

Chapter 7 - Confidence Intervals and Sample Size

Note: Answers may vary due to rounding.

EXERCISE SET 7-1

1.

A point estimate of a parameter specifies a specific value such as $\mu = 87$, whereas an interval estimate specifies a range of values for the parameter such as $84 < \mu < 90$.

The advantage of an interval estimate is that a specific confidence level (say 95%) can be selected, and one can be 95% confident that the parameter being estimated lies in the interval.

2.

The standard deviation of the population must be known or it must be estimated or specified in terms of E. Sample size must be specified, and the degree of confidence must be selected.

3.

The margin of error is the likely range of values above or below the statistic that may contain the parameter.

4.

A 95% confidence interval means that one can be 95% confident that the parameter being estimated will be contained within the limits of the interval.

5.

A good estimator should be unbiased, consistent, and relatively efficient.

6.

\bar{X}

7.

- a. 2.58 d. 1.65
- b. 2.33 e. 1.88
- c. 1.96

8.

To determine sample size, the margin of error and the degree of confidence must be specified and the population standard deviation must be known.

9.

For 95% confidence, $z_{\frac{\alpha}{2}} = 1.96$

$\bar{X} = 28.1$ $\sigma = 4.7$

$$28.1 - 1.96\left(\frac{4.7}{\sqrt{40}}\right) < \mu < 28.1 + 1.96\left(\frac{4.7}{\sqrt{40}}\right)$$

$$28.1 - 1.5 < \mu < 28.1 + 1.5$$

$$26.6 < \mu < 29.6$$

10.

For 98% confidence, $z_{\frac{\alpha}{2}} = 2.33$

$\bar{X} = \$18.21$ $\sigma = \$5.92$

$$18.21 - 2.33\left(\frac{5.92}{\sqrt{50}}\right) < \mu < 18.21 + 2.33\left(\frac{5.92}{\sqrt{50}}\right)$$

$$18.21 - 1.95 < \mu < 18.21 + 1.95$$

$$\$16.26 < \mu < \$20.16$$

11.

a. $\bar{X} = 30$ is the point estimate for μ .

$$b. \bar{X} - z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$30 - (1.96)\left(\frac{4.2}{\sqrt{60}}\right) < \mu < 30 + (1.96)\left(\frac{4.2}{\sqrt{60}}\right)$$

$$30 - 1.06 < \mu < 30 + 1.06$$

$$28.94 < \mu < 31.06 \text{ or } 28.9 < \mu < 31.1$$

$$c. 30 - (2.58)\left(\frac{4.2}{\sqrt{60}}\right) < \mu < 30 + (2.58)\left(\frac{4.2}{\sqrt{60}}\right)$$

$$30 - 1.40 < \mu < 30 + 1.40$$

$$28.6 < \mu < 31.4$$

d. The 99% confidence interval is larger because the confidence level is larger.

12.

a. $\bar{X} = 7.2$ is the point estimate for μ .

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12. continued

$$b. 7.2 - 1.96\left(\frac{2.1}{\sqrt{50}}\right) < \mu < 7.2 + 1.96\left(\frac{2.1}{\sqrt{50}}\right)$$

$$7.2 - 0.58 < \mu < 7.2 + 0.58$$

$$6.62 < \mu < 7.78 \text{ or } 6.6 < \mu < 7.8$$

c.

$$7.2 - 2.58\left(\frac{2.1}{\sqrt{50}}\right) < \mu < 7.2 + 2.58\left(\frac{2.1}{\sqrt{50}}\right)$$

$$7.2 - 0.77 < \mu < 7.2 + 0.77$$

$$6.43 < \mu < 7.97 \text{ or } 6.4 < \mu < 8.0$$

d. The 95% confidence is smaller since there is less of a chance that the mean is contained in the 95% interval as opposed to the 99% confidence interval.

13.

For 92% confidence, $z_{\frac{\alpha}{2}} = 1.75$

$$\bar{X} = 346.25 \quad \sigma = 165.1$$

$$346.25 - 1.75\left(\frac{165.1}{\sqrt{32}}\right) < \mu < 346.25 + 1.75\left(\frac{165.1}{\sqrt{32}}\right)$$

$$346.25 - 51.08 < \mu < 346.25 + 51.08$$

$$295.2 < \mu < 397.3$$

14.

$$\bar{X} = 2.82 \quad \sigma = 0.62$$

$$2.82 - 2.58\left(\frac{0.62}{\sqrt{36}}\right) < \mu < 2.82 + 2.58\left(\frac{0.62}{\sqrt{36}}\right)$$

$$2.82 - 0.27 < \mu < 2.82 + 0.27$$

$$2.55 < \mu < 3.09$$

15.

$$\bar{X} = 58.17 \quad \sigma = 6.46$$

$$58.17 - 1.96\left(\frac{6.46}{\sqrt{35}}\right) < \mu < 58.17 + 1.96\left(\frac{6.46}{\sqrt{35}}\right)$$

$$58.17 - 2.14 < \mu < 58.17 + 2.14$$

$$56.1 < \mu < 60.3$$

16.

$$\bar{X} = 43.45 \quad \sigma = 31$$

$$43.45 - 1.65\left(\frac{31}{\sqrt{31}}\right) < \mu < 43.45 + 1.65\left(\frac{31}{\sqrt{31}}\right)$$

$$43.45 - 9.2 < \mu < 43.45 + 9.2$$

$$34.3 < \mu < 52.7$$

$$(TI \text{ answer: } 34.3 < \mu < 52.6)$$

17.

$$\bar{X} = 749 \quad \sigma = 32$$

$$749 - 1.96\left(\frac{32}{\sqrt{36}}\right) < \mu < 749 + 1.96\left(\frac{32}{\sqrt{36}}\right)$$

$$738.5 < \mu < 759.5 \text{ or } 739 < \mu < 760$$

803 gallons per year does not seem

reasonable since it is outside this interval.

