

Chapter 5 - Discrete Probability Distributions

Note: Answers may vary due to rounding, TI-83's or computer programs.

EXERCISE SET 5-1

1. A random variable is a variable whose values are determined by chance.

Examples will vary.

2. If the values a random variable can assume are countable, then the variable is called discrete; otherwise, it is called a continuous variable.

3. The number of commercials a radio station plays during each hour.

The number of times a student uses his or her calculator during a mathematics exam. The number of leaves on a specific type of tree. (Answers will vary.)

4. The weights of strawberries grown in a specific plot.

The heights of all seniors at a specific college.

The times it takes students to complete a mathematics exam. (Answers will vary.)

5. For continuous variables, examples are length of home run, length of game, temperature at game time, pitcher's ERA, batting average., etc. For discrete variables, examples are number of hits, number of pitches, number of seats in each row, etc.

6. A probability distribution is a distribution which consists of the values a random variable can assume along with the corresponding probabilities of these values. (Examples will vary.)

7. No; probabilities cannot be negative and the sum of the probabilities is not 1.

8. No; probability values cannot be negative.

9. Yes

10. Yes

11. No; the sum of the probabilities is greater than 1.

12. No; the sum of the probabilities does not equal to 1.

13. Discrete

14. Discrete

15. Continuous

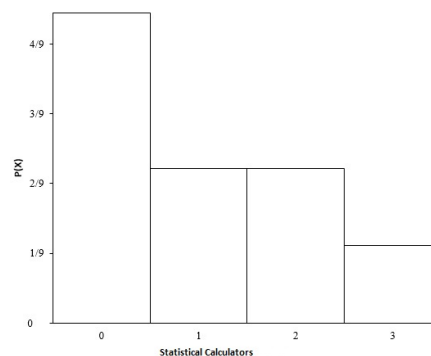
16. Continuous

17. Discrete

18. Continuous

19.

X	0	1	2	3
P(X)	$\frac{4}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

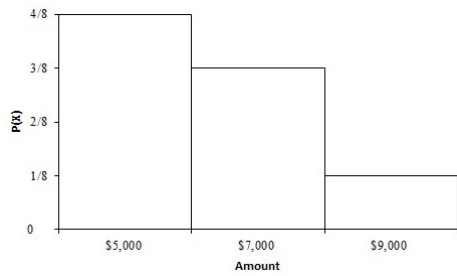


20.

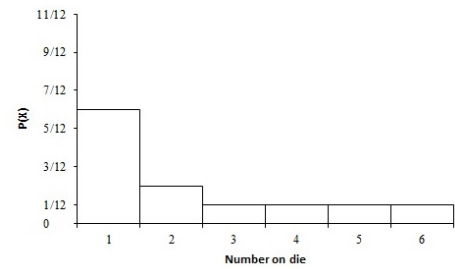
X	\$5000	\$7000	\$9000
P(X)	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$

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20. continued

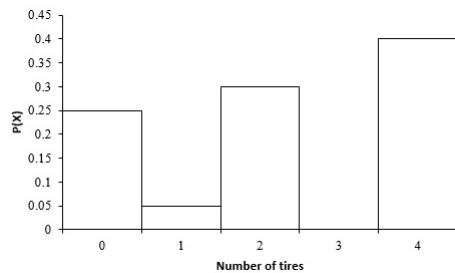


23. continued



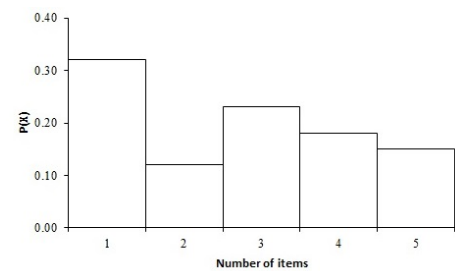
21.

X	0	1	2	3	4
P(X)	0.25	0.05	0.30	0.00	0.40



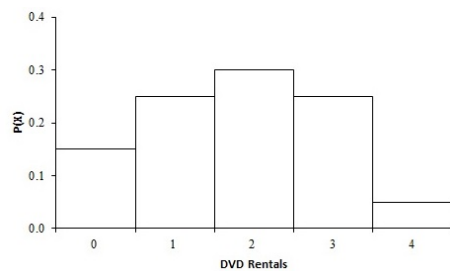
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X	1	2	3	4	5
P(X)	0.32	0.12	0.23	0.18	0.15



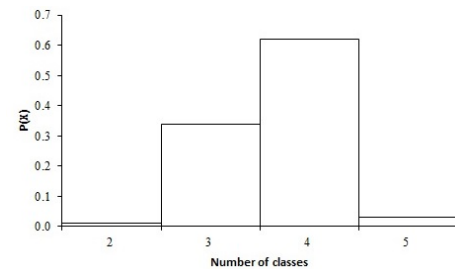
22.

X	0	1	2	3	4
P(X)	0.15	0.25	0.3	0.25	0.05



25.

X	2	3	4	5
P(X)	0.01	0.34	0.62	0.03



23.

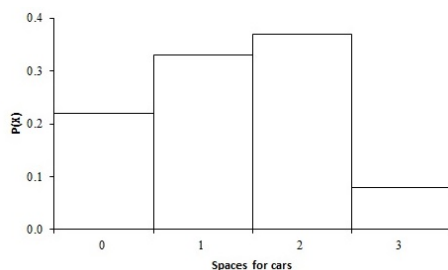
X	1	2	3	4	5	6
P(X)	1/2	1/6	1/12	1/12	1/12	1/12

26.

X	0	1	2	3
P(X)	0.22	0.33	0.37	0.08

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26. continued



27.

X	4	7	9	11	13	16	18
P(X)	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{15}$

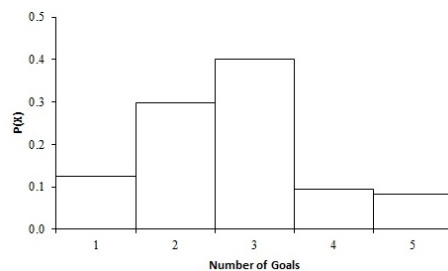
X	21	22	24	25	27	31	36
P(X)	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{1}{15}$

28.

X	Computer	Dress-Up	Blocks	Paint
P(X)	0.45	0.27	0.18	0.1

29.

X	1	2	3	4	5
P(X)	0.124	0.297	0.402	0.094	0.083



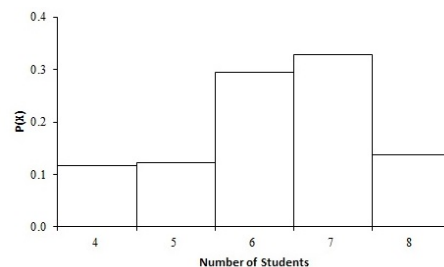
30.

$$P(4 - 7 \text{ students}) = 0.117 + 0.123 + 0.295 + 0.328 = 0.863$$

$$P(8 \text{ students}) = 1 - 0.863 = 0.137$$

X	4	5	6	7	8
P(X)	0.117	0.123	0.295	0.328	0.137

30. continued



31.

X	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$

Yes.

32.

X	0.2	0.3	0.5
P(X)	0.2	0.3	0.5

Yes.

33.

X	3	4	7
P(X)	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{7}{6}$

No, the sum of the probabilities is greater than one and $P(7) = \frac{7}{6}$ which is also greater than 1.

34.

X	0.1	0.02	0.04
P(X)	0.2	0.12	0.14

No, the sum of the probabilities is less than 1.

35.

X	1	2	4
P(X)	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{4}{7}$

Yes.

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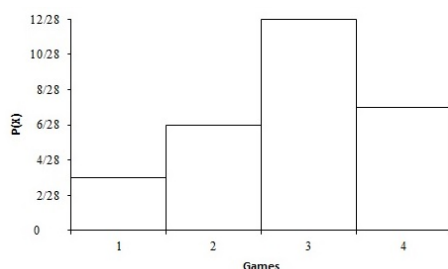
36.

X	0	1	2
P(X)	0	$\frac{1}{3}$	$\frac{1}{2}$

No, the sum of the probabilities is less than 1.

37.

X	1	2	3	4
P(X)	$\frac{3}{28}$	$\frac{6}{28}$	$\frac{12}{28}$	$\frac{7}{28}$



EXERCISE SET 5-2

1.

