2.1
7	Savings Account	4.0%	none	none	-0.3
8	Total amount available	\$100,000		Limit	100,000
9					
10		Amount			Total weighted
11	Decision variables	invested			risk
12	Life Insurance	\$ 5,000.00			146,000.00
13	Bond mutual funds	\$ 50,000.00			
14	Stock mutual funds	\$ 30,000.00			Total expected
15	Savings Account	\$ 15,000.00			return
16	Total amount invested	\$ 100,000.00			\$ 7,650.00

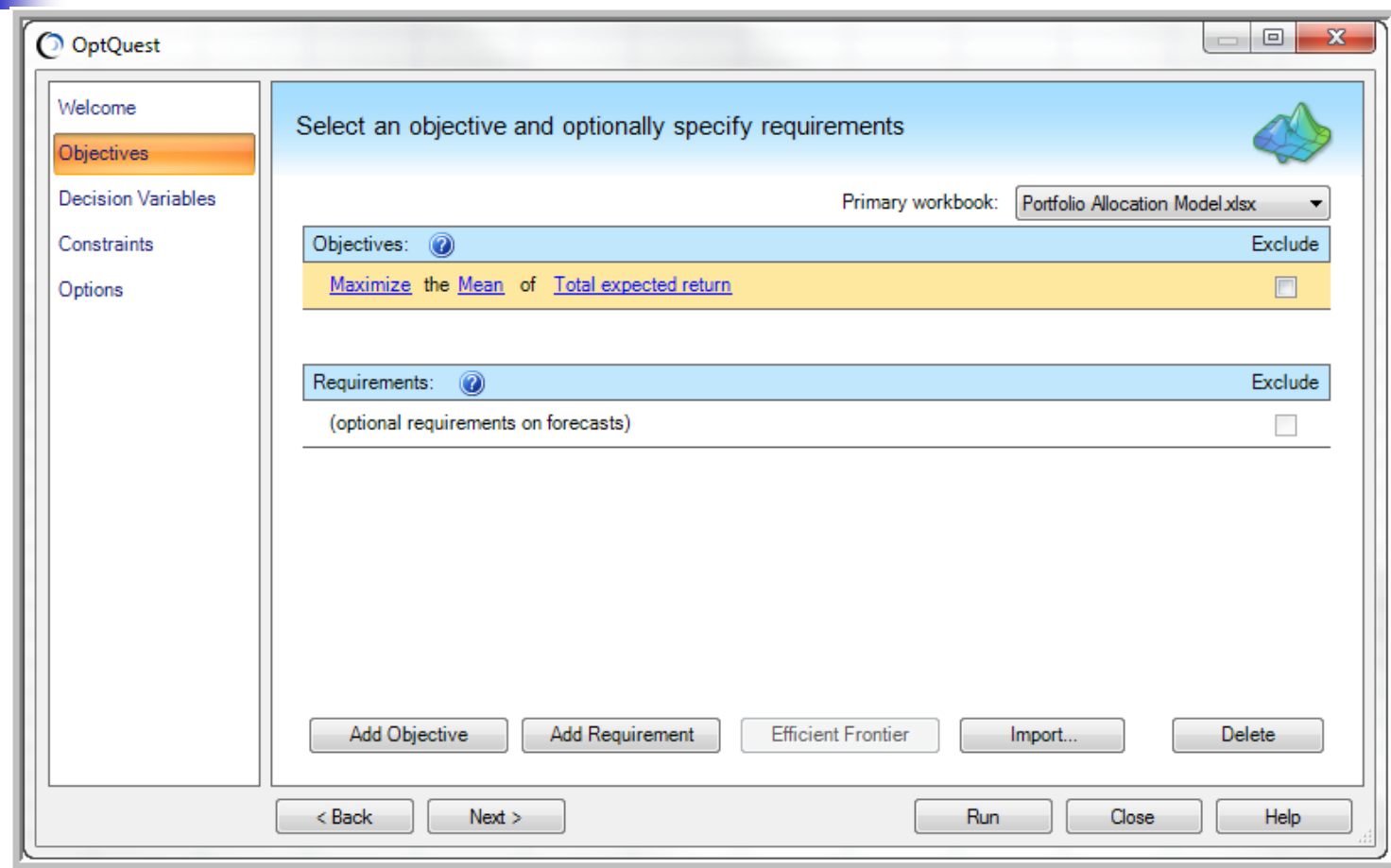
Define Decision Variable Dialog



The dialog box is titled "Define Decision Variable: Cell B12". It contains the following fields and options:

- Name:** A text field containing "Life Insurance".
- Bounds:** A section containing two text fields: "Lower: \$2,500.00" and "Upper: \$5,000.00".
- Type:** A section containing two radio buttons: "Continuous" (selected) and "Discrete".
- Step:** A text field next to the "Discrete" radio button, currently empty.
- Buttons:** "OK", "Cancel", and "Help" buttons at the bottom.

OptQuest Objectives Screen



The screenshot shows the OptQuest software interface. On the left is a vertical navigation pane with the following options: Welcome, Objectives (highlighted in orange), Decision Variables, Constraints, and Options. The main area is titled "Select an objective and optionally specify requirements" and features a 3D surface plot icon in the top right. Below the title, there is a "Primary workbook:" dropdown menu set to "Portfolio Allocation Model.xlsx". The "Objectives:" section contains a text field with the text "Maximize the Mean of Total expected return" and an "Exclude" checkbox to its right. The "Requirements:" section contains a text field with the text "(optional requirements on forecasts)" and an "Exclude" checkbox to its right. At the bottom of the main area are five buttons: "Add Objective", "Add Requirement", "Efficient Frontier", "Import...", and "Delete". At the very bottom of the window are four buttons: "< Back", "Next >", "Run", and "Close", with a "Help" button located to the right of the "Close" button.