18.

$$\$3987 - 1.65\left(\frac{630}{\sqrt{50}}\right) < \mu < \$987 + 1.65\left(\frac{630}{\sqrt{50}}\right)$$

$$\$3840 < \mu < \$4134$$

\$3800 would be a reasonable amount to charge for tuition.

19.

$$\bar{X} - z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$61.2 - 1.96\left(\frac{7.9}{\sqrt{84}}\right) < \mu < 61.2 + 1.96\left(\frac{7.9}{\sqrt{84}}\right)$$

$$61.2 - 1.69 < \mu < 61.2 + 1.69$$

$$59.5 < \mu < 62.9$$

20.

$$190.7 - 1.96\left(\frac{54.2}{\sqrt{35}}\right) < \mu < 190.7 + 1.96\left(\frac{54.2}{\sqrt{35}}\right)$$

$$190.7 - 18.0 < \mu < 190.7 + 18.0$$

$$172.7 < \mu < 208.7$$

21.

$$n = \left[\frac{z_{\frac{\alpha}{2}} \sigma}{E} \right]^2 = \left[\frac{(1.96)(7.5)}{2} \right]^2$$

$$n = (7.35)^2 = 54.0225 \text{ or } 55$$

22.

$$58.0 - 1.65\left(\frac{4.8}{\sqrt{171}}\right) < \mu < 58.0 + 1.65\left(\frac{4.8}{\sqrt{171}}\right)$$

$$58.0 - 0.61 < \mu < 58.0 + 0.61$$

$$57.4 < \mu < 58.6$$

23.

$$n = \left[\frac{z_{\frac{\alpha}{2}} \sigma}{E} \right]^2 = \left[\frac{(1.65)(42)}{10} \right]^2$$

$$n = (6.93)^2 = 48.02 \text{ or } 49 \text{ minutes}$$

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24.

$$n = \left[\frac{z_{\frac{\alpha}{2}} \sigma}{E} \right]^2 = \left[\frac{(1.96)(0.26)}{0.15} \right]^2$$

$$n = (3.397)^2 = 11.5 \text{ or } 12$$

25.

$$n = \left[\frac{z_{\frac{\alpha}{2}} \sigma}{E} \right]^2 = \left[\frac{(2.58)(5.29)}{3} \right]^2$$

$$n = (4.55)^2 = 20.7025 \text{ or } 21 \text{ days}$$

26.

$$n = \left[\frac{z_{\frac{\alpha}{2}} \sigma}{E} \right]^2 = \left[\frac{(2.58)(1.2)}{0.25} \right]^2$$

$$n = (12.384)^2 = 153.4 \text{ or } 154$$

EXERCISE SET 7-2

1.

The characteristics of the t-distribution

are: It is bell-shaped, symmetrical about the mean, and never touches the x-axis.

The mean, median, and mode are equal to 0 and are located at the center of the distribution. The variance is greater than 1. The t-distribution is a family of curves based on degrees of freedom. As sample size increases the t-distribution approaches the standard normal distribution.

2.

The degrees of freedom are the number of values free to vary after a sample statistic has been computed.

3.

a. 2.898 where d. f. = 17

b. 2.074 where d. f. = 22

c. 2.624 where d. f. = 14

d. 1.833 where d. f. = 9

e. 2.093 where d. f. = 19

4.

The t-distribution should be used when σ is unknown.

5.

$$\bar{X} - t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$$

$$\bar{X} = 44.2 \quad s = 2.6$$

$$44.2 - (1.761) \left(\frac{2.6}{\sqrt{15}} \right) < \mu < 44.2 + (1.761) \left(\frac{2.6}{\sqrt{15}} \right)$$

$$44.2 - 1.18 < \mu < 44.2 + 1.18$$

$$43 < \mu < 45$$

6.

$$\bar{X} = 842.6 \quad s = 534.30$$

$$\bar{X} - t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$$

$$842.6 - 2.262 \left(\frac{534.30}{\sqrt{10}} \right) < \mu < 842.6 + 2.262 \left(\frac{534.30}{\sqrt{10}} \right)$$

$$842.6 - 382.19 < \mu < 842.6 + 382.19$$

$$460.41 < \mu < 1224.79$$

Assume the variable is normally distributed.

7.

$$\bar{X} = 33.4 \quad s = 28.7$$

$$\bar{X} - t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$$

$$33.4 - 1.746 \left(\frac{28.7}{\sqrt{17}} \right) < \mu < 33.4 + 1.746 \left(\frac{28.7}{\sqrt{17}} \right)$$

$$33.4 - 12.2 < \mu < 33.4 + 12.2$$

$$21.2 < \mu < 45.6$$

The point estimate is 33.4 and is close to the population mean of 32, which is within the 90% confidence interval. The mean may not be the best estimate since the data value 132 is large and possibly an outlier.

8.

$$\bar{X} = 43.49 \quad s = 9.24$$

$$\bar{X} - t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$$

$$43.49 - 1.796 \left(\frac{9.24}{\sqrt{12}} \right) < \mu < 43.49 + 1.796 \left(\frac{9.24}{\sqrt{12}} \right)$$

$$43.49 - 4.79 < \mu < 43.49 + 4.79$$

$$38.70 < \mu < 48.28$$

Yes, this interval contains the national average of 44.7 cents.

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9.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$\bar{X} = 243.2 \quad s = 23.8$$

$$243.2 - 2.650\left(\frac{23.8}{\sqrt{14}}\right) < \mu < 243.2 + 2.650\left(\frac{23.8}{\sqrt{14}}\right)$$

$$243.2 - 16.9 < \mu < 243.2 + 16.9$$

$$226.3 < \mu < 260.1$$

10.

$$\bar{X} = 29.84 \quad s = 6.18$$

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$29.84 - 2.878\left(\frac{6.18}{\sqrt{19}}\right) < \mu < 29.84 + 2.878\left(\frac{6.18}{\sqrt{19}}\right)$$

$$29.84 - 4.08 < \mu < 29.84 + 4.08$$

$$25.76 < \mu < 33.92 \text{ or } 25.8 < \mu < 33.9$$

11.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$12,300 - 2.365\left(\frac{22}{\sqrt{8}}\right) < \mu < 12,300 + 2.365\left(\frac{22}{\sqrt{8}}\right)$$

$$12,300 - 18 < \mu < 12,300 + 18$$

$$12,282 < \mu < 12,318$$

The population mean for the weights of adult elephants is $12,282 < \mu < 12,318$.

12.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$15 - 3.055\left(\frac{1.7}{\sqrt{13}}\right) < \mu < 15 + 3.055\left(\frac{1.7}{\sqrt{13}}\right)$$

$$15 - 1.4 < \mu < 15 + 1.4$$

$$13.6 < \mu < 16.4$$

The highest speed would be 16.4 mph.

13.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$98 - 2.11\left(\frac{5.6}{\sqrt{18}}\right) < \mu < 98 + 2.11\left(\frac{5.6}{\sqrt{18}}\right)$$

$$98 - 3 < \mu < 98 + 3$$

$$95 < \mu < 101$$

14.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$13.1 - 2.365\left(\frac{4.1}{\sqrt{8}}\right) < \mu < 13.1 + 2.365\left(\frac{4.1}{\sqrt{8}}\right)$$

$$13.1 - 3.4 < \mu < 13.1 + 3.4$$

$$9.7 < \mu < 16.5$$

15.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$109 - 3.106\left(\frac{4}{\sqrt{12}}\right) < \mu < 109 + 3.106\left(\frac{4}{\sqrt{12}}\right)$$

$$109 - 4 < \mu < 109 + 4$$

$$105 < \mu < 113$$

16.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$41.6 - 2.069\left(\frac{7.5}{\sqrt{24}}\right) < \mu < 41.6 + 2.069\left(\frac{7.5}{\sqrt{24}}\right)$$

$$41.6 - 3.17 < \mu < 41.6 + 3.17$$

$$38.4 < \mu < 44.8$$

17.

$$\bar{X} = 51.5 \quad s = 45.98$$

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$51.5 - 1.746\left(\frac{45.98}{\sqrt{17}}\right) < \mu < 51.5 + 1.746\left(\frac{45.98}{\sqrt{17}}\right)$$

$$51.5 - 19.5 < \mu < 51.5 + 19.5$$

$$32.0 < \mu < 71.0$$

Assume a normal distribution.

18.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$\bar{X} = 224.357 \quad s = 1.432$$

$$224.357 - 2.365\left(\frac{1.432}{\sqrt{8}}\right) < \mu < 224.357 + 2.365\left(\frac{1.432}{\sqrt{8}}\right)$$

$$224.357 - 1.197 < \mu < 224.357 + 1.197$$

$$223.160 < \mu < 225.554$$

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19.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$\bar{X} = 40.494 \quad s = 31.9$$

$$40.494 - 2.145\left(\frac{31.9}{\sqrt{15}}\right) < \mu < 40.494 + 2.145\left(\frac{31.9}{\sqrt{15}}\right)$$

$$40.494 - 17.667 < \mu < 40.494 + 17.667$$

$$22.827 < \mu < 58.161$$

20.

$$\bar{X} = 33.5 \quad s = 27.678$$

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$33.5 - 2.821\left(\frac{27.678}{\sqrt{10}}\right) < \mu < 33.5 + 2.821\left(\frac{27.678}{\sqrt{10}}\right)$$

$$33.5 - 24.691 < \mu < 33.5 + 24.691$$

$$8.8 < \mu < 58.2$$

21.

$$\bar{X} = 2.175 \quad s = 0.585$$

$$\text{For } \mu > \bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right):$$

$$\mu > 2.175 - 1.729\left(\frac{0.585}{\sqrt{20}}\right)$$

$$\mu > 2.175 - 0.226$$

Thus, $\mu > \$1.95$ means that one can be 95% confident that the mean revenue is greater than \$1.95.

$$\text{For } \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right):$$

$$\mu < 2.175 + 1.729\left(\frac{0.585}{\sqrt{20}}\right)$$

$$\mu < 2.175 + 0.226$$

Thus, $\mu < \$2.40$ means that one can be 95% confident that the mean revenue is less than \$2.40.