X	0	1	2	3	4
P(X)	0.31	0.42	0.21	0.04	0.02

$$\mu = \sum X \cdot P(X) = 0(0.31) + 1(0.42) + 2(0.21) + 3(0.04) + 4(0.02) = 1.04$$

$$\sigma^2 = \sum X^2 \cdot P(X) - \mu^2 = [0^2(0.31) + 1^2(0.42) + 2^2(0.21) + 3^2(0.04) + 4^2(0.02)] - 1.04^2$$

$$\sigma^2 = 0.858$$

$$\sigma = \sqrt{0.858} = 0.926$$

X	P(X)	X · P(X)	X ² · P(X)
0	0.31	0	0
1	0.42	0.42	0.42
2	0.21	0.42	0.84
3	0.04	0.12	0.36
4	0.02	<u>0.08</u>	<u>0.32</u>
		$\mu = 1.04$	1.94

2.

$$\mu = 19(0.2) + 20(0.2) + 21(0.3) + 22(0.2) + 23(0.1) = 20.8$$

$$\sigma^2 = [19^2(0.2) + 20^2(0.2) + 21^2(0.3) + 22^2(0.2) + 23^2(0.1)] - 20.8^2 = 1.56 \text{ or } 1.6$$

$$\sigma = \sqrt{1.56} = 1.25 \text{ or } 1.3$$

The manager would need to purchase 20.8(5) = 104 suits.

X	P(X)	X · P(X)	X ² · P(X)
8	0.1	0.8	6.4
9	0.2	1.8	16.2
10	0.2	2.0	20.0
11	0.3	3.3	36.3
12	0.2	<u>2.4</u>	<u>28.8</u>
		$\mu = 10.3$	107.7

3.

$$\mu = \sum X \cdot P(X) = 0(0.42) + 1(0.35) + 2(0.20) + 3(0.03) = 0.84 \text{ or } 0.8$$

$$\sigma^2 = \sum X^2 \cdot P(X) - \mu^2 = [0^2(0.42) + 1^2(0.35) + 2^2(0.20) + 3^2(0.03)] - 0.84^2 = 0.71 \text{ or } 0.7$$

$$\sigma = \sqrt{0.71} = 0.85 \text{ or } 0.9$$

X	P(X)	X · P(X)	X ² · P(X)
0	0.42	0	0
1	0.35	0.35	0.35
2	0.20	0.40	0.80
3	0.03	<u>0.09</u>	<u>0.27</u>
		$\mu = 0.84$	1.42

4.

$$\mu = \sum X \cdot P(X) = 5(0.05) + 6(0.2) + 7(0.4) + 8(0.1) + 9(0.15) + 10(0.1) = 7.4$$

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4. continued

$$\sigma^2 = \sum X^2 \cdot P(X) - \mu^2 = [5^2(0.05) + 6^2(0.2) + 7^2(0.4) + 8^2(0.1) + 9^2(0.15) + 10^2(0.1)] - 7.4^2 = 1.84 \text{ or } 1.8$$

$$\sigma = \sqrt{1.8} = 1.4$$

X	P(X)	X · P(X)	X ² · P(X)
5	0.05	0.25	1.25
6	0.20	1.20	7.20
7	0.40	2.80	19.6
8	0.10	0.80	6.40
9	0.15	1.35	12.15
10	0.1	<u>1.00</u>	<u>10.00</u>
		$\mu = 7.4$	56.6

5.

$$\mu = \sum X \cdot P(X) = 6(0.30) + 7(0.40) + 8(0.25) + 9(0.05) = 7.05$$

$$\begin{aligned}\sigma^2 &= \sum X^2 \cdot P(X) - \mu^2 \\ &= [6^2(0.30) + 7^2(0.40) + 8^2(0.25) + 9^2(0.05)] - 7.05^2 \\ &= 0.75\end{aligned}$$

$$\sigma = \sqrt{0.75} = 0.86$$

X	P(X)	X · P(X)	X ² · P(X)
6	0.30	1.8	10.8
7	0.40	2.8	19.6
8	0.25	2.0	16.0
9	0.05	<u>0.45</u>	<u>4.05</u>
		$\mu = 7.05$	50.45

6.

$$\mu = \sum X \cdot P(X) = 0(0.4) + 1(0.2) + 2(0.2) + 3(0.1) + 4(0.1) = 1.3$$

$$\begin{aligned}\sigma^2 &= \sum X^2 \cdot P(X) - \mu^2 = [0^2(0.4) + 1^2(0.2) + 2^2(0.2) + 3^2(0.1) + 4^2(0.1)] - 1.3^2 \\ &= 1.81 \text{ or } 1.8\end{aligned}$$

6. continued

$$\sigma = \sqrt{1.8} = 1.3$$

X	P(X)	X · P(X)	X ² · P(X)
0	0.4	0	0
1	0.2	0.2	0.2
2	0.2	0.4	0.8
3	0.1	0.3	0.9
4	0.1	<u>0.4</u>	<u>1.6</u>
		$\mu = 1.3$	3.5

7.

$$\begin{aligned}\mu &= \sum X \cdot P(X) = 0(0.1) + 1(0.2) + 2(0.4) + 3(0.2) + 4(0.1) = 2.0 \\ \mu &= 2\end{aligned}$$

$$\begin{aligned}\sigma^2 &= \sum X^2 \cdot P(X) - \mu^2 = [0^2(0.1) + 1^2(0.2) + 2^2(0.4) + 3^2(0.2) + 4^2(0.1)] - 2.0^2 = 1.2 \\ \sigma^2 &= 1.2\end{aligned}$$

$$\sigma = \sqrt{1.2} = 1.1$$

X	P(X)	X · P(X)	X ² · P(X)
0	0.1	0	0
1	0.2	0.2	0.2
2	0.4	0.8	1.6
3	0.2	0.6	1.8
4	0.1	<u>0.4</u>	<u>1.6</u>
		$\mu = 2.0$	5.2

8.

$$\begin{aligned}\mu &= \sum X \cdot P(X) = 1(0.301) + 2(0.176) + 3(0.125) + 4(0.097) + 5(0.079) + 6(0.067) \\ &\quad + 7(0.058) + 8(0.051) + 9(0.046) = 3.441\end{aligned}$$

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8. continued

X	P(X)	X · P(X)
1	0.301	0.301
2	0.176	0.352
3	0.125	0.375
4	0.097	0.388
5	0.079	0.395
6	0.067	0.402
7	0.058	0.406
8	0.051	0.408
9	0.046	<u>0.414</u>

$$\mu = 3.441$$

9.

$$\mu = \sum X \cdot P(X) = 1(0.27) + 2(0.46) + 3(0.21) + 4(0.05) + 5(0.01) = 2.07 \text{ or } 2.1$$

$$\sigma^2 = \sum X^2 \cdot P(X) - \mu^2 = [1^2(0.27) + 2^2(0.46) + 3^2(0.21) + 4^2(0.05) + 5^2(0.01)] - 2.1^2 = 0.765 \text{ or } 0.80$$

$$\sigma = \sqrt{0.80} = 0.89 \text{ or } 0.90$$

X	P(X)	X · P(X)	X ² · P(X)
1	0.27	0.27	0.27
2	0.46	0.92	1.84
3	0.21	0.63	1.89
4	0.05	0.20	0.80
5	0.01	<u>0.05</u>	<u>0.25</u>

$$\mu = 2.1 \quad 5.05$$

10.

$$\mu = \sum X \cdot P(X) = 35(0.1) + 36(0.2) + 37(0.3) + 38(0.3) + 39(0.1) = 37.1$$

$$\sigma^2 = \sum X^2 \cdot P(X) - \mu^2 = [35^2(0.1) + 36^2(0.2) + 37^2(0.3) + 38^2(0.3) + 39^2(0.1)] - 37.1^2 = 1.29 \text{ or } 1.3$$

$$\sigma = \sqrt{1.39} = 1.136 \text{ or } 1.1$$

10. continued

X	P(X)	X · P(X)	X ² · P(X)
35	0.1	3.5	122.5
36	0.2	7.2	259.2
37	0.3	11.1	410.7
38	0.3	11.4	433.2
39	0.1	<u>3.9</u>	<u>152.1</u>

$$\mu = 37.1 \quad 1377.7$$

It could happen (perhaps on a Super Bowl Sunday), but it is highly unlikely.

11.

Bag Value	X = Net Amt Won	P(X)
\$1	− \$1	$\frac{10}{20}$
\$2	\$0	$\frac{6}{20}$
\$3	\$1	$\frac{4}{20}$

$$E(X) = \sum X \cdot P(X)$$

$$E(X) = [-\$1(\frac{10}{20}) + \$0(\frac{6}{20}) + \$1(\frac{4}{20})] = -\$0.30$$

12.

$$\mu = 1(0.2) + 2(0.3) + 3(0.4) + 4(0.1) = 2.4$$

$$\text{Expected Profit} = 2.4(\$3000) = \$7200$$

13.

$$E(X) = \sum X \cdot P(X) = \$5.00(\frac{1}{6}) = \$0.83$$

He should pay about \$0.83.