OptQuest

Welcome
Objectives
Decision Variables
Constraints
Options

Select an objective and optionally specify requirements

Primary workbook: Portfolio Allocation Model.xlsx

Objectives: ☐ Exclude

Maximize the Mean of Total expected return

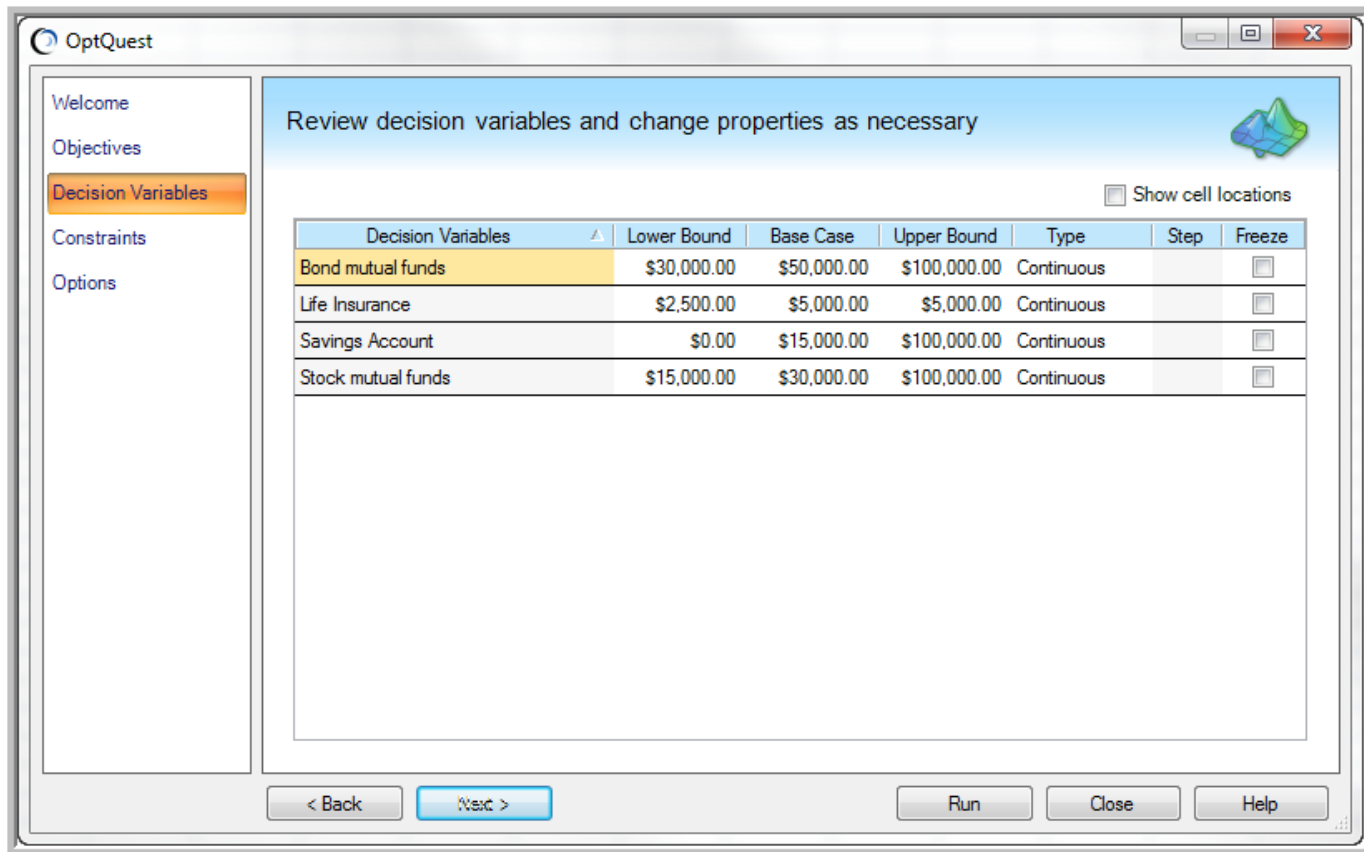
Requirements: ☐ Exclude

(optional requirements on forecasts)

Add Objective Add Requirement Efficient Frontier Import... Delete

< Back Next > Run Close Help

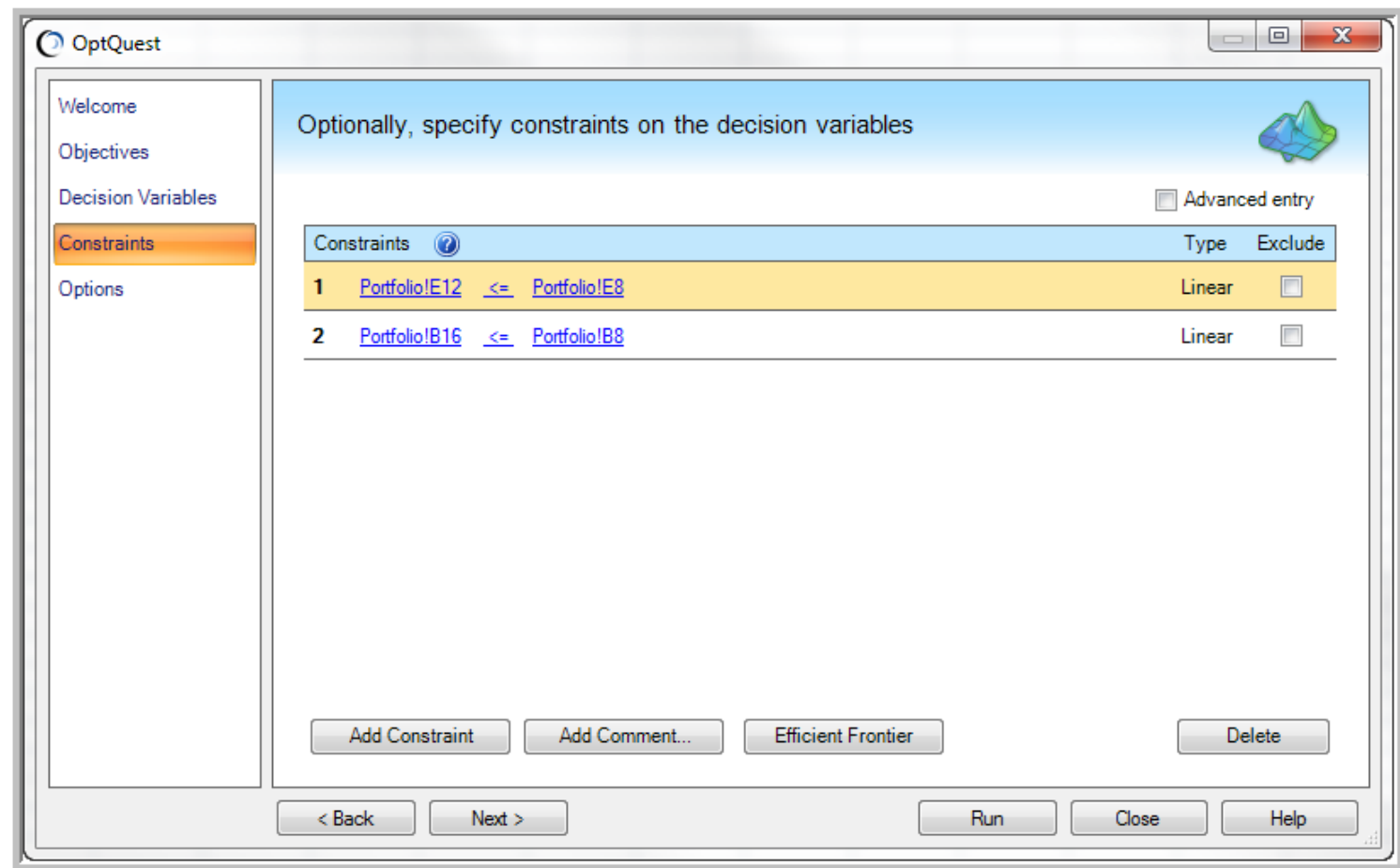
OptQuest Decision Variable Selection Screen



