Chapter 3 - Data Description

Note: Answers may vary due to rounding, TI 83's, or computer programs.

EXERCISE SET 3-1

1.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{1457}{14} = 104.1$

b. MD: 102

c. MR: $\frac{160+50}{2} = 105$

d. Mode: 50, 95, 102, 160

2.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{47{,}619}{15} = 3174.6$

b. MD $= 1479$

c. Mode: no mode

d. MR: $\frac{203+9822}{2} = 5012.5$

3.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{1312}{6} = 218.7$

b. MD: 221

c. MR $= \frac{180+251}{2} = 215.5$

d. Mode: no mode

4.

For Observers:

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{3804}{10} = 380.4$

b. MD: $\frac{352+378}{2} = 365$

c. Mode: no mode

d. MR $= \frac{484+302}{2} = 393$

For Visits:

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{2769}{10} = 276.9$

b. MD : $\frac{194+219}{2} = 206.5$

c. Mode: no mode

d. MR $= \frac{114+634}{2} = 374$

The values are higher for observers.

5.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{10{,}671{,}300}{10} = 1{,}067{,}130$

b. MD $= \frac{1{,}100{,}000 + 1{,}210{,}000}{2} = 1{,}155{,}000$

c. Mode: 1,340,000

d. MR $= \frac{298{,}000 + 2{,}000{,}000}{2} = 1{,}149{,}000$

6.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{247}{13} = \$19$ million

b. MD: \$10 million

c. Mode: \$7 million

d. MR $= \frac{7+50}{2} = \$28.5$ million

The data is positively skewed since the mean is much higher than the median or mode.

7.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{289}{11} = 26.3$

b. MD $= 28$

c. MR $= \frac{10+38}{2} = 24$

d. Mode: 30

The mean, median, and midrange are all very close.

8.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{1221.1}{50} = \$24.42$

b. MD $= \frac{23.2+23.7}{2} = \$23.45$

c. Mode: 16.9, 17.2, 18, 19.1, 24, 25.2, 31.7

d. MR $= \frac{16.5+47.7}{2} = 32.1$

It appears that the mean and median are good measures of average.

9.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{398.2}{13} = 30.6$

b. MD $= 10$

c. MR $= \frac{3.1+143.8}{2} = 73.45$

d. Mode: no mode

10.

New England States:

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{14{,}709}{6} = 2451.5$

b. MD $= \frac{1112+1795}{2} = 1453.5$

c. Mode: none

Northwestern States:

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{3419}{6} = 569.8$

b. MD $= \frac{172+620}{2} = 396$

c. Mode: none

The measures of central tendency are much larger for New England compared to those for the Northwestern states.

11.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{486.2}{13} = 37.4$

b. MD $= 33.7$

c. Mode: no mode

d. MR $= \frac{4.4+87.9}{2} = 46.15$

12.

a. $\overline{X} = \frac{\Sigma X}{n} = \frac{209}{21} = 9.952 \approx 10$

b. MD $= 9$

c. Mode $= 8$ and 9

d. MR $= \frac{6+14}{2} = 10$

13.

Boundaries	X_m	f	$f \cdot X_m$
47.5 - 54.5	51	3	153
54.5 - 61.5	58	2	116
61.5 - 68.5	65	9	585
68.5 - 75.5	72	13	936
75.5 - 82.5	79	8	632
82.5 - 89.5	86	3	258
89.5 - 96.5	93	2	186
		40	2866

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{2866}{40} = 71.65$

b. modal class: 68.5 - 75.5

14.

Class Limits	*Boundaries*	X_m	f	$f \cdot X_m$
2.48 - 7.48	2.475 - 7.485	4.98	7	34.86
7.49 - 12.49	7.485 - 12.495	9.99	3	29.97
12.50 - 17.50	12.495 - 17.505	15.00	1	15.00
17.51 - 22.51	17.505 - 22.515	20.01	7	140.07
22.52 - 27.52	2.515 - 27.525	25.02	5	125.10
27.53 - 32.53	27.525 - 32.535	30.03	5	150.15
			28	495.15

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{495.15}{28} = 17.68$

b. modal class: $2.48 - 7.48$ and $17.51 - 22.51$

The grouped mean is less.

15.

Class Limits	*Boundaries*	X_m	f	$f \cdot X_m$
14 - 20	13.5 - 20.5	17	10	170
21 - 27	20.5 - 27.5	24	11	264
28 - 34	27.5 - 34.5	31	6	186
35 - 41	34.5 - 41.5	38	8	304
42 - 48	41.5 - 48.5	45	4	180
49 - 55	48.5 - 55.5	52	1	52
			40	1156

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{1156}{40} = 28.9$

b. modal class: $21 - 27$

16.

Percentage	*Boundaries*	X_m	f	$f \cdot X_m$
0.8 - 4.4	0.75 - 4.45	2.6	26	67.6
4.5 - 8.1	4.45 - 8.15	6.3	11	69.3
8.2 - 11.8	8.15 - 11.85	10.0	4	40.0
11.9 - 15.5	11.85 - 15.55	13.7	5	68.5
15.6 - 19.2	15.55 - 19.25	17.4	2	34.8
19.3 - 22.9	19.25 - 22.95	21.1	1	21.1
23.0 - 26.6	22.95 - 26.65	24.8	1	24.8
			50	326.1

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{326.1}{50} = 6.5$

b. modal class: $0.8 - 4.4$

The mean is probably not the best measure of central tendency for this data because the data is "bottom heavy."

17.

Percentage	*Boundaries*	X_m	f	$f \cdot X_m$
15.2 - 19.6	15.15 - 19.65	17.4	3	52.2
19.7 - 24.1	19.65 - 24.15	21.9	15	328.5
24.2 - 28.6	24.15 - 28.65	26.4	19	501.6
28.7 - 33.1	28.65 - 33.15	30.9	6	185.4
33.2 - 37.6	33.15 - 37.65	35.4	7	247.8
37.7 - 42.1	37.65 - 42.15	39.9	0	0
42.2 - 46.6	42.15 - 46.65	44.4	1	44.4
			51	1359.9

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{1359.9}{51} = 26.66$ or 26.7

b. modal class: $24.2 - 28.6$

18.

Class Limits	*Boundaries*	X_m	f	$f \cdot X_m$
$10 - 20$	$9.5 - 20.5$	15	2	30
$21 - 31$	$20.5 - 31.5$	26	8	208
$32 - 42$	$31.5 - 42.5$	37	15	555
$43 - 53$	$42.5 - 53.5$	48	7	336
$54 - 64$	$53.5 - 64.5$	59	10	590
$65 - 75$	$64.5 - 75.5$	70	3	210
			45	1929

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{1929}{45} = 42.9$

b. modal class: $32 - 42$

19.

Boundaries	X_m	f	$f \cdot X_m$
0.5 - 19.5	10	12	120
19.5 - 38.5	29	7	203
38.5 - 57.5	48	5	240
57.5 - 76.5	67	3	201
76.5 - 95.5	86	3	258
		30	1022

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{1022}{30} = 34.1$

b. modal class: $0.5 - 19.5$

20.

Class Limits	*Boundaries*	X_m	f	$f \cdot X_m$
$150 - 158$	$149.5 - 158.5$	154	5	770
$159 - 167$	$158.5 - 167.5$	163	16	2608
$168 - 176$	$167.5 - 176.5$	172	20	3440
$177 - 185$	$176.5 - 185.5$	181	21	3801
$186 - 194$	$185.5 - 194.5$	190	20	3800
$195 - 203$	$194.5 - 203.5$	199	15	2985
$204 - 212$	$203.5 - 212.5$	208	3	624
			100	18,028

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{18{,}028}{100} = 180.3$

b. modal class: $177 - 185$

21.

Children	f	$f \cdot X_m$
0	6	0
1	6	6
2	10	20
3	6	18
4	6	24
5	4	20
6	4	24
7	2	14
8	2	16
9	0	0
10	1	10
	47	152

21. continued

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{152}{47} = 3.23$

b. mode: 2

22.

Limits	*Boundaries*	X_m	f	$f \cdot X_m$
1013 - 1345	1012.5 - 1345.5	1179	11	12969
1346 - 1678	1345.5 - 1678.5	1512	4	6048
1679 - 2011	1678.5 - 2011.5	1845	7	12915
2012 - 2344	2011.5 - 2344.5	2178	3	6534
2345 - 2677	2344.5 - 2677.5	2511	5	12555
2678 - 3010	2677.5 - 3010.5	2844	3	8532
			33	59553

$$\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{59553}{33} = 1804.6$$

modal class: $1013 - 1345$

23.

$$\overline{X} = \frac{\sum w \cdot X}{\sum w} = \frac{8(10{,}000) + 10(12{,}000) + 12(8{,}000)}{8 + 10 + 12}$$
$$= \frac{296{,}000}{8 + 10 + 12}$$
$$= \frac{296{,}000}{30}$$
$$= \$\,9866.67$$

24.

$$\overline{X} = \frac{\sum w \cdot X}{\sum w} = \frac{3(3.33) + 3(3.00) + 2(2.5) + 2.5(4.4) + 4(1.75)}{3 + 3 + 2 + 2.5 + 4}$$
$$= \frac{41.99}{14.5} = 2.896$$

25.

$$\overline{X} = \frac{\sum w \cdot X}{\sum w} = \frac{40(1000) + 30(3000) + 50(800)}{1000 + 3000 + 800} = 35.4\%$$

26.

$$\overline{X} = \frac{\sum w \cdot X}{\sum w} = \frac{9(427000) + 6(365000) + 12(725000)}{9 + 6 + 12}$$
$$= \frac{14{,}733{,}000}{27} = \$545,666.67$$

27.

$$\overline{X} = \frac{\sum w \cdot X}{\sum w} = \frac{20(83) + 30(72) + 50(90)}{100} = 83.2$$

28.

$$\overline{X} = \frac{\sum w \cdot X}{\sum w} = \frac{1(62) + 1(83) + 1(97) + 1(90) + 2(82)}{6}$$
$$= \frac{496}{6} = 82.7$$

29.

a. Mode
b. Median
c. Median
d. Mode
e. Mean
f. Median

30.

a. Median
b. Mean
c. Mode
d. Mode
e. Mode
f. Mean

31.

Roman letters, $\overline{X}$
Greek letters, μ

32. Both could be true since one could be using the mean for the average salary, and the other could be using the mode for the average.

33.

$5 \cdot 64 = 320$

34.

$5 \cdot 8.2 = 41$
$6 + 10 + 7 + 12 + x = 41$
$x = 6$

35.

The mean of the original data is 30.
The means will be:

a. 40
b. 20
c. 300
d. 3
e. The results will be the same as adding, subtracting, multiplying, and dividing the mean by 10.

36.

a. $\frac{2}{\frac{1}{30}+\frac{1}{45}} = 36$ mph

b. $\frac{2}{\frac{1}{40}+\frac{1}{25}} = 30.8$ mph

c. $\frac{2}{\frac{1}{50}+\frac{1}{10}} = \$16.67$

37.

a. $\sqrt[3]{(1.35)(1.24)(1.18)} = 1.2547 \approx 1.255$

Average growth rate: $1.255 - 1 = 0.255$ or 25.5%

b. $\sqrt[4]{(1.08)(1.06)(1.04)(1.05)} = 1.057397$

Average growth rate: $1.057 - 1 = 0.057$ or 5.7%

c. $\sqrt[5]{(1.10)(1.08)(1.12)(1.09)(1.03)} = 1.084$

Average growth rate: $1.084 - 1 = 0.084$ or 8.4%

d. $\sqrt[3]{(1.01)(1.03)(1.055)} = \sqrt[3]{1.0975165} = 1.032$

Average growth rate: $1.032 - 1 = 0.032$ or 3.2%

38.

$\sqrt{\frac{8^2+6^2+3^2+5^2+4^2}{5}} = \sqrt{30} = 5.477$

39. MD $= \frac{\frac{50}{2}-0}{26}(3.7) + 0.75 = 4.31$

EXERCISE SET 3-2

1. The square root of the variance is equal to the standard deviation.

2. One extremely high or low data value would influence the range.

3. σ^2, σ

4. s^2, s

5. When the sample size is less than 30, the formula for the variance of the sample will underestimate the population variance.

6.

a. s = 4.320

b. s = 5.066

c. s = 6.00

Data set A is least variable and data set C is the most variable.

7.

$R = 110.8 - 20.1 = 90.7$

$s^2 = \frac{n\sum X^2-(\sum X)^2}{n(n-1)} = \frac{10(28{,}948.44)-457.4^2}{10(10-1)}$

$= \frac{80{,}269.64}{90} = 891.9$

$s = \sqrt{891.88} = 29.9$

8.

$R = 70 - 8 = 62$

$s^2 = \frac{n\sum X^2-(\sum X)^2}{n(n-1)} = \frac{17(30{,}324)-652^2}{17(17-1)} = 332.4$

$s = \sqrt{332.4} = 18.2$

Using the range rule of thumb, $s \approx \frac{70-8}{4} = 15.5$

This is close to the actual standard deviation of 18.2.

9.

Silver:

$R = 35.42 - 7.34 = 27.9$

$s^2 = \frac{n\sum X^2-(\sum X)^2}{n(n-1)} = \frac{9(3998.77)-172.46^2}{9(9-1)} = \frac{6246.45}{72}$

$= 86.75$

$s = \sqrt{86.8} = 9.314$

Tin:

$R = 15.75 - 4.83 = 10.92$

$s^2 = \frac{n\sum X^2-(\sum X)^2}{n(n-1)} = \frac{9(1079.58)-93.51^2}{9(9-1)} = \frac{972.14}{72}$

$= 13.5$

9. continued

$s = \sqrt{13.5} = 3.67$

The prices of silver are more variable.

10.
Eastern states:
$R = 37{,}741 - 20{,}966 = 16{,}775$

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{6(5{,}830{,}685{,}308) - 183{,}684^2}{6(6-1)}$$
$$= 41,476,666.4$$

$s = \sqrt{41,476,666.4} = 6440.2$

Western states:
$R = 101{,}510 - 54{,}339 = 47{,}171$

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{6(31{,}891{,}035{,}030) - 428{,}362^2}{6(6-1)}$$
$$= 261,740,237.9$$

$s = \sqrt{261,740,237.9} = 16,178.4$

Western states are more variable.

11.
Triplets:
$R = 7110 - 5877 = 1233$

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{10(427{,}765{,}643) - 65267^2}{10(10-1)}$$
$$= \frac{17{,}875{,}141}{90} = 198,612.7$$

$s = \sqrt{198,612.7} = 445.7$

Quadruplets:
$R = 512 - 345 = 167$

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{10(1{,}925{,}217) - 4347^2}{10(10-1)}$$
$$= \frac{355{,}761}{90} = 3952.9$$

$s = \sqrt{3952.9} = 62.9$

Quintuplets:
$R = 91 - 46 = 45$

11. continued

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{10(56{,}535) - 741^2}{10(10-1)} = \frac{16{,}269}{90}$$
$$= 180.8$$

$s = \sqrt{180.8} = 13.4$

The data for triplets are most variable.

12.
Europe:
$R = \$48{,}704 - \$27{,}789 = \$20{,}915$

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{7(8{,}745{,}505{,}887) - 242{,}459^2}{7(7-1)}$$
$$= 57{,}908{,}917.4$$

$s = \sqrt{57,908,917.33} = \$7609.8$

Asia:
$R = \$26{,}852 - \$5862 = \$20{,}990$

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{6(1{,}923{,}668{,}064) - 97{,}958^2}{6(6-1)}$$
$$= 64{,}874{,}620.7$$

$s = \sqrt{64{,}874{,}620.67} = \$8054.5$

The data for Asia are more variable.

13.
$R = 46 - 26 = 20$
Using the range rule of thumb, $s \approx \frac{20}{4} = 5$

14.
$R = 71 - 49 = 22$

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{12(38{,}359) - 675^2}{12(12-1)} = 35.48 \text{ or } 35.5$$

$s = \sqrt{35.5} = 5.96 \approx 6$

15.
$R = 580 - 283 = 297$

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{8(1{,}552{,}471) - 3457^2}{8(8-1)} = 8373.6$$

$s = \sqrt{8373.6} = 91.5$

16.
$R = 2786 - 65 = 2721$

$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = 355{,}427.57$ or $355{,}427.6$

$s = \sqrt{355,427.6} = 596.2$

17.
$R = 156 - 26 = 130$

$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = \frac{25(271{,}995) - 2471^2}{25(25-1)} = 1156.7$

$s = \sqrt{1156.7} = 34.0$

18.
For unemployment:

$s = 29.9 \qquad \frac{\text{Range}}{4} = \frac{90.7}{4} = 22.7$

For executions:

$s = 91.5 \qquad \frac{\text{Range}}{4} = \frac{297}{4} = 74.3$

For precipitation:

$s = 34.0 \qquad \frac{\text{Range}}{4} = \frac{130}{4} = 32.5$

The closest estimate is for precipitation.
The estimate for unemployment is also close.