14.

X	Win	P(X)
1	\$0	$\frac{1}{6}$
2	\$0	$\frac{1}{6}$
3	\$1	$\frac{1}{6}$
4	\$2	$\frac{1}{6}$
5	\$3	$\frac{1}{6}$
6	\$4	$\frac{1}{6}$

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14. continued

$$\begin{aligned} E(X) &= \sum X \cdot P(X) = \left[\$1\left(\frac{1}{6}\right) + \$2\left(\frac{1}{6}\right) \right. \\ &\quad \left. + \$3\left(\frac{1}{6}\right) + \$4\left(\frac{1}{6}\right) \right] - \$2 \\ &= -\$0.33 \text{ or } -33\text{¢} \end{aligned}$$

No; the game favors the house.

15.

$$\begin{aligned} E(X) &= \sum X \cdot P(X) = \$1000\left(\frac{1}{1000}\right) + \\ &\quad \$500\left(\frac{1}{1000}\right) + \$100\left(\frac{5}{1000}\right) - \$3.00 \\ E(X) &= -\$1.00 \end{aligned}$$

Alternate Solution:

$$\begin{aligned} E(X) &= 997\left(\frac{1}{1000}\right) + 497\left(\frac{1}{1000}\right) \\ &\quad + 97\left(\frac{5}{1000}\right) - 3\left(\frac{993}{1000}\right) \\ &= -\$1.00 \end{aligned}$$

16.

$$E(X) = 2(-1.00) = -\$2.00$$

17.

$$\begin{aligned} E(X) &= \sum X \cdot P(X) = \$500\left(\frac{1}{1000}\right) - \$1.00 \\ E(X) &= -\$0.50 \end{aligned}$$

Alternate Solution:

$$E(X) = \$499\left(\frac{1}{1000}\right) - 1\left(\frac{999}{1000}\right) = -\$0.50$$

There are 6 possibilities when a number with all different digits is boxed,
($3 \cdot 2 \cdot 1 = 6$).

$$E(X) = \$80\left(\frac{6}{1000}\right) - \$1.00$$

$$E(X) = \$0.48 - \$1.00 = -\$0.52$$

Alternate Solution:

$$E(X) = 79\left(\frac{6}{1000}\right) - 1\left(\frac{994}{1000}\right) = -\$0.52$$

18.

$$E(X) = \$360(0.999057) -$$

$$\$ (100,000 - 360)(0.000943) = \$265.70$$

19.

$$\text{a. } P(\text{red}) = \frac{18}{38}(\$1) + \frac{20}{38}(-\$1)$$

$$P(\text{red}) = -\$0.0526$$

19. continued

$$\begin{aligned} \text{b. } P(\text{even}) &= \frac{18}{38}(\$1) + \frac{20}{38}(-\$1) \\ P(\text{even}) &= -\$0.0526 \end{aligned}$$

$$\begin{aligned} \text{c. } P(00) &= \frac{1}{38}(\$35) + \frac{37}{38}(-\$1) \\ P(00) &= -\$0.0526 \end{aligned}$$

$$\begin{aligned} \text{d. } P(\text{any single number}) &= \\ \frac{1}{38}(\$35) + \frac{37}{38}(-\$1) &= -\$0.0526 \end{aligned}$$

$$\begin{aligned} \text{e. } P(0 \text{ or } 00) &= \\ \frac{2}{38}(\$17) + \frac{36}{38}(-\$1) &= -\$0.0526 \end{aligned}$$

20.

X	2	3	4	5	6
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$

X	7	8	9	10	11	12
P(X)	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$\begin{aligned} \mu &= \sum X \cdot P(X) = 2\left(\frac{1}{36}\right) + 3\left(\frac{2}{36}\right) + 4\left(\frac{3}{36}\right) + \\ &\quad 5\left(\frac{4}{36}\right) + 6\left(\frac{5}{36}\right) + 7\left(\frac{6}{36}\right) + 8\left(\frac{5}{36}\right) + 9\left(\frac{4}{36}\right) + \\ &\quad 10\left(\frac{3}{36}\right) + 11\left(\frac{2}{36}\right) + 12\left(\frac{1}{36}\right) = 7 \end{aligned}$$

$$\begin{aligned} \sigma^2 &= \sum X^2 \cdot P(X) - \mu^2 = \left[2^2\left(\frac{1}{36}\right) + 3^2\left(\frac{2}{36}\right) \right. \\ &\quad \left. + 4^2\left(\frac{3}{36}\right) + 5^2\left(\frac{4}{36}\right) + 6^2\left(\frac{5}{36}\right) + 7^2\left(\frac{6}{36}\right) + \right. \\ &\quad \left. 8^2\left(\frac{5}{36}\right) + 9^2\left(\frac{4}{36}\right) + 10^2\left(\frac{3}{36}\right) + 11^2\left(\frac{2}{36}\right) + \right. \\ &\quad \left. 12^2\left(\frac{1}{36}\right) \right] - 7^2 = 5.83 \text{ or } 5.8 \end{aligned}$$

$$\sigma = \sqrt{5.83} = 2.4$$

21.

The expected value for a single die is 3.5, and since 3 die are rolled, the expected value is $3(3.5) = 10.5$

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22.

$$\begin{aligned}\sigma^2 &= \sum (X - \mu)^2 \cdot P(X) \\ \sigma^2 &= \sum (X^2 - 2\mu X + \mu^2)P(X) \\ \sigma^2 &= \sum [X^2 P(X) - 2\mu X P(X) + \mu^2 P(X)] \\ \sigma^2 &= \sum x^2 P(X) - 2\mu \sum x P(X) + \mu^2 \sum P(X) \\ \sigma^2 &= \sum X^2 \cdot P(X) - 2\mu \cdot \mu + \mu^2(1) \\ \sigma^2 &= \sum X^2 \cdot P(X) - 2\mu^2 + \mu^2 \\ \sigma^2 &= \sum X^2 \cdot P(X) - \mu^2\end{aligned}$$

23.

Let $x = P(4)$. Then $\frac{2}{3}x = P(6)$.

$$0.23 + 0.18 + x + \frac{2}{3}x + 0.015 = 1$$

$$0.425 + \frac{5}{3}x = 1$$

$$\frac{5}{3}x = 0.575$$

$$x = P(4) = 0.345$$

$$\text{Then } \frac{2}{3}x = P(6) = 0.23$$

X	P(X)	X · P(X)	X ² · P(X)
1	0.23	0.23	0.23
2	0.18	0.36	0.72
4	0.345	1.38	5.52
6	0.23	1.38	8.28
9	0.015	<u>0.135</u>	<u>1.215</u>
		$\mu = 3.485$	15.965

$$\begin{aligned}\mu &= 1(0.23) + 2(0.18) + 4(0.345) + 6(0.23) \\ &\quad + 9(0.015) = 3.485\end{aligned}$$

$$\begin{aligned}\sigma^2 &= [1^2(0.23) + 2^2(0.18) + 4^2(0.345) + \\ &\quad 6^2(0.23) + 9^2(0.015)] - 3.485^2 = 3.819\end{aligned}$$

$$\sigma = \sqrt{3.819} = 1.954$$

24.

Answers will vary.

25.

Answers will vary.

26.

$$\begin{aligned}E(X) &= [\$100,000 \cdot \frac{1}{1,000,000} \\ &\quad + 10,000 \cdot \frac{2}{50,000} + 1000 \cdot \frac{5}{10,000} \\ &\quad + 100 \cdot \frac{10}{1000}] - \$0.47 \\ &= \$0.10 + \$0.40 + \$0.50 + \$1.00 - \$0.47 \\ &= \$1.53 \text{ (with the cost of a stamp = \$0.47)}\end{aligned}$$

27.

