

Chapter 6 - The Normal Distribution

Note to instructors: Graphs are not to scale and are intended to convey a general idea.

Answers are generated using Table E.

Answers generated using the TI calculator will vary slightly. Some TI calculator answers are shown.

EXERCISE SET 6-1

1.

The characteristics of the normal distribution are:

- It is bell-shaped.
- It is symmetric about the mean.
- The mean, median, and mode are equal.
- It is continuous.
- It never touches the X-axis.
- The area under the curve is equal to 1.
- It is unimodal.
- About 68% of the area lies within 1 standard deviation of the mean, about 95% within 2 standard deviations, and about 99.7% within 3 standard deviations of the mean.

2.

Many variables are normally distributed, and the distribution can be used to describe these variables.

3.

1 or 100%.

4.

50% of the area lies below the mean, and 50% lies above the mean.

5.

68%, 95%, 99.7%

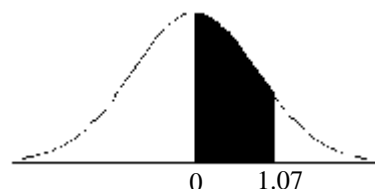
6.

Gaussian distribution and bell curve

7.

The area is found by looking up $z = 1.07$ in Table E and subtracting 0.5.

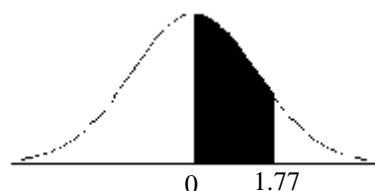
$$\text{Area} = 0.8577 - 0.5 = 0.3577$$



8.

The area is found by looking up $z = 1.77$ in Table E and subtracting 0.5.

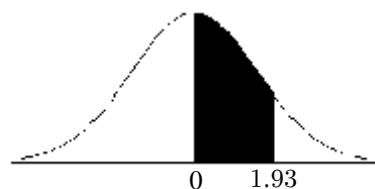
$$\text{Area} = 0.9616 - 0.5 = 0.4616$$



9.

The area is found by looking up $z = 1.93$ in Table E and subtracting 0.5.

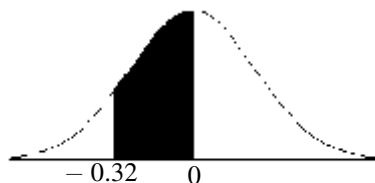
$$\text{Area} = 0.9732 - 0.5 = 0.4732$$



10.

The area is found by looking up $z = -0.32$ in Table E and subtracting from 0.5.

$$\text{Area} = 0.5 - 0.3745 = 0.1255$$

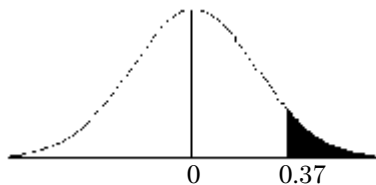


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11.

The area is found by looking up $z = 0.37$ in Table E and subtracting from 1.

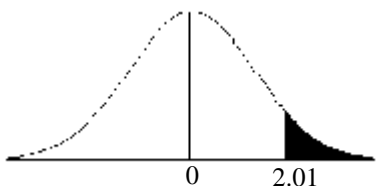
$$\text{Area} = 1 - 0.6443 = 0.3557$$



12.

The area is found by looking up $z = 2.01$ in Table E and subtracting from 1.

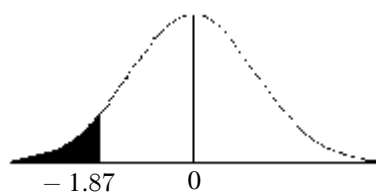
$$\text{Area} = 1 - 0.9778 = 0.0222$$



13.

The area is found by looking up $z = -1.87$ in Table E.

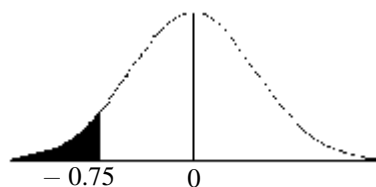
$$\text{Area} = 0.0307$$



14.

The area is found by looking up $z = -0.75$ in Table E.

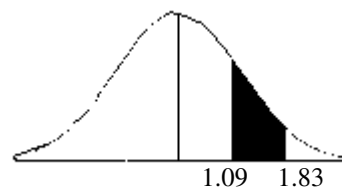
$$\text{Area} = 0.2266$$



15.

The area is found by looking up the values 1.09 and 1.83 in Table E and subtracting the areas.

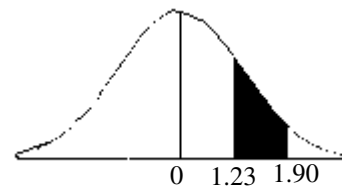
$$\text{Area} = 0.9664 - 0.8621 = 0.1043$$



16.

The area is found by looking up the values 1.23 and 1.90 in Table E and subtracting the areas.

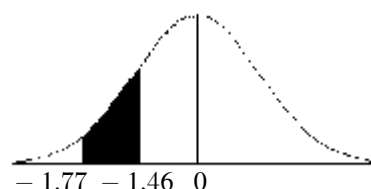
$$\text{Area} = 0.9713 - 0.8907 = 0.0806$$



17.

The area is found by looking up the values -1.46 and -1.77 in Table E and subtracting the areas.

$$\text{Area} = 0.0721 - 0.0384 = 0.0337$$



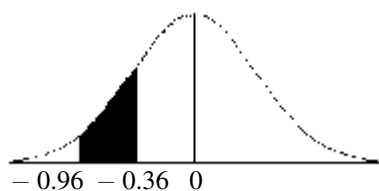
18.

The area is found by looking up the values -0.96 and -0.36 in Table E and subtracting the areas.

$$\text{Area} = 0.3594 - 0.1685 = 0.1909$$

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18. continued



19.

The area is found by looking up the values -1.46 and -1.98 in Table E and subtracting the areas.

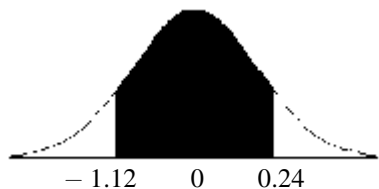
$$\text{Area} = 0.0721 - 0.0239 = 0.0482$$



20.

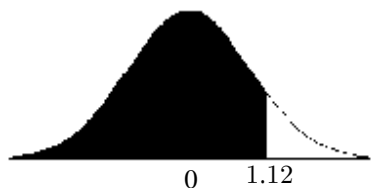
The area is found by looking up the values -1.12 and 0.24 and subtracting the areas.

$$\text{Area} = 0.5948 - 0.1314 = 0.4634$$



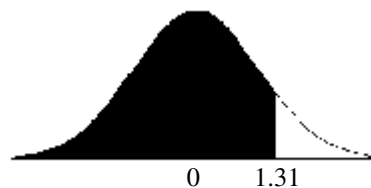
21.

The area is found by looking up 1.12 in Table E. $\text{Area} = 0.8686$



22.

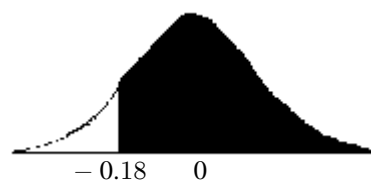
The area is found by looking up 1.31 in Table E. $\text{Area} = 0.9049$



23.

The area is found by looking up -0.18 in Table E and subtracting it from 1.

$$1 - 0.4286 = 0.5714$$



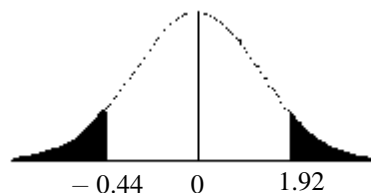
24.

The area is found by looking up -1.92 in Table E and subtracting the area from 1. $\text{Area} = 1 - 0.0274 = 0.9726$



25.

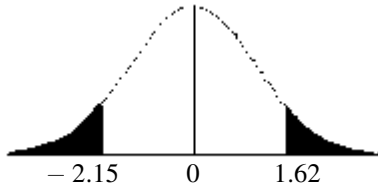
For $z = -0.44$, the area is 0.3300 . For $z = 1.92$, the area is $1 - 0.9726 = 0.0274$. $\text{Area} = 0.3300 + 0.0274 = 0.3574$



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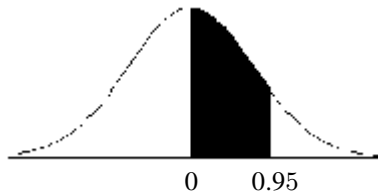
26.

For $z = -2.15$, the area is 0.0158. For $z = 1.62$, the area is $1 - 0.9474 = 0.0526$. Area = $0.0158 + 0.0526 = 0.0684$



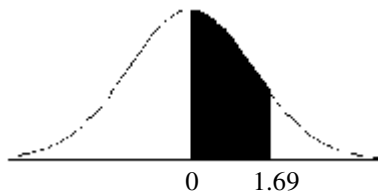
27.

Area = $0.8289 - 0.5 = 0.3289$



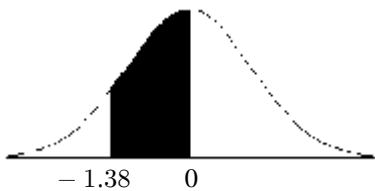
28.

Area = $0.9750 - 0.5 = 0.4750$



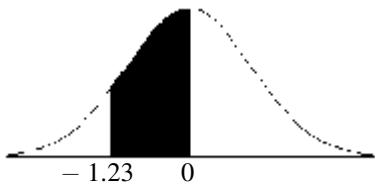
29.

Area = $0.5 - 0.0838 = 0.4162$



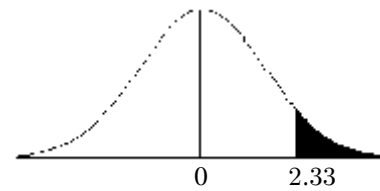
30.

Area = $0.5 - 0.1093 = 0.3907$



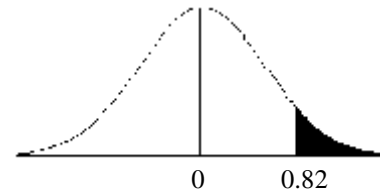
31.

Area = $1 - 0.9901 = 0.0099$



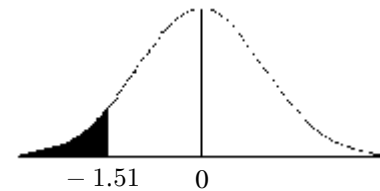
32.

Area = $1 - 0.7939 = 0.2061$



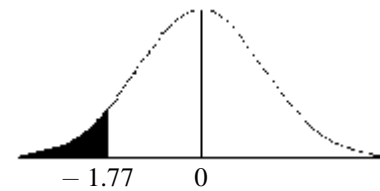
33.

Area = 0.0655



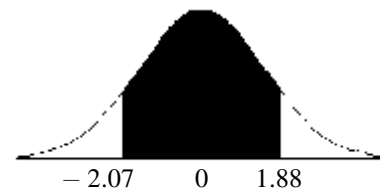
34.

Area = 0.0384



35.

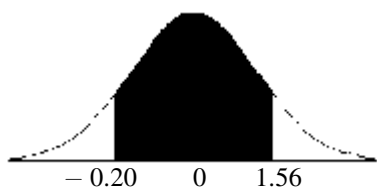
Area = $0.9699 - 0.0192 = 0.9507$



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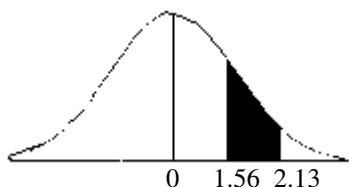
36.

$$\text{Area} = 0.9406 - 0.4207 = 0.5199$$



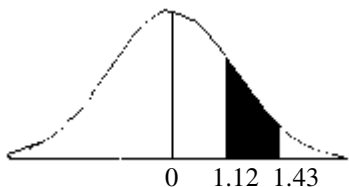
37.

$$\text{Area} = 0.9845 - 0.9947 = 0.0428$$



38.

$$\text{Area} = 0.9236 - 0.8686 = 0.055$$



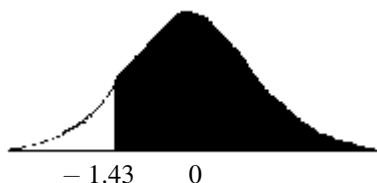
39.

$$\text{Area} = 0.9222$$



40.

$$\text{Area} = 1 - 0.0764 = 0.9236$$



41.

