

## Chapter 10 - Correlation and Regression

Note: Graphs are not to scale and are intended to convey a general idea.

Answers may vary due to rounding, TI-83's, or computer programs.

### EXERCISE SET 10-1

1.

Two variables are related when there exists a discernible pattern between them.

2.

Relationships are measured by the correlation coefficient,  $r$ . When  $r$  is near  $+1$ , there is a strong positive linear relationship between the variables. When  $r$  is near  $-1$ , there is a strong negative linear relationship. When  $r$  is near zero, there is no linear relationship between the variables.

3.

$r$ ,  $\rho$  (rho)

4.

The range of  $r$  is from  $-1$  to  $+1$ .

5.

A positive relationship means that as  $x$  increases,  $y$  also increases.

A negative relationship means that as  $x$  increases,  $y$  decreases.

6.

Answers will vary.

7.

The diagram is called a scatter plot. It shows the nature of the relationship.

8.

Pearson's Product Moment Correlation Coefficient.

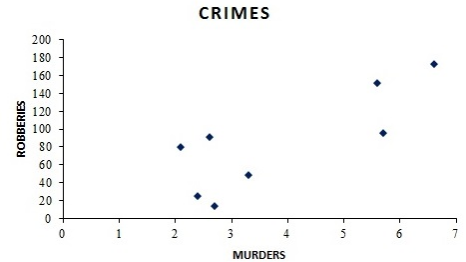
9.

$t$  test

10.

There are many other possibilities, such as chance, relationship to a third variable, etc.

11.



$$\sum x = 31$$

$$\sum y = 680.1$$

$$\sum x^2 = 142.52$$

$$\sum y^2 = 80,033.99$$

$$\sum xy = 3202.71$$

$$n = 8$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{8(3202.71) - (31)(680.1)}{\sqrt{[8(142.52) - 31^2][8(80,033.99) - 680.1^2]}}$$

$$r = 0.804$$

$$H_0: \rho = 0$$

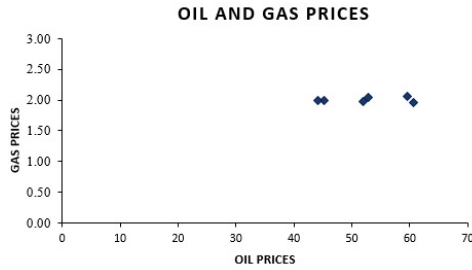
$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.707 \quad d. f. = 6$$

Decision: Reject. There is a significant linear relationship between the number of murders and the number of robberies per 100,000 people in a random sample of states.

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12.



$$\begin{aligned}\sum x &= 314.31 & \sum y^2 &= 24.0878 \\ \sum y &= 12.02 & \sum xy &= 629.8826 \\ \sum x^2 &= 16,704.9823 \\ n &= 6\end{aligned}$$

$$r = \frac{6(629.8826) - (314.31)(12.02)}{\sqrt{[6(16,704.9823) - 314.31^2][6(24.0878) - 12.02^2]}}$$

$$r = 0.158$$

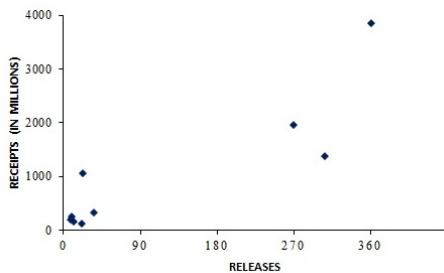
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.811 \quad d. f. = 4$$

Decision: Do not reject. There is no significant linear relationship between the price of oil and the price of gasoline.

13.



$$\begin{aligned}\sum x &= 1045 \\ \sum y &= 9283 \\ \sum x^2 &= 299,315 \\ \sum y^2 &= 21,881,839 \\ \sum xy &= 2,380,435 \\ n &= 9\end{aligned}$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

13. continued

$$r = \frac{9(2,380,435) - (1045)(9283)}{\sqrt{[9(299,315) - 1045^2][9(21,881,839) - 9283^2]}}$$

$$r = 0.880$$

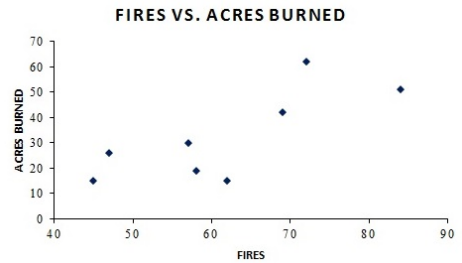
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.666 \quad d. f. = 7$$

Decision: Reject. There is a significant linear relationship between number of movie releases and gross receipts.

14.



$$\begin{aligned}\sum x &= 494 \\ \sum y &= 260 \\ \sum x^2 &= 31,692 \\ \sum y^2 &= 10,596 \\ \sum xy &= 17,285 \\ n &= 8\end{aligned}$$

$$r = \frac{8(17,285) - (494)(260)}{\sqrt{[8(31,692) - 494^2][8(10,596) - 260^2]}}$$

$$r = 0.771$$

$$H_0: \rho = 0$$

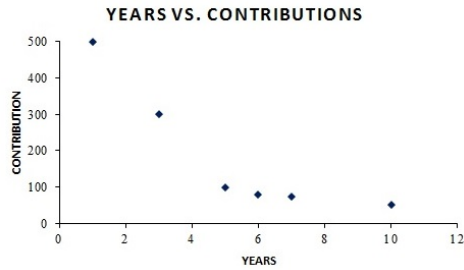
$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.707 \quad d. f. = 6$$

Decision: Reject. There is a significant linear relationship between the number of forest fires and the number of acres burned.

## Chapter 10 - Correlation and Regression

15.



$$\sum x = 32$$

$$\sum y = 1105$$

$$\sum x^2 = 220$$

$$\sum y^2 = 364,525$$

$$\sum xy = 3405$$

$$n = 6$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{6(3405) - (32)(1105)}{\sqrt{[6(220) - 32^2][6(364525) - 1105^2]}}$$

$$r = -0.883$$

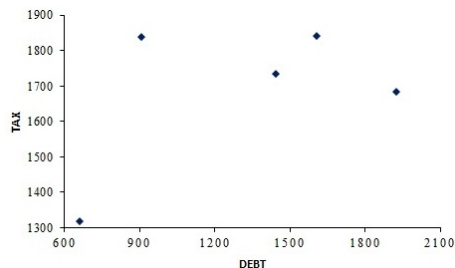
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.811 \quad d. f. = 4$$

Decision: Reject. There is a significant linear relationship between the number of years a person has been out of school and his or her contributions.

16.



$$\sum x = 6545$$

$$\sum y = 8416$$

$$\sum x^2 = 9,635,035$$

$$\sum y^2 = 14,351,678$$

$$\sum xy = 11,247,109$$

16. continued

$$n = 5$$

$$r = \frac{5(11,247,109) - (6545)(8416)}{\sqrt{[5(9,635,035) - 6545^2][5(14,351,678) - 8416^2]}}$$

$$r = 0.518$$

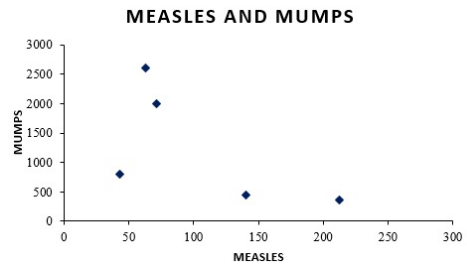
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.878 \quad d. f. = 3$$

Decision: Do not reject. There is no significant linear relationship between per capita debt and tax.

17.



$$\sum x = 529$$

$$\sum y = 6227$$

$$\sum x^2 = 75,403$$

$$\sum y^2 = 11,769,641$$

$$\sum xy = 482,317$$

$$n = 5$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{5(482,317) - (529)(6227)}{\sqrt{[5(75,403) - 529^2][5(11,769,641) - 6227^2]}}$$

$$r = -0.632$$

$$H_0: \rho = 0$$

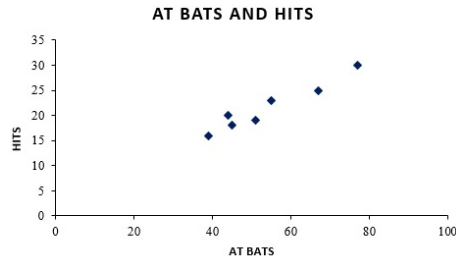
$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.878 \quad d. f. = 3$$

Decision: Do not reject. There is no significant linear relationship between the number of cases of measles and mumps.

