Note: Graphs are not to scale and are intended to convey a general idea. Answers may vary due to rounding.

EXERCISE SET 12-1

1.
The analysis of variance using the F-test can be used to compare 3 or more means.

2.
a. Comparing two means at a time ignores all other means.
b. The probability of type I error is larger than $\alpha$ when multiple t-tests are used.
c. The more sample means, the more t-tests are needed.

3.
The populations from which the samples were obtained must be normally distributed. The samples must be independent of each other. The variances of the populations must be equal, and the samples should be random.

4.
The between-group variance estimates the population variance using the means. The within-group variance estimates the population variance using all the data values.

5.
$H_0$: $\mu_1 = \mu_2 = ... = \mu_n$
$H_1$: At least one mean is different from the others.

6.
1

7.
$H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
$H_1$: At least one mean is different from the others. (claim)

C. V. = 3.01 $\alpha = 0.05$
d. f. N. = 3 d. f. D. = 24

$\overline{X}_1 = 52.286$ $s_1 = 67.243$
$\overline{X}_2 = 19.571$ $s_2 = 12.541$
$\overline{X}_3 = 35.286$ $s_3 = 16.849$
$\overline{X}_4 = 25.429$ $s_4 = 15.490$

$\overline{X}_{GM} = \frac{928}{28} = 33.143$

$$s_B^2 = \frac{\sum n_i(\overline{X}_i - \overline{X}_{GM})^2}{k - 1}$$

$$s_B^2 = \frac{7(52.286-33.143)^2+7(19.571-33.143)^2+7(35.286-33.143)^2+7(25.429-33.143)^2}{3}$$

$s_B^2 = 1434.421$

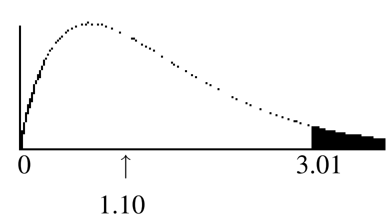
$$s_W^2 = \frac{\sum(n_i-1)s_i^2}{\sum(n_i-1)}$$

$$s_W^2 = \frac{6(67.243)^2+6(12.541)^2+6(16.849)^2+6(15.490)^2}{6+6+6+6}$$

$s_W^2 = 1300.682$

$$F = \frac{s_B^2}{s_W^2}$$

$$F = \frac{1434.421}{1300.682} = 1.10$$




Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

8.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 3.52 $\alpha = 0.05$

d. f. N = 2 d. f. D = 19

$\bar{X}_1 = 165.714$ $s_1^2 = 5695.238$

$\bar{X}_2 = 245.714$ $s_2^2 = 3928.571$

$\bar{X}_3 = 237.5$ $s_3^2 = 7335.714$

$\bar{X}_{GM} = \frac{4780}{22} = 217.273$

$s_B^2 =$

$\frac{7(165.714-217.273)^2+7(245.714-217.273)^2+8(237.5-217.273)^2}{2}$

$s_B^2 = 13{,}771.799$

$s_W^2 = \frac{6(5695.238) + 6(3928.571) + 7(7335.714)}{6+6+7}$

$s_W^2 = 5741.729$

$F = \frac{s_B^2}{s_W^2} = \frac{13771.799}{5741.729} = 2.3985 \text{ or } 2.40$

Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different.

9.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 4.26 $\alpha = 0.05$

d. f. N. = 2 d. f. D. = 9

$\bar{X}_1 = 1.888$ $s_1 = 0.535$

$\bar{X}_2 = 2.224$ $s_2 = 0.1328$

$\bar{X}_3 = 3.525$ $s_3 = 0.1344$

$\bar{X}_{GM} = \frac{27.61}{12} = 2.301$

$s_B^2 = \frac{5(1.888-2.301)^2+5(2.224-2.301)^2+2(3.525-2.301)^2}{3-1}$

$s_B^2 = 1.9394$

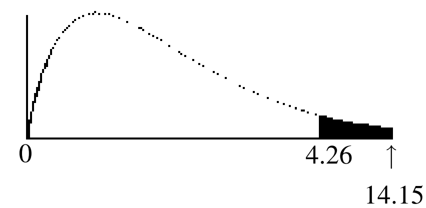
9. continued

$s_W^2 = \frac{4(0.535)^2+4(0.1328)^2+1(0.1344)^2}{4+4+1}$

$s_W^2 = 0.1371$

$F = \frac{1.9394}{0.1371} = 14.146 \text{ or } 14.15$

(TI: F = 14.1489)




Reject the null hypothesis. There is enough evidence to conclude that at least one mean is different from the others.

10.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 3.68 $\alpha = 0.05$

d. f. N = 2 d. f. D = 15

$\bar{X}_1 = 1.592$ $s_1 = 0.8127$

$\bar{X}_2 = 2.5$ $s_2 = 0.8944$

$\bar{X}_3 = 3.667$ $s_3 = 0.9092$

$\bar{X}_{GM} = \frac{\sum X}{n} = \frac{46.55}{18} = 2.586$

$s_B^2 = \frac{6(1.592-2.586)^2+6(2.5-2.586)^2+6(3.667-2.586)^2}{3-1}$

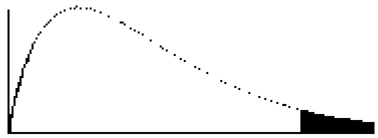
$s_B^2 = 6.492$

$s_W^2 = \frac{5(0.8127)^2 + 5(0.8944)^2 + 5(0.9092)^2}{5+5+5}$

$s_W^2 = 0.7624$

$F = \frac{6.492}{0.7624} = 8.515 \text{ or } 8.52$

10. continued




Reject the null hypothesis. There is enough evidence to conclude that at least one mean differs from the others.

11.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 3.89 $\alpha = 0.05$

d. f. N = 2 d. f. D = 12

$\overline{X}_1 = 32.10$ $s_1 = 16.88$

$\overline{X}_2 = 19.96$ $s_2 = 2.07$

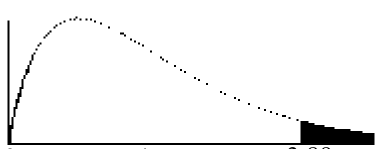
$\overline{X}_3 = 25.98$ $s_3 = 1.65$

$$\overline{X}_{GM} = \frac{390.2}{15} = 26.01$$

$$s_B^2 = 184.23$$

$$s_W^2 = 97.31$$

$$F = \frac{184.23}{97.31} = 1.89$$




Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

12.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

12. continued

$$\overline{X}_{GM} = \frac{\sum X}{n} = \frac{99}{14} = 7.07$$

$$s_B^2 = \frac{101.095}{2} = 50.548$$

$$s_W^2 = \frac{71.833}{11} = 6.530$$

$$F = \frac{s_B^2}{s_W^2} = \frac{50.548}{6.530} = 7.74$$

P-value = 0.00797

Reject since P-value < 0.05. There is enough evidence to support the claim that at least one mean is different from the others.

13.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different. (claim)

k = 3 N = 18 d.f.N. = 2 d.f.D. = 15

CV = 3.68

$\overline{X}_1 = 7$ $s_1^2 = 1.37$

$\overline{X}_2 = 8.12$ $s_2^2 = 0.64$

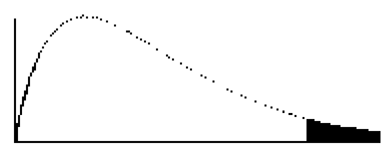
$\overline{X}_3 = 5.23$ $s_3^2 = 2.66$

$\overline{X}_{GM} = 6.7833$

$$s_B^2 = \frac{6(7-6.78)^2}{2} + \frac{6(8.12-6.78)^2}{2} + \frac{6(5.23-6.78)^2}{2} = 12.7$$

$$s_W^2 = \frac{5(1.37)+5(0.64)+5(2.66)}{5+5+5} = 1.56$$

$$F = \frac{12.7}{1.56} = 8.14$$




Reject the null hypothesis. There is enough evidence to support the claim that at least one mean is different.

