

Part I: Summary of Confidence Intervals:

(I) CI for the Population Mean μ :

Case 1: Large sample ($n \geq 30$):

CI: $(\bar{x} - E, \bar{x} + E)$

$$E = z_c \frac{s}{\sqrt{n}}$$

$$z_c = NORM.S.INV\left(\frac{1+c}{2}\right)$$

Determining the ample size: $n = \left(\frac{z_c s}{E}\right)^2$

Case 2: Small sample, Population is normal, σ is given: Same as case 1.

Case 3: Small sample, Population is normal, σ is unknown:

CI: $(\bar{x} - E, \bar{x} + E)$

$$E = t_c \frac{s}{\sqrt{n}}$$

$$DF = n - 1;$$

$$t_c = T.INV\left(\frac{1+c}{2}, DF\right)$$

(II) CI for the Population Proportion p :

First ensure that $n \hat{p} \geq 10$ and $n \hat{q} \geq 10$; where : $\hat{p} = \frac{x}{n}$ is the sample proportion and $\hat{q} = 1 - \hat{p}$.

Next,

CI: $(\hat{p} - E, \hat{p} + E)$

$$E = z_c \sqrt{\frac{\hat{p} \hat{q}}{n}}$$

$$z_c = NORM.S.INV\left(\frac{1+c}{2}\right).$$

Determining the sample size: $n = \hat{p} \hat{q} \left(\frac{z_c}{E}\right)^2$.

(III) CI for the Population Standard Deviation σ :

CI Lower Limit for σ : $\sqrt{\frac{(n-1) s^2}{\chi_R^2}}$

CI Upper Limit for σ : $\sqrt{\frac{(n-1) s^2}{\chi_L^2}}$

$$\chi_R^2 := CHISQ.INV\left(\frac{1+c}{2}, DF\right)$$

$$\chi_L^2 := CHISQ.INV\left(\frac{1-c}{2}, DF\right)$$

$$DF = n - 1;$$

Part II Hypothesis Testing Summary:

(I) Hypothesis testing for a population mean μ : (large sample: $n \geq 30$)

| | Right Tailed Problem | Left-Tailed Problem | Two Tailed Problem: |
|--|---|--|--|
| Hypotheses | Null: $H_a : \mu \leq \mu_o$ Alternative: $H_a : \mu > \mu_o$ | Null: $H_a : \mu \geq \mu_o$ Alternative: $H_a : \mu < \mu_o$ | Null: $H_a : \mu = \mu_o$ Alternative: $H_a : \mu \neq \mu_o$ |
| Test Statistic z | $z = \frac{(\bar{x} - \mu_o)\sqrt{n}}{s}$ | $z = \frac{(\bar{x} - \mu_o)\sqrt{n}}{s}$ | $z = \frac{(\bar{x} - \mu_o)\sqrt{n}}{s}$ |
| <u>P-Value:</u> | $P(Z \geq z)$ <u>Excel:</u> $= 1 - \text{norm.s.dist}(z, 1)$ | $P(Z \leq z)$ <u>Excel:</u> $= \text{norm.s.dist}(z, 1)$ | $\begin{cases} 2P(Z \geq z) & \text{if } z \text{ is positive} \\ 2P(Z \leq z) & \text{if } z \text{ is negative} \end{cases}$ <u>Excel:</u> $\begin{cases} 2 * (1 - \text{norm.s.dist}(z, 1)) & \text{if } z \geq 0 \\ 2 * \text{norm.s.dist}(z, 1) & \text{if } z < 0 \end{cases}$ |
| <u>Critical Value(s)</u> | Z_c with an area of α on its right <u>Excel:</u> $= \text{norm.s.inv}(1 - \alpha)$ | Z_c with an area of α on its left <u>Excel:</u> $= \text{norm.s.inv}(\alpha)$ | Critical 1: Z_c with an area of $\alpha/2$ on its left <u>Excel:</u> $= \text{norm.s.inv}(\alpha/2)$ Critical 2: Z_c with an area of $\alpha/2$ on its right <u>Excel:</u> $= \text{norm.s.inv}(1 - \alpha/2)$ |
| Decision (Using the Critical values): | Reject H_o if $z \geq Z_c$ | Reject H_o if $z \leq Z_c$ | $\begin{cases} \text{if } z < 0, & \text{reject } H_o \text{ if } z \leq \text{Critical 1} \\ \text{If } z > 0, & \text{reject } H_o \text{ if } z \geq \text{Critical 2} \end{cases}$ |
| Decision (Using the P-values): | Reject H_o if $P\text{value} \leq \alpha$ | Reject H_o if $P\text{value} \leq \alpha$ | Reject H_o if $P\text{value} \leq \alpha$ |

(II) Hypothesis testing for a population mean μ : (small sample, population is Normal, σ is unknown)

| | Right Tailed Problem | Left-Tailed Problem | Two Tailed Problem: |
|--|--|---|--|
| Hypotheses | Null: $H_a : \mu \leq \mu_o$ Alternative: $H_a : \mu > \mu_o$ | Null: $H_a : \mu \geq \mu_o$ Alternative: $H_a : \mu < \mu_o$ | Null: $H_a : \mu = \mu_o$ Alternative: $H_a : \mu \neq \mu_o$ |
| Test Statistic t | $t = \frac{(\bar{x} - \mu_o)\sqrt{n}}{s}$ | $t = \frac{(\bar{x} - \mu_o)\sqrt{n}}{s}$ | $t = \frac{(\bar{x} - \mu_o)\sqrt{n}}{s}$ |
| <u>P-Value:</u> | $P(T \geq t)$ <u>Excel:</u> $= 1 - \text{T.DIST}(t, df, 1)$ $df = n - 1$ | $P(T \leq t)$ <u>Excel:</u> $= \text{T.DIST}(t, df, 1)$ | $\begin{cases} 2P(T \geq t) & \text{if } t \text{ is positive} \\ 2P(T \leq t) & \text{if } t \text{ is negative} \end{cases}$ <u>Excel:</u> $\begin{cases} 2 * (1 - \text{T.DIST}(t, df, 1)) & \text{if } t \geq 0 \\ 2 * \text{T.DIST}(t, df, 1) & \text{if } t < 0 \end{cases}$ |
| <u>Critical Value(s)</u> | T_c with an area of α on its right <u>Excel:</u> $= \text{T.INV}(1 - \alpha, df)$ | T_c with an area of α on its left <u>Excel:</u> $= \text{T.INV}(\alpha, df)$ | Critical 1: T_c with an area of $\alpha/2$ on its left <u>Excel:</u> $= \text{T.INV}(\alpha/2, df)$ Critical 2: T_c with an area of $\alpha/2$ on its right <u>Excel:</u> $= \text{T.INV}(1 - \alpha/2, df)$ |
| Decision (Using the Critical values): | Reject H_o if $t \geq T_c$ | Reject H_o if $t \leq T_c$ | $\begin{cases} \text{if } t < 0, \text{ reject } H_o \text{ if } t \leq \text{Critical 1} \\ \text{If } t > 0, \text{ reject } H_o \text{ if } t \geq \text{Critical 2} \end{cases}$ |
| Decision (Using the P-values): | Reject H_o if $P\text{value} \leq \alpha$ | Reject H_o if $P\text{value} \leq \alpha$ | Reject H_o if $P\text{value} \leq \alpha$ |

(III) Hypothesis testing for a population Proportion p :

| | Right Tailed Problem | Left-Tailed Problem | Two Tailed Problem: |
|--|--|---|---|
| Hypotheses | Null: $H_a : p \leq p_o$ Alternative: $H_a : p > p_o$ | Null: $H_a : p \geq p_o$ Alternative: $H_a : p < p_o$ | Null: $H_a : p = p_o$ Alternative: $H_a : p \neq p_o$ |
| Test Statistic z | $z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}}$ | $z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}}$ | $z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o q_o}{n}}}$ |
| <u>P-Value:</u> | $P(Z \geq z)$ <u>Excel:</u> = 1 - norm.s.dist(z , 1) | $P(Z \leq z)$ <u>Excel:</u> = norm.s.dist(z , 1) | $\begin{cases} 2P(Z \geq z) & \text{if } z \text{ is positive} \\ 2P(Z \leq z) & \text{if } z \text{ is negative} \end{cases}$ <u>Excel:</u> $\begin{cases} 2 * (1 - \text{norm.s.dist}(z, 1)) & \text{if } z \geq 0 \\ 2 * \text{norm.s.dist}(z, 1) & \text{if } z < 0 \end{cases}$ |
| <u>Critical Value(s)</u> | Z_c with an area of α on its right <u>Excel:</u> = norm.s.inv(1 - α) | Z_c with an area of α on its left <u>Excel:</u> = norm.s.inv(α) | Critical 1: Z_c with an area of $\alpha/2$ on its left <u>Excel:</u> = norm.s.inv($\alpha/2$) Critical 2: Z_c with an area of $\alpha/2$ on its right <u>Excel:</u> = norm.s.inv(1 - $\alpha/2$) |
| Decision (Using the Critical values): | Reject H_o if $z \geq Z_c$ | Reject H_o if $z \leq Z_c$ | $\begin{cases} \text{if } z < 0, & \text{reject } H_o \text{ if } z \leq \text{Critical 1} \\ \text{If } z > 0, & \text{reject } H_o \text{ if } z \geq \text{Critical 2} \end{cases}$ |
| Decision (Using the P-values): | Reject H_o if $P\text{value} \leq \alpha$ | Reject H_o if $P\text{value} \leq \alpha$ | Reject H_o if $P\text{value} \leq \alpha$ |

(IV) Hypothesis testing for a population Standard Deviation σ :

| | Right Tailed Problem | Left-Tailed Problem | Two Tailed Problem: |
|--|---|--|---|
| Hypotheses | Null: $H_a : \sigma \leq \sigma_o$ Alternative: $H_a : \sigma > \sigma_o$ | Null: $H_a : \sigma \geq \sigma_o$ Alternative: $H_a : \sigma < \sigma_o$ | Null: $H_a : \sigma = \sigma_o$ Alternative: $H_a : \sigma \neq \sigma_o$ |
| Test Statistic χ^2 | $\chi^2 = \frac{(n-1)s^2}{\sigma_o^2}$ | $\chi^2 = \frac{(n-1)s^2}{\sigma_o^2}$ | $\chi^2 = \frac{(n-1)s^2}{\sigma_o^2}$ |
| P-Value: | $P(X^2 \geq \chi^2)$ <u>Excel:</u> $= 1 - \text{chisq.dist}(\chi^2, df, 1)$ $df = n - 1$ | $P(X^2 \leq \chi^2)$ <u>Excel:</u> $= \text{chisq.dist}(\chi^2, df, 1)$ | The smaller of $2 P(X^2 \geq \chi^2)$ or $2 P(X^2 \leq \chi^2)$ <u>Excel:</u> The smaller of $= 2 * (1 - \text{chisq.dist}(\chi^2, df, 1))$ Or $= 2 * \text{chisq.dist}(\chi^2, df, 1)$ |
| <u>Critical Value(s)</u> | χ_{c^2} with an area of α on its right <u>Excel:</u> $= \text{chisq.inv}(1 - \alpha, df)$ | χ_{c^2} with an area of α on its left <u>Excel:</u> $= \text{chisq.inv}(\alpha, df)$ | Critical 1: χ_{c^2} with an area of $\alpha/2$ on its left <u>Excel:</u> $= \text{chisq.inv}(\alpha/2, df)$ Critical 2: χ_{c^2} with an area of $\alpha/2$ on its right <u>Excel:</u> $= \text{chisq.inv}(1 - \alpha/2, df)$ |
| Decision (Using the Critical values): | Reject H_o if $\chi^2 \geq \chi_{c^2}$ | Reject H_o if $\chi^2 \leq \chi_{c^2}$ | Reject H_o if either $\chi^2 \leq \text{Critical 1}$, or if $\chi^2 \geq \text{Critical 2}$ |
| Decision (Using the P-values): | Reject H_o if $P\text{value} \leq \alpha$ | Reject H_o if $P\text{value} \leq \alpha$ | Reject H_o if $P\text{value} \leq \alpha$ |

Part III: Summary of Continuous Probability Formulas in Excel

1. The Normal Distribution:

(i) Standard Normal Distribution Z

$$P(Z \leq z): \quad \underline{= NORM.S.DIST(z, 1)}$$

$$z: \quad \underline{= NORM.S.INV(P(Z \leq z))}$$

$P(Z \leq z)$ represents the area on the left of Z

(ii) Non-Standard Normal Distribution X

$$P(X \leq x): \quad \underline{= NORM.DIST(x, \mu, \sigma, 1)}$$

$$x: \quad \underline{= NORM.INV(P(X \leq x), \mu, \sigma)}$$

$P(X \leq x)$ represents the area on the left of x

2. The T Distribution T (DF = $n - 1$)

$$P(T \leq t): \quad \underline{= T.DIST(t, df)}$$

$$t: \quad \underline{= T.INV(P(T \leq t), df)}$$

$P(T \leq t)$ represents the area on the left of t

3. The Chi-squared Distribution X^2 (DF = $n - 1$)

$$P(X^2 \leq \chi^2): \quad \underline{= CHISQ.DIST(\chi^2, df, 1)}$$

$$\chi^2: \quad \underline{= CHISQ.INV(P(X^2 \leq \chi^2), df)}$$

$P(X^2 \leq \chi^2)$ represents the area on the left of χ^2