The screenshot shows the OptQuest software interface. On the left is a sidebar with navigation links: Welcome, Objectives, Decision Variables (highlighted), Constraints, and Options. The main area is titled 'Review decision variables and change properties as necessary' and contains a table of decision variables. A checkbox 'Show cell locations' is present. At the bottom are buttons for '< Back', 'Next >', 'Run', 'Close', and 'Help'.

Decision Variables	Lower Bound	Base Case	Upper Bound	Type	Step	Freeze
Bond mutual funds	\$30,000.00	\$50,000.00	\$100,000.00	Continuous		<input type="checkbox"/>
Life Insurance	\$2,500.00	\$5,000.00	\$5,000.00	Continuous		<input type="checkbox"/>
Savings Account	\$0.00	\$15,000.00	\$100,000.00	Continuous		<input type="checkbox"/>
Stock mutual funds	\$15,000.00	\$30,000.00	\$100,000.00	Continuous		<input type="checkbox"/>

OptQuest Constraints Screen



The screenshot shows the OptQuest software interface. On the left is a vertical navigation pane with the following items: Welcome, Objectives, Decision Variables, Constraints (highlighted in orange), and Options. The main area has a light blue header with the text "Optionally, specify constraints on the decision variables" and a 3D surface plot icon. Below this is a table of constraints. The table has three columns: "Constraints" (with a help icon), "Type", and "Exclude". There are two rows of constraints. Row 1: "1 Portfolio!E12 <= Portfolio!E8", "Linear", and an unchecked "Exclude" checkbox. Row 2: "2 Portfolio!B16 <= Portfolio!B8", "Linear", and an unchecked "Exclude" checkbox. To the right of the table is an unchecked checkbox labeled "Advanced entry". At the bottom of the main area are four buttons: "Add Constraint", "Add Comment...", "Efficient Frontier", and "Delete". The bottom of the window has a navigation bar with buttons: "< Back", "Next >", "Run", "Close", and "Help".