Since the z score is on the left side of 0, use the negative z table. Areas in the negative z table are in the tail, so we will use $0.5 - 0.4175 = 0.0825$ as the area. The closest z score corresponding to an area of 0.0825 is $z = -1.39$.

(TI answer = -1.3885)

42.

Since the z score is on the right side of 0, use the positive z score table. Areas for positive z scores include the left side of the curve, which has an area of 0.5. Hence, we must use $0.5 + 0.4066 = 0.9066$ as the area. The z score corresponding to an area of 0.9066 is $z = 1.32$.

(TI answer = 1.3201)

43.

$z = -2.08$, found by using the negative z table.

(TI answer = -2.0792)

44.

Using the positive z table,

$1 - 0.0239 = 0.9761$, thus $z = +1.98$.

(TI answer = 1.9791)

45.

Use the negative z table and

$1 - 0.8962 = 0.1038$ for the area. The z score is $z = -1.26$.

(TI answer = -1.2602)

46.

$z = +1.84$, found by using the positive z table.

(TI answer = 1.8398)

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47.

a. Using the negative z table,
 $\text{area} = 1 - 0.9887 = 0.0113$. Hence
 $z = -2.28$.
 (TI answer = -2.2801)

b. Using the negative z table,
 $\text{area} = 1 - 0.8212 = 0.1788$. Hence
 $z = -0.92$.
 (TI answer = -0.91995)

c. Using the negative z table,
 $\text{area} = 1 - 0.6064 = 0.3936$. Hence
 $z = -0.27$.
 (TI answer = -0.26995)

48.

a. $z = 0.12$ for $\text{area} = 0.5478$
 (TI answer = 0.1201)

b. $z = 0.52$ for $\text{area} = 0.6985$
 (TI answer = 0.5201)

c. $z = 1.18$ for $\text{area} = 0.8810$

49.

a. For total area = 0.05 , there will be
 $\text{area} = 0.025$ in each tail. The z scores are
 ± 1.96 .
 (TI answer = ± 1.95996)

b. For total area = 0.10 , there will be
 $\text{area} = 0.05$ in each tail. The z scores are
 $z = \pm 1.645$.
 (TI answer = ± 1.64485)

c. For total area = 0.01 , there will be
 $\text{area} = 0.005$ in each tail. The z scores are
 $z = \pm 2.58$.
 (TI answer = ± 2.57583)

50.

For a middle area of 48% , 24% lies on
 each side of 0 . To find the z score on the
 left side, use $\text{area} = 0.5 - 0.24 = 0.26$.
 The z score closest to an area of 0.26 is $-$
 0.64 . Since the curve is symmetrical
 about the mean (or center line), the z
 score on the right side is $+0.64$.
 (TI answer = ± 0.6433)

51.

$$P(-1 < z < 1) = 0.8413 - 0.1587 \\ = 0.6826$$

$$P(-2 < z < 2) = 0.9772 - 0.0228 \\ = 0.9544 \quad (\text{TI answer} = 0.9545)$$

$$P(-3 < z < 3) = 0.9987 - 0.0013 \\ = 0.9974 \quad (\text{TI answer} = 0.9973)$$

They are very close.

52.

For the 75th percentile $z = 0.67$
 (TI answer = 0.6745)

For the 80th percentile $z = 0.84$
 (TI answer = 0.8416)

For the 92th percentile $z = 1.41$
 (TI answer = 1.40507)

53.

For $z = -1.2$, $\text{area} = 0.1151$
 $\text{Area (left side)} = 0.5 - 0.1151 = 0.3849$
 $0.8671 - 0.3849 = 0.4822$
 $\text{Area (right side)} = 0.4822 + 0.5 =$
 0.9822
 For $\text{area} = 0.9822$, $z = 2.10$ Thus,
 $P(-1.2 < z < 2.10) = 0.8671$

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54.

For $z = 2.5$, area = 0.9938

Area (right side) = $0.9938 - 0.5 = 0.4938$

$0.7672 - 0.4938 = 0.2734$

Area (left side) = $0.5 - 0.2734 = 0.2266$

For area = 0.2266, $z = -0.75$

Thus, $P(-0.75 < z < 2.5) = 0.7672$

55.

For $z = -0.5$, area = 0.3085

$0.3085 - 0.2345 = 0.074$

For area = 0.074, $z = -1.45$

Thus, $P(-1.45 < z < -0.5) = 0.2345$

For $z = -0.5$, area = 0.3085

$0.5 - 0.3085 = 0.1915$

$0.2345 - 0.1915 = 0.043$

$0.5 + 0.043 = 0.543$

For area = 0.543, $z = 0.11$

Thus, $P(-0.5 < z < 0.11) = 0.2345$

56.

$0.76 \div 2 = 0.38$ on each side.

Area (right side) = $0.5 + 0.38 = 0.88$

$z = 1.175$

Area (left side) = $0.5 - 0.38 = 0.12$

$z = -1.175$

Thus, $P(-1.175 < z < 1.175) = 0.76$

(TI answer = ± 1.17499)

57.

$$y = \frac{e^{-\frac{(x-0)^2}{2(1)^2}}}{1\sqrt{2\pi}} = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

58.

Each x value (-2 , -1.5 , etc.) is

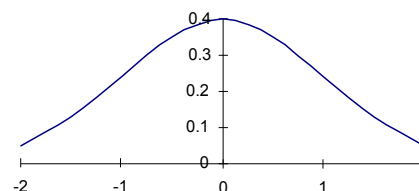
substituted in the formula $y = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$ to get

the corresponding y value. The pairs are then plotted as shown below.

58. continued

$$\begin{aligned} \text{For } x = -2, y &= \frac{e^{-\frac{(-2)^2}{2}}}{\sqrt{2\pi}} = \frac{e^{-2}}{\sqrt{6.28}} \\ &= \frac{0.1353}{\sqrt{6.28}} = 0.05 \end{aligned}$$

x	y
-2.0	0.05
-1.5	0.13
-1.0	0.24
-0.5	0.35
0	0.40
0.5	0.35
1.0	0.24
1.5	0.13
2.0	0.05



59.

Since the area under the curve to the left of $z = 2.3$ and the area under the curve to the right of $z = -1.2$ are overlapping areas, this covers the entire area under the curve. Thus, the total area is 1.00.

60.

Since the area under the curve to the right of $z = 2.3$ does not overlap the area to the left of $z = -1.2$, the area is 0.

EXERCISE SET 6-2

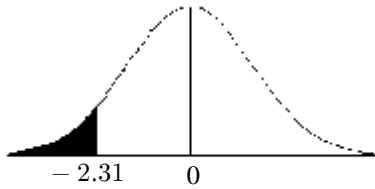
1.

$$z = \frac{40 - 43.7}{1.6} = -2.31$$

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1. continued

$$P(z < -2.31) = 0.0104 \text{ or } 1.04\%$$



2.

$$\text{a. } z = \frac{35,000 - 47,750}{5680} = -2.24$$

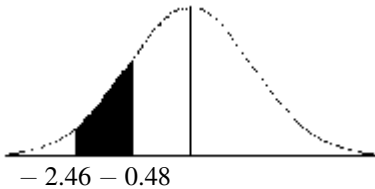
$$z = \frac{45,000 - 47,750}{5680} = -0.48$$

$$P(-2.24 < z < -0.62)$$

$$= 0.3156 - 0.0125$$

$$P = 0.3031 \text{ or } 30.31\% \text{ (TI answer}$$

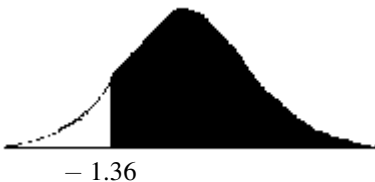
$$= 0.3017)$$



$$\text{b. } z = \frac{40,000 - 47,750}{5680} = -1.36$$

$$P(z > -1.36) = 1 - 0.0869 = 0.9131$$

$$\text{(TI answer} = 0.91378)$$



c. Not too happy! It's really at the bottom of the heap!

$$z = \frac{31,000 - 47,750}{5680} = -2.95$$

$$P(z \leq -2.95) = 0.0016$$

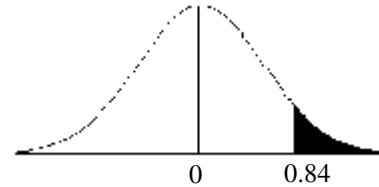
Only 0.16% of salaries are below \$31,000.

3.

$$\text{a. } z = \frac{750,000 - 706,242}{52,145} = 0.84$$

$$P(z > 0.84) = 1 - 0.7995 = 0.2005 \text{ or}$$

$$20.05\% \text{ (TI answer} = 0.2007)$$

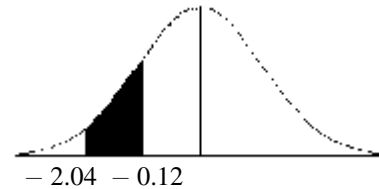


$$\text{b. } z = \frac{600,000 - 706,242}{52,145} = -2.04$$

$$z = \frac{700,000 - 706,242}{52,145} = -0.12$$

$$P(-2.04 < z < -0.12) = 0.4522 - 0.0207$$

$$P = 0.4315 \text{ or } 43.15\% \text{ (TI answer} = 0.4316)$$



4.

For the 90th percentile, area = 0.4 and

$$z = 1.28$$

$$x = 1.28(92) + 1028$$

$$x = 1145.8 \text{ or } 1146$$

$$\text{For a score of } 1200, z = \frac{1200 - 1028}{92} = 1.87$$

$$P(z > 1.87) = 1 - 0.9693 = 0.0307 \text{ or } 3.07\%$$

5.

$$\text{a. } z = \frac{200 - 225}{10} = -2.5$$

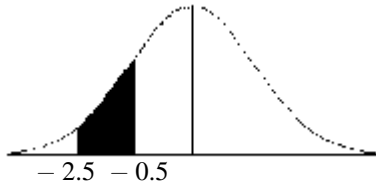
$$z = \frac{220 - 225}{10} = -0.5$$

$$P(-2.5 < z < -0.5) =$$

$$0.3085 - 0.0062 = 0.3023 \text{ or } 30.23\%$$

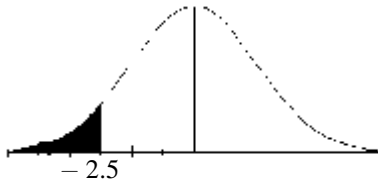
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5. continued



b. $z = -2.5$

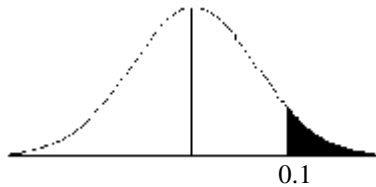
$P(z < -2.5) = 0.0062$ or 0.62%



6.

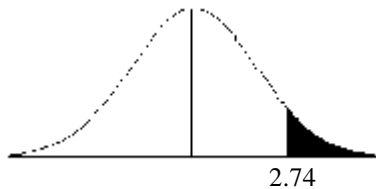
a. $z = \frac{1000-982}{180} = 0.1$

$P(z > 0.1) = 1 - 0.5398 = 0.4602$ or 46.02%



b. $z = \frac{1475-982}{180} = 2.74$

$P(z > 2.74) = 1 - 0.9969 = 0.0031$ or 0.31%

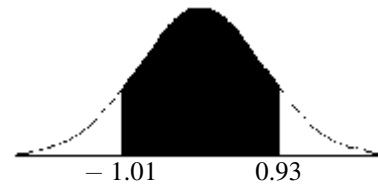


c. $z = \frac{800-982}{180} = -1.01$

$z = \frac{1150-982}{180} = 0.93$

$P(-1.01 < z < 0.93) = 0.8238 - 0.1562 = 0.6676$ or 66.76%

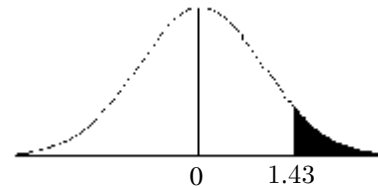
6. continued



7.

