

Chapter 4 - Probability and Counting Rules

Note: Answers may vary due to rounding, TI-83's or computer programs.

EXERCISE SET 4-1

1. A probability experiment is a chance process which leads to well-defined outcomes.

2. The set of all possible outcomes of a probability experiment is called a sample space.

3. An outcome is the result of a single trial of a probability experiment, whereas an event can consist of one or more outcomes.

4. Equally likely events have the same probability of occurring.

5. The range of values is $0 \leq P(E) \leq 1$.

6. 1

7. 0

8. 1

9.

$$1 - 0.20 = 0.80$$

Since the probability that it won't rain is 80%, you could leave your umbrella at home and be fairly safe.

10. c, d, e, h

11.

- a. Empirical c. Empirical
- b. Classical d. Classical

12.

- a. Empirical c. Subjective
- b. Empirical d. Subjective

13.

- a. 0 c. 1
- b. $\frac{1}{2}$ d. $\frac{1}{2}$

14.

- a. 1 c. $\frac{1}{3}$
- b. $\frac{2}{3}$ d. 0

15. There are 6^2 or 36 outcomes.

a. There are 4 ways to get a sum of 5. They are (4,1), (3,2), (2,3), and (1,4). The probability then is $\frac{4}{36} = \frac{1}{9}$.

b. There are 4 ways to get a sum of 9 and 3 ways to get a sum of 10. They are (6,4), (5,5), (4,6), (6,3), (5,4), (4,5), and (3,6).

The probability then is $\frac{7}{36}$.

c. There are 6 ways to get doubles. They are (1,1), (2,2), (3,3), (4,4), (5,5), and (6,6). The probability then is $\frac{6}{36} = \frac{1}{6}$.

16.

a. To get a sum less than nine, one must roll a 2, 3, 4, 5, 6, 7, or 8. There are 26 ways to get a sum less than 9. The probability then is $\frac{26}{36} = \frac{13}{18}$.

b. To get a sum greater than or equal to 10, one must roll a 10, 11, or 12. There are six ways to do this. They are (6,4), (5,5), (4,6), (6,5), (5,6), and (6,6). The probability is $\frac{6}{36} = \frac{1}{6}$.

c. There are 11 ways to get a 3 on one or both die. The probability is $\frac{11}{36}$.

17.

- a. $\frac{1}{13}$ d. $\frac{2}{13}$
- b. $\frac{1}{4}$ e. $\frac{6}{13}$
- c. $\frac{1}{52}$

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18.

- a. $\frac{1}{52}$ d. $\frac{1}{2}$
 b. $\frac{1}{26}$ e. $\frac{1}{2}$
 c. $\frac{1}{26}$

19. There are 20 possible outcomes.

a. $P(\text{winning \$10}) = P(\text{rolling a 1})$

$$P(\text{rolling a 1}) = \frac{2}{20} = \frac{1}{10} = 0.1$$

b. $P(\text{winning \$5 or \$10}) = P(\text{rolling either a 1 or 2})$

$$P(1 \text{ or } 2) = \frac{4}{20} = \frac{1}{5} = 0.2$$

c. $P(\text{winning a coupon}) = P(\text{rolling either a 3 or 4})$

$$P(3 \text{ or } 4) = \frac{16}{20} = \frac{4}{5} = 0.8$$

20.

a. $P(\text{begins with M}) = \frac{8}{50} = \frac{4}{25}$

b. $P(\text{begins with a vowel}) = \frac{12}{50} = \frac{6}{25}$

$$P(\text{not a vowel}) = 1 - \frac{6}{25} = \frac{19}{25}$$

21.

a. $P(\text{type B}) = 0.12$ or 12%

b. $P(\text{type AB or O}) = 0.05 + 0.43 = 0.48$
or 48%

c. $P(\text{not type O}) = 1 - P(\text{type O})$
 $= 1 - 0.43$
 $= 0.57$ or 57%

22.

$P(\text{other}) = 1 - P(1989 \text{ or Frozen or Lonely Hour})$

$$P(\text{other}) = 1 - (0.25 + 0.241 + 0.082) = 0.427 \text{ or } 42.7\%$$

23.

a. $P(\text{odd prime number}) = \frac{24}{25} = 0.96$

b. $P(\text{sum of the digits is odd}) = \frac{12}{25} = 0.48$

c. $P(\text{greater than 70}) = \frac{6}{25} = 0.24$

24.

a. $P(60 \text{ or } 70 \text{ mph}) = \frac{19}{50} = 0.38$

b. $P(\text{greater than 65 mph}) = \frac{31}{50} = 0.62$

c. $P(70 \text{ mph or less}) = \frac{37}{50} = 0.74$

25.

The sample space is BBBB, BBGB, BGGB, GBBB, GGBB, GBGB, BGGG, GGGB, BBBG, BBGG, BGBG, GBBG, GGBG, GBGG, BGGG, and GGGG.

a. All girls is the outcome GGGG; hence

$$P(\text{all girls}) = \frac{1}{16}.$$

b. Exactly two girls and two boys would be

GGBB, GBGB, BGGB, BBGG, BGBG, GBBG; hence, $P(\text{exactly two girls and two boys}) = \frac{6}{16} = \frac{3}{8}.$

c. At least one child who is a girl would be all outcomes, apart from BBBB. The

probability then is $\frac{15}{16}.$

d. At least one child of each gender means

at least one boy or at least one girl. The

outcomes are BBGB, BGGB, GBBB, GGBB, GBGB, BGGB, GGGB, BBBG, BBGG, BGBG, GBBG, GGBG, GBGG, BGGG. Hence the probability is $\frac{14}{16} = \frac{7}{8}.$

26.

a. $P(\text{not oil}) = 1 - 0.39 = 0.61$

b. $P(\text{natural gas or oil}) = 0.39 + 0.24 = 0.63$

c. $P(\text{nuclear}) = 0.08$

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27.

The outcomes for 7 or 11 are (1,6), (2,5), (3,4), (4,3), (5,2), (6,1), (5,6), and (6,5);

hence, $P(7 \text{ or } 12) = \frac{8}{36}$.

The outcomes for 2, 3, or 12 are (1,1), (1,2), (2,1), and (6,6); hence, $P(2, 3, \text{ or } 12) =$

$$\frac{1+2+1}{36} = \frac{4}{36}.$$

$$\begin{aligned} P(\text{game will last only one roll}) &= \frac{8}{36} + \frac{4}{36} \\ &= \frac{12}{36} \\ &= \frac{1}{3} \end{aligned}$$

28.

a. $P(50 \text{ or fewer}) = \frac{24,501}{83,057} = 0.295$

b. $P(\text{more than } 100) = \frac{34,803}{83,057} = 0.419$

c. $P(\text{no more than } 20) = \frac{7760}{83,057} = 0.093$

29.

a. $P(\text{debt is less than } \$5001) = 27\%$.

b. $P(\text{debt is more than } \$20,000) =$
 $P(\$20,001 \text{ to } \$50,000) + P(\$50,000+) =$
 $19\% + 14\% = 33\%$

c. $P(\text{debt is between } \$1 \text{ and } \$20,000) =$
 $P(\$1 \text{ to } \$5000) + P(\$5001 \text{ to } \$20,000) =$
 $27\% + 40\% = 67\%$

d. $P(\text{debt is more than } \$50,000) = 14\%$

30.

$P(\text{native Hawaiian or Pacific Islander}) = 0.095$

$P(\text{Asian}) = 0.377$

$P(\text{White}) = 0.227$

31.

$P(\text{motor vehicle theft}) = \frac{275}{2500} = 0.11$

$P(\text{not an assault}) = 1 - P(\text{assault})$

$P(\text{not an assault}) = 1 - \frac{200}{2500} = 0.92$

32.

$P(\text{both parents}) = \frac{51,823}{74,719} = 0.694$

$P(\text{mother present}) = P(\text{both parents and mother only})$

$P(\text{mother present}) = \frac{17,283}{74,719} = 0.925$

33.

$P(\text{either a truck or a motorcycle}$

$\text{accident}) = \frac{5,200,000 + 178,000}{18,878,000} = 0.285$

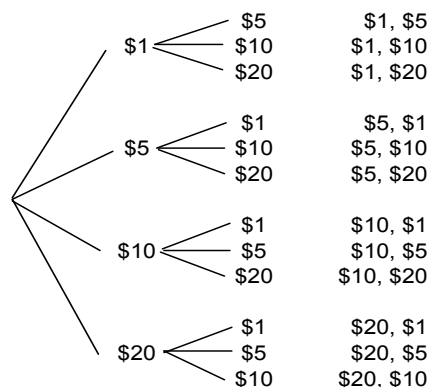
$P(\text{not a truck accident}) = 1 - P(\text{truck}$

$\text{accident}) = 1 - \frac{5,200,000}{18,878,000} = 0.725$

34.

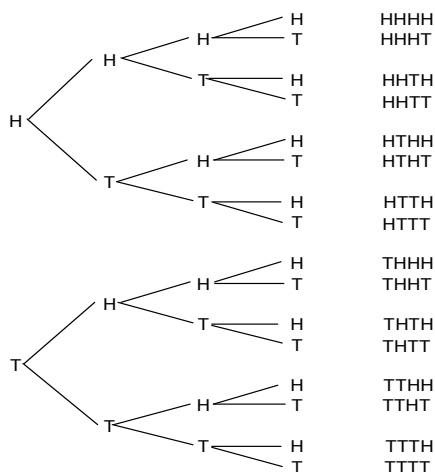
$P(\text{individual or corporate taxes}) = 0.60$

35.

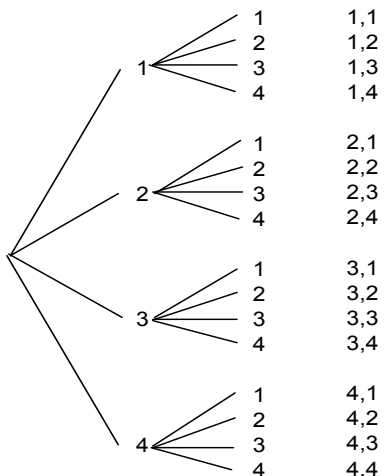


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36.

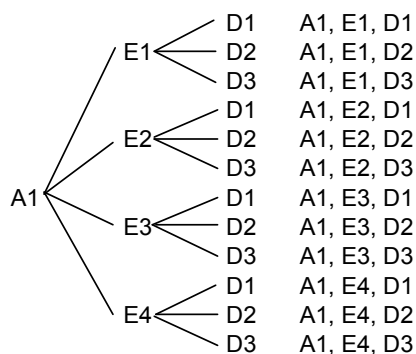


37.