14.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 3.89 $\alpha = 0.05$

d. f. N = 2 d. f. D = 12

$\bar{X}_1 = 65.276$ $s_1 = 16.158$

$\bar{X}_2 = 66.1$ $s_2 = 37.165$

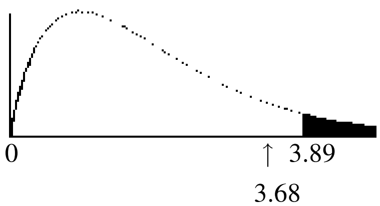
$\bar{X}_3 = 57.864$ $s_3 = 11.03$

$$\bar{X}_{GM} = \frac{946.2}{15} = 63.08$$

$$s_B^2 = 771.14$$

$$s_W^2 = 209.71$$

$$F = \frac{s_B^2}{s_W^2} = \frac{771.14}{209.71} = 3.677 \text{ or } 3.68$$




Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one of the means is different from the others.

15.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 2.64 $\alpha = 0.10$

d. f. N = 2 d. f. D = 17

$\bar{X}_1 = 96.33$ $s_1 = 43.80$

$\bar{X}_2 = 30$ $s_2 = 6.34$

$\bar{X}_3 = 27$ $s_3 = 15.70$

$$\bar{X}_{GM} = \frac{983}{20} = 49.15$$

15. continued

$$s_B^2 = \frac{\sum n_i(\bar{X}_i - \bar{X}_{GM})^2}{k-1}$$

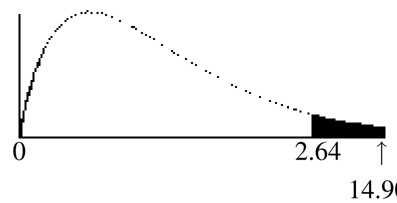
$$s_B^2 = \frac{6(96.33-49.15)^2 + 9(30-49.15)^2 + 5(27-49.15)^2}{2}$$

$$= 9554.665$$

$$s_W^2 = \frac{\sum (n_i - 1)s_i^2}{\sum (n_i - 1)}$$

$$= \frac{5(43.80)^2 + 8(6.34)^2 + 4(15.70)^2}{5+8+4} = 641.16$$

$$F = \frac{s_B^2}{s_W^2} = \frac{9554.665}{641.16} = 14.90$$




Reject the null hypothesis. There is enough evidence to support the claim that at least one mean is different from the others.

16.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others.

C. V. = 3.98 $\alpha = 0.05$

d. f. N = 2 d. f. D = 11

$\bar{X}_1 = 8705.8$ $s_1 = 1279.912$

$\bar{X}_2 = 7376.6$ $s_2 = 1568.072$

$\bar{X}_3 = 7179.25$ $s_3 = 1927.062$

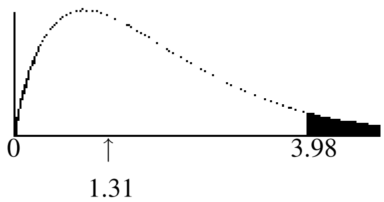
$$X_{GM} = \frac{109{,}129}{14} = 7794.929$$

$$s_B^2 = \frac{6{,}539{,}668.18}{2} = 3{,}269{,}834.09$$

$$s_W^2 = \frac{27{,}528{,}808.8}{11} = 2{,}502{,}618.98$$

$$F = \frac{s_B^2}{s_W^2} = \frac{3{,}269{,}834.09}{2{,}502{,}618.98} = 1.3066 \text{ or } 1.31$$

16. continued




Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

17.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

$\overline{X}_1 = 233.33$ $s_1 = 28.225$

$\overline{X}_2 = 203.125$ $s_2 = 39.364$

$\overline{X}_3 = 155.625$ $s_3 = 28.213$

$\overline{X}_{GM} = 194.091$

$$s_B^2 = \frac{21{,}729.735}{2} = 10{,}864.8675$$

$$s_W^2 = \frac{20{,}402.083}{19} = 1073.794$$

$$F = \frac{s_B^2}{s_W^2} = \frac{10{,}864.8675}{1073.794} = 10.12$$

P-value = 0.00102

Reject since P-value < 0.10. There is enough evidence to conclude that at least one mean is different from the others.

18.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 4.10 $\alpha = 0.05$

d. f. N = 2 d. f. D = 10

$\overline{X}_1 = 850$ $s_1 = 108.01$

$\overline{X}_2 = 914$ $s_2 = 79.56$

$\overline{X}_3 = 575$ $s_3 = 110.91$

18. continued

$$\overline{X}_{GM} = \frac{10{,}270}{13} = 790$$

$$s_B^2 = \frac{276{,}180}{2} = 138{,}090$$

$$s_W^2 = \frac{97{,}220}{10} = 9722$$

$$F = \frac{s_B^2}{s_W^2} = \frac{138{,}090}{9722} = 14.20$$

Reject the null hypothesis. There is enough evidence to support the claim that at least one mean is different from the others.

19.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean differs from the others. (claim)

C. V. = 3.01 $\alpha = 0.05$

d. f. N = 2 d. f. D = 9

$\overline{X}_1 = 20.5$ $s_1 = 3.416$

$\overline{X}_2 = 26.25$ $s_2 = 3.5$

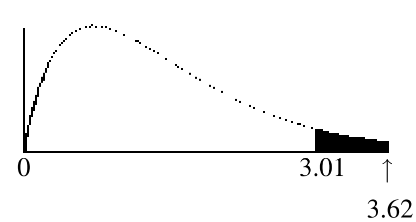
$\overline{X}_3 = 22.5$ $s_3 = 2.082$

$$\overline{X}_{GM} = \frac{277}{12} = 23.083$$

$$s_B^2 = \frac{68.167}{2} = 34.083$$

$$s_w^2 = \frac{84.75}{9} = 9.417$$

$$F = \frac{34.083}{9.417} = 3.62$$




Reject the null hypothesis. There is enough evidence to support the claim that at least one mean is different from the others.

20.

$H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_1$: At least one mean differs from the others. (claim)

C. V. = 3.24 $\quad \alpha = 0.05$

d. f. N = 3 $\quad$ d. f. D = 16

$\overline{X}_1 = 14{,}394.6 \quad s_1 = 1253.424$

$\overline{X}_2 = 14{,}668.2 \quad s_2 = 2367.532$

$\overline{X}_3 = 14{,}275.2 \quad s_3 = 821.006$

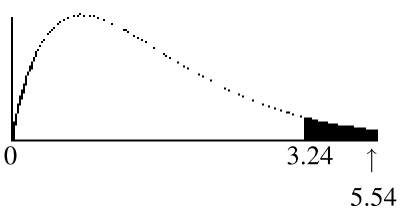
$\overline{X}_4 = 18{,}327 \quad s_4 = 2415.376$

$$\overline{X}_{GM} = \frac{308{,}325}{20} = 15{,}416.25$$

$$s_B^2 = \frac{56{,}889{,}041}{3} = 18{,}963{,}013.67$$

$$s_w^2 = \frac{54{,}737{,}476.8}{16} = 3{,}421{,}092.3$$

$$F = \frac{18{,}963{,}013.67}{3{,}421{,}092.3} = 5.54$$




Reject the null hypothesis. There is enough evidence to support the claim that at least one mean is different from the others.