## Chapter 10 - Correlation and Regression

18.



$$\begin{aligned}\sum x &= 378 \\ \sum y &= 151 \\ \sum x^2 &= 21,526 \\ \sum y^2 &= 3395 \\ \sum xy &= 8533 \\ n &= 7\end{aligned}$$

$$r = \frac{7(8533) - (378)(151)}{\sqrt{[7(21,526) - 378^2][7(3395) - 151^2]}}$$

$$r = 0.968$$

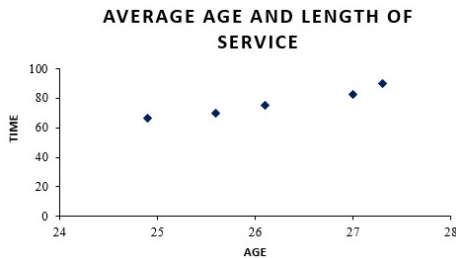
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.754 \quad d. f. = 5$$

Decision: Reject. There is a significant linear relationship between the number of hits a World Series player gets and the number of times at bat the player has.

19.



$$\begin{aligned}\sum x &= 130.9 \\ \sum y &= 383.5 \\ \sum x^2 &= 3430.87 \\ \sum y^2 &= 29,768.21 \\ \sum xy &= 10,076.41 \\ n &= 5\end{aligned}$$

$$r = \frac{5(10,076.41) - (130.9)(383.5)}{\sqrt{[5(3430.87) - 130.9^2][5(29,768.21) - 383.5^2]}}$$

19. continued

$$r = 0.978$$

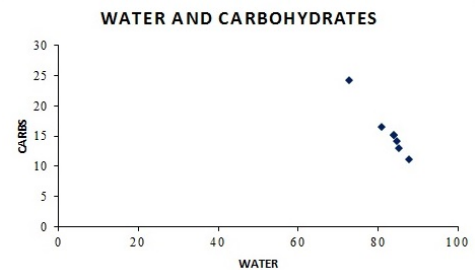
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.878 \quad d. f. = 3$$

Decision: Reject. There is a significant linear relationship between the average age and length of service in months.

20.



$$\begin{aligned}\sum x &= 578.82 \\ \sum y &= 109.42 \\ \sum x^2 &= 47,999.8288 \\ \sum y^2 &= 1815.937 \\ \sum xy &= 8927.9664 \\ n &= 7\end{aligned}$$

$$r = \frac{7(8927.9664) - (578.82)(109.42)}{\sqrt{[7(47,999.8288) - 578.82^2][7(1815.937) - 109.42^2]}}$$

$$r = -0.993$$

$$H_0: \rho = 0$$

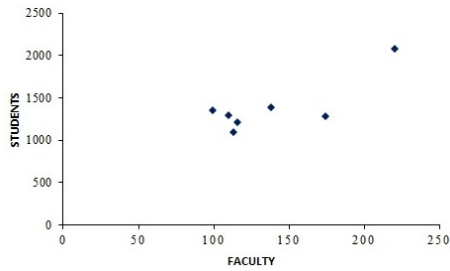
$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.754 \quad d. f. = 5$$

Decision: Reject. There is a significant linear relationship between the number of grams of water and the number of grams of carbohydrates in 100-gram servings of fruits and vegetables.

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21.



$$\sum x = 970$$

$$\sum y = 9689$$

$$\sum x^2 = 145,846$$

$$\sum y^2 = 14,023,529$$

$$\sum xy = 1,410,572$$

$$n = 7$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{7(1,410,572) - (970)(9689)}{\sqrt{[7(145,846) - 970^2][7(14,023,529) - 9689^2]}}$$

$$r = 0.812$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.754 \quad d. f. = 5$$

Decision: Reject. There is a significant linear relationship between the number of faculty and the number of students.

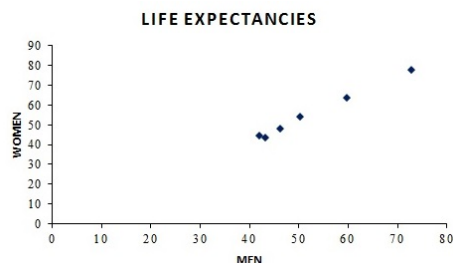
For  $x$  = Students and  $y$  = Faculty:

$$r = \frac{7(1,410,572) - (9689)(970)}{\sqrt{[7(14,023,529) - 9689^2][7(145,846) - 970^2]}}$$

$$r = 0.812$$

The results are the same when  $x$  and  $y$  are switched. Students would be the more likely independent variable.

22.



22. continued

$$\sum x = 314.2$$

$$\sum y = 331.9$$

$$\sum x^2 = 17,164.88$$

$$\sum y^2 = 19,244.67$$

$$\sum xy = 18,171.89$$

$$n = 6$$

$$r = \frac{6(18,171.89) - (314.2)(331.9)}{\sqrt{[6(17,164.88) - 314.2^2][6(19,244.67) - 331.9^2]}}$$

$$r = 0.997$$

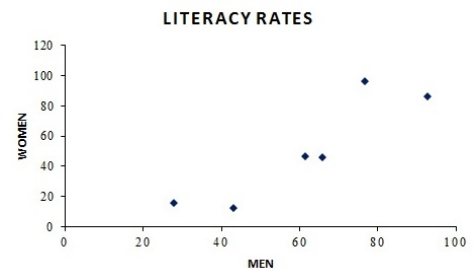
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.811 \quad d. f. = 4$$

Decision: Reject. There is a significant linear relationship between the life expectancy of men and the life expectancy of women.

23.



$$\sum x = 367.5$$

$$\sum y = 302.7$$

$$\sum x^2 = 25,192.41$$

$$\sum y^2 = 21,346.59$$

$$\sum xy = 22,207.31$$

$$n = 6$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{6(22,207.31) - (367.5)(302.7)}{\sqrt{[6(25,192.41) - 367.5^2][6(21,346.59) - 302.7^2]}}$$

$$r = 0.908$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

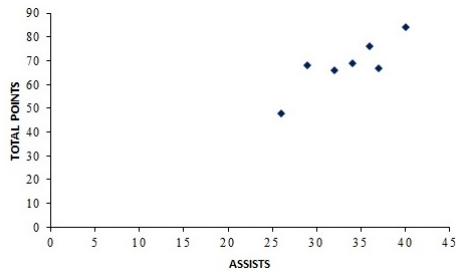
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23. continued

$$C. V. = \pm 0.811 \quad d. f. = 4$$

Decision: Reject. There is a significant linear relationship between the literacy rates of men and the literacy rates of women.

24.



$$\sum x = 234$$

$$\sum y = 478$$

$$\sum x^2 = 7962$$

$$\sum y^2 = 33,366$$

$$\sum xy = 16,253$$

$$n = 7$$

$$r = \frac{7(16,253) - (234)(478)}{\sqrt{[7(7962) - 234^2][7(33,366) - 478^2]}}$$

$$r = 0.861$$

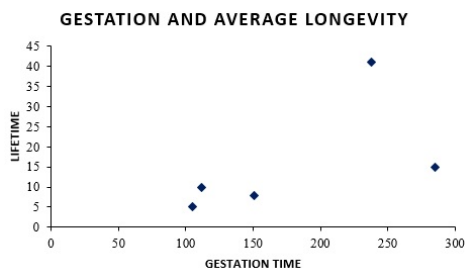
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.754 \quad d. f. = 5$$

Decision: Reject. There is a significant linear relationship between the number of assists and the number of total points.