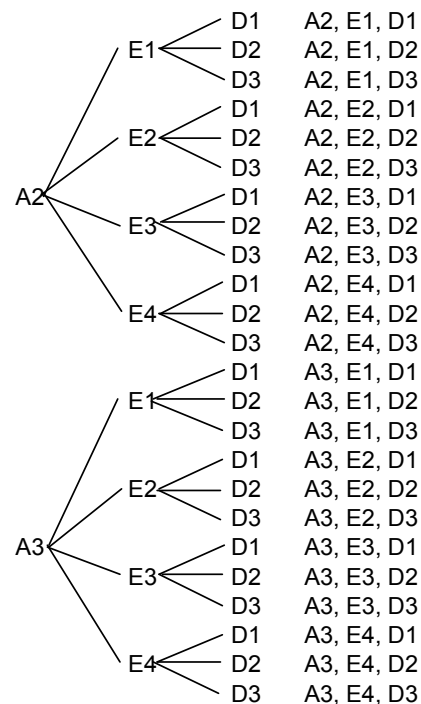


38.

Appetizers Entrees Desserts

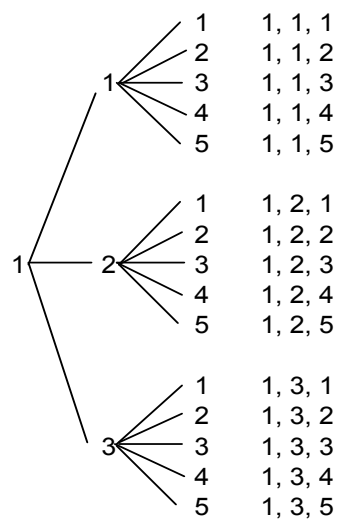


38. continued



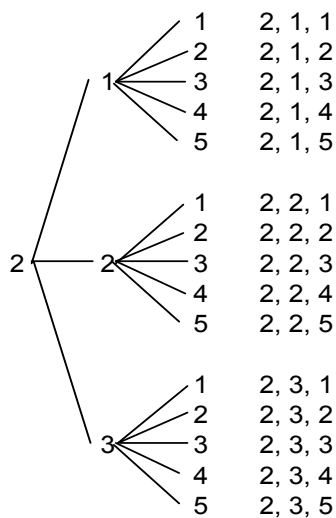
39.

English Math Elective

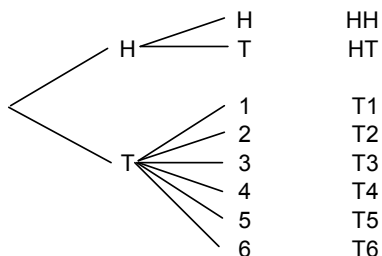


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39. continued



40.



41.

- 0.08
- 0.01
- $0.08 + 0.27 = 0.35$
- $0.01 + 0.24 + 0.11 = 0.36$

42. Probably

43. The statement is probably not based on empirical probability and probably not true.

44. The outcomes will be:

0,0	0,1	0,2	0,3	0,4
1,0	1,1	1,2	1,3	1,4
2,0	2,1	2,2	2,3	2,4
3,0	3,1	3,2	3,3	3,4
4,0	4,1	4,2	4,3	4,4

44. continued

- $\frac{6}{25}$
- $\frac{10}{25} = \frac{2}{5}$
- $\frac{9}{25}$
- $\frac{12}{25}$
- $\frac{5}{25} = \frac{1}{5}$

45. Actual outcomes will vary, however the probabilities of 0, 1, 2, or 3 heads should be approximately $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{8}$, and $\frac{1}{8}$ respectively.

46. Actual outcomes will vary; however, the probabilities of 0, 1, and 2 heads will be approximately $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ respectively.

47.

- 1:5, 5:1
- 1:1, 1:1
- 1:3, 3:1
- 1:1, 1:1
- 1:12, 12:1
- 1:3, 3:1
- 1:1, 1:1

EXERCISE SET 4-2

1. Two events are mutually exclusive if they cannot occur at the same time.
Examples will vary.

2. Addition rule 2 can be used because $P(A \text{ and } B) = 0$ when A and B are mutually exclusive.

3.

a. Not mutually exclusive
You can get the 6 of spades.

b. Mutually exclusive

c. Mutually exclusive

d. Not mutually exclusive

Some sophomore students are male.

4.

- Yes
- No
- Yes
- Yes

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5.

a. $\frac{1,348,503}{1,907,172} = 0.707$

b. $\frac{46,024}{1,907,172} + \frac{1,098,371}{1,907,172} - \frac{21,683}{1,907,172} =$

$\frac{1,122,712}{1,907,172} = 0.589$

c. $\frac{21,683}{1,907,172} = 0.011$

d. $\frac{1,394,527}{1,907,172} = 0.731$

6.

$P(\text{John has company}) = 0.23 + 0.4 = 0.63$

7.

a. $P(\text{pathologist}) = \frac{7}{38}$ or 0.184

b. $P(\text{orthopedist or MD}) = \frac{22}{38} + \frac{33}{38} - \frac{20}{38}$

$P(\text{orthopedist or MD}) = \frac{35}{38}$ or 0.921

8.

a. Since 73% are 20 years and over and 13.1% are 65 years and over, $73\% - 13.1\%$ or 59.9% are between 20 years and 64 years old.

$P(20 \text{ years to } 64 \text{ years}) = 0.599$

b. $P(\text{under 20 or 65 and over}) = 0.27 + 0.131 = 0.401$

c. $P(\text{not 65 and over}) = 1 - P(65 \text{ and over})$

$P(\text{not 65 and over}) = 1 - 0.131 = 0.869$

9.

$\frac{8}{16} + \frac{2}{16} = \frac{10}{16} = \frac{5}{8}$

10.

$\frac{310}{980} + \frac{150}{980} = \frac{460}{980}$ or $\frac{23}{49}$

The probability of the event is slightly less than 0.5, which makes it about equally likely to occur or not to occur.

11.

	Cheese Pizzas	Pizzas with one or more toppings	Total
Eaten at work	12	10	22
Not eaten at work	<u>12</u>	<u>6</u>	<u>18</u>
Total	24	16	40

a. $P(\text{a cheese pizza eaten at}$

$\text{work}) = \frac{12}{40} = \frac{3}{10} = 0.30$

b. $P(\text{a pizza with either one or more toppings, and it was not eaten at}$

$\text{work}) = \frac{3}{4} = 0.75$

c. $P(\text{a cheese pizza or a pizza eaten at work})$

$= \frac{24}{40} + \frac{22}{40} - \frac{12}{40} = \frac{34}{40} = \frac{17}{20} = 0.85$

12.

	Fiction	Non-Fiction	Total
Adult	30	70	100
Children	100	60	160
	130	130	260

a. $P(\text{fiction}) = \frac{130}{260} = \frac{1}{2}$ or 0.5

b. $P(\text{children's nonfiction}) = \frac{60}{260} = \frac{3}{13}$

$P(\text{not a children's nonfiction}) =$

$1 - \frac{3}{13} = \frac{10}{13}$ or 0.769

c. $P(\text{adult book or children's nonfiction}) =$

$\frac{100}{260} + \frac{60}{260} = \frac{160}{260}$ or $\frac{8}{13}$ or 0.615

13.

	18 - 24	25 - 34	Total
Male	7922	2534	10,456
Female	5779	995	6,774
Total	13,701	3529	17,230

a. $P(\text{female aged } 25 - 34) = \frac{995}{17,230} = 0.058$

b. $P(\text{male or aged } 18 - 24) =$

$\frac{10,456}{17,230} + \frac{13,701}{17,230} - \frac{7922}{17,230} = \frac{16,235}{17,230} = 0.942$

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13. continued

c. $P(\text{under 25 years and not male}) =$

$$\frac{5779}{17,230} = 0.335$$

14.

	Endangered - US	Endangered - Foreign	
Mammals	68	251	
Birds	77	175	
Reptiles	14	64	
Amphibians	11	8	
Total	170	498	

	Threatened - US	Threatened - Foreign	Total
Mammals	10	20	349
Birds	13	6	271
Reptiles	22	16	116
Amphibians	10	1	30
Total	55	43	766

a. $P(\text{threatened and in the US}) = \frac{55}{776} = 0.072$

b. $P(\text{an endangered foreign bird}) =$

$$\frac{175}{766} = 0.228$$

c. $P(\text{a mammal or a threatened species}) =$

$$\frac{349}{766} + \frac{43}{766} - \frac{20}{766} = \frac{372}{766} = 0.486$$

15.

Total = 136,238 multiple births

a. $P(\text{more than two babies}) =$

$$\frac{7663}{136,328} = 0.056$$

b. $P(\text{quads or quints}) = \frac{553}{136,328} = 0.004$

c. The total number of babies who are

triplets = 21,330

The total number of babies from multiple births = 280,957

$$P(\text{baby is a triplet}) = \frac{21,330}{280,957} = 0.076$$

16.

Age	Male	Female	Total
19 and under	4746	4517	9263
20	1625	1553	3178
21	1679	1627	3306
Total	8050	7697	15,747

a. $P(\text{male and 19 or under}) = \frac{4746}{15,747} = 0.301$

b. $P(20 \text{ or female}) =$

$$\frac{3178}{15,747} + \frac{7697}{15,747} - \frac{1553}{15,747} = \frac{9322}{15,747} = 0.592$$

c. $P(\text{at least 20}) = \frac{6484}{15,747} = 0.412$

17.

Age	High School	College	Neither	Total
Under 30	53	107	450	610
30 and over	27	32	367	426
Total	80	139	817	1036

a. $P(\text{The prisoner does not take}$

$$\text{classes}) = \frac{817}{1036} = 0.789$$

b. $P(\text{under 20 and is taking either a high school class or a college class})$

$$= \frac{53}{1036} + \frac{107}{1036} = 0.154$$

c. $P(\text{over 30 and is taking either a high school class or a college class})$

$$= \frac{27}{1036} + \frac{32}{1036} = 0.057$$

18.

	1st Class	Ad	Magazine	Total
Home	325	406	203	934
Business	<u>732</u>	<u>1021</u>	<u>97</u>	<u>1850</u>
Total	1057	1427	300	2784

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18. continued

$$a. P(\text{home}) = \frac{934}{2784} = \frac{467}{1392}$$

$$b. P(\text{advertisement or business}) = P(\text{ad}) + P(\text{business}) - P(\text{business and ad}) = \frac{1427}{2784} + \frac{1850}{2784} - \frac{1021}{2784} = \frac{2256}{2784} = \frac{47}{58}$$

$$c. P(\text{1st class or home}) = P(\text{1st class}) + P(\text{home}) - P(\text{1st class and home}) = \frac{1057}{2784} + \frac{934}{2784} - \frac{325}{2784} = \frac{1666}{2784} = \frac{833}{1392}$$

19.

The total of the frequencies is 30.

$$a. \frac{2}{30} = \frac{1}{15}$$

$$b. \frac{2+3+5}{30} = \frac{10}{30} = \frac{1}{3}$$

$$c. \frac{12+8+2+3}{30} = \frac{25}{30} = \frac{5}{6}$$

$$d. \frac{12+8+2+3}{30} = \frac{25}{30} = \frac{5}{6}$$

$$e. \frac{8+2}{30} = \frac{10}{30} = \frac{1}{3}$$

20.

