Chapter 14: Integer, Nonlinear, and Advanced Optimization Methods

Statistics, Data Analysis, and Decision Modeling, Fifth Edition

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Integer Optimization Models

- Integer linear optimization model (integer program): some or all decision variables are restricted to integer values
  - If only a subset of variables are integer, we have a mixed integer model

Nonlinear Optimization Models

- The objective function and/or constraint functions are nonlinear functions of the decision variables
  - Terms cannot be written as a constant times a variable

Example: Cutting Stock Problem

- Suppose that a company makes standard 110-inch-wide rolls of thin sheet metal, and slits them into smaller rolls to meet customer orders for widths of 12, 15, and 30 inches. The demands for these widths vary from week to week. Demands this week are 500 12" rolls, 715 15" rolls, and 630 30" rolls.
- Cutting patterns:

	Size of End Item			
Pattern	**12"**	**15"**	**30"**	**Scrap**
1	0	7	0	5"
2	0	1	3	5"
3	1	0	3	8"
4	9	0	0	2"
5	2	1	2	11"
6	7	1	0	11"

IP Model

- Define Xi to be the number of 110” rolls to cut using cutting pattern i, for i = 1,…,6.

Min $5X1 + 5X2 + 8X3 + 2X4 + 11X5 + 11X6$

$0X1 + 0X2 + 1X3 + 9X4 + 2X5 + 7X6 \geq 500$ (12” rolls)

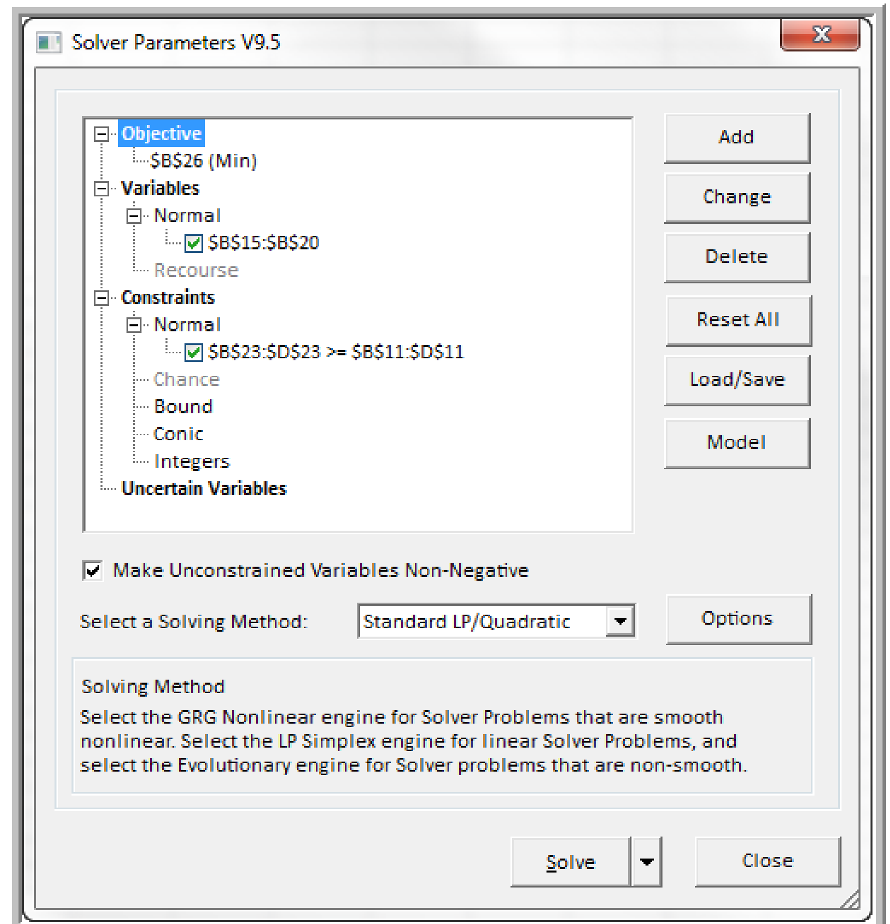
$7X1 + 1X2 + 0X3 + 0X4 + 1X5 + 1X6 \geq 715$ (15” rolls)

$0X1 + 3X2 + 3X3 + 0X4 + 2X5 + 0X6 \geq 630$ (30” rolls)

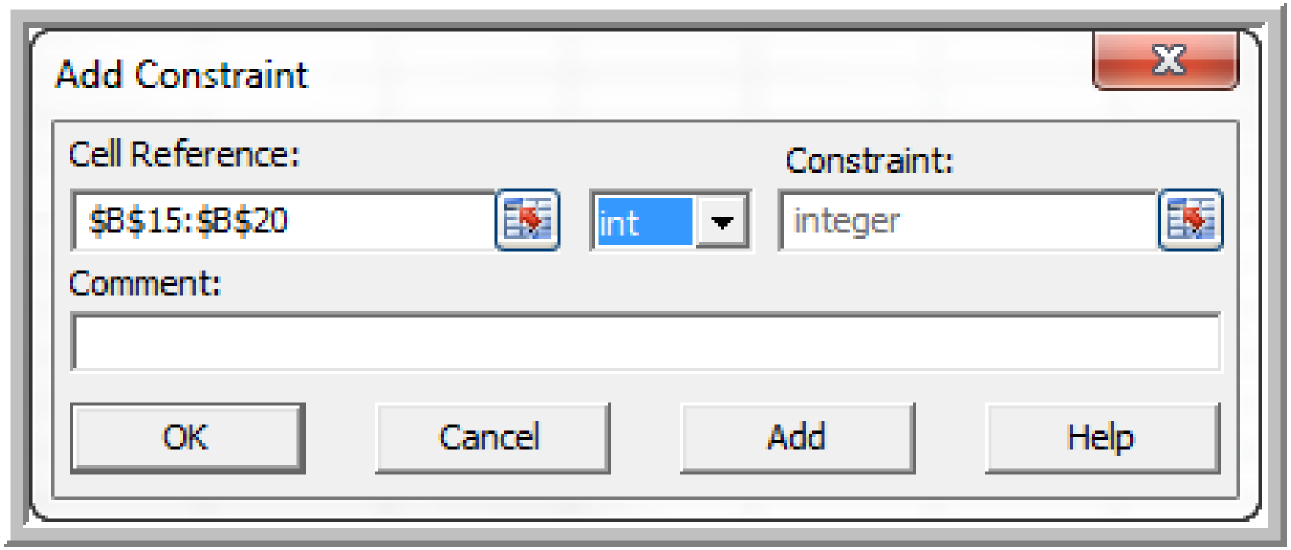
$Xi \geq 0$ and integer

Spreadsheet and *Solver* Models

	A	B	C	D	E
1	**Cutting Stock Problem**				
2					
3	**Data**				
4	Pattern	12-in rolls	15-in rolls	30-in rolls	Scrap
5	1	0	7	0	5
6	2	0	1	3	5
7	3	1	0	3	8
8	4	9	0	0	2
9	5	2	1	2	11
10	6	7	1	0	11
11	Demand	500	715	630	
12					
13	**Model**				
14		No. of rolls			
15	Pattern 1	72.14			
16	Pattern 2	210.00			
17	Pattern 3	0.00			
18	Pattern 4	55.56			
19	Pattern 5	0.00			
20	Pattern 6	0.00			
21					
22		12-in rolls	15-in rolls	30-in rolls	
23	Number produced	500	715	630	
24					
25		Total			
26	Scrap	1521.8254			
27					



Adding Integer Restrictions in *Solver*



Optimal Integer Solution

	A	B	C	D	E
1	**Cutting Stock Problem**				
2					
3	**Data**				
4	Pattern	12-in rolls	15-in rolls	30-in rolls	Scrap
5	1	0	7	0	5
6	2	0	1	3	5
7	3	1	0	3	8
8	4	9	0	0	2
9	5	2	1	2	11
10	6	7	1	0	11
11	Demand	500	715	630	
12					
13	**Model**				
14		No. of rolls			
15	Pattern 1	73.00			
16	Pattern 2	210.00			
17	Pattern 3	0.00			
18	Pattern 4	56.00			
19	Pattern 5	0.00			
20	Pattern 6	0.00			
21					
22		12-in rolls	15-in rolls	30-in rolls	
23	Number produced	504	721	630	
24					
25		Total			
26	Scrap	1527			

IP Models With Binary Variables

- A binary variable x is simply a general integer variable that is restricted to being between 0 and 1:

$$0 \leq x \leq 1 \text{ and integer}$$

- We usually just write this as

$$x = 0 \text{ or } 1$$

Example: Project Selection

TABLE 14.1 Project Selection Data

	Project 1	Project 2	Project 3	Project 4	Project 5	Available Resources
Expected return (NPV)	\$180,000	\$220,000	\$150,000	\$140,000	\$200,000	
Cash requirements	\$55,000	\$83,000	\$24,000	\$49,000	\$61,000	\$150,000
Personnel requirements	5	3	2	5	3	12

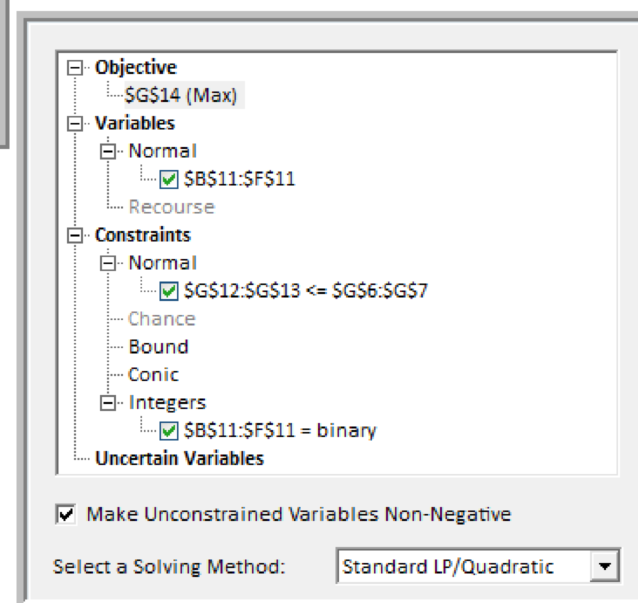
Maximize $\$180{,}000x_1 + \$220{,}000x_2 + \$150{,}000x_3 + \$140{,}000x_4 + \$200{,}000x_5$

$\$55{,}000x_1 + \$83{,}000x_2 + \$24{,}000x_3 + \$49{,}000x_4 + \$61{,}000x_5 \leq \$150{,}000$ (cash limitation)

$5x_1 + 3x_2 + 2x_3 + 5x_4 + 3x_5 \leq 12$ (personnel limitation)

Spreadsheet and Solver Models

	A	B	C	D	E	F	G
1	**Project Selection Model**						
2							
3	**Data**						
4		**Project 1**	**Project 2**	**Project 3**	**Project 4**	**Project 5**	**Available**
5	**Expected Return (NPV)**	$ 180,000	$ 220,000	$ 150,000	$ 140,000	$ 200,000	**Resources**
6	Cash requirements	$ 55,000	$ 83,000	$ 24,000	$ 49,000	$ 61,000	$ 150,000
7	Personnel requirements	5	3	2	5	3	12
8							
9	**Model**						
10							
11	**Project selection decisions**	1	0	1	0	1	**Total**
12	Cash Used	$ 55,000	$ -	$ 24,000	$ -	$ 61,000	$ 140,000
13	Personnel Used	5	0	2	0	3	10
14	Return	$ 180,000	$ -	$ 150,000	$ -	$ 200,000	$ 530,000



Site Location Model

- Response times from fire stations to districts

From/To	1	2	3	4	5	6	7
1	0	2	10	6	12	5	8
2	2	0	6	9	11	7	10
3	10	6	0	5	5	12	6
4	6	9	5	0	9	4	3
5	12	11	5	9	0	10	8
6	5	7	12	4	10	0	6
7	8	10	6	3	8	6	0