19.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
10	2	20	200
13	20	260	3380
16	18	288	4608
19	7	133	2527
22	2	44	968
25	1	25	625
	50	770	12,308

$s^2 = \frac{n\sum f \cdot X_m^2 - (\sum f \cdot X_m)^2}{n(n-1)} = \frac{50(12{,}308) - 770^2}{50(50-1)} = 9.2$

$s = \sqrt{9.18} = 3.0$

20.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
10	3	30	300
15	5	75	1125
20	15	300	6000
25	5	125	3125
30	2	60	1800
	30	590	12,350

$s^2 = \frac{30(12{,}350) - 590^2}{30(30-1)} = 25.7$
$s = \sqrt{25.7} = 5.07$ or 5.1

21.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
65	13	845	54,925
128	2	256	32,768
191	0	0	0
254	5	1270	322,580
317	1	317	100,489
380	1	380	144,400
443	0	0	0
506	1	506	256,036
569	2	1138	647,522
	25	4712	1,558,720

$s^2 = \frac{n\sum f \cdot X_m^2 - (\sum f \cdot X_m)^2}{n(n-1)} = \frac{25(1{,}558{,}720) - 4712^2}{25(25-1)}$
$= 27{,}941.76$

$s = \sqrt{27941.76} = 167.16$ or 167.2

22.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
2.4	12	28.8	69.12
3.1	13	40.3	124.93
3.8	7	26.6	101.08
4.5	5	22.5	101.25
5.2	2	10.4	54.08
5.9	1	5.9	34.81
	40	134.5	485.27

22. continued

$s^2 = \frac{40(485.27) - 134.5^2}{40(40-1)} = 0.85$

$s = \sqrt{0.85} = 0.92$

23.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
5	5	25	125
14	7	98	1372
23	10	230	5290
32	3	96	3072
41	3	123	5043
50	2	100	5000
	30	672	19,902

$s^2 = \frac{n\sum f \cdot X_m^2 - (\sum f \cdot X_m)^2}{n(n-1)} = \frac{30(19{,}902) - 672^2}{30(30-1)} = 167.2$

$s = \sqrt{167.2} = 12.9$

24.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
8	8	64	512
15	5	75	1125
22	7	154	3388
29	1	29	841
36	1	36	1296
43	3	129	5547
	25	487	12709

$s^2 = \frac{25(12{,}709) - 487^2}{25(25-1)} = 134.26$ or 134.3

$s = \sqrt{134.3} = 11.6$

25.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
119	8	952	113,288
252	11	2772	698,544
385	2	770	296,450
518	1	518	268,324
651	4	2604	1,695,204
784	2	1568	1,229,312
	28	9184	4,301,122

25. continued

$s^2 = \frac{n\sum f \cdot X_m^2 - (\sum f \cdot X_m)^2}{n(n-1)} = \frac{28(4{,}301{,}122) - 9184^2}{28(28-1)}$

$= 47{,}732.22$

$s = \sqrt{47{,}732.22} = 218.5$

26.

For National League:

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
0.244	3	0.732	0.178608
0.249	6	1.494	0.372006
0.254	1	0.254	0.064516
0.259	11	2.849	0.737891
0.264	11	2.904	0.766656
0.269	1	0.269	0.072361
	33	8.502	2.192038

$s^2 = \frac{33(2.192038) - 8.502^2}{33(33-1)} = 0.00005$

$s = \sqrt{0.00005} = 0.0071$

For American League:

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
0.2465	3	0.7395	0.13364450
0.2525	6	1.5150	0.34980125
0.2585	2	0.5170	0.29268100
0.2645	1	0.2645	0.15290450
0.2705	3	0.8115	0.079 80625
0.2765	0	0	0
	15	3.8475	0.98794

$s^2 = \frac{15(0.98794) - 3.8475^2}{15(15-1)} = 0.000075$

$s = \sqrt{0.000075} = 0.0087$

The averages for American League are more variable.

27.

C. Var $= \frac{s}{\overline{X}} = \frac{2.3}{11} = 0.209 = 20.9\%$

C. Var $= \frac{s}{\overline{X}} = \frac{1.8}{8} = 0.225 = 22.5\%$

The factory workers' data are more variable.

28.

For US: $\overline{X} = 3386.6$, $s = 693.9$;

C. Var $= \frac{s}{\overline{X}} = \frac{693.9}{3386.6} = 0.2049$ or 20.49%

For World: $\overline{X} = 4997.8$, $s = 803.2$;

C. Var $= \frac{s}{\overline{X}} = \frac{803.2}{4997.8} = 0.1607$ or 16.07%

The data for US is more variable.

29.

C. Var $= \frac{s}{\overline{X}} = \frac{10.5}{80.2} = 0.131 = 13.1\%$

C. Var $= \frac{s}{\overline{X}} = \frac{18.3}{120.6} = 0.152 = 15.2\%$

The waiting time for people who are discharged is more variable.

30.

C. Var $= \frac{s}{\overline{X}} = \frac{6}{26} = 0.231 = 23.1\%$

C. Var $= \frac{s}{\overline{X}} = \frac{4000}{31{,}000} = 0.129 = 12.9\%$

Age is more variable.

31.

a. $1 - \frac{1}{2^2} = \frac{3}{4}$ or 75%

b. $1 - \frac{1}{1.5^2} = 0.56$ or 56%

32.

a. $1 - \frac{1}{5^2} = 0.96$ or 96%

b. $1 - \frac{1}{4^2} = 0.9375$ or 93.75%

33.

$\frac{120}{160} = 0.75 = 75\%$ so $k = 2$

$72 + 2s = 77$

$s = 2.5$

$72 + 2.5k = 82$

$k = 4$

$1 - \frac{1}{4^2} = 0.9375$ or at least 93.75%.

34.

$\overline{X} = 240$ and $s = 38$

At least 75% of the data values will fall withing two standard deviations of the mean; hence, $2(38) = 76$ and $240 - 76 = 164$ and $240 + 76 = 316$. Hence at least 75% of the data values will fall between 164 and 316 calories.

35.

$1 - \frac{1}{k^2} = 0.8889$ $\qquad k = 3$

$\overline{X} = 3$ hours or 180 minutes and $s = 32$ minutes

$180 - 3(32) = 84$ minutes; $180 + 3(32) = 276$

At least 88.89% of the data values will fall between 84 and 276 minutes.

36.

$1 - \frac{1}{k^2} = 0.8889$ $\qquad k = 3$

$\overline{X} = 640$and $s = 85$

At least 88.89% of the data values will fall within 3 standard deviations of the mean, hence $640 - 3(85) = 385$ and $640 + 3(85) = 895$. Therefore at least 88.89% of the data values will fall between 385 and 895 pounds.

37.

$1 - \frac{1}{k^2} = 0.75$ $\qquad k = 2$

$\overline{X} = \$258{,}100$ and $s = \$48{,}500$

$\$258{,}100 - 2\ (\$48{,}500) = \$161{,}100$ and $\$258{,}100 + 2\ (\$48{,}500) = \$355{,}100$. At least 75% of the homes will fall between \$161,100 and \$355,100.

38.
$\overline{X} = 12$ and $s = 3$
$20 - 12 = 8$ and $8 \div 3 = 2.67$

Hence, $1 - \frac{1}{k^2} = 1 - \frac{1}{2.67^2}$
$= 1 - 0.14 = 0.86 = 86\%$

At least 86% of the data values will fall between 4 and 20.

39.
$\overline{X} = 504$ and $s = 55.7$
$504 + 55.7k = 896.57$ so $k = 7.05$
$1 - \frac{1}{k^2} = 1 - \frac{1}{7.05^2} = 0.98$ or at least 98%

40.
$26.8 + 1(4.2) = 31$
By the Empirical Rule, 68% of consumption is within 1 standard deviation of the mean. Then $\frac{1}{2}$ of 32%, or 16%, of consumption would be more than 31 pounds of citrus fruit per year.

41.
By the Empirical Rule, 68% of scores are within 1 standard deviation of the mean.
Thus, $538 - 1(48) = 490$ and $538 + 1(48) = 586$.
Therefore, 68% of the scores would fall between 490 and 586.

To find the percentage of scores above 634, first find k:
$538 + k(48) = 634$
$48k = 96$
$k = 2$
By the Empirical Rule, 95% of the data are within $k = 2$ standard deviations of the mean. This means that $100\% - 95\% = 5\%$ of the scores would be above and below 2 standard deviations of the mean. Thus, $\frac{1}{2}$ of 5% or 2.5% of the data are above 634.

42.
(a) $53 + 4.2k = 58.6$
$4.2k = 5.6$
$k = 2$
By Chebyshev's Theorem, $1 - \frac{1}{2^2} = .75$ or 75% of hours worked are within 2 standard deviations of the mean. Because we do not know anything about symmetry, we can say that not more than 25% of faculty members work more than 58.6 hours per week.

(b) By the Empirical Rule, $k = 2$ standard deviations of the mean is 95% of hours worked. Then $\frac{1}{2}$ of 5%, or 2.5%, work more than 58.6 hours per week.

43.
The average price of an instrument at a small music store is \$325. The standard deviation of the price is \$52. The Owner decides to raise the price of all the instruments by \$20.
The new mean of prices is
$\overline{X} = \$325 + \$20 = \$345$, and the new standard deviation of prices is $s = \$52$.

44.
The mean and standard deviation of the number of hours the employees work in the music store per week are 18.6 and 3.2 hours respectively. The Owner increases the number of hours each employee works per week by 4 hours.
The new mean of number of hours is

$$\overline{X} = 18.6 + 4 = 22.6$$

and the new standard deviation of the number of hours is $s = 3.2$.