The total of the frequencies is 32.

$$a. P(\text{more than 10}) = \frac{9}{32} = 0.281$$

$$b. P(\text{at least one}) = \frac{30}{32} = 0.938$$

$$c. P(1 - 5 \text{ or more than } 15) = \frac{18}{32} = 0.563$$

21.

The total of the frequencies is 30.

$$a. \frac{4}{30} = \frac{2}{15}$$

$$b. \frac{11+9+5}{30} = \frac{25}{30} = \frac{5}{6}$$

$$c. \frac{9}{30} + \frac{5}{30} = \frac{14}{30} = \frac{7}{15}$$

$$d. \frac{11+9+5}{30} = \frac{25}{30} = \frac{5}{6}$$

21. continued

$$e. \frac{4+1}{30} = \frac{5}{30} = \frac{1}{6}$$

22.

	High Chol.	Normal Chol.	Total
Alcoholic	87	13	100
Non-Alcoholic	<u>43</u>	<u>157</u>	<u>200</u>
Total	56	244	300

$$a. P(\text{alcoholic with elevated cholesterol}) = \frac{87}{300} = \frac{29}{100}$$

$$b. P(\text{non-alcoholic}) = \frac{200}{300} = \frac{2}{3}$$

$$c. P(\text{non-alcoholic with normal cholesterol}) = \frac{157}{300}$$

23.

a. There are 4 sevens, 4 eights, and 4 nines; hence, $P(\text{seven or eight or nine}) = \frac{12}{52} = \frac{3}{13}$

b. There are 13 spades, 4 kings, and 4 queens, but the king and queen of spades were counted twice.

Hence, $P(\text{spade or king or queen}) = P(\text{spade}) + P(\text{king}) + P(\text{queen}) - P(\text{king and queen of spades}) = \frac{13}{52} + \frac{4}{52} + \frac{4}{52} - \frac{2}{52} = \frac{19}{52}$

c. There are 13 clubs, and 12 face cards, but the face card of clubs was counted twice.

Hence, $P(\text{club or face}) = P(\text{club}) + P(\text{face}) - P(\text{face card of clubs}) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26}$

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23. continued

d. There are 4 aces, 13 diamonds, and 13 hearts. There is one ace of diamonds and one ace of hearts.

Hence, $P(\text{ace or diamond or heart}) = P(\text{ace}) +$

$P(\text{diamond}) + P(\text{heart}) - P(\text{ace of hearts and ace of diamonds})$

$$= \frac{4}{52} + \frac{13}{52} + \frac{13}{52} - \frac{2}{52} = \frac{28}{52} = \frac{7}{13}$$

e. There are 4 nines, 4 tens, 13 spades, and 13 clubs. There is one nine of spades, one ten of spades, one nine of clubs and one ten of clubs. Hence, $P(9 \text{ or } 10 \text{ or spade or club}) = P(9) + P(10) + P(\text{spades}) + P(\text{club})$

$$- P(9 \text{ and } 10 \text{ of clubs and spades})$$

$$= \frac{4}{52} + \frac{4}{52} + \frac{13}{52} + \frac{13}{52} - \frac{4}{52} = \frac{30}{52} = \frac{15}{26}$$

24.

a. $P(\text{sum of } 8) + P(\text{sum of } 9) + P(\text{sum of } 10) = \frac{5}{36} + \frac{4}{36} + \frac{3}{36} = \frac{12}{36} \text{ or } \frac{1}{3}$

b. $P(\text{doubles}) + P(\text{sum of } 7) = \frac{6}{36} + \frac{6}{36} = \frac{12}{36} \text{ or } \frac{1}{3}$

c. $P(\text{sum} > 9) + P(\text{sum} < 4) = \frac{6}{36} + \frac{3}{36} = \frac{9}{36} \text{ or } \frac{1}{4}$

d. The event in part c is least likely to occur since it has the lowest probability.

25.

$$P(\text{apple juice or apple sauce}) = \frac{4.4}{11} + \frac{1}{11} = 0.491$$

26.

There are $6^3 = 216$ possible outcomes.

a. $\frac{6}{216} = \frac{1}{36}$ since there are 6 triples: (1,1,1), (2,2,2), ..., (6,6,6).

26. continued

b. $\frac{6}{216} = \frac{1}{36}$ since there are six possible outcomes summing to 5: (1,2,2), (2,1,2), (2,2,1), (1,1,3), (1,3,1), and (3,1,1).

27.

$P(\text{mushrooms or pepperoni}) =$

$$P(\text{mushrooms}) + P(\text{pepperoni}) - P(\text{mushrooms and pepperoni})$$

Let $X = P(\text{mushrooms and pepperoni})$

$$\text{Then } 0.55 = 0.32 + 0.17 - X$$

$$X = 0.06$$

28.

$P(\text{one or two car garage}) =$

$$0.20 + 0.70 = 0.90$$

$$\text{Hence, } P(\text{no garage}) = 1 - 0.90 = 0.10$$

29.

$$P(\text{not a two-car garage}) = 1 - 0.70 = 0.30$$

30.

No. $P(A \cap B) \neq 0$

31.

$$P(A \text{ or } B) = \frac{m}{2m+n} + \frac{n}{2m+n}$$

$$P(A \text{ or } B) = \frac{m+n}{2m+n}$$

32.

a. Since $P(A \text{ or } B) = P(A) + P(B)$ when A and B are mutually exclusive, then

$$0.653 = 0.392 + P(B)$$

$$0.261 = P(B)$$

b. $P(\text{not } A) = 1 - P(A)$

$$P(\text{not } A) = 1 - 0.392$$

$$P(\text{not } A) = 0.608$$

c. Since A and B are mutually exclusive, $P(A \text{ and } B) = 0$.

Chapter 4 - Probability and Counting Rules

EXERCISE SET 4-3

1.
 - a. Independent c. Dependent
 - b. Dependent d. Dependent

2.
 - a. Independent c. Dependent
 - b. Dependent d. Independent

3.
 - a. $P(\text{none play video or computer games}) = (0.31)^4 = 0.009$ or 0.9%

 - b. $P(\text{all four play video or computer games}) = (0.69)^4 = 0.227$ or 22.7%

4.

$$P(\text{all 4 used a seat belt}) = (.52)^4 = 7.3\%$$

5.

$$P(\text{making a sale}) = 0.21$$

$$P(\text{making 4 sales}) = (0.21)^4 = 0.0019$$
 or 0.002
 The event is unlikely to occur since the probability is small.

6.

$$P(\text{two inmates are not citizens}) = (0.25)^2 = 0.0625$$
 or 6.3%

7.
 - a. If 66% of law enforcement workers are sworn officers, and 88.4% of those workers are male, then $100\% - 88.4\% = 11.6\%$ of 66% are female sworn officers. Thus, $P(\text{female sworn officer}) = 0.116(66\%) = 7.656\%$ or 0.07656

7. continued

b. If 66% of law enforcement workers are sworn officers, then $100\% - 66\% = 34\%$ are civilian workers, both male and female. Likewise, if 60.7% of civilian workers are female, then $100\% - 60.7\% = 39.3\%$ are male civilian workers. Thus, $P(\text{male civilian employee}) = 0.393(34\%) = 13.362\%$ or 0.13362

c. The total of male employees, both sworn officers and civilian, is 71.706%. The total of civilian employees is 34%. Thus, $P(\text{male or civilian}) = P(\text{male}) + P(\text{civilian}) - P(\text{both})$
 $P(\text{male or civilian}) = 0.71706 + 0.34 - 0.13362$
 $P(\text{male or civilian}) = 0.92344$

8.

a. $P(\text{at least one doesn't use a computer at work}) = 1 - P(\text{none of the women don't use a computer at work})$
 $P(\text{at least one doesn't use a computer}) = 1 - (0.72)^5 = 0.807$

b. $P(\text{all 5 use a computer}) = (0.72)^5 = 0.193$

9.

$P(\text{none are mothers}) = (0.25)^3 = 0.016$

10.

a. $P(\text{red 1st and white 2nd}) = \frac{9}{23} \cdot \frac{8}{22} = \frac{36}{253}$ or 0.142

b. $P(\text{both red or both white or both blue}) = \frac{9}{23} \cdot \frac{8}{22} + \frac{8}{23} \cdot \frac{7}{22} + \frac{6}{23} \cdot \frac{5}{22} = \frac{79}{253}$ or 0.312

c. $P(\text{2nd marble is blue}) = P(\text{red or white 1st and blue 2nd}) + P(\text{blue 1st and blue 2nd})$
 $= \frac{17}{23} \cdot \frac{6}{22} + \frac{6}{23} \cdot \frac{5}{22}$
 $= \frac{66}{253}$ or 0.261

Chapter 4 - Probability and Counting Rules

11.

a. $P(\text{none of the three households had a smart TV}) = (1 - 0.45)^3 = 0.166375$

b. $P(\text{all three households had a smart TV}) = (0.45)^3 = 0.091125$

c. $P(\text{at least one of the three households had a smart TV})$
 $= 1 - 0.166375$
 $= 0.833625$

12.

$P(\text{both are defective}) = \frac{2}{6} \cdot \frac{1}{5} = \frac{1}{15}$

13.

a. $\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} = \frac{1}{270,725} = 0.00000369$

b. $\frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} \cdot \frac{23}{49} = \frac{358800}{6497400} = \frac{46}{833} = 0.055$

c.

$\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} = \frac{17160}{6,497,400} = \frac{11}{4165} = 0.00264$

14.

$\frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{5}{28}$

15.

a. $P(\text{both are nines}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$

b. $P(\text{both are the same suit}) = \frac{4}{4} \cdot \frac{12}{51} = \frac{4}{17}$

c. $P(\text{both are spades}) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$

16.

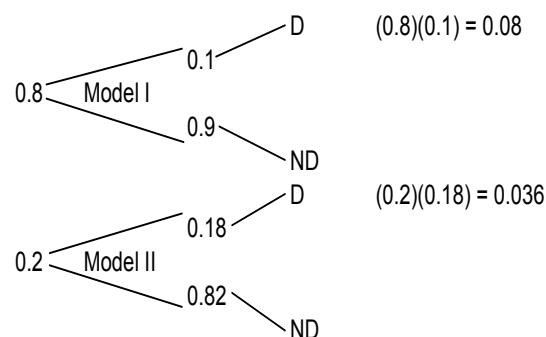
$P(\text{both prizes are won by men}) =$
 $\frac{10}{30} \cdot \frac{9}{29} = \frac{90}{870}$ or $\frac{3}{29}$ unlikely

17.