- Find best location to reach all districts within 8 minutes

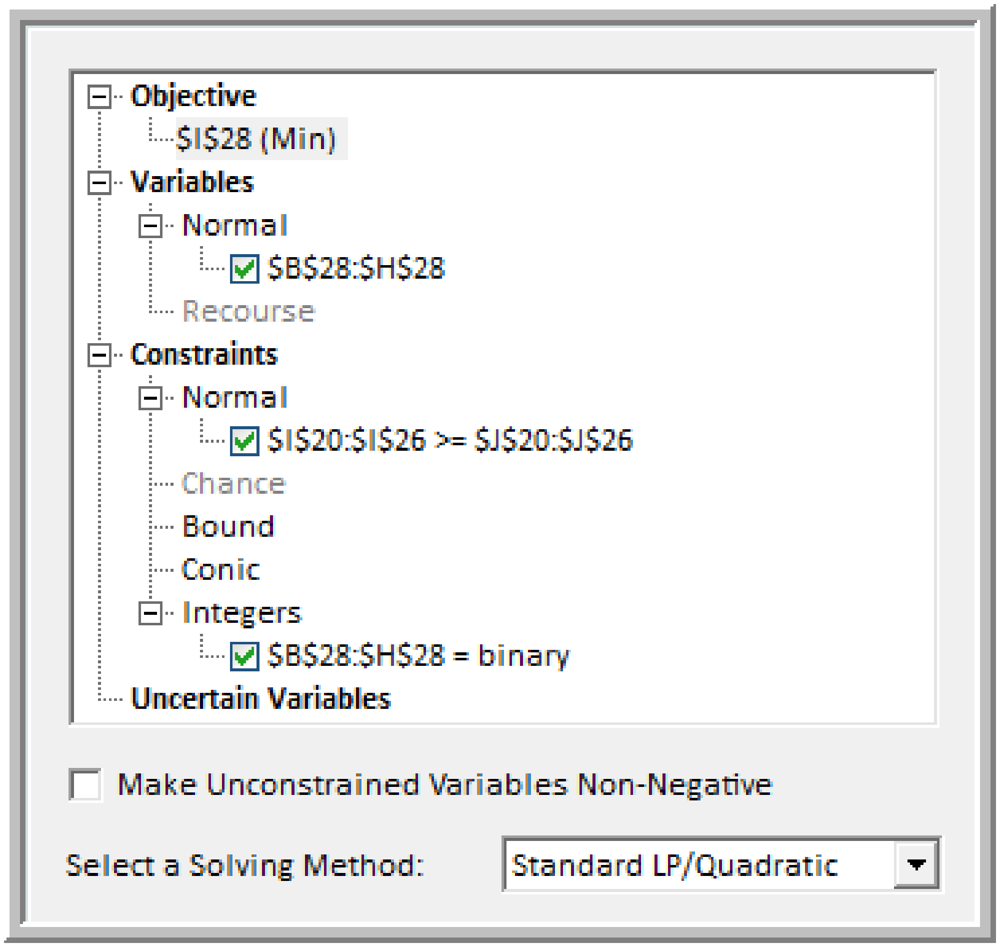
IP Model

- Min X1 + X2 + X3 + X4 + X5 + X6 + X7
- Meet response time requirement for each district:
  - $X1 + X2 + X4 + X6 + X7 \geq 1$
  - $X1 + X2 + X3 + X6 \geq 1$
  - $X2 + X3 + X4 + X5 + X7 \geq 1$
  - $X1 + X3 + X4 + X6 + X7 \geq 1$
  - $X1 + X2 + X4 + X6 + X7 \geq 1$
  - $X1 + X3 + X4 + X5 + X6 + X7 \geq 1$

Spreadsheet Model

	A	B	C	D	E	F	G	H	I	J
1	**Fire Station Location Model**									
2										
3	**Data**									
4										
5	**Response time**		8							
6										
7	Response Times									
8	From/To	1	2	3	4	5	6	7		
9	1	0	2	10	6	12	5	8		
10	2	2	0	6	9	11	7	10		
11	3	10	6	0	5	5	12	6		
12	4	6	9	5	0	9	4	3		
13	5	12	11	5	9	0	10	8		
14	6	5	7	12	4	10	0	6		
15	7	8	10	6	3	8	6	0		
16										
17	**Model**									
18										
19	From/To	1	2	3	4	5	6	7	Covered?	Requirement
20	1	1	1	0	1	0	1	1	1	1
21	2	1	1	1	0	0	1	0	1	1
22	3	0	1	1	1	1	0	1	2	1
23	4	1	0	1	1	0	1	1	2	1
24	5	0	0	1	0	1	0	1	2	1
25	6	1	1	0	1	0	1	1	1	1
26	7	1	0	1	1	1	1	1	2	1
27									Total	
28	Location	0	0	1	0	0	0	1	2	

Solver Model



Modeling Logical Conditions

TABLE 14.2 Modeling Logical Conditions Using Binary Variables

Logical Condition	Constraint Model Form
If A, then B	$B \geq A$ or $B - A \geq 0$
If not A, then B	$B \geq 1 - A$ or $A + B \geq 1$
If A, then not B	$B \leq 1 - A$ or $B + A \leq 1$
At most one of A and B	$A + B \leq 1$
If A, then B and C	$(B \geq A$ and $C \geq A)$ or $B + C \geq 2A$
If A and B, then C	$C \geq A + B - 1$ or $A + B - C \leq 1$

Supply Chain Facility Location

- X_{ij} = 1 if customer zone j is assigned to DC i, and 0 if not, and Y_i = 1 if CD i is chosen from among a set of k potential locations.
- C_{ij} = the total cost of satisfying the demand in customer zone j from DC i.

$$\text{Min } \sum C_{ij} X_{ij}$$

$$\sum X_{ij} = 1, \text{ for every } j \text{ (summed over } i\text{)}$$

$$\sum Y_i = k \text{ (summed over } i\text{)}$$

$$X_{ij} \leq Y_i, \text{ for every } i \text{ and } j$$

Plant Location Model

TABLE 14.3 Plant Location Data

Plant	Distribution Center: Cleveland	Baltimore	Chicago	Phoenix	Capacity
Marietta	$12.60	$14.35	$11.52	$17.58	1,200
Minneapolis	$9.75	$16.26	$8.11	$17.92	800
Fayetteville	$10.41	$11.54	$9.87	$11.64	1,500
Chico	$13.88	$16.95	$12.51	$8.32	1,500
Demand	300	500	700	1,800	

Select a new plant from among Fayetteville and Chico

IP Model

Minimize $12.60X_{11} + 14.35X_{12} + 11.52X_{13} + 17.58X_{14} + 9.75X_{21} + 16.26X_{22} + 8.11X_{23} + 17.92X_{24} + 10.41X_{31} + 11.54X_{32} + 9.87X_{33} + 11.64X_{34} + 13.88X_{41} + 16.95X_{42} + 12.51X_{43} + 8.32X_{44}$

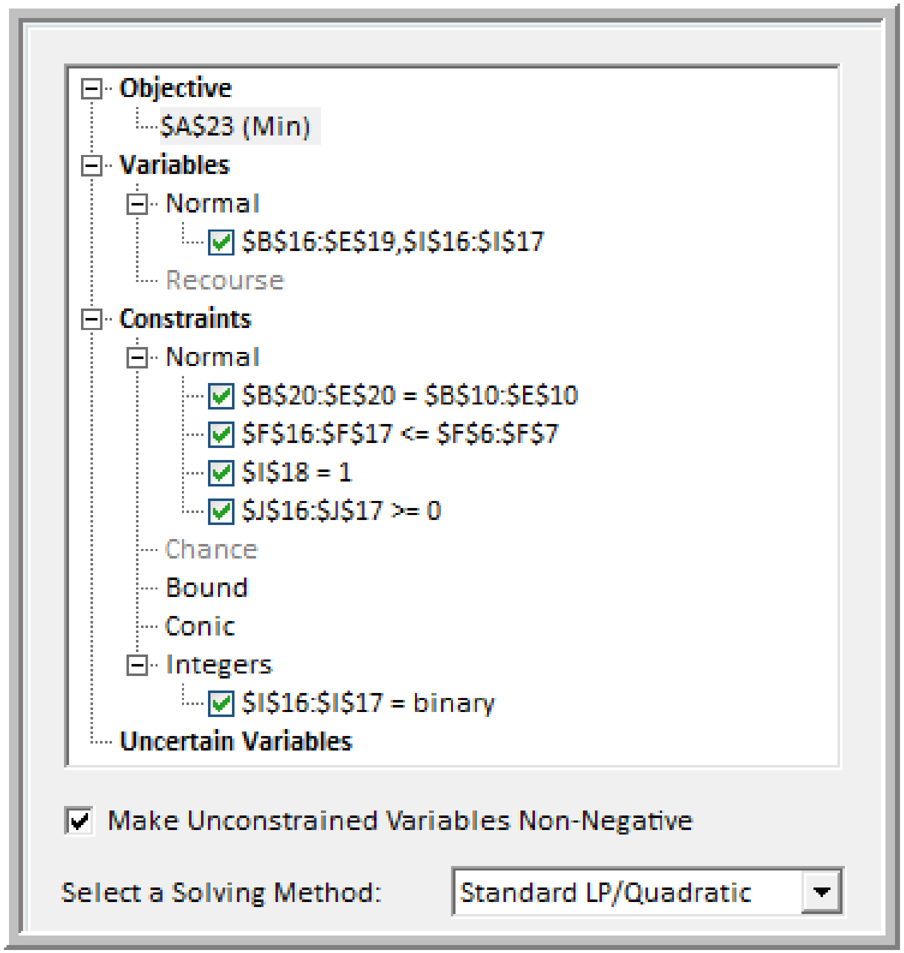
$$X_{11} + X_{12} + X_{13} + X_{14} \leq 1200$$
$$X_{21} + X_{22} + X_{23} + X_{24} \leq 800$$
$$X_{31} + X_{32} + X_{33} + X_{34} \leq 1500Y_1$$
$$X_{41} + X_{42} + X_{43} + X_{44} \leq 1500Y_2$$
$$X_{11} + X_{21} + X_{31} + X_{41} = 300$$
$$X_{12} + X_{22} + X_{32} + X_{42} = 500$$
$$X_{13} + X_{23} + X_{33} + X_{43} = 700$$
$$X_{14} + X_{24} + X_{34} + X_{44} = 1800$$
$$Y_1 + Y_2 = 1$$
$$X_{ij} \geq 0, \text{ for all i and j}$$
$$Y_1, Y_2 = 0,1$$

Ensures that exactly one DC is selected. Y_1 corresponds to Fayetteville; Y_2 corresponds to Chico

Spreadsheet Model

	A	B	C	D	E	F	G	H	I	J
1	**Plant Location Model**									
2										
3	**Data**									
4			**Distribution Center**							
5	**Plant**	Cleveland	Baltimore	Chicago	Phoenix	**Capacity**				
6	Marietta	$ 12.60	$ 14.35	$ 11.52	$ 17.58	1200				
7	Minneapolis	$ 9.75	$ 16.26	$ 8.11	$ 17.92	800				
8	Fayetteville	$ 10.41	$ 11.54	$ 9.87	$ 11.64	1500				
9	Chico	$ 13.88	$ 16.95	$ 12.51	$ 8.32	1500				
10	**Demand**	300	500	700	1800					
11										
12	**Model**									
13										
14	Amount Shipped		**Distribution Center**							
15	**Plant**	Cleveland	Baltimore	Chicago	Phoenix	**Total shipped**			New Plant Chosen	Surplus Capacity
16	Marietta	200	500	0	300	1000		Fayetteville	0	0
17	Minneapolis	100	0	700	0	800		Chico	1	0
18	Fayetteville	0	0	0	0	0		Total	1	
19	Chico	0	0	0	1500	1500				
20	**Demand met**	300	500	700	1800					
21										
22	**Total cost**									
23	$ 34,101									