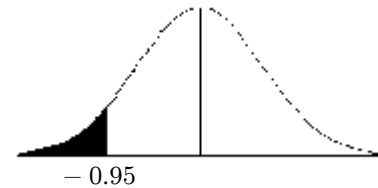
a. $z = \frac{18-15}{2.1} = 1.43$

$P(z > 1.43) = 1 - 0.9236 = 0.0764$ or 7.64% (TI answer = 0.0764)



b. $z = \frac{13-15}{2.1} = -0.95$

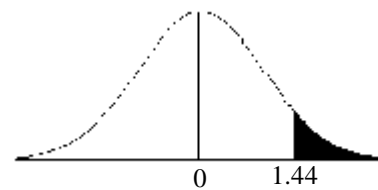
$P(z < -0.95) = 0.1711$ or 17.11% (TI answer = 0.1711)



8.

a. $z = \frac{15,000-12,837}{1500} = 1.44$

$P(z > 1.44) = 1 - 0.9251 = 0.0749$ or 7.49%



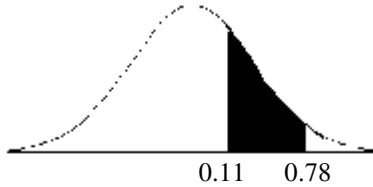
b. $z = \frac{13,000-12,837}{1500} = 0.11$

$z = \frac{14,000-12,837}{1500} = 0.78$

Chapter 6 - The Normal Distribution

8. continued

$$P(0.11 < z < 0.78) = 0.7823 - 0.5438 \\ = 0.2385 \text{ or } 23.85\%$$

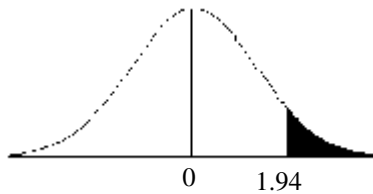


9.

For $x \geq 15,000$ miles:

$$z = \frac{15,000 - 12,494}{1290} = 1.94$$

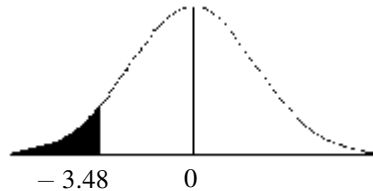
$$P(z > 1.94) = 1 - 0.9738 = 0.0262 \\ (\text{TI answer} = 0.02603)$$



For $x < 8000$ miles:

$$z = \frac{8000 - 12,494}{1290} = -3.48$$

$$P(z < -3.48) = 0.0003 \\ (\text{TI answer} = 0.00025)$$



For $x < 6000$ miles:

$$z = \frac{6000 - 12,494}{1290} = -5.03$$

$$P(z < -5.03) = 0.0001$$

Maybe it would be good to know why it had only been driven less than 6000 miles.

10.

$$z = \frac{30 - 25}{6.1} = 0.82$$

$$P(z > 0.82) = 1 - 0.7939 = 0.2061 \text{ or } 20.61\%$$

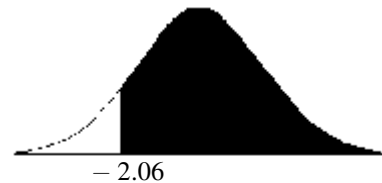
$$z = \frac{18 - 25}{6.1} = -1.15$$

$$P(z < -1.15) = 0.1251 \text{ or } 12.51\%$$

11.

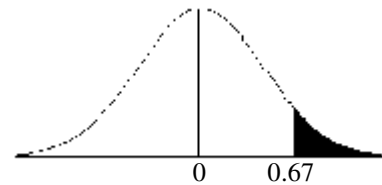
$$\text{a. } z = \frac{1000 - 3262}{1100} = -2.06$$

$$P(z \geq -2.06) = 1 - 0.0197 = 0.9803 \text{ or } 98.03\% \text{ (TI answer} = 0.9801)$$



$$\text{b. } z = \frac{4000 - 3262}{1100} = 0.67$$

$$P(z > 0.67) = 1 - 0.7486 = 0.2514 \text{ or } 25.14\% \text{ (TI answer} = 0.2511)$$



$$\text{c. } z = \frac{3000 - 3262}{1100} = -0.24$$

$$P(-0.24 < z < 0.67) = 0.7486 - 0.4052 = 0.3434 \text{ or } 34.34\% \\ (\text{TI answer} = 0.3430)$$



Chapter 6 - The Normal Distribution

12.

$$P(x < \$3.00) = P(z < ?)$$

For area = 0.15, $z = -1.04$

$$\text{Using } z = \frac{X - \bar{X}}{s}:$$

$$-1.04 = \frac{3.00 - 3.42}{s}$$

$$-1.04s = 3.00 - 3.42$$

$$-1.04s = -0.42$$

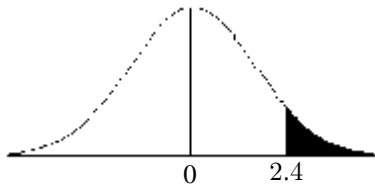
$$s = 0.4038 \text{ or } \approx 40.38 \text{ cents}$$

13.

$$\text{a. } z = \frac{142 - 130}{5} = 2.4$$

$$P(z > 2.4) = 1 - 0.9918 = 0.0082$$

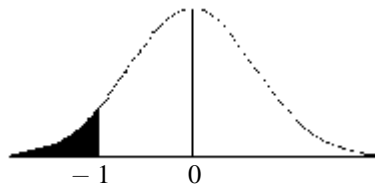
(TI answer = 0.0082)



$$\text{b. } z = \frac{125 - 130}{5} = -1$$

$$P(z < -1) = 0.1587$$

(TI answer = 0.1587)



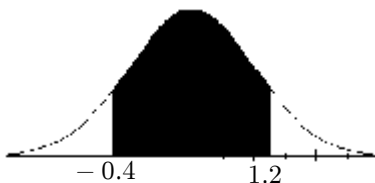
$$\text{c. } z = \frac{136 - 130}{5} = 1.2$$

$$z = \frac{128 - 130}{5} = -0.4$$

$$P(-0.4 < z < 1.2) =$$

$$0.8849 - 0.3446 = 0.5403$$

(TI answer = 0.5403)

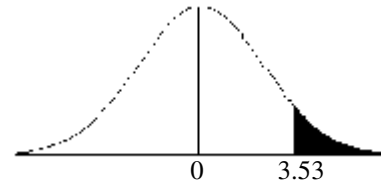


14.

$$z = \frac{384 - 225}{45} = 3.53$$

$$P(z > 3.53) = 1 - 0.9999 = 0.0001$$

The probability is less than 0.0001.



15.

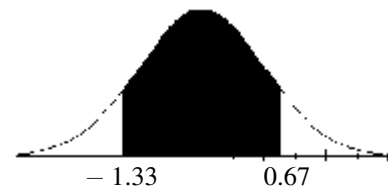
$$\text{a. } z = \frac{74 - 72}{3} = 0.67$$

$$z = \frac{68 - 72}{3} = -1.33$$

$$P(-1.33 < z < 0.67) =$$

$$0.7486 - 0.0918 = 0.6568$$

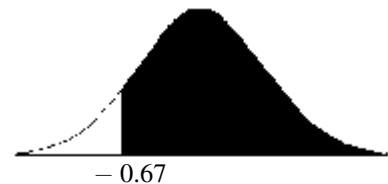
(TI answer = 0.6568)



b.

$$z = \frac{70 - 72}{3} = -0.67$$

$$P(z > -0.67) = 1 - 0.2514 = 0.7486$$



$$\text{c. } z = \frac{75 - 72}{3} = 1$$

$$P(z < 1) = 0.8413$$



Chapter 6 - The Normal Distribution

16.

P_{80} corresponds to $z = 0.84$

For male professors:

$$x = 0.84(5200) + 99,685$$

$$x = \$104,053$$

For female professors:

$$x = 0.84(5200) + 90,330$$

$$x = \$94,698$$

17.

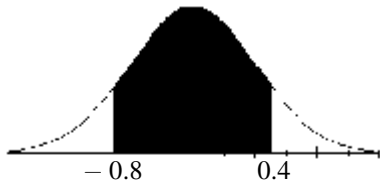
$$z = \frac{38-36}{5} = 0.4$$

$$z = \frac{32-36}{5} = -0.8$$

$$P(-0.8 < z < 0.4) =$$

$$0.6554 - 0.2119 = 0.4435$$

(TI answer = 0.4435)



18.

The middle 50% means that 25% of the area will be on either side of the mean.

Thus, area = 0.25 and $z = \pm 0.67$.

$$x = 0.67(103) + 792 = 861.01$$

$$x = -0.67(103) + 792 = 722.99$$

The contributions are between \$723 and \$861.



19.

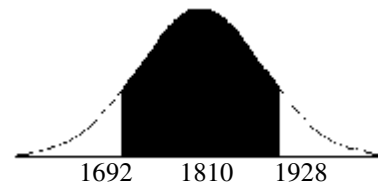
The middle 80% means that 40% of the area will be on either side of the mean.

The corresponding z scores will be ± 1.28 .

$$x = -1.28(92) + 1810 = 1692.24 \text{ sq. ft.}$$

$$x = 1.28(92) + 1810 = 1927.76 \text{ sq. ft.}$$

(TI answers: 1927.90 maximum, 1692.10 minimum)



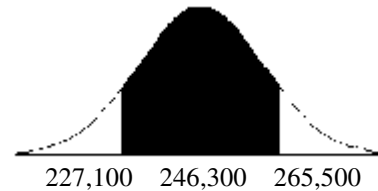
20.

The middle 80% means that 40% of the area will be on either side of the mean.

Thus, $z = \pm 1.28$

$$x = -1.28(15,000) + 246,300 = \$227,100$$

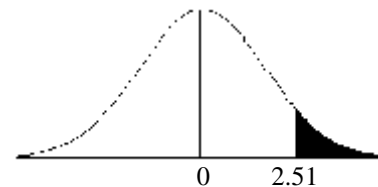
$$x = 1.28(15,000) + 246,300 = \$265,500$$



21.

$$z = \frac{1200-949}{100} = 2.51$$

$$P(z > 2.51) = 1 - 0.9940 = 0.006 \text{ or } 0.6\%$$



For the least expensive 10%, the area is 0.4 on the left side of the curve. Thus,

$$z = -1.28.$$

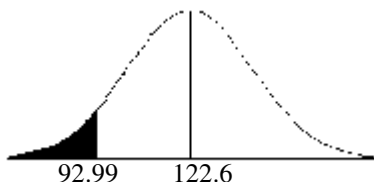
$$x = -1.28(100) + 949 = \$821$$

Chapter 6 - The Normal Distribution

22.

The bottom 5% (area) is in the left tail of the normal curve. The corresponding z score is found using area = 0.05. Thus, $z = -1.645$.

$$x = -1.645(18) + 122.6 = 92.99 \text{ or } 93$$



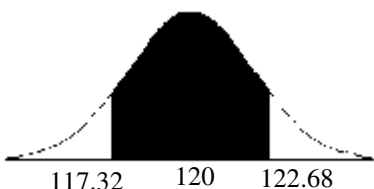
23.

The middle 50% means that 25% of the area will be on either side of the mean. The corresponding z scores will be ± 0.67 .

$$x = -0.67(4) + 120 = 117.32$$

$$x = 0.67(4) + 120 = 122.68$$

(TI answer: $117.32 < \mu < 122.68$)



24.

For the oldest 20%, the area is 0.8.

Thus, $z = 0.84$.

$$22.8 = 0.84s + 19.4$$

$$s = 4.048 \text{ or } 4.05 \text{ years}$$

25.

For the longest 10%, the area is 0.90.

Thus, $z = 1.28$

Since $\sigma^2 = 2.1$, $\sigma = \sqrt{2.1} = 1.449$

$$x = 1.28(1.449) + 4.8$$

$$x = 6.65 \text{ or } 6.7 \text{ days}$$

(TI answer = 6.657)

25. continued

For the shortest 30%, the area is 0.30.

Thus, $z = -0.52$.

$$x = -0.52(1.449) + 4.8$$

$$x = 4.047 \text{ days or } 4.05 \text{ days}$$

(TI answer = 4.040)

26.

a. For the top 3%, the area is 0.97.