$P(\text{both are dead}) = \frac{2}{12} \cdot \frac{1}{11} = \frac{1}{66} \approx 0.015$

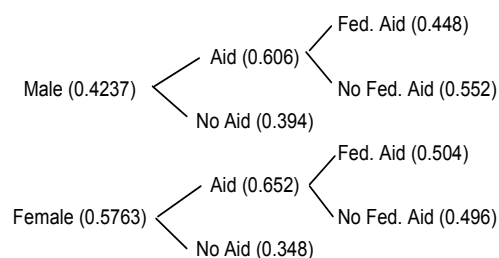
Highly unlikely

18.



$P(\text{defective}) = 0.08 + 0.036 = 0.116$

19.



a. $P(\text{male student without aid}) = 0.4237(0.394) = 0.0167$

b. $P(\text{male student | student has aid}) =$

$\frac{P(\text{aid \& male})}{P(\text{aid})} = \frac{0.4237(0.606)}{0.4237(0.606) + 0.5763(0.652)} = 0.406$

c. $P(\text{female student or a student who receives federal aid}) =$

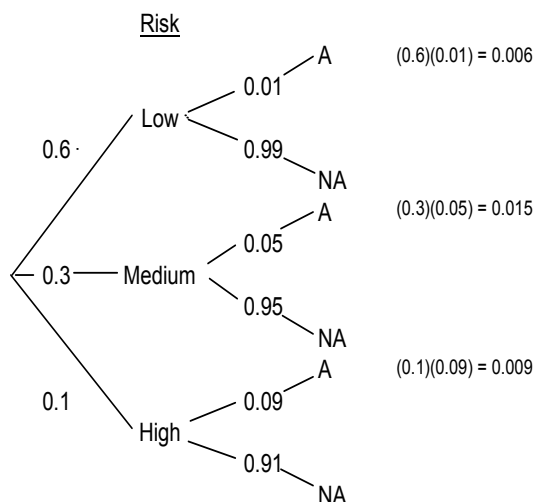
$P(\text{female}) + P(\text{federal aid}) - P(\text{female with federal aid}) =$
 $0.5763 + (0.115 + 0.1894) - 0.1894 = 0.69$

20.

$P(\text{red}) = \frac{1}{3} \cdot \frac{5}{8} + \frac{1}{3} \cdot \frac{3}{4} + \frac{1}{3} \cdot \frac{4}{6} = \frac{49}{72}$

Chapter 4 - Probability and Counting Rules

21.



$$P(\text{accident}) = .006 + .015 + .009 = 0.03$$

22.

$$P(\text{defective}) = 0.15$$

$$P(\text{defective \& misclassified}) = (0.15)(0.1) = 0.015$$

$$P(\text{good \& correctly classified}) = (0.85)(0.9) = 0.765$$

$$P(\text{good}) = 0.765 + 0.015 = 0.78$$

$$P(\text{defective} \mid \text{classified good}) = \frac{0.015}{(0.015 + 0.765)} = 0.019$$

23.

$$P(\text{female} \mid \text{adult}) = \frac{0.07}{0.99} = 0.071$$

24.

$$P(\leq 9 \text{ 1st roll and } \leq 9 \text{ 2nd roll and } > 9 \text{ 3rd roll}) = \frac{30}{36} \cdot \frac{30}{36} \cdot \frac{6}{36} = 0.116$$

25.

$$P(\text{ischemic death} \mid \text{heart disease})$$

$$= \frac{P(\text{heart disease and ischemic})}{P(\text{heart disease})}$$

$$= \frac{0.164}{0.25} = 0.656$$

$$P(\text{at least one from heart disease}) =$$

$$1 - P(\text{none are from heart disease})$$

$$P(\text{at least one from heart disease}) =$$

$$1 - 0.75^2 = 0.4375 \text{ or } 0.438$$

26.

$$P(\text{swim} \mid \text{bridge}) = \frac{P(\text{play bridge and swim})}{P(\text{play bridge})}$$

$$= \frac{0.73}{0.82} = 0.89 \text{ or } 89\%$$

27.

$$P(\text{calculus} \mid \text{dean's list}) = \frac{0.042}{0.21} = 0.2$$

28.

a.

$$P(\text{Dem and expires in 2015}) = \frac{20}{78} \text{ or } 0.256$$

b.

$$P(\text{Rep or expires in 2013}) = \frac{36}{78} + \frac{29}{78} - \frac{8}{78}$$

$$P(\text{Rep or expires in 2013}) = \frac{57}{78} \text{ or } 0.731$$

c.

$$P(\text{Rep} \mid \text{expires in 2017}) =$$

$$\frac{P(\text{expires in 2017 and Rep})}{P(\text{expires in 2017})}$$

$$= \frac{13/78}{14/78} = \frac{13}{14} \text{ or } 0.929$$

No. Some Republicans have terms that expire in 2017.

29.

$$P(\text{salad} \mid \text{pizza}) = \frac{0.65}{0.95} = 0.684 \text{ or } 68.4\%$$

30.

a. $P(\text{coffee or candy}) =$

$$\frac{43}{77} + \frac{22}{77} - \frac{10}{77} = 0.714$$

b. $P(\text{tea} \mid \text{contains mugs}) = \frac{10/77}{23/77} = 0.435$

c. $P(\text{tea and cookies}) = \frac{12}{77} = 0.156$

31.

a. $P(O^-) = 0.06$

b. $P(\text{type O} \mid \text{Rh}^+) = \frac{0.37}{0.85} = 0.435$

c. $P(A^+ \text{ or } AB^-) = 0.34 + 0.01 = 0.35$

d. $P(\text{Rh}^- \mid \text{type B}) = \frac{0.02}{0.12} = 0.167$

Chapter 4 - Probability and Counting Rules

32.

$$a. P(\text{male} \mid \text{pediatrician}) = \frac{\frac{33,020}{124,645}}{\frac{66,371}{124,645}} = 0.498$$

$$b. P(\text{pathologist} \mid \text{female}) = \frac{5604}{51,247} = 0.109$$

$$c. \text{No. } P(\text{path} \mid \text{female}) \neq P(\text{path})$$

33.

$$a. P(\text{tree} \mid \text{after 2000}) = \frac{\frac{77}{623}}{\frac{388}{623}} = 0.198$$

$$b. P(\text{camping and before 2001}) = \frac{117}{623} = 0.188$$

$$c. P(\text{camping} \mid \text{before 2001}) = \frac{\frac{117}{623}}{\frac{235}{623}} = 0.498$$

34.

$$a. P(\text{Ethiopia} \mid 2013) = \frac{P(2013 \text{ and Ethiopia})}{P(2013)}$$

$$P(\text{Ethiopia} \mid 2013) = \frac{993/7014}{3737/7014} = \frac{993}{3737} = 0.2657$$

$$b. P(\text{Ukraine and 2014}) = \frac{521}{7014} = 0.074$$

$$c. P(\text{not 2014 and not Ethiopia}) = \frac{2306 + 438}{7014} = 0.3912$$

$$d. P(\text{from China}) = \frac{4346}{7014} = 0.6196$$

$$P(\text{both from China}) = (0.6196)^2 = 0.3839$$

35.

$$a. P(\text{all 3 get enough exercise}) = (0.27)^3 = 0.0197 \text{ or } 0.020$$

$$b. P(\text{at least one gets enough exercise}) = 1 - (0.73)^3 = 0.611$$

36.

$$P(5 \text{ buy at least 1}) = \frac{90}{120} \cdot \frac{89}{119} \cdot \frac{88}{118} \cdot \frac{87}{117} \cdot \frac{86}{116} = 0.231$$

37.

$$a. P(\text{none have been married}) = (0.703)^5 = 0.172$$

$$b. P(\text{at least one has been married}) = 1 - P(\text{none have been married}) = 1 - 0.1717 = 0.828$$

38.

$$a. P(\text{all three caused by driver error}) = (0.54)^3 = 0.157$$

$$b. P(\text{none caused by driver error}) = (0.46)^3 = 0.097$$

$$c. P(\text{at least one caused by driver error}) = 1 - P(\text{none by driver error}) = 1 - 0.097 = 0.903$$

39.

$$P(\text{at least one not on time}) = 1 - P(\text{none not on time}) = 1 - P(\text{all 5 on time}) = 1 - (0.855)^5 = 0.5431$$

40.

$$a. P(\text{all 4 on time}) = (0.9)(0.8)(0.5)(0.9) \\ P(\text{all 4 on time}) = 0.324$$

$$b. P(\text{at least one not on time}) = 1 - P(\text{none are not on time}) = 1 - P(\text{all 4 on time}) = 1 - 0.324 = 0.676$$

$$c. P(\text{at least 1 on time}) = 1 - P(\text{none on time}) = 1 - P(\text{all 4 not on time}) = 1 - (0.1)(0.2)(0.5)(0.1) \\ P(\text{at least 1 on time}) = 0.999$$

$$d. \text{The events in parts a and b are complementary.}$$

Chapter 4 - Probability and Counting Rules

41.

If $P(\text{read to}) = 0.58$, then

$$P(\text{not being read to}) = 1 - 0.58 = 0.42$$

$P(\text{at least one is read to}) = 1 - P(\text{none are read to})$

$$= 1 - P(\text{all five are not read to})$$

$$= 1 - (0.42)^5 = 0.987$$

42.

a. $P(\text{all three have assistantships}) = (0.6)^3 = 0.216$

b. $P(\text{none have assistantships}) = (0.4)^3 = 0.064$

c. $P(\text{at least one has an assistantship}) = 1 - P(\text{none have assistantships})$
 $= 1 - 0.064 = 0.936$

43.

$P(\text{at least one diamond})$

$$= 1 - P(\text{no diamond})$$

$$= 1 - \frac{39}{52} \cdot \frac{38}{51} \cdot \frac{37}{50} \cdot \frac{36}{49} \cdot \frac{35}{48} = 1 - \frac{69,090,840}{311,875,200}$$

$$= \frac{242,784,360}{311,875,200} = \frac{7,411}{9,520}$$

44.

$$P(\text{autism}) = \frac{1}{88}$$

$$P(\text{does not have autism}) = \frac{87}{88}$$

a. $P(\text{none of 3 have autism}) = \left(\frac{87}{88}\right)^3 = 0.966$

b. $P(\text{at least 1 of 3 has autism}) = 1 - P(\text{none of 3 has autism})$
 $1 - \left(\frac{87}{88}\right)^3 = 0.034$

c. $P(\text{at least 1 of 10 has autism}) = 1 - P(\text{none of 10 has autism})$
 $1 - \left(\frac{87}{88}\right)^{10} = 0.108$

45.

a. $P(\text{no video games are rated mature}) = 1 - 0.155 = 0.845$

$P(\text{none of the five were rated mature}) = (0.845)^5 = 0.4308$

b. $P(\text{at least one of the five was rated mature}) = 1 - 0.4308 = 0.5692$

46.

$P(\text{at least one will not improve}) = 1 -$

$$P(\text{all will improve}) = 1 - (0.75)^{12}$$

$$= 0.968 \text{ or } 96.8\%$$

47.