EXERCISE SET 12-2

1.
The Scheffe' and Tukey tests are used.

2.
The Scheffe' test is usually used when sample sizes are not the same. The Tukey test is usually used when the sample sizes are equal.

3.
Scheffe´ Test
C. V. = 8.52

3. continued

$$F_s = \frac{(\overline{X}_i - \overline{X}_j)^2}{s_W^2\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$$

For $\overline{X}_1$ vs $\overline{X}_2$

$$F_S = \frac{(1.888 - 2.224)^2}{0.13707\left(\frac{1}{5} + \frac{1}{5}\right)} = \frac{0.112896}{0.054828} = 2.10$$

For $\overline{X}_1$ vs $\overline{X}_3$

$$F_S = \frac{(1.888 - 3.525)^2}{0.13707\left(\frac{1}{5} + \frac{1}{2}\right)} = \frac{2.679769}{0.095949} = 27.93$$

For $\overline{X}_2$ vs $\overline{X}_3$

$$F_S = \frac{(2.224 - 3.525)^2}{0.13707\left(\frac{1}{5} + \frac{1}{2}\right)} = \frac{1.692601}{0.0959504} = 17.64$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_3$ and between $\overline{X}_2$ and $\overline{X}_3$.

4.
Scheffe´ Test
C. V. = 7.96

For $\overline{X}_1$ vs $\overline{X}_2$

$$F_S = \frac{(5 - 10.167)^2}{6.530\left(\frac{1}{4} + \frac{1}{6}\right)} = \frac{26.698}{2.721} = 9.81$$

For $\overline{X}_1$ vs $\overline{X}_3$

$$F_S = \frac{(5 - 4.5)^2}{6.530\left(\frac{1}{4} + \frac{1}{4}\right)} = \frac{0.25}{3.265} = 0.08$$

For $\overline{X}_2$ vs $\overline{X}_3$

$$F_S = \frac{(10.167 - 4.5)^2}{6.530\left(\frac{1}{6} + \frac{1}{4}\right)} = \frac{32.115}{2.721} = 11.80$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_2$ and between $\overline{X}_2$ and $\overline{X}_3$.

5.
Tukey Test:
C. V. = 3.67

$\overline{X}_1 = 7.0$

$\overline{X}_2 = 8.12$

$\overline{X}_3 = 5.23$

5. continued

$\overline{X}_1$ vs $\overline{X}_2$:

$$q = \frac{7 - 8.12}{\sqrt{\frac{1.56}{6}}} = -2.20$$

$\overline{X}_1$ vs $\overline{X}_3$:

$$q = \frac{7 - 5.23}{\sqrt{\frac{1.56}{6}}} = 3.47$$

$\overline{X}_2$ vs $\overline{X}_3$:

$$q = \frac{8.12 - 5.23}{\sqrt{\frac{1.56}{6}}} = 5.67$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_3$ and between $\overline{X}_2$ and $\overline{X}_3$. One reason for the difference might be that students are enrolled in cyber schools with different fees.

6.
Scheffe' Test
C. V. = 5.22

$\overline{X}_1$ vs $\overline{X}_2$:

$$F_s = \frac{(\overline{X}_i - \overline{X}_j)^2}{s_W^2(\frac{1}{n_i} + \frac{1}{n_j})} = \frac{(233.33 - 203.125)^2}{1073.776(\frac{1}{6} + \frac{1}{8})}$$

$F_s = 2.91$

$\overline{X}_1$ vs $\overline{X}_3$:

$$F_s = \frac{(233.33 - 155.625)^2}{1073.776(\frac{1}{6} + \frac{1}{8})} = 19.30$$

$\overline{X}_2$ vs $\overline{X}_3$ :

$$F_s = \frac{(203.125 - 155.625)^2}{1073.776(\frac{1}{8} + \frac{1}{8})} = 8.40$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_3$ and between $\overline{X}_2$ and $\overline{X}_3$.

7.
Scheffe' Test
C. V. = 8.20

$\overline{X}_1$ vs $\overline{X}_2$:

$$F_s = \frac{(\overline{X}_i - \overline{X}_j)^2}{s_W^2(\frac{1}{n_i} + \frac{1}{n_j})} = \frac{(850 - 914)^2}{9722(\frac{1}{4} + \frac{1}{5})}$$

$F_s = 0.94$

7. continued

$\overline{X}_1$ vs $\overline{X}_3$:

$$F_s = \frac{(850 - 575)^2}{9722(\frac{1}{4} + \frac{1}{4})} = 15.56$$

$\overline{X}_2$ vs $\overline{X}_3$ :

$$F_s = \frac{(914 - 575)^2}{9722(\frac{1}{4} + \frac{1}{5})} = 26.27$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_3$ and between $\overline{X}_2$ and $\overline{X}_3$.

8.
Tukey Test:
C. V. = 3.65

$\overline{X}_1$ vs $\overline{X}_2$:

$$q = \frac{(14{,}394.6 - 14{,}668.2)}{\sqrt{\frac{3{,}421{,}092.3}{5}}} = -0.331$$

$\overline{X}_1$ vs $\overline{X}_3$:

$$q = \frac{(14{,}394.6 - 14{,}275.2)}{\sqrt{\frac{3{,}421{,}092.3}{5}}} = 0.144$$

$\overline{X}_1$ vs $\overline{X}_4$:

$$q = \frac{(14{,}394.6 - 18{,}327)}{\sqrt{\frac{3{,}421{,}092.3}{5}}} = -4.75$$

$\overline{X}_2$ vs $\overline{X}_3$ :

$$q = \frac{(14{,}668.2 - 14{,}275.2)}{\sqrt{\frac{3{,}421{,}092.3}{5}}} = 0.475$$

$\overline{X}_2$ vs $\overline{X}_4$ :

$$q = \frac{(14{,}668.2 - 18{,}327)}{\sqrt{\frac{3{,}421{,}092.3}{5}}} = -4.42$$

$\overline{X}_3$ vs $\overline{X}_4$ :

$$q = \frac{(14{,}275.2 - 18{,}327)}{\sqrt{\frac{3{,}421{,}092.3}{5}}} = -4.898$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_4$, $\overline{X}_2$ and $\overline{X}_4$, and $\overline{X}_3$ and $\overline{X}_4$

9.
$H_0$: $\mu_1 = \mu_2 = \mu_3$
$H_1$: At least one mean is different from the others. (claim)

9. continued

C. V. $= 3.68$ $\quad \alpha = 0.05$
d. f. N $= 2$ $\quad$ d. f. D $= 15$

$\overline{X}_1 = 32.333$ $\quad s_1 = 8.140$
$\overline{X}_2 = 27.833$ $\quad s_2 = 5.529$
$\overline{X}_3 = 22.5$ $\quad s_3 = 4.370$
$\overline{X}_{GM} = 27.556$

$$s_B^2 = \frac{290.778}{2} = 145.389$$

$$s_W^2 = \frac{579.667}{15} = 38.644$$

$$F = \frac{s_B^2}{s_W^2} = \frac{145.389}{38.644} = 3.76$$

Reject the null hypothesis. At least one mean is different from the others.