OptQuest

Welcome
Objectives
Decision Variables
Constraints
Options

Optionally, specify constraints on the decision variables

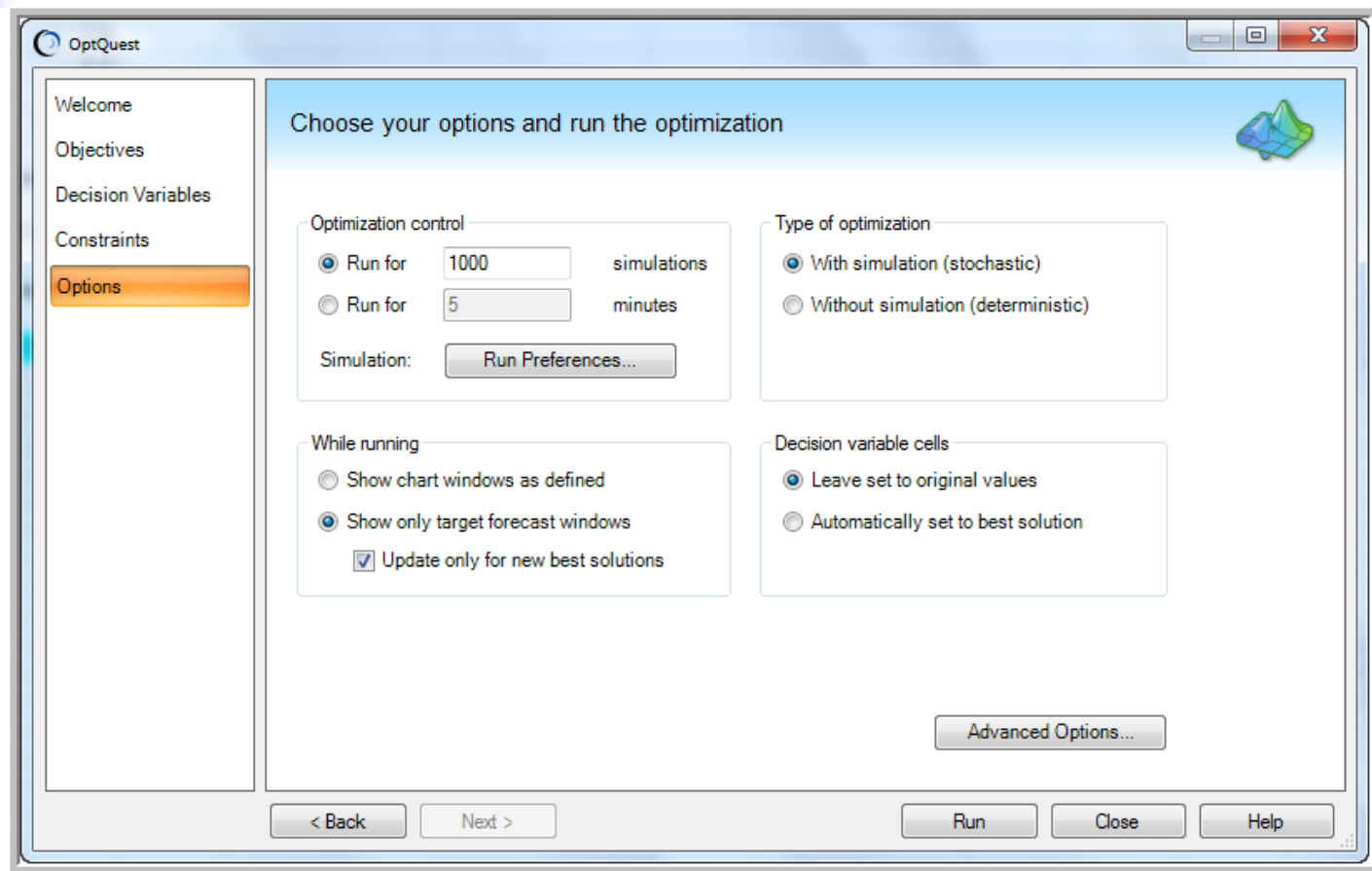
☐ Advanced entry

Constraints	Type	Exclude
1 Portfolio!E12 <= Portfolio!E8	Linear	<input type="checkbox"/>
2 Portfolio!B16 <= Portfolio!B8	Linear	<input type="checkbox"/>

Add Constraint Add Comment... Efficient Frontier Delete

< Back Next > Run Close Help

OptQuest Options Screen

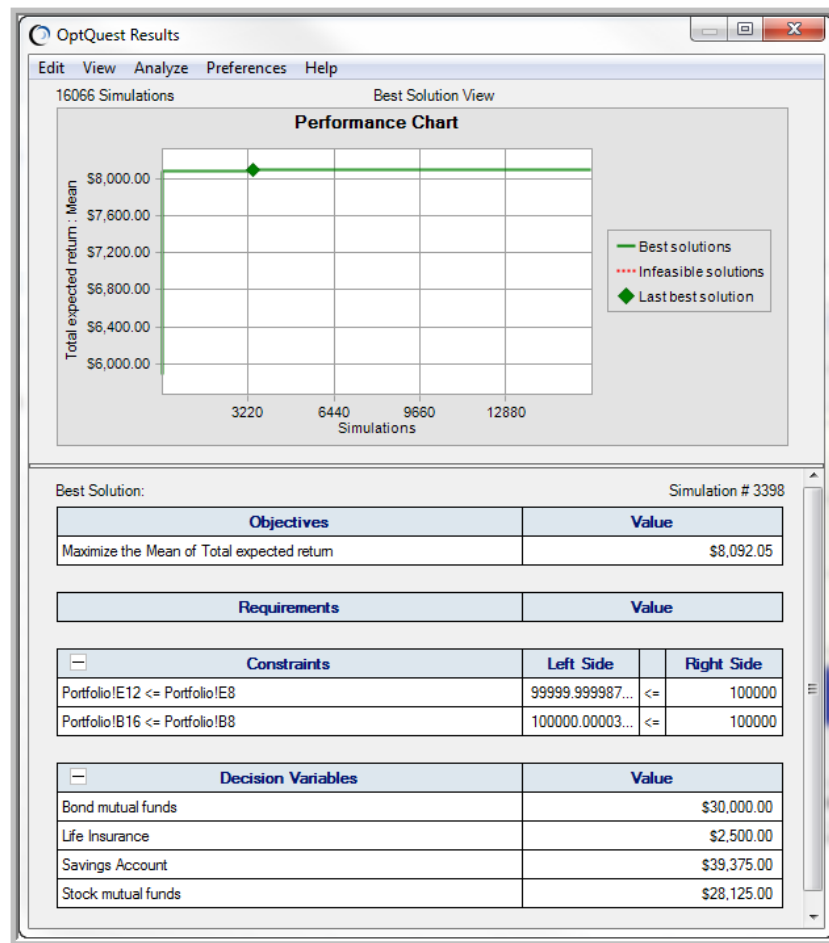


The screenshot shows the OptQuest Options Screen. On the left is a vertical navigation pane with the following items: Welcome, Objectives, Decision Variables, Constraints, and Options (which is highlighted with an orange background). The main area is titled "Choose your options and run the optimization" and contains several settings:

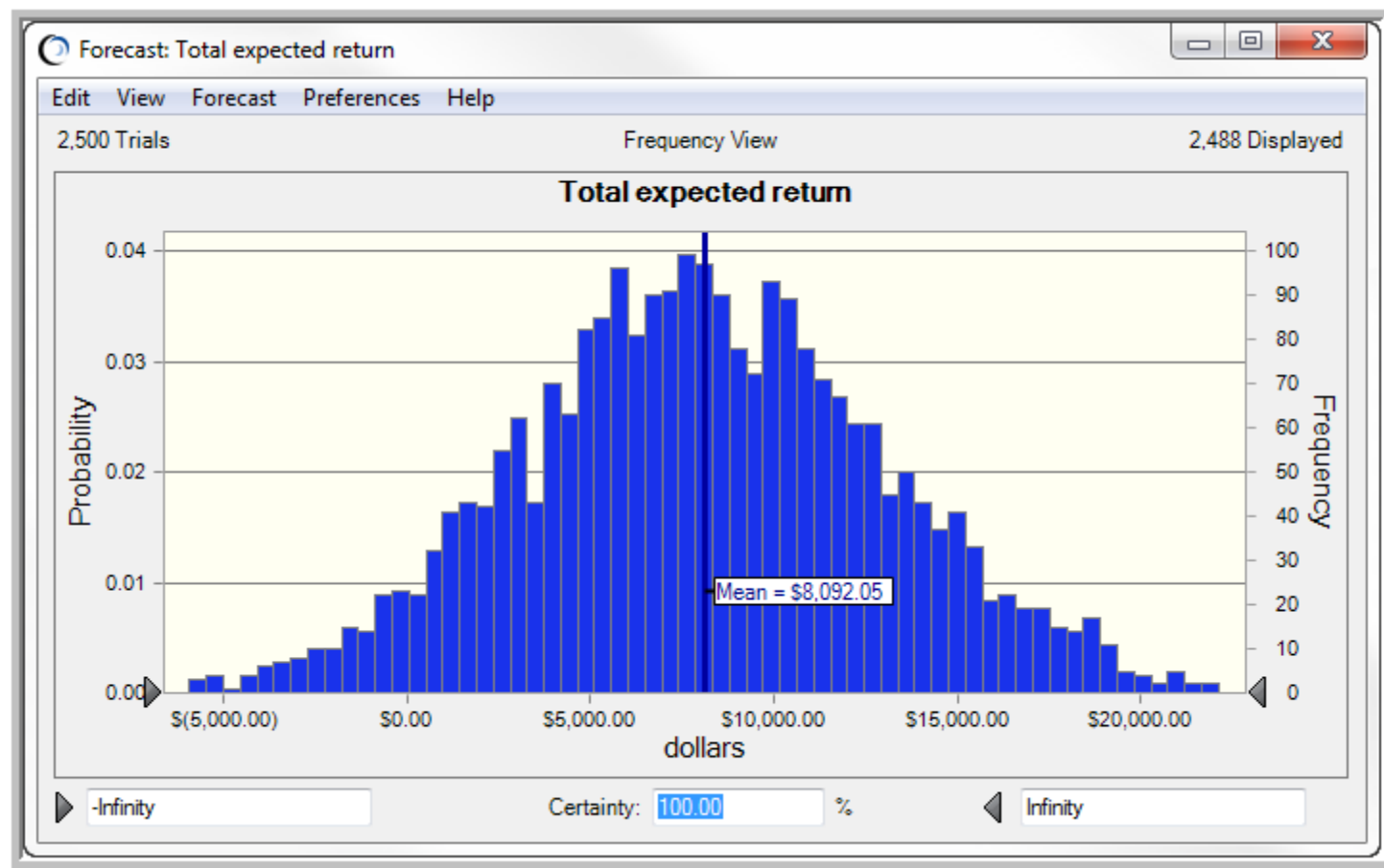
- Optimization control:**
 - ☒ Run for 1000 simulations
 - ☐ Run for 5 minutes
 - Simulation: Run Preferences...
- Type of optimization:**
 - ☒ With simulation (stochastic)
 - ☐ Without simulation (deterministic)
- While running:**
 - ☐ Show chart windows as defined
 - ☒ Show only target forecast windows
 - ☒ Update only for new best solutions
- Decision variable cells:**
 - ☒ Leave set to original values
 - ☐ Automatically set to best solution

At the bottom right of the main area is a button labeled "Advanced Options...". At the bottom of the window are four buttons: "< Back", "Next >", "Run", and "Close". A "Help" button is also present in the bottom right corner.

OptQuest Results



Crystal Ball Results



Adding a Requirement

The screenshot shows the OptQuest software window. On the left is a sidebar with a tree view containing 'Welcome', 'Objectives' (highlighted in orange), 'Decision Variables', 'Constraints', and 'Options'. The main area has a title bar 'OptQuest' and standard window controls. Below the title bar is a light blue header with the text 'Select an objective and optionally specify requirements' and a 3D surface plot icon. Below this, there's a 'Primary workbook:' dropdown menu set to 'Portfolio Allocation Model.xlsx'. The 'Objectives' section shows a table with one row: 'Maximize the Mean of Total expected return'. The 'Requirements' section shows a table with one row: 'The Standard Deviation of Total expected return must be less than or equal to \$1,000.00 dollars'. At the bottom of the main area are buttons: 'Add Objective', 'Add Requirement', 'Efficient Frontier', 'Import...', and 'Delete'. At the very bottom of the window are navigation buttons: '< Back', 'Next >', 'Run', 'Close', and 'Help'.

OptQuest

Welcome
Objectives
Decision Variables
Constraints
Options

Select an objective and optionally specify requirements

Primary workbook: Portfolio Allocation Model.xlsx

Objectives: ? Exclude

Maximize the Mean of Total expected return

Requirements: ? Exclude

The Standard Deviation of Total expected return must be less than or equal to \$1,000.00 dollars

Add Objective Add Requirement Efficient Frontier Import... Delete

< Back Next > Run Close Help

OptQuest Results with Requirement