List the possible outcomes with the sums of the numbers on the balls:

(1,2) = 3	(4,1) = 5
(1,4) = 5	(4,2) = 6
(1,7) = 8	(4,7) = 11
(1,*) = 2	(4,*) = 8
(2,1) = 3	(7,1) = 8
(2,4) = 6	(7,2) = 9
(2,7) = 9	(7,4) = 11
(2,*) = 4	(7,*) = 14

(*,1) = 2
(*,2) = 4
(*,4) = 8
(*,7) = 14

Totals	f	P(X)
2	2	0.1
3	2	0.1
4	2	0.1
5	2	0.1
6	2	0.1
8	4	0.2
9	2	0.1
11	2	0.1
14	2	0.1

$$\begin{aligned}\mu &= 2(0.1) + 3(0.1) + 4(0.1) + 5(0.1) + \\ &\quad 6(0.1) + 8(0.2) + 9(0.1) + 11(0.1) + \\ &\quad 14(0.1) = 7\end{aligned}$$

$$\begin{aligned}\sigma^2 &= [2^2(0.1) + 3^2(0.1) + 4^2(0.1) + 5^2(0.1) \\ &\quad + 6^2(0.1) + 8^2(0.2) + 9^2(0.1) + 11^2(0.1) + \\ &\quad 14^2(0.1)] - 7^2 = 12.6 \\ \sigma &= \sqrt{12.6} = 3.55\end{aligned}$$

Chapter 5 - Discrete Probability Distributions

EXERCISE SET 5-3

1.

- a. Yes
- b. Yes
- c. Yes
- d. No, there are more than two outcomes.
- e. No, there are more than two outcomes.

2.

- a. Yes
- b. Yes
- c. Yes
- d. No, there are more than two outcomes.
- e. Yes

3.

- a. 0.420
- b. 0.346
- c. 0.590
- d. 0.251
- e. 0.000

4.

- a. 0.250
- b. 0.418
- c. 0.176
- d. 0.246

5.

$$a. P(X) = \frac{n!}{(n-X)! X!} \cdot p^X \cdot q^{n-X}$$

$$P(X) = \frac{6!}{3! \cdot 3!} \cdot (0.03)^3 (0.97)^3 = 0.0005$$

$$b. P(X) = \frac{4!}{2! \cdot 2!} \cdot (0.18)^2 \cdot (0.82)^2 = 0.131$$

$$c. P(X) = \frac{5!}{2! \cdot 3!} \cdot (0.63)^3 \cdot (0.37)^2 = 0.342$$

6.

$$a. P(X) = \frac{9!}{9! \cdot 0!} \cdot (0.42)^0 \cdot (0.58)^9 = 0.007$$

$$b. P(X) = \frac{10!}{5! \cdot 5!} \cdot (0.37)^5 \cdot (0.63)^5 = 0.173$$

7.

$$n = 10, p = 0.10$$

$$a. P(\text{at least two}) = 1 - P(\text{none}) - P(\text{one})$$

$$P(X \geq 2) = 1 - 0.349 - 0.387 = 0.264$$

b.

$$P(X = 2 \text{ or } 3) = 0.194 + 0.057 = 0.251$$

c.

$$P(X = 1) = 0.387$$

8.

$$n = 10, p = 0.2, X \geq 15$$

$$P(X) = 0.000$$

The event is unlikely to occur because the probability is extremely small (≈ 0).

9.

$$a. n = 16, p = 0.20, X = 0$$

$$P(X) = \frac{16!}{0! 16!} (0.20)^0 (0.80)^{16} = 0.028$$

$$b. n = 16, P = 0.20, X = 8$$

$$P(X) = \frac{16!}{8! 8!} (0.20)^8 (0.80)^8 = 0.006$$

$$c. n = 16, p = 0.20, X = 4$$

$$P(X) = \frac{16!}{4! 12!} (0.20)^4 (0.80)^{12} = 0.200$$

10.

$$a. n = 10, p = 0.103, X = 0, 1, \text{ or } 2$$

$$P(X) = \frac{10!}{10! 0!} (0.103)^0 (0.897)^{10} +$$

$$\frac{10!}{9! 1!} (0.103)^1 (0.897)^9 + \frac{10!}{8! 2!} (0.103)^2 (0.897)^8$$

$$= 0.337 + 0.387 + 0.2 = 0.925$$

$$b. n = 10, p = 0.897, X \geq 6$$

$$P(X) = \frac{10!}{4! 6!} (0.897)^6 (0.103)^4 +$$

$$\frac{10!}{3! 7!} (0.897)^7 (0.103)^3 + \frac{10!}{2! 8!} (0.897)^8 (0.103)^2$$

$$+ \frac{10!}{1! 9!} (0.897)^9 (0.103)^1 + \frac{10!}{0! 10!} (0.897)^{10} (0.103)^0$$

$$P(X) = 0.0123 + 0.0613 + 0.2001 +$$

$$0.3872 + 0.3372 = 0.998$$

Chapter 5 - Discrete Probability Distributions

10. continued

c. $n = 10, p = 0.897, X = 10$

$$P(X) = \frac{10!}{0!10!}(0.897)^{10}(0.103)^0 = 0.337$$

11.

$n = 9, p = \frac{3}{4} = 0.75, X = 5$

$$P(X) = \frac{9!}{5!4!}(0.75)^5(0.25)^4 = 0.117$$

12.

a. $n = 15, p = 0.789, q = 0.211, X = 5$

$$P(X) = \frac{15!}{10!5!}(0.789)^5(0.211)^{10} = 0.000$$

b. $n = 15, p = 0.789, q = 0.211, X \geq 5$

$$P(X) = \frac{15!}{10!5!}(0.789)^5(0.211)^{10} +$$

$$\frac{15!}{9!6!}(0.789)^6(0.211)^9 + \dots +$$

$$\frac{15!}{0!15!}(0.789)^0(0.211)^{15} = 0.999$$

c. $n = 15, p = 0.211, q = 0.789, X \geq 9$

$$P(X) = \frac{15!}{9!6!}(0.211)^9(0.789)^6 +$$

$$\frac{15!}{10!5!}(0.211)^{10}(0.789)^5 + \dots +$$

$$\frac{15!}{0!15!}(0.211)^0(0.789)^{15} = 0.001$$

13.

$n = 5, p = 0.40$

a. $X = 3, P(X) = 0.230$

b. $X = 0, 1, 2, 3, \text{ or } 4 \text{ people}$

$$P(X) = 0.078 + 0.259 + 0.346 + 0.230 \\ + 0.077 = 0.99$$

c. $n = 5, p = 0.40, X \geq 3$

$$P(X) = 0.230 + 0.077 + 0.01 = 0.317$$

d. $X = 0, \text{ or } 1 \text{ person}$

$$P(X) = 0.078 + 0.259 = 0.337$$

14.

a. $n = 12, p = 0.26, X = 6$

$$P(X) = 0.047$$

b. $n = 12, p = 0.26, X \geq 6$

$$P(X) = 0.065$$

c. $n = 12, p = 0.26, X < 5$

$$P(X) = 0.821$$

15.

a. $n = 10, p = 0.53, X = 5$

$$P(X) = \frac{10!}{5!5!}(0.53)^5(0.47)^5 = 0.242$$

b. $n = 10, p = 0.47, X \geq 5$

$$P(X) = \frac{10!}{5!5!}(0.47)^5(0.53)^5 +$$

$$\frac{10!}{6!4!}(0.47)^6(0.53)^4 + \frac{10!}{7!3!}(0.47)^7(0.53)^3 +$$

$$\frac{10!}{8!2!}(0.47)^8(0.53)^2 + \frac{10!}{9!1!}(0.47)^9(0.53)^1 +$$

$$\frac{10!}{10!0!}(0.47)^{10}(0.53)^0 = 0.548$$

c. $n = 10, p = 0.53, X < 5$

$$P(X) = \frac{10!}{5!5!}(0.53)^5(0.47)^5 +$$

$$\frac{10!}{4!6!}(0.53)^4(0.47)^6 + \frac{10!}{3!7!}(0.53)^3(0.47)^7 +$$

$$\frac{10!}{2!8!}(0.53)^2(0.47)^8 + \frac{10!}{1!9!}(0.53)^1(0.47)^9$$

$$\frac{10!}{0!10!}(0.53)^0(0.47)^{10} = 0.306$$

16.

a. $n = 5, p = 0.05, X = 2$

$$P(X) = 0.021 \text{ (TI} = 0.0214)$$

b. $n = 5, p = 0.05, X > 2$

$$P(X) = 0.001 \text{ (TI} = 0.001158)$$

c. $n = 6, p = 0.05, X = 5$

$$P(X) = 0 \text{ (TI} = 0.0000003)$$

Chapter 5 - Discrete Probability Distributions

16. continued

d. The answers are reasonable because the probability any component will fail is very small (0.05). The probabilities of more than one part failing get increasingly smaller.