25.



$$\sum x = 891$$

$$\sum y = 79$$

$$\sum x^2 = 184,239$$

25. continued

$$\sum y^2 = 2095$$

$$\sum xy = 16,886$$

$$n = 5$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{5(16,886) - (891)(79)}{\sqrt{[5(184,239) - 891^2][5(2095) - 79^2]}}$$

$$r = 0.605$$

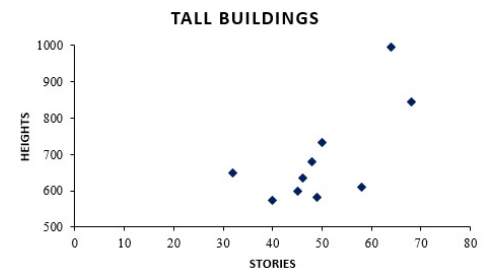
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.878 \quad d. f. = 3$$

Decision: Do not reject. There is not enough evidence to support the claim that there is a significant relationship between the gestation time and the lifetime.

26.



$$\sum x = 500$$

$$\sum y = 6899$$

$$\sum x^2 = 26,054$$

$$\sum y^2 = 4,922,673$$

$$\sum xy = 354,077$$

$$n = 10$$

$$r = \frac{10(354,077) - (500)(6899)}{\sqrt{[10(26,054) - 500^2][10(4,922,673) - 6899^2]}}$$

$$r = 0.696$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.632 \quad d. f. = 8$$

Decision: Reject. There is enough evidence to support the claim that there is a relationship between the number of stories in a building and its height.

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27.



$$\sum x = 77$$

$$\sum y = 513$$

$$\sum x^2 = 1149$$

$$\sum y^2 = 43,969$$

$$\sum xy = 6495$$

$$n = 6$$

$$r = \frac{6(6495) - (77)(513)}{\sqrt{[6(1149) - 77^2][6(43,969) - 513^2]}}$$

$$r = -0.673$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.811 \quad d. f. = 4$$

Decision: Do not reject. There is not a significant linear relationship between class size and average grade.

28.

$$\bar{x} = 12.8333$$

$$\bar{y} = 85.5$$

$$\sum (x - \bar{x})(y - \bar{y}) = -88.5$$

$$s_x = 5.6716$$

$$s_y = 4.6368$$

$$r = \frac{-88.5}{(6-1)(5.6716)(4.6368)} = -0.673$$

They are the same. Any slight difference is due to rounding.

29.

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

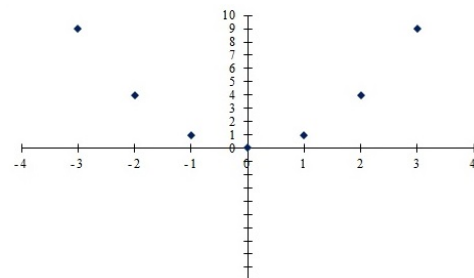
$$r = \frac{5(180) - (15)(50)}{\sqrt{[5(55) - 15^2][5(590) - 50^2]}} = 1$$

$$r = \frac{5(180) - (50)(15)}{\sqrt{[5(590) - 50^2][5(55) - 15^2]}} = 1$$

29. continued

The value of  $r$  does not change when the values for  $x$  and  $y$  are interchanged.

30.



$$\sum x = 0$$

$$\sum y = 28$$

$$\sum x^2 = 28$$

$$\sum y^2 = 196$$

$$\sum xy = 0$$

$$n = 7$$

$$r = \frac{7(0) - (0)(28)}{\sqrt{[7(28) - 0^2][7(196) - 28^2]}} = 0$$

The relationship is non-linear, as shown in the scatter plot.

### EXERCISE 10-2

1.

Draw the scatter plot and test the significance of the correlation coefficient.

2. The assumptions are:

a. The sample is a random sample.

b. For any specific value of the independent variable  $x$ , the value of the dependent variable  $y$  must be normally distributed about the regression line.

c. The standard deviation of each of the dependent variables must be the same for each value of the independent variable.

3.

$$y' = a + bx$$

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4.

$b, a$

5.

It is the line that is drawn on the scatter plot such that the sum of the squares of the vertical distances each point is from the line is at a minimum.

6.

$r$  would equal  $+1$  or  $-1$

7.

When  $r$  is positive,  $b$  will be positive.

When  $r$  is negative,  $b$  will be negative.

8.

They would be clustered closer to the line.

9.

The closer  $r$  is to  $+1$  or  $-1$ , the more accurate the predicted value will be.

10.

When  $r$  is not significant, the mean of the  $y$  values should be used to predict  $y$ .

11.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(680.1)(142.52) - (31)(3202.71)}{8(142.52) - (31)^2} = -13.151$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{8(3202.71) - (31)(680.1)}{8(142.52) - (31)^2} = 25.333$$

$$y' = a + bx$$

$$y' = -13.151 + 25.333x$$

$$y' = -13.151 + 25.333(4.5) = 100.848$$

robberies

12.

Since  $r$  is not significant, no regression should be done.

13.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(9283)(299,315) - (1045)(2,380,435)}{9(299,315) - (1045)^2} = 181.661$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{9(2,380,435) - (1045)(9283)}{9(299,315) - (1045)^2} = 7.319$$

$$y' = a + bx$$

$$y' = 181.661 + 7.319x$$

$$y' = 181.661 + 7.319(200) = \$1645.5$$

million gross receipts

14.

$$a = \frac{(260)(31,692) - (494)(17,285)}{8(31,692) - (494)^2}$$

$$a = -31.460$$

$$b = \frac{8(17,285) - (494)(260)}{8(31,692) - (494)^2}$$

$$b = 1.036$$

$$y' = -31.46 + 1.036x$$

$$y' = -31.46 + 1.036(60) = 30.7 \text{ acres}$$

15.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(1105)(220) - (32)(3405)}{6(220) - (32)^2}$$

$$a = \frac{243100 - 108960}{1320 - 1024} = 453.176$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{6(3405) - (32)(1105)}{6(220) - (32)^2}$$

$$b = -50.439$$

$$y' = a + bx$$

$$y' = 453.176 - 50.439x$$

$$y' = 453.176 - 50.439(4) = \$251.42$$

16.

Since  $r$  is not significant, no regression should be done.  $\bar{y}$  can be used for the prediction.



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17.

Since  $r$  is not significant, no regression should be done.  $\bar{y}$  can be used for the prediction.

18.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(151)(21,526) - (378)(8533)}{7(21,526) - (378)^2}$$

$$a = 3.200$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{7(8533) - (378)(151)}{7(21,526) - (378)^2}$$

$$b = 0.340$$

$$y' = a + bx$$

$$y' = 3.2000 + 0.340x$$

$$y' = 3.2000 + 0.340(60) = 23.6 \text{ or } 24 \text{ hits}$$

19.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(383.5)(3430.87) - (130.9)(10,076.41)}{5(3430.87) - (130.9)^2}$$

$$a = -167.012$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{5(10,076.41) - (130.9)(383.5)}{5(3430.87) - (130.9)^2}$$

$$b = 9.309$$

$$y' = a + bx$$

$$y' = -167.012 + 9.309x$$

$$y' = -167.012 + 9.309(26.8) = 82.5$$

months

20.

$$a = \frac{(109.42)(47,999.8288) - (578.82)(8927.9664)}{7(47,999.8288) - (578.82)^2}$$

$$a = 87.409$$

$$b = \frac{7(8927.9664) - (578.82)(109.42)}{7(47,999.8288) - (578.82)^2}$$

$$b = -0.868$$

20. continued

$$y' = 87.409 - 0.868x$$

$$y' = 87.409 - 0.868(75) = 22.309 \text{ grams}$$

21.

For  $x$  = Students and  $y$  = Faculty:

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(970)(14,023,529) - (9689)(1,410,572)}{7(14,023,529) - (9689)^2}$$

$$a = -14.974$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{7(1,410,572) - (9689)(970)}{7(14,023,529) - (9689)^2} = 0.111$$

$$y' = a + bx$$

$$y' = -14.974 + 0.111x$$

22.

$$a = \frac{(331.9)(17,164.88) - (314.2)(18,171.89)}{6(17,164.88) - (314.2)^2}$$

$$a = -2.949$$

$$b = \frac{6(18,171.89) - (314.2)(331.9)}{6(17,164.88) - (314.2)^2}$$

$$b = 1.113$$

$$y' = a + bx$$

$$y' = -2.949 + 1.113x$$

$$y' = -2.949 + 1.113(60) = 63.831 \text{ years}$$

23.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(302.7)(25,192.41) - (367.5)(22,207.31)}{6(25,192.41) - (367.5)^2}$$

$$a = -33.261$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{6(22,207.31) - (367.5)(302.7)}{6(25,192.41) - (367.5)^2}$$

$$b = 1.367$$

$$y' = a + bx$$

$$y' = -33.261 + 1.367x$$

$$y' = -33.261 + 1.367(80) = 76.099 \text{ or } 76.1\%$$

## Chapter 10 - Correlation and Regression

24.

$$a = \frac{(478)(7962) - (234)(16,253)}{7(7962) - (234)^2} = 2.693$$

$$b = \frac{7(16,253) - (234)(478)}{7(7962) - (234)^2} = 1.962$$

$$y' = a + bx$$

$$y' = 2.693 + 1.962x$$

$$y' = 2.693 + 1.962(30)$$

$$y' = 61.6 \approx 62 \text{ points}$$

25.

Since  $r$  is not significant, no regression should be done.  $\bar{y}$  can be used for the prediction.

26.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(6899)(26,054) - (500)(354,077)}{10(26,054) - (500)^2}$$

$$a = 256.930$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{10(354,077) - (500)(6899)}{10(26,054) - (500)^2} = 8.659$$

$$y' = 256.930 + 8.659x$$

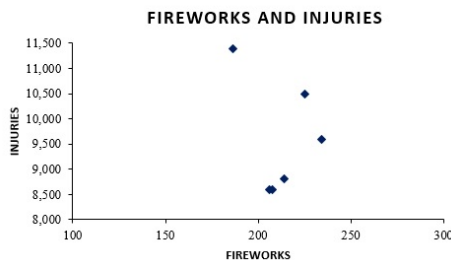
$$y' = 256.930 + 8.659(200)$$

$$y' = 733 \text{ feet}$$

27.

Since  $r$  is not significant, no regression should be done.  $\bar{y}$  can be used for the prediction.

28.



28. continued

$$\sum x = 1273.1$$

$$\sum y = 57,500$$

$$\sum x^2 = 271,512.13$$

$$\sum y^2 = 557,730,000$$

$$\sum xy = 12,175,530$$

$$n = 6$$

$$r = \frac{6(12,175,530) - (1273.1)(57,500)}{\sqrt{[6(271,512.13) - (1273.1)^2][6(557,730,000) - (57,500)^2]}}$$

$$r = -0.260$$

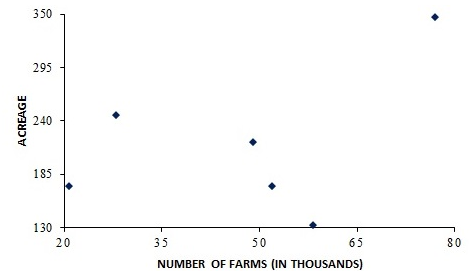
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.811 \quad d. f. = 4$$

Decision: Do not reject. There is no significant relationship between the number of fireworks in use and the number of related injuries. No regression should be done.

29.



$$\sum x = 285$$

$$\sum y = 1289$$

$$\sum x^2 = 15,637.88$$

$$\sum y^2 = 305,731$$

$$\sum xy = 64,565.8$$

$$n = 6$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{6(64,565.8) - (285)(1289)}{\sqrt{[6(15,637.88) - (285)^2][6(305,731) - (1289)^2]}}$$

$$r = 0.429$$

## Chapter 10 - Correlation and Regression

29. continued

$$H_0: \rho = 0$$

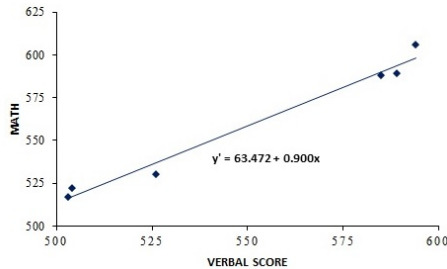
$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.811 \quad d. f. = 4$$

Decision: Do not reject

There is no significant relationship between the number of farms and acreage.

30.



$$\sum x = 3301$$

$$\sum y = 3352$$

$$\sum x^2 = 1,825,683$$

$$\sum y^2 = 1,880,574$$

$$\sum xy = 1,852,784$$

$$n = 6$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{6(1,852,784) - (3301)(3352)}{\sqrt{[6(1,825,683) - (3301)^2][6(1,880,574) - (3352)^2]}}$$

$$r = 0.990$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.811 \quad d. f. = 4$$

Decision: Reject. There is a significant linear relationship between verbal and math scores.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(3352)(1,825,683) - (3301)(1,852,784)}{6(1,825,683) - (3301)^2} = 63.472$$

30. continued

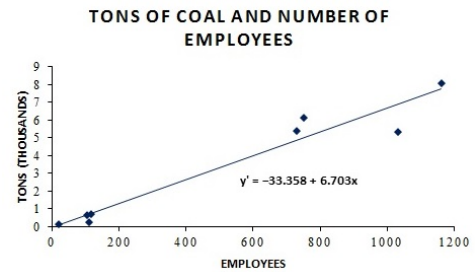
$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{6(1,852,784) - (3301)(3352)}{6(1,825,683) - (3301)^2} = 0.900$$

$$y' = a + bx$$

$$y' = 63.472 + 0.900x$$

31.



$$\sum x = 4027$$

$$\sum y = 26,728$$

$$\sum x^2 = 3,550,103$$

$$\sum y^2 = 162,101,162$$

$$\sum xy = 23,663,669$$

$$n = 8$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{8(23,663,669) - (4027)(26,728)}{\sqrt{[8(3,550,103) - (4027)^2][8(162,101,162) - (26,728)^2]}}$$

$$r = 0.970$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.707 \quad d. f. = 6$$

Decision: Reject. There is a significant relationship between the number of tons of coal produced and the number of employees.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(26,728)(3,550,103) - (4027)(23,663,669)}{8(3,550,103) - (4027)^2}$$

## Chapter 10 - Correlation and Regression

31. continued

$$a = -33.358$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

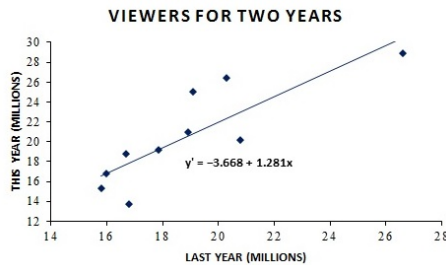
$$b = \frac{8(23663669) - (4027)(26728)}{8(3550103) - (4027)^2} = 6.703$$

$$y' = a + bx$$

$$y' = -33.358 + 6.703x$$

$$y' = -33.358 + 6.703(500) = 3318.142$$

32.



$$\sum x = 188.85$$

$$\sum y = 205.3$$

$$\sum x^2 = 3659.7025$$

$$\sum y^2 = 4432.31$$

$$\sum xy = 3996.6$$

$$n = 10$$

$$r = \frac{10(3996.6) - (188.85)(205.3)}{\sqrt{[10(3659.7025) - (188.85)^2][10(4432.31) - (205.3)^2]}}$$

$$r = 0.839$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C.V. = \pm 0.632 \quad d.f. = 8$$

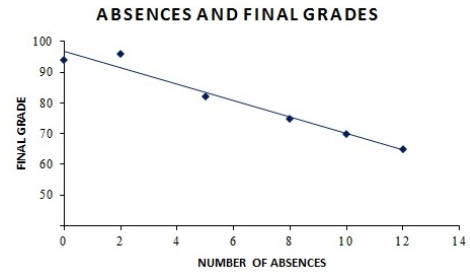
Decision: Reject. There is a significant linear relationship between the number of viewers of last year's show and the number of viewers of the same shows this year.