$P(\text{at least one odd number}) = 1 -$

$P(\text{no odd number})$

$$= 1 - \left(\frac{1}{2}\right)^3 = 1 - \frac{1}{8} = 0.875$$

The event is likely to occur since the probability is high.

48.

$P(\text{at least one X}) = 1 - P(\text{no X's})$

$$1 - \left(\frac{25}{26}\right)^3 = 1 - \frac{15,625}{17,576} = \frac{1951}{17,576} \text{ or } 0.111$$

The event is unlikely to occur since the probability is only about 11%.

49.

$$P(\text{at least one 6}) = \frac{11}{36} = 0.306$$

50.

a. $P(\text{all were waiting for a kidney}) = (0.814)^6 = 0.2909$

b. $P(\text{none were waiting for a kidney}) = (1 - 0.814)^6 = 0.00004$

c. $P(\text{at least one is waiting for a kidney}) = 1 - 0.00004 = 0.99996$

Chapter 4 - Probability and Counting Rules

51.

$P(\text{at least one will consider himself lucky}) = 1 - P(\text{no one will consider himself lucky})$
 $= 1 - (0.88)^3 = 0.319$

52.

$P(\text{at least one rose}) = 1 - P(\text{no roses})$
 $1 - \frac{26}{34} \cdot \frac{25}{33} \cdot \frac{24}{32} \cdot \frac{23}{31} = 1 - \frac{7475}{23,188} = 0.678$

Yes; the event is a little more likely to occur than not since the probability is about 68%.

53.

No, because $P(A \cap B) = 0$ and $P(A \cap B) \neq P(A) \cdot P(B)$

54.

If independent, then $P(\text{compact} \mid \text{domestic}) = P(\text{compact})$

$$P(\text{compact}) = \frac{150}{300} = \frac{1}{2}$$

$$P(\text{compact} \mid \text{domestic}) = \frac{P(\text{domestic and compact})}{P(\text{domestic})}$$

$$= \frac{\frac{100}{300}}{\frac{210}{300}} = \frac{100}{210} \text{ or } \frac{10}{21}$$

Thus, $P(\text{compact} \mid \text{domestic}) \neq P(\text{compact})$
 since $\frac{1}{2} \neq \frac{10}{21}$.

55.

Yes.

$$P(\text{enroll}) = 0.55$$

$P(\text{enroll} \mid \text{DW}) > P(\text{enroll})$ which indicates that DW has a positive effect on enrollment.

$P(\text{enroll} \mid \text{LP}) = P(\text{enroll})$ which indicates that LP has no effect on enrollment.

$P(\text{enroll} \mid \text{MH}) < P(\text{enroll})$ which indicates that MH has a low effect on enrollment.

Thus, all students should meet with DW.

56.

$$P(\text{buy}) = 0.35$$

a. If $P(\text{buy} \mid \text{ad}) = 0.20$, then the commercial adversely affects the probability of buying since the events are dependent and the probability that a person buys the product is less than 0.35. The events are dependent.

b. If $P(\text{buy} \mid \text{ad}) = 0.35$, then the commercial has no effect on buying the product. The events are independent.

c. If $P(\text{buy} \mid \text{ad}) = 0.55$, then the commercial has an effect on buying the product. The events are dependent.

57.

The Addition Rule states that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ and if A and B are mutually exclusive, $P(A \text{ and } B) = 0$.

$$\text{Then } 0.601 = 0.342 + 0.279 - P(A \text{ and } B)$$

$$0.601 = 0.621 - P(A \text{ and } B)$$

$$P(A \text{ and } B) = 0.02$$

Therefore, events A and B are not mutually exclusive.

If A and B are independent,

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

$$0.02 \neq (0.342)(0.279)$$

Therefore, A and B are not independent.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.02}{0.279} = 0.072$$

$$P(\text{not } B) = 1 - 0.279 = 0.721$$

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58.

There are $6 \cdot 6$ possible outcomes for the roll of both die. Using a tree diagram to list the outcomes (Hare, Tortoise), the only outcomes in which the tortoise has a higher score than the hare are:
(1, 2), (1, 2), (1, 3), and (2, 3)

Thus, $P(\text{tortoise ahead of hare}) = \frac{4}{36}$ or $\frac{1}{9}$

59.

$$P(\text{black} \mid \text{bag 1 or black} \mid \text{bag 2}) = \frac{2}{15}$$

$$P(\text{black} \mid \text{bag 1}) + P(\text{black} \mid \text{bag 2}) = \frac{2}{15}$$

$$\frac{1}{2} \cdot \frac{1}{10} + \frac{1}{2} \cdot \frac{1}{1+x} = \frac{2}{15}$$

$$\frac{1}{20} + \frac{1}{2+2x} = \frac{2}{15}$$

$$\frac{2x+22}{20(2+2x)} = \frac{2}{15}$$

$$30x + 330 = 80 + 80x$$

$$-50x = -250$$

$$x = 5$$

There are 5 white marbles in Bag #2.

EXERCISE SET 4-4

1.

$$10^5 = 100,000$$

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30,240$$

2.

MATH ATHM TAMH HATM
MAHT ATMH TAHM HAMT
MTAH AMTH THMA HTAM
MTHA AMHT THAM HTMA
MHAT AHMT TMAH HMAT
MHTA AHTM TMHA HMTA

3.

$$7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$$

4.

$$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362,880$$

5.

$$10^5 = 100,000$$

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30,240$$

6.

$$5! = 120$$

$$3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1 = 12$$

7.

$$6^5 = 7776$$

8.

$$2 \cdot 25 \cdot 24 \cdot 23 = 27,600$$

$$2 \cdot 26 \cdot 26 \cdot 26 = 35,152$$

9.

$$8 \cdot 3 \cdot 5 = 120$$

10.

If repetitions are permitted: $6^4 = 1296$

If repetitions are not permitted:

$$6 \cdot 5 \cdot 4 \cdot 3 = 360$$

11.

$$\frac{12!}{(12-7)!} = 3,991,680$$

$$\frac{8!}{(8-3)!} \cdot 4! = 8064$$

12.

$$2 \cdot 4 = 8$$

13.

$$\begin{aligned} \text{a. } 11! &= 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 39,916,800 \end{aligned}$$

$$\begin{aligned} \text{b. } 9! &= 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 362,880 \end{aligned}$$

$$\text{c. } 0! = 1$$

$$\text{d. } 1! = 1$$

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13. continued

$$e. {}_6P_4 = \frac{6!}{(6-4)!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 360$$

$$f. {}_{12}P_8 = \frac{12!}{(12-8)!} \\ = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 19,958,400$$

$$g. {}_7P_7 = \frac{7!}{(7-7)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1} = 5040$$

$$h. {}_4P_0 = \frac{4!}{(4-0)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 1$$

$$i. {}_9P_2 = \frac{9!}{(9-2)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 72$$

$$j. {}_{11}P_3 = \frac{11!}{(11-3)!} \\ = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 990$$

14.

$$a. 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

$$b. 11! = 39,916,800$$

$$c. 2! = 2$$

$$d. 9! = 362,880$$

$$e. {}_9P_6 = \frac{9!}{(9-6)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 60,480$$

$$f. {}_{11}P_4 = \frac{11!}{(11-4)!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 7920$$

$$g. {}_8P_0 = \frac{8!}{(8-0)!} = 1$$

$$h. {}_{10}P_2 = \frac{10!}{(10-2)!} = \frac{10 \cdot 9 \cdot 8!}{8!} = 90$$

15.

$${}_4P_4 = \frac{4!}{(4-4)!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{0!} = 24$$

16.

$${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4!} = 210$$

17.

$${}_9P_3 = \frac{9!}{(9-3)!} = 504$$

18.

$${}_{10}P_5 = \frac{10!}{(10-5)!} = \frac{10!}{5!} = 30,240$$

19.

$${}_7P_4 = \frac{7!}{(7-4)!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 840$$

20.

$$4 \cdot 6 \cdot 5 = 120$$

21.

$${}_{10}P_6 = \frac{10!}{(10-6)!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1} = 151,200$$

22.

$$\text{Exactly 3 samples: } {}_{13}C_3 = \frac{13!}{(13-3)!3!} = 286$$

Up to 3 samples:

$${}_{13}C_0 + {}_{13}C_1 + {}_{13}C_2 + {}_{13}C_3 = 378$$

23.

$${}_{50}P_4 = \frac{50!}{(50-4)!} = \frac{50!}{46!} = 5,527,200$$

24.

$${}_{11}C_4 = \frac{11!}{(11-4)!4!} = 330$$

25.

$$\text{Same task: } {}_{12}C_4 = \frac{12!}{(12-4)!4!} = 495$$

$$\text{Different tasks: } {}_{12}P_4 = \frac{12!}{(12-4)!} = 11,880$$

26.

$${}_7P_5 = \frac{7!}{(7-5)!} = \frac{7!}{2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2!}{2!} = 2520$$

27.

$$\frac{7!}{3!2!2!} = 210$$

28.

The word Massachusetts has the following letters:

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28. continued

1 - M 1 - H

2 - A's 1 - U

4 - S's 1 - E

1 - C 2 - T's

$$\frac{13!}{1! \cdot 2! \cdot 4! \cdot 1! \cdot 1! \cdot 1! \cdot 1! \cdot 2!} = 64,864,800$$

29.

$$\frac{9!}{4! \cdot 3! \cdot 2!} = 1260$$

30.

$$\frac{12!}{5! \cdot 3! \cdot 4!} = 27,720$$

31.

$$\frac{12!}{6! \cdot 3! \cdot 3!} = 18,480$$

32.

The word Cincinnati has the following letters:

2 - C's

3 - I's

2 - N's

1 - A

2 - T's

$$\frac{10!}{2! \cdot 3! \cdot 2! \cdot 1! \cdot 2!} = 75,600$$

33.

a. $\frac{5!}{3! \cdot 2!} = 10$

b. $\frac{8!}{5! \cdot 3!} = 56$

c. $\frac{7!}{3! \cdot 4!} = 35$

d. $\frac{6!}{4! \cdot 2!} = 15$

e. $\frac{6!}{2! \cdot 4!} = 15$

34.

a. $\frac{3!}{3! \cdot 0!} = 1$

b. $\frac{3!}{0! \cdot 3!} = 1$

c. $\frac{9!}{2! \cdot 7!} = 36$

34. continued

d. $\frac{12!}{10! \cdot 2!} = 66$

e. $\frac{4!}{1! \cdot 3!} = 4$

35.

$${}_{50}C_5 = \frac{50!}{45! \cdot 5!} = 2,118,760$$

36.

$$\begin{aligned} {}_{12}C_4 \cdot {}_9C_3 &= \frac{12!}{8! \cdot 4!} \cdot \frac{9!}{6! \cdot 3!} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 3 \cdot 2 \cdot 1} = 41,580 \end{aligned}$$

37.

$${}_{12}C_4 = \frac{12!}{8! \cdot 4!} = 495$$

38.

$${}_{10}C_3 = \frac{10!}{7! \cdot 3!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 3 \cdot 2 \cdot 1} = 120$$

39.

$${}_{10}C_3 \cdot {}_6C_2 = \frac{10!}{7! \cdot 3!} \cdot \frac{6!}{4! \cdot 2!} = 1800$$

40.

$${}_{16}C_4 = \frac{16!}{12! \cdot 4!} = 1820$$

If 4 insist on playing together, the remaining 12 will make up the other tables.