Solver Model



Modeling Fixed Costs

- Multiperiod Production Planning Model
  - Y_A = 1 if production occurs during the Autumn, and 0 if not
  - Y_W = 1 if production occurs during the Winter, and 0 if not
  - Y_S = 1 if production occurs during the Spring, and 0 if not

Minimize $11P_A + 14P_W + 12.50P_S + 1.20I_A + 1.20I_W + 1.20I_S + 65(Y_A + Y_W + Y_S)$

$$P_A - I_A = 150$$
$$P_W + I_A - I_W = 400$$
$$P_S + I_W - I_S = 50$$
$$P_A \leq 600Y_A$$
$$P_W \leq 600Y_W$$
$$P_S \leq 600Y_S$$

Spreadsheet Implementation

	A	B	C	D
1	**Kristin's Kreations Fixed Cost Model**			
2				
3	Cost	Quarter 1	Quarter 2	Quarter 3
4	Production	$ 11.00	$ 14.00	$ 12.50
5	Inventory	$ 1.20	$ 1.20	$ 1.20
6	Demand	150	400	50
7	Fixed cost	$ 65.00	$ 65.00	$ 65.00
8				
9		Quarter 1	Quarter 2	Quarter 3
10	Production	600	0	0
11	Inventory	450	50	0
12	Binary	1	0	0
13				
14	Binary constraints	600	0	0
15	Net production	150	400	50
16				
17		Cost		
18	Total	$ 7,265.00		

	A	B	C	D
1	**Kristin's Kreations**			
2				
3	Cost	Quarter 1	Quarter 2	Quarter 3
4	Production	11	14	12.5
5	Inventory	1.2	1.2	1.2
6	Demand	150	400	50
7	Fixed cost	65	65	65
8				
9		Quarter 1	Quarter 2	Quarter 3
10	Production	600	0	0
11	Inventory	450	50	0
12	Binary	1	0	0
13				
14	Binary constraints	=600*B12	=600*C12	=600*D12
15	Net production	=B10-B11	=C10-C11+B11	=D10-D11+C11
16				
17		Cost		
18	Total	=SUMPRODUCT(B4:D5,B10:D11) + 65*(B12+C12+D12)		

Nonlinear Optimization

- Either objective function or constraint functions are not linear
- Models are unique in structure
- Solution techniques are different from linear and integer optimization

Hotel Pricing With Elastic Demand

- A 450-room hotel has the following history:

Room Type	Rate	Daily Avg. No. Sold	Revenue
Standard	$85	250	$21,250
Gold	$98	100	$9,800
Platinum	$139	50	$6,950
		Total revenue	$38,000

Room Type	Price Elasticity of Demand
Standard	–1.5
Gold	–2.0
Platinum	–1.0

Model Development

- Projected number of rooms of a given type sold =

(Historical Average Number of Rooms Sold) + (Elasticity)(New Price - Current Price)(Historical Average Number of Rooms Sold)/(Current Price)

- Define S = price of a standard room, G = price of a gold room, and P = price of a platinum room.

Total Revenue = $S(625 - 4.41176S) + G(300 - 2.04082G) + P(100 - 0.35971P)$

$= 625S + 300G + 100P - 4.41176S^2 - 2.04082G^2 - 0.35971P^2$

Model

Maximize $625S + 300G + 100P - 4.41176S^2 - 2.04082G^2 - 0.35971P^2$

$70 \leq S \leq 90$ (price range restrictions)

$90 \leq G \leq 110$

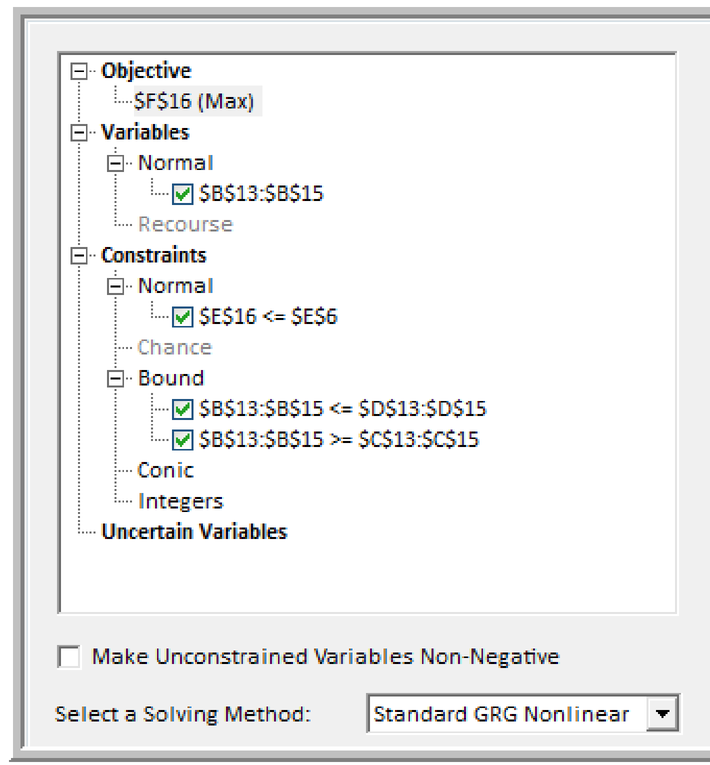
$120 \leq P \leq 149$

$(625 - 4.41176S) + (300 = 2.04082G) + (100 = 0.35971P) \leq 450$

or $1025 - 4.41176S - 2.04082G - 0.35971P \leq 450$ (room limitation)

Spreadsheet and Solver Model

	A	B	C	D	E	F
1	**Marquis Hotel**					
2						
3	**Data**					
4		**Current**	**Average**		**Total Room**	
5	**Room type**	**Rate**	**Daily Sold**	**Elasticity**	**Capacity**	
6	Standard	$ 85.00	250	-1.5	450	
7	Gold	$ 98.00	100	-2		
8	Platinum	$ 139.00	50	-1		
9						
10	**Model**				**Projected**	
11					**Rooms**	**Projected**
12	**Room type**	**New Price**	**Price Range**		**Sold**	**Revenue**
13	Standard	$ 76.87	$ 70.00	$ 90.00	286	$ 21,974.39
14	Gold	$ 90.00	$ 90.00	$ 110.00	116	$ 10,469.39
15	Platinum	$ 145.04	$ 120.00	$ 149.00	48	$ 6,936.87
16				Totals	450	$ 39,380.65



Solving Nonlinear Models

- Select Standard GRG Nonlinear in Premium Solver as the solution procedure
- Sensitivity report is different for nonlinear models
  - Reduced gradient is analogous to reduced cost, but more difficult to interpret
  - Lagrange multipliers are similar to shadow prices, but give only approximate rates of change

Solver Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$F$16	Totals Revenue	0	39380.65104

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$13	Standard New Price	$ -	$ 76.87
$B$14	Gold New Price	$ -	$ 90.00
$B$15	Platinum New Price	$ -	$ 145.04

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$16	Totals Sold	450.0000004	$E$16<=$E$6	Binding	0
$B$13	Standard New Price	$ 76.87	$B$13>=$C$13	Not Binding	6.87476046
$B$14	Gold New Price	$ 90.00	$B$14>=$C$14	Binding	0
$B$15	Platinum New Price	$ 145.04	$B$15>=$C$15	Not Binding	25.0414271
$B$13	Standard New Price	$ 76.87	$B$13<=$D$13	Not Binding	13.1252395
$B$14	Gold New Price	$ 90.00	$B$14<=$D$14	Not Binding	20
$B$15	Platinum New Price	$ 145.04	$B$15<=$D$15	Not Binding	3.95857289

Solver Sensitivity Report

	A	B	C	D	E
4					
5	Target Cell (Max)				
6		**Cell**	**Name**	**Final Value**	
7		$F$16	Totals Revenue	39380.65104	
8					
9	Adjustable Cells				
10				**Final**	**Reduced**
11		**Cell**	**Name**	**Value**	**Gradient**
12		$B$13	Standard New Price	$ 76.87	$ -
13		$B$14	Gold New Price	$ 90.00	$ (42.69)
14		$B$15	Platinum New Price	$ 145.04	$ -
15					
16	Constraints				
17				**Final**	**Lagrange**
18		**Cell**	**Name**	**Value**	**Multiplier**
19		$E$16	Totals Sold	450.0000004	12.08293216

Markowitz Portfolio Model

- Select stocks to minimize portfolio variance

$$\sum_{i=1}^{k} s_i^2 x_i^2 + \sum_{i=1}^{k} \sum_{j>i} 2 s_{ij} x_i x_j$$

and ensure a specified expected return

  - s_i^2 =the sample variance in the return of stock *i*
  - s_{ij} = the sample covariance between stocks *i* and *j*

Example

Variance-Covariance Matrix

	Stock 1	Stock 2	Stock 3
Stock 1	.025	.015	-.002
Stock 2		.030	.005
Stock 3			.004
Exp. return	10%	12%	7%

Minimize Variance =

$$.025x_1^2 + .030x_2^2 + .004x_3^2 + 0.03x_1x_2 - 0.004x_1x_3 + 0.010x_2x_3$$

$x_1 + x_2 + x_3 = 1$

$10x_1 + 12x_2 + 7x_3 \geq 10$ (required return)

$x_1, x_2, x_3 \geq 0$

Spreadsheet Model

	A	B	C	D	E	F	G
1	**Markowitz Model**						
2							
3	**Data**						
4		Expected			**Variance-Covariance Matrix**		
5		Return			Stock 1	Stock 2	Stock 3
6	Stock 1	10%		Stock 1	0.025	0.015	-0.002
7	Stock 2	12%		Stock 2		0.03	0.005
8	Stock 3	7%		Stock 3			0.004
9	Target Return	10%					
10							
11	**Model**						
12					Variance Calculations		
13		Allocation			Squared Terms	Cross-Products	
14	Stock 1	0.25			0.001579256	0.003387	
15	Stock 2	0.45			0.006053361	-0.000301067	
16	Stock 3	0.30			0.000358718	0.001345191	
17	Total	1					
18							
19							
20		Return	Variance				
21	Portfolio	10.0%	0.012				

Sensitivity Report

	A	B	C	D	E
4					
5	Objective Cell (Min)				
6		**Cell**	**Name**	**Final Value**	
7		$C$21	Portfolio Variance	0.01242246	
8					
9	Decision Variable Cells				
10				**Final**	**Reduced**
11		**Cell**	**Name**	**Value**	**Gradient**
12		$B$14	Stock 1 Allocation	0.25	0.00
13		$B$15	Stock 2 Allocation	0.45	0.00
14		$B$16	Stock 3 Allocation	0.30	0.00
15					
16	Constraints				
17				**Final**	**Lagrange**
18		**Cell**	**Name**	**Value**	**Multiplier**
19		$B$17	Total Allocation	1	-0.038363637
20		$B$21	Portfolio Return	10.0%	63.2%

Risk versus Return Profile

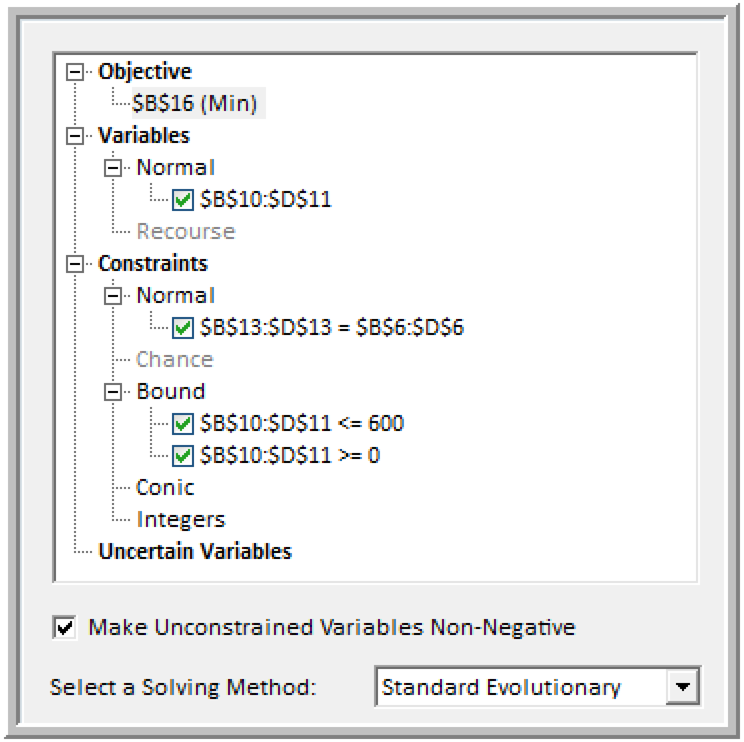
Evolutionary Solver for Nonsmooth Optimization

- Used for difficult nonlinear problems and models using Excel functions such as IF, ABS, MIN, and MAX
- *Evolutionary Solver* uses heuristics—intelligent rules for systematically searching among solutions—that remember the best solutions they find, then modifying or combining them in attempting to find better solutions.