EXERCISE SET 7-3

1.

$$\text{a. } \hat{p} = \frac{40}{80} = 0.5 \quad \hat{q} = \frac{40}{80} = 0.5$$

$$\text{b. } \hat{p} = \frac{90}{200} = 0.45 \quad \hat{q} = \frac{110}{200} = 0.55$$

$$\text{c. } \hat{p} = \frac{60}{130} = 0.46 \quad \hat{q} = \frac{70}{130} = 0.54$$

1. continued

$$\text{d. } \hat{p} = 0.25 \quad \hat{q} = 0.75$$

$$\text{e. } \hat{p} = 0.42 \quad \hat{q} = 0.58$$

2.

$$\text{a. } \hat{p} = \frac{35}{60} = 0.58 \quad \hat{q} = \frac{25}{60} = 0.42$$

$$\text{b. } \hat{p} = \frac{43}{95} = 0.45 \quad \hat{q} = \frac{52}{95} = 0.55$$

$$\text{c. } \hat{p} = 0.68 \quad \hat{q} = 1 - 0.68 = 0.32$$

$$\text{d. } \hat{p} = 0.55 \quad \hat{q} = 1 - 0.55 = 0.45$$

$$\text{e. } \hat{p} = 0.12 \quad \hat{q} = 1 - 0.12 = 0.88$$

3.

$$\hat{p} = 0.33 \quad \hat{q} = 0.67$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.33 - (1.96)\sqrt{\frac{(0.33)(0.67)}{1000}} < p <$$

$$0.33 + (1.96)\sqrt{\frac{(0.33)(0.67)}{1000}}$$

$$0.33 - 0.029 < p < 0.33 + 0.029$$

$$0.301 < p < 0.359$$

4.

$$\hat{p} = \frac{26}{122} = 0.213 \quad \hat{q} = \frac{96}{122} = 0.787$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.213 - 1.65\sqrt{\frac{(0.213)(0.787)}{122}} < p <$$

$$0.213 + 1.65\sqrt{\frac{(0.213)(0.787)}{122}}$$

$$0.213 - 0.061 < p < 0.213 + 0.061$$

$$0.152 < p < 0.274$$

5.

$$\hat{p} = 0.68$$

$$\hat{q} = 1 - 0.68 = 0.32$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.68 - 1.65\sqrt{\frac{(0.68)(0.32)}{100}} < p < 0.68 + 1.65\sqrt{\frac{(0.68)(0.32)}{100}}$$

$$0.68 - 0.077 < p < 0.68 + 0.077$$

$$0.603 \text{ or } 60.3\% < p < 0.757 \text{ or } 75.7\%$$

$$(\text{TI answer: } 0.603 < p < 0.757)$$

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6.

$$\hat{p} = 0.317 \quad \hat{q} = 0.683$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.317 - 2.58\sqrt{\frac{(0.317)(0.683)}{205}} < p < 0.317 + 2.58\sqrt{\frac{(0.317)(0.683)}{205}}$$

$$0.317 - 0.084 < p < 0.317 + 0.084$$

$$0.233 < p < 0.401$$

7.

$$\hat{p} = 0.84 \quad \hat{q} = 0.16$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.84 - 1.65\sqrt{\frac{(0.84)(0.16)}{200}} < p < 0.84 + 1.65\sqrt{\frac{(0.84)(0.16)}{200}}$$

$$0.84 - 0.043 < p < 0.84 + 0.043$$

$$0.797 < p < 0.883$$

8.

$$\hat{p} = \frac{x}{n} = \frac{329}{763} = 0.431$$

$$\hat{q} = 1 - 0.431 = 0.569$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.431 - 1.75\sqrt{\frac{(0.431)(0.569)}{763}} < p < 0.431 + 1.75\sqrt{\frac{(0.431)(0.569)}{763}}$$

$$0.431 - 0.031 < p < 0.431 + 0.031$$

$$0.400 < p < 0.462$$

$$(TI \text{ answer: } 0.3998 < p < 0.4626)$$

9.

$$\hat{p} = 0.65 \quad \hat{q} = 0.36$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.65 - 1.96\sqrt{\frac{(0.65)(0.35)}{300}} < p < 0.65 + 1.96\sqrt{\frac{(0.65)(0.35)}{300}}$$

$$0.65 - 0.054 < p < 0.65 + 0.054$$

$$0.596 < p < 0.704$$

10.

$$\hat{p} = \frac{x}{n} = \frac{154}{200} = 0.77$$

$$\hat{q} = 1 - 0.77 = 0.23$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.77 - 1.65\sqrt{\frac{(0.77)(0.23)}{200}} < p < 0.77 + 1.65\sqrt{\frac{(0.77)(0.23)}{200}}$$

$$0.77 - 0.049 < p < 0.77 + 0.049$$

$$0.721 < p < 0.819 \text{ or } 72.1\% < p < 81.9\%$$

Answers will vary.