${}_{12}C_4 = 495$ ways the other 12 can be grouped together in tables of 4. Add the table consisting of the 4 who insist on playing together for a total of 496 ways.

41.

$$5 \cdot 5 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 6400$$

42.

$$\begin{aligned} {}_4C_2 \cdot {}_{12}C_5 \cdot {}_7C_3 &= \frac{4!}{2! \cdot 2!} \cdot \frac{12!}{7! \cdot 5!} \cdot \frac{7!}{4! \cdot 3!} \\ &= \frac{4 \cdot 3 \cdot 2!}{2! \cdot 2 \cdot 1} \cdot \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2 \cdot 1} \\ &= 6 \cdot 792 \cdot 35 = 166,320 \end{aligned}$$

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43.

$${}_{12}C_4 = 495$$

$${}_7C_2 \cdot {}_5C_2 = 21 \cdot 10 = 210$$

$${}_7C_2 \cdot {}_5C_2 + {}_7C_3 \cdot {}_5C_1 + {}_7C_4$$

$$= 21 \cdot 10 + 35 \cdot 5 + 35$$

$$= 210 + 175 + 35 = 420$$

44.

$${}_{10}C_3 \cdot {}_{10}C_3 = \frac{10!}{7!3!} \cdot \frac{10!}{7!3!}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 3 \cdot 2 \cdot 1} \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 3 \cdot 2 \cdot 1} = 120 \cdot 120 = 14,400$$

45.

The possibilities are CVV or VCV or VVV.

Assuming the same vowel can't be used

twice in a "word":

$$7 \cdot 5 \cdot 4 + 5 \cdot 7 \cdot 4 + 5 \cdot 4 \cdot 3 = 340$$

Assuming the same vowel can be used twice

in a "word":

$$7 \cdot 5 \cdot 5 + 5 \cdot 7 \cdot 5 + 5 \cdot 5 \cdot 5 = 475$$

46.

$${}_{12}C_6 \cdot {}_{10}C_6 = \frac{12!}{6!6!} \cdot \frac{10!}{4!6!}$$

$$= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 924 \cdot 210 = 194,040$$

47.

The possibilities are 2 men and 2 women, 4 men and no women, or no men and 4 women.

$${}_6C_2 \cdot {}_4C_2 + {}_6C_4 \cdot {}_4C_0 + {}_6C_0 \cdot {}_4C_4 =$$

$$\frac{6!}{4!2!} \cdot \frac{4!}{2!2!} + \frac{6!}{2!4!} \cdot \frac{4!}{4!0!} + \frac{6!}{6!0!} \cdot \frac{4!}{0!4!} =$$

$$90 + 15 + 1 = 106$$

48.

$${}_{25}C_5 = \frac{25!}{20!5!} = \frac{25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20!}{20! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 53,130$$

49.

There are ${}_7C_2 = 21$ tiles with unequal numbers and 7 tiles with equal numbers.

Thus, the total number of tiles is 28.

50.

$${}_{16}C_4 \cdot {}_{15}C_2 = \frac{16!}{(16-4)!4!} \cdot \frac{15!}{(15-2)!2!}$$

$$= 191,100$$

51.

$${}_{12}C_2 \cdot {}_8C_2 \cdot {}_6C_2 = \frac{12!}{10!2!} \cdot \frac{8!}{6!2!} \cdot \frac{6!}{4!2!} = 27,720$$

52.

$${}_{13}C_8 = \frac{13!}{5!8!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1287$$

53.

$${}_{10}C_3 \cdot {}_6C_2 \cdot {}_5C_1 = \frac{10!}{7!3!} \cdot \frac{6!}{4!2!} \cdot \frac{5!}{4!1!} = 9,000$$

54.

$${}_{17}C_8 = \frac{17!}{9!8!} = \frac{17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$= 24,310$$

55.

$${}_{20}C_8 = \frac{20!}{(20-8)!8!}$$

$$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 125,970$$

56.

$${}_6P_3 = \frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 120$$

57.

$${}_9C_5 = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5!} = 126$$

58.

$${}_{11}C_2 \cdot {}_8C_3 = \frac{11!}{(11-2)!2!} \cdot \frac{8!}{(8-3)!3!} = 3080$$

59.

$${}_{17}C_2 = \frac{17!}{(17-2)!2!} = \frac{17 \cdot 16 \cdot 15!}{15! \cdot 2 \cdot 1} = 136$$

60.

$${}_{10}C_8 = \frac{10!}{(10-8)!8!} = \frac{10 \cdot 9 \cdot 8!}{2 \cdot 1 \cdot 8!} = 45$$

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61.

$${}_{11}C_3 = \frac{11!}{8!3!} = \frac{11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 3 \cdot 2 \cdot 1} = 165$$

62.

$${}_5P_3 + {}_5P_4 + {}_5P_5 = \frac{5!}{2!} + \frac{5!}{1!} + \frac{5!}{0!} = 300$$

63.

$${}_6C_3 \cdot {}_5C_2 = \frac{6!}{3!3!} \cdot \frac{5!}{2!3!} = 200$$

64.

$${}_9C_5 = \frac{9!}{4!5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 5!} = 126$$

65.

$${}_8P_3 = \frac{8!}{5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 336$$

66.

$${}_{12}P_4 = \frac{12!}{8!} = 11,880$$

$${}_4P_1 \cdot {}_4P_1 \cdot {}_4P_1 \cdot {}_9P_1 =$$

$$4 \cdot 4 \cdot 4 \cdot 9 = 576$$

67.

$${}_4C_1 + {}_4C_2 + {}_4C_3 + {}_4C_4 =$$

$$\frac{4!}{3!1!} + \frac{4!}{2!2!} + \frac{4!}{1!3!} + \frac{4!}{0!4!} =$$

$$4 + 6 + 4 + 1 = 15$$

68.

$$1 \cdot 2 \cdot 1 = 2$$

$$1 \cdot 3 \cdot 2 \cdot 1 = 6$$

$$1 \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1 = (n-1)!$$

69.

- a. $2! \cdot 4! = 48$
- b. 60 ways
- Using a table, list the number of ways in each column and multiply:

69. continued

B	C	3	2	1	=	6
B	3	C	2	1	=	6
B	3	2	C	1	=	6
B	3	2	1	C	=	6
3	B	C	2	1	=	6
3	B	2	C	1	=	6
3	B	2	1	C	=	6
3	2	B	C	1	=	6
3	2	B	1	C	=	6
3	2	1	B	C	=	<u>6</u>
						60

c. $5! - 2 \cdot 4! = 72$

70.

- a. ${}_4C_1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 4$
- b. ${}_{10}C_1 \cdot {}_4C_1 - {}_4C_1 = 36$
- c. ${}_{13}C_1 \cdot {}_{12}C_1 \cdot {}_4C_1 = 624$
- d. ${}_{13}C_1 \cdot {}_4C_3 \cdot {}_{12}C_1 \cdot {}_4C_2 = 3744$

71.

$$(x+2)C_x = \frac{(x+2)!}{(x+2-x)! \cdot x!}$$

$$= \frac{(x+2)(x+1)(x)(x-1)\dots(3)(2)(1)}{2! \cdot x!}$$

$$= \frac{(x+2)(x+1)x!}{2 \cdot x!} = \frac{(x+2)(x+1)}{2}$$

72.

In a deck of cards, there are 4 aces, 4 twos, 4 threes, etc. up to 4 kings. Thus, there are 13 sets of "matches" in a deck of 52 cards.

The number of two-card matches is

$${}_{13}C_2 = \frac{13!}{11!2!} = 78 \text{ matches.}$$

EXERCISE SET 4-5

1.

$$P(2 \text{ face cards}) = \frac{12}{52} \cdot \frac{11}{51} = \frac{11}{221}$$

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2.

There are a total of $10!$ ways to deal the ten cards. There are 2 ways the cards could be dealt in order, either 1 - 10 or 10 - 1.

$$P(\text{all 10 in order}) = \frac{2}{10!} = 0.0000005$$

3.

a. There are ${}_5C_3$ ways of selecting 3 men and ${}_9C_3$ total ways to select 3 people;

$$\text{hence, } P(\text{all men}) = \frac{{}_5C_3}{{}_9C_3} = \frac{10}{84} = \frac{5}{42}.$$

b. There are ${}_4C_3$ ways of selecting 3 women;

$$\text{hence, } P(\text{all women}) = \frac{{}_4C_3}{{}_9C_3} = \frac{4}{84} = \frac{1}{21}.$$

c. There are ${}_5C_2$ ways of selecting 2 men and ${}_4C_1$ ways of selecting one woman;

$$\begin{aligned} \text{hence, } P(2 \text{ men and 1 woman}) &= \frac{{}_5C_2 \cdot {}_4C_1}{{}_9C_3} \\ &= \frac{10}{21}. \end{aligned}$$

d. There are ${}_4C_2$ ways to select two women and ${}_5C_1$ ways of selecting one man; hence,

$$P(2 \text{ women and 1 man}) = \frac{{}_4C_2 \cdot {}_5C_1}{{}_9C_3} = \frac{5}{14}.$$

4. There are ${}_{54}C_3$ ways to select 3

Republicans; hence, $P(3 \text{ Republicans})$

$$= \frac{{}_{54}C_3}{{}_{100}C_3} = \frac{24,804}{161,700} = 0.1534$$

There are ${}_{44}C_3$ ways to select 3

Democrats; hence, $P(\text{Democrats})$

$$= \frac{{}_{44}C_3}{{}_{100}C_3} = \frac{13,244}{161,700} = 0.0819$$

There are 54 ways to select one Republican, 2 ways to select one Independent, and 44 ways to select one Democrat; hence

$P(\text{one from each arty})$

$$= \frac{54}{100} \cdot \frac{2}{99} \cdot \frac{44}{98} = \frac{4752}{97,020} = 0.0049$$

5.

$$\text{a. } P(\text{both are men}) = \frac{{}_6C_2 \cdot {}_7C_0}{{}_{13}C_2} = \frac{15}{78} = 0.192$$

$$\text{b. } P(\text{both are women}) = \frac{{}_6C_0 \cdot {}_7C_2}{{}_{13}C_2} = \frac{21}{78} = 0.269$$

c. $P(\text{one man and one woman}) =$

$$\frac{{}_6C_1 \cdot {}_7C_1}{{}_{13}C_2} = \frac{42}{78} = 0.538$$

$$\text{d. } P(\text{twins}) = \frac{1}{78} = 0.013$$

6.

$$\text{a. } P(\text{no defective resistors}) = \frac{{}_9C_4}{{}_{12}C_4} = \frac{126}{495} = \frac{14}{55}$$

$$\text{b. } P(1 \text{ defective resistor}) = \frac{{}_3C_1 \cdot {}_9C_3}{{}_{12}C_4} = \frac{252}{495} = \frac{28}{55}$$

$$\text{c. } P(3 \text{ defective resistors}) = \frac{{}_3C_3 \cdot {}_9C_1}{{}_{12}C_4} = \frac{1}{55}$$

7.

$$\frac{{}_3C_2}{{}_{10}C_2} = \frac{3}{45} = \frac{1}{15}$$

8.