Thus, $z = 1.88$.

$$x = 1.88(100) + 400$$

$x = 588$ minimum score to receive the award.

b. For the bottom 1.5%, the area is 0.015. Thus, $z = -2.17$.

$$x = -2.17(100) + 400$$

$$x = 183$$

The minimum score needed to avoid summer school is 184 since a score of 183 would be included in the summer school group.

27.

The bottom 18% area is 0.18. Thus,

$z = -0.92$.

$$x = -0.92(6256) + 24,596 = \$18,840.48$$

(TI answer = \$18,869.48)

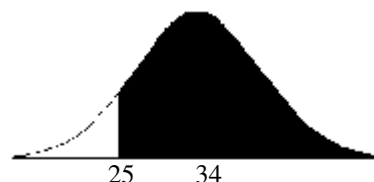
28.

For $x > 25$ gallons:

$$z = \frac{25 - 34}{2.7} = -3.33$$

$$P(z > -3.33) = 1 - 0.0004 = 0.9996$$

(TI answer = 0.9996)



Chapter 6 - The Normal Distribution

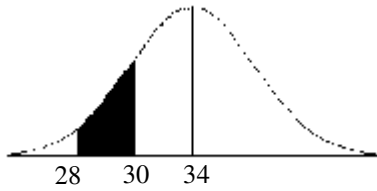
28. continued

For $28 < x < 30$ gallons:

$$z = \frac{28 - 34}{2.7} = -2.22$$

$$z = \frac{30 - 34}{2.7} = -1.48$$

$$\begin{aligned} P(-2.22 \leq z \leq -1.48) \\ = 0.0694 - 0.0132 = 0.0562 \\ (\text{TI answer} = 0.0562) \end{aligned}$$

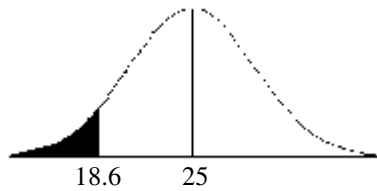


29.

The 10% to be exchanged would be at the left, or bottom, of the curve; therefore,

area = 0.10 and the corresponding z score will be -1.28 .

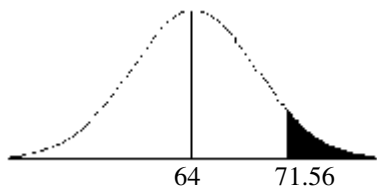
$$x = -1.28(5) + 25 = 18.6 \text{ months.}$$



30.

The top 20% means that area = 0.80. The corresponding z score is 0.84.

$$x = 0.84(9) + 64 = 71.56 \approx 72$$



31.

a. $\mu = 120 \quad \sigma = 20$

b. $\mu = 15 \quad \sigma = 2.5$

c. $\mu = 30 \quad \sigma = 5$

32.

No. Any subgroup would not be a perfect representation of the seniors; therefore, the mean and standard deviation would be different.

33.

For temperature of at least 85° , area is $1 - 0.05 = 0.95$. Then $z = 1.645$

$$85 = 1.645s + 73$$

$$s = 7.29^\circ$$

34.

No. The shape of the distributions would be the same, since z scores are raw scores scaled by the standard deviation.

35.

For payments above \$1255.94, area is $1 - 0.25 = 0.75$. Then $z = 0.67$

$$1255.94 = 0.67(120) + x$$

$$x = \$1175.54$$

36.

3.75% area in the left tail means that area = 0.0375. Thus, $z = -1.78$.

$$-1.78 = \frac{85 - \mu}{6}$$

$$-1.78(6) = 85 - \mu$$

$$\mu = 95.68$$

37.

Since $P(13.1 < x < 23.5) = 0.95$, the area on each side of the mean is 0.475.

Thus, $z = \pm 1.96$.

$$1.96 = \frac{23.5 - 18.3}{s}$$

$$s = 2.653$$

$$z = \frac{15 - 18.3}{2.653} = -1.24$$

$$P(z < -1.24) = 0.1075$$

Chapter 6 - The Normal Distribution

38.

The cutoff for the A's and F's would be:

$$x = \mu + z\sigma$$

$$x = 60 + 1.65(10)$$

$$x = 76.5 \text{ for the A's}$$

$$x = 60 + (-1.65)(10)$$

$$x = 43.5 \text{ for the F's}$$

For the B's and D's:

$$x = 60 + (0.84)(10)$$

$$x = 68.4 \text{ for the B's}$$

$$x = 60 + (-0.84)(10)$$

$$x = 51.6 \text{ for the D's}$$

The grading scale would be:

77 and up A

68 – 76 B

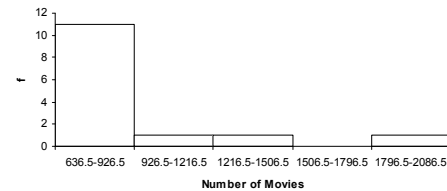
52 – 67 C

44 – 51 D

0 – 43 F

39.

Histogram:



The histogram shows a positive skew.

$$PI = \frac{3(970.2-853.5)}{376.5} = 0.93$$

$$IQR = Q_3 - Q_1 = 910 - 815 = 95$$

$$1.5(IQR) = 1.5(95) = 142.5$$

$$Q_1 - 142.5 = 672.5$$

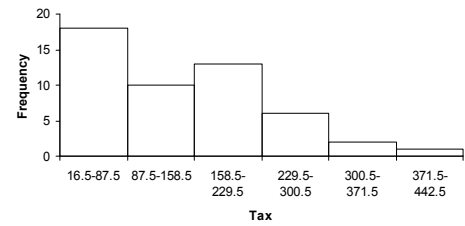
$$Q_3 + 142.5 = 1052.5$$

There are several outliers.

Conclusion: The distribution is not normal.

40.

Histogram:



The histogram shows a positive skew.

$$PI = \frac{3(147.04-138.5)}{93.55} = 0.27$$

$$IQR = Q_3 - Q_1 = 200 - 62 = 138$$

$$1.5(IQR) = 1.5(138) = 207$$

$$Q_1 - 207 = -145$$

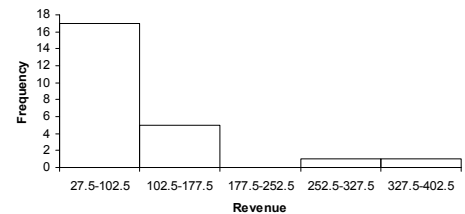
$$Q_3 + 207 = 407$$

There is one outlier.

Conclusion: The distribution is not normal.

41.

Histogram:



The histogram shows a positive skew.

$$PI = \frac{3(90-59)}{89.598} = 1.04$$

$$IQR = Q_3 - Q_1 = 111 - 32 = 79$$

$$1.5(IQR) = 1.5(79) = 118.5$$

$$Q_1 - 118.5 = -86.5$$

$$Q_3 + 118.5 = 229.5$$

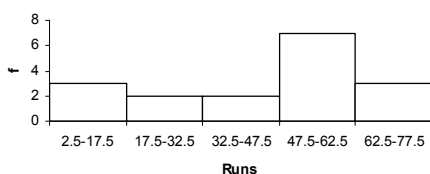
There are two outliers.

Conclusion: The distribution is not normal.

Chapter 6 - The Normal Distribution

42.

Histogram:



The histogram shows a negative skew.

$$PI = \frac{3(45.2 - 52)}{20.58} = -0.99$$

$$IQR = Q_3 - Q_1 = 60.5 - 29.5 = 31$$

$$1.5(IQR) = 1.5(31) = 46.5$$

$$Q_1 - 46.5 = -17$$

$$Q_3 + 46.5 = 107$$

There are no outliers.

Conclusion: The distribution is not normal.

43. Answers will vary.

EXERCISE SET 6-3

1.

The distribution is called the sampling distribution of sample means.

2.

The sample is not a perfect representation of the population. The difference is due to what is called sampling error.

3.

The mean of the sample means is equal to the population mean.

4.

The standard deviation of the sample means is called the standard error of the mean.

$$\sigma_X = \frac{\sigma}{\sqrt{n}}$$

5.

The distribution will be approximately normal when sample size is large.

6.

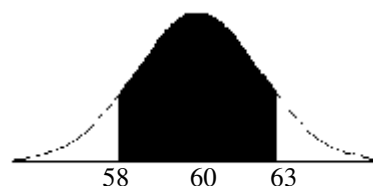
$$z = \frac{X - \mu}{\sigma}$$

7.

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{63 - 60}{\frac{8}{\sqrt{30}}} = 2.05$$

$$z = \frac{58 - 60}{\frac{8}{\sqrt{30}}} = -1.37$$

$$P(-1.37 < z < 2.05) = 0.9798 - 0.0853 = 0.8945 \text{ or } 89.45\%$$

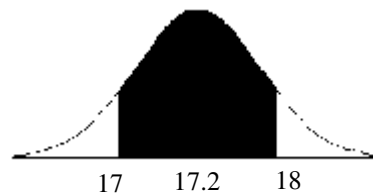


8.

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{17 - 17.2}{\frac{2.5}{\sqrt{55}}} = -0.59$$

$$z = \frac{18 - 17.2}{\frac{2.5}{\sqrt{55}}} = 2.37$$

$$P(-0.59 < z < 2.37) = 0.9911 - 0.2776 = 0.7135 \text{ or } 71.35\%$$

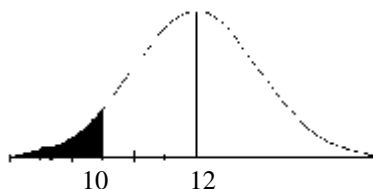


9.

$$a. z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{10 - 12}{\frac{3.2}{\sqrt{36}}} = -3.75$$

$$P(z < -3.75) = 0.00009$$

(TI answer = 0.00009)



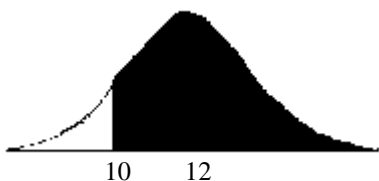
Chapter 6 - The Normal Distribution

9. continued

$$b. z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{10 - 12}{\frac{3.2}{\sqrt{36}}} = -3.75$$

$$P(z > -3.75) = 1 - 0.00009 = 0.99991$$

(TI answer = 0.99991)



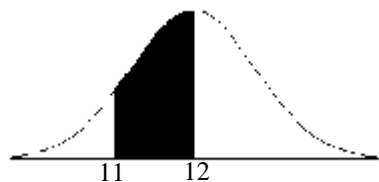
$$c. z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{12 - 12}{\frac{3.2}{\sqrt{36}}} = 0$$

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{11 - 12}{\frac{3.2}{\sqrt{36}}} = -1.88$$

$$P(-1.88 < z < 0) = 0.50 - 0.0301$$

$$= 0.4699$$

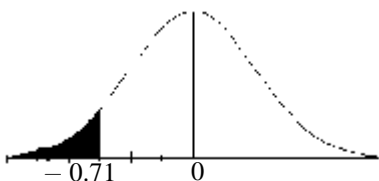
(TI answer = 0.4699)



10.

$$a. z = \frac{\$52,000 - \$57,337}{\$7500} = -0.71$$

$$P(z < -0.71) = 0.2389 \text{ or } 23.89\%$$

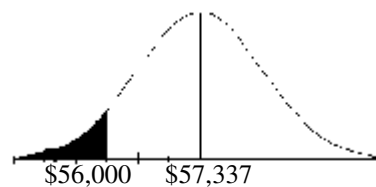


$$b. z = \frac{\$56,000 - \$57,337}{\frac{\$7500}{\sqrt{100}}} = -1.78$$

$$P(z < -1.78) = 0.0375 \text{ or } 3.75\%$$

(TI answer = 0.0373)

10. continued



11.