$$a = \frac{(205.3)(3659.7025) - (188.85)(3996.6)}{10(3659.7025) - (188.85)^2} = -3.668$$

$$b = \frac{10(3996.6) - (188.85)(205.3)}{10(3659.7025) - (188.85)^2} = 1.281$$

$$y' = -3.668 + 1.281x$$

33.



$$\sum x = 37$$

$$\sum y = 482$$

$$\sum x^2 = 337$$

$$\sum y^2 = 39526$$

$$\sum xy = 2682$$

$$n = 6$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{6(2682) - (37)(482)}{\sqrt{[6(337) - (37)^2][6(39526) - (482)^2]}}$$

$$r = -0.981$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C.V. = \pm 0.811 \quad d.f. = 4$$

Decision: Reject. There is a significant relationship between the number of absences and the final grade.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(482)(337) - (37)(2682)}{6(337) - (37)^2} = 96.784$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

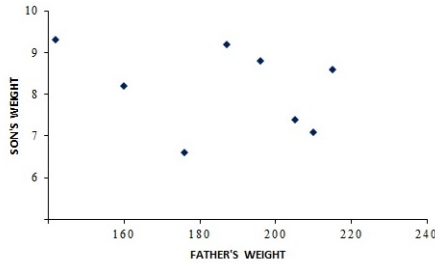
$$b = \frac{6(2682) - (37)(482)}{6(337) - (37)^2} = -2.668$$

$$y' = a + bx$$

$$y' = 96.784 - 2.668x$$

## Chapter 10 - Correlation and Regression

34.



$$\sum x = 1491$$

$$\sum y = 65.2$$

$$\sum x^2 = 282,475$$

$$\sum y^2 = 538.5$$

$$\sum xy = 12,096.4$$

$$n = 8$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{8(12096.4) - (1491)(65.2)}{\sqrt{[8(282475) - (1491)^2][8(538.5) - (65.2)^2]}}$$

$$r = -0.306$$

$$H_0: \rho = 0$$

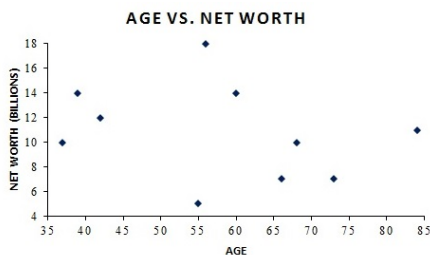
$$H_1: \rho \neq 0$$

$$t = -0.787;$$

$$0.20 < \text{P-value} < 0.50 \text{ (0.462)}$$

Decision: Do not reject since P-value > 0.05. There is no significant relationship between the weights of the fathers and sons. Since  $r$  is not significant, no regression analysis should be done.

35.



35. continued

$$\sum x = 580$$

$$\sum y = 108$$

$$\sum x^2 = 35,780$$

$$\sum y^2 = 1304$$

$$\sum xy = 6120$$

$$n = 10$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{10(6120) - (580)(108)}{\sqrt{10(35,780) - 580^2][10(1304) - 108^2]}}$$

$$r = -0.265$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$t = -0.777; \text{ P-value} > 0.05 \text{ (0.459)}$$

Decision: Do not reject since P-value > 0.05. There is no significant linear relationship between the ages of billionaires and their net worth. Since  $r$  is not significant, no regression should be done.

36.

For Exercise 13:

$$\bar{x} = 116.11$$

$$\bar{y} = 1031.44$$

$$y' = 181.661 + 7.319x$$

$$y' = 181.661 + 7.319(116.11)$$

$$y' = 181.661 + 849.81$$

$$y' = 1031.47$$

$$\bar{y} = y' \text{ (difference is due to rounding)}$$

For Exercise 15:

$$\bar{x} = 5.333$$

$$\bar{y} = 184.167 \text{ (or 184)}$$

$$y' = 453.176 - 50.439x$$

$$y' = 453.176 - 50.439(5.333)$$

$$y' = 184.185 \text{ (or 184)}$$

$$\bar{y} = y'$$

## Chapter 10 - Correlation and Regression

36. continued

For Exercise 21:

$$\bar{x} = 1384.1$$

$$\bar{y} = 138.6$$

$$y' = -14.974 + 0.111x$$

$$y' = -14.974 + 0.111(1384.1)$$

$$y' = 138.7$$

$$\bar{y} = y' \text{ (difference is due to rounding)}$$

In all cases  $\bar{y} = y'$ , hence the regression line will always pass through the point  $(\bar{x}, \bar{y})$ .

Slight differences that occur between  $\bar{y}$  and  $y'$  are due to rounding.

37.

For Exercise 15:

$$\bar{x} = 5.3333$$

$$\bar{y} = 184.1667$$

$$b = -50.439$$

$$a = \bar{y} - b\bar{x}$$

$$a = 184.1667 - (-50.439)(5.3333)$$

$$a = 184.1667 + 269.0063$$

$$a = 453.173 \text{ (differs due to rounding)}$$

For Exercise 16:

Since  $r$  is not significant, no regression should be done.

38.

For Exercise 18:

$$b = 0.3402$$

$$s_x = 13.6260$$

$$s_y = 4.7909$$

$$r = \frac{bs_x}{s_y} = \frac{0.3402(13.6260)}{4.7909} = 0.968$$

For Exercise 20:

$$b = -0.868$$

$$s_x = 4.796$$

$$s_y = 4.194$$

$$r = \frac{-0.868(4.796)}{4.194} = -0.993$$

### EXERCISE SET 10-3

1.

Explained variation is the variation due to the relationship and is computed by  $\sum(y' - \bar{y})^2$ .

2.

Unexplained variation is the variation due to chance and is computed by  $\sum(y - y')^2$ .

3.

Total variation is the sum of the squares of the vertical distances of the points from the mean. It is computed by  $\sum(y - \bar{y})^2$ .

4.

The coefficient of determination is a measure of variation of the dependent variable that is explained by the regression line and the independent variable.

5.

It is found by squaring  $r$ .

6.

It is the percent of the variation in  $y$  that is not due to the variation in  $x$ .

7.

The coefficient of non-determination is  $1 - r^2$ .

8.

For  $r = 0.62$ ,  $r^2 = 0.3844$  and  $1 - r^2 = 0.6156$ . Thus 38.44% of the variation of  $y$  is due to the variation of  $x$ , and 61.56% of the variation of  $y$  is due to chance.

## Chapter 10 - Correlation and Regression

9.

For  $r = 0.44$ ,  $r^2 = 0.1936$  and

$1 - r^2 = 0.8064$ .

Thus 19.36% of the variation of  $y$  is due to the variation of  $x$ , and 80.64% of the variation of  $y$  is due to chance.

10.

For  $r = 0.51$ ,  $r^2 = 0.2601$  and

$1 - r^2 = 0.7399$ . Thus 26.01% of the variation of  $y$  is due to the variation of  $x$ , and 73.99% of the variation of  $y$  is due to chance.

11.

For  $r = 0.97$ ,  $r^2 = 0.9409$  and

$1 - r^2 = 0.0591$ . Thus 94.09% of the variation of  $y$  is due to the variation of  $x$ , and 5.91% of the variation of  $y$  is due to chance.

12.

For  $r = 0.12$ ,  $r^2 = 0.0144$  and

$1 - r^2 = 0.9856$ . Thus 1.44% of the variation of  $y$  is due to the variation of  $x$ , and 98.56% of the variation of  $y$  is due to chance.

13.

For  $r = 0.15$ ,  $r^2 = 0.0225$  and

$1 - r^2 = 0.9775$ . Thus 2.25% of the variation of  $y$  is due to the variation of  $x$ , and 97.75% of the variation of  $y$  is due to chance.

14.

The standard error of estimate is the standard deviation of the observed  $y$  values about the predicted  $y'$  values. It can be used when one is using the  $t$  distribution.