11.

$$\hat{p} = 0.68 \quad \hat{q} = 0.32$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.68 - 2.58\sqrt{\frac{(0.68)(0.32)}{50}} < p < 0.68 + 2.58\sqrt{\frac{(0.68)(0.32)}{50}}$$

$$0.68 - 0.170 < p < 0.68 + 0.170$$

$$0.510 < p < 0.850$$

12.

$$\hat{p} = \frac{x}{n} = 0.238$$

$$\hat{q} = 1 - 0.238 = 0.762$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.238 - 2.33\sqrt{\frac{(0.238)(0.762)}{400}} < p < 0.238 + 2.33\sqrt{\frac{(0.238)(0.762)}{400}}$$

$$0.238 - 0.05 < p < 0.238 + 0.05$$

$$0.188 < p < 0.288$$

Yes, the interval contains 20.4.

13.

$$\hat{p} = 0.86 \quad \hat{q} = 0.14$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.86 - 2.58\sqrt{\frac{(0.86)(0.14)}{349}} < p < 0.86 + 2.58\sqrt{\frac{(0.86)(0.14)}{349}}$$

$$0.86 - 0.048 < p < 0.86 + 0.048$$

$$0.812 < p < 0.908$$

Chapter 7 - Confidence Intervals and Sample Size

14.

$$\begin{aligned}\hat{p} &= \frac{157}{180} = 0.872 \quad \hat{q} = \frac{23}{180} = 0.128 \\ \hat{p} - (z_{\frac{\alpha}{2}}) \sqrt{\frac{\hat{p}\hat{q}}{n}} &< p < \hat{p} + (z_{\frac{\alpha}{2}}) \sqrt{\frac{\hat{p}\hat{q}}{n}} \\ 0.872 - 1.65 \sqrt{\frac{(0.872)(0.128)}{180}} &< p < \\ 0.872 + 1.65 \sqrt{\frac{(0.872)(0.128)}{180}} & \\ 0.872 - 0.041 < p < 0.872 + 0.041 & \\ 0.831 < p < 0.913 &\end{aligned}$$

15.

$$\begin{aligned}\hat{p} &= \frac{40}{200} = 0.2 \quad \hat{q} = 0.8 \\ n &= \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2 = (0.2)(0.8) \left[\frac{1.96}{0.04} \right]^2 \\ n &= 384.16 \text{ or } 385\end{aligned}$$

If no estimate of the sample proportion is available, use $\hat{p} = 0.5$:

$$\begin{aligned}\hat{p} &= 0.5 \quad \hat{q} = 0.5 \\ n &= \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2 = (0.5)(0.5) \left[\frac{1.96}{0.04} \right]^2 \\ n &= 600.25 \text{ or } 601\end{aligned}$$

16.

$$\begin{aligned}\text{a. } \hat{p} &= 0.29 \quad \hat{q} = 0.71 \\ n &= \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2 = (0.29)(0.71) \left[\frac{1.65}{0.05} \right]^2 \\ n &= 224.2251 \text{ or } 225\end{aligned}$$

$$\begin{aligned}\text{b. } \hat{p} &= 0.5 \quad \hat{q} = 0.5 \\ n &= \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2 = (0.5)(0.5) \left[\frac{1.65}{0.05} \right]^2 \\ n &= 272.25 \text{ or } 273\end{aligned}$$

17.

$$\begin{aligned}\text{a. } \hat{p} &= 0.25 \quad \hat{q} = 0.75 \\ n &= \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2 = (0.25)(0.75) \left[\frac{1.96}{0.03} \right]^2 \\ n &= 800.33 \text{ or } 801\end{aligned}$$

$$\begin{aligned}\text{b. } \hat{p} &= 0.5 \quad \hat{q} = 0.5 \\ n &= \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2 = (0.5)(0.5) \left[\frac{1.96}{0.03} \right]^2 \\ n &= 1067.11 \text{ or } 1068\end{aligned}$$

18.

$$\begin{aligned}\text{a. } \hat{p} &= 0.5 \quad \hat{q} = 0.5 \\ n &= \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2 = (0.5)(0.5) \left[\frac{1.96}{0.0055} \right]^2 \\ n &= 31,749\end{aligned}$$

19.

$$\begin{aligned}\hat{p} &= 0.352 \quad \hat{q} = 0.648 \\ n &= \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2 \\ n &= (0.352)(0.648) \left[\frac{1.65}{0.025} \right]^2 \\ n &= 994\end{aligned}$$

20.

$$\begin{aligned}n &= \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2 \\ n &= (0.27)(0.73) \left[\frac{1.96}{0.02} \right]^2 \\ n &= 1892.9 \text{ or } 1893\end{aligned}$$

21.

$$\begin{aligned}600 &= (0.5)(0.5) \left[\frac{z}{0.04} \right]^2 \\ 600 &= 156.25 z^2 \\ 3.84 &= z^2 \\ \sqrt{3.84} &= 1.96 = z \\ 1.96 &\text{ corresponds to a 95\% degree of confidence.}\end{aligned}$$

22.

$$\begin{aligned}1015 &= (0.68)(0.32) \left[\frac{z}{0.03} \right]^2 \\ 1015 &= 241.78 z^2 \\ 4.198 &= z^2 \\ \sqrt{4.198} &= 2.05 = z \\ \text{For } z &= 2.05, \text{ the area under the normal curve is } 0.9798. \text{ Since} \\ 1 - 0.9798 &= 0.0202, \frac{\alpha}{2} = 0.0202. \text{ Then} \\ \alpha &= 0.0404 \text{ or } 4\%, \text{ and } 100\% - 4\% = 96\%. \\ \text{Thus, } 2.05 &\text{ corresponds to a 96\% degree of confidence.}\end{aligned}$$

Chapter 7 - Confidence Intervals and Sample Size

EXERCISE SET 7-4

1.