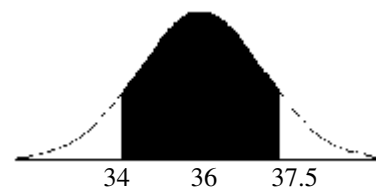
$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{37.5 - 36}{\frac{3.6}{\sqrt{35}}} = 2.47$$

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{34 - 36}{\frac{3.6}{\sqrt{35}}} = -3.29$$

$$P(-3.29 < z < 2.47) = 0.9932 - 0.0005$$

$$= 0.9927 \text{ or } 99.27\%$$

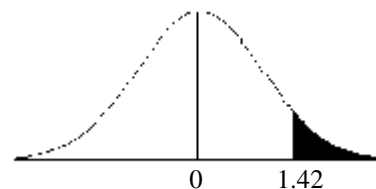
(TI answer = 0.9927)



12.

$$a. z = \frac{\$45,000 - \$37,764}{\$5100} = 1.42$$

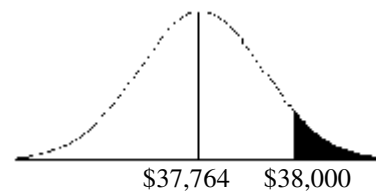
$$P(z > 1.42) = 1 - 0.9222 = 0.0778 \text{ or } 7.78\%$$



$$b. z = \frac{\$38,000 - \$37,764}{\frac{\$5100}{\sqrt{75}}} = 0.40$$

$$P(z > 0.40) = 1 - 0.6554 = 0.3446$$

$$\text{or } 34.46\%$$



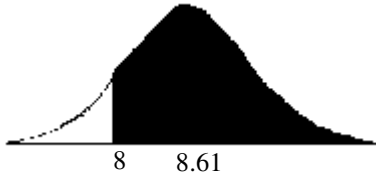
Chapter 6 - The Normal Distribution

13.

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{8 - 8.61}{\frac{1.39}{\sqrt{50}}} = -3.10$$

$$P(z > -3.1) = 1 - 0.001 = 0.9990$$

(TI answer = 0.9990)



14.

Since $n \geq 30$, we can use the normal distribution.

$$z = \frac{1050 - 1028}{\frac{100}{\sqrt{200}}} = 3.11$$

$$P(z \geq 3.11) = 1 - 0.9991 = 0.0009$$

or 0.001

(TI answer = 0.0009)

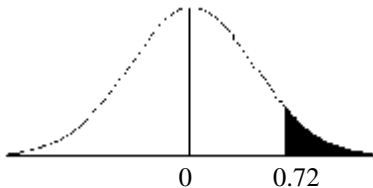
Thus, we would be surprised to get a sample mean of 1050 since the probability is very small.

15.

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3000 - 2708}{\frac{405}{\sqrt{30}}} = 0.72$$

$$P(z > 0.72) = 1 - 0.7642 = 0.2358$$

(TI answer = 0.2355)



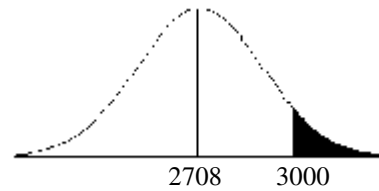
$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3000 - 2708}{\frac{405}{\sqrt{30}}} = 3.95$$

$$P(z > 3.95) = 1 - 0.9999 = 0.0001$$

$$P(z > 3.95) < 0.0001$$

(TI answer = 0.000039)

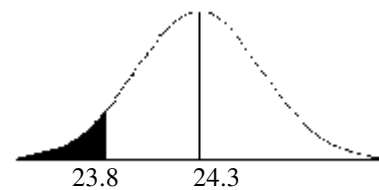
15. continued



16.

$$z = \frac{23.8 - 24.3}{\frac{2.6}{\sqrt{33}}} = -1.10$$

$$P(z < -1.10) = 0.1357 \text{ or } 13.57\%$$



17.

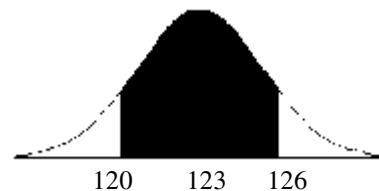
$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{120 - 123}{\frac{21}{\sqrt{15}}} = -0.55$$

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{126 - 123}{\frac{21}{\sqrt{15}}} = 0.55$$

$$P(-0.55 < z < 0.55) = 0.7088 - 0.2912$$

$$= 0.4176 \text{ or } 41.76\%$$

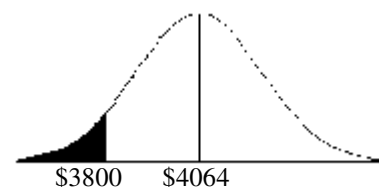
(TI answer = 0.4199)



18.

$$\text{a. } z = \frac{\$3800 - \$4064}{\frac{460}{\sqrt{20}}} = -2.57$$

$$P(z < -2.57) = 0.0051 \text{ or } 0.51\%$$

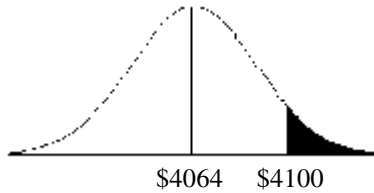


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18. continued

$$b. z = \frac{\$4100 - \$4064}{\frac{460}{\sqrt{20}}} = 0.35$$

$$P(z > 0.35) = 1 - 0.6368 = 0.3632$$

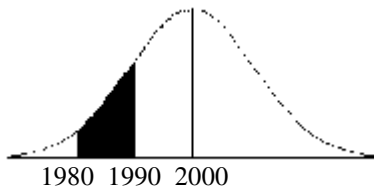


19.

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1980 - 2000}{\frac{187.5}{\sqrt{50}}} = -0.75$$

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1990 - 2000}{\frac{187.5}{\sqrt{50}}} = -0.38$$

$$P(-0.75 < z < -0.38) \\ = 0.3520 - 0.2266 = 0.1254 \\ (\text{TI answer} = 0.12769)$$

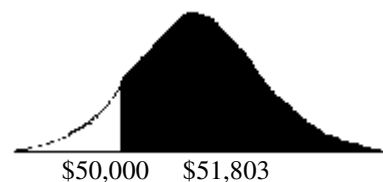


20.

For $x > \$50,000$:

$$z = \frac{X - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\$50,000 - \$51,803}{\frac{\$4850}{\sqrt{34}}} = -2.17$$

$$P(z > -2.17) = 1 - 0.0150 = 0.985 \text{ or } 98.5\%$$



For $x < \$48,000$:

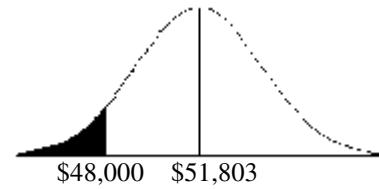
$$z = \frac{\$48,000 - \$51,803}{\frac{\$4850}{\sqrt{34}}} = -4.57$$

20. continued

$$P(z < -4.57) = 0.0001 \text{ or } 0.01\%$$

(TI answer = 0.0000024)

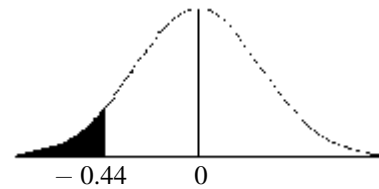
The probability is less than 0.0001.



21.

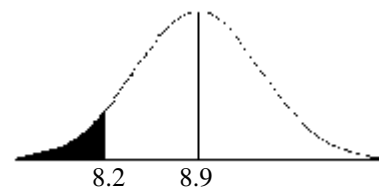
$$a. z = \frac{X - \mu}{\sigma} = \frac{8.2 - 8.9}{1.6} = -0.44$$

$$P(z < -0.44) = 0.33 \text{ or } 33\%$$



$$b. z = \frac{8.2 - 8.9}{\frac{1.6}{\sqrt{10}}} = -1.38$$

$$P(z < -1.38) = 0.0838 \text{ or } 8.38\%$$



c. Yes, since the probability is slightly more than 30%.

d. Yes, but not as likely.

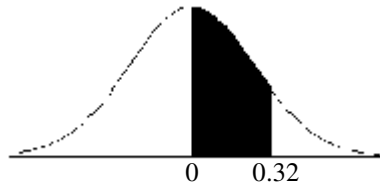
22.

$$a. z = \frac{121.8 - 120}{5.6} = 0.32$$

$$z = \frac{120 - 120}{5.6} = 0$$

$$P(0 < z < 0.32) = 0.6255 - 0.5 = 0.1255 \\ \text{or } 12.55\%$$

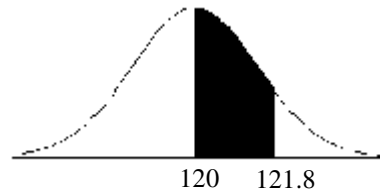
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$$\text{b. } z = \frac{121.8 - 120}{\frac{5.6}{\sqrt{30}}} = 1.76$$

$$z = \frac{120 - 120}{\frac{5.6}{\sqrt{30}}} = 0$$

$$P(0 < z < 1.76) = 0.9608 - 0.5 \\ = 0.4608 \text{ or } 46.08\%$$



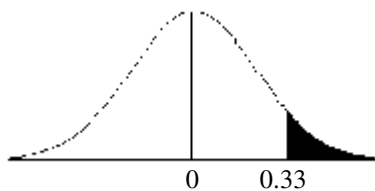
c. Sample means are less variable than individual data.

23.

$$\text{a. } z = \frac{220 - 215}{15} = 0.33$$

$$P(z > 0.33) = 1 - 0.6293 = 0.3707 \text{ or } 37.07\%$$

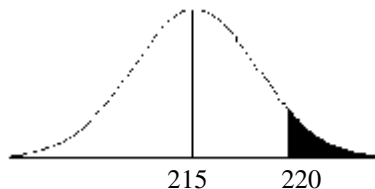
(TI answer = 0.3694)



$$\text{b. } z = \frac{220 - 215}{\frac{15}{\sqrt{25}}} = 1.67$$

$$P(z > 1.67) = 1 - 0.9525 = 0.0475 \text{ or } 4.75\%$$

(TI answer = 0.04779)

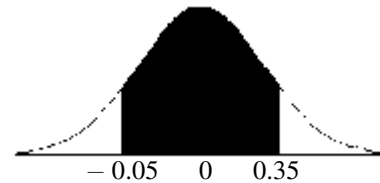


24.

$$\text{a. } z_1 = \frac{36 - 36.2}{3.7} = -0.05$$

$$z_2 = \frac{37.5 - 36.2}{3.7} = 0.35$$

$$P(-0.05 < z < 0.35) = 0.6368 - 0.4801 \\ = 0.1567 \text{ or } 15.67\%$$

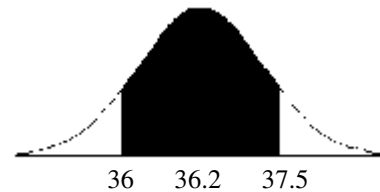


$$\text{b. } z_1 = \frac{36 - 36.2}{\frac{3.7}{\sqrt{15}}} = -0.21$$

$$z_2 = \frac{37.5 - 36.2}{\frac{3.7}{\sqrt{15}}} = 1.36$$

$$P(-0.21 < z < 1.36) = 0.9131 - 0.4168 \\ = 0.4963 \text{ or } 49.63\%$$

(TI answer = 0.04779)



25.

$$1 - 0.0985 = 0.9015$$

The z score corresponding to an area of 0.9015 is 1.29.

$$1.29 = \frac{520 - 508}{\frac{72}{\sqrt{n}}}$$

$$1.29 = \frac{12\sqrt{n}}{72}$$

$$92.88 = 12\sqrt{n}$$

$$7.74 = \sqrt{n}$$

$$59.9 = n$$

The sample size is approximately 60.

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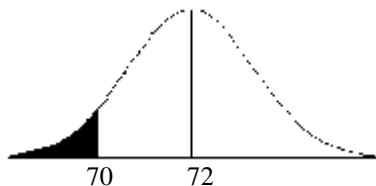
26.

Since $50 > 0.05(500)$ or 25, the correction factor must be used.

$$\text{It is } \sqrt{\frac{500-50}{500-1}} = 0.950$$

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{n-1}}} = \frac{70-72}{\frac{5.3}{\sqrt{50}} \cdot (0.95)} = -2.81$$

$$P(z < -2.81) = 0.0025$$



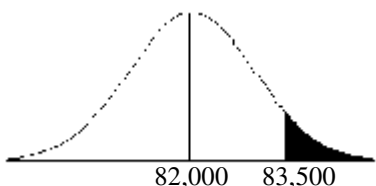
27.