14. continued

Note: For Exercises 15 – 18, values for  $a$  and  $b$  are rounded to 3 decimal places according to the textbook's rounding rule for intercept and slope of the regression equation. Where these answers differ from the text, additional decimal places are included to show consistency with text answers.

15.

$$S_{\text{est}} = \sqrt{\frac{\sum y^2 - a \sum y - b \sum xy}{n-2}}$$

$$S_{\text{est}} = \sqrt{\frac{21,881,839 - 181.661(9283) - (7.319)(2,380,435)}{9-2}}$$

$$S_{\text{est}} = \sqrt{396,153.7389} = 629.41$$

Using  $a = 181.661102$  and  $b = 7.318708$ ,

$$S_{\text{est}} = 629.4862$$

16.

$$S_{\text{est}} = \sqrt{\frac{10,596 - (-31.46)(260) - (1.036)(17,285)}{8-2}}$$

$$S_{\text{est}} = \sqrt{144.723333} = 12.03$$

Using  $a = -31.46$  and  $b = 1.035789$ ,

$$S_{\text{est}} = 12.0553 \text{ or } 12.06$$

17.

$$S_{\text{est}} = \sqrt{\frac{\sum y^2 - a \sum y - b \sum xy}{n-2}}$$

$$S_{\text{est}} = \sqrt{\frac{364525 - (453.176)(1105) - (-50.439)(3405)}{6-2}}$$

$$S_{\text{est}} = 94.22$$

18.

Since  $r$  is not significant, the standard error should not be calculated.

## Chapter 10 - Correlation and Regression

19.

$$y' = 181.661 + 7.319x$$

$$y' = 181.661 + 7.319(200)$$

$$y' = 1645.461$$

$$y' - t_{\frac{\alpha}{2}} \cdot s_{\text{est}} \sqrt{1 + \frac{1}{n} + \frac{n(x - \bar{X})^2}{n\sum x^2 - (\sum x)^2}} < y <$$

$$y' + t_{\frac{\alpha}{2}} \cdot s_{\text{est}} \sqrt{1 + \frac{1}{n} + \frac{n(x - \bar{X})^2}{n\sum x^2 - (\sum x)^2}}$$

$$1645.461 - (1.895)(629.4862)$$

$$\sqrt{1 + \frac{1}{9} + \frac{9(200 - 116.11)^2}{9(299,315) - 1045^2}}$$

$$< y < 1645.461 +$$

$$(1.895)(629.4862) \sqrt{1 + \frac{1}{9} + \frac{9(200 - 116.11)^2}{9(299,315) - 1045^2}}$$

$$1645.461 - 1279.580227 < y <$$

$$1645.461 + 1279.580227$$

$$365.88 < y < 2925.04$$

20.

$$y' = -31.46 + 1.036x$$

$$y' = -31.46 + 1.036(60)$$

$$y' = 30.7$$

$$30.7 - (2.447)(12.06) \sqrt{1 + \frac{1}{8} + \frac{8(60 - 61.75)^2}{8(31,692) - (494)^2}} < y <$$

$$30.7 + (2.447)(12.06) \sqrt{1 + \frac{1}{8} + \frac{8(60 - 61.75)^2}{8(31,692) - (494)^2}}$$

$$30.7 - (2.447)(12.06)(1.062) < y <$$

$$30.7 + (2.447)(12.06)(1.062)$$

$$-0.64 < y < 62.04 \text{ or } 0 < y < 62$$

21.

$$y' = 453.176 - 50.439x$$

$$y' = 453.176 - 50.439(4)$$

$$y' = 251.42$$

21. continued

$$y' - t_{\frac{\alpha}{2}} \cdot s_{\text{est}} \sqrt{1 + \frac{1}{n} + \frac{n(x - \bar{X})^2}{n\sum x^2 - (\sum x)^2}} < y <$$

$$y' + t_{\frac{\alpha}{2}} \cdot s_{\text{est}} \sqrt{1 + \frac{1}{n} + \frac{n(x - \bar{X})^2}{n\sum x^2 - (\sum x)^2}}$$

$$251.42 - 2.132(94.22) \sqrt{1 + \frac{1}{6} + \frac{6(4 - 5.33)^2}{6(220) - 32^2}}$$

$$< y < 251.42 + 2.132(94.22) \sqrt{1 + \frac{1}{6} + \frac{6(4 - 5.33)^2}{6(220) - 32^2}}$$

$$251.42 - (2.132)(94.22)(1.1) < y <$$

$$251.42 + (2.132)(94.22)(1.1)$$

$$\$30.46 < y < \$472.38$$

22.

Since  $r$  is not significant, the prediction interval should not be calculated.

### EXERCISE SET 10-4

1.

Simple linear regression has one independent variable and one dependent variable. Multiple regression has one dependent variable and two or more independent variables.

2.

$$y' = a + b_1x_1 + b_2x_2 + \cdots + b_nx_n;$$

$a$  represents the slope and the  $b$ 's represent the partial regression coefficients.

3.

The relationship would include all variables in one equations.

4.

The variables must be normally distributed, variances must be equal, independent, not collinear, and linear.



## Chapter 10 - Correlation and Regression

5.

The multiple correlation coefficient  $R$  is always higher than the individual correlation coefficients. Also, the value of  $R$  can range from 0 to +1.

6.

$$y' = -34,127 + 132(32) + 20,805(3.4)$$

$$y' = \$40,834$$

7.

$$y' = 0.217 + 0.0654x_1 + 0.32x_2$$

$$y' = 0.217 + 0.0654(72) + 0.32x_2(8) = 7.5$$

8.

$$y' = -26.279 + 14.855(3) + 3.1035(48) + 0.73079(40)$$

$$y' = \$196.49$$

9.

$$y' = -14.9 + 0.93359x_1 + 0.99847x_2 + 5.3844x_3$$

$$y' = -14.9 + 0.93359(8) + 0.99847(34) + 5.3844(11)$$

$$y' = 85.75 \text{ (grade) or } 86$$

10.

$$y' = 97.7 + 0.691(35) + 219(194) - 299(142)$$

$$y' = 149.885 \approx 150$$

11.

$R$  is a measure of the strength of the relationship between the dependent variables and all the independent variables.

12.

0 to +1

13.

$R^2$  is the coefficient of multiple determination.  $R^2_{\text{adj}}$  is adjusted for sample size and the number of predictors.

14.

$$H_0: \rho = 0 \text{ and } H_1: \rho \neq 0$$

15.

The  $F$  test is used to test the significance of  $R$ .

16.

The adjusted  $R^2$  is the adjusted coefficient of multiple determination. It is computed when sample size is small and is a better estimate since  $R^2$  is larger when sample size is small. ( $n < k$ )

### REVIEW EXERCISES - CHAPTER 10

1.



$$\sum x = 31$$

$$\sum y = 383$$

$$\sum x^2 = 211$$

$$\sum y^2 = 30,543$$

$$\sum xy = 2460$$

$$n = 6$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{6(2460) - (31)(383)}{\sqrt{[6(211) - (31)^2][6(30,543) - (383)^2]}}$$

$$r = 0.864$$

## Chapter 10 - Correlation and Regression

1. continued

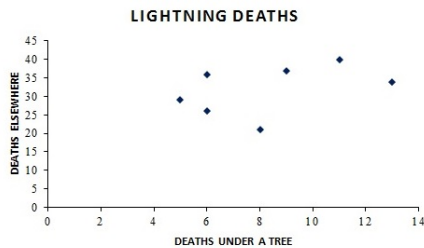
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.917 \text{ at } \alpha = 0.01 \quad d. f. = 4$$

Decision: Do not reject. There is no significant linear relationship between customer satisfaction and the amount customers spend. No regression should be done since  $r$  is not significant.