Fixed Cost Model

The objective function in cell B16 is =SUMPRODUCT(B4:D5,B10:D11) + IF(B10>0,B7,0)+IF(C10>0,C7,0)+IF(D10>0,D7,0).

	A	B	C	D
1	**Kristin's Kreations Evolutionary Solver Model**			
2				
3	Cost	Quarter 1	Quarter 2	Quarter 3
4	Production	$ 11.00	$ 14.00	$ 12.50
5	Inventory	$ 1.20	$ 1.20	$ 1.20
6	Demand	150	400	50
7	Fixed cost	$ 65.00	$ 65.00	$ 65.00
8				
9		Quarter 1	Quarter 2	Quarter 3
10	Production	550	0	50
11	Inventory	400	0	0
12				
13	Net production	150	400	50
14				
15		Cost		
16	Total	$ 7,285.00		



Location Model

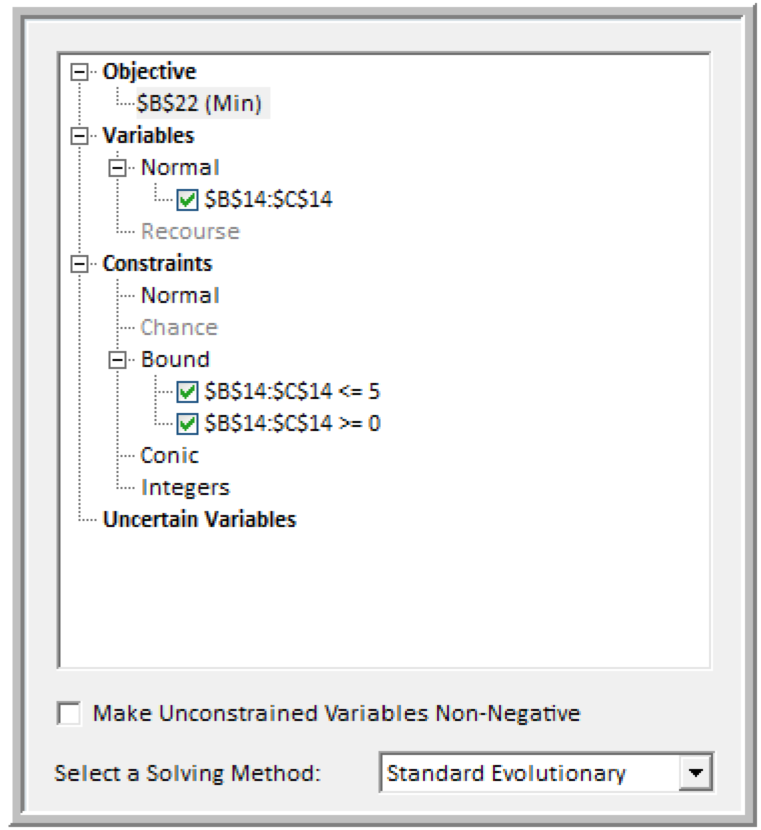
- Locate a tool bin to minimize the weighted distance between the location coordinates and each production cell

Cell	*X*-coordinate	*Y*-coordinate	Demand
Fabrication	1	4	12
Paint	1	2	24
Subassembly 1	2.5	2	13
Subassembly 2	3	5	7
Assembly	4	4	17

Minimize $12(|X - 1| + |Y - 4|) + 24(|X - 1| + |Y - 2|) + 13(|X - 2.5| + |Y - 2|) + 7(|X - 3| + |Y - 5) + 17(|X - 4| + |Y - 4|)$

Spreadsheet and *Solver* Models

	A	B	C	D
1	**Edwards Manufacturing**			
2				
3	**Data**			
4				
5	Cell	x-coordinate	y-coordinate	Demand
6	Fabrication	1	4	12
7	Paint	1	2	24
8	Subassembly 1	2.5	2	13
9	Subassembly 2	3	5	7
10	Assembly	4	4	17
11	Maximum	4	5	
12				
13	**Model**			
14	Tool bin location	2.499997179	2.489551412	
15				
16	Cell	Weighted Distance		
17	Fabrication	36.1253492		
18	Paint	47.7491662		
19	Subassembly 1	6.364205031		
20	Subassembly 2	21.07315986		
21	Assembly	51.17767394		
22	Total	162.4895542		



Job Sequencing

- A custom manufacturing company has ten jobs waiting to be processed. Each job i has an estimated processing time (P_i) and a due date (D_i) that was requested by the customer, as shown in the table below:

Job	1	2	3	4	5	6	7	8	9	10
Time	8	7	6	4	10	8	10	5	9	5
Due date	20	27	39	28	23	40	25	35	29	30

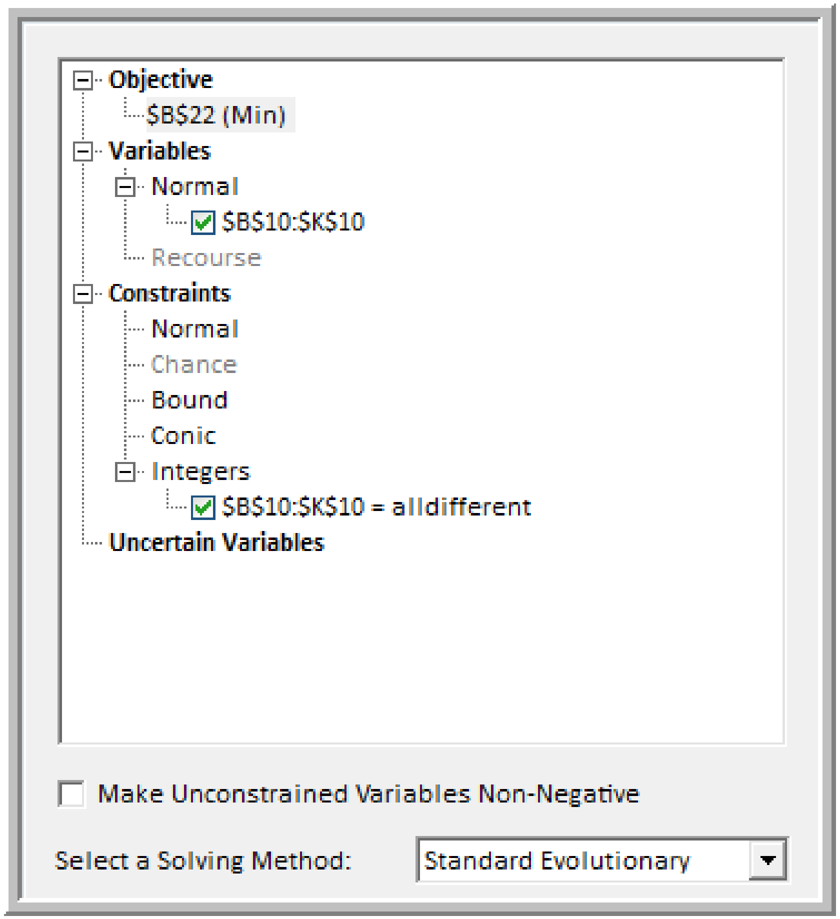
Performance Measures

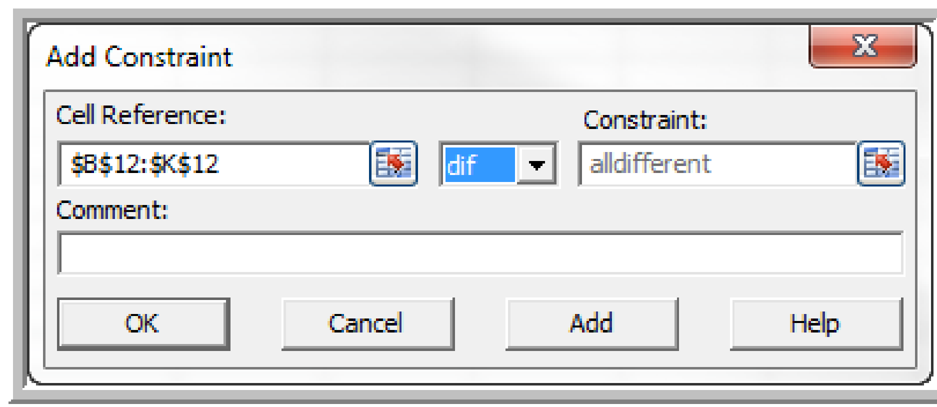
- *Lateness* (L_i) is the difference between the completion time and the due date (either positive or negative).
- *Tardiness* (T_i) is the amount of time by which the completion time exceeds the due date; thus tardiness is zero if a job is completed early.

Spreadsheet Model

	A	B	C	D	E	F	G	H	I	J	K
1	Job Sequencing										
2											
3	Data										
4	Job	1	2	3	4	5	6	7	8	9	10
5	Time	8	7	6	4	10	8	10	5	9	5
6	Due date	26	27	39	28	23	40	25	35	29	30
7											
8	Model										
9	Sequence	1	2	3	4	5	6	7	8	9	10
10	Job Assigned	5	7	1	2	4	9	10	8	3	6
11	Processing time	10	10	8	7	4	9	5	5	6	8
12	Completion time	10	20	28	35	39	48	53	58	64	72
13	Due Date	23	25	26	27	28	29	30	35	39	40
14	Lateness	-13	-5	2	8	11	19	23	23	25	32
15	Tardiness	0	0	2	8	11	19	23	23	25	32
16											
17	Average Completion Time	42.7									
18	Maximum Number Tardy	8									
19	Total Lateness	125									
20	Average Lateness	12.5									
21	Variance of Lateness	188.85									
22	Total Tardiness	143									
23	Average Tardiness	14.3									
24	Variance of Tardiness	121.21									