17.

$$\begin{aligned} \text{a. } \mu &= 100(0.75) = 75 \\ \sigma^2 &= 100(0.75)(0.25) = 18.75 \text{ or } 18.8 \\ \sigma &= \sqrt{18.75} = 4.33 \text{ or } 4.3 \end{aligned}$$

$$\begin{aligned} \text{b. } \mu &= 300(0.3) = 90 \\ \sigma^2 &= 300(0.3)(0.7) = 63 \\ \sigma &= \sqrt{63} = 7.94 \text{ or } 7.9 \end{aligned}$$

$$\begin{aligned} \text{c. } \mu &= 20(0.5) = 10 \\ \sigma^2 &= 20(0.5)(0.5) = 5 \\ \sigma &= \sqrt{5} = 2.236 \text{ or } 2.2 \end{aligned}$$

$$\begin{aligned} \text{d. } \mu &= 10(0.8) = 8 \\ \sigma^2 &= 10(0.8)(0.2) = 1.6 \\ \sigma &= \sqrt{1.6} = 1.265 \text{ or } 1.3 \end{aligned}$$

18.

$$\begin{aligned} \text{a. } \mu &= 1000(0.1) = 100 \\ \sigma^2 &= 1000(0.1)(0.9) = 90 \\ \sigma &= \sqrt{90} = 9.49 \text{ or } 9.5 \end{aligned}$$

$$\begin{aligned} \text{b. } \mu &= 500(0.25) = 125 \\ \sigma^2 &= 500(0.25)(0.75) = 93.75 \text{ or } 93.8 \\ \sigma &= \sqrt{93.75} = 9.68 \text{ or } 9.7 \end{aligned}$$

$$\begin{aligned} \text{c. } \mu &= 50\left(\frac{2}{5}\right) = 20 \\ \sigma^2 &= 50\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) = 12 \\ \sigma &= \sqrt{12} = 3.464 \text{ or } 3.5 \end{aligned}$$

$$\begin{aligned} \text{d. } \mu &= 36\left(\frac{1}{6}\right) = 6 \\ \sigma^2 &= 36\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = 5 \\ \sigma &= \sqrt{5} = 2.236 \text{ or } 2.2 \end{aligned}$$

19.

$$\begin{aligned} n &= 300, p = 0.25 \\ \mu &= 300(0.25) = 75 \\ \sigma^2 &= 300(0.25)(0.75) = 56.25 \\ \sigma &= \sqrt{56.25} = 7.5 \end{aligned}$$

20.

$$\begin{aligned} n &= 10, p = \frac{1}{2} \text{ or } 0.5 \\ \mu &= 10(0.5) = 5 \\ \sigma^2 &= 10(0.5)(0.5) = 2.5 \\ \sigma &= \sqrt{2.5} = 1.58 \end{aligned}$$

21.

If 13.1% are foreign-born, 86.9% are native Americans. In a sample of 60, one could expect $60(0.869) = 52.1$ or 52 approximately to be American-born.

$$\begin{aligned} n &= 60, p = 0.131 \\ \mu &= 60(0.131) = 7.86 \\ \sigma^2 &= 60(0.131)(0.869) = 6.8 \\ \sigma &= \sqrt{6.8} = 2.6 \end{aligned}$$

22.

$$\begin{aligned} n &= 200, p = 0.83 \\ \mu &= 200(0.83) = 166 \\ \sigma^2 &= 200(0.83)(0.17) = 28.2 \\ \sigma &= \sqrt{28.2} = 5.3 \end{aligned}$$

23.

$$\begin{aligned} n &= 1000, p = 0.21 \\ \mu &= 1000(0.21) = 210 \\ \sigma^2 &= 1000(0.21)(0.79) = 165.9 \\ \sigma &= \sqrt{165.9} = 12.9 \end{aligned}$$

24.

$$\begin{aligned} n &= 120, p = 0.85 \\ \mu &= 120(0.85) = 102 \\ \sigma^2 &= 120(0.85)(0.15) = 15.3 \\ \sigma &= \sqrt{15.3} = 3.9 \end{aligned}$$

Chapter 5 - Discrete Probability Distributions

25.

$$n = 18, p = 0.25, X = 5$$

$$P(X) = \frac{18!}{13! 5!} (0.25)^5 (0.75)^{13} = 0.199$$

26.

$$n = 14, p = 0.63, X = 9$$

$$P(X) = \frac{14!}{5! 9!} (0.63)^9 (0.37)^5 = 0.217$$

27.

$$n = 10, p = \frac{1}{3}, X = 0, 1, 2, 3$$

$$P(X) = \frac{10!}{10! 0!} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^{10} + \frac{10!}{9! 1!} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^9 \\ + \frac{10!}{8! 2!} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^8 + \frac{10!}{7! 3!} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^7 = 0.559$$

28.

$$n = 200, p = 0.32$$

$$\mu = 200(0.32) = 64$$

$$\sigma^2 = (200)(0.32)(0.68) = 43.52$$

$$\sigma = \sqrt{43.52} = 6.597 \text{ or } 6.6$$

29.

$$n = 20, p = 0.58, X = 14$$

$$P(X) = \frac{20!}{6! 14!} (0.58)^{14} (0.42)^6 = 0.104$$

30.

$$n = 5, p = 0.13, X = 3, 4, 5$$

$$P(X) = \frac{5!}{2! 3!} (0.13)^3 (0.87)^2 + \\ \frac{5!}{1! 4!} (0.13)^4 (0.87)^1 + \frac{5!}{0! 5!} (0.13)^5 (0.87)^0 \\ = 0.018$$

31.

$$n = 7, p = 0.14, X = 2 \text{ or } 3$$

$$P(X) = \frac{7!}{5! 2!} (0.14)^2 (0.86)^5 + \\ \frac{7!}{4! 3!} (0.14)^3 (0.86)^4 = 0.246$$

32.

Yes. $P(3) = 0.216$. This implies that $p = 0.6$ and then $q = 0.4$.

Using the binomial table, $P(0) = 0.064$, $P(1) = 0.288$, and $P(2) = 0.432$.

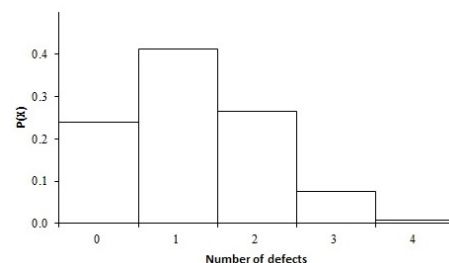
33.

X	0	1	2	3
P(X)	0.125	0.375	0.375	0.125

34.

$$n = 4, p = 0.3, X = 0, 1, 2, 3, 4$$

X	0	1	2	3	4
P(X)	0.240	0.412	0.265	0.076	0.008



35.

If $n = 3$, the outcomes and their corresponding probabilities are:

$$X = 0; P(0) = \frac{3!}{3! 0!} (p)^0 (q)^3 = q^3$$

$$X = 1; P(1) = \frac{3!}{2! 1!} (p)^1 (q)^2 = 3pq^2$$

$$X = 2; P(2) = \frac{3!}{1! 2!} (p)^2 (q)^1 = 3p^2q$$

$$X = 3; P(3) = \frac{3!}{0! 3!} (p)^3 (q)^0 = p^3$$

Chapter 5 - Discrete Probability Distributions

35. continued

$$\mu = \sum X \cdot P(X)$$

$$\mu = 0(q^3) + 1(3pq^2) + 2(3p^2q) + 3(p^3)$$

$$\mu = 3pq^2 + 6p^2q + 3p^3$$

$$\mu = 3p(q^2 + 2pq + p^2)$$

$$\mu = 3p(q + p)^2$$

$$\mu = 3p(1) = 3p$$

EXERCISE SET 5-4

1.

$$\begin{aligned} \text{a. } P(M) &= \frac{6!}{3!2!1!}(0.5)^3(0.3)^2(0.2)^1 \\ &= 0.135 \end{aligned}$$

$$\begin{aligned} \text{b. } P(M) &= \frac{5!}{1!2!2!}(0.3)^1(0.6)^2(0.1)^2 \\ &= 0.0324 \end{aligned}$$

$$\begin{aligned} \text{c. } P(M) &= \frac{4!}{1!1!2!}(0.8)^1(0.1)^1(0.1)^2 \\ &= 0.0096 \end{aligned}$$

2.

$$\begin{aligned} \text{a. } P(M) &= \frac{3!}{1!1!1!}(0.5)^1(0.3)^1(0.2)^1 \\ &= 0.18 \end{aligned}$$

$$\begin{aligned} \text{b. } P(M) &= \frac{5!}{1!3!1!}(0.7)^1(0.2)^3(0.1)^1 \\ &= 0.0112 \end{aligned}$$

$$\begin{aligned} \text{c. } P(M) &= \frac{7!}{2!3!2!}(0.4)^2(0.5)^3(0.1)^2 \\ &= 0.042 \end{aligned}$$