2.



$$\sum x = 58$$

$$\sum y = 223$$

$$\sum x^2 = 532$$

$$\sum y^2 = 7379$$

$$\sum xy = 1900$$

$$n = 7$$

$$r = \frac{7(1900) - (58)(223)}{\sqrt{[7(532) - 58^2][7(7379) - 223^2]}}$$

$$r = 0.440$$

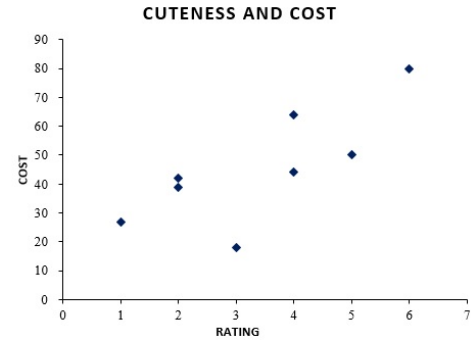
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.875 \quad d. f. = 5$$

Decision: Do not reject. There is not enough evidence to say that a significant linear relationship exists between the number of lightning deaths occurring under a tree and the number of lightning deaths occurring in other locations. Since  $r$  is not significant, no regression should be done. Use  $\bar{y} = 31.9$  as a predicted value for any  $x$ .

3.



$$\sum x = 27$$

$$\sum y = 364$$

$$\sum x^2 = 111$$

$$\sum y^2 = 19,270$$

$$\sum xy = 1405$$

$$n = 8$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{8(1405) - (27)(364)}{\sqrt{[8(111) - (27)^2][8(19,270) - (364)^2]}}$$

$$r = 0.761$$

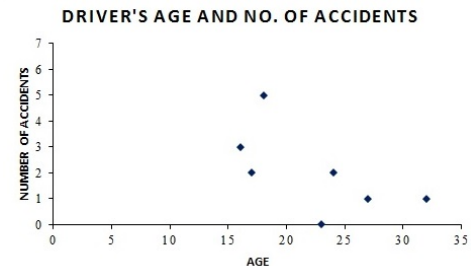
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.834 \quad d. f. = 6$$

Decision: Do not reject. There is no significant linear relationship between the cuteness of a puppy and its cost. No regression should be done since  $r$  is not significant.

4.



## Chapter 10 - Correlation and Regression

4. continued

$$\begin{aligned}\sum x &= 157 \\ \sum y &= 14 \\ \sum x^2 &= 3727 \\ \sum y^2 &= 44 \\ \sum xy &= 279 \\ n &= 7\end{aligned}$$

$$r = \frac{7(279) - (157)(14)}{\sqrt{[7(3727) - (157)^2][7(44) - (14)^2]}}$$

$$r = -0.610$$

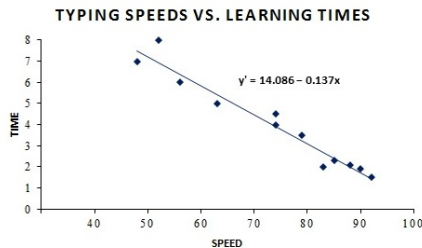
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.875 \quad d. f. = 5$$

Decision: Do not reject. There is not a significant relationship between age and the number of accidents a person has. No regression analysis should be done since the null hypothesis has not been rejected.

5.



$$\begin{aligned}\sum x &= 884 \\ \sum y &= 47.8 \\ \sum x^2 &= 67,728 \\ \sum y^2 &= 242.06 \\ \sum xy &= 3163.8 \\ n &= 12\end{aligned}$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{12(3163.8) - (884)(47.8)}{\sqrt{[12(67728) - (884)^2][12(242.06) - (47.8)^2]}}$$

$$r = -0.974$$

5. continued

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.708 \quad d. f. = 10$$

Decision: Reject. There is a significant relationship between speed and time.

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2}$$

$$a = \frac{(47.8)(67728) - (884)(3163.8)}{12(67728) - (884)^2}$$

$$a = 14.086$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2}$$

$$b = \frac{12(3163.8) - (884)(47.8)}{12(67728) - (884)^2}$$

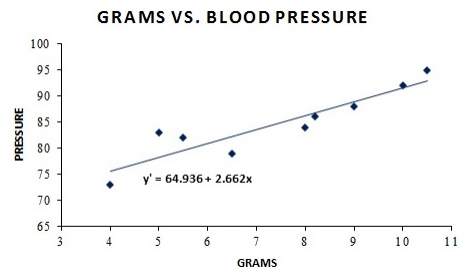
$$b = -0.137$$

$$y' = a + bx$$

$$y' = 14.086 - 0.137x$$

$$y' = 14.086 - 0.137(72) = 4.2 \text{ hours}$$

6.



$$\begin{aligned}\sum x &= 66.7 \\ \sum y &= 762 \\ \sum x^2 &= 535.99 \\ \sum y^2 &= 64,868 \\ \sum xy &= 5758.2 \\ n &= 9\end{aligned}$$

$$r = \frac{9(5758.2) - (66.7)(762)}{\sqrt{[9(535.99) - (66.7)^2][9(64868) - (762)^2]}}$$

$$r = 0.916$$

## Chapter 10 - Correlation and Regression

6. continued

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.798 \quad d. f. = 7$$

Decision: Reject. There is a significant relationship between grams and pressure.

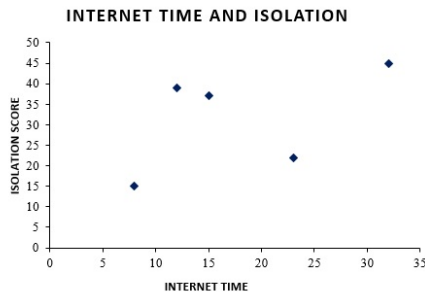
$$a = \frac{(762)(535.99) - (66.7)(5758.2)}{9(535.99) - 66.7^2} = 64.936$$

$$b = \frac{9(5758.2) - (66.7)(762)}{9(535.99) - 66.7^2} = 2.662$$

$$y' = 64.936 + 2.662x$$

$$y' = 64.936 + 2.662(8) = 86.23$$

7.



$$\sum x = 90$$

$$\sum y = 158$$

$$\sum x^2 = 1986$$

$$\sum y^2 = 5624$$

$$\sum xy = 3089$$

$$n = 5$$

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$

$$r = \frac{5(3089) - (90)(158)}{\sqrt{[5(1986) - (90)^2][5(5624) - (158)^2]}}$$

$$r = 0.510$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.959 \quad d. f. = 3$$

7. continued

Decision: Do not reject. There is no significant linear relationship between internet use and isolation. No regression should be done since  $r$  is not significant.

8.

(For calculation purposes only, since no regression should be done.)

$$S_{est} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}}$$

$$S_{est} = \sqrt{\frac{44 - (5.816)(14) - (-0.1701)(279)}{7-2}}$$

$$S_{est} = \sqrt{\frac{10.0339}{5}} = \sqrt{2.00678} = 1.417$$

9.

$$S_{est} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}}$$

$$S_{est} = \sqrt{\frac{242.06 - 14.086(47.8) + 0.137(3163.8)}{12-2}}$$

$$S_{est} = \sqrt{\frac{2.1898}{10}} = \sqrt{0.21898} = 0.468$$

(Note: TI-83 calculator answer is 0.513)

10.

$$S_{est} = \sqrt{\frac{\sum y^2 - a\sum y - b\sum xy}{n-2}}$$

$$S_{est} = \sqrt{\frac{64,868 - (64.936)(762) - (2.662)(5758.2)}{9-2}}$$

$$S_{est} = \sqrt{\frac{64,868 - 49,481.232 - 15,328.328}{7}}$$

$$S_{est} = \sqrt{\frac{58.44}{7}} = \sqrt{8.349} = 2.89$$

(Note: TI-83 calculator answer is 2.845)

11.