There are ${}_4C_3$ ways of getting 3 of a kind for one denomination and there are 13 denominations. There are ${}_4C_2$ ways of getting two of a kind and 12 denominations left. There are ${}_{52}C_5$ ways to get five cards; hence,

$$P(\text{full house}) = \frac{{}_{13}C_3 \cdot {}_{12}C_2}{{}_{52}C_5} = \frac{6}{4165}$$

9.

$P(\text{at least one U.S}) = 1 - P(\text{none are U.S})$

$$\begin{aligned} &= 1 - \frac{{}_7C_0 \cdot {}_{13}C_5}{{}_{20}C_5} \\ &= 1 - \frac{1287}{15,504} = 0.917 \end{aligned}$$

$P(\text{at least two U.S}) = 1 - P(\text{none or one U.S})$

$$\begin{aligned} &= 1 - \left(\frac{{}_7C_0 \cdot {}_{13}C_5 + {}_7C_1 \cdot {}_{13}C_4}{{}_{20}C_5} \right) \\ &= 1 - \frac{6292}{15,504} = 0.594 \end{aligned}$$

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9. continued

$$P(\text{all five U.S}) = \frac{{}^7C_5 \cdot {}^{13}C_0}{{}^{20}C_5} = \frac{21}{15,504} = 0.001$$

10.

There are 6 red face cards and 16 black cards numbered 2 - 9, for a total of 22 cards.

$$\text{a. } P(\text{all 4 red}) = \frac{{}^6C_4 \cdot {}^{16}C_0}{{}^{22}C_4} = 0.002$$

$$\text{b. } P(2 \text{ red and 2 black}) = \frac{{}^6C_2 \cdot {}^{16}C_2}{{}^{22}C_4} = 0.246$$

$$\text{c. } P(\text{at least one red}) = 1 - P(\text{none red})$$

$$P(\text{at least one red}) = 1 - \frac{{}^6C_0 \cdot {}^{16}C_4}{{}^{22}C_4} = 0.751$$

$$\text{d. } P(\text{all 4 black}) = \frac{{}^{16}C_4 \cdot {}^6C_0}{{}^{22}C_4} = 0.249$$

11.

$$\text{a. } P(\text{red}) = \frac{{}^{11}C_2}{{}^{19}C_2} = \frac{55}{171} = 0.322$$

$$\text{b. } P(\text{black}) = \frac{{}^8C_2}{{}^{19}C_2} = \frac{28}{171} = 0.164$$

$$\text{c. } P(\text{unmatched}) = \frac{{}^{11}C_1 \cdot {}^8C_1}{{}^{19}C_2} = \frac{88}{171} = 0.515$$

d. It probably got lost in the wash!

12.

$$\frac{{}^8C_3 \cdot {}^9C_4}{{}^{17}C_7} = \frac{56 \cdot 126}{19,448} = \frac{7056}{19,448} = \frac{882}{2431}$$

13.

There are $6^3 = 216$ ways of tossing three dice, and there are 10 ways of getting a sum of 6 such as (1, 1, 4), (1, 2, 3), (2, 2, 2), (1, 4, 1), etc. Hence, the probability of rolling a sum of 6 is $\frac{10}{216} = \frac{5}{108}$.

14.

a. P(all 4 seniors)

$$= \frac{{}^{10}C_4 \cdot {}^{20}C_0 \cdot {}^{20}C_0 \cdot {}^{15}C_0}{{}^{65}C_4} = 0.0003$$

14. continued

b. P(one of each)

$$= \frac{{}^{20}C_1 \cdot {}^{20}C_1 \cdot {}^{15}C_1 \cdot {}^{10}C_1}{{}^{65}C_4} = 0.089$$

c. P(2 sophomores and 2 freshmen)

$$= \frac{{}^{20}C_2 \cdot {}^{20}C_2 \cdot {}^{15}C_0 \cdot {}^{10}C_0}{{}^{65}C_4} = 0.053$$

d. P(at least 1 senior)

$$= 1 - P(\text{none are seniors})$$

$$= 1 - \frac{{}^{55}C_4}{{}^{65}C_4} = 0.496$$

15.

There are $5! = 120$ ways to arrange 5 washers in a row and 2 ways to have them in correct order, small to large or large to small; hence, the probability is $\frac{2}{120} = \frac{1}{60}$.

16.

There are ${}_{52}C_5 = \frac{52!}{47!5!} = 2,598,960$ possible hands.

$$\text{a. } \frac{4}{2,598,960}$$

$$\text{b. } \frac{36}{2,598,960}$$

$$\text{c. } \frac{624}{2,598,960}$$

17.

P(berries are produced) = P(1 or 2 males)

$$P(1 \text{ or } 2 \text{ males}) = \frac{{}^8C_2 \cdot {}^4C_1}{{}^{12}C_3} + \frac{{}^8C_1 \cdot {}^4C_2}{{}^{12}C_3} = 0.509 + 0.218 = 0.727$$

REVIEW EXERCISES - CHAPTER 4

1.

$$\text{a. } \frac{1}{8} = 0.125 \quad \text{b. } \frac{3}{8} = 0.375$$

$$\text{c. } \frac{4}{8} = 0.50$$

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2.

a. $\frac{13}{52} = \frac{1}{4}$ d. $\frac{4}{52} = \frac{1}{13}$

b. $\frac{11}{26}$ e. $\frac{26}{52} = \frac{1}{2}$

c. $\frac{1}{52}$

3.

a. $P(\text{not used for taxes}) = P(\text{virus or other})$

$P(\text{virus or other}) = \frac{5}{10} + \frac{2}{10} = 0.7$

b. $P(\text{taxes or other use}) = \frac{3}{10} + \frac{2}{10} = 0.5$

4.

a. $P(\text{US}) = \frac{7632}{42,857} = 0.178$

b. $P(\text{not Asia}) = 1 - P(\text{Asia})$

$1 - P(\text{Asia}) = 1 - \frac{29,525}{42,857} = 0.311$

c. $P(\text{Germany or Japan}) = \frac{14,897}{42,857} = 0.348$

5.

$P(\text{neither}) = 1 - (\text{either})$

$1 - P(\text{either}) = 1 - (0.32 + 0.41 - 0.06)$

$P(\text{neither}) = 0.33$

6.

Refer to the sample space for tossing two dice.

a. There are 4 ways to roll a 5 and 5 ways to roll a 6; hence, $P(5 \text{ or } 6) = \frac{4}{36} + \frac{5}{36} = \frac{1}{4}$

b. There are 3 ways to get a 10, 2 ways to get an 11 and 1 way to get a 12; hence,

$P(\text{sum greater than } 9) = \frac{3}{36} + \frac{2}{26} + \frac{1}{36} = \frac{1}{6}$

c. A sum less than 4 means 3 or 2, and greater than 9 means 10, 11, 12; hence, the probability is $\frac{2+1+3+2+1}{36} = \frac{9}{36} = \frac{1}{4}$.

d. Four, 8, and 12 are divisible by 4; hence, the probability of rolling a 4, 8, or 12 is

$\frac{3+5+1}{36} = \frac{9}{36} = \frac{1}{4}$.

6. continued

e. Since this is impossible, the answer is 0.

f. Since this is the entire sample space, the probability is $\frac{36}{36} = 1$.

7.

$P(\text{either backup or GPS})$
 $= 0.6 + 0.4 - 0.2 = 0.8$

$P(\text{neither backup nor GPS})$
 $= 1 - 0.8 = 0.2$

8.

$P(\text{preferred juice}) = \frac{13}{60}$

9.

$P(\text{either a lawnmower or a weed wacker}) = 0.7 + 0.5 - 0.3 = 0.9$

10.

$P(\text{table games | slot machines})$

$= \frac{P(\text{table games and slot machines})}{P(\text{slot machines})}$
 $= \frac{0.15}{0.85} = 0.176$

11.

$P(\text{enrolled in an online course}) = \frac{1}{6} \text{ or } 0.167$

a. $P(\text{all 5 took an online course}) = (\frac{1}{6})^5 = 0.0001$

b. $P(\text{none took an online course}) = (\frac{5}{6})^5 = 0.402$

c. $P(\text{at least one took an online course})$
 $= 1 - P(\text{none took an online course})$
 $= 1 - (\frac{5}{6})^5 = 0.598$

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12.

a. $P(\text{blue}) = \frac{9}{35}$

b. $P(\text{yellow or white}) = \frac{7}{35} + \frac{16}{35} = \frac{23}{35}$

c. $P(\text{red, blue, or yellow}) = \frac{3}{35} + \frac{9}{35} + \frac{7}{35} = \frac{19}{35}$

d. $P(\text{not white}) = 1 - P(\text{white})$
 $P(\text{not white}) = 1 - \frac{16}{35} = \frac{19}{35}$

13.

a. $\frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} = \frac{2}{17}$

b. $\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} = \frac{33}{2550} = \frac{11}{850}$

c. $\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} = \frac{1}{5525}$

14.

a. $\frac{1}{2} \cdot \frac{4}{52} = \frac{1}{26}$

b. $\frac{1}{2} \cdot \frac{26}{52} = \frac{1}{4}$

c. $\frac{1}{2} \cdot \frac{13}{52} = \frac{1}{8}$

15.

Total number of movie releases = 1384

a. $P(\text{European}) = \frac{834}{1384} = 0.603$

b. $P(\text{US}) = \frac{471}{1384} = 0.340$

c. $P(\text{German or French}) = \frac{316}{1384} + \frac{132}{1384}$
 $= \frac{448}{1384} \text{ or } 0.324$

d. $P(\text{German} | \text{European})$
 $= \frac{P(\text{European and German})}{P(\text{European})} = \frac{\frac{316}{1384}}{\frac{834}{1384}} = 0.379$

16.