3.

$$\begin{aligned} P(M) &= \\ \frac{12!}{2!2!2!2!2!2!}(0.13)^2(0.13)^2(0.14)^2(0.16)^2 \\ (0.2)^2(0.24)^2 &= 0.0025 \end{aligned}$$

4.

$$\begin{aligned} P(M) &= \frac{5!}{3!1!1!}(0.5)^3(0.4)^1(0.1)^1 \\ &= 0.1 \end{aligned}$$

5.

$$\begin{aligned} P(M) &= \frac{10!}{3!3!2!2!}(0.5)^3(0.3)^3(0.15)^2(0.05)^2 \\ &= 0.0048 \end{aligned}$$

6.

$$\begin{aligned} P(M) &= \frac{8!}{1!3!3!1!}\left(\frac{9}{16}\right)^1\left(\frac{3}{16}\right)^3\left(\frac{3}{16}\right)^3\left(\frac{1}{16}\right)^1 \\ &= 0.0017 \text{ or } 0.002 \end{aligned}$$

7.

$$\text{a. } P(6; 4) = 0.1042$$

$$\text{b. } P(2; 5) = 0.0842$$

$$\text{c. } P(7; 3) = 0.0216$$

8.

$$\text{a. } P(10; 7) = 0.071$$

$$\text{b. } P(9; 8) = 0.1241$$

$$\text{c. } P(3; 4) = 0.1954$$

9.

$$\begin{aligned} p &= \frac{1}{20,000} = 0.00005 \\ \lambda &= n \cdot p = 80,000(0.00005) = 4 \end{aligned}$$

$$\text{a. } P(0; 4) = 0.0183$$

$$\text{b. } P(1; 4) = 0.0733$$

$$\text{c. } P(2; 4) = 0.1465$$

$$\begin{aligned} \text{d. } P(3 \text{ or more}; 4) &= 1 - [P(0; 4) + P(1; 4) \\ &\quad + P(2; 4)] \\ &= 1 - (0.0183 + 0.0733 + 0.1465) \\ &= 0.7619 \end{aligned}$$

10.

$$\begin{aligned} \lambda &= \frac{200}{400} = 0.5 \\ P(1, 0.5) &= 0.3033 \end{aligned}$$

11.

$$\begin{aligned} p &= 0.05 \\ \lambda &= n \cdot p = (200) \cdot (0.05) = 10 \\ P(14; 10) &= 0.0521 \end{aligned}$$

Chapter 5 - Discrete Probability Distributions

12.

$$p = \frac{5}{500} = 0.01$$

$$\lambda = n \cdot p = 100 \cdot (0.01) = 1$$

$$\begin{aligned} P(\text{at least } 2; 1) &= 1 - [P(0; 1) + P(1; 1)] \\ &= 1 - (0.3679 + 0.3679) = 0.2642 \end{aligned}$$

13.

$$\lambda = 200(0.015) = 3$$

$$P(0; 3) = 0.0498$$

14.

$$\lambda = 0.03(90) = 2.7$$

$$P(3; 2.7) = 0.2205$$

15.

$$P(5; 4) = 0.1563$$

16.

$$p = 0.004, n = 150, \lambda = 0.004(150) = 0.6$$

$$P(5; 6) = 0.0004$$

17.

P(one from each class)

$$= \frac{{}_5C_1 \cdot {}_4C_1 \cdot {}_5C_1 \cdot {}_7C_1}{{}_{21}C_4} = \frac{700}{5985} = 0.117$$

18.

P(at least 2 with defective pages) =

$$1 - [P(0 \text{ with defective pages}) + P(1 \text{ with defective pages})]$$

$$P(0) = \frac{{}_5C_0 \cdot {}_{20}C_5}{{}_{25}C_5} = \frac{2584}{8855} = 0.292$$

$$P(1) = \frac{{}_5C_1 \cdot {}_{20}C_4}{{}_{25}C_5} = \frac{1615}{3542} = 0.456$$

P(at least 2 with defective pages) =

$$1 - (0.292 + 0.456) = 0.252$$

19.

P(three of them have a PhD)

$$= \frac{{}_6C_3}{{}_{12}C_3} = 0.0909 \text{ or } 0.1$$

20.

P(at least one defective) = 1 -

P(0 defective)

$$a = 6, b = 18, n = 4, X = 0$$

$$P(0) = \frac{{}_6C_0 \cdot {}_{18}C_4}{{}_{24}C_4} = \frac{3060}{10,626} = 0.288$$

P(at least one defective) =

$$1 - 0.288 = 0.712$$

21.

P(at least 1 defective) = 1 - P(0 defectives)

$$a = 6, b = 18, n = 3, X = 0$$

$$P(0) = \frac{{}_6C_0 \cdot {}_{18}C_3}{{}_{24}C_3} = \frac{102}{253} = 0.403$$

$$P(\text{at least 1 defective}) = 1 - 0.403 = 0.597$$

22.

$$P(3 \text{ college graduates}) = \frac{{}_5C_3 \cdot {}_5C_0}{{}_{10}C_3} = 0.083$$

23.

$$P(\text{yes}) = 0.33 \quad P(\text{no}) = 1 - 0.33 = 0.67$$

$$P(\text{"yes" fourth}) = (0.67)^3(0.33) = 0.099$$

24.

$$\text{a. } P(\text{win on first purchase}) = \frac{1}{6}$$

$$\text{b. } P(\text{win on third purchase}) = \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)$$

$$P(\text{win on third purchase}) = \frac{25}{216} \text{ or } 0.116$$

$$\text{c. } P(\text{no wins in five purchases}) = \left(\frac{5}{6}\right)^5 = 0.402$$

25.

$$P(\text{bull's eye}) = 0.4$$

$$P(\text{no bull's eye}) = 0.6$$

$$P(\text{bull's eye on third shot}) = (0.6)^2(0.4)$$

$$= 0.144$$

26.

$$P(\text{prize on third shot}) = (0.2)^2(0.8) = 0.032$$

Chapter 5 - Discrete Probability Distributions

27.

$$P(\text{club}) = p = \frac{13}{52} \text{ or } \frac{1}{4}$$

$$k = 3$$

$$\mu = \frac{3}{\frac{1}{4}} = 12$$

28.

$$P(5) = \frac{1}{8}$$

$$k = 2$$

$$\mu = \frac{2}{\frac{1}{8}} = 16$$

29.

$$P(\text{face card}) = \frac{12}{52} \text{ or } \frac{3}{13}$$

$$k = 4$$

$$\mu = \frac{4}{\frac{3}{13}} = 17.33 \text{ or } 18$$

30.

$$P(AB) = 0.04$$

$$k = 1$$

$$\mu = \frac{1}{0.04} = 25$$

31.

$$P(\text{prefer shower}) = p = \frac{4}{5} \text{ or } 0.8$$

$$q = 1 - 0.8 = 0.2$$

$$\mu = \frac{1}{0.8} = 1.25$$

$$\sigma = \sqrt{\frac{0.2}{(0.8)^2}} = 0.559$$

32.

$$p = \frac{2}{3} \quad q = \frac{1}{3}$$

$$\mu = \frac{1}{\frac{2}{3}} = 1.5$$

$$\sigma = \sqrt{\frac{1/3}{(2/3)^2}} = 0.866$$

33.

$$p = \frac{1}{5} \text{ or } 0.2 \quad q = \frac{4}{5} \text{ or } 0.8$$

$$\mu = \frac{1}{0.2} = 5$$

$$\sigma = \sqrt{\frac{0.8}{(0.2)^2}} = 4.472$$

34.

$$p = \frac{1}{4} \text{ or } 0.25 \quad q = 0.75$$

$$\mu = \frac{1}{0.25} = 4$$

$$\sigma = \sqrt{\frac{0.75}{(0.25)^2}} = 3.464$$

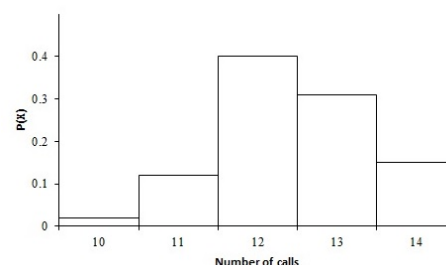
REVIEW EXERCISES - CHAPTER 5

1. Yes

2. No, the sum of the probabilities is not equal to 1.

3. No, the sum of the probabilities is greater than 1.

4.



5.