Solver Model with *Alldifferent* Constraint

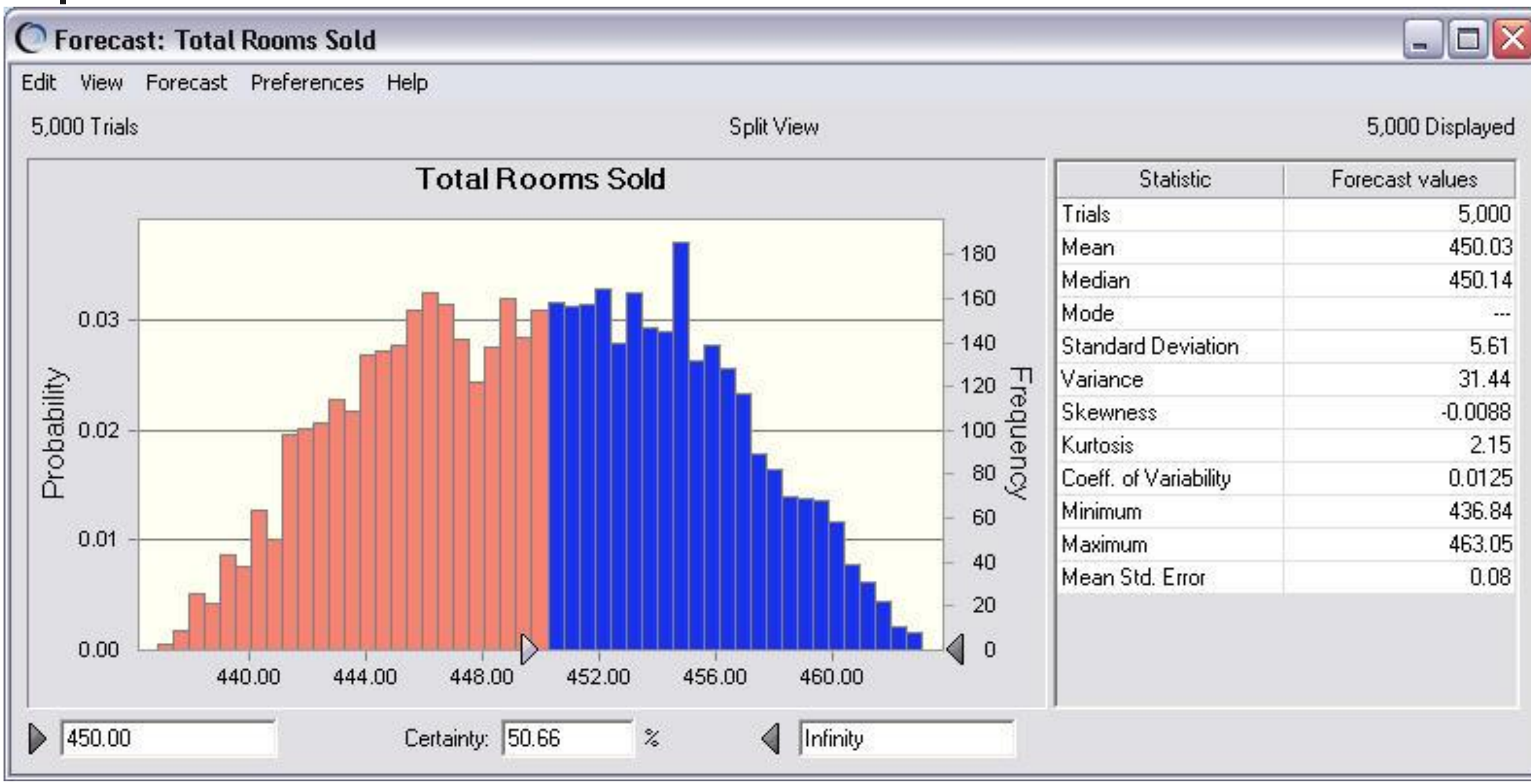




Minimum Total Tardiness Solution Using *Evolutionary Solver*

	A	B	C	D	E	F	G	H	I	J	K
1	**Job Sequencing**										
2											
3	**Data**										
4	**Job**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
5	**Time**	8	7	6	4	10	8	10	5	9	5
6	**Due date**	26	27	39	28	23	40	25	35	29	30
7											
8	**Model**										
9	**Sequence**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
10	**Job Assigned**	2	5	1	4	10	8	3	6	9	7
11	**Processing time**	7	10	8	4	5	5	6	8	9	10
12	**Completion time**	7	17	25	29	34	39	45	53	62	72
13	**Due Date**	27	23	26	28	30	35	39	40	29	25
14	**Lateness**	-20	-6	-1	1	4	4	6	13	33	47
15	**Tardiness**	0	0	0	1	4	4	6	13	33	47
16											
17	**Average Completion Time**	38.3									
18	**Maximum Number Tardy**	7									
19	**Total Lateness**	81									
20	**Average Lateness**	8.1									
21	**Variance of Lateness**	331.69									
22	**Total Tardiness**	108									
23	**Average Tardiness**	10.8									
24	**Variance of Tardiness**	236.96									

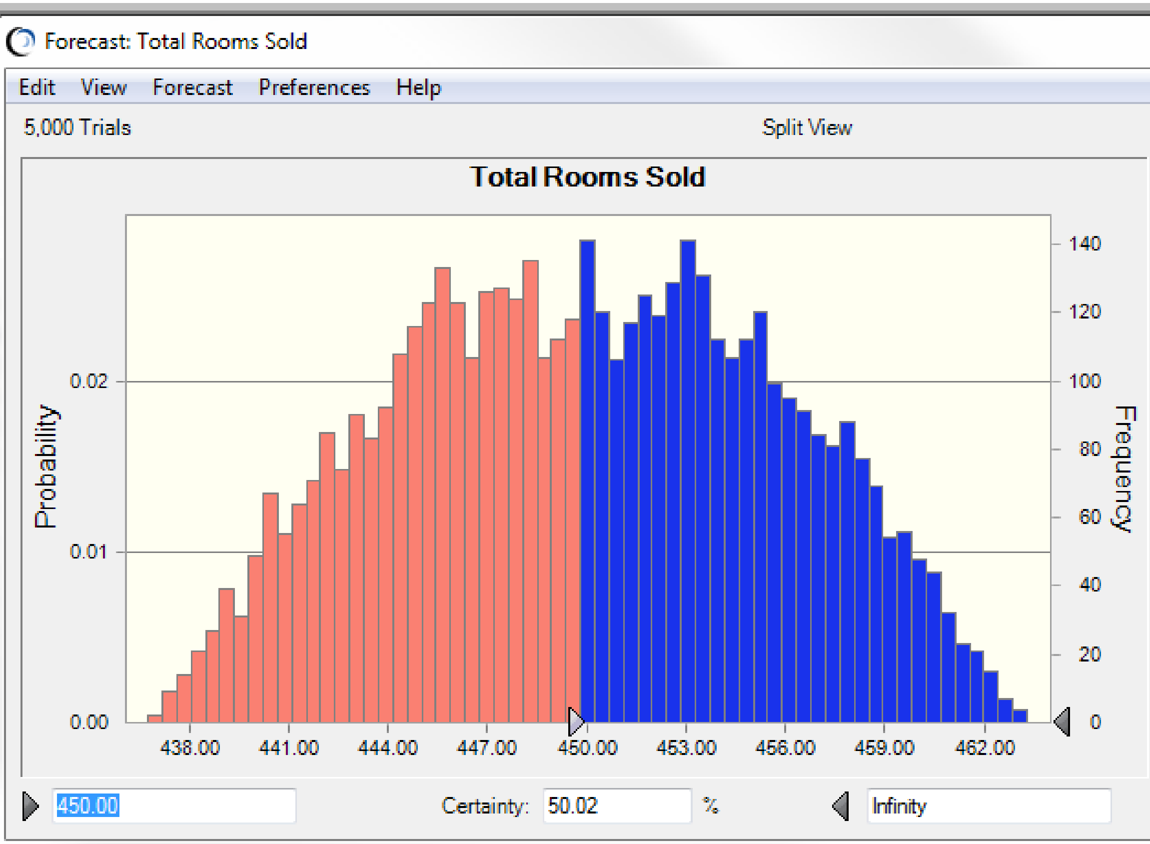
Crystal Ball Results



Risk Analysis and Optimization

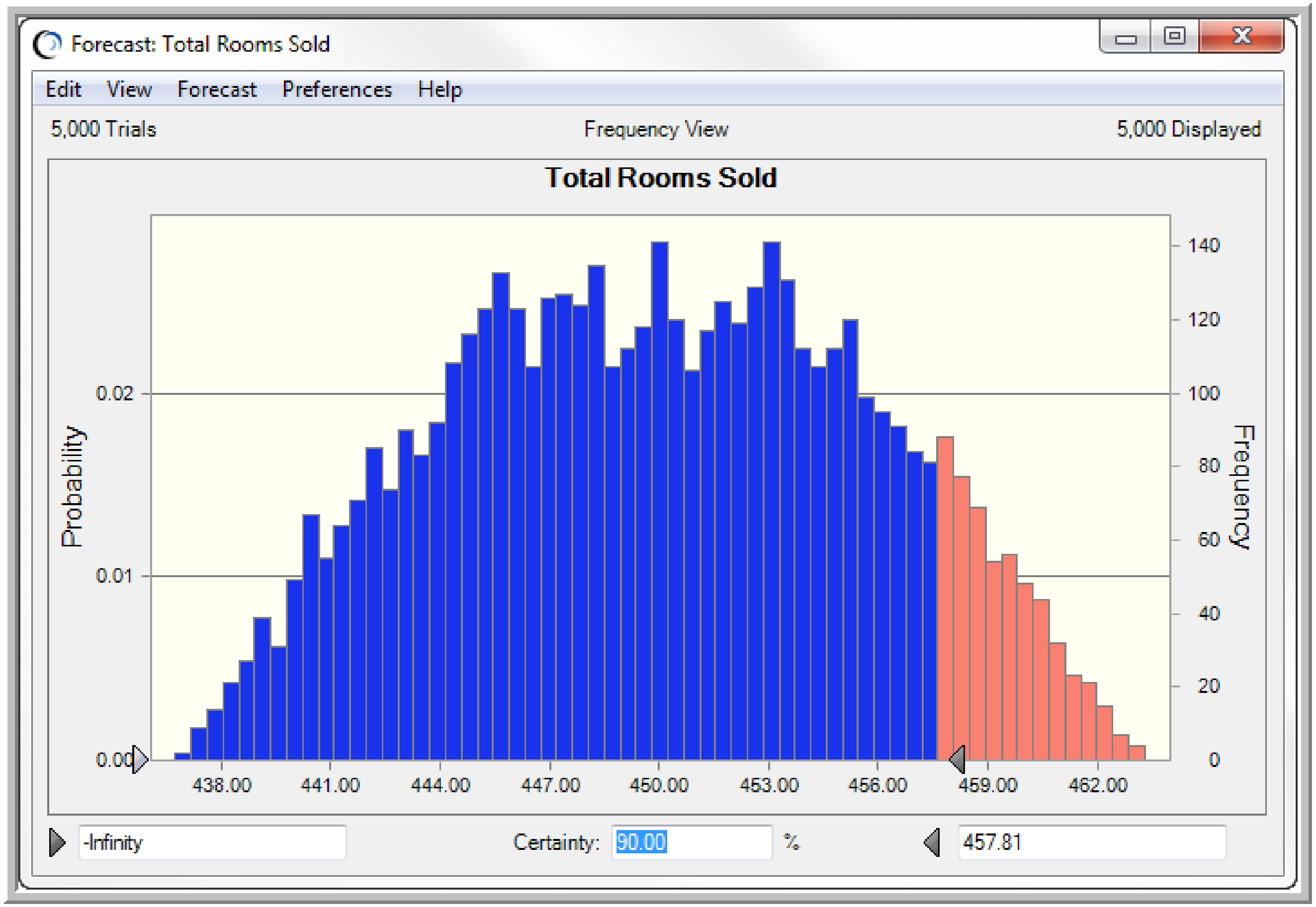
- *Crystal Ball* may be used to conduct post-optimality risk analysis to understand the impact of uncertainty of optimization model parameters.
- Example – Hotel Pricing Model
  - Assume price-demand elasticities may vary by plus or minus 25%

Crystal Ball Results



Statistic	Forecast values
Trials	5,000
Mean	450.01
Median	450.02
Mode	---
Standard Deviation	5.75
Variance	33.01
Skewness	0.0015
Kurtosis	2.16
Coeff. of Variability	0.0128
Minimum	436.74
Maximum	463.29
Mean Std. Error	0.08