Chi-square (χ^2)

2.

The variable must be normally distributed.

3.

	χ_{left}^2	χ_{right}^2
a.	3.816	21.920
b.	10.117	30.144
c.	13.844	41.923
d.	0.412	16.750
e.	26.509	55.758

4.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{23(4.8)^2}{35.172} < \sigma^2 < \frac{23(4.8)^2}{13.091}$$

$$15.1 < \sigma^2 < 40.5$$

$$3.9 < \sigma < 6.4$$

Yes. The lifetimes seem to be somewhat consistent; deviation is between 3.9 and 6.4 months.

5.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{16(10.1)^2}{28.845} < \sigma^2 < \frac{16(10.1)^2}{6.908}$$

$$56.6 < \sigma^2 < 236.3$$

$$7.5 < \sigma < 15.4$$

6.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{5(4.1)^2}{16.750} < \sigma^2 < \frac{5(4.1)^2}{0.412}$$

$$5.0 < \sigma^2 < 204.0$$

$$2.2 < \sigma < 14.3$$

7.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{19(1.6)^2}{32.852} < \sigma^2 < \frac{19(1.6)^2}{8.907}$$

$$1.48 < \sigma^2 < 5.46$$

$$1.22 < \sigma < 2.34$$

Yes, the estimate is reasonable.

8.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{23(2.3)^2}{35.172} < \sigma^2 < \frac{23(2.3)^2}{13.091}$$

$$3.5 < \sigma^2 < 9.3$$

$$1.9 < \sigma < 3$$

9.

$$s = 120.82$$

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{14(120.82)^2}{23.685} < \sigma^2 < \frac{14(120.82)^2}{6.571}$$

$$8,628.44 < \sigma^2 < 31,100.99$$

$$\$92.89 < \sigma < \$176.35$$

10.

$$s^2 = 411.46$$

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{19(411.46)}{30.144} < \sigma^2 < \frac{19(411.46)}{10.117}$$

$$259.3 < \sigma^2 < 772.7$$

$$16.1 < \sigma < 27.8$$

11.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{10(53)^2}{25.188} < \sigma^2 < \frac{10(53)^2}{2.156}$$

$$1115.21 < \sigma^2 < 13,028.76$$

$$\$33.39 < \sigma < \$114.14$$

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12.

$$s = 4.448$$

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}}$$

$$\frac{7(4.448)^2}{20.278} < \sigma^2 < \frac{7(4.448)^2}{0.989}$$

$$6.83 < \sigma^2 < 140.03$$

$$2.61 < \sigma < 11.83$$

13.

$$s = 23.827$$

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}}$$

$$\frac{13(23.827)^2}{24.736} < \sigma^2 < \frac{13(23.827)^2}{5.009}$$

$$298.368 < \sigma^2 < 1473.435$$

$$17.3 < \sigma < 38.4$$

14.

$$s = 124.875$$

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}}$$

$$\frac{11(124.875)^2}{26.757} < \sigma^2 < \frac{11(124.875)^2}{2.603}$$

$$6410.7 < \sigma^2 < 65,897.6$$

15.

$$s = 43.072$$

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}}$$

$$\frac{14(43.072)^2}{26.119} < \sigma^2 < \frac{14(43.072)^2}{5.629}$$

$$994.401 < \sigma^2 < 4614.099$$

$$31.5 < \sigma < 67.9$$

16.

$$s - z_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{2n}} \right) < \sigma < s + z_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{2n}} \right)$$

$$18 - 1.96 \left(\frac{18}{\sqrt{400}} \right) < \sigma < 18 + 1.96 \left(\frac{18}{\sqrt{400}} \right)$$

$$16.2 < \sigma < 19.8$$

REVIEW EXERCISES - CHAPTER 7

1.

$\bar{X} = 25$ is the point estimate of μ .

$$\bar{X} - z_{\frac{\alpha}{2}} \left(\frac{\sigma}{\sqrt{n}} \right) < \mu < \bar{X} + z_{\frac{\alpha}{2}} \left(\frac{\sigma}{\sqrt{n}} \right)$$

$$25 - 1.65 \left(\frac{4}{\sqrt{49}} \right) < \mu < 25 + 1.65 \left(\frac{4}{\sqrt{49}} \right)$$

$$25 - 0.9429 < \mu < 25 + 0.9429$$

$$24.06 < \mu < 25.94 \text{ or } 24 < \mu < 26$$

(TI answer: $24.06 < \mu < 25.94$)

2.