	X	Y	Z	Total
TV	18	32	15	65
Stereo	<u>6</u>	<u>20</u>	<u>13</u>	<u>39</u>
Total	24	52	28	104

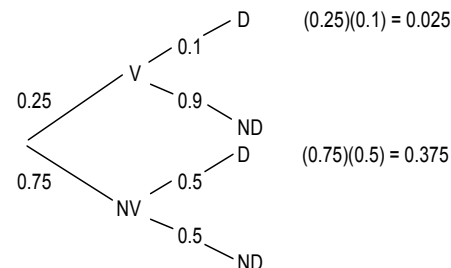
16. continued

a. $\frac{24}{104} + \frac{39}{104} - \frac{6}{104} = \frac{57}{104}$

b. $\frac{52}{104} + \frac{28}{104} = \frac{80}{104} = \frac{10}{13}$

c. $\frac{65}{104} + \frac{28}{104} - \frac{15}{104} = \frac{78}{104} = \frac{3}{4}$

17.



$P(\text{disease}) = 0.025 + 0.375 = 0.4$

18.

$P(\text{defective and from factory A})$

$A) = (0.05)(0.6) = 0.03$

$P(\text{defective and from factory B})$

$B) = (0.06)(0.4) = 0.024$

$P(\text{defective}) = 0.03 + 0.024 = 0.054$

$P(\text{factory A} | \text{defective})$

$= \frac{P(\text{defective and factory A})}{P(\text{defective})}$
 $= \frac{(0.05)(0.6)}{0.054} = 0.556$

19.

$P(\text{NC} | \text{C}) = \frac{P(\text{NC and C})}{P(\text{C})} = \frac{0.37}{0.73} = 0.507$

20.

$P(\text{all 4 correct}) = \frac{1}{24} = 0.042$

$P(3 \text{ are correct}) = 0$, since if 3 labels are correct, the 4th label must also be correct.

$P(2 \text{ are correct}) = \frac{6}{24} = 0.25$

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20. continued

$P(\text{at least one correct}) = 1 - P(\text{none correct})$

$P(\text{at least one correct}) = 1 - P(\text{all 4 labels are wrong})$

$$P(\text{at least one correct}) = 1 - \frac{9}{24} = \frac{15}{24} \text{ or } 0.625$$

21.

$$\frac{0.43}{0.75} = 0.573 \text{ or } 57.3\%$$

22.

$P(\text{bus late} \mid \text{bad weather}) =$

$$\frac{P(\text{bus late and bad weather})}{P(\text{bad weather})} = \frac{0.023}{0.40} = 0.058$$

23.

	<4 yrs HS	HS	College	Total
Smoker	6	14	19	39
Non-Smoker	<u>18</u>	<u>7</u>	<u>25</u>	<u>50</u>
Total	24	21	44	89

a. There are 44 college graduates and 19 of them smoke; hence, the probability is $\frac{19}{44}$.

b. There are 24 people who did not graduate from high school, 6 of whom do not smoke; hence, the probability is $\frac{6}{24} = \frac{1}{4}$.

24.

$P(\text{veteran}) = 0.11$; $P(\text{not a veteran}) = 0.89$

$P(\text{none of 5 are veterans}) = (0.89)^5 = 0.558$

$P(\text{at least one is a veteran}) = 1 - 0.558 = 0.442$

25.

$P(\text{at least one household has a television set})$

$= 1 - P(\text{none have a television set})$

$$= 1 - (0.02)^4 = 0.99999984$$

26.

$P(\text{at least one has chronic sinusitis})$

$= 1 - P(\text{none has chronic sinusitis})$

$$= 1 - (0.85)^5 = 0.556 \text{ or } 55.6\%$$

27.

If repetitions are allowed:

$$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 676,000$$

If repetitions are not allowed:

$${}_{26}P_2 \cdot {}_{10}P_3 = \frac{26 \cdot 25 \cdot 24!}{24!} \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 468,000$$

If repetitions are allowed in the digits but not in the letters:

$$10 \cdot 10 \cdot 10 \cdot {}_{26}P_2 = 650,000$$

28.

$$5 \cdot 11 \cdot 2 \cdot 2 = 220 \text{ different types of paper}$$

29.

$${}_5C_3 \cdot {}_7C_4 = \frac{5!}{2!3!} \cdot \frac{7!}{3!4!} = 10 \cdot 35 = 350$$

30.

$${}_9C_3 = 84$$

31.

Although there are a total of 20 names, the names Ethan, Jacob and Noah appear on both lists. There are 17 different names to choose from.

${}_{17}C_5 = 6188$ different ways to choose 5 names.

32.

$${}_6C_3 \cdot {}_5C_2 \cdot {}_4C_1 = \frac{6!}{3!3!} \cdot \frac{5!}{2!2!} \cdot \frac{4!}{3!1!} = 20 \cdot 10 \cdot 4 = 800$$

33.

$$100!$$

34.

$$5 \cdot 3 \cdot 2 = 30$$

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35.

$${}_{12}C_4 = \frac{12!}{8!4!} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 8!} = 495$$

36.

$${}_{13}C_3 = \frac{13!}{10!3!} = \frac{13 \cdot 12 \cdot 11 \cdot 10!}{10! \cdot 3 \cdot 2 \cdot 1} = 286$$

37.

$$\frac{6!}{2!1!3!} = 60$$

38.

For the word MATHEMATICS, there are 2 Ms, 2 As, and 2 Ts. All other letters occur one time. The number of permutations is

$$\frac{11!}{2!2!1!1!1!1!1!1!} = 4,989,600$$

For the word PROBABILITY, there are 2 Bs and 2 Is. All other letters occur one time. The number of permutations is

$$\frac{11!}{2!2!1!1!1!1!1!1!1!} = 9,979,200$$

There are 4, 989,600 more, or twice as many, permutations in the word PROBABILITY.

39.

$${}_{16}C_6 = \frac{16!}{10!6!} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{10! \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 8008$$

40.

$$3 \cdot 5 \cdot 4 = 60$$

41.

Total catalog number of outcomes:

$$26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 676,000$$

Total number of ways for ID followed by a number divisible by 5:

$$26 \cdot 26 \cdot 10 \cdot 10 \cdot 2 = 135,200$$

$$\text{Hence, } P = \frac{135,200}{676,000} = 0.2$$

42.

Total number of outcomes:

$$26 \cdot 25 \cdot 24 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 156,000,000$$

42. continued

There are 24 ways for the three letters to occur in alphabetical orders.

$$\text{Hence } P = 0.022$$

43.

Total number of territories = 45

P(3 French or 3 UK or 3

$$\text{US}) = \frac{{}_{16}C_3}{{}_{45}C_3} + \frac{{}_{15}C_3}{{}_{45}C_3} + \frac{{}_{14}C_3}{{}_{45}C_3}$$

$$= \frac{560}{14,190} + \frac{455}{14,190} + \frac{364}{14,190}$$

$$= \frac{1379}{14,190} = 0.097$$

44.

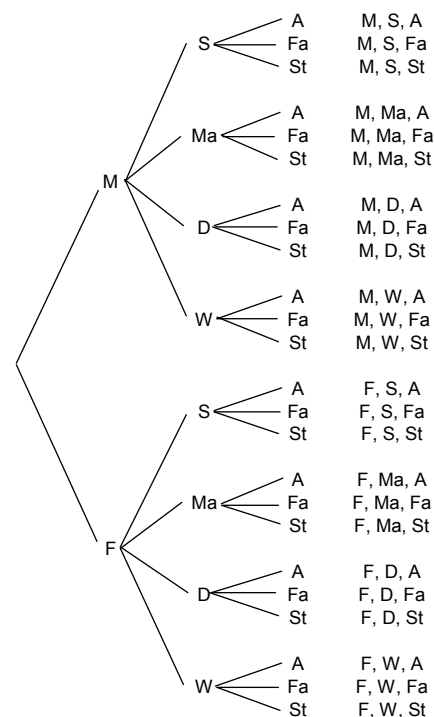
P(Yahtzee on first roll) =

$$\frac{6}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = 0.00077 \text{ or } 0.0008$$

P(Yahtzee on two successive rolls) =

$$(0.000772)^2 = 0.0000006$$

45.



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CHAPTER 4 QUIZ

1. False, subjective probability can be used when other types of probabilities cannot be found.
2. False, empirical probability uses frequency distributions.
3. True
4. False, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
5. False, the probabilities can be different.
6. False, complementary events cannot occur at the same time.
7. True
8. False, order does not matter in combinations.
9. b
10. b and d
11. d
12. b
13. c
14. b
15. d
16. b
17. b
18. Sample space
19. 0, 1
20. 0
21. 1
22. Mutually exclusive

23. a. $\frac{4}{52} = \frac{1}{13}$ c. $\frac{16}{52} = \frac{4}{13}$

b. $\frac{4}{52} = \frac{1}{13}$

24. a. $\frac{13}{52} = \frac{1}{4}$ d. $\frac{4}{52} = \frac{1}{13}$

b. $\frac{4+13-1}{52} = \frac{4}{13}$ e. $\frac{26}{52} = \frac{1}{2}$

c. $\frac{1}{52}$

25. a. $\frac{12}{31}$ c. $\frac{27}{31}$

b. $\frac{12}{31}$ d. $\frac{24}{31}$

26. a. $\frac{11}{36}$ d. $\frac{1}{3}$

b. $\frac{5}{18}$ e. 0

c. $\frac{11}{36}$ f. $\frac{11}{12}$

27. $(0.75 - 0.16) + (0.25 - 0.16) = 0.68$

28. $(0.3)^5 = 0.002$

29. a. $\frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} \cdot \frac{23}{49} \cdot \frac{22}{48} = \frac{253}{9996}$

b. $\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} = \frac{33}{66,640}$

c. 0

30. $\frac{0.35}{0.65} = 0.538$

31. $\frac{0.16}{0.3} = 0.533$

32. $\frac{0.57}{0.7} = 0.814$

33. $\frac{0.028}{0.5} = 0.056$

34. a. $\frac{1}{2}$ b. $\frac{3}{7}$

35. $1 - (0.45)^6 = 0.992$

36. $1 - (\frac{5}{6})^4 = 0.518$

37. $1 - (0.15)^6 = 0.9999886$

38. 2,646

39. 40,320

40. 1,365

41. 1,188,137,600; 710,424,000

42. 720

43. 33,554,432

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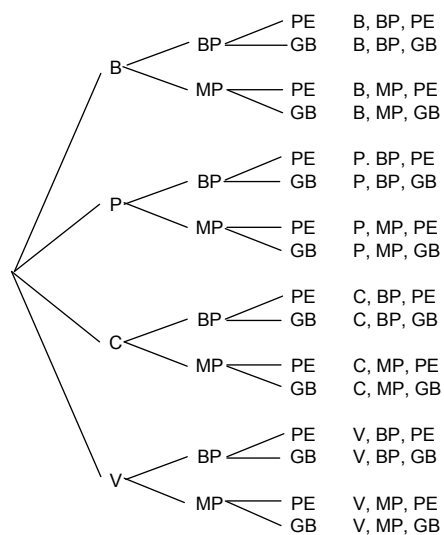
44. 56

45. $\frac{1}{4}$

46. $\frac{3}{14}$

47. $\frac{12}{55}$

48.



49. 120,120

50. 210