Since $50 > 0.05(800)$ or 40, the correction factor is necessary.

$$\text{It is } \sqrt{\frac{800-50}{800-1}} = 0.969$$

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{N-n}{n-1}}} = \frac{83,500-82,000}{\frac{5000}{\sqrt{50}} \cdot (0.969)} = 2.19$$

$$P(z > 2.19) = 1 - 0.9857 = 0.0143 \text{ or } 1.43\%$$



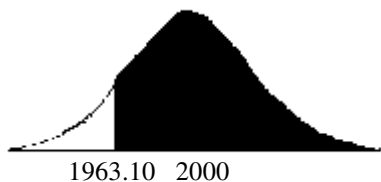
28.

The upper 95% is the same as 5% in the left tail. For area = 0.05 in the left tail, the corresponding z score is -1.65 .

$$-1.65 = \frac{\bar{X} - 2000}{\frac{100}{\sqrt{20}}}$$

$$-1.65\left(\frac{100}{\sqrt{20}}\right) + 2000 = \bar{X}$$

$$\bar{X} = 1963.10$$



29.

$$\sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{100}} = 1.5$$

$$2(1.5) = \frac{15}{\sqrt{n}}$$

$$3 \cdot \sqrt{n} = 15$$

$$\sqrt{n} = 5$$

$n = 25$, the sample size necessary to double the standard error.

30.

$$\frac{1.5}{2} = \frac{15}{\sqrt{n}}$$

$$0.75 \cdot \sqrt{n} = 15$$

$$\sqrt{n} = \frac{15}{0.75} = 20$$

$n = 400$, the sample size necessary to cut the standard error in half.

EXERCISE SET 6-4

1.

When p is approximately 0.5, and as n increases, the shape of the binomial distribution becomes similar to the normal distribution.

2.

The normal approximation should be used only when $n \cdot p$ and $n \cdot q$ are both greater than or equal to 5.

3.

The correction for continuity is necessary because the normal distribution is continuous and the binomial is discrete.

4.

When p is close to 0 or 1 and n is small, the normal distribution should not be used as an approximation to the binomial distribution. That is, when $np < 5$ and $nq < 5$, the normal distribution should not be used to approximate the binomial distribution.

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5.

For each problem use the following formulas:

$$\mu = np \quad \sigma = \sqrt{npq} \quad z = \frac{\bar{X} - \mu}{\sigma}$$

Be sure to correct each X for continuity.

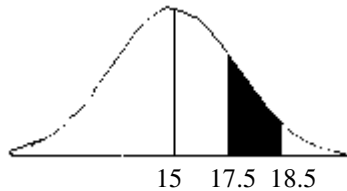
a. $\mu = 0.5(30) = 15$

$$\sigma = \sqrt{(0.5)(0.5)(30)} = 2.74$$

$$z = \frac{17.5 - 15}{2.74} = 0.91 \quad \text{area} = 0.8186$$

$$z = \frac{18.5 - 15}{2.74} = 1.28 \quad \text{area} = 0.8997$$

$$P(17.5 < X < 18.5) = 0.8997 - 0.8186 \\ = 0.0811 = 8.11\%$$



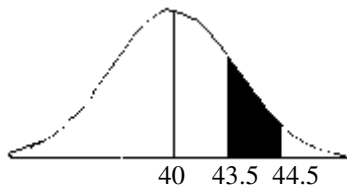
b. $\mu = 0.8(50) = 40$

$$\sigma = \sqrt{(50)(0.8)(0.2)} = 2.83$$

$$z = \frac{43.5 - 40}{2.83} = 1.24 \quad \text{area} = 0.8925$$

$$z = \frac{44.5 - 40}{2.83} = 1.59 \quad \text{area} = 0.9441$$

$$P(43.5 < X < 44.5) = 0.9441 - 0.8925 \\ = 0.0516 \text{ or } 5.16\%$$



c. $\mu = 0.1(100) = 10$

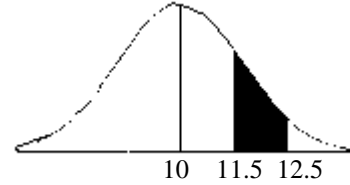
$$\sigma = \sqrt{(0.1)(0.9)(100)} = 3$$

$$z = \frac{11.5 - 10}{3} = 0.50 \quad \text{area} = 0.6915$$

$$z = \frac{12.5 - 10}{3} = 0.83 \quad \text{area} = 0.7967$$

5. continued

$$P(11.5 < X < 12.5) = 0.7967 - 0.6915 \\ = 0.1052 \text{ or } 10.52\%$$



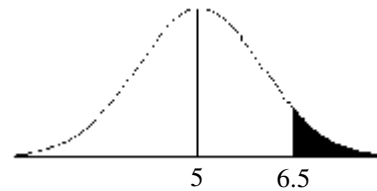
6.

a. $\mu = 10(0.5) = 5$

$$\sigma = \sqrt{(0.5)(0.5)(10)} = 1.58$$

$$z = \frac{6.5 - 5}{1.58} = 0.95 \quad \text{area} = 0.8289$$

$$P(X \geq 6.5) = 1 - 0.8289 = 0.1711 \text{ or } 17.11\%$$

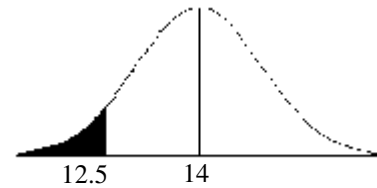


b. $\mu = 20(0.7) = 14$

$$\sigma = \sqrt{(20)(0.7)(0.3)} = 2.05$$

$$z = \frac{12.5 - 14}{2.05} = -0.73 \quad \text{area} = 0.2327$$

$$P(X \leq 12.5) = 0.2327 \text{ or } 23.27\%$$



c. $\mu = 50(0.6) = 30$

$$\sigma = \sqrt{(50)(0.6)(0.4)} = 3.46$$

$$z = \frac{40.5 - 30}{3.46} = 3.03 \quad \text{area} = 0.9988$$

$$P(X \leq 40.5) = 0.9988 \text{ or } 99.88\%$$

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6. continued



7.

- a. $np = 20(0.50) = 10 \geq 5$ Yes
 $nq = 20(0.50) = 10 \geq 5$
- b. $np = 10(0.60) = 6 \geq 5$ No
 $nq = 10(0.40) = 4 < 5$
- c. $np = 40(0.90) = 36 \geq 5$ No
 $nq = 40(0.10) = 4 < 5$

8.

- a. $np = 50(0.20) = 10 \geq 5$ Yes
 $nq = 50(0.80) = 40 \geq 5$
- b. $np = 30(0.80) = 24 \geq 5$ Yes
 $nq = 30(0.20) = 6 \geq 5$
- c. $np = 20(0.85) = 17 \geq 5$ No
 $nq = 20(0.15) = 3 < 5$

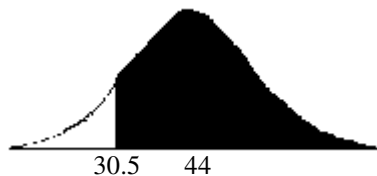
9.

$$\mu = 200(0.22) = 44$$

$$\sigma = \sqrt{(200)(0.22)(0.78)} = 5.8583$$

$$z = \frac{30.5 - 44}{5.8583} = -2.30 \quad \text{area} = 0.0107$$

$$P(X > 30.5) = 1 - 0.0107 = 0.9893$$



10.

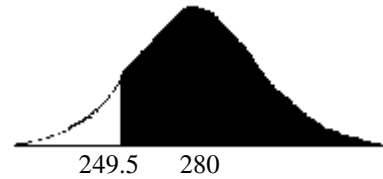
$$\mu = 500(0.56) = 280$$

$$\sigma = \sqrt{(500)(0.56)(0.44)} = 11.1$$

$$z = \frac{249.5 - 280}{11.1} = -2.75 \quad \text{area} = 0.0030$$

10. continued

$$P(X > 249.5) = 1 - 0.0030 = 0.9970 \text{ or } 99.7\%$$



11.

$$\mu = 120(0.659) = 79.08$$

$$\sigma = \sqrt{(120)(0.659)(0.341)} = 5.1929$$

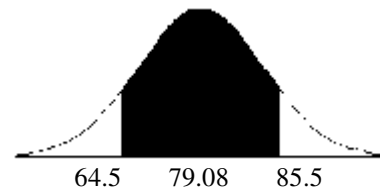
$$z = \frac{64.5 - 79.08}{5.1929} = -2.81 \quad \text{area} = 0.0025$$

$$z = \frac{85.5 - 79.08}{5.1929} = 1.24 \quad \text{area} = 0.8925$$

$$P(64.5 < X < 85.5) = 0.8925 - 0.0025$$

$$P(64.5 \leq X \leq 85.5) = 0.8900$$

(TI answer = 0.8893)



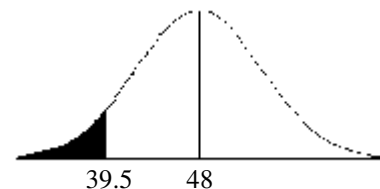
12.

$$\mu = 600(0.08) = 48$$

$$\sigma = \sqrt{(600)(0.08)(0.92)} = 6.65$$

$$z = \frac{39.5 - 48}{6.65} = -1.28$$

$$P(X < 39.5) = 0.1003 \text{ or } 10.03\%$$



13.

$$\mu = 60(0.76) = 45.6$$

$$\sigma = \sqrt{(60)(0.76)(0.24)} = 3.3082$$

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13. continued

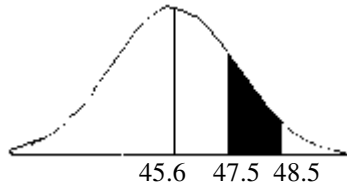
$$z = \frac{48.5 - 45.6}{3.3082} = 0.88 \quad \text{area} = 0.8106$$

$$z = \frac{47.5 - 45.6}{3.3082} = 0.57 \quad \text{area} = 0.7157$$

$$P(47.5 < X < 48.5) = 0.8106 - 0.7157$$

$$P(47.5 < X < 48.5) = 0.0949$$

(TI answer = 0.0949)



14.

$$\mu = 180(0.72) = 129.6$$

$$\sigma = \sqrt{(180)(0.72)(0.28)} = 6.024$$

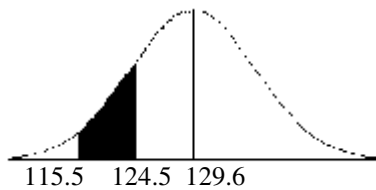
$$z = \frac{124.5 - 129.6}{6.024} = -0.85 \quad \text{area} = 0.1977$$

$$z = \frac{115.5 - 129.6}{6.024} = -2.34 \quad \text{area} = 0.0096$$

$$P(115.5 < X < 124.5) = 0.1977 - 0.0096$$

$$P(115.5 < X < 124.5) = 0.1881$$

(TI answer = 0.1890)



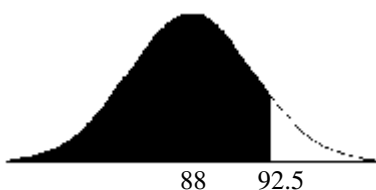
15.

$$p = 0.22 \quad \mu = 400(0.22) = 88$$

$$\sigma = \sqrt{(400)(0.22)(0.78)} = 8.2849$$

$$z = \frac{92.5 - 88}{8.2849} = 0.54$$

$$P(X \leq 92.5) = 0.7054 \text{ or } 70.54\%$$



16.

$$\mu = 150(0.289) = 43.35$$

$$\sigma = \sqrt{(150)(0.289)(0.711)} = 5.55$$

$$z = \frac{50.5 - 43.35}{5.55} = 1.29 \quad \text{area} = 0.9015$$

$$P(X > 50.5) = 1 - 0.9015 = 0.0985$$



17.

$$\mu = 200(0.125) = 25$$

$$\sigma = \sqrt{(200)(0.125)(0.875)} = 4.6771$$

$$z = \frac{21.5 - 25}{4.6771} = -0.75$$

$$P(X \geq 21.5) = 1 - 0.2266 = 0.7734$$

(TI answer = 0.7734)

Yes, it is very likely.