$\bar{X} = 7.5$ is the point estimate of μ .

$$\bar{X} - z_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + z_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$$

$$7.5 - 1.96 \left(\frac{0.8}{\sqrt{1500}} \right) < \mu < 7.5 + 1.96 \left(\frac{0.8}{\sqrt{1500}} \right)$$

$$7.46 < \mu < 7.54$$

3.

$$n = \left[\frac{z_{\frac{\alpha}{2}} \sigma}{E} \right]^2 = \left[\frac{1.65(4.8)}{2} \right]^2$$

$$n = (3.96)^2 = 15.68 \text{ or } 16 \text{ female students}$$

4.

$\bar{X} = \$23.45$ is the point estimate of μ .

$$\bar{X} - t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$$

$$\$23.45 - 1.65 \left(\frac{\$2.80}{\sqrt{49}} \right) < \mu < \$23.45 + 1.65 \left(\frac{\$2.80}{\sqrt{49}} \right)$$

$$\$23.45 - \$0.66 < \mu < \$23.45 + \$0.66$$

$$\$22.79 < \mu < \$24.11$$

5.

$\bar{X} = 82.64$ is the point estimate of μ .

$$\bar{X} - t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$$

$$82.64 - 2.228 \left(\frac{8.49}{\sqrt{11}} \right) < \mu < 82.64 + 2.228 \left(\frac{8.49}{\sqrt{11}} \right)$$

$$82.64 - 5.7 < \mu < 82.64 + 5.7$$

$$76.9 < \mu < 88.3$$

Chapter 7 - Confidence Intervals and Sample Size

6.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$28 - 2.132\left(\frac{3}{\sqrt{5}}\right) < \mu < 28 + 2.132\left(\frac{3}{\sqrt{5}}\right)$$

$$25 < \mu < 31$$

7.

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.34 - 1.96\sqrt{\frac{(0.34)(0.66)}{1000}} < P <$$

$$0.34 + 1.96\sqrt{\frac{(0.34)(0.66)}{1000}}$$

$$0.34 - 0.029 < p < 0.34 + 0.029$$

$$0.311 < p < 0.369$$

8.

$$\hat{p} = 0.42 \quad \hat{q} = 0.58$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.42 - 1.96\sqrt{\frac{(0.42)(0.58)}{1500}} < p <$$

$$0.42 + 1.96\sqrt{\frac{(0.42)(0.58)}{1500}}$$

$$0.42 - 1.96(0.013) < p < 0.42 + 1.96(0.013)$$

$$0.395 < p < 0.445$$

9.

$$\hat{p} = \frac{316}{600} = 0.5267 \quad \hat{q} = \frac{284}{600} = 0.4733$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.5267 - 2.58\sqrt{\frac{(0.5267)(0.4733)}{600}} < p <$$

$$0.5267 + 2.58\sqrt{\frac{(0.5267)(0.4763)}{600}}$$

$$0.5267 - 0.053 < p < 0.5267 + 0.053$$

$$0.474 < p < 0.580$$

10.

$$\hat{p} = 0.4725 \quad \hat{q} = 0.5275$$

$$\hat{p} - (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}})\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.4725 - 2.33\sqrt{\frac{(0.4725)(0.5275)}{400}} < p <$$

$$0.4725 + 2.33\sqrt{\frac{(0.4725)(0.5275)}{400}}$$

$$0.4725 - 0.0582 < p < 0.4725 + 0.0582$$

$$0.414 < p < 0.531$$

11.

$$\hat{p} = 0.84 \quad \hat{q} = 0.16$$

$$n = \hat{p}\hat{q}\left[\frac{z_{\frac{\alpha}{2}}}{E}\right]^2 = (0.84)(0.16)\left[\frac{1.65}{0.03}\right]^2$$

$$n = 406.56 \text{ or } 407$$

12.

$$\hat{p} = 0.73 \quad \hat{q} = 0.27$$

$$n = \hat{p}\hat{q}\left[\frac{z_{\frac{\alpha}{2}}}{E}\right]^2 = (0.73)(0.27)\left[\frac{1.96}{0.03}\right]^2$$

$$n = 841.3 \text{ or } 842$$

With no prior knowledge, use $\hat{p} = 0.5$ and $\hat{q} = 0.5$. Then,

$$n = \hat{p}\hat{q}\left[\frac{z_{\frac{\alpha}{2}}}{E}\right]^2 = (0.5)(0.5)\left[\frac{1.96}{0.03}\right]^2$$

$$n = 1067.1 \text{ or } 1068$$

13.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{(18-1)(0.29)^2}{30.191} < \sigma^2 < \frac{(18-1)(0.29)^2}{7.564}$$

$$0.0474 < \sigma^2 < 0.1890$$

$$0.218 < \sigma < 0.435 \text{ or } 0.22 < \sigma < 0.44$$

Yes; it seems that there is a large standard deviation.

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14.

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}}$$

$$\frac{(22-1)(2.6)}{35.479} < \sigma^2 < \frac{(22-1)(2.6)}{10.283}$$

$$1.5 < \sigma^2 < 5.3$$

15.