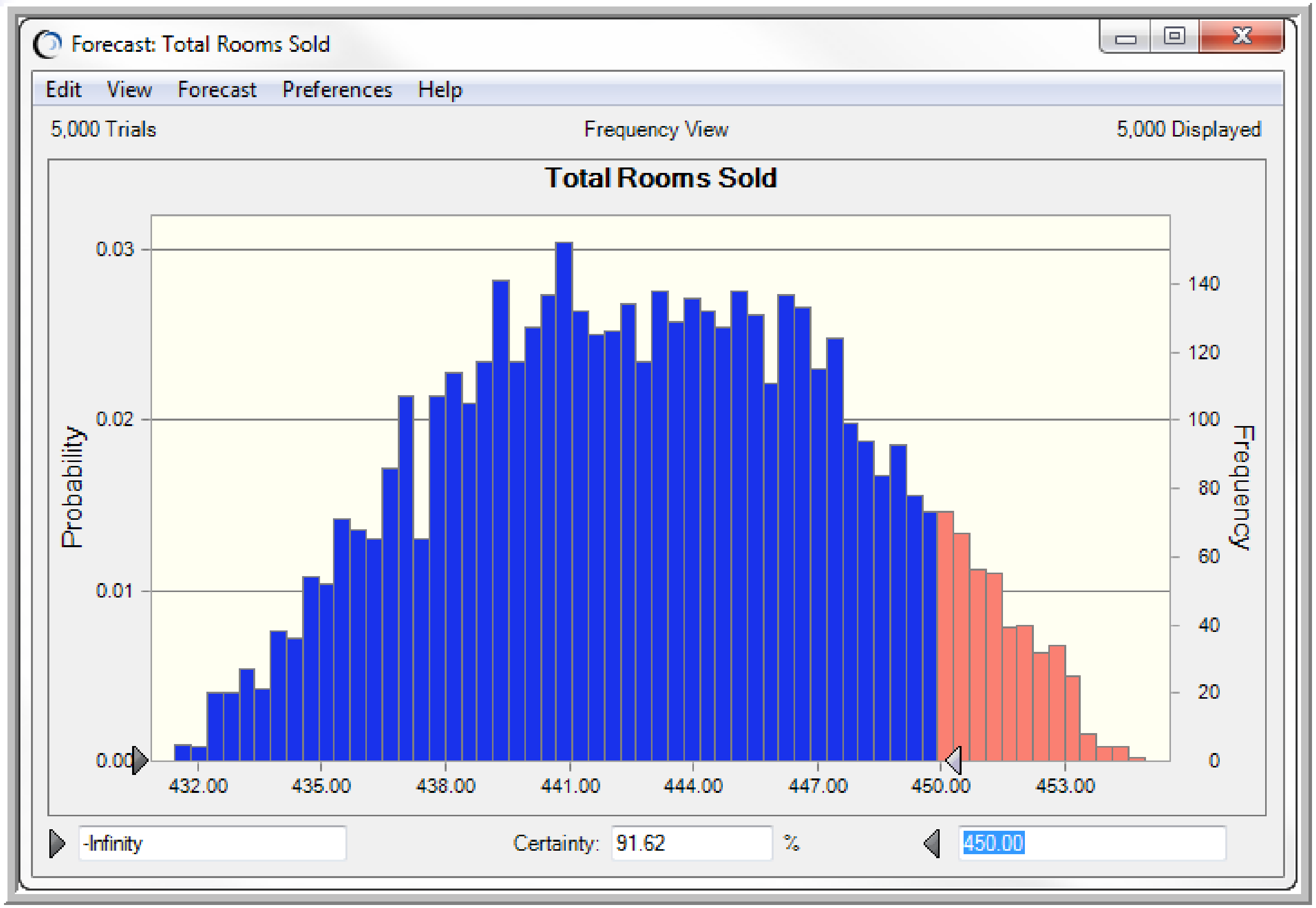
Forecast Chart for a 10% Risk of Exceeding Capacity



Solver Solution for 443 Room Capacity

	A	B	C	D	E	F
1	**Marquis Hotel**					
2						
3	**Data**					
4		**Current**	**Average**		**Total Room**	
5	**Room type**	**Rate**	**Daily Sold**	**Elasticity**	**Capacity**	
6	Standard	$ 85.00	250	-1.5	443	
7	Gold	$ 98.00	100	-2		
8	Platinum	$ 139.00	50	-1		
9						
10	**Model**				**Projected**	
11					**Rooms**	**Projected**
12	**Room type**	**New Price**	**Price Range**		**Sold**	**Revenue**
13	Standard	$ 78.34	$ 70.00	$ 90.00	279	$ 21,886.69
14	Gold	$ 90.00	$ 90.00	$ 110.00	116	$ 10,469.39
15	Platinum	$ 146.51	$ 120.00	$ 149.00	47	$ 6,929.72
16				Totals	443	$ 39,285.80

Crystal Ball Confirmation Run



OptQuest: Combining Optimization and Simulation

- *OptQuest* searches for optimal solutions within *Crystal Ball* simulation model spreadsheets.
- *OptQuest* is also designed to find solutions that satisfy a wide variety of constraints or a set of goals that you may define.

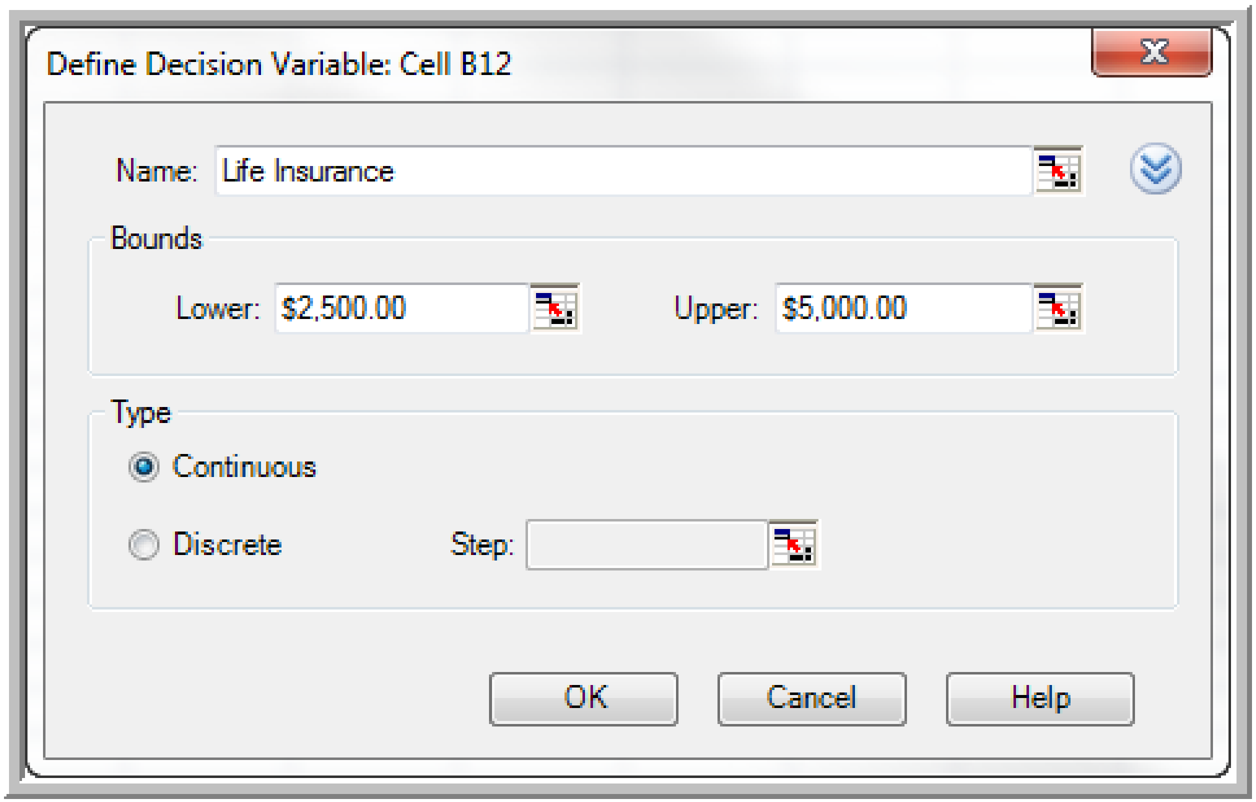
Using *OptQuest*

1. Create a *Crystal Ball* model of the decision problem.
2. Define the decision variables within *Crystal Ball.*
3. Run *OptQuest* from the *Crystal Ball* Tools group.
4. Specify objectives, decision variables, constraints, and other options as appropriate.
5. Solve the optimization problem.

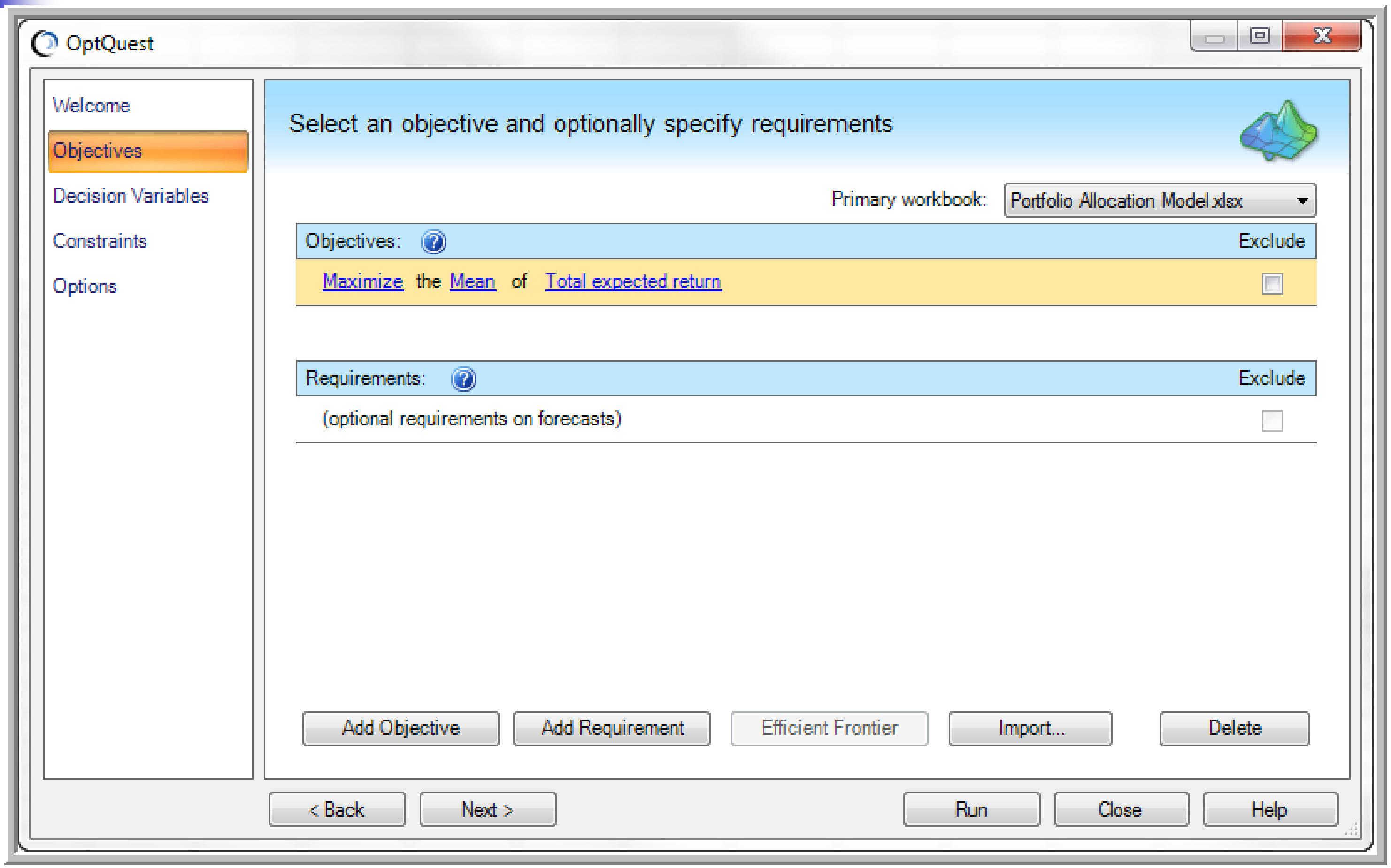
Portfolio Allocation Model

	A	B	C	D	E
1	**Portfolio Allocation Model**				
2		**Annual**			**Risk factor**
3	**Investment**	**return**	**Minimum**	**Maximum**	**per dollar**
4	Life Insurance	5.0%	$ 2,500.00	$ 5,000.00	-0.5
5	Bond mutual funds	7.0%	$ 30,000.00	none	1.8
6	Stock mutual funds	11.0%	$ 15,000.00	none	2.1
7	Savings Account	4.0%	none	none	-0.3
8	Total amount available	$100,000		**Limit**	100,000
9					
10		**Amount**			**Total weighted**
11	**Decision variables**	**invested**			**risk**
12	Life Insurance	$ 5,000.00			146,000.00
13	Bond mutual funds	$ 50,000.00			
14	Stock mutual funds	$ 30,000.00			**Total expected**
15	Savings Account	$ 15,000.00			**return**
16	Total amount invested	$ 100,000.00			$ 7,650.00

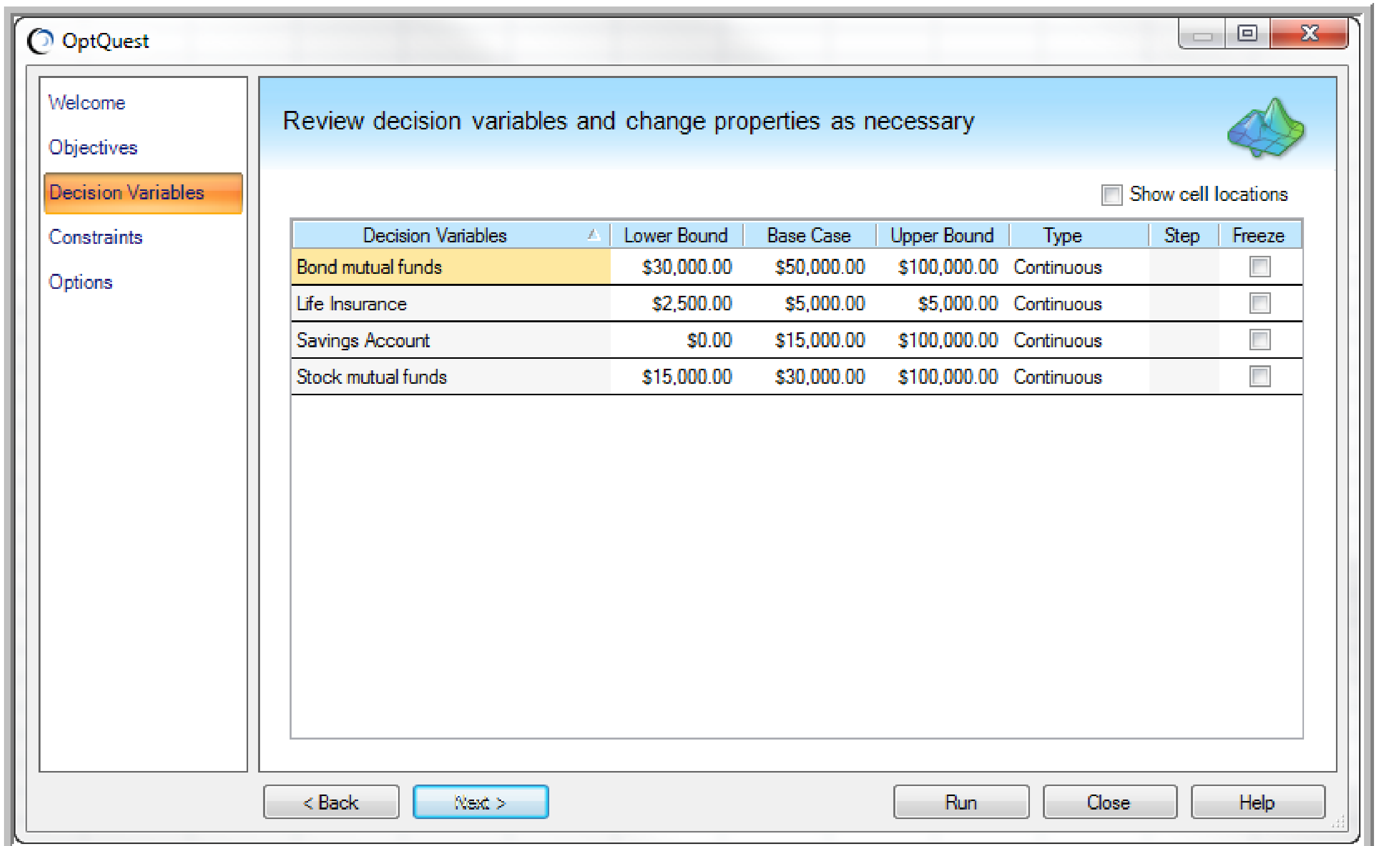
Define Decision Variable Dialog



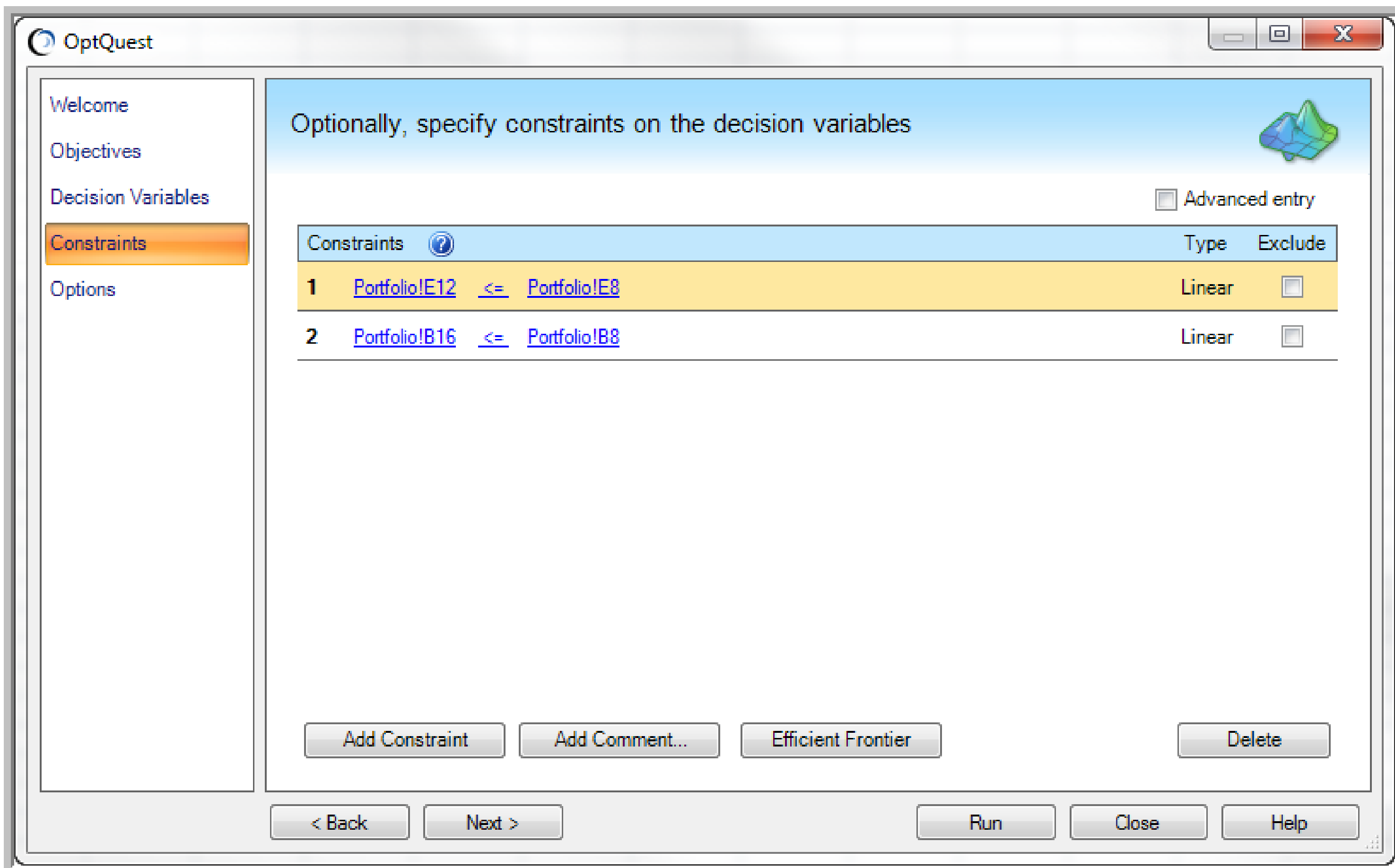
OptQuest Objectives Screen



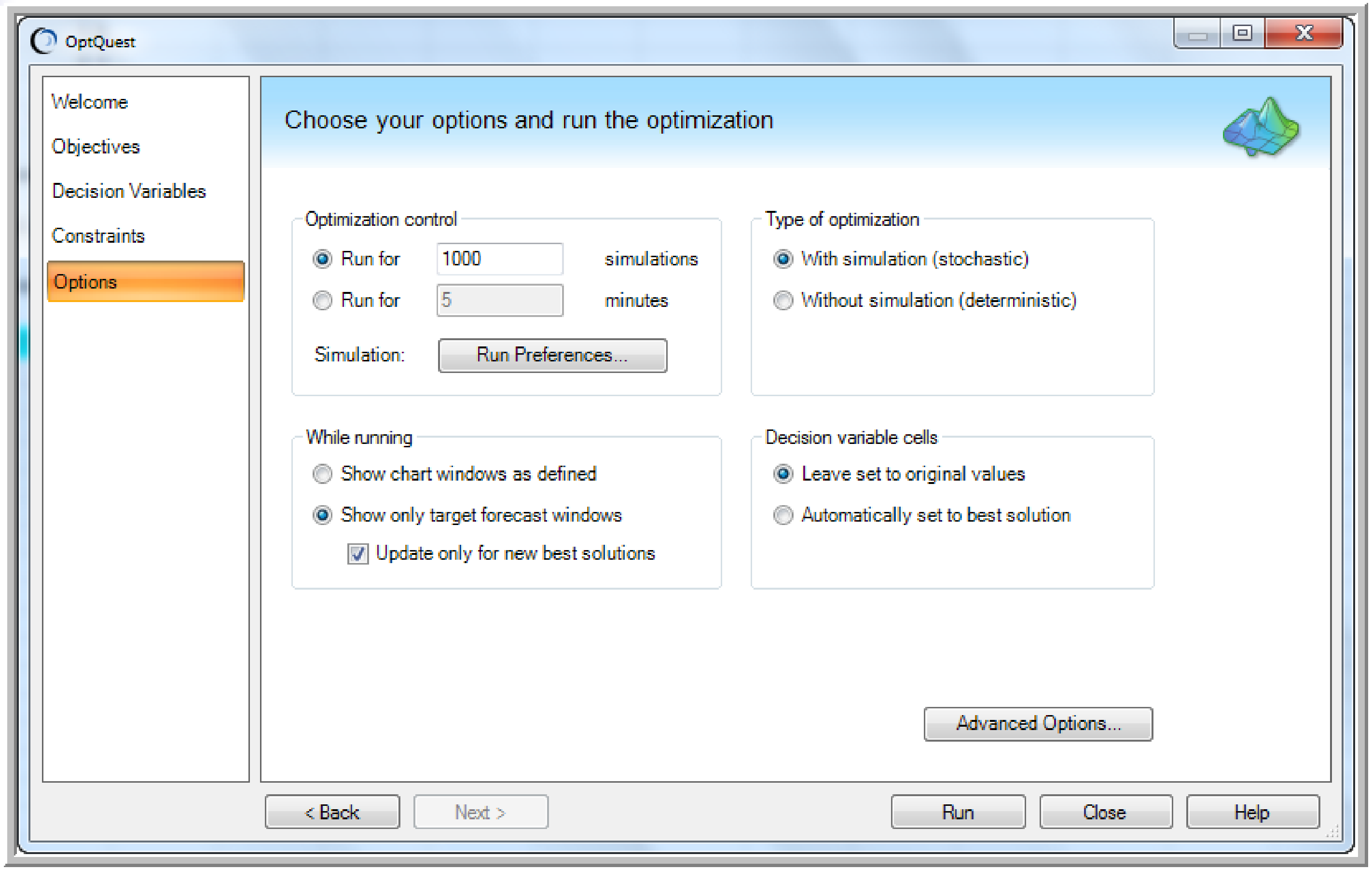
OptQuest Decision Variable Selection Screen



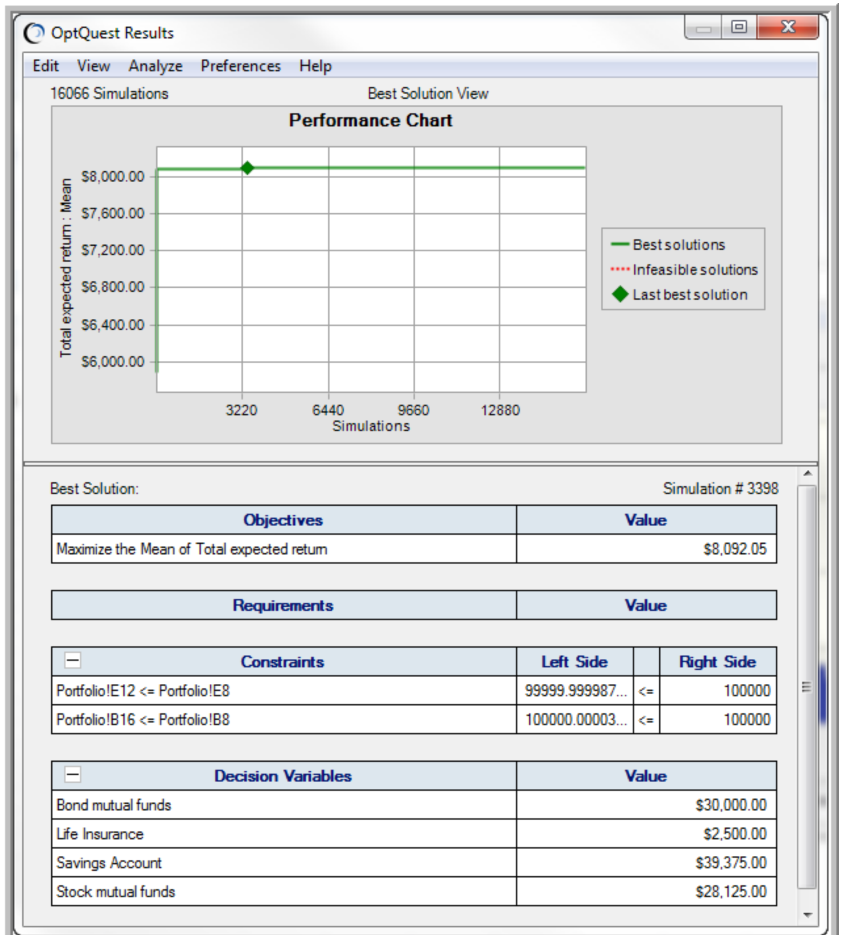
OptQuest Constraints Screen



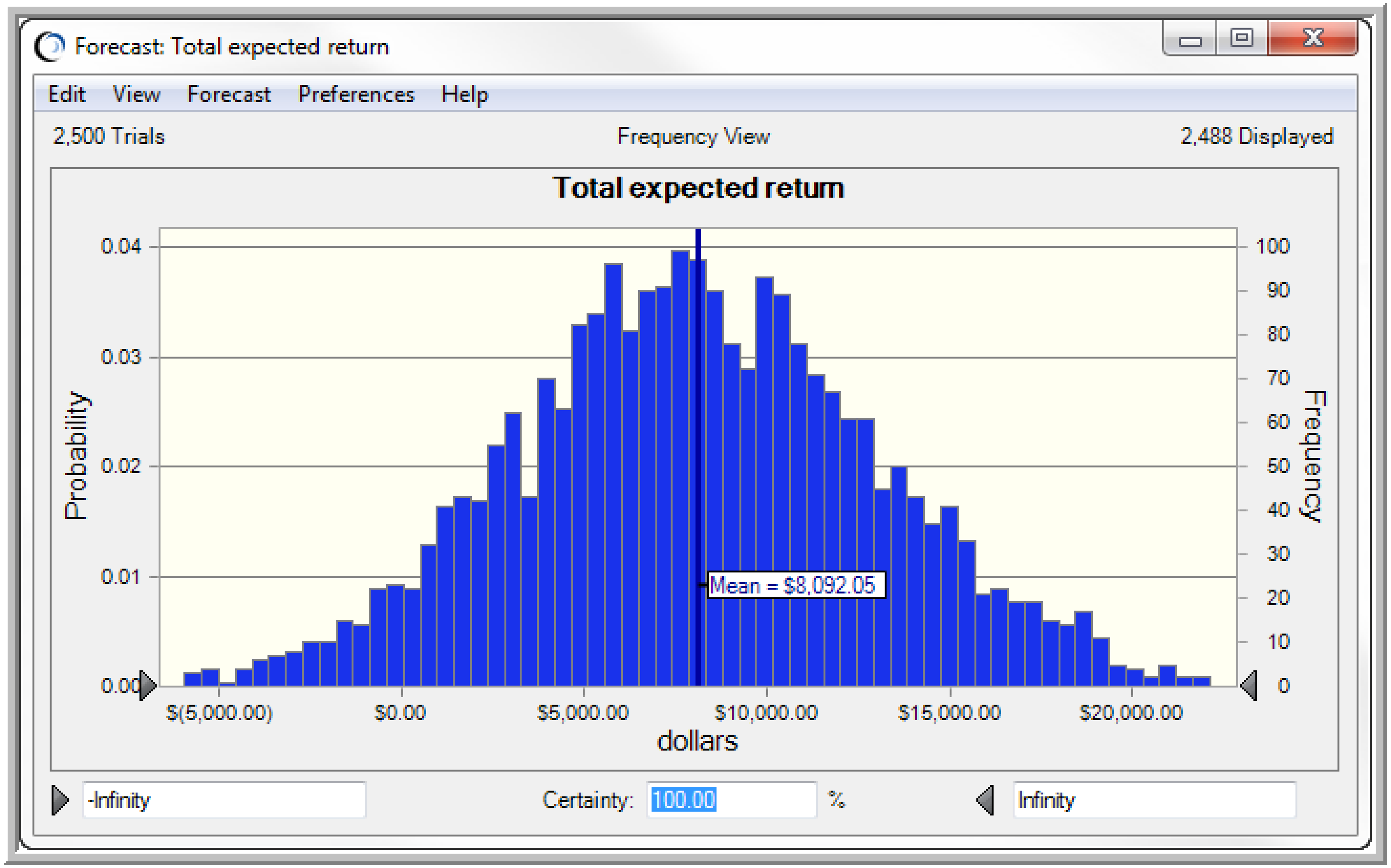
OptQuest Options Screen



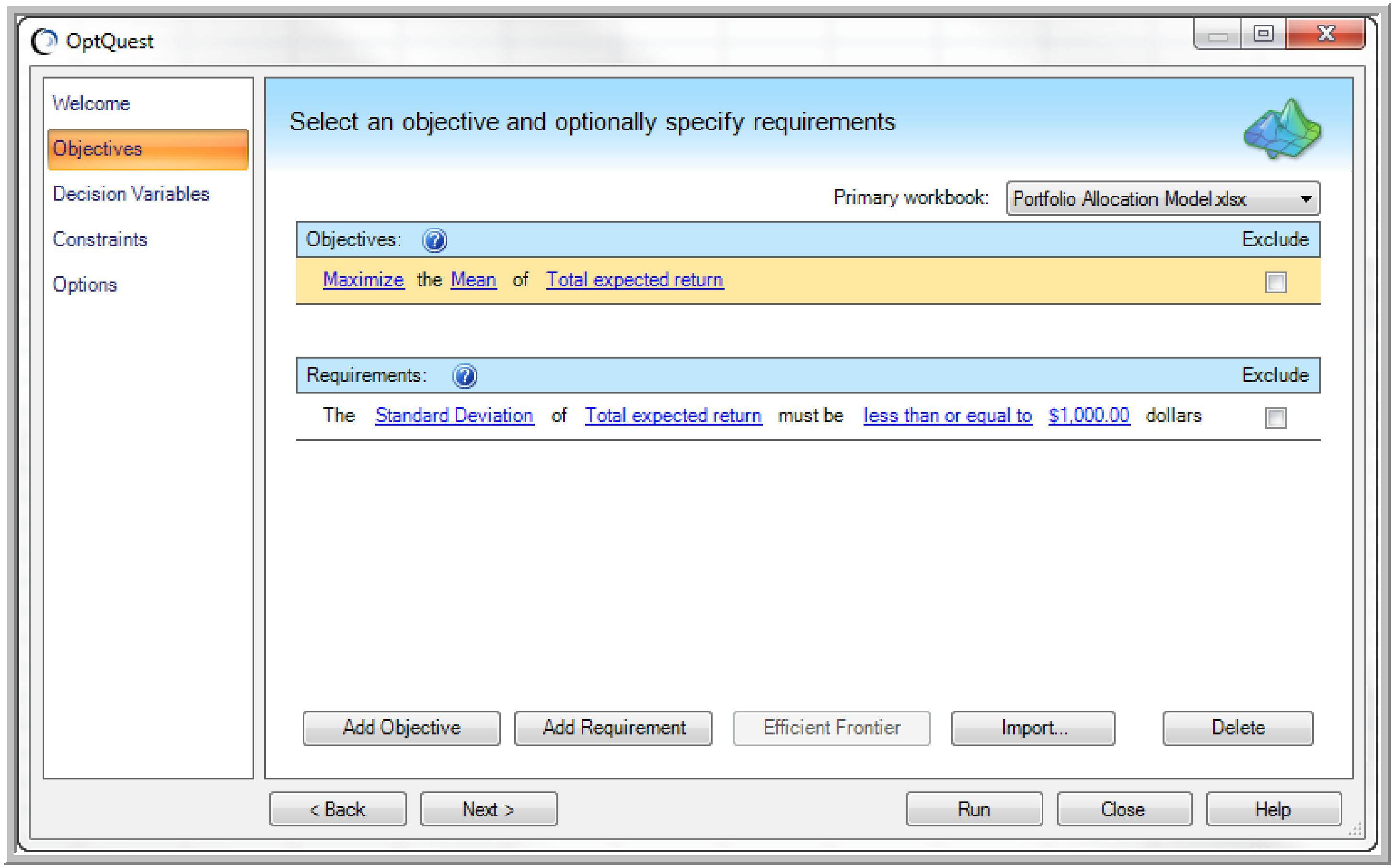
OptQuest Results



Crystal Ball Results



Adding a Requirement

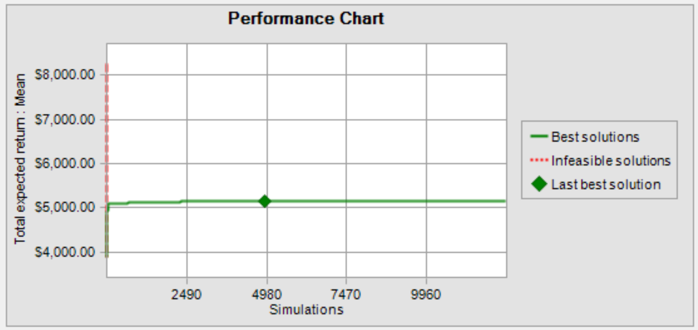


OptQuest Results with Requirement

OptQuest Results

Edit View Analyze Preferences Help

12406 Simulations Best Solution View



Best Solution: Simulation # 4916

Objectives	Value
Maximize the Mean of Total expected return	$5,146.34

Requirements	Value
The Standard Deviation of Total expected return must be less than or e...	$999.32

Constraints	Left Side		Right Side
Portfolio!E12 <= Portfolio!E8	100000	<=	100000
Portfolio!B16 <= Portfolio!B8	64863.447523...	<=	100000

Decision Variables	Value
Bond mutual funds	$37,495.41
Life Insurance	$5,000.00
Savings Account	$4,985.26
Stock mutual funds	$17,382.78