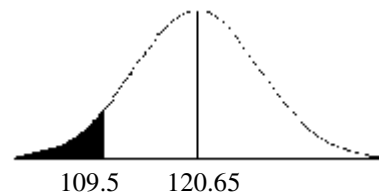
18.

$$\mu = 250(0.4826) = 120.65$$

$$\sigma = \sqrt{(250)(0.4826)(0.5174)} = 7.9009$$

$$z = \frac{109.5 - 120.65}{7.9009} = -1.41 \quad \text{area} = 0.0793$$

$$P(X < 109.5) = 0.0793$$



Chapter 6 - The Normal Distribution

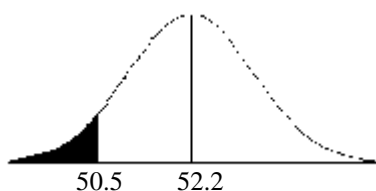
19.

$$\mu = 200(0.261) = 52.2$$

$$\sigma = \sqrt{(200)(0.261)(0.739)} = 6.21$$

$$z = \frac{50.5 - 52.2}{6.21} = -0.27$$

$$P(X \leq 50.5) = 0.3936$$



20.

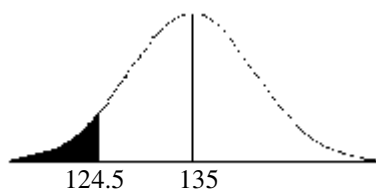
$$\mu = 200(0.675) = 135$$

$$\sigma = \sqrt{200(0.675)(0.325)} = 6.62$$

$$z = \frac{124.5 - 135}{6.62} = -1.59 \quad \text{area} = 0.0559$$

$$P(X < 124.5) = 0.0559 \text{ or } 5.59\%$$

(TI answer = 0.0565)



21.

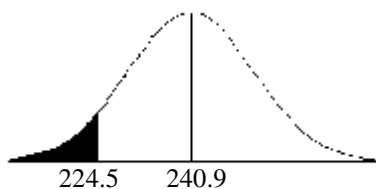
$$\mu = 300(0.803) = 240.9$$

$$\sigma = \sqrt{(300)(0.803)(0.197)} = 6.89$$

$$X < \frac{3}{4}(300) \text{ or } X < 225$$

$$z = \frac{224.5 - 240.9}{6.89} = -2.38$$

$$P(X < 224.5) = 0.0087$$



22.

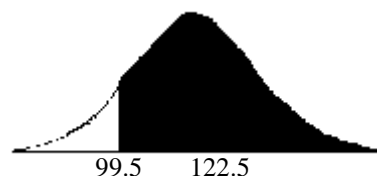
$$\mu = 350(0.35) = 122.5$$

$$\sigma = \sqrt{(350)(0.35)(0.65)} = 8.92$$

$$z = \frac{99.5 - 122.5}{8.92} = -2.58 \quad \text{area} = 0.0049$$

$$P(X > 99.5) = 1 - 0.0049 = 0.9951 \text{ or } 99.51\% \quad (\text{TI answer} = 0.9950)$$

Yes; it is likely that 100 or more people would favor the parking lot.



23.

$$\text{a. } n(0.1) \geq 5 \quad n \geq 50$$

$$\text{b. } n(0.3) \geq 5 \quad n \geq 17$$

$$\text{c. } n(0.5) \geq 5 \quad n \geq 10$$

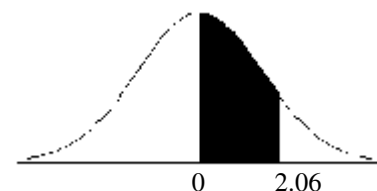
$$\text{d. } n(0.2) \geq 5 \quad n \geq 25$$

$$\text{e. } n(0.1) \geq 5 \quad n \geq 50$$

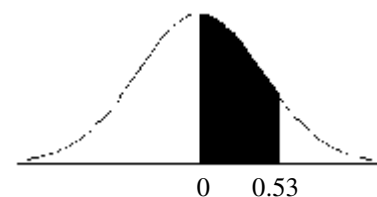
REVIEW EXERCISES - CHAPTER 6

1.

$$\text{a. } 0.9803 - 0.5 = 0.4803$$



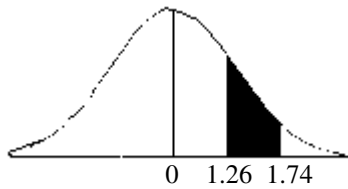
$$\text{b. } 0.7019 - 0.5 = 0.2019$$



Chapter 6 - The Normal Distribution

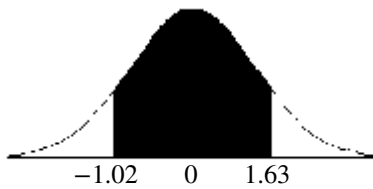
1. continued

c. $0.9591 - 0.8962 = 0.0629$

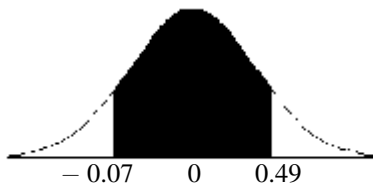


d. $0.9484 - 0.1539 = 0.7945$

(TI answer = 0.7945)

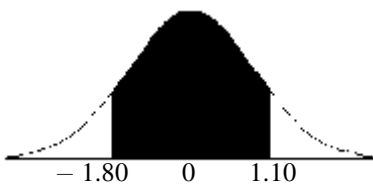


e. $0.6879 - 0.4721 = 0.2158$

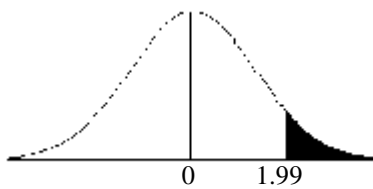


2.

a. $0.8643 - 0.0359 = 0.8284$

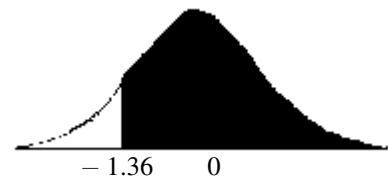


b. $1 - 0.9767 = 0.0233$

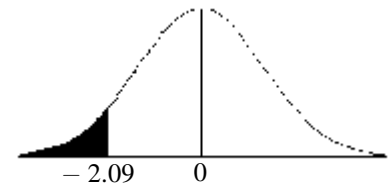


2. continued

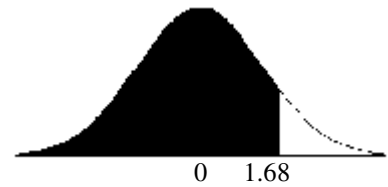
c. $1 - 0.0869 = 0.9131$



d. 0.0183

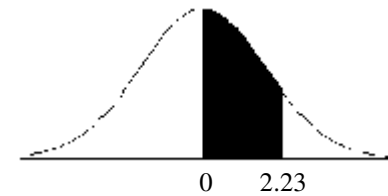


e. 0.9535

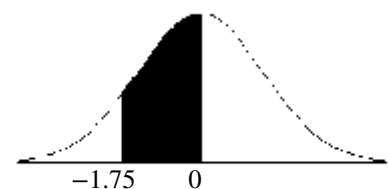


3.

a. $0.9871 - 0.5 = 0.4871$



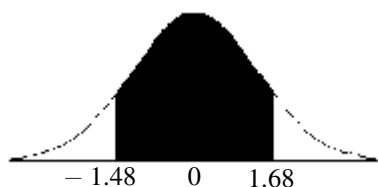
b. $0.5 - 0.0401 = 0.4599$



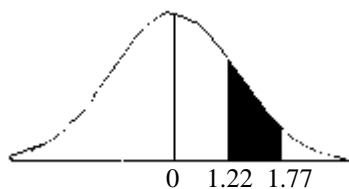
Chapter 6 - The Normal Distribution

3. continued

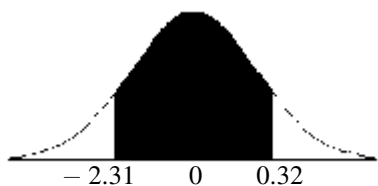
c. $0.9535 - 0.0694 = 0.8841$



d. $0.9616 - 0.8888 = 0.0728$

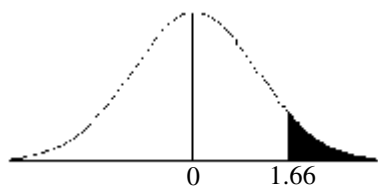


e. $0.6255 - 0.0104 = 0.6151$

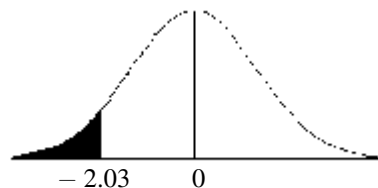


4.

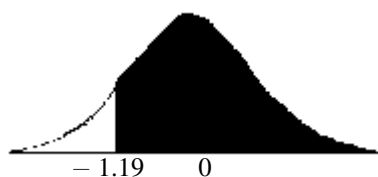
a. $1 - 0.9515 = 0.0485$



b. 0.0212



c. $1 - 0.1170 = 0.8830$

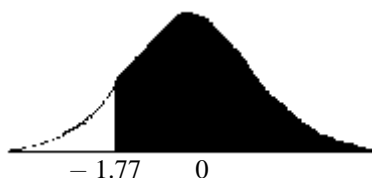


4. continued

d. 0.9732



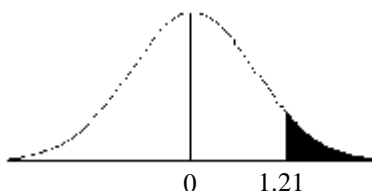
e. $1 - 0.0384 = 0.9616$



5.

$$z = \frac{\$6000 - \$5274}{\$600} = 1.21$$

$$P(z > 1.21) = 1 - 0.8869 = 0.1131$$



For the middle 50%, 25% of the area is on each side of 0. Thus, $z = \pm 0.67$

$$x = 0.67(600) + 5274 = \$5676$$

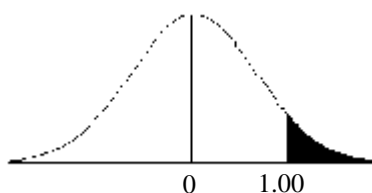
$$x = -0.67(600) + 5274 = \$4872$$

(TI answers: \$4869.31 to \$5678.69)

6.

a. $z = \frac{68,000 - 63,000}{5000} = 1.00$ area = 0.8413

$$P(z > 1.00) = 1 - 0.8413 = 0.1587$$

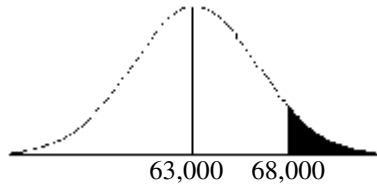


Chapter 6 - The Normal Distribution

6. continued

$$b. z = \frac{68,000 - 63,000}{\frac{5000}{\sqrt{9}}} = 3.00 \quad \text{area} = 0.9987$$

$$P(z > 3.00) = 1 - 0.9987 = 0.0013$$



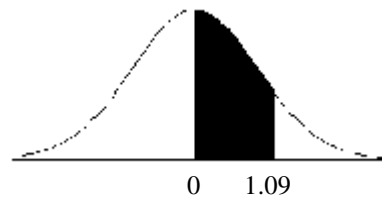
7.

$$a. z = \frac{476 - 476}{22} = 0$$

$$z = \frac{500 - 476}{22} = 1.09$$

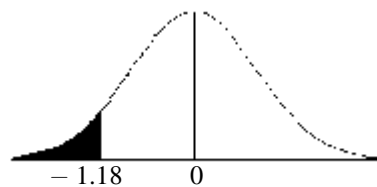
$$P(0 < z < 1.09) = 0.8621 - 0.5 = 0.3621$$

or 36.21%



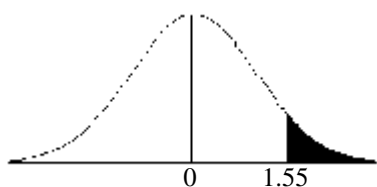
$$b. z = \frac{450 - 476}{22} = -1.18$$

$$P(z < -1.18) = 0.1190 \text{ or } 11.9\%$$



$$c. z = \frac{510 - 476}{22} = 1.55$$

$$P(z > 1.55) = 1 - 0.9394 = 0.0606 \text{ or } 6.06\%$$

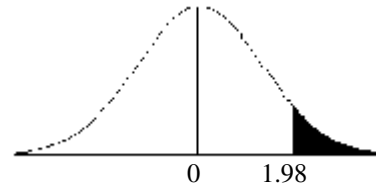


8.