45.
The mean price of the fish in a pet shop is \$2.17, and the standard deviation of the price is \$0.55.
The Owner decides to triple the prices.
The new mean of prices is
$\overline{X} = \$2.17 \times 3 = \$6.51$
and the new standard deviation of prices is
$s = \$0.55 \times 3 = \$1.65$.

46.
The mean and standard deviation of the bonuses that the employees of a company received 10 years ago were \$2000 and \$325 respectively. Today, the amount of the bonuses is 5 times of what it was 10 years ago.
The mean of new bonuses is
$\overline{X} = \$2000 \times 5 = \$10{,}000$,
and the standard deviation of new bonuses is $s = \$325 \times 5 = \$1625$.

47.
$n = 30 \quad \overline{X} = 215 \quad s = 20.8$ At least 75% of the data values will fall between $\overline{X} \pm 2s$.
$\overline{X} - 2(20.8) = 215 - 41.6 = 173.4$ and
$\overline{X} + 2(20.8) = 215 + 41.6 = 256.6$
In this case all 30 values fall within this range.

48.
$n = 30 \quad \overline{X} = 34.5 \quad s = 13.3$
$\overline{X} - 2s = 34.5 - 2(13.3) = 7.9$ and
$\overline{X} + 2s = 34.5 + 2(13.3) = 61.1$
In this case 28 out of 30 data values fall within the range of 7.9 to 61.1. This is 93.3% which
is consistent with Chebyshev's Theorem.

49.
For $k = 1.5,\ 1 - \frac{1}{1.5^2} = 1 - 0.44$
$= 0.56$ or 56%

49. continued
For $k = 2,\ 1 - \frac{1}{2^2} = 1 - 0.25$
$= 0.75$ or 75%
For $k = 2.5,\ 1 - \frac{1}{2.5^2} = 1 - 0.16$
$= 0.84$ or 84%
For $k = 3,\ 1 - \frac{1}{3^2} = 1 - 0.1111$
$= .8889$ or 89%
For $k = 3.5,\ 1 - \frac{1}{3.5^2} = 1 - 0.08$
$= 0.92$ or 92%

50.
a. $s = 15.81$
b. $s = 15.81$
c. $s = 15.81$
d. $s = 79.06$
e. $s = 3.16$
f. The standard deviation is unchanged by adding or subtracting a specific number to each data value. If each data value is multiplied by a number the standard deviation increases by the number times the original standard deviation. For division the standard deviation is divided by the number.
g. When adding or subtracting the same number to each data value the mean will increase or decrease by that number, but the standard deviation will remain unchanged. When multiplying each data value by the same number the mean or standard deviation will be equal to that number times the original mean or standard deviation. When dividing each data value by the same number the mean or standard deviation will be equal to the original mean or standard deviation divided by that number.

51.
$\overline{X} = 13.3$
Mean Dev
$= \frac{|5-13.3|+|9-13.3|+|10-13.3|+|11-13.3|+|11-13.3|}{10}$

$+ \frac{|12-13.3|+|15-13.3|+|18-13.3|+|20-13.3|+|22-13.3|}{10}$
$= 4.36$

52.

a. $\text{Sk} = \frac{3(10-8)}{3} = 2$
positively skewed

b. $\text{Sk} = \frac{3(42-45)}{4} = -2.25$
negatively skewed

c. $\text{Sk} = \frac{3(18.6-18.6)}{1.5} = 0$
symmetric

d. $\text{Sk} = \frac{3(98-97.6)}{4} = 0.3$
positively skewed

53.

For $n = 25$, $\overline{X} = 50$, and $s = 3$:

$s\sqrt{n-1} = 3\sqrt{25-1} = 14.7$
$\overline{X} + s\sqrt{n-1} = 50 + 14.7 = 64.7$

67 may be an incorrect data value, since it is beyond the range using the formula $s\sqrt{n-1}$.

EXERCISE SET 3-3

1. A z score tells how many standard deviations the data value is above or below the mean.

2. A percentile rank indicates the percentage of data values that fall below the specific rank.

3. A percentile is a relative measure while a percent is an absolute measure of the part to the total.

4. A quartile is a relative measure of position obtained by dividing the data set into quarters.

5. $Q_1 = P_{25},\ Q_2 = P_{50},\ Q_3 = P_{75}$

6. A decile is a relative measure of position obtained by dividing the data set into tenths.

7. $D_1 = P_{10},\ D_2 = P_{20},\ D_3 = P_{30}$, etc

8. $P_{50},\ Q_2,\ D_5$

9.

For Canada:
$z = \frac{X-\overline{X}}{s} = \frac{26-29.4}{8.6} = -0.40$
For Italy:
$z = \frac{X-\overline{X}}{s} = \frac{42-29.4}{8.6} = 1.47$
For US:
$z = \frac{X-\overline{X}}{s} = \frac{13-29.4}{8.6} = -1.91$

10.

For Senator who is 48 years old:
$z = \frac{X-\overline{X}}{s} = \frac{48-61.7}{10.6} = -1.29$
For Senator who is 66 years old:
$z = \frac{X-\overline{X}}{s} = \frac{66-61.7}{10.6} = 0.41$

11.

a. $z = \frac{X-\overline{X}}{s} = \frac{27-24.6}{3.2} = 0.75$

b. $z = \frac{22-24.6}{3.2} = -0.8125$

c. $z = \frac{31-24.6}{3.2} = 2$

d. $z = \frac{18-24.6}{3.2} = -2.0625$

e. $z = \frac{26-24.6}{3.2} = 0.4375$

12.

If $z = \frac{X-\overline{X}}{s}$ then $X = zs + \overline{X}$

a. $X = 2(10{,}200) + 54{,}166$
$= \$74{,}566$

b. $X = -1(10{,}200) + 54{,}166$
$= \$43{,}966$

c. $X = 0(10{,}200) + 54{,}166$
$= \$54{,}166$

12. continued

d. $X = 2.5(10{,}200) + 54{,}166$
$= \$79{,}666$

e. $X = -1.6(10{,}200) + 54{,}166$
$= \$37{,}846$

13.

For the geography test: $z = \frac{83 - 72}{6}$
$= 1.83$

For the accounting test: $z = \frac{61 - 55}{3.5}$
$= 1.71$

The geography test score is relatively higher than the accounting test score.

14.

For student #1:
$z = \frac{9650-8455}{1865} = 0.64$

For student #2:
$z = \frac{12360-10326}{2143} = 0.95$

The student from the university (student #2) has a higher relative debt.

15.

a. $z = \frac{16{,}000 - 14{,}090}{3500} = 0.55$
b. $z = \frac{10{,}000 - 14{,}090}{3500} = -1.17$

c. To find the number of miles, use
$X = zs + \overline{X}$
$X = 1.6(3500) + 14{,}090 = 19{,}690$ miles
$X = -0.5(3500) + 14{,}090 = 12{,}340$ miles
$X = 0(3500) + 14{,}090 = 14{,}090$ miles

16.

a. $z = \frac{3.2-4.6}{1.5} = -0.93$
b. $z = \frac{630-800}{200} = -0.85$
c. $z = \frac{43-50}{5} = -1.4$

The score in part b is the highest.

17.

a. For the 40th percentile:
$c = \frac{(27)(40)}{100} = 10.8$ or 11th value,
which is the data value of 21.

b. For the 75th percentile:
$c = \frac{(27)(75)}{100} = 20.25$ or 21st value,
which is the data value of 43.

c. For the 90th percentile:
$c = \frac{(27)(90)}{100} = 24.3$ or 25th value,
which is the data value of 97.

d. For the 30th percentile:
$c = \frac{(27)(30)}{100} = 8.1$ or 9th value,
which is the data value of 19.

a. For 27:
$P = \frac{15 + 0.5}{27}$
$= 0.574$ or the 57th percentile.

b. For 40:
$P = \frac{19 + 0.5}{27}$
$= 0.722$ or the 72nd percentile.

c. For 58:
$P = \frac{21 + 0.5}{27}$
$= 0.796$ or the 80th percentile.

d. For 67:
$P = \frac{23 + 0.5}{27}$
$= 0.870$ or the 87th percentile.

18.

a. \$5833 b. \$6,563
c. \$7579 d. \$8625
e. 24^{th} f. 67^{th}
g. 48^{th} h. 88^{th}

19.

a. 6^{th} b. 24^{th}
c. 68^{th} d. 76^{th}
e. 94^{th} f. 234
g. 251 h. 263
i. 274 j. 284

20.

a. 375 b. 389
c. 433 d. 477
e. 504 f. 13^{th}
g. 40^{th} h. 54^{th}
i. 76^{th} j. 92^{nd}

21.

$$\text{Percentile} = \frac{\text{number of values below} + 0.5}{\text{total number of values}} \cdot 100\%$$

Data: 228, 489, 524, 597, 623, 659, 736, 777, 804

For 228, $\frac{0+.5}{9} \cdot 100\% = 6^{th}$ percentile
For 489, $\frac{1+.5}{9} \cdot 100\% = 17^{th}$ percentile

For 524, $\frac{2+.5}{9} \cdot 100\% = 28^{th}$ percentile
For 597, $\frac{3+.5}{9} \cdot 100\% = 39^{th}$ percentile

For 623, $\frac{4+.5}{9} \cdot 100\% = 50^{th}$ percentile
For 659, $\frac{5+.5}{9} \cdot 100\% = 61^{st}$ percentile

For 736, $\frac{6+.5}{9} \cdot 100\% = 72^{nd}$ percentile
For 777, $\frac{7+.5}{9} \cdot 100\% = 83^{rd}$ percentile

For 804, $\frac{8+.5}{9} \cdot 100\% = 94^{th}$ percentile

$c = \frac{9(40)}{100} = 3.6$ or 4^{th} data value,
which is 597

22.

For 12, $\frac{0+.5}{7} \cdot 100\% = 7^{th}$ percentile
For 28, $\frac{1+.5}{7} \cdot 100\% = 21^{st}$ percentile

22. continued

For 35, $\frac{2+.5}{7} \cdot 100\% = 36^{th}$ percentile
For 42, $\frac{3+.5}{7} \cdot 100\% = 50^{th}$ percentile

For 47, $\frac{4+.5}{7} \cdot 100\% = 64^{th}$ percentile
For 49, $\frac{5+.5}{7} \cdot 100\% = 79^{th}$ percentile

For 50, $\frac{6+.5}{7} \cdot 100\% = 93^{rd}$ percentile

$c = \frac{n \cdot p}{100} = \frac{7(60)}{100} = 4.2$ or 5

Hence, 47 is the closest value to the 60^{th} percentile.

23.

$$\text{Percentile} = \frac{\text{number of values below} + 0.5}{\text{total number of values}} \cdot 100\%$$

Data: 1.1, 1.7, 1.9, 2.1, 2.2, 2.5, 3.3, 6.2, 6.8, 20.3

For 1.1, $\frac{0+.5}{10} \cdot 100\% = 5^{th}$ percentile
For 1.7, $\frac{1+.5}{10} \cdot 100\% = 15^{th}$ percentile

For 1.9, $\frac{2+.5}{10} \cdot 100\% = 25^{th}$ percentile
For 2.1, $\frac{3+.5}{10} \cdot 100\% = 35^{th}$ percentile

For 2.2, $\frac{4+.5}{10} \cdot 100\% = 45^{th}$ percentile
For 2.5, $\frac{5+.5}{10} \cdot 100\% = 55^{th}$ percentile

For 3.3, $\frac{6+.5}{10} \cdot 100\% = 65^{th}$ percentile
For 6.2, $\frac{7+.5}{10} \cdot 100\% = 75^{th}$ percentile

For 6.8, $\frac{8+.5}{10} \cdot 100\% = 85^{th}$ percentile
For 20.3, $\frac{9+.5}{10} \cdot 100\% = 95^{th}$ percentile

$c = \frac{10(40)}{100} = 4$
average the 4th and 5th values:
$P_{40} = \frac{2.1+2.2}{2} = 2.15$

24.

$$\text{Percentile} = \frac{\text{number of values below} + 0.5}{\text{total number of values}} \cdot 100\%$$

24. continued

Data: 5, 12, 15, 16, 20, 21

For 5, $\frac{0+.5}{6} \cdot 100\% = 8^{th}$ percentile

For 12, $\frac{1+.5}{6} \cdot 100\% = 25^{th}$ percentile

For 15, $\frac{3+.5}{6} \cdot 100\% = 42^{nd}$ percentile

For 16, $\frac{4+.5}{6} \cdot 100\% = 58^{th}$ percentile

For 20, $\frac{5+.5}{6} \cdot 100\% = 75^{th}$ percentile

For 21, $\frac{5+.5}{6} \cdot 100\% = 92^{nd}$ percentile

$c = \frac{6(33)}{100} = 1.98$ or 2^{nd} data value, which is 12.

25.
To find Q_1, find P_{25}:

$c = \frac{(10)(25)}{100} = 2.5$, round up to 3.

Q_1 is at the 3rd value, which is 11.

To find Q_3, find P_{75}:

$c = \frac{(10)(75)}{100} = 7.5$, round up to 8.

Q_3 is at the 8th value, which is 32.

$IQR = Q_3 - Q_1 = 32 - 11 = 21.$

26.
To find Q_1, find P_{25}:

$c = \frac{(12)(25)}{100} = 3$, average the 3rd and 4th values.

$Q_1 = \frac{349{,}026 + 358{,}208}{2} = 353{,}617$

To find Q_3, find P_{75}:

$c = \frac{(12)(75)}{100} = 9$, average the 9th and 10th values.

$Q_3 = \frac{506{,}809 + 518{,}868}{2} = 512{,}838.5$

27.
To find Q_1, find P_{25}:

$c = \frac{(11)(25)}{100} = 2.75$, round up to 3.

Q_1 is at the 3rd value, which is 19.7.

To find Q_3, find P_{75}:

$c = \frac{(11)(75)}{100} = 8.25$, round up to 9.

Q_3 is at the 9th value, which is 78.8.

$IQR = Q_3 - Q_1 = 78.8 - 19.7 = 59.1.$

28.
To find Q_1, find P_{25}:

$c = \frac{(9)25)}{100} = 2.25$, round up to 3.

Q_1 is at the 3rd value, which is 6.
Note: TI83 answer is 4.5.

To find Q_3, find P_{75}:

$c = \frac{(9)(75)}{100} = 6.75$, round up to 7.

Q_3 is at the 7th value, which is 7.
Note: TI83 answer is 24.

29.
a. 19 21 25 28 29 32 34 46

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ Q_1 & MD & Q_3 \end{array}$$

$MD = \frac{28+29}{2} = 28.5$

For Q_1: $Q_1 = \frac{21+25}{2} = 23$

For Q_3: $Q_3 = \frac{32+34}{2} = 33$

$Q_3 - Q_1 = 33 - 23 = 10$ and $10(1.5) = 15$.
$23 - 15 = 8$ and $33 + 15 = 48$.

Since all the values fall within the range of 8 to 48, there are no outliers.

29. continued

b. 65 82 89 90 93 94 97 100 101

$\uparrow$ Q$_1$ (82), $\uparrow$ MD (93), $\uparrow$ Q$_3$ (97)

MD = 93

For Q_1: $Q_1 = \frac{82+89}{2} = 85.5$

For Q_3: $Q_3 = \frac{97+100}{2} = 98.5$

$Q_3 - Q_1$: $98.5 - 85.5 = 13$ and
$13(1.5) = 19.5$. $85.5 - 19.5 = 66$ and
$98.5 + 19.5 = 118$.
Only the value 65 lies outside the range of 66 to 118 and is a suspected outlier.

c. 175 371 489 527 1007

$\uparrow$ Q$_1$ (371), $\uparrow$ MD (489), $\uparrow$ Q$_3$ (527)

MD = 489

For Q_1: $Q_1 = \frac{175+371}{2} = 273$

For Q_3: $Q_3 = \frac{527+1007}{2} = 767$

$Q_3 - Q_1$: $767 - 273 = 494$
and $494(1.5) = 741$.
$273 - 741 = -468$
and $767 + 741 = 1508$.

Since all the values fall within the range of -468 to 1508, there are no outliers.

30.

a. 72 84 85 86 88 97 100

$\uparrow$ Q$_1$ (84), $\uparrow$ MD (86), $\uparrow$ Q$_3$ (97)

MD = 86
For Q_1: $Q_1 = 84$
For Q_3: $Q_3 = 97$.
$Q_3 - Q_1$: $97 - 84 = 13$
and $13(1.5) = 19.5$.

30. continued

$84 - 19.5 = 64.5$
and $97 + 19.5 = 116.5$.

Since all values fall within the range of 64.5 to 116.5, there are no outliers.

b. 116 118 119 122 125 145

$\uparrow$ Q$_1$ (118), $\uparrow$ MD, $\uparrow$ Q$_3$ (125)

$MD = \frac{119+122}{2} = 120.5$
For Q_1: $Q_1 = 118$.
For Q_3: $Q_3 = 125$.
$Q_3 - Q_1$: $125 - 118 = 7$ and $7(1.5) = 10.5$.
$118 - 10.5 = 107.5$ and $125 + 10.5 = 125.5$.

Only the value 145 is outside the range of 107.5 to 135.5 and is a suspected outlier.

c. 13 14 15 16 18 19 20 27 36

$\uparrow$ Q$_1$, $\uparrow$ MD (18), $\uparrow$ Q$_3$

MD = 18
For Q_1: $Q_1 = \frac{14+15}{2} = 14.5$
For Q_3: $Q_3 = \frac{20+27}{2} = 23.5$
$Q_3 - Q_1$: $23.5 - 14.5 = 9$
and $9(1.5) = 13.5$.
$14.5 - 13.5 = 1$
and $23.5 + 13.5 = 37$.

Since all values fall within the range of 1 to 37, there are no outliers.

31.

a. 5, 12, 16, 25, 32, 38
$Q_1 = 12$, $Q_2 = 20.5$, $Q_3 = 32$
Midquartile $= \frac{12+32}{2} = 22$
Interquartile range: $32 - 12 = 20$

b. 53, 62, 78, 94, 96, 99, 103

31. continued

$Q_1 = 62$, $Q_2 = 94$, $Q_3 = 99$
Midquartile $= \frac{62+99}{2} = 80.5$
Interquartile range: $99 - 62 = 37$

32.
If $s^2 = 250$, then $s = \sqrt{250} = 15.81$
Using a score of 142:
$142 = -0.5(15.81) + \overline{X}$
$149.9 \approx \overline{X}$

33.
Tom's score is 158. Harry's score can be calculated based on his z score:
$X = 2(18) + 125 = 161$.

Since the data are normally distributed, 95% fall within 2 standard deviations of the mean (using the Empirical Rule).
Thus, $125 \pm 2(18)$ gives a range of 89 to 161, and a score of 161 is the 95th percentile. Since Dick scored in the 98th percentile, his raw score must be higher than 161.

Therefore, Tom's score is the lowest followed by Harry, with Dick's score being the highest.

EXERCISE SET 3-4

1. Data arranged in order:
6, 8, 12, 19, 27, 32, 54
Minimum: 6
Q_1: 8
Median: 19
Q_3: 32
Maximum: 54
Interquartile Range: $32 - 8 = 24$

2. Data arranged in order:
7, 16, 19, 22, 48
Minimum: 7

2. continued

Q_1: $\frac{7+16}{2} = 11.5$
Median: 19
Q_3: $\frac{22+48}{2} = 35$
Maximum: 48
Interquartile Range: $35 - 11.5 = 23.5$

3. Data arranged in order:
188, 192, 316, 362, 437, 589
Minimum: 188
Q_1: 192
Median: $\frac{316+362}{2} = 339$
Q_3: 437
Maximum: 589
Interquartile Range: $437 - 192 = 245$

4. Data arranged in order:
147, 156, 243, 303, 543, 632
Minimum: 147
Q_1: 156
Median: $\frac{243+303}{2} = 273$
Q_3: 543
Maximum: 632
Interquartile Range: $543 - 156 = 387$

5. Data arranged in order:
14.6, 15.5, 16.3, 18.2, 19.8
Minimum: 14.6
Q_1: $\frac{14.6+15.5}{2} = 15.05$
Median: 16.3
Q_3: $\frac{18.2+19.8}{2} = 19.0$
Maximum: 19.8
Interquartile Range: $19.0 - 15.05 = 3.95$

6. Data arranged in order:
2.2, 3.7, 3.8, 4.6, 6.2, 9.4, 9.7
Minimum: 2.2
Q_1: 3.7
Median: 4.6
Q_3: 9.4
Maximum: 9.7
Interquartile Range: $9.4 - 3.7 = 5.7$

7. Minimum: 3

Q_1: 5

Median: 8

Q_3: 9

Maximum: 11

Interquartile Range: $9 - 5 = 4$

8. Minimum: 200

Q_1: 225

Median: 275

Q_3: 300

Maximum: 325

Interquartile Range: $300 - 225 = 75$

9. Minimum: 55

Q_1: 65

Median: 70

Q_3: 90

Maximum: 95

Interquartile Range: $90 - 65 = 25$

10. Minimum: 2000

Q_1: 3000

Median: 4000

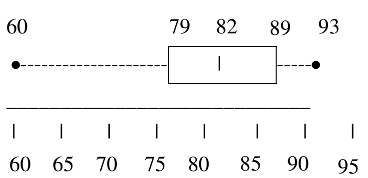
Q_3: 5000

Maximum: 6000

Interquartile Range: $5000 - 3000 = 2000$

11.

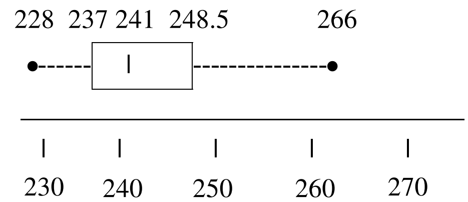
$MD = 82$ $Q_1 = 79$ $Q_3 = 89$



The distribution is right-skewed.

12.

$MD = 241$ $Q_1 = 237$ $Q_3 = 248.5$

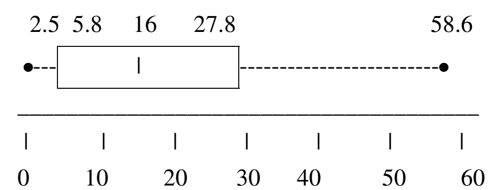


The distribution is slightly right-skewed.

13.

$MD = \frac{14+18}{2} = 16$ $Q_1 = 5.8$

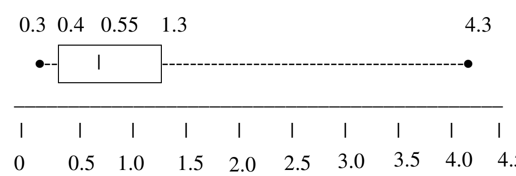
$Q_3 = 27.8$



The box plot of the data is somewhat positively skewed.

14.

$MD = \frac{0.5+0.6}{2} = 0.55$ $Q_1 = 0.4$ $Q_3 = 1.3$



The box plot of the data is somewhat positively skewed.

15.

For Baltic Sea:

$MD = 1154$ $Q_1 = \frac{228+610}{2} = 419$

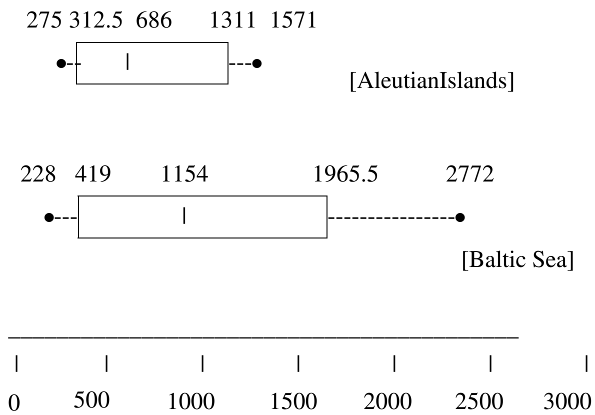
$Q_3 = \frac{1159+2772}{2} = 1965.5$

15. continued

For Aleutian Islands:

$\text{MD} = 686 \qquad Q_1 = \frac{275+350}{2} = 312.5$

$Q_3 = \frac{1051+1571}{2} = 1311$

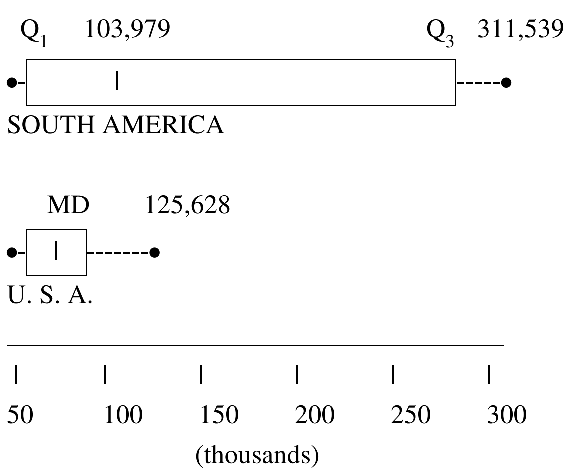


The areas of the islands in the Baltic Sea are more variable than the ones in the Aleutian Islands. Also, they are in general larger in area.

16.

For USA: min $= 50{,}000$, max $= 125{,}628$, MD $= 72{,}100$, $Q_1 = 57{,}642.5$, and $Q_3 = 85{,}004$

For South America: min $= 46{,}563$, max $= 311{,}539$, MD $= 103{,}979$, $Q_1 = 56{,}242$, and $Q_3 = 274{,}026$



The range and variation of the capacity of the dams in South America is considerably larger than those of the United States.

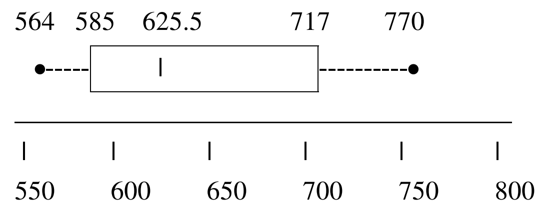
17.

$\text{MD} = \frac{606+645}{2} = 625.5 \qquad Q_1 = 585$

$Q_3 = 717$

Lowest value = 564 Highest value = 770

IQR = 132



18.

(a)

For April: $\overline{X} = 126.67$

For May: $\overline{X} = 247.33$

For June: $\overline{X} = 196.67$

For July: $\overline{X} = 91.67$

The month with the highest mean number of tornadoes is May.

(b)

For 2013: $\overline{X} = 125.5$

For 2014: $\overline{X} = 158.25$

For 2015: $\overline{X} = 213$

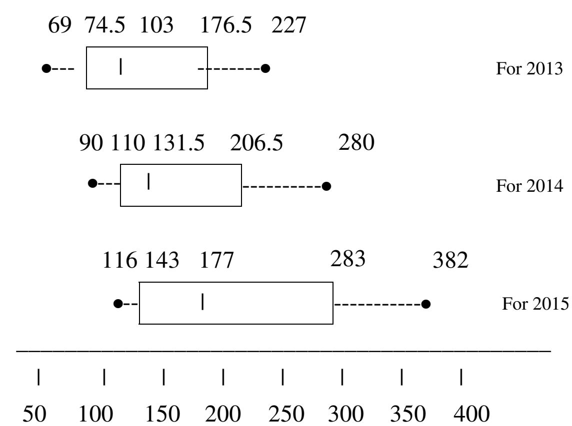
The year with the highest mean number of tornadoes is 2015.

(c) The 5-number summaries for each year are:

For 2013: 69, 74.5, 103, 176.5, 227

For 2014: 90, 110, 131.5, 206.5, 280

For 2015: 116, 143, 177, 283, 382

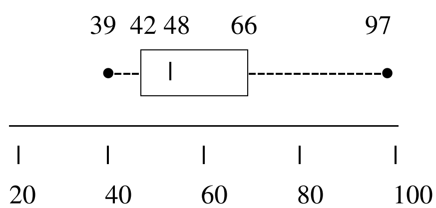


18. continued

The distribution for 2013, 2014, and 2015 are positively skewed. The data for 2013 appears to be the least variable.

19. Data arranged in order: 39, 39, 42, 43, 43, 53, 54, 66, 91, 97
Minimum: 39
Q_1: 42
Median: $\frac{43+53}{2} = 48$
Q_3: 66
Maximum: 97
Interquartile Range: $66 - 42 = 24$

$1.5(24) = 36$ for mild outliers;
$3(24) = 72$ for extreme outliers
There are no outliers.



REVIEW EXERCISES - CHAPTER 3

1.
$$\overline{X} = \frac{\sum X}{n} = \frac{548}{15} = 36.5$$

Data arranged in order: 0, 0, 3, 3, 4, 4, 7, 11, 14, 24, 30, 51, 92, 148, 157
MD = 11

Mode = 0, 3, and 4

$$MR = \frac{0+157}{2} = 78.5$$

2.
Attacks:
$$\overline{X} = \frac{318}{5} = 63.6$$

2. continued

Data arranged in order: 57, 61, 64, 65, 71
MD = 64
no mode
$$MR = \frac{57+71}{2} = 64$$

Deaths:
$$\overline{X} = \frac{20}{5} = 4$$

Data arranged in order: 1, 4, 4, 4, 7
MD = 4

Mode = 4

$$MR = \frac{1+7}{2} = 4$$

3.

Class	X_m	f	$f \cdot X_m$
105 - 109	107	2	214
110 - 114	112	5	560
115 - 119	117	6	702
120 - 124	122	8	976
125 - 129	127	8	1016
130 - 134	132	1	132
		30	3600

$$\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{3600}{30} = 120$$

Modal Classes = $120 - 124$ or $119.5 - 124.5$ and $125 - 129$ or $124.5 - 129.5$

4.

X_m	f	$f \cdot X_m$	$f \cdot X_m^2$
491	4	1964	964,324
518	6	3108	1,609,944
545	2	1090	594,050
572	2	1144	654,368
599	2	1198	717,602
	16	8504	4,540,288

$$\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{8504}{16} = 531.5$$

Modal Class $= 505 - 531$

5.

$$\overline{X} = \frac{\sum w \cdot X}{\sum w} = \frac{1.6(1.4) + 0.8(0.8) + 0.4(0.3) + 1.8(1.6)}{1.4 + 0.8 + 0.3 + 1.6}$$

$= 1.43$ viewers per household

6.

$$\overline{X} = \frac{0.3(10{,}000) + 0.5(3000) + 0.2(1000)}{0.3 + 0.5 + 0.2} = \$4{,}700.00$$

7.

Range $= 212 - 37 = 175$

$$s^2 = \frac{n\sum f \cdot X_m^2 - (\sum f \cdot X_m)^2}{n(n-1)} = \frac{12(110{,}077) - 989^2}{12(12-1)}$$

$$= 2596.99$$

$$s = \sqrt{2596.99} = 51.0$$

8.

Range $= 75 - 47 = 28$

$$s^2 = \frac{n\sum f \cdot X_m^2 - (\sum f \cdot X_m)^2}{n(n-1)} = \frac{13(41{,}379) - 725^2}{13(13-1)} = 78.85$$

$$s = \sqrt{78.9} = 8.9$$

9.

Class Boundaries	X_m	f	$f \cdot X_m$	$f \cdot X_m^2$	*cf*
12.5 - 27.5	20	6	120	2400	6
27.5 - 42.5	35	3	105	3675	9
42.5 - 57.5	50	5	250	12,500	14
57.5 - 72.5	65	8	520	33,800	22
72.5 - 87.5	80	6	480	38,400	28
87.5 - 102.5	95	2	190	18,050	30
		30	1665	108,825	

a. $\overline{X} = \frac{\sum f \cdot X_m}{n} = \frac{1665}{30} = 55.5$

b. Modal class $= 57.5 - 72.5$

c. $s^2 = \frac{n\sum f \cdot X_m^2 - (\sum f \cdot X_m)^2}{n(n-1)} = \frac{30(108{,}825) - 1665^2}{30(30-1)}$

$= 566.1$

d. $s = \sqrt{566.1} = 23.8$

10.

Class	X_m	f	$f \cdot X_m$	$f \cdot X_m^2$	*cf*
10 - 12	11	6	66	726	6
13 - 15	14	4	56	784	10
16 - 18	17	14	238	4046	24
19 - 21	20	15	300	6000	39
22 - 24	23	8	184	4232	47
25 - 27	26	2	52	1352	49
28 - 30	29	1	29	841	50
		50	925	17981	

a. $\overline{X} = \frac{925}{50} = 18.5$

b. Modal Class $= 19 - 21$

c. $s^2 = \frac{50(17{,}981) - 925^2}{50(50-1)} = 17.7$

d. $s = \sqrt{17.7} = 4.2$

11. $s \approx \frac{24}{4} = 6$

12. $s \approx \frac{56}{4} = 14$

13.

Textbooks: C. Var $= \frac{5}{16} = 0.3125$ or 31.25%

Ages: C. Var $= \frac{8}{43} = 0.186$ or 18.6%

The number of books is more variable.

14.

Magazines: C. Var $= \frac{s}{\overline{X}} = \frac{12}{56} = 0.214$ or 21.4%

Year: C. Var $= \frac{s}{\overline{X}} = \frac{2.5}{6} = 0.417$ or 41.7%

The number of years is more variable.

15.

$\overline{X} = 0.32 \quad s = 0.03 \quad k = 2$

$0.32 - 2(0.03) = 0.26$ and $0.32 + 2(0.03) = 0.38$

At least 75% of the values will fall between \$0.26 and \$0.38.

16.
$\overline{X} = \$58{,}500 \quad s = \$11{,}200$
a. $58{,}500 + 11{,}200k = 69{,}700$
$k = 1$
Since Chebyshev's Theorem is appropriate only for $k > 1$, no information can be obtained about the percentage of workers earning between \$47,300 and \$69,700.

b. $58{,}500 + 11{,}200k = 80{,}900$
$k = 2$
$1 - \frac{1}{2^2} = 0.75$ or 75%
Hence at most $100\% - 75\% = 25\%$ earn more than \$80,900.

c. $58{,}500 + 11{,}200k = 100{,}000$
$k = 3.7054$
$1 - \frac{1}{3.7054^2} = 0.927$ or 92.7%
Hence at most $100\% - 92.7\% = 7.3\%$ earn more than \$100,000.

17.
$\overline{X} = 54 \quad s = 4 \quad 60 - 54 = 6 \quad k = \frac{6}{4} = 1.5$
$1 - \frac{1}{1.5^2} = 1 - 0.44 = 0.56$ or 56%

18.
$\overline{X} = 231 \quad s = 5 \quad 243 - 231 = 12 \quad k = \frac{12}{5} = 2.4$
$1 - \frac{1}{2.4^2} = 0.83$ or 83%

19. By the Empirical Rule, 68% of the scores will be within 1 standard deviation of the mean.
$21 + 1(4) = 25$
$21 - 1(4) = 17$
Then, 68% of the haircut cost is between \$17 and \$25.

20.
Since the data are normally distributed, the Empirical Rule can be used. For 95%, use $k = 2$ standard deviations.
$\overline{X} \pm 2s = 44 \pm 2(9)$
Thus, 95% of the times are between 26 and 62 minutes.

21.
$\overline{X} = 14.64$
$s = 17.24$

a. $z = \frac{10 - 14.64}{17.24} = -0.27$

b. $z = \frac{28 - 14.64}{17.24} = 0.77$

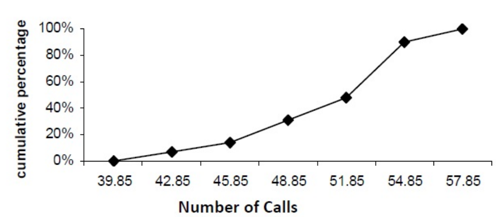
c. $z = \frac{41 - 14.64}{17.24} = 1.53$

22.

a. $z = \frac{82 - 85}{6} = -0.5$

b. $z = \frac{56 - 60}{5} = -0.8$
The exam in part a has a better relative position.

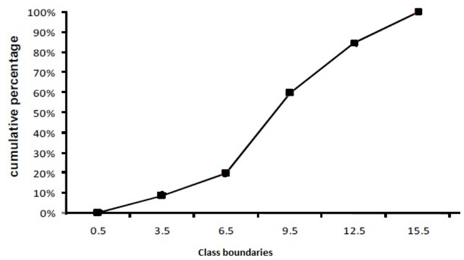
23.
a.



b. $P_{35} = 50$; $P_{65} = 53$; $P_{85} = 55$
(answers are approximate)
c. $44 = 10^{th}$ percentile; $48 = 26^{th}$ percentile; $54 = 78^{th}$ percentile (answers are approximate)

24.

a.



b. $P_{20} = 6.5$; $P_{50} = 9.4$; $P_{70} = 14.3$ (answers are approximate)

c. 5 = 13th percentile; 10 = 67th percentile; 14 = 93rd percentile (answers are approximate)

25.

a. 400 506 511 514 517 521

$\uparrow$ Q_1 (506) $\uparrow$ Q_3 (517)

For Q_1: $c = \frac{np}{100} = \frac{6(25)}{100} = 1.5$ round up to 2

$Q_1 = 506$

For Q_3: $c = \frac{np}{100} = \frac{6(75)}{100} = 4.5$ round up to 5

$Q_3 = 517$

$Q_3 - Q_1 = 517 - 506 = 11$; $11(1.5) = 16.5$;
$506 - 16.5 = 489.5$ and $517 + 16.5 = 533.5$
Therefore, only the value 400 lies outside the range of 489.5 to 533.5 and is a suspected outlier.

b. 3 6 7 8 9 10 12 14 16 20

$\uparrow$ Q_1 (7) $\uparrow$ Q_3 (14)

For Q_1: $c = \frac{np}{100} = \frac{10(25)}{100} = 2.5$ round up to 3

$Q_1 = 7$

For Q_3: $c = \frac{np}{100} = \frac{10(75)}{100} = 7.5$ round up to 8

$Q_3 = 14$

25. continued

$Q_3 - Q_1 = 14 - 7 = 7$; $7(1.5) = 10.5$;
$7 - 10.5 = -3.5$ and $14 + 10.5 = 24.5$
Since all values fall within the range of -3.5 to 24.5, there are no outliers.

26.

a. 5 13 14 18 19 25 26 27

$\uparrow$ $Q_1 = 13.5$ $\uparrow$ $Q_3 = 25.5$

For Q_1: $c = \frac{np}{100} = \frac{8(25)}{100} = 2.0$ Use the value between the 2nd and 3rd position:
$Q_1 = \frac{13+14}{2} = 13.5$

For Q_3: $c = \frac{np}{100} = \frac{8(75)}{100} = 6.0$ Use the value between the 6th and 7th position:
$Q_3 = \frac{25+26}{2} = 25.5$

$Q_3 - Q_1 = 25.5 - 13.5 = 12$; $12(1.5) = 18$:
$13.5 - 18 = -4.5$ and $25.5 + 18 = 43.5$
Since all values fall within the range of -4.5 to 43.5, there are no outliers.

b. 112 116 129 131 153 157 192

$\uparrow$ Q_1 (116) $\uparrow$ Q_3 (157)

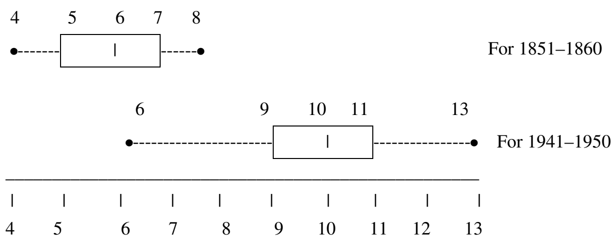
For Q_1: $c = \frac{np}{100} = \frac{7(25)}{100} = 1.75$ Round up to 2.
$Q_1 = 116$
For Q_3: $c = \frac{np}{100} = \frac{7(75)}{100} = 5.25$ Round up to 6.
$Q_3 = 157$
$Q_3 - Q_1 = 157 - 116 = 41$; $41(1.5) = 61.5$:
$116 - 61.5 = 54.5$ and $157 + 61.5 = 218.5$
Since all values fall within the range of 54.5 to 218.5, there are no outliers.

27.
For years 1851–1860:
$MD = 6 \quad Q_1 = 5 \quad Q_3 = 7$

27. continued

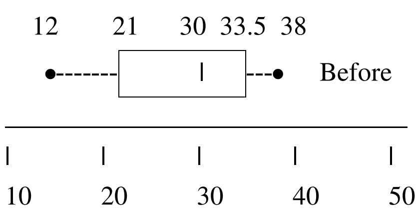
For 1941–1950:

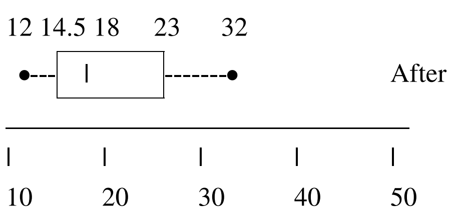
MD = 10 $Q_1 = 9$ $Q_3 = 11$



The data for years 1941–1950 have a higher median and are more variable.

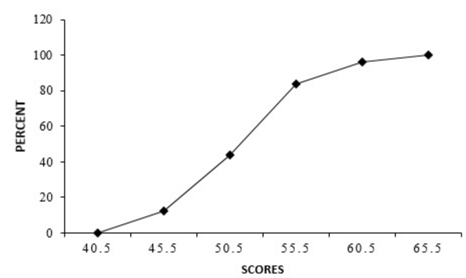
28. The five-number summaries are:
Before: 12, 21, 30, 33.5, 38
After: 12, 14.5, 18, 23, 32





CHAPTER 3 QUIZ

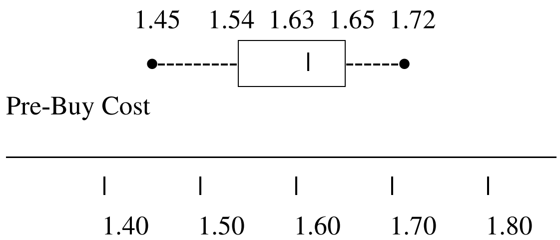
1. True
2. True
3. False
4. False
5. False
6. False
7. False
8. False
9. False
10. c
11. c
12. a and b
13. b
14. d
15. b
16. Statistic
17. Parameters, statistics
18. Standard deviation
19. σ
20. Midrange
21. Positively
22. Outlier
23. a. 15.3   b. 15.5   c. 15, 16, 17   d. 15
    e. 6   f. 3.57   g. 1.9
24. a. 6.4   b. $6 - 8$ c. 11.6   d. 3.4
25. 4.46 or 4.5
26. 0.107 or 10.7%, 0.114 or 11.4%; newspapers sold in a convenience store are more variable
27. 88.89%
28. For above 1129: 16%; For above 799: 97.5%
29. $s \approx \frac{18}{4} = 4.5$
30. -0.75; -1.67; science
31.
    a.

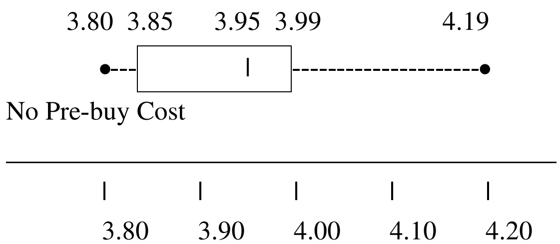


    b. 47; 55; 64
    c. 56th percentile; 6th percentile; 99th percentile

32.
For Pre-buy:
MD = 1.625 or 1.63 $Q_1 = 1.54$
$Q_3 = 1.65$
For No Pre-buy:
MD = 3.95 $Q_1 = 3.85$ $Q_3 = 3.99$

32. continued





The cost of pre-buy gas is much less than to return the car without filling it with gas. The variability of the return without filling with gas is larger than the variability of the pre-buy gas.