(For calculation purposes only, since no regression should be done.)

$$y' = 14.086 - 0.137x$$

$$y' = 14.086 - 0.137(72) = 4.222$$

## Chapter 10 - Correlation and Regression

11. continued

$$y' - t_{\frac{\alpha}{2}} \cdot S_{\text{est}} \sqrt{1 + \frac{1}{n} + \frac{n(\bar{x} - \bar{X})^2}{n\sum x^2 - (\sum x)^2}} < y <$$

$$y' + t_{\frac{\alpha}{2}} \cdot S_{\text{est}} \sqrt{1 + \frac{1}{n} + \frac{n(\bar{x} - \bar{X})^2}{n\sum x^2 - (\sum x)^2}}$$

$$4.222 - 1.812(0.468) \sqrt{1 + \frac{1}{12} + \frac{12(72 - 73.667)^2}{12(67,728) - 884^2}}$$

$$< y < 4.222 + 1.812(0.468) \sqrt{1 + \frac{1}{12} + \frac{12(72 - 73.667)^2}{12(67,728) - 884^2}}$$

$$4.222 - 1.812(0.468)(1.041) < y < 4.222 + 1.812(0.468)(1.041)$$

$$3.34 < y < 5.10$$

12.

$$y' = 64.936 + 2.662x$$

$$y' = 64.936 + 2.662(8) = 86.232$$

$$86.232 - 2.365(2.89) \sqrt{1 + \frac{1}{9} + \frac{9(8 - 7.411)^2}{9(535.99) - 66.7^2}}$$

$$< y < 86.232 + 2.365(2.89) \sqrt{1 + \frac{1}{9} + \frac{9(8 - 7.411)^2}{9(535.99) - 66.7^2}}$$

$$86.232 - 2.365(2.89)(1.058) < y < 86.232 + 2.365(2.89)(1.058)$$

$$79.0 < y < 93.5$$

13.

$$y' = 12.8 + 2.09X_1 + 0.423X_2$$

$$y' = 12.8 + 2.09(4) + 0.423(2) = 22.006 \text{ or } 22.01$$

14.

$$R = \sqrt{\frac{(0.681)^2 + (0.872)^2 - 2(0.681)(0.872)(0.746)}{1 - (0.746)^2}}$$

$$R = \sqrt{0.7624799} = 0.873$$

15.

$$R_{\text{adj}}^2 = 1 - \left[ \frac{(1 - R^2)(n - 1)}{n - k - 1} \right]$$

$$R_{\text{adj}}^2 = 1 - \left[ \frac{(1 - 0.873^2)(10 - 1)}{10 - 3 - 1} \right]$$

$$R_{\text{adj}}^2 = 1 - \left[ \frac{2.1408}{6} \right] = 0.643$$

## CHAPTER 10 QUIZ

1. False, the y variable would decrease.

2. True

3. True

4. False, the relationship may be affected by another variable, or by chance.

5. False, a relationship may be caused by chance.

6. False, there are several independent variables and one dependent variable.

7. a

8. a

9. d

10. c

11. b

12. Scatter plot

13. Independent

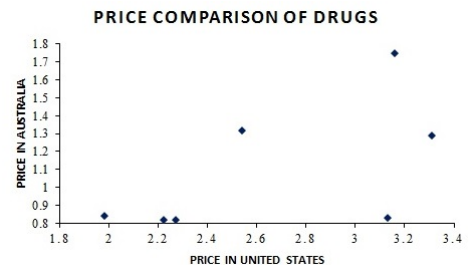
14. 1, + 1

15. b (slope)

16. Line of best fit

17. + 1, - 1

18.



## Chapter 10 - Correlation and Regression

18. continued

$$\begin{aligned}\sum x &= 18.61 \\ \sum x^2 &= 51.1919 \\ \sum y &= 7.67 \\ \sum y^2 &= 9.2083 \\ \sum xy &= 21.0956 \\ n &= 7\end{aligned}$$

$$r = 0.600$$

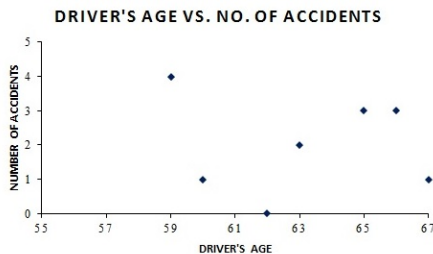
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C.V. = \pm 0.754 \quad d.f. = 5$$

Do not reject. There is no significant linear relationship between the price of the same drugs in the United States and in Australia. No regression should be done.

19.



$$\begin{aligned}\sum x &= 442 \\ \sum x^2 &= 27,964 \\ \sum y &= 14 \\ \sum y^2 &= 40 \\ \sum xy &= 882 \\ n &= 7\end{aligned}$$

$$r = -0.078$$

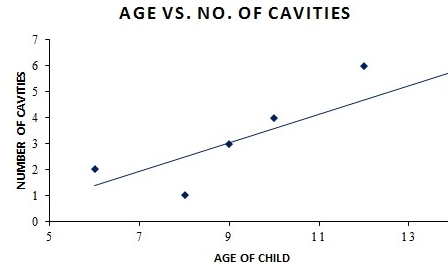
$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C.V. = \pm 0.754 \quad d.f. = 5$$

Decision: Do not reject. There is not a significant relationship between age and number of accidents. No regression should be done.

20.



$$\begin{aligned}\sum x &= 59 \\ \sum x^2 &= 621 \\ \sum y &= 21 \\ \sum y^2 &= 91 \\ \sum xy &= 229 \\ n &= 6\end{aligned}$$

$$r = 0.842$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C.V. = \pm 0.811 \quad d.f. = 4$$

Decision: Reject. There is a significant linear relationship between age and number of cavities.

$$a = -1.918 \quad b = 0.551$$

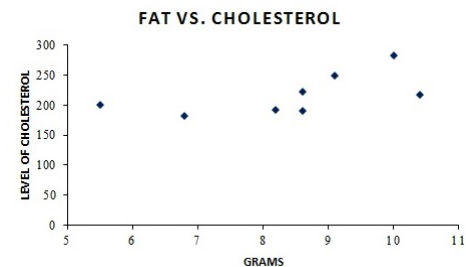
$$y' = -1.918 + 0.551x$$

When  $x = 11$ :

$$y' = -1.918 + 0.551(11)$$

$$y' = 4.14 \text{ or } 4 \text{ cavities}$$

21.



$$\begin{aligned}\sum x &= 67.2 \\ \sum x^2 &= 582.62 \\ \sum y &= 1740 \\ \sum y^2 &= 386,636\end{aligned}$$

## Chapter 10 - Correlation and Regression

21. continued

$$\sum xy = 14847.9$$

$$n = 8$$

$$r = 0.602$$

$$H_0: \rho = 0$$

$$H_1: \rho \neq 0$$

$$C. V. = \pm 0.707 \quad d. f. = 6$$

Decision: Do not reject. There is no significant linear relationship between fat and cholesterol. No regression should be done.

22.

$$S_{est} = \sqrt{\frac{91 - (-1.918)(21) - 0.551(229)}{6-2}}$$

$$S_{est} = 1.129^*$$

23.

(For calculation purposes only, since no regression should be done.)

$$S_{est} = \sqrt{\frac{386,636 - 110.12(1740) - 12.784(14,847.9)}{8-2}}$$

$$S_{est} = 29.47^*$$

24.

$$y' = -1.918 + 0.551(7) = 1.939 \text{ or } 2$$

$$2 - 2.132(1.129)\sqrt{1 + \frac{1}{6} + \frac{6(11-9.833)^2}{6(621)-59^2}} < y$$

$$< 2 + 2.132(1.129)\sqrt{1 + \frac{1}{6} + \frac{6(11-9.833)^2}{6(621)-59^2}}$$

$$2 - 2.132(1.129)(1.095) < y < 2 + 2.132(1.129)(1.095)$$

$$-0.6 < y < 4.6 \text{ or } 0 < y < 5^*$$

25.

Since no regression should be done, the average of the  $y'$  values is used:  $\bar{y} = 217.5$

26.

$$y' = 98.7 + 3.82(3) + 6.51(1.5) = 119.9^*$$

27.

$$R = \sqrt{\frac{(0.561)^2 + (0.714)^2 - 2(0.561)(0.714)(0.625)}{1 - (0.625)^2}}$$

$$R = 0.729^*$$

28.

$$R_{adj}^2 = 1 - \left[ \frac{(1 - 0.774^2)(8-1)}{(8-2-1)} \right]$$

$$R_{adj}^2 = 0.439^*$$

\*These answers may vary due to method of calculation and/or rounding.