Tukey Test:
C. V. $= 3.67$

$$q = \frac{\overline{X}_i - \overline{X}_j}{\sqrt{\frac{s_W^2}{n}}}$$

$\overline{X}_1$ vs $\overline{X}_2$:

$$q = \frac{(32.333 - 27.833)}{\sqrt{\frac{38.644}{6}}} = 1.77$$

$\overline{X}_1$ vs $\overline{X}_3$:

$$q = \frac{(32.333 - 22.5)}{\sqrt{\frac{38.644}{6}}} = 3.87$$

$\overline{X}_2$ vs $\overline{X}_3$ :

$$q = \frac{(27.833 - 22.5)}{\sqrt{\frac{38.644}{6}}} = 2.10$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_3$.

10.
$H_0$: $\mu_1 = \mu_2 = \mu_3$ (claim)
$H_1$: At least one mean is different from the others.

C. V. $= 7.21$ $\quad \alpha = 0.01$
d. f. N $= 2$ $\quad$ d. f. D $= 11$

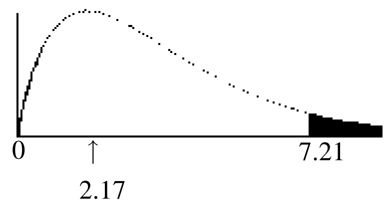
10. continued

$\overline{X}_1 = 329.6$ $\quad s_1 = 142.451$
$\overline{X}_2 = 164.8$ $\quad s_2 = 86.0186$
$\overline{X}_3 = 359.25$ $\quad s_3 = 225.114$
$\overline{X}_{GM} = 279.214$

$$s_B^2 = \frac{103{,}769.607}{2} = 51{,}884.8035$$

$$s_W^2 = \frac{262{,}794.75}{11} = 23{,}890.4318$$

$$F = \frac{s_B^2}{s_W^2} = \frac{51{,}884.8035}{23{,}890.4318} = 2.17$$




Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

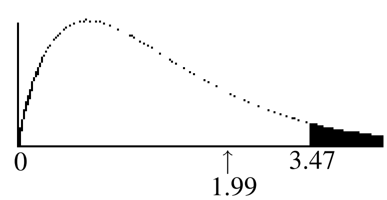
11.
$H_0$: $\mu_1 = \mu_2 = \mu_3$
$H_1$: At least one mean is different from the others. (claim)

C. V. $= 3.47$ $\quad \alpha = 0.05$
d. f. N. $= 2$ $\quad$ d. f. D. $= 21$

$\overline{X}_{GM} = 4.554$ $\quad s_B^2 = 9.82113$

$s_W^2 = 4.93225$

$$F = \frac{9.82113}{4.93225} = 1.99$$




Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

12.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 4.10 $\alpha = 0.05$

d. f. N = 2 d. f. D = 10

$\overline{X}_1 = 6091.4$ $s_1^2 = 667{,}494.3$

$\overline{X}_2 = 6519.75$ $s_2^2 = 425{,}494.25$

$\overline{X}_3 = 6831.5$ $s_3^2 = 1{,}881{,}561.667$

$X_{GM} = 6450.923$

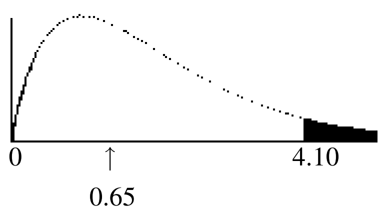
$$s_B^2 = \frac{5(6091.4 - 6450.923)^2 + 4(6519.75 - 6450.923)^2}{3-1}$$

$$+ \frac{4(6831.5 - 6450.923)^2}{3-1} = 622{,}293.987$$

$$s_W^2 = \frac{4(667494.3) + 3(425494.25) + 3(1881561.667)}{4+3+3}$$

$$s_W^2 = 959114.495$$

$$F = \frac{s_B^2}{s_W^2} = \frac{622293.987}{959114.495} = 0.65$$




Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

13.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different. (claim)

C. V. = 3.68 $\alpha = 0.05$

d. f. N = 2 d. f. D = 15

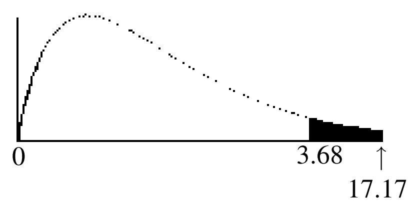
$$\overline{X}_{GM} = \frac{4666}{18} = 259.22$$

$$s_B^2 = \frac{39{,}374.111}{2} = 19{,}687.056$$

13. continued

$$s_w^2 = \frac{17{,}197}{15} = 1146.467$$

$$F = \frac{19{,}687.056}{1146.467} = 17.17$$




Reject the null hypothesis. There is enough evidence to support the claim that at least one mean is different.

Tukey Test:

C. V. = 3.67

$\overline{X}_1$ vs $\overline{X}_2$:

$$q = \frac{(208.17 - 321.17)}{\sqrt{\frac{1146.467}{6}}} = -8.17$$

$\overline{X}_1$ vs $\overline{X}_3$:

$$q = \frac{(208.17 - 248.33)}{\sqrt{\frac{1146.467}{6}}} = -2.91$$

$\overline{X}_2$ vs $\overline{X}_3$ :

$$q = \frac{(321.17 - 248.33)}{\sqrt{\frac{1146.467}{6}}} = 5.27$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_2$ and between $\overline{X}_2$ and $\overline{X}_3$.

14.

$H_0$: $\mu_2 = \mu_3$

$H_1$: $\mu_2 \neq \mu_3$ (claim)

C.V. is somewhere between 2.821 and 3.250.

$$\alpha = \frac{0.05}{3} = 0.016$$

d. f. = 12 − 3 = 9

$$t = \frac{\overline{X}_2 - \overline{X}_3}{\sqrt{s_W^2\left(\frac{1}{n_2} + \frac{1}{n_3}\right)}} = 2.456$$

14. continued

Since $2.456 < 2.821$, do not reject the null hypothesis. There is not a significant difference between $\mu_2$ and $\mu_3$. Also, the P-value is 0.075, which is larger than 0.05.

EXERCISE SET 12-3

1.
The two-way ANOVA allows the researcher to test the effects of two independent variables and a possible interaction effect. The one-way ANOVA can test the effects of one independent variable only.

2.
The main effects are the effects of the independent variables taken separately. The interaction effect occurs when one independent variable effects the dependent variable differently at different levels of the other independent variable.

3.
The mean square values are computed by dividing the sum of squares by the corresponding degrees of freedom.

4.
The F test value is computed by dividing the mean square for the variable by the mean square for the within (error) term.

5.
a. d. $f._A = (3 - 1) = 2$ for factor A
b. d. $f._B = (2 - 1) = 1$ for factor B
c. d. $f._{AxB} = (3 - 1)(2 - 1) = 2$
d. d. $f._{within} = 3 \cdot 2(5 - 1) = 24$

6.
a. d. $f._A = (6 - 1) = 5$
b. d. $f._B = (5 - 1) = 4$
c. d. $f._{AxB} = (6 - 1)(5 - 1) = 20$
d. d. $f._{within} = 6 \cdot 5(7 - 1) = 180$

7.
The two types of interactions that can occur are ordinal and disordinal.

8.
The main effects can be interpreted independently when the interaction effect is not significant or the interaction is ordinal.

9.
For interaction:
$H_0$: There is no interaction between the amount of glycerin additive and the soap concentration.
$H_1$: There is an interaction between the amount of glycerin additives.

For glycerin additives:
$H_0$: There is no difference in the means of the glycerin additives.
$H_1$: There is a difference in the means of the glycerin additives.

For soap concentrations:
$H_0$: There is no difference in the means of the soap concentrations.
$H_1$: There is a difference in the means of the soap concentrations.

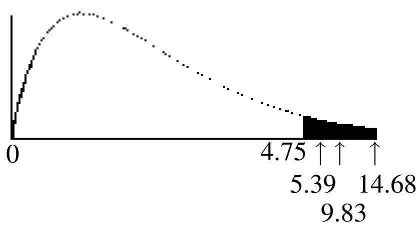
ANOVA SUMMARY TABLE

| Source | SS | d. f. | M. S. | F |
|---|---|---|---|---|
| Soap additive | 100.00 | 1 | 100.00 | 5.39 |
| Glycerin | 182.25 | 1 | 182.25 | 9.83 |
| Interaction | 272.25 | 1 | 272.25 | 14.68 |
| Within | 222.50 | 12 | 18.54 | |
| Total | 777.0 | 15 | | |

9. continued

The critical value at $\alpha = 0.05$ with d. f. N = 1 and d. f. D = 12 is 4.75 for $F_A$, $F_B$ and $F_{AxB}$.

All F test values exceed the critical value, so the decision is to reject all null hypotheses. There is a significant difference at $\alpha = 0.05$ for interaction, for soap additive, and for glycerin concentration.




10.
For interaction:

$H_0$: There is no interaction effect between the strength of the Grow-light and the plant food supplement.

$H_1$: There is an interaction effect between the Grow-light strength and the plant food supplement.

For plant food:

$H_0$: There is no difference between mean growth and the type of to plant food supplement.

$H_1$: There is a difference between mean growth and the type of plant food supplement.

For grow-light:

$H_0$: There is no difference between the mean growth and the strength of the Grow-light.

$H_1$: There is a difference between the mean growth and the strength of the Grow-light.

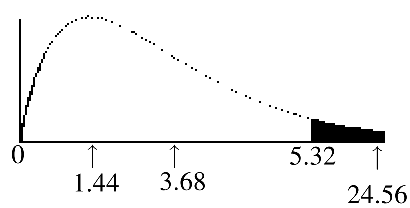
10. continued

ANOVA SUMMARY TABLE

| Source | SS | d. f. | M. S. | F |
|---|---|---|---|---|
| Plant food | 12.813 | 1 | 12.813 | 24.56 |
| Grow-light | 1.92 | 1 | 1.92 | 3.68 |
| Interaction | 0.75 | 1 | 0.75 | 1.44 |
| Within | 4.173 | 8 | 0.522 | |
| Total | 19.656 | 11 | | |

The critical value at $\alpha = 0.05$ with d. f. N = 1 and d. f. D = 8 is 5.32 for $F_A$, $F_B$ and $F_{AxB}$.

Since the F test value for the plant food, 24.56, is greater than the critical value, 5.32, the decision is to reject the null hypothesis for the plant food. It can be concluded that there is a significant difference in the mean growth for the plant food. Plant light strength and the interaction have no effect.




11.
For interaction:

$H_0$: There is no interaction effect between temperature and level of humidity.

$H_1$: There is an interactive effect between temperature and level of humidity.

For humidity:

$H_0$: There is no difference in mean length of effectiveness with respect to humidity.

$H_1$: There is a difference in mean length of effectiveness with respect to humidity.

11. continued

For temperature:

$H_0$: There is no difference in mean length of effectiveness based on temperature.

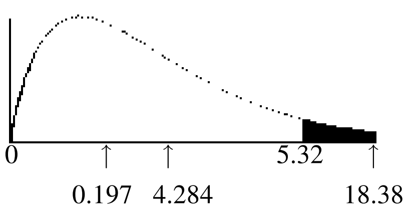
$H_1$: There is a difference in mean length of effectiveness based on temperature.

ANOVA SUMMARY TABLE

| Source | SS | d. f. | MS | F | *P*-value |
|---|---|---|---|---|---|
| Humidity | 280.3333 | 1 | 280.3333 | 18.38 | 0.003 |
| Temperature | 3 | 1 | 3 | 0.197 | 0.669 |
| Interaction | 65.3333 | 1 | 65.3333 | 4.284 | 0.0722 |
| Within | 122 | 8 | 15.25 | | |
| Total | 470.6667 | 11 | | | |

The critical value at $\alpha = 0.05$ with d. f. N = 1 and d. f. D = 8 is 5.32 for $F_A$, $F_B$, and $F_{AxB}$.

Since the only F test value that exceeds the critical value is the one for humidity, there is sufficient evidence to conclude that there is a difference in mean length of effectiveness based on the humidity level. The temperature and interaction effects are not significant.




12.

For interaction:

$H_0$: There is no interaction effect between the subcontractors and the types of homes they build on the times it takes to build the homes.

$H_1$: There is an interaction effect between the subcontractors and the types of homes they build on the times it takes to build the homes.

12. continued

For subcontractors:

$H_0$: There is no difference in the means of the times it takes the subcontractors to build the homes.

$H_1$: There is a difference in the means of the times it takes the subcontractors to build the homes.

For types of homes:

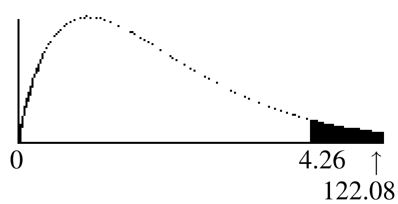
$H_0$: There is no difference among the means of the times for the types of homes built.

$H_1$: There is a difference among the means of the times for the types of homes built.

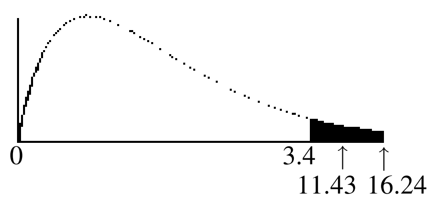
ANOVA SUMMARY TABLE

| Source | SS | d. f. | MS | F |
|---|---|---|---|---|
| Subcontractor | 1672.553 | 1 | 1672.553 | 122.08 |
| Home Type | 444.867 | 2 | 222.434 | 16.24 |
| Interaction | 313.267 | 2 | 156.634 | 11.43 |
| Within | 328.800 | 24 | 13.700 | |
| Total | 2759.487 | 29 | | |

The critical values at $\alpha = 0.05$: For the subcontractor with d. f. N = 1, d. f. D = 24, C. V. = 4.26.




For the home type and interaction d. f. N = 2 and d. f. D = 24, C. V. = 3.40.



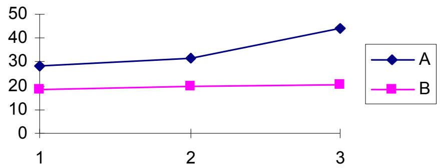

All F test values exceed the critical values and all of the null hypotheses are rejected. Since there is a significant interaction effect the means of the cells must be computed and graphed to determine the type of interaction.

12. continued

The cell means are:

HOME TYPE

| Contractor | I | II | III |
|---|---|---|---|
| A | 28 | 31.4 | 44.4 |
| B | 18.6 | 20.0 | 20.4 |




Since all of the three means for the home types for contractor A are greater than the three means for contractor B, and the differences are not equal, there is an ordinal interaction. Hence it can be concluded that there is a difference in means for the subcontractors, home types, and also an interaction effect is present.

13.

For interaction:

$H_0$: There is no interaction effect on the durability rating between the dry additives and the solution-based additives.

$H_1$: There is an interaction effect on the durability rating between the dry additives and the solution-based additives.

For solution-based additive:

$H_0$: There is no difference in the mean durability rating with respect to the solution-based additives.

$H_1$: There is a difference in the mean durability rating with respect to the solution-based additives.

For dry additives:

$H_0$: There is no difference in the mean durability rating with respect to the dry additive.

13. continued

$H_1$: There is a difference in the mean durability rating with respect to the dry additive.

ANOVA SUMMARY TABLE

| Source | SS | d. f. | MS | F | *P*-value |
|---|---|---|---|---|---|
| Solution | 1.563 | 1 | 1.563 | 0.50 | 0.494 |
| Dry | 0.063 | 1 | 0.063 | 0.020 | 0.890 |
| Interaction | 1.563 | 1 | 1.563 | 0.50 | 0.494 |
| Within | 37.750 | 12 | 3.146 | | |
| Total | 40.939 | 15 | | | |

The critical value at $\alpha = 0.05$ with d. f. N = 1 and d. f. D = 12 is 4.75. F = 0.50 for the solution-based additive and F = 0.020 for the dry additive. There is not enough evidence to conclude an effect on the mean durability based on either type of additive. For interaction, there is also not a significant interaction effect.

14.

For interaction:

$H_0$: There is no interaction effect between the type of paint and the geographic location on the lifetimes of the paint.

$H_1$: There is an interaction effect between the type of paint and the geographic location on the lifetimes of the paint.

For lifetimes:

$H_0$: There is no difference between the means of the lifetimes of the two types of paints.

$H_1$: There is a difference between the means of the lifetimes of the two types of paints.

For locations:

$H_0$: There is no difference among the means of the lifetimes of the paints used in different geographic locations.

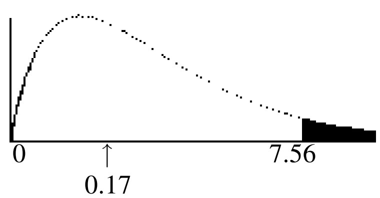
$H_1$: There is a difference in the means of the lifetimes of the paints used in different geographic locations.

14. continued

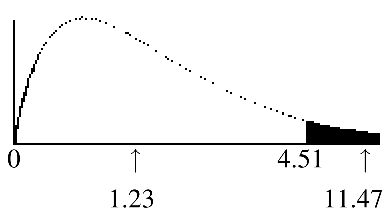
ANOVA SUMMARY TABLE

| Source | SS | d. f. | MS | F |
|---|---|---|---|---|
| Paint Type | 12.1 | 1 | 12.1 | 0.17 |
| Location | 2501.0 | 3 | 833.667 | 11.47 |
| Interaction | 268.1 | 3 | 89.367 | 1.23 |
| Within | 2326.8 | 32 | 72.713 | |
| Total | 5108.0 | 39 | | |

For $\alpha = 0.01$ the critical values are:
For the paint type, d. f. N = 1, d. f. D = 32 (use 30), and C. V. = 7.56




For the location and interaction , d. f. N = 3, d. f. D = 32 (use 30), and C. V. = 4.51




Since the only F test value that exceeds the critical value is the one for the location, it can be concluded that there is a difference in the means for the geographic locations., but not for paint types.

15.
For interaction:
$H_0$: There is no interaction effect between the ages of the salespersons and the products they sell on the monthly sales.
$H_1$: There is an interaction effect between the ages of the salespersons and the products they sell on the monthly sales.

15. continued

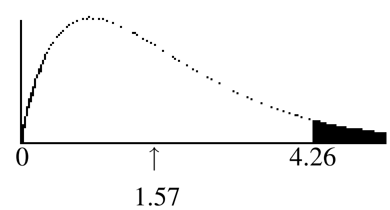
For age:
$H_0$: There is no difference in the means of the monthly sales of the two age groups.
$H_1$: There is a difference in the means of the monthly sales of the two age groups.

For products:
$H_0$: There is no difference among the means of the sales for the different products.
$H_1$: There is a difference among the means of the sales for the different products.

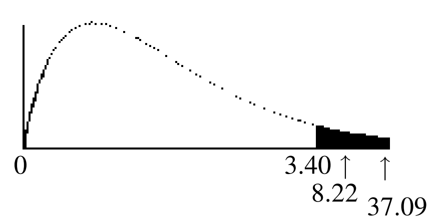
ANOVA SUMMARY TABLE

| Source | SS | d. f. | MS | F |
|---|---|---|---|---|
| Age | 168.033 | 1 | 168.033 | 1.57 |
| Product | 1762.067 | 2 | 881.034 | 8.22 |
| Interaction | 7955.267 | 2 | 3977.634 | 37.09 |
| Within | 2574.000 | 24 | 107.250 | |
| Total | 12459.367 | 29 | | |

At $\alpha = 0.05$, the critical values are:
For age, d. f. N = 1, d. f. D = 24, C. V. = 4.26




For product and interaction, d. f. N = 2, d. f. D = 24, and C. V. = 3.40

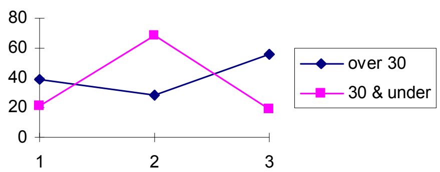



The null hypotheses for the interaction effect and for the type of product sold are rejected since the F test values exceed the critical value, 3.40.

15. continued

The cell means are:

| Age | Pools | Spas | Saunas |
|---|---|---|---|
| over 30 | 38.8 | 28.6 | 55.4 |
| 30 & under | 21.2 | 68.6 | 18.8 |




Since the lines cross, there is a disordinal interaction hence there is an interaction effect between the age of the sales person and the type of products sold.

REVIEW EXERCISES - CHAPTER 12

1.

$H_0$: $\mu_1 = \mu_2 = \mu_3$ (claim)

$H_1$: At least one mean is different from the others.

C. V. = 5.39 $\alpha = 0.01$

d. f. N = 2 d. f. D = 33

$\bar{X}_1 = 620.5$ $s_1^2 = 5445.91$

$\bar{X}_2 = 610.17$ $s_2^2 = 22{,}108.7$

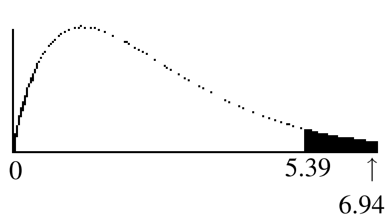
$\bar{X}_3 = 477.83$ $s_3^2 = 5280.33$

$$\bar{X}_{GM} = \frac{20{,}502}{36} = 569.5$$

$$s_B^2 = \frac{151{,}890.667}{2} = 75{,}945.333$$

$$s_W^2 = \frac{361{,}184.333}{33} = 10{,}944.9798$$

$$F = \frac{s_B^2}{s_W^2} = \frac{75{,}945.333}{10{,}944.9798} = 6.94$$




1. continued

Reject. At least one mean is different.

Tukey Test C. V. = 4.45

$\bar{X}_1$ vs $\bar{X}_2$

$$q = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_W^2}{n}}} = \frac{620.5 - 610.17}{\sqrt{\frac{10{,}944.98}{12}}} = 0.34$$

$\bar{X}_1$ vs $\bar{X}_3$

$$q = \frac{620.5 - 477.83}{\sqrt{\frac{10{,}944.98}{12}}} = 4.72$$

$\bar{X}_2$ vs $\bar{X}_3$

$$q = \frac{610.17 - 477.83}{\sqrt{\frac{10{,}944.98}{12}}} = 4.38$$

There is a significant difference between $\bar{X}_1$ and $\bar{X}_3$.

2.

$H_0$: $\mu_1 = \mu_2 = \mu_3$ (claim)

$H_1$: At least one mean is different from the others.