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}}$$

$$\frac{(15-1)(8.6)}{23.685} < \sigma^2 < \frac{(15-1)(8.6)}{6.571}$$

$$5.1 < \sigma^2 < 18.3$$

16.

$$\frac{(n-1)s^2}{\chi^2_{\text{right}}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{\text{left}}}$$

$$\frac{(11-1)(8.4885)^2}{25.188} < \sigma^2 < \frac{(11-1)(8.4885)^2}{2.156}$$

$$28.61 < \sigma^2 < 334.2$$

$$5.3 < \sigma < 18.3$$

CHAPTER 7 QUIZ

1. True

2. True

3. False, it is consistent if, as sample size increases, the estimator approaches the parameter being estimated.

4. True

5. b

6. a

7. b

8. Unbiased, consistent, relatively efficient

9. Margin of error

10. Point

11. 90, 95, 99

12.

$\bar{X} = \$121.60$ is the point estimate for μ .

$$\bar{X} - z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$\$121.60 - 1.65\left(\frac{6.36}{\sqrt{36}}\right) < \mu < \$121.60 + 1.65\left(\frac{6.36}{\sqrt{36}}\right)$$

$$\$119.85 < \mu < \$123.35$$

13.

$\bar{X} = \$44.80$ is the point estimate for μ .

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$\$44.80 - 2.093\left(\frac{3.53}{\sqrt{20}}\right) < \mu < \$44.80 + 2.093\left(\frac{3.53}{\sqrt{20}}\right)$$

$$\$43.15 < \mu < \$46.45$$

14.

$\bar{X} = 4150$ is the point estimate for μ .

$$\bar{X} - z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + z_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$\$4150 - 2.58\left(\frac{480}{\sqrt{40}}\right) < \mu < \$4150 + 2.58\left(\frac{480}{\sqrt{40}}\right)$$

$$\$3954 < \mu < \$4346$$

15.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$48.6 - 2.262\left(\frac{4.1}{\sqrt{10}}\right) < \mu < 48.6 + 2.262\left(\frac{4.1}{\sqrt{10}}\right)$$

$$45.7 < \mu < 51.5$$

16.

$$\bar{X} - t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{X} + t_{\frac{\alpha}{2}}\left(\frac{s}{\sqrt{n}}\right)$$

$$438 - 3.499\left(\frac{16}{\sqrt{8}}\right) < \mu < 438 + 3.499\left(\frac{16}{\sqrt{8}}\right)$$

$$418 < \mu < 458$$

Chapter 7 - Confidence Intervals and Sample Size

17.

$$\bar{X} - t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right) < \mu < \bar{X} + t_{\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}} \right)$$

$$31 - 2.353 \left(\frac{4}{\sqrt{4}} \right) < \mu < 31 + 2.353 \left(\frac{4}{\sqrt{4}} \right)$$

$$26 < \mu < 36$$

18.

$$n = \left[\frac{z_{\frac{\alpha}{2}} \sigma}{E} \right]^2 = \left[\frac{2.58(2.6)}{0.5} \right]^2$$

$$n = 179.98 \text{ or } 180$$

19.

$$n = \left[\frac{z_{\frac{\alpha}{2}} \sigma}{E} \right]^2 = \left[\frac{1.65(900)}{300} \right]^2$$

$$n = 24.5 \text{ or } 25$$

20.

$$\hat{p} - (z_{\frac{\alpha}{2}}) \sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}}) \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$\hat{p} = 0.43 \quad \hat{q} = 0.57$$

$$0.43 - 1.96 \sqrt{\frac{(0.43)(0.57)}{300}} < p <$$

$$0.43 + 1.96 \sqrt{\frac{(0.43)(0.57)}{300}}$$

$$0.374 < p < 0.486$$

21.

$$p\hat{p} - (z_{\frac{\alpha}{2}}) \sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}}) \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.36 - 1.65 \sqrt{\frac{(0.36)(0.64)}{150}} < p <$$

$$0.36 + 1.65 \sqrt{\frac{(0.36)(0.64)}{150}}$$

$$0.295 < p < 0.425$$

22.

$$\hat{p} - (z_{\frac{\alpha}{2}}) \sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + (z_{\frac{\alpha}{2}}) \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.4444 - 1.96 \sqrt{\frac{(0.4444)(0.5556)}{90}} < p <$$

$$0.4444 + 1.96 \sqrt{\frac{(0.4444)(0.5556)}{90}}$$

22. continued

$$0.342 < p < 0.547$$

23.

$$n = \hat{p} \hat{q} \left[\frac{z_{\frac{\alpha}{2}}}{E} \right]^2$$

$$n = (0.15)(0.85) \left[\frac{1.96}{0.03} \right]^2$$

$$n = 544.22 \text{ or } 545$$

24.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{24(9)^2}{39.364} < \sigma^2 < \frac{24(9)^2}{12.401}$$

$$49.4 < \sigma^2 < 156.8$$

$$7 < \sigma < 13$$

25.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{26(6.8)^2}{38.885} < \sigma^2 < \frac{26(6.8)^2}{15.379}$$

$$30.9 < \sigma^2 < 78.2$$

$$5.6 < \sigma < 8.8$$

26.

$$\frac{(n-1)s^2}{\chi_{\text{right}}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{\text{left}}^2}$$

$$\frac{19(2.3)^2}{30.144} < \sigma^2 < \frac{19(2.3)^2}{10.177}$$

$$3.33 < \sigma^2 < 10$$

$$1.8 < \sigma < 3.2$$