For more than \$15 per month:

$$z = \frac{\$15 - \$10.15}{\$2.45} = 1.98$$

$$P(z > 1.98) = 1 - 0.9761 = 0.0239$$



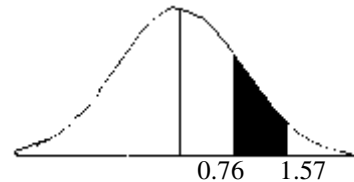
For between \$12 and \$14 per month:

$$z = \frac{\$12 - \$10.15}{\$2.45} = 0.76$$

$$z = \frac{\$14 - \$10.15}{\$2.45} = 1.57$$

$$P(0.76 < z < 1.57) = 0.9418 - 0.7764 = 0.1654$$

(TI answer = 0.16705)



9.

For 15% costs, area = 0.85

$$z = 1.04$$

$$X = 1.04(10.50) + 120 = \$130.92$$

10.

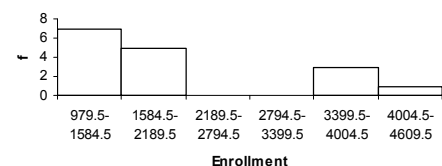
For 15% costs, area = 0.85

$$z = -1.04$$

$$X = -1.04(\$750) + \$8000 = \$7220$$

11.

Histogram:



The histogram shows a positive skew.

Chapter 6 - The Normal Distribution

11. continued

$$PI = \frac{3(2136.1 - 1755)}{1171.7} = 0.98$$

$$IQR = Q_3 - Q_1$$

$$IQR = 2827 - 1320 = 1507$$

$$1.5(IQR) = 1.5(1507) = 2260.5$$

$$Q_1 - 2260.5 = -940.5$$

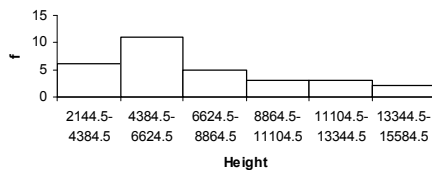
$$Q_3 + 2260.5 = 5087.5$$

There are no outliers.

Conclusion: The distribution is not normal.

12.

Histogram:



The histogram shows a positive skew.

$$PI = \frac{3(6972.2 - 5931.5)}{3458.85} = 0.90$$

$$IQR = Q_3 - Q_1$$

$$IQR = 9348 - 5135 = 4213$$

$$1.5(IQR) = 1.5(4213) = 6319.5$$

$$Q_1 - 6319.5 = -1184.5$$

$$Q_3 + 6319.5 = 15,667.5$$

There are no outliers.

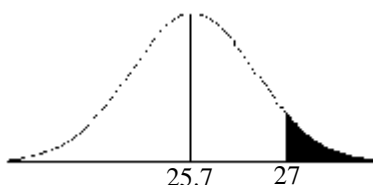
Conclusion: The distribution is not normal.

13.

$$a. z = \frac{X - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{27 - 25.7}{\frac{3.75}{\sqrt{40}}} = 2.19$$

$$P(\bar{X} > 27) = 1 - 0.9857 = 0.0143$$

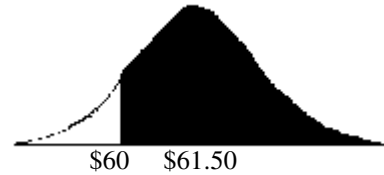
(TI answer = 0.0142)



13. continued

$$b. z = \frac{X - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{\$60 - \$61.50}{\frac{\$5.89}{\sqrt{50}}} = -1.80$$

$$P(\bar{X} > 60) = 1 - 0.0359 = 0.9641$$

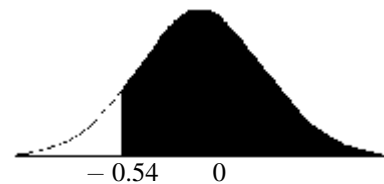


14.

$$a. z = \frac{18 - 19.32}{2.44} = -0.54$$

$$P(z > -0.54) = 1 - 0.2946 = 0.7054$$

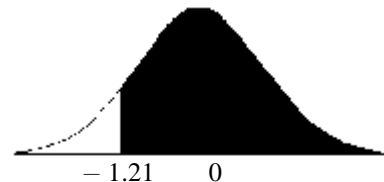
(TI answer = 0.7057)



$$b. z = \frac{18 - 19.32}{\frac{2.44}{\sqrt{5}}} = -1.21$$

$$P(z > -1.21) = 1 - 0.1131 = 0.8869$$

(TI answer = 0.8868)

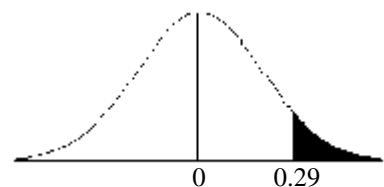


15.

$$a. z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{670 - 660}{35} = 0.29$$

$$P(z > 0.29) = 1 - 0.6141 = 0.3859$$

(TI answer = 0.3875)



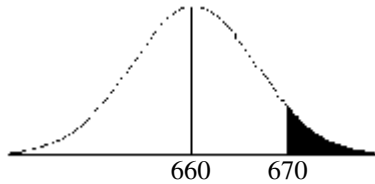
Chapter 6 - The Normal Distribution

15. continued

$$b. \ z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{670 - 660}{\frac{35}{\sqrt{10}}} = 0.90$$

$$P(z > 0.90) = 1 - 0.8159 = 0.1841$$

(TI answer = 0.1831)

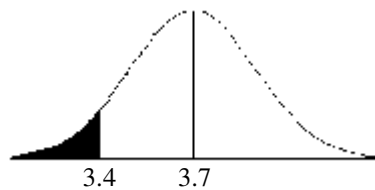


c. Individual values are more variable than means.

16.

$$z = \frac{3.4 - 3.7}{\frac{0.6}{\sqrt{32}}} = -2.83$$

$$P(\bar{X} < 3.4) = 1 - 0.9977 = 0.0023 \text{ or } 0.23\%$$



Yes, since the probability is less than 1%.

17.

$$\mu = 120(0.173) = 20.76$$

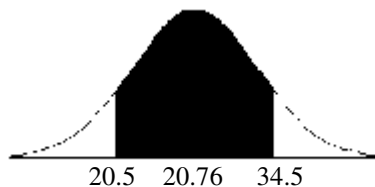
$$\sigma = \sqrt{120(0.173)(0.827)} = 4.14$$

$$z = \frac{20.5 - 20.76}{4.14} = -0.06$$

$$z = \frac{34.5 - 20.76}{4.14} = 3.32$$

$$P(20.5 < X < 34.5) = 0.9995 - 0.4761 = 0.5234$$

(TI answer = 0.52456)



18.

$$\mu = np = 500(0.05) = 25$$

$$\sigma = \sqrt{npq} = \sqrt{(500)(0.05)(0.95)} = 4.87$$

$$z = \frac{30.5 - 25}{4.87} = 1.13$$

$$z = \frac{29.5 - 25}{4.87} = 0.92$$

$$P(29.5 < X < 30.5) = 0.8708 - 0.8212 = 0.0496 \text{ or } 4.96\%$$



19.

For fewer than 10 holding multiple jobs:

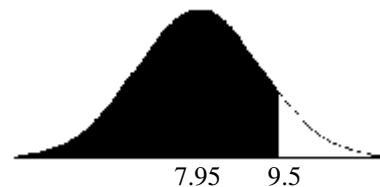
$$\mu = 150(0.053) = 7.95$$

$$\sigma = \sqrt{(150)(0.053)(0.947)} = 2.744$$

$$z = \frac{9.5 - 7.95}{2.74} = 0.56$$

$$P(X < 9.5) = 0.7123$$

(TI answer = 0.7139)



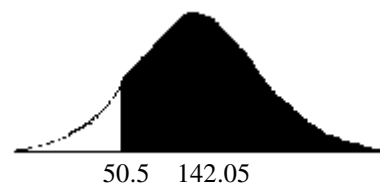
For more than 50 not holding multiple jobs: $\mu = 150(0.947) = 142.05$

$$\sigma = \sqrt{150(0.947)(0.053)} = 2.744$$

$$z = \frac{50.5 - 142.05}{2.744} = -33.37$$

$$P(X > 50.5) = 1 - 0.0001 = 0.9999$$

(TI answer = 0.9999)



Chapter 6 - The Normal Distribution

20.

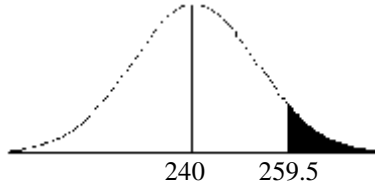
$$\mu = 800(0.30) = 240$$

$$\sigma = \sqrt{(800)(0.3)(0.7)} = 12.96$$

$$z = \frac{259.5 - 240}{12.96} = 1.50$$

$$P(X \geq 259.5) = 1 - 0.9332 = 0.0668$$

or 6.68%



21.

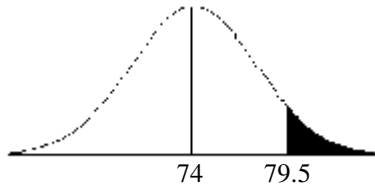
$$\mu = 200(0.37) = 74$$

$$\sigma = \sqrt{(200)(0.37)(0.63)} = 6.8279$$

$$z = \frac{79.5 - 74}{6.8279} = 0.81$$

$$P(X \geq 79.5) = 1 - 0.7910 = 0.2090$$

or 20.90%



22.

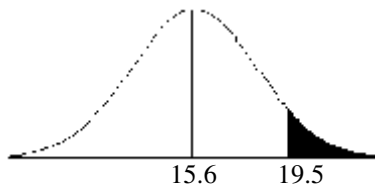
$$\mu = 60(0.26) = 15.6$$

$$\sigma = \sqrt{(60)(0.26)(0.74)} = 3.3976$$

$$z = \frac{19.5 - 15.6}{3.3976} = 1.15$$

$$P(X \geq 19.5) = 1 - 0.8749 = 0.1251$$

or 12.51%



CHAPTER 6 QUIZ

1. False, the total area is equal to one.
2. True
3. True
4. True
5. False, the area is positive.
6. False, it applies to means taken from the same population.
7. a
8. a
9. b
10. b
11. c
12. 0.5
13. Sampling error
14. The population mean
15. The standard error of the mean
16. 5
17. 5%
18. The areas are:

a. 0.4332	f. 0.8284
b. 0.3944	g. 0.0401
c. 0.0344	h. 0.8997
d. 0.1029	i. 0.017
e. 0.2912	j. 0.9131
19. The probabilities are:

a. 0.4846	f. 0.0384
b. 0.4693	g. 0.0089
c. 0.9334	h. 0.9582
d. 0.0188	i. 0.9788
e. 0.7461	j. 0.8461

Chapter 6 - The Normal Distribution

20. The probabilities are:

- a. 0.7734
- b. 0.0516
- c. 0.3837

d. Any rainfall above 65 inches could be considered an extremely wet year since this value is two standard deviations above the mean.

21. The probabilities are:

- a. 0.0668 c. 0.4649
- b. 0.0228 d. 0.0934

22. The probabilities are:

- a. 0.4525 c. 0.3707
- b. 0.3707 d. 0.019

23. The probabilities are:

- a. 0.0013 c. 0.0081
- b. 0.5 d. 0.5511

24. The probabilities are:

- a. 0.0037 c. 0.5
- b. 0.0228 d. 0.3232

25. 8.804 cm

26. The lowest acceptable score is 121.24.

27. 0.015

28. 0.9738

29. 0.0495; no

30. 0.0455 or 4.55%

31. 0.0614

32. 0.0495

33. The distribution is not normal.

34. The distribution is approximately normal.