

The evolution of reciprocal sharing networks and norms under risk and uncertainty

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Abstract

We model norms that prescribe the sharing of resources under environmental stochasticity and show that foragers with pessimistic risk attitudes maximize fitness by sharing with each other at a cost under reciprocal sharing norms that enable stable growth in the long run. The resulting risk-pooling networks will be at their most stable when foragers maintain a similar number of partners to the rest of their associates and when individuals preferentially assort with others in similar socioeconomic conditions. The optimal network size and the limits to network growth are given by relationships between the costs of maintaining connections, the stake/package size and the variability in resource production, while optimal sharing norms depend on benefit package size and sharing cluster density. Our results are compatible with

1 Introduction

When we ask ourselves how cooperative groups form and maintain themselves, we are engaging in a twofold problem. First, it is necessary to ask what sort of incentive structures lead people to join groups and adopt their norms over the other options provided by their environments and personal capacities. At the center of this concern is the question: why join a group if I could be on my own? Second, it is important to consider the psychology of decision-makers and how it interacts with the aforementioned incentive structures. Rational decision-making posits that agents choose strategies such that they maximize their expected utility. However, decisions do not happen in a vacuum, and decision-makers might factor their uncertainty about the world into their choices in order to deal with possibilities like unknown risks or incomplete knowledge about the consequences of choices. To an observer expecting rational action, this might seem like an irrational or inefficient use of the information at hand. However, from an evolutionary point of view, being pessimistic in the face of environmental uncertainty can be an ecologically rational course of action.

In this work we model the emergence of risk-pooling networks under reciprocal sharing norms. Agents (be they individuals or households) enter pairwise contracts with other agents that prescribe the sharing of benefits earned through each agent’s individual exploitation of their environment, within the context of a wider community where the proportion of shared resources in dyadic interactions is an agreed-upon norm. Sharing and reciprocity are ubiquitous in human societies, and often continuing relationships (like kinship ties and friendships) are maintained by stable benefit transfers like the sharing of workloads, reciprocal gift-giving and resource transfers, all practices which tend to develop in either explicit or implicit normative backgrounds that aid in setting the individual expectations needed to facilitate trust and coordination. In stochastic environments, reciprocal sharing enables tapping into other individuals’ securities in ways that lead to variance-reduction in expected returns (risk-pooling), while possibly requiring the payment of costs of interaction maintenance that cut down on the direct benefits earned by individual agents during each time period.

Previous work on the risk-mitigating benefits of reciprocal sharing has concentrated on showing how sharing behaviors can propagate and persist in different scenarios through reduction of payoff variance (McCloskey, 1991; Winterhalder, 1986; Alderman & Paxson, 1994; Blumenstock, Fafchamps, & Eagle, 2011). However, the way in which costs of maintaining reciprocal sharing ties constrain the formation of dyadic and/or community-wide arrangements has not been explored in depth. We think this is important, as reciprocity and resource-sharing has been extensively documented in societies around the world (CITE REVIEW), but sharing arrangements everywhere are constrained to clusters of individuals instead of encompassing whole populations. While the reasons for this are likely several (and are also likely to interact with one other in non-trivial ways), here we strive to show how considering per-connection costs can make risk-pooling models more realistic in their predicted outcomes. In this vein, we aim to explore the following questions:

1. What kind of network properties and assortment preferences afford the emergence of normative risk-pooling relationships?
2. From an individual’s point of view, how large and/or dense can sharing networks grow before losing their risk-pooling benefits?
3. What are the optimal sharing norms to adopt at the group level? What sort of dynamic group processes can lead to the evolution of these sharing norms?

We attempt to answer these questions by building a model in which agents perform some productive activity with an uncertain return (i.e. foraging), consisting of either a baseline payoff (the low-value outcome) or a surplus payoff (the high-value outcome) with an associated risk and success rate, respectively. Loners are agents that embark on this productive effort alone, while Sharers pay a per-connection cost in order to cluster in reciprocal sharing networks. By

using agreed-upon sharing norms that stipulate what proportion of the surplus payoff Sharers break up and distribute among their network peers (when they are successful in procuring it), clusters of Sharers can mitigate environmental risk by potentially reducing the variance of their returns. Community pooling is a special case of this setup, where networks are cliques and individuals share out their whole surplus payoff when successful.

Successful risk-pooling networks grow as they attract more individuals. As they do, focal Sharers must “keep up” with the increase in density of their personal networks by increasing their number of connections. If they don’t, they run the risk of getting less shared benefits from peers who do choose to keep up and accumulate more connections relative to the focal agent. This means Sharers have to be sensitive to the average number of connections in their personal network, and increase their number of connections accordingly, paying additional costs for every new connection. As they do so, they reap higher benefits from variability-reduction in resource returns, but the increase in this benefit diminishes with every acquired connection. Assuming groups admit free entry, this can lead to a runaway increase in network density, resulting in average payoffs that first increase, then peak, and eventually decay. Once the payoffs become close to the payoff of a Loner, we assume individuals abandon them (either alone or as a group through a fission event, where some individuals choose to abandon the community and form their own, smaller, cluster of Sharers). The resulting flow of individuals through clusters and the formation of “offspring” communities can therefore allow individuals to reap the benefits of membership to non-over-crowded sharing clusters. Additionally, we can assume that fission events introduce variability in the adopted sharing norms of the offspring clusters (perhaps the members wish to distinguish themselves from the previous group by innovating on top of the previous norm). The process then repeats, and this iterated dynamic of growth, migration and fission leads to the population-level evolution of sharing cluster densities and sharing norms. An illustration of this proposed process can be appreciated in Figure 1.

Throughout this paper, we use the term “cluster” to refer to networked communities of individuals under the same sharing norm, while we reserve the term network to refer to the personal/ego networks of particular individuals, which are embedded in the aforementioned clusters. With cluster density we refer to a cluster’s mean degree, which we also call cluster size. This is not to be confused with the number of cluster members, as this is a measure that only arises in the case of full community pooling on a clique, where it converges to our definition of cluster/network density.

By using game-theoretic methods from evolutionary biology and economics, we can explore the conditions for which Sharers achieve better results than Loners, allowing us to analytically derive approximate conditions for the maximum density of a sharing cluster (the density at which the average payoff of cluster Sharers equals the payoff of a Loner) given per-connection costs, stake/package sizes and environmental variability. We also find approximate expressions for the optimal cluster density and sharing norms, motivating a discussion on what sorts of dynamics can lead towards or away from optimality in network sizes, as

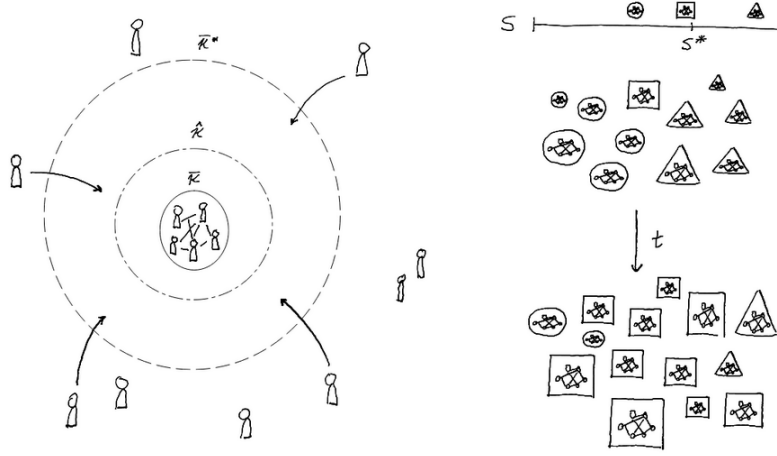


Figure 1: Schematic illustration of the sharing game and the evolution of sharing norms through the growth and propagation of sharing clusters. Agents form new clