Northeastern University
College of Professional Studies
Analytics Program

ALY 6050 Project 6

Problem 1

Josh Steele manages a professional choir in a major city. His marketing plan is focused on generating additional local demand for concerts and increasing ticket revenue, and also gaining attention at the national level to build awareness of the ensemble across the country.

He has $20,000 to spend on media advertising. The goal of the advertisement campaign is to generate as much local recognition as possible while reaching at least 4,000 units of national exposure. He has set a limit of 100 total ads. Additional information is shown below.

Media	Price	Local Exposure	National Exposure	Limit
FM radio spot	$80.00	110	40	30
AM radio spot	$65.00	55	20	30
Cityscape ad	$250.00	80	5	24
MetroWeekly ad	$225.00	65	8	24
Hometown paper ad	$500.00	400	70	10
Neighborhood paper ad	$300.00	220	40	10
Downtown magazine ad	$55.00	35	0	15
Choir journal ad	$350.00	10	75	12
Professional organization magazine ad	$300.00	20	65	12

The last column sets limits on the number of ads to ensure that the advertising markets do not become saturated.

a. Find the optimal number of ads of each type to run to meet the choir's goals by developing and solving an integer optimization model.
b. What if he decides to use no more than six different types of ads? Modify the model in part (a) to answer this question.

Hints:

- For part (b), add binary variables along with proper constraints to select the six media options.
- Use multiple successive iterations to reach a converged solution, using each ending iteration solution as the input to the next one. This should help convergence to optimality.

Problem 2

The Hodrick-Prescott Filter (Decomposition) is a mathematical method used in real business cycle theory in economics to decompose a time series into its cyclical and trend components. Its formulation is based on the following quadratic programming problem: Let $\boldsymbol{X_t}$ be the logarithm of a time series (hence, itself a time series). The Hodrick-Prescott filtering decomposes $\boldsymbol{X_t}$ into its cyclic component $\boldsymbol{C_t}$ and its trend component $\boldsymbol{T_t}$ (that is, $\boldsymbol{X_t} = \boldsymbol{C_t} + \boldsymbol{T_t}$), such that the following quadratic objective functional is minimized:

$$Z = \sum_{t=1}^{N} {C_t}^2 + \omega \sum_{t=2}^{N-1} (T_{t+1} - 2T_t + T_{t-1})^2$$

As for the multiplier $\boldsymbol{\omega}$, Hodrick and Prescott recommend $\boldsymbol{\omega} = \mathbf{100}$ for an annual time series, $\boldsymbol{\omega} = \mathbf{1600}$ for a quarterly time series, and $\boldsymbol{\omega} = \mathbf{14400}$ for a monthly time series.

- **a.** Perform a research about the Hodrick-Prescott decomposition and provide insights about its historical development.
- **b.** Interpret each term of the Hodrick-Prescott objective function and discuss a few advantages and disadvantages of this decomposition method.
- **c.** Consider the quarterly time series of HON stock prices (courtesy of https://finance.yahoo.com given in the Excel workbook: Week 6 Project-part 2-Data.xlsx. Apply the Hodrick-Prescott optimization method to decompose the logarithm of the given time series into its cyclic and trend components. Use the Excel Solver to solve the problem
- **d.** Interpret the results obtained from part (3) above and discuss the merits of your decomposition; if any.
- **e.** Plot the line plots of the original time series along with its trend component on the same chart.
- **f.** Use the results of your decomposition method and write a summary conclusion of your findings.

Hint: Note that the Hodrick-Prescott decomposition of the logarithm of a time series is an additive decomposition; thus, this decomposition becomes a multiplicative type of decomposition for the original time series. In other words, if $\boldsymbol{y}$ is the original time series and if $\boldsymbol{x} = \boldsymbol{log}(\boldsymbol{y})$, then the decomposition $\boldsymbol{x} = \boldsymbol{c} + \boldsymbol{t}$ for $\boldsymbol{x}$ implies that $\boldsymbol{y} = \mathbf{10}^{c+t} = (\mathbf{10}^{c})\,(\mathbf{10}^{t})$.