Chapter 7: Forecasting

Statistics, Data Analysis, and Decision Modeling, Fifth Edition

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Forecasting Techniques

- Qualitative and judgmental
- Statistical time series models
- Explanatory/causal models

Qualitative and Judgmental Methods

- Historical analogy – comparative analysis with a previous situation
- Delphi Method – response to a sequence of questionnaires by a panel of experts

Indicators and Indexes

- Indicators – measures believed to influence the behavior of a variable we wish to forecast
  - Leading indicators
  - Lagging indicators
- Index – a weighted combination of indicators
- Indicators and indexes are often used in economic forecasting

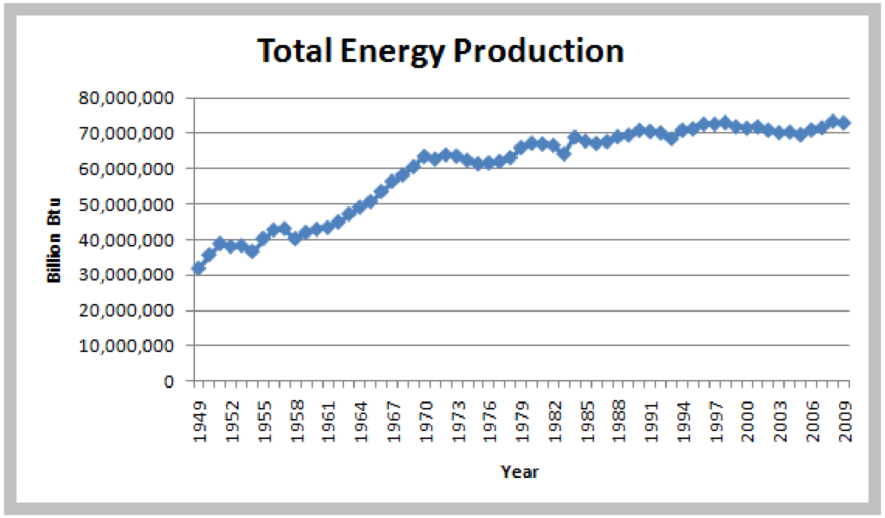
Example Dept. of Commerce Index of Leading Indicators

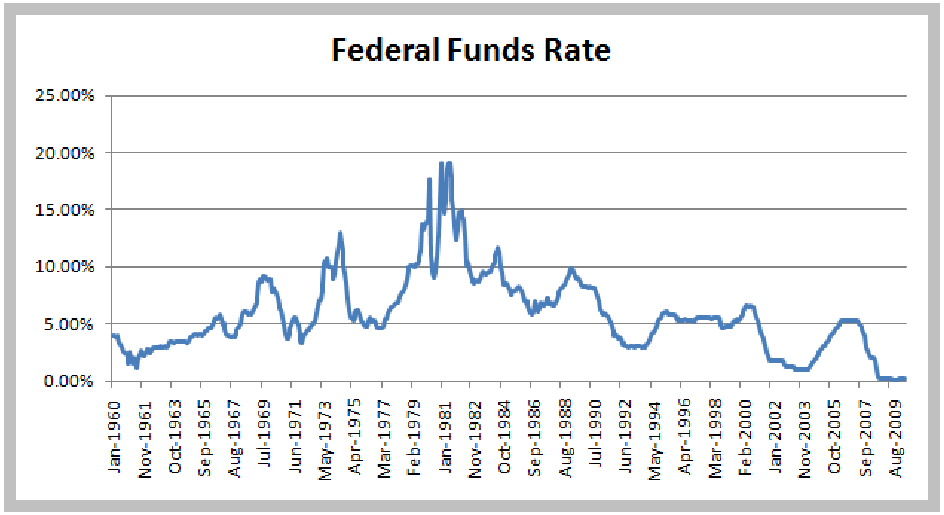
- Average weekly hours, manufacturing
- Average weekly initial claims, unemployment insurance
- New orders, consumer goods and materials
- Vendor performance—slower deliveries
- New orders, nondefense capital goods
- Building permits, private housing
- Stock prices, 500 common stocks (Standard & Poor)
- Money supply
- Interest rate spread
- Index of consumer expectations (University of Michigan)

Time Series

- A time series is a stream of historical data
- Components of time series
  - Trend
  - Short-term seasonal effects
  - Longer-term cyclical effects

Examples of Time Series





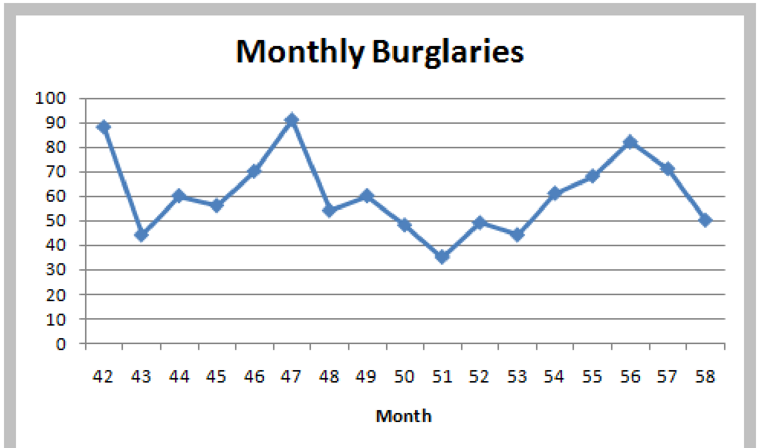
Statistical Forecasting Methods

- Moving average
- Exponential smoothing
- Regression analysis

Simple Moving Average

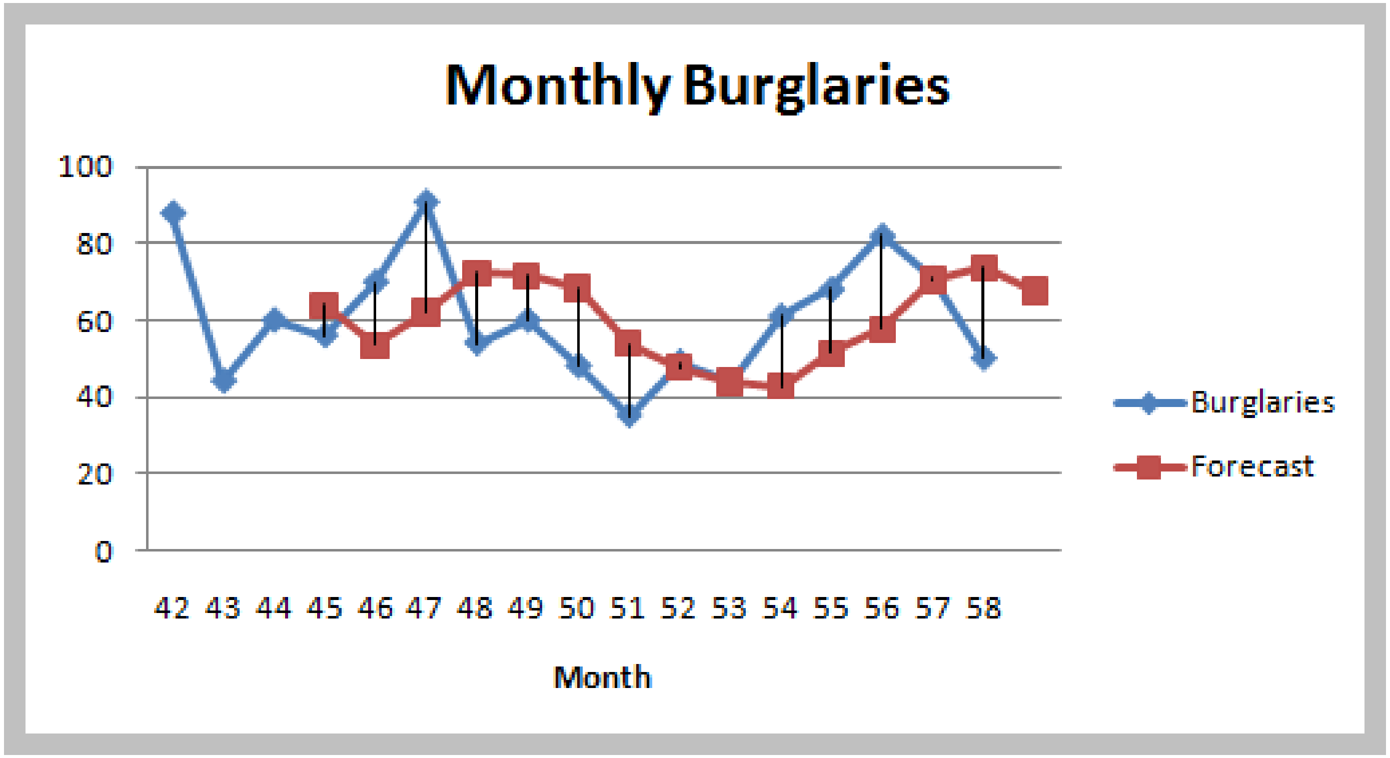
- Average random fluctuations in a time series to infer short-term changes in direction
- Assumption: future observations will be similar to recent past
- Moving average for next period = average of most recent k observations

Example: Moving Average Forecast With k = 3



	C	D	E	F
1	After Citizen-Police Program		Moving Average	
2	Month	Monthly burglaries	Forecast	
3	42	88		
4	43	44		
5	44	60		
6	45	56	64.00	Forecast for month 45 =AVERAGE(D3:D5)
7	46	70	53.33	
8	47	91	62.00	
9	48	54	72.33	
10	49	60	71.67	
11	50	48	68.33	
12	51	35	54.00	
13	52	49	47.67	
14	53	44	44.00	
15	54	61	42.67	
16	55	68	51.33	
17	56	82	57.67	
18	57	71	70.33	
19	58	50	73.67	
20	59		67.67	Forecast for month 59 =AVERAGE(D17:D19)
21				

Time Series Data and Moving Averages



Alternative Moving Average Models

	A	B	C	D	E	F	G	H
1	After Citizen-Police Program						3 Period	
2	Month	Monthly burglaries	k = 2	Error	k = 3	Error	Weighted	Error
3	42	88						
4	43	44						
5	44	60	66.00	-6.00				
6	45	56	52.00	4.00	64.00	-8.00	58.00	-2.00
7	46	70	58.00	12.00	53.33	16.67	56.00	14.00
8	47	91	63.00	28.00	62.00	29.00	64.80	26.20
9	48	54	80.50	-26.50	72.33	-18.33	81.20	-27.20
10	49	60	72.50	-12.50	71.67	-11.67	66.70	-6.70
11	50	48	57.00	-9.00	68.33	-20.33	61.30	-13.30
12	51	35	54.00	-19.00	54.00	-19.00	52.20	-17.20
13	52	49	41.50	7.50	47.67	1.33	41.40	7.60
14	53	44	42.00	2.00	44.00	0.00	44.70	-0.70
15	54	61	46.50	14.50	42.67	18.33	44.60	16.40
16	55	68	52.50	15.50	51.33	16.67	54.70	13.30
17	56	82	64.50	17.50	57.67	24.33	63.50	18.50
18	57	71	75.00	-4.00	70.33	0.67	75.70	-4.70
19	58	50	76.50	-26.50	73.67	-23.67	74.00	-24.00
20	59		60.50		67.67		59.50	

Error Metrics and Forecast Accuracy

- Mean absolute deviation (MAD)

$$\text{MAD} = \frac{\sum_{i=1}^{n} \left| A_t - F_t \right|}{n}$$

- Mean square error (MSE)

$$\text{MSE} = \frac{\sum_{i=1}^{n} \left(A_t - F_t \right)^2}{n}$$

- Mean absolute percentage error (MAPE)

$$\text{MAPE} = \frac{\sum_{i=1}^{n} \left| \frac{A_t - F_t}{A_t} \right|}{n}$$

Summary of Error Metrics for Burglary Data

TABLE 7.2 Error Metrics for Moving Average Models of Burglary Data

	$k = 2$	$k = 3$	Three-Period Weighted
MAD	13.63	14.86	13.70
MSE	254.38	299.84	256.31
RMSE	15.95	17.32	16.01
MAPE	23.63%	26.53%	24.46%

Exponential Smoothing

- Exponential smoothing model:

$$F_{t+1} = (1-\alpha)F_t + \alpha A_t$$
$$= F_t + \alpha(A_t - F_t)$$

- F_{t+1} is the forecast for time period t+1,
- F_t is the forecast for period t,
- A_t is the observed value in period t, and
- α is a constant between 0 and 1, called the smoothing constant.

Exponential Smoothing Example

	C	D	E	F	G	H	I	J	K	L	M
1	**After Citizen-Police Program**		**Smoothing Constant**								
2	**Month**	**Monthly burglaries**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
3	42	88	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00
4	43	44	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00	88.00
5	44	60	83.60	79.20	74.80	70.40	66.00	61.60	57.20	52.80	48.40
6	45	56	81.24	75.36	70.36	66.24	63.00	60.64	59.16	58.56	58.84
7	46	70	78.72	71.49	66.05	62.14	59.50	57.86	56.95	56.51	56.28
8	47	91	77.84	71.19	67.24	65.29	64.75	65.14	66.08	67.30	68.63
9	48	54	79.16	75.15	74.37	75.57	77.88	80.66	83.53	86.26	88.76
10	49	60	76.64	70.92	68.26	66.94	65.94	64.66	62.86	60.45	57.48
11	50	48	74.98	68.74	65.78	64.17	62.97	61.87	60.86	60.09	59.75
12	51	35	72.28	64.59	60.45	57.70	55.48	53.55	51.86	50.42	49.17
13	52	49	68.55	58.67	52.81	48.62	45.24	42.42	40.06	38.08	36.42
14	53	44	66.60	56.74	51.67	48.77	47.12	46.37	46.32	46.82	47.74
15	54	61	64.34	54.19	49.37	46.86	45.56	44.95	44.70	44.56	44.37
16	55	68	64.00	55.55	52.86	52.52	53.28	54.58	56.11	57.71	59.34
17	56	82	64.40	58.04	57.40	58.71	60.64	62.63	64.43	65.94	67.13
18	57	71	66.16	62.83	64.78	68.03	71.32	74.25	76.73	78.79	80.51
19	58	50	66.65	64.47	66.65	69.22	71.16	72.30	72.72	72.56	71.95
20	59		64.98	61.57	61.65	61.53	60.58	58.92	56.82	54.51	52.20
21		**MAD**	19.33	17.16	16.15	15.36	14.93	14.71	14.72	14.88	15.36
22		**MSE**	496.07	390.84	359.18	346.56	340.77	338.41	339.03	343.32	352.38
23		**RMSE**	22.273	19.77	18.952	18.616	18.46	18.396	18.413	18.529	18.772
24		**MAPE**	38.28%	32.71%	30.12%	28.36%	27.54%	27.09%	27.09%	27.38%	28.23%

Forecasting Models With Linear Trends

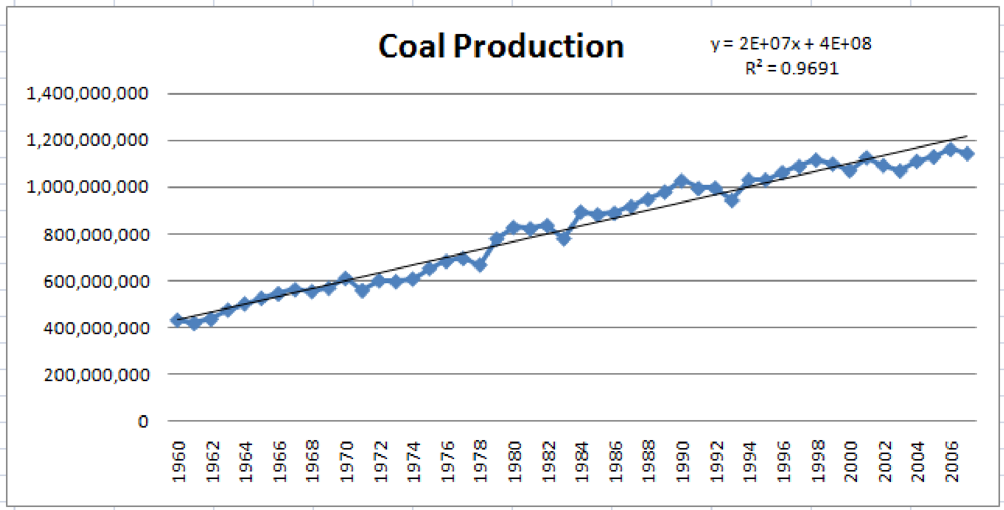
- Double Moving Average
- Double Exponential Smoothing
- Based on the linear trend equation

$$F_{t+k} = a_t + b_t k$$

Regression-Based Forecasting

Coal Production

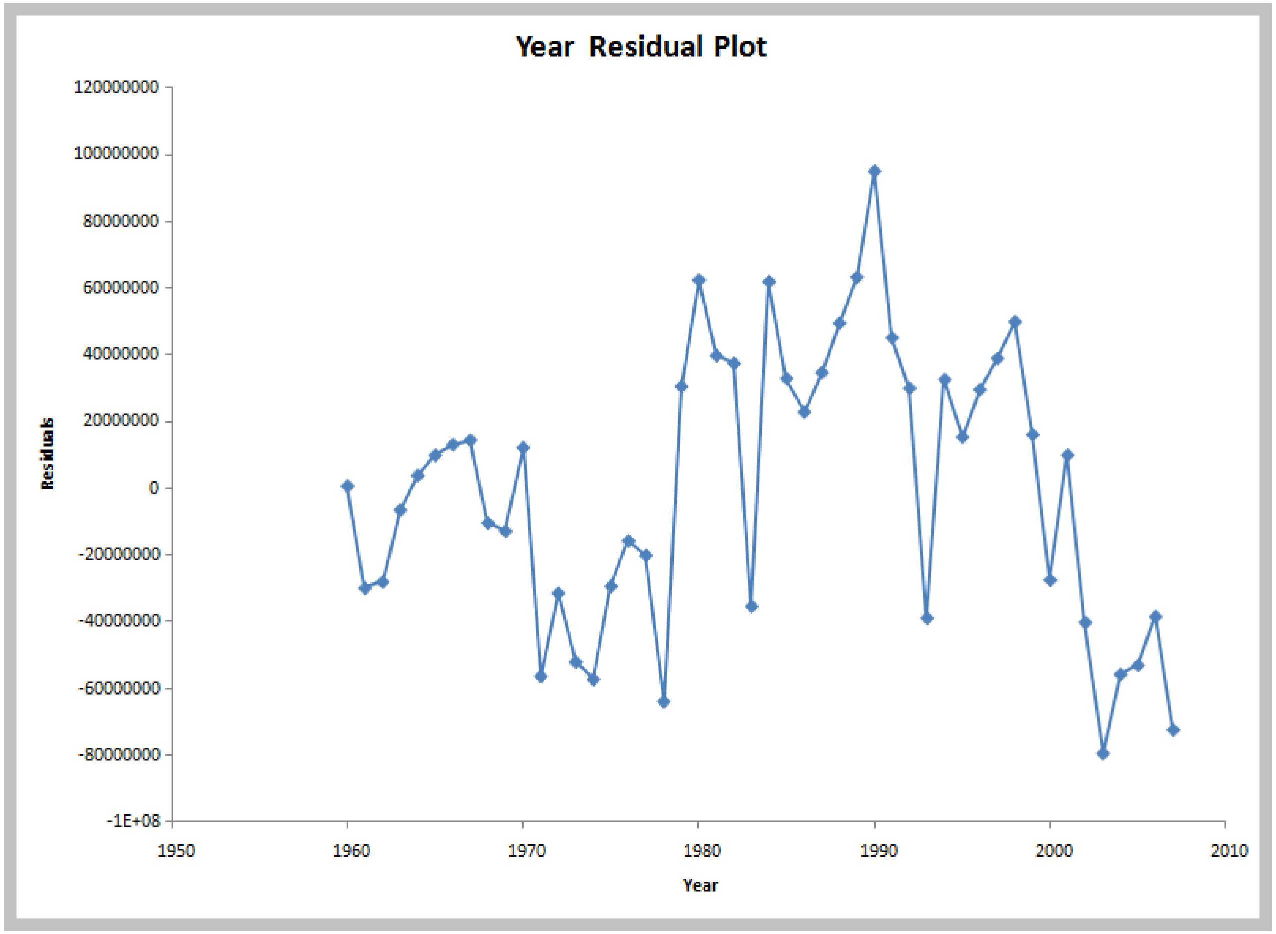
Year	Total Tons
1960	434,329,000
1961	420,423,000
1962	439,043,000
1963	477,195,000
1964	504,182,000
1965	526,954,000
1966	546,822,000
1967	564,882,000
1968	556,706,000
1969	570,978,000
1970	612,661,000
1971	560,919,000
1972	602,492,000
1973	598,568,000
1974	610,023,000
1975	654,641,000
1976	684,913,000
1977	697,205,000
1978	670,164,000
1979	781,134,000



A forecast for 2008 would be:

Tons = 416,896,322.7 + 16,685,398.57 * (49) = 1,234,480,853

Autocorrelated Data



Autoregressive Models

- First-order autoregressive model
  - $Y_i = a_0 + a_1 Y_{i-1} + \delta_i$
- Second-order autoregressive model
  - $Y_i = a_0 + a_1 Y_{i-1} + a_2 Y_{i-2} + \delta_i$

Portion of Coal Production File for Autoregessive Modeling

	A	B	C	D	E
1	**Coal Production**				
2					
3	**Year**	**Total Tons**	**Year - 1**	**Year - 2**	**Year - 3**
4	1960	434,329,000			
5	1961	420,423,000	434,329,000		
6	1962	439,043,000	420,423,000	434,329,000	
7	1963	477,195,000	439,043,000	420,423,000	434,329,000
8	1964	504,182,000	477,195,000	439,043,000	420,423,000
9	1965	526,954,000	504,182,000	477,195,000	439,043,000
10	1966	546,822,000	526,954,000	504,182,000	477,195,000

Third-Order Autoregressive Model

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2							
3	*Regression Statistics*						
4	Multiple R	0.988073439					
5	R Square	0.97628912					
6	Adjusted R Square	0.974554178					
7	Standard Error	35247696.59					
8	Observations	45					
9							
10	ANOVA						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
12	Regression	3	2.09738E+18	6.99125E+17	562.721337	2.49811E-33	
13	Residual	41	5.09384E+16	1.2424E+15			
14	Total	44	2.14831E+18				
15							
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17	Intercept	50332718.33	20917609.66	2.406236618	0.020709694	8088749.075	92576687.59
18	Year - 1	0.565292456	0.154196228	3.666058917	0.000700825	0.25388686	0.876698052
19	Year - 2	0.247260179	0.174243467	1.41904993	0.163442473	-0.104631637	0.599151996
20	Year - 3	0.156914847	0.151580327	1.035192694	0.306646344	-0.14920783	0.463037525

Second-Order Autogressive Model

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2							
3	*Regression Statistics*						
4	Multiple R	0.988567258					
5	R Square	0.977265224					
6	Adjusted R Square	0.976207792					
7	Standard Error	34986337.08					
8	Observations	46					
9							
10	ANOVA						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
12	Regression	2	2.26249E+18	1.13125E+18	924.1877675	4.66498E-36	
13	Residual	43	5.26339E+16	1.22404E+15			
14	Total	45	2.31513E+18				
15							
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17	Intercept	43087157.38	19471726.51	2.212806213	0.032269724	3818678.935	82355635.83
18	Year - 1	0.63224491	0.142019472	4.451818489	5.95528E-05	0.345835353	0.918654467
19	Year - 2	0.341258327	0.141128508	2.418067984	0.019912562	0.056645569	0.625871085

Forecasting Models With Seasonality

- Additive model

$$F_{t+k} = a_t + S_{t-s+k}$$

- Multiplicative model

$$F_{t+k} = a_t \, S_{t-s+k}$$

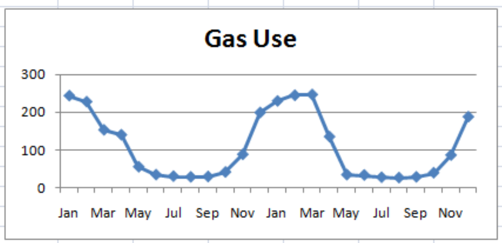
Incorporating Seasonality into Regression Models

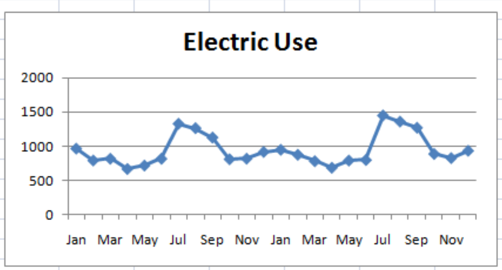
- Use dummy variables. Example:

$$\text{Gas usage} = \beta_0 + \beta_1\,\text{Time} + \beta_2\,\text{February} + \beta_3\,\text{March} + \beta_4\,\text{April} + \beta_5\,\text{May} + \beta_6\,\text{June} + \beta_7\,\text{July} + \beta_8\,\text{August} + \beta_9\,\text{September} + \beta_{10}\,\text{October} + \beta_{11}\,\text{November} + \beta_{12}\,\text{December}$$

Gas and Electric Usage

Month	Gas Use	Electric Use
Jan	244	967
Feb	228	795
Mar	153	820
Apr	140	672
May	55	722
Jun	34	820
Jul	30	1326
Aug	28	1262
Sep	29	1126
Oct	41	814
Nov	88	821
Dec	199	918
Jan	230	950
Feb	245	878
Mar	247	785
Apr	135	690
May	34	794
Jun	33	802
Jul	27	1445
Aug	26	1357
Sep	28	1268
Oct	39	889
Nov	86	830
Dec	188	935





Data Matrix

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
3	**Month**	**Gas Use**	**Time**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
4	Jan	244	1	0	0	0	0	0	0	0	0	0	0	0
5	Feb	228	2	1	0	0	0	0	0	0	0	0	0	0
6	Mar	153	3	0	1	0	0	0	0	0	0	0	0	0
7	Apr	140	4	0	0	1	0	0	0	0	0	0	0	0
8	May	55	5	0	0	0	1	0	0	0	0	0	0	0
9	Jun	34	6	0	0	0	0	1	0	0	0	0	0	0
10	Jul	30	7	0	0	0	0	0	1	0	0	0	0	0
11	Aug	28	8	0	0	0	0	0	0	1	0	0	0	0
12	Sep	29	9	0	0	0	0	0	0	0	1	0	0	0
13	Oct	41	10	0	0	0	0	0	0	0	0	1	0	0
14	Nov	88	11	0	0	0	0	0	0	0	0	0	1	0
15	Dec	199	12	0	0	0	0	0	0	0	0	0	0	1
16	Jan	230	13	0	0	0	0	0	0	0	0	0	0	0
17	Feb	245	14	1	0	0	0	0	0	0	0	0	0	0
18	Mar	247	15	0	1	0	0	0	0	0	0	0	0	0
19	Apr	135	16	0	0	1	0	0	0	0	0	0	0	0
20	May	34	17	0	0	0	1	0	0	0	0	0	0	0
21	Jun	33	18	0	0	0	0	1	0	0	0	0	0	0
22	Jul	27	19	0	0	0	0	0	1	0	0	0	0	0
23	Aug	26	20	0	0	0	0	0	0	1	0	0	0	0
24	Sep	28	21	0	0	0	0	0	0	0	1	0	0	0
25	Oct	39	22	0	0	0	0	0	0	0	0	1	0	0
26	Nov	86	23	0	0	0	0	0	0	0	0	0	1	0
27	Dec	188	24	0	0	0	0	0	0	0	0	0	0	1

Regression Model Results

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2							
3	*Regression Statistics*						
4	Multiple R	0.985480895					
5	R Square	0.971172595					
6	Adjusted R Square	0.948997667					
7	Standard Error	19.54432831					
8	Observations	24					
9							
10	ANOVA						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
12	Regression	10	167292.2083	16729.22083	43.79597661	2.33344E-08	
13	Residual	13	4965.75	381.9807692			
14	Total	23	172257.9583				
15							
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17	Intercept	236.75	9.772164157	24.22697738	3.33921E-12	215.6385229	257.8614771
18	Mar	-36.75	16.92588482	-2.171230656	0.04901621	-73.31615098	-0.183849024
19	Apr	-99.25	16.92588482	-5.863799799	5.55744E-05	-135.816151	-62.68384902
20	May	-192.25	16.92588482	-11.35834268	4.02824E-08	-228.816151	-155.683849
21	Jun	-203.25	16.92588482	-12.00823485	2.07264E-08	-239.816151	-166.683849
22	Jul	-208.25	16.92588482	-12.30364038	1.54767E-08	-244.816151	-171.683849
23	Aug	-209.75	16.92588482	-12.39226204	1.41949E-08	-246.316151	-173.183849
24	Sep	-208.25	16.92588482	-12.30364038	1.54767E-08	-244.816151	-171.683849
25	Oct	-196.75	16.92588482	-11.62420766	3.05791E-08	-233.316151	-160.183849
26	Nov	-149.75	16.92588482	-8.847395666	7.30451E-07	-186.316151	-113.183849
27	Dec	-43.25	16.92588482	-2.555257847	0.023953114	-79.81615098	-6.683849024

Model

$$\text{Gas Usage} = 236.75 - 36.75\text{ March} - 99.25\text{ April} - 192.25\text{ May} - 203.25\text{ June} - 208.25\text{ July} - 209.75\text{ August} - 208.25\text{ September} - 196.75\text{ October} - 149.75\text{ November} - 43.25\text{ December}$$

Models for Trend and Seasonality

- Holt-Winters Additive Model

$$F_{t+1} = a_t + b_t + S_{t-s+1}$$

- Holt-Winters Multiplicative Model

$$F_{t+1} = (a_t + b_t)S_{t-s+1}$$

Model Choice

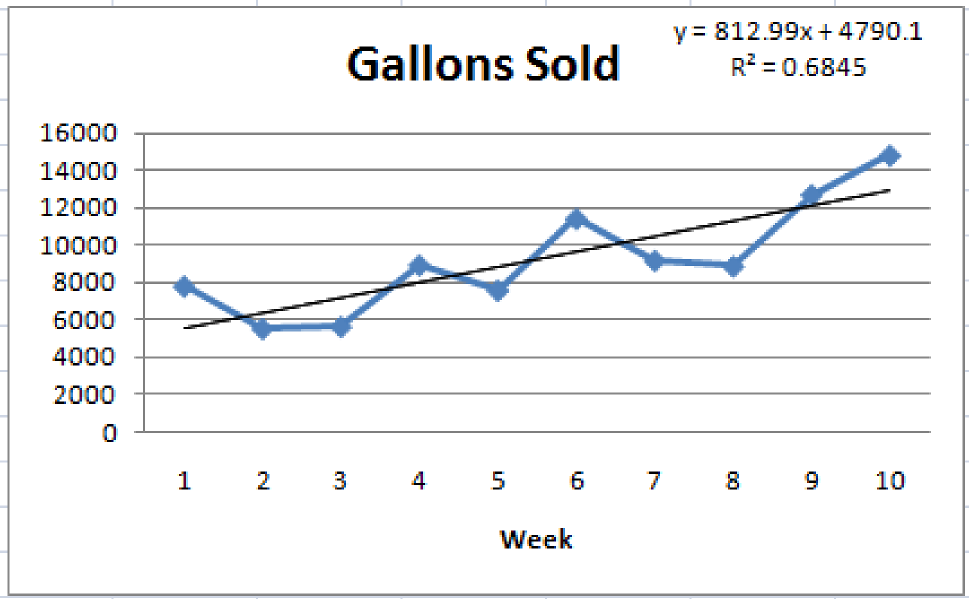
TABLE 7.3 Forecasting Model Choice

	No Seasonality	Seasonality
No Trend	Single moving average or single exponential smoothing	Seasonal additive or seasonal multiplicative model
Trend	Double moving average or double exponential smoothing	Holt–Winters additive or Holt–Winters multiplicative model

Regression Forecasting With Causal Variables

Gasoline Sales

Gallons Sold	Week	Price/Gallon
7815	1	$ 3.95
5541	2	$ 4.20
5650	3	$ 4.12
8949	4	$ 3.98
7600	5	$ 4.01
11430	6	$ 3.92
9190	7	$ 4.03
8889	8	$ 3.98
12721	9	$ 3.92
14830	10	$ 3.90
	11	$ 3.80



Sales (week 11) = 4790.1 + 812.00(11) = 13,733 gallons

Causal Model

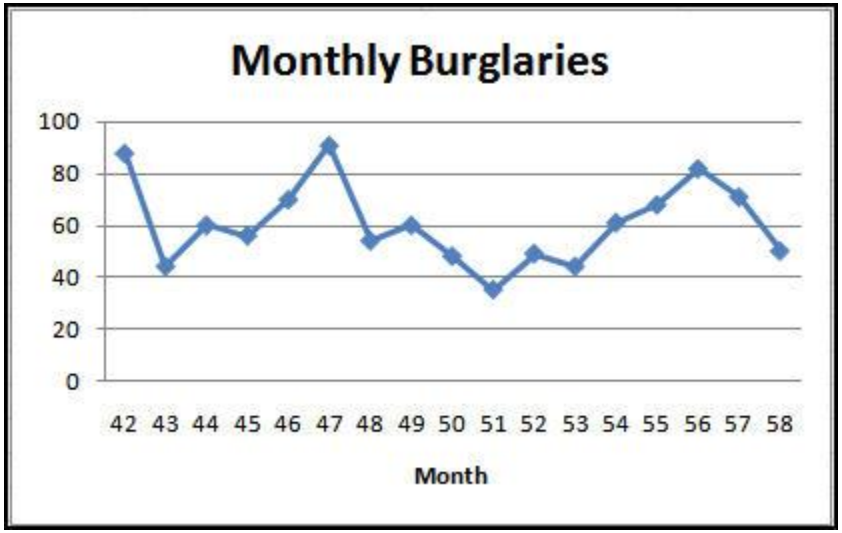
- Sales = β_0 + β_1Week + β_2 Price/Gallon
- Sales = 72333.08 + 508.67 Week – 16463.20 Price/Gallon

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2							
3	*Regression Statistics*						
4	Multiple R	0.930528528					
5	R Square	0.865883342					
6	Adjusted R Square	0.827564297					
7	Standard Error	1235.400329					
8	Observations	10					
9							
10	ANOVA						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
12	Regression	2	68974748.7	34487374.35	22.59668368	0.000883465	
13	Residual	7	10683497.8	1526213.972			
14	Total	9	79658246.5				
15							
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17	Intercept	72333.08447	21969.92267	3.292368642	0.013259225	20382.47253	124283.6964
18	Week	508.6681395	168.1770861	3.024598364	0.019260863	110.9925233	906.3437558
19	Price/Gallon	-16463.19901	5351.082403	-3.076611005	0.017900405	-29116.49823	-3809.899789

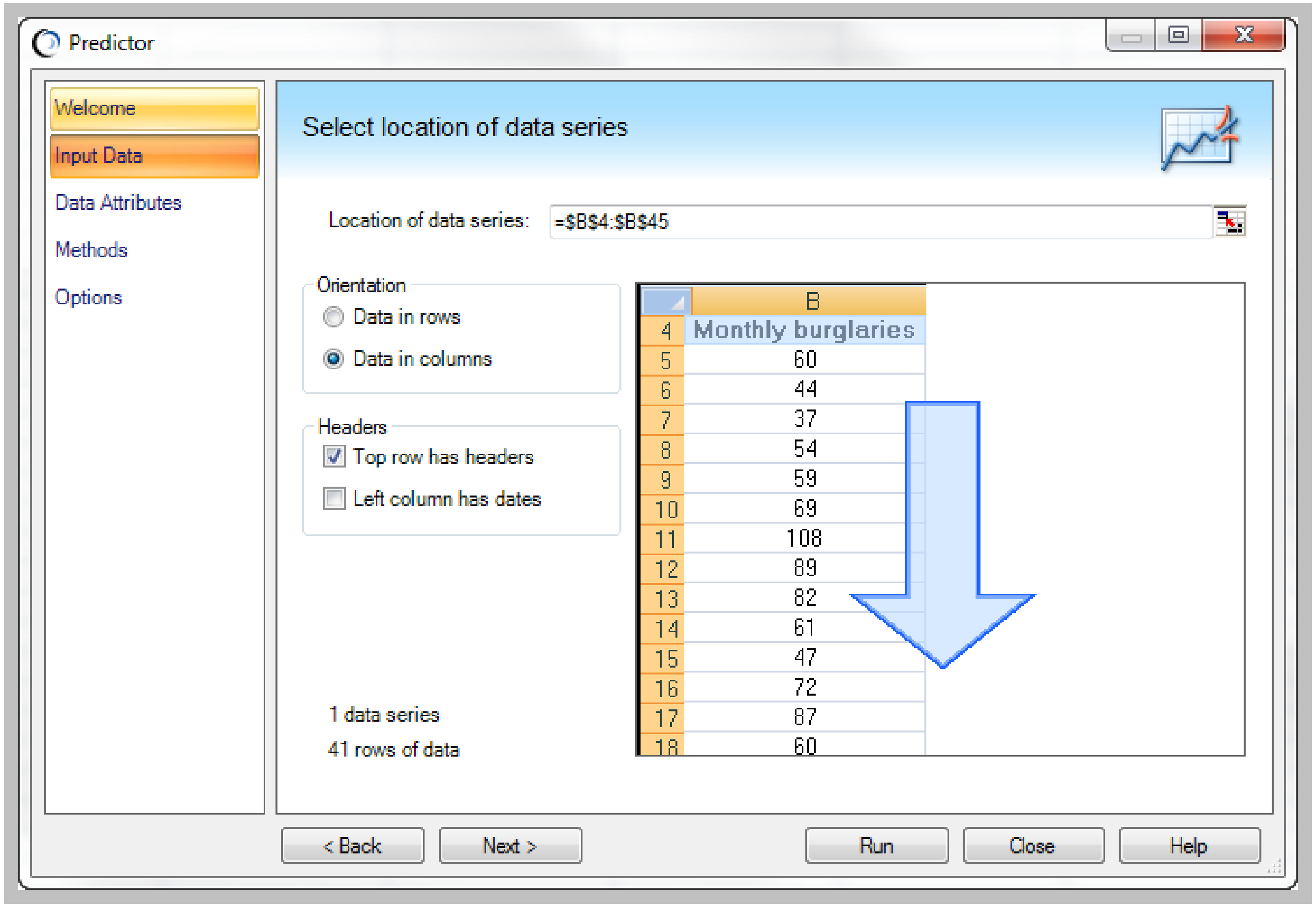
Sales (week 11) = 72333.08 + 508.67(11) - 16463.2(3.80) = 13,733 gallons

CB Predictor

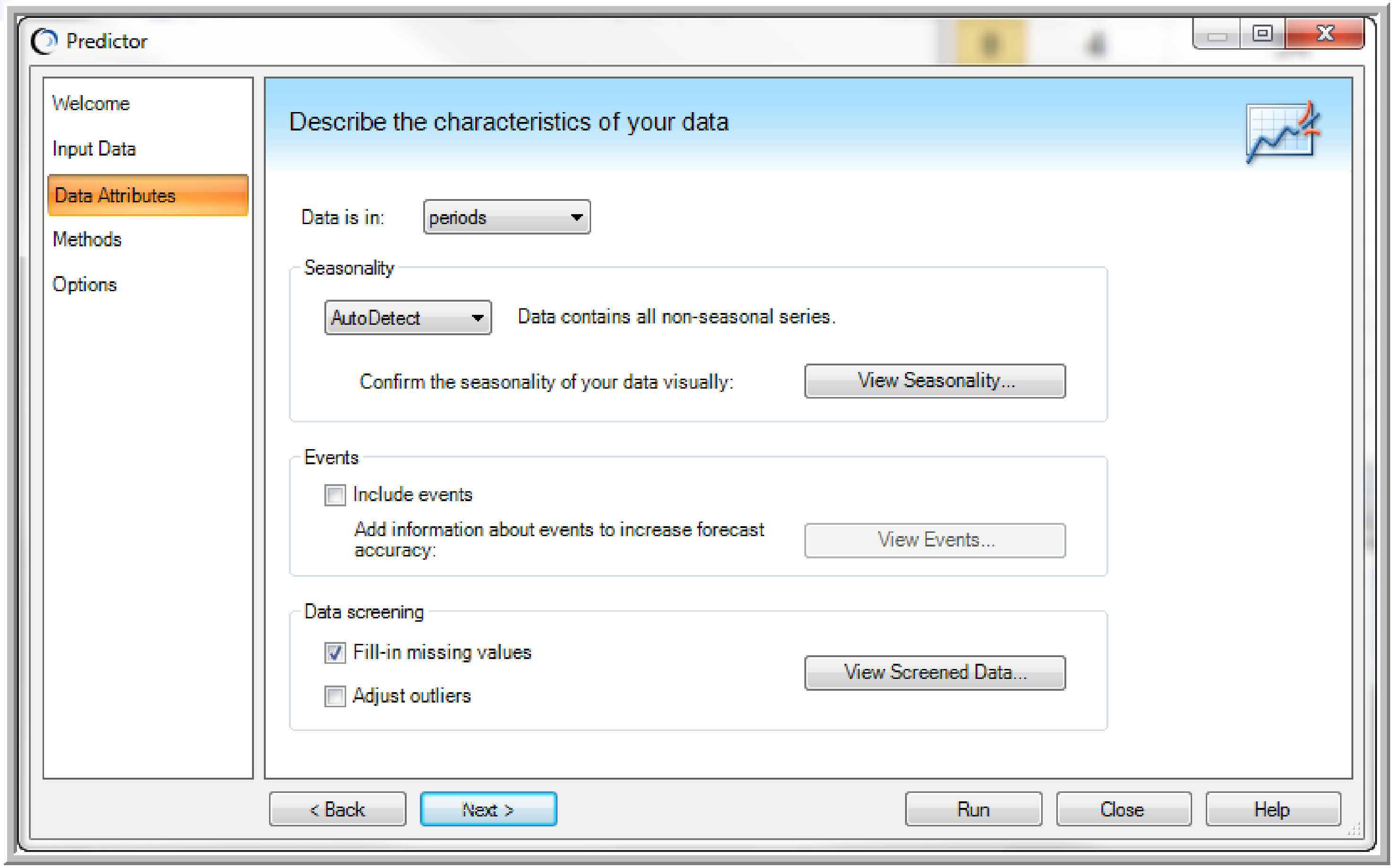
- Excel add-in for forecasting
- Integrated with *Crystal Ball* software
- Example: Burglary Data



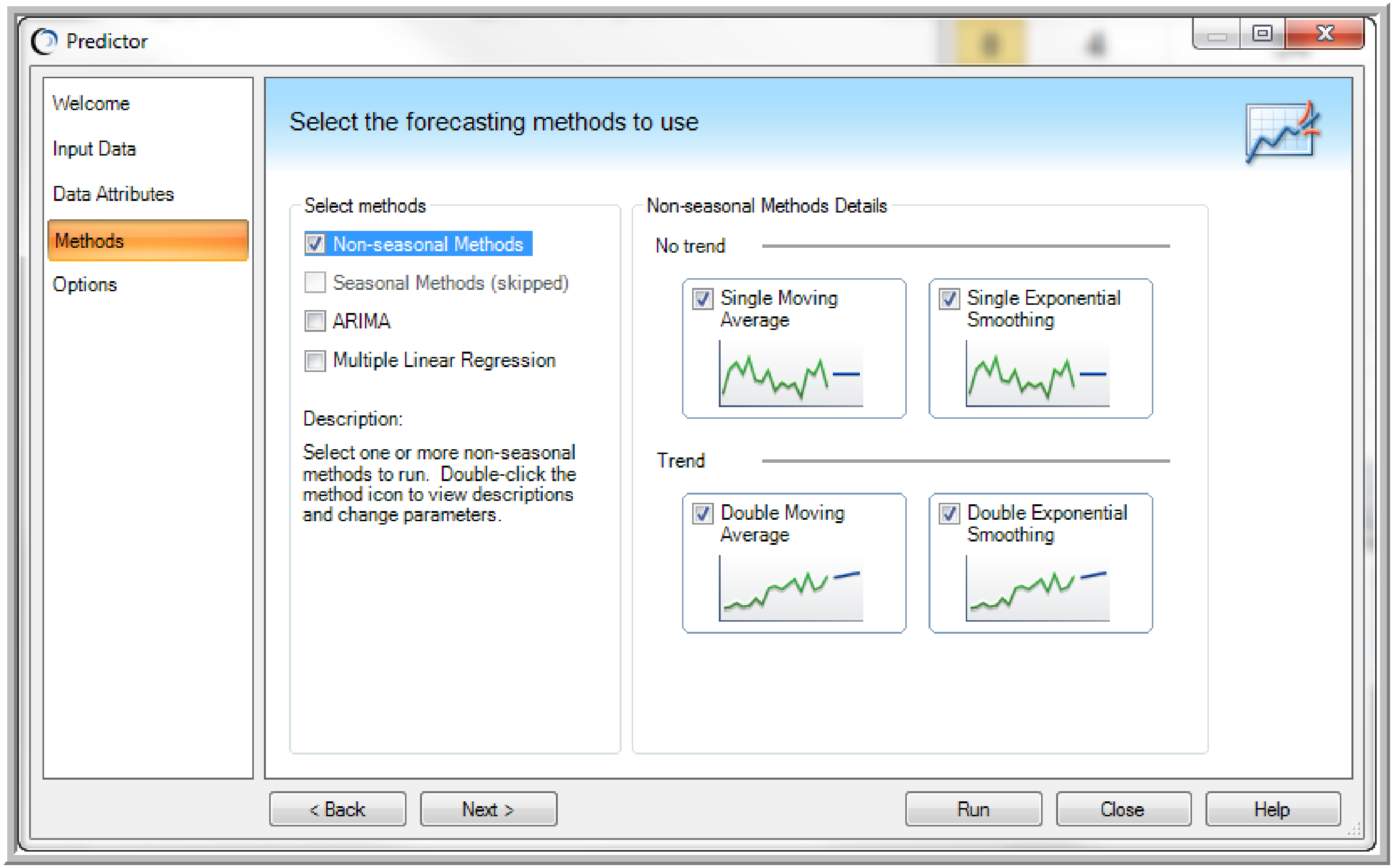
Input Data Dialog



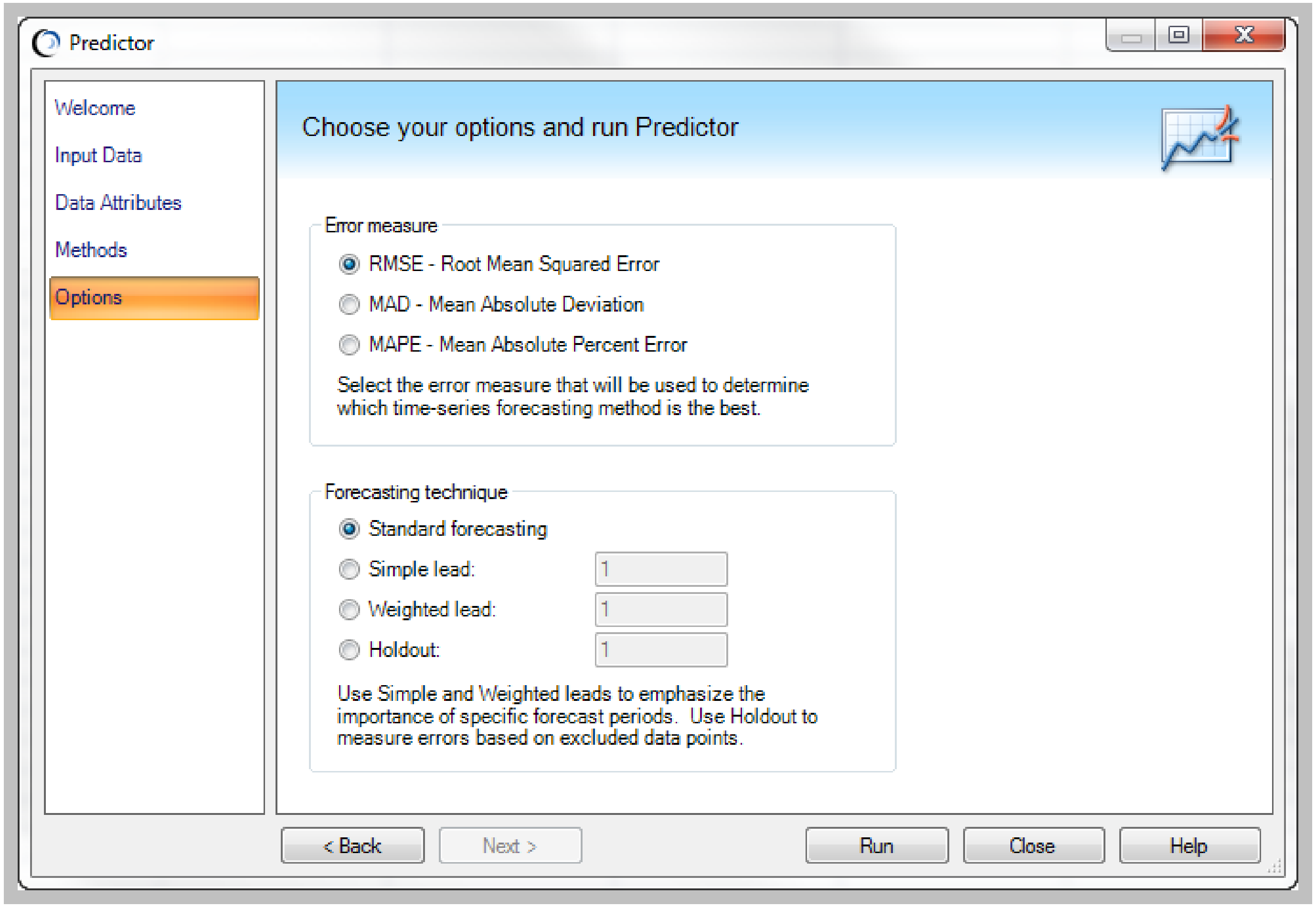
Data Attributes Dialog



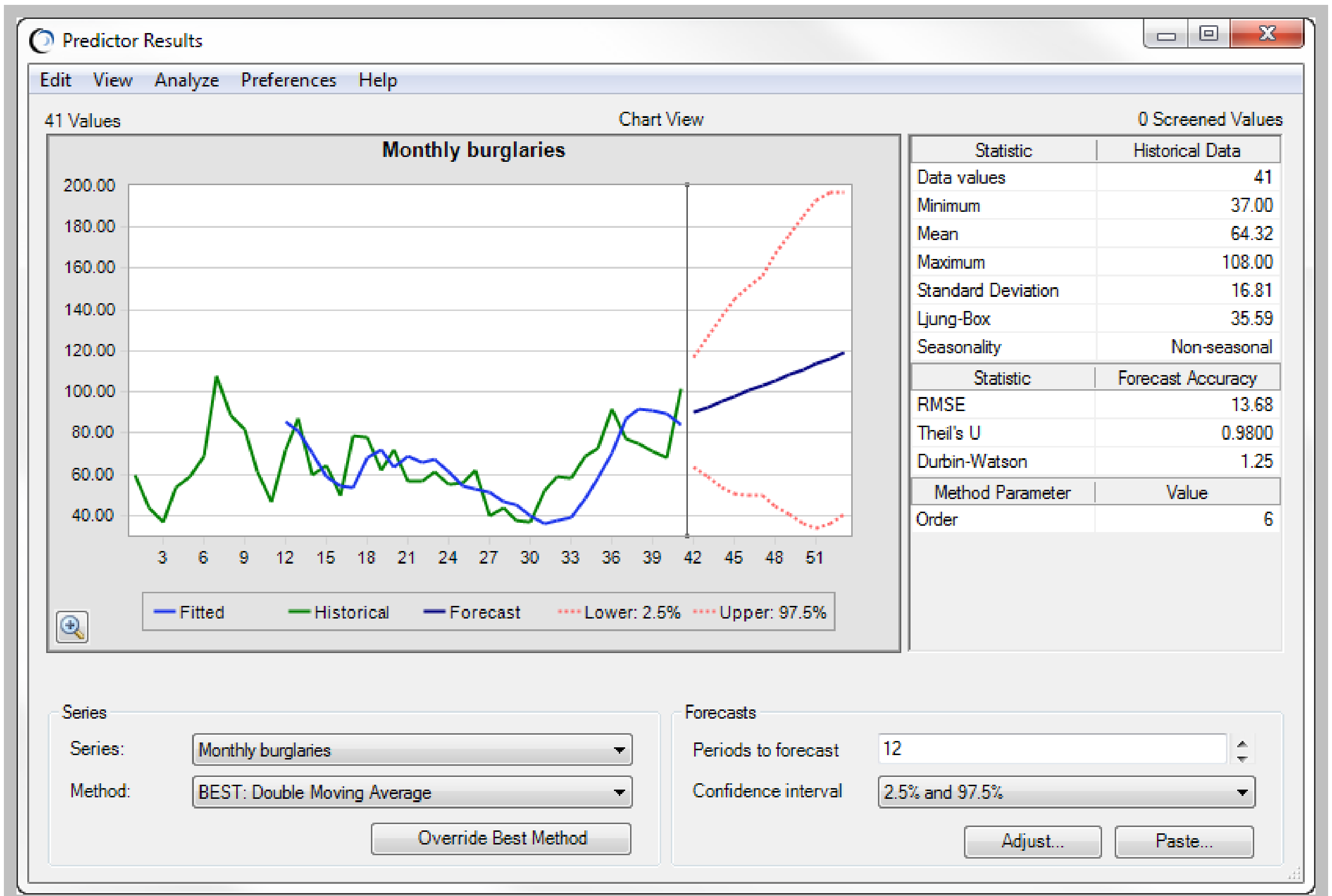
Methods Dialog



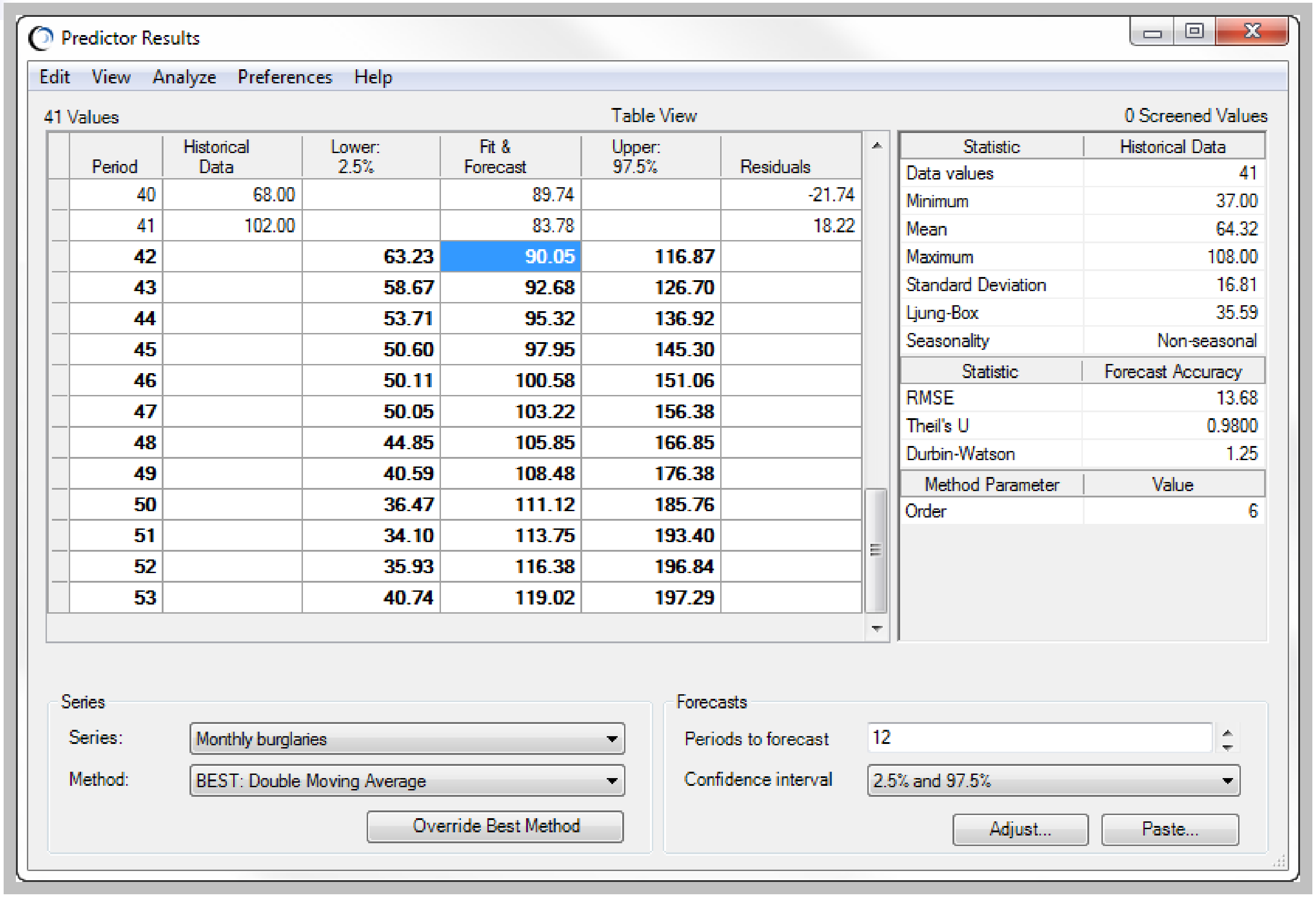
Options Dialog



CB Predictor Results for Forecasting Burglaries



CB Predictor Table View



Predictor Results

Edit View Analyze Preferences Help

41 Values | Table View | 0 Screened Values

Period	Historical Data	Lower: 2.5%	Fit & Forecast	Upper: 97.5%	Residuals
40	68.00		89.74		-21.74
41	102.00		83.78		18.22
42		**63.23**	**90.05**	**116.87**	
43		**58.67**	**92.68**	**126.70**	
44		**53.71**	**95.32**	**136.92**	
45		**50.60**	**97.95**	**145.30**	
46		**50.11**	**100.58**	**151.06**	
47		**50.05**	**103.22**	**156.38**	
48		**44.85**	**105.85**	**166.85**	
49		**40.59**	**108.48**	**176.38**	
50		**36.47**	**111.12**	**185.76**	
51		**34.10**	**113.75**	**193.40**	
52		**35.93**	**116.38**	**196.84**	
53		**40.74**	**119.02**	**197.29**	

Statistic	Historical Data
Data values	41
Minimum	37.00
Mean	64.32
Maximum	108.00
Standard Deviation	16.81
Ljung-Box	35.59
Seasonality	Non-seasonal

Statistic	Forecast Accuracy
RMSE	13.68
Theil's U	0.9800
Durbin-Watson	1.25

Method Parameter	Value
Order	6

Series

Series: Monthly burglaries

Method: BEST: Double Moving Average

Override Best Method

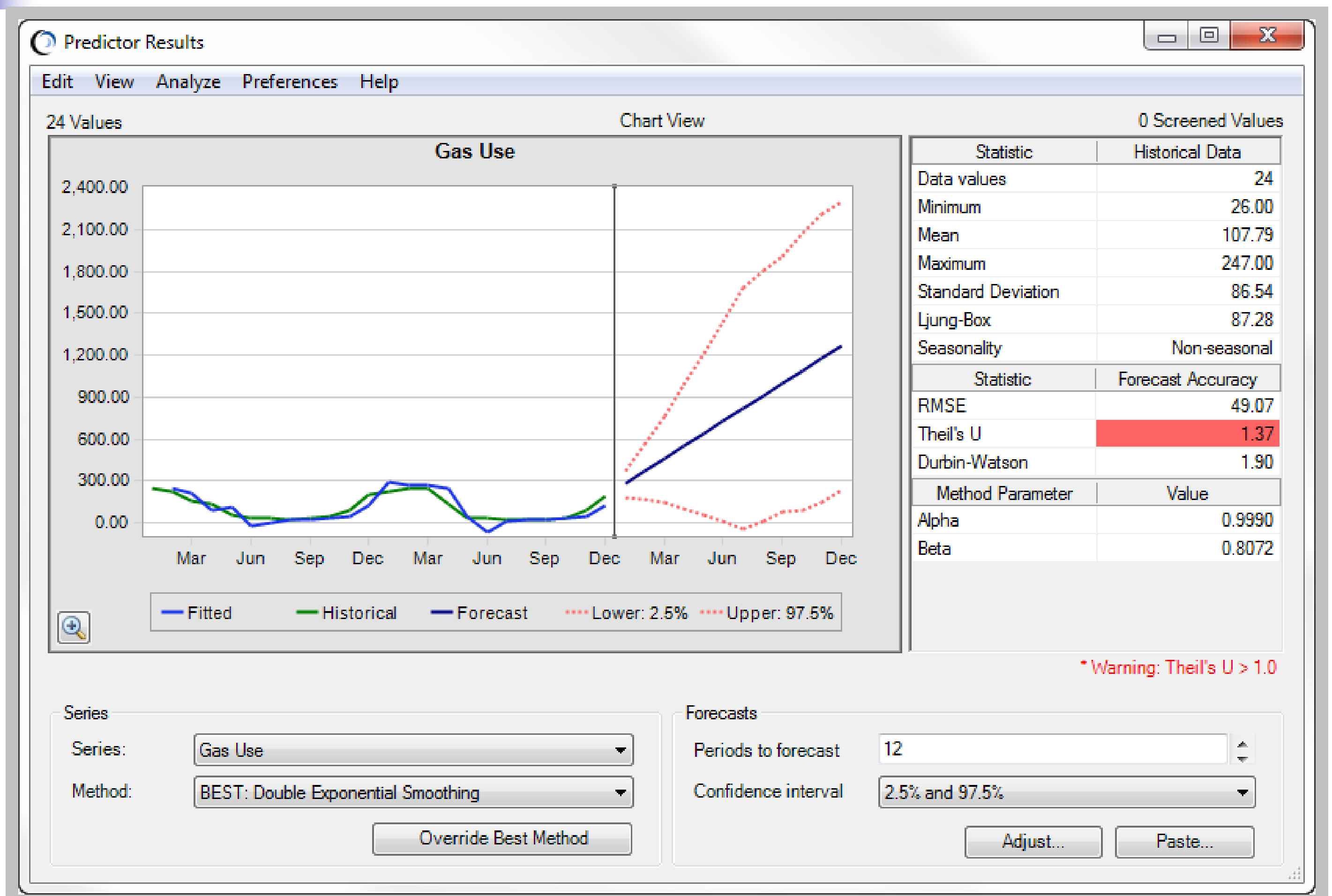
Forecasts

Periods to forecast: 12

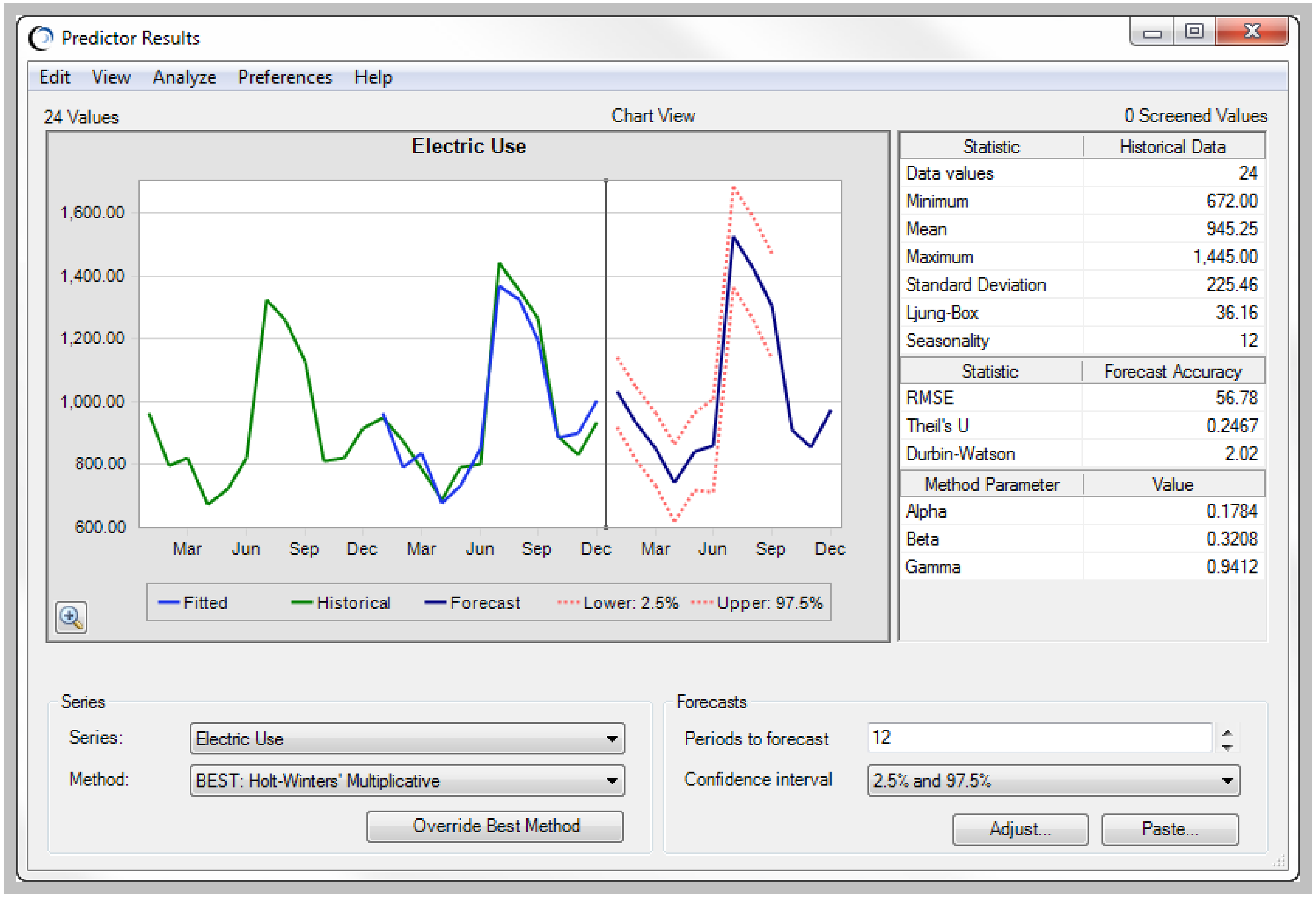
Confidence interval: 2.5% and 97.5%

Adjust... Paste...

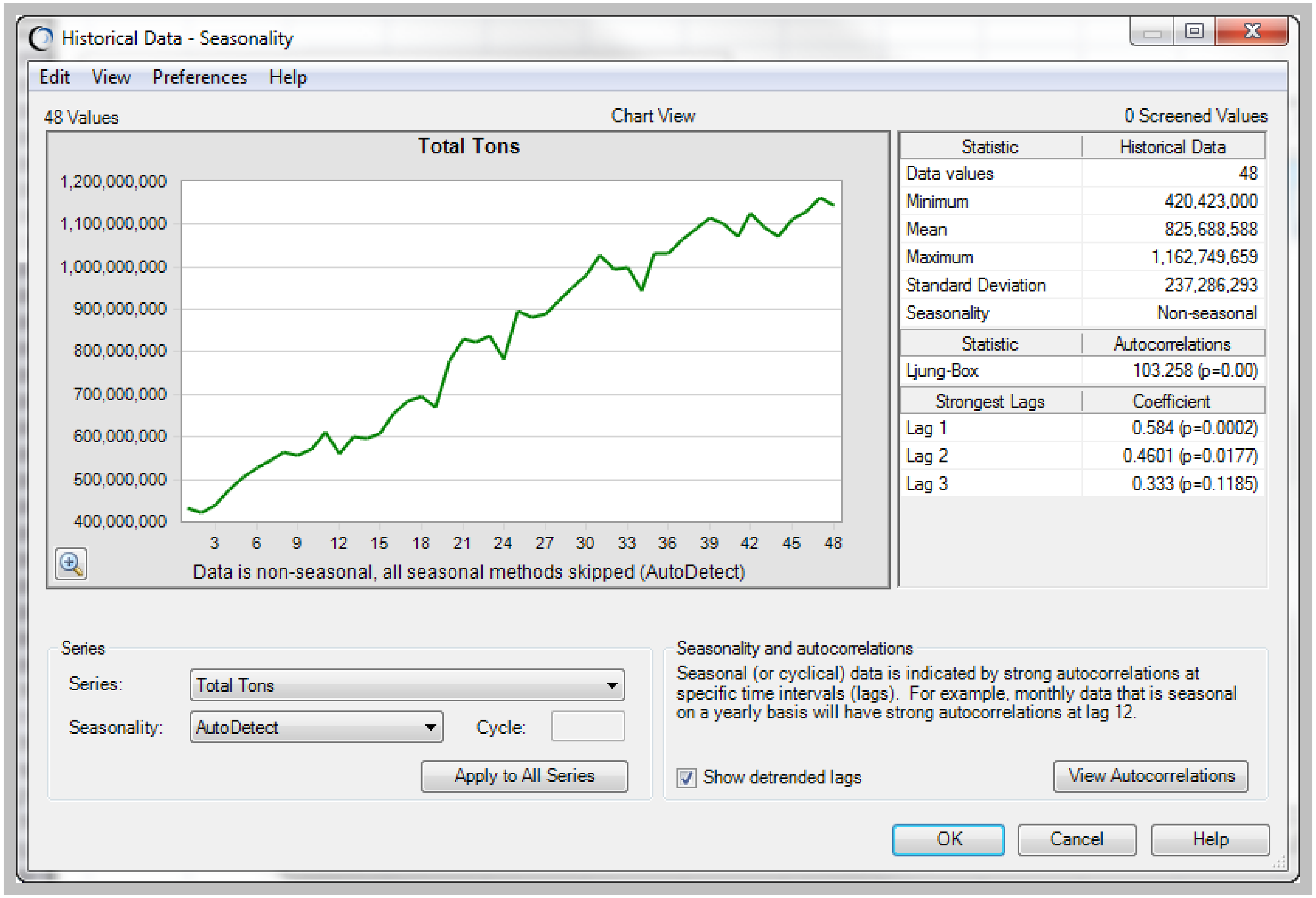
CB Predictor Results for Gas Use Example



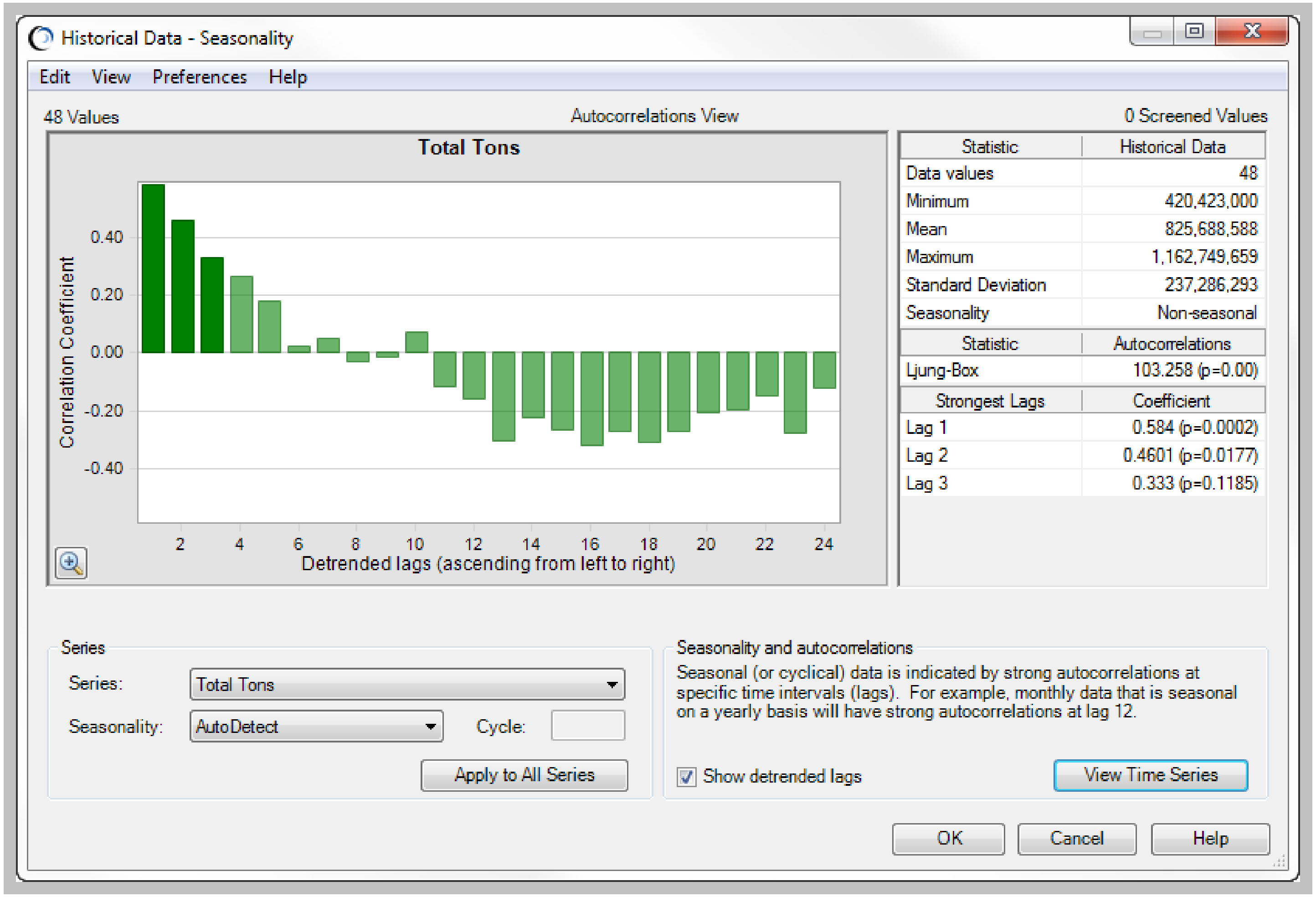
CB Predictor Results for Electric Use Example



CB Predictor Seasonality Dialog



Autocorrelations in *CB Predictor*



CB Predictor ARIMA Model for Coal Production