X	0	1	2	3	4	5
P(X)	0.27	0.28	0.2	0.15	0.08	0.02

a. $P(2 \text{ or } 3 \text{ applications}) = 0.2 + 0.15 = 0.35$

b.

$$\mu = 0(0.27) + 1(0.28) + 2(0.2) + 3(0.15) + 4(0.08) + 5(0.02) = 1.55$$

$$\sigma^2 = [0^2(0.27) + 1^2(0.28) + 2^2(0.2) + 3^2(0.15) + 4^2(0.08) + 5^2(0.02)] - 1.55^2 = 1.8075 \text{ or } 1.808$$

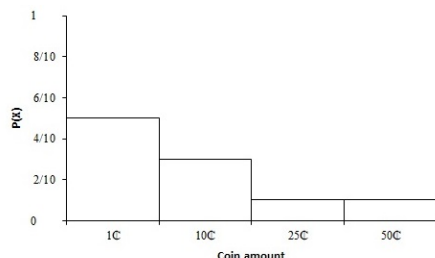
$$\sigma = \sqrt{1.8075} = 1.3444$$

X	P(X)	X · P(X)	X ² · P(X)
0	0.27	0	0
1	0.28	0.28	0.28
2	0.2	0.40	0.80
3	0.15	0.45	1.35
4	0.08	0.32	1.28
5	0.02	<u>0.10</u>	<u>0.50</u>
		$\mu = 1.55$	4.21

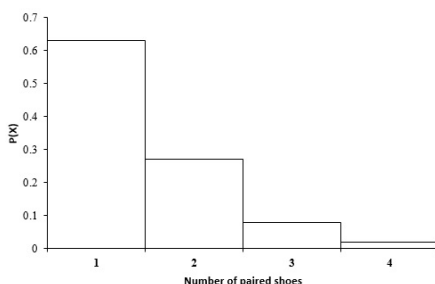
Chapter 5 - Discrete Probability Distributions

6.

X	\$0.01	\$0.10	\$0.25	\$0.50
P(X)	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{1}{10}$



7.



8.

$$\mu = \sum X \cdot P(X) = 0(0.12) + 1(0.20) + 2(0.31) + 3(0.25) + 4(0.12) = 2.05 \text{ or } 2.1$$

$$\sigma^2 = \sum X^2 \cdot P(X) - \mu^2 = [0^2(0.12) + 1^2(0.2) + 2^2(0.31) + 3^2(0.25) + 4^2(0.12)] - 2.05^2 = 1.4075 \text{ or } 1.4$$

$$\sigma = \sqrt{1.4075} = 1.186 \text{ or } 1.2$$

X	P(X)	X · P(X)	X ² · P(X)
0	0.12	0.00	0.00
1	0.20	0.20	0.20
2	0.31	0.62	1.24
3	0.25	0.75	2.25
4	0.12	<u>0.48</u>	<u>1.92</u>
		$\mu = 2.05$	5.61

9.

$$\mu = \sum X \cdot P(X) = 5(0.14) + 6(0.21) + 7(0.24) + 8(0.18) + 9(0.16) + 10(0.07) = 7.22$$

$$\sigma^2 = \sum X^2 \cdot P(X) - \mu^2 = [5^2(0.14) + 6^2(0.21) + 7^2(0.24) + 8^2(0.18) + 9^2(0.16) + 10^2(0.07)] - 7.22^2 = 2.1716 \text{ or } 2.2$$

$$\sigma = \sqrt{2.1716} = 1.47 \text{ or } 1.5$$

X	P(X)	X · P(X)	X ² · P(X)
5	0.14	0.7	3.5
6	0.21	1.26	7.56
7	0.24	1.68	11.76
8	0.18	1.44	11.52
9	0.16	1.44	12.96
10	0.07	<u>0.7</u>	<u>7.00</u>
		$\mu = 7.22$	54.3

10.

$$\mu = \sum X \cdot P(X) = 1(0.42) + 2(0.27) + 3(0.15) + 4(0.1) + 5(0.06) = 2.11 \text{ or } 2.1$$

$$\sigma^2 = \sum X^2 \cdot P(X) - \mu^2 = [1^2(0.42) + 2^2(0.27) + 3^2(0.15) + 4^2(0.1) + 5^2(0.06)] - 2.11^2 = 1.4979 \text{ or } 1.5$$

$$\sigma = \sqrt{1.4979} = 1.22 \text{ or } 1.2$$

X	P(X)	X · P(X)	X ² · P(X)
1	0.42	0.42	0.42
2	0.27	0.54	1.08
3	0.15	0.45	1.35
4	0.10	0.40	1.60
5	0.06	<u>0.30</u>	<u>1.50</u>
		$\mu = 2.11$	5.95

Chapter 5 - Discrete Probability Distributions

11.

$$\mu = \sum X \cdot P(X) = 0(0.15) + 1(0.43) + 2(0.32) + 3(0.06) + 4(0.04) = 1.4$$

$$\sigma^2 = \sum X^2 \cdot P(X) - \mu^2 = [0^2(0.15) + 1^2(0.43) + 2^2(0.32) + 3^2(0.06) + 4^2(0.04)] - 1.4^2 = 0.90$$

$$\sigma = \sqrt{0.90} = 0.95$$

X	P(X)	X · P(X)	X ² · P(X)
0	0.15	0	0
1	0.43	0.43	0.43
2	0.32	0.64	1.28
3	0.06	0.18	0.54
4	0.04	<u>0.16</u>	<u>0.64</u>
		$\mu = 1.4$	2.89

Two people at most should be employed.

12.

$$\mu = \$15,000(0.7) - \$8000(0.3) = \$8100$$

13.

Let x = cost to play the game

$$P(\text{ace}) = \frac{1}{13} \quad P(\text{face card}) = \frac{3}{13}$$

$$P(2 - 10) = \frac{9}{13}$$

For a fair game, $E(X) = 0$.

$$0 = -20\left(\frac{1}{13}\right) + 10\left(\frac{3}{13}\right) + 2\left(\frac{9}{13}\right) - x$$

$$x = \$2.15$$

14.

$$\begin{aligned} 0 &= -20\left(\frac{1}{13}\right) + 10\left(\frac{3}{13}\right) + 2\left(\frac{1}{13}\right) + 3\left(\frac{1}{13}\right) + \\ &4\left(\frac{1}{13}\right) + 5\left(\frac{1}{13}\right) + 6\left(\frac{1}{13}\right) + 7\left(\frac{1}{13}\right) + 8\left(\frac{1}{13}\right) + \\ &9\left(\frac{1}{13}\right) + 10\left(\frac{1}{13}\right) - x \\ x &= \$4.92 \end{aligned}$$

15.

$$n = 12, p = 0.3$$

$$\text{a. } P(X = 8) = 0.008$$

$$\text{b. } P(X < 5) = 0.724$$

$$\text{c. } P(X \geq 10) = 0.0002$$

$$\text{d. } P(4 < x \leq 9) = 0.275$$

16.

$$n = 10, p = 0.14$$

$$P(X = 2) = 0.2639 \text{ or } 0.264$$

$$P(X > 2) = 0.155$$

17.

$$\mu = n \cdot p = 250(0.58) = 145$$

$$\sigma^2 = n \cdot p \cdot q = 250(0.58)(0.42) = 60.9$$

$$\sigma = \sqrt{60.9} = 7.804$$

18.

$$n = 300, p = 0.63$$

$$\mu = n \cdot p = 300(0.63) = 189$$

$$\sigma^2 = n \cdot p \cdot q = 300(0.63)(0.37) = 69.93$$

$$\sigma = \sqrt{69.93} = 8.3624 \text{ or } 8.4$$

19.

$$n = 8, p = 0.25$$

$$P(X \leq 3) = \frac{8!}{8! 0!} (0.25)^0 (0.75)^8 +$$

$$\frac{8!}{7! 1!} (0.25)^1 (0.75)^7 + \frac{8!}{6! 2!} (0.25)^2 (0.75)^6 +$$

$$\frac{8!}{5! 3!} (0.25)^3 (0.75)^5 = 0.8862 \text{ or } 0.886$$

20.

$$N = 500, p = 0.27$$

$$\mu = 500(0.27) = 135$$

$$\sigma^2 = 500(0.27)(0.73) = 98.55 \text{ or } 98.6$$

$$\sigma = \sqrt{98.55} = 9.9$$

21.

$$n = 20, p = 0.75, X = 16$$

$$P(16 \text{ have eaten pizza for breakfast}) =$$

$$\frac{20!}{4! 16!} (0.75)^{16} (0.25)^4 = 0.1897 \text{ or } 0.190$$

22.