C. V. = 3.98 $\alpha = 0.05$

d. f.N = 2 d. f. D = 11

$\bar{X}_1 = 44.2$ $s_1 = 15.3199$

$\bar{X}_2 = 33.4$ $s_2 = 8.3546$

$\bar{X}_3 = 58$ $s_3^2 = 33.9803$

$\bar{X}_{GM} = 44.2857$

$s_B^2 = 672.4286$

$s_W^2 = 425.6364$

$$F = \frac{s_B^2}{s_W^2} = \frac{672.4286}{425.6364} = 1.580$$

Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

3.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 3.55 $\quad \alpha = 0.05$

d. f. N = 2 $\quad$ d. f. D = 18

$\overline{X}_1 = 29.625 \qquad s_1^2 = 59.125$

$\overline{X}_2 = 29\, s_2^2 = 63.333$

$\overline{X}_3 = 28.5 \qquad s_3^2 = 37.1$

$\overline{X}_{GM} = 29.095$

$$s_B^2 = \frac{\sum n_i(\overline{X}_i - \overline{X}_{GM})^2}{k-1}$$

$$s_B^2 = \frac{8(29.625-29.095)^2}{2} + \frac{7(29-29.095)^2}{2} + \frac{6(28.5-29.095)^2}{2} = 2.21726$$

$$s_W^2 = \frac{\sum(n_i-1)s_i^2}{\sum(n_i-1)}$$

$$s_W^2 = \frac{7(59.125)+6(63.333)+5(37.1)}{7+6+5}$$

$$s_W^2 = 54.509611$$

$$F = \frac{s_B^2}{s_W^2} = \frac{2.21726}{54.509611} = 0.04$$

Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

4.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 6.01 $\quad \alpha = 0.01$

d. f. N = 2 $\quad$ d. f. D = 18

$\overline{X}_1 = 17.28571 \qquad s_1^2 = 11.2381$

$\overline{X}_2 = 19.57143 \qquad s_2^2 = 9.28571$

$\overline{X}_3 = 19.42857 \qquad s_3^2 = 32.28571$

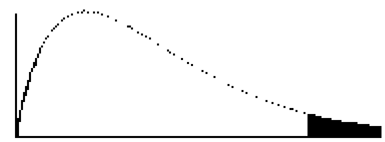
4. continued

$$\overline{X}_{GM} = \frac{\sum X}{N} = 18.7619$$

$$s_B^2 = \frac{7(17.28571-18.7619)^2 + 7(19.57143-18.7619)^2}{3-1} + \frac{7(19.42857-18.7619)^2}{3-1} = 11.476$$

$$s_W^2 = \frac{6(11.2381) + 6(9.28571) + 6(32.28571)}{6+6+6} = 17.603$$

$$F = \frac{s_B^2}{s_W^2} = \frac{11.476}{17.603} = 0.65$$



0 $\qquad \uparrow \qquad$ 6.01

0.65

Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

5.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different. (claim)

C. V. = 2.61 $\quad \alpha = 0.10$

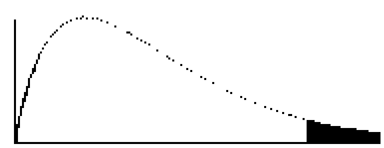
d. f. N = 2 $\quad$ d. f. D = 19

$\overline{X}_{GM} = 3.8591$

$s_B^2 = 1.65936$

$s_W^2 = 3.40287$

$$F = \frac{1.65936}{3.40287} = 0.49$$



0 $\qquad \uparrow \qquad$ 2.61

0.49

Do not reject. There is not enough evidence to support the claim that at least one mean is different from the others.

6.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others.

C. V. = 3.89 $\alpha = 0.05$

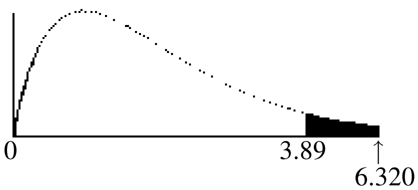
d. f. N = 2 d. f. D = 12

$\overline{X}_{GM} = 62$

$s_B^2 = 1399.4$

$s_W^2 = 221.433$

$F = \frac{1399.4}{221.433} = 6.320$




Reject the null hypothesis. There is enough evidence to conclude that there is a difference in means.

Tukey Test:

C. V. = 3.77

$\overline{X}_1$ vs $\overline{X}_2$:

$$q = \frac{(44.2-77.4)}{\sqrt{\frac{221.43}{5}}} = -5.0$$

$\overline{X}_1$ vs $\overline{X}_3$:

$$q = \frac{(44.2-64.4)}{\sqrt{\frac{221.43}{5}}} = -3.04$$

$\overline{X}_2$ vs $\overline{X}_3$:

$$q = \frac{(77.4-64.4)}{\sqrt{\frac{221.43}{5}}} = 2.00$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_2$.

7.

$H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_1$: At least one mean is different from the others. (claim)

7. continued

C. V. = 3.59 $\alpha = 0.05$

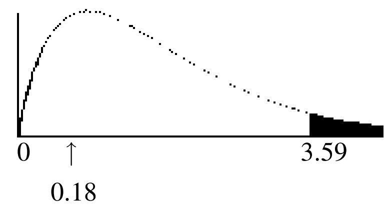
d. f. N = 3 d. f. D = 11

$\overline{X}_{GM} = 12.267$

$s_B^2 = 21.422$

$s_W^2 = 117.697$

$F = \frac{21.422}{117.697} = 0.18$




Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others.

8.

$H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 2.46 $\alpha = 0.05$

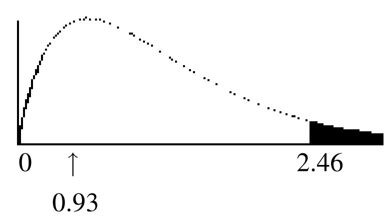
d. f. N =3 d. f. D = 11

$\overline{X}_{GM} = 24.867$

$s_B^2 = 374.867$

$s_W^2 = 19.333$

$F = \frac{374.867}{19.333} = 0.93$




Do not reject the null hypothesis. There is enough evidence to conclude that there is a difference in means.

8. continued

Tukey Test:
C. V. = 5.05

$\overline{X}_1$ vs $\overline{X}_2$:

$$q = \frac{(28.4 - 31.2)}{\sqrt{\frac{19.333}{5}}} = -1.42$$

$\overline{X}_1$ vs $\overline{X}_3$:

$$q = \frac{(28.4 - 15)}{\sqrt{\frac{19.333}{5}}} = 6.82$$

$\overline{X}_2$ vs $\overline{X}_3$:

$$q = \frac{(31.2 - 15)}{\sqrt{\frac{19.333}{5}}} = 8.24$$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_3$ and between $\overline{X}_2$ and $\overline{X}_3$.

9.

$H_0$: There is no interaction effect between type of formula delivery system and review organization.
$H_1$: There is an interaction effect between type of formula delivery system and review organization.

$H_0$: There is no difference in mean scores based on who leads the review.
$H_1$: There is a difference in mean scores based on who leads the review.

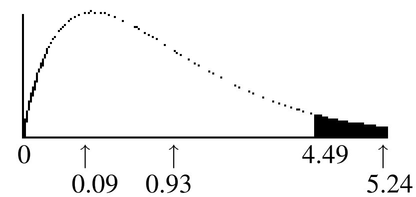
$H_0$: There is no difference in mean scores based on who provides the formulas.
$H_1$: There is a difference in mean scores based on who provides the formulas.

ANOVA SUMMARY TABLE

| Source | SS | d. f. | MS | F | *P*-value |
|---|---|---|---|---|---|
| Leaders | 288.8 | 1 | 288.8 | 5.24 | 0.036 |
| Formulas | 51.2 | 1 | 51.2 | 0.93 | 0.349 |
| Interaction | 5 | 1 | 5 | 0.09 | 0.767 |
| Within | 881.2 | 16 | 55.075 | | |
| Total | 1226.2 | 19 | | | |

9. continued

At $\alpha = 0.05$ the d. f. N = 1 and the d. f. D = 16. The critical value is 4.49.




There is sufficient evidence to conclude a difference in mean scores based on who leads the review.

10.

$H_0$: There is no interaction effect between the type of exercise program and the type of diet on a person's glucose level.
$H_1$: There is an interaction effect between the type of exercise program and the type of diet on a person's glucose level.

$H_0$: There is no difference in the means for the glucose levels of the persons in the two exercise programs.
$H_1$: There is a difference in the means for the glucose levels of the persons in the two exercise programs.

$H_0$: There is no difference in the means for the glucose levels of the persons in the two diet programs.
$H_1$: There is a difference in the means for the glucose levels of the persons in the two diet programs.

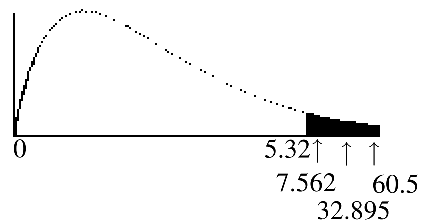
ANOVA SUMMARY TABLE

| Source | SS | d. f. | MS | F |
|---|---|---|---|---|
| Exercise | 816.750 | 1 | 816.750 | 60.50 |
| Diet | 102.083 | 1 | 102.083 | 7.56 |
| Interaction | 444.083 | 1 | 444.083 | 32.90 |
| Within | 108.000 | 8 | 13.500 | |
| Total | 1470.916 | 11 | | |

10. continued

At $\alpha = 0.05$ and d. f. N = 1 and d. f. D = 8 the critical value is 5.32 for each $F_A$, $F_B$, and $F_{AxB}$.

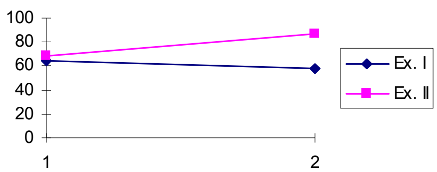
Hence all three null hypotheses are rejected.




The cell means should be calculated.

Diet

| Exercise | A | B |
|---|---|---|
| I | 64.000 | 57.667 |
| II | 68.333 | 86.333 |




Since the means for the Exercise Program I are both smaller than those for Exercise Program II and the vertical differences are not the same, the interaction is ordinal. Hence one can say that there is a difference for exercise, diet; and that an interaction effect is present.

CHAPTER 12 QUIZ

1. False, there could be a significant difference between only some of the means.

2. False, degrees of freedom are used to find the critical value.

3. False, the null hypothesis should not be rejected.

4. True

5. d

6. a

7. a

8. c

9. ANOVA

10. Tukey

11. $H_0$: $\mu_1 = \mu_2 = \mu_3$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 8.02 $\alpha = 0.01$

$s_B^2 = 0.30451$ $s_W^2 = 0.00392$

$F = \frac{0.30451}{0.00392} = 77.68$

Reject the null hypothesis. There is enough evidence to support the claim that at least one mean is different from the others.

Tukey Test:

C. V. = 5.43

$\overline{X}_1 = 3.195$

$\overline{X}_2 = 3.633$

$\overline{X}_3 = 3.705$

$\overline{X}_1$ vs $\overline{X}_2$: $q = -13.99$

$\overline{X}_1$ vs $\overline{X}_3$: $q = -16.29$

$\overline{X}_2$ vs $\overline{X}_3$: $q = -2.30$

There is a significant difference between $\overline{X}_1$ and $\overline{X}_2$ and between $\overline{X}_1$ and $\overline{X}_3$.

12. $H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_1$: At least one mean is different from the others. (claim)

C. V. = 3.49

12. continued

$s_B^2 = 116.646 \qquad s_W^2 = 36.132$

$F = \frac{116.646}{36.132} = 3.23$

Do not reject the null hypothesis. There is not enough evidence to support the claim that the means are different.

13. $H_0$: $\mu_1 = \mu_2 = \mu_3$
$H_1$: At least one mean is different from the others. (claim)

C. V. = 6.93 $\qquad \alpha = 0.01$

$s_B^2 = 119.467 \qquad s_W^2 = 34.167$

$F = \frac{119.467}{34.167} = 3.497$

Do not reject the null hypothesis. There is not enough evidence to support the claim that at least one mean is different from the others. Writers would want to target their material to the age group of the viewers.

14. $H_0$: $\mu_1 = \mu_2 = \mu_3$
$H_1$: At least one mean is different from the others. (claim)

C. V. = 4.26 $\qquad \alpha = 0.05$
$s_B^2 = 9.6658 \qquad s_W^2 = 0.9642$

$F = \frac{9.6658}{0.9642} = 10.025$
Reject the null hypothesis. There is enough evidence to support the claim that at least one mean is different from the others.

Tukey Test:
C. V. = 3.95
$\overline{X}_1 = 7.015$
$\overline{X}_2 = 7.64$
$\overline{X}_3 = 4.69$
$\overline{X}_1$ vs $\overline{X}_2$: $q = -1.28$
$\overline{X}_1$ vs $\overline{X}_3$: $q = 4.74$
$\overline{X}_2$ vs $\overline{X}_3$: $q = 6.02$
There is a significant difference between $\overline{X}_1$ and $\overline{X}_3$ and between $\overline{X}_2$ and $\overline{X}_3$.

15. $H_0$: $\mu_1 = \mu_2 = \mu_3$ (claim)
$H_1$: At least one mean is different from the others.

C. V. = 4.46 $\alpha = 0.05$
$s_B^2 = 2114.985 \qquad s_W^2 = 317.958$

$F = \frac{2114.985}{317.958} = 6.65$
Reject the null hypothesis. There is enough evidence to support the claim that the means are not the same.

Scheffe´ Test:
C. V. = 8.90

For $\overline{X}_1$ vs $\overline{X}_2$, $F_S = 9.32$
For $\overline{X}_1$ vs $\overline{X}_3$, $F_S = 10.13$
For $\overline{X}_2$ vs $\overline{X}_3$, $F_S = 0.13$
There is a significant difference between $\overline{X}_1$ and $\overline{X}_2$ and between $\overline{X}_1$ and $\overline{X}_3$.

16. $H_0$: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
$H_1$: At least one mean is different from the others. (claim)

C. V. = 3.07 $\qquad \alpha = 0.05$
$s_B^2 = 15.3016 \qquad s_W^2 = 33.5283$

$F = \frac{15.3016}{33.5283} = 0.46$

Do not reject. There is not enough evidence to support the claim that at least one mean is different.

17.
a. two-way ANOVA

b. diet and exercise program

c. 2

d. $H_0$: There is no interaction effect between the type of exercise program and the type of diet on a person's weight loss.
$H_1$: There is an interaction effect between the type of exercise program and the type of diet on a person's weight loss.

17. continued

$H_0$: There is no difference in the means of the weight losses for those in the exercise programs.
$H_1$: There is a difference in the means of the weight losses for those in the exercise programs.

$H_0$: There is no difference in the means of the weight losses for those in the diet programs.
$H_1$: There is a difference in the means of the weight losses for those in the diet programs.

e. Diet: $F = 21.0$, significant
Exercise Program: $F = 0.429$, not significant
Interaction: $F = 0.429$, not significant

f. Reject the null hypothesis for the diets.