$$n = 250, p = 0.246 \text{ (have been married)}$$

$$\mu = 250(0.246) = 61.5$$

$$\sigma^2 = 250(0.246)(0.754) = 46.371 \text{ or } 46.4$$

$$\sigma = \sqrt{46.371} = 6.8096 \text{ or } 6.8$$

Chapter 5 - Discrete Probability Distributions

23.

$$P(M) = \frac{10!}{5! 3! 1! 1!} (0.46)^5 (0.41)^3 (0.09)^1 (0.04)^1 \\ = 0.026$$

24.

$$P(M) = \frac{12!}{8! 3! 1!} (0.9)^8 (0.06)^3 (0.04)^1 = 0.007$$

25.

$$P(M) = \frac{20!}{9! 6! 3! 2!} (0.50)^9 (0.28)^6 (0.15)^3 (0.07)^2 \\ = 0.012$$

26.

$$\lambda = n \cdot p = 400 \left(\frac{8.25}{1000} \right) = 3.3 \\ P(5; 3.3) = 0.120$$

27.

$$\text{a. } P(6 \text{ or more}; 6) = 1 - P(5 \text{ or less}; 6) \\ = 1 - (0.0025 + 0.0149 + 0.0446 + \\ 0.0892 + 0.1339 + 0.1606) = 0.5543$$

$$\text{b. } P(4 \text{ or more}; 6) = 1 - P(3 \text{ or less}; 6) \\ = 1 - (0.0025 + 0.0149 + 0.0446 + \\ 0.0892) = 0.8488$$

$$\text{c. } P(5 \text{ or less}; 6) = P(0; 6) + \dots + P(6; 6) \\ = 0.4457$$

28.

$$\lambda = n \cdot p = 1000(0.003) = 3 \\ P(6; 3) = 0.0504$$

29.

$$a = 13, b = 39, n = 5, X = 2 \\ P(2) = \frac{{}_{13}C_2 \cdot {}_{39}C_3}{{}_{52}C_5} = \frac{9,139}{33,320} = 0.27$$

30.

$$a = 10, b = 40, n = 5, X = 2 \\ P(2) = \frac{{}_{10}C_2 \cdot {}_{40}C_3}{{}_{50}C_5} = \frac{22,230}{105,938} = 0.21$$

31.

$$P(1 \text{ vegetable \& 2 fruit}) = \\ \frac{{}_{10}C_0 \cdot {}_8C_1 \cdot {}_8C_2}{{}_{26}C_3} = \frac{224}{2600} = 0.086$$

32.

$$P(3 \text{ on fourth roll}) = \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right) = \frac{125}{1296}$$

33.

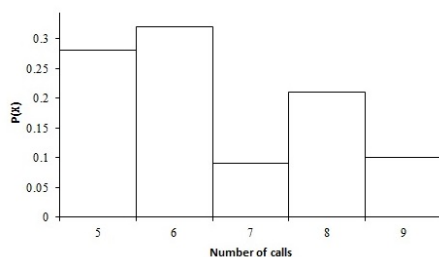
$$P(\text{face card on fourth draw}) = \\ \left(\frac{40}{52}\right)^3 \left(\frac{12}{52}\right) = 0.105$$

CHAPTER 5 QUIZ

1. True
2. False, it is a discrete random variable.
3. False, the outcomes must be independent.
4. True
5. Chance
6. $\mu = n \cdot p$
7. One
8. c
9. c
- 10.d
- 11.No, the sum of the probabilities is greater than one.
- 12.Yes
- 13.Yes
- 14.Yes

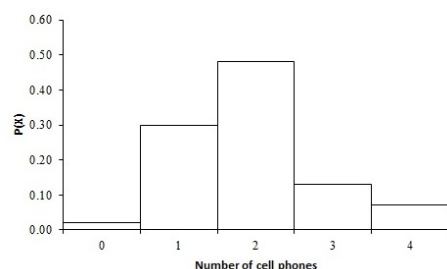
Chapter 5 - Discrete Probability Distributions

15.



16.

X	0	1	2	3	4
P(X)	0.02	0.30	0.48	0.13	0.07



17.

$$\mu = 0(0.10) + 1(0.23) + 2(0.31) + 3(0.27) + 4(0.09) = 2.02 \text{ or } 2$$

$$\sigma^2 = [0^2(0.10) + 1^2(0.23) + 2^2(0.31) + 3^2(0.27) + 4^2(0.09)] - 2.02^2 = 1.3$$

$$\sigma = \sqrt{1.3} = 1.1$$

18.

$$\mu = 30(0.05) + 31(0.21) + 32(0.38) + 33(0.25) + 34(0.11) = 32.16 \text{ or } 32.2$$

$$\sigma^2 = [30^2(0.05) + 31^2(0.21) + 32^2(0.38) + 33^2(0.25) + 34^2(0.11)] - 32.16^2 = 1.07 \text{ or } 1.1$$

$$\sigma = \sqrt{1.07} = 1.0$$

19.

$$\mu = 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 10\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 7\left(\frac{1}{6}\right) = 5.17 \text{ or } 5.2$$

20.

$$\mu = \$2\left(\frac{1}{2}\right) + \$10\left(\frac{5}{26}\right) + \$25\left(\frac{3}{26}\right) + \$100\left(\frac{1}{26}\right) = \$9.65$$

21.

$$n = 20, p = 0.40, X = 5$$

$$P(5) = 0.124$$

22.

$$n = 20, p = 0.60$$

$$\text{a. } P(15) = 0.075$$

$$\text{b. } P(10, 11, \dots, 20) = 0.872$$

$$\text{c. } P(0, 1, 2, 3, 4, 5) = 0.126$$

23.

$$n = 300, p = 0.80$$

$$\mu = 300(0.80) = 240$$

$$\sigma^2 = 300(0.80)(0.20) = 48$$

$$\sigma = \sqrt{48} = 6.9$$

24.

$$n = 75, p = 0.12$$

$$\mu = 75(0.12) = 9$$

$$\sigma^2 = 75(0.12)(0.88) = 7.9$$

$$\sigma = \sqrt{7.9} = 2.8$$

25.

$$P(M)$$

$$= \frac{30!}{15! 8! 5! 2!} (0.5)^{15} (0.3)^8 (0.15)^5 (0.05)^2$$

$$= 0.008$$

26.

$$P(M) = \frac{16!}{9! 4! 3!} (0.88)^9 (0.08)^4 (0.04)^3$$

$$= 0.0003$$

27.

$$P(M) = \frac{12!}{5! 4! 3!} (0.45)^5 (0.35)^4 (0.2)^3$$

$$= 0.061$$

Chapter 5 - Discrete Probability Distributions

28.

$$\lambda = 100(0.08) = 8, \quad X = 6$$

$$P(6; 8) = 0.122$$

29.

$$\lambda = 8$$

$$\begin{aligned} \text{a. } P(X \geq 8; 8) &= 0.1396 + \dots + 0.0001 \\ &= 0.547 \end{aligned}$$

$$\begin{aligned} \text{b. } P(X \geq 3; 8) &= 1 - P(0, 1, \text{ or } 2 \text{ calls}) \\ &= 1 - (0.0003 + 0.0027 + 0.0107) \\ &= 1 - 0.0137 = 0.9863 \end{aligned}$$

$$\begin{aligned} \text{c. } P(X \leq 7; 8) &= 0.0003 + \dots + 0.1396 \\ &= 0.4529 \end{aligned}$$

30.

$$a = 12, b = 36, n = 6, X = 3$$

$$P(A) = \frac{{}_{12}C_3 \cdot {}_{36}C_3}{{}_{48}C_6} = \frac{\frac{12!}{9!3!} \cdot \frac{36!}{33!3!}}{\frac{48!}{42!6!}} = 0.128$$

31.

$$\text{a. } \frac{{}_6C_3 \cdot {}_8C_1}{{}_{14}C_4} = \frac{\frac{6!}{3!3!} \cdot \frac{8!}{7!1!}}{\frac{14!}{10!4!}} = 0.160$$

$$\text{b. } \frac{{}_6C_2 \cdot {}_8C_2}{{}_{14}C_4} = \frac{\frac{6!}{4!2!} \cdot \frac{8!}{6!2!}}{\frac{14!}{10!4!}} = 0.42$$

$$\text{c. } \frac{{}_6C_0 \cdot {}_8C_4}{{}_{14}C_4} = \frac{\frac{6!}{6!0!} \cdot \frac{8!}{4!4!}}{\frac{14!}{10!4!}} = 0.07$$

32.

$$P(\text{AB sixth person}) = (0.96)^5(0.04) = 0.033$$

33.

$$P = (0.65)^9(0.35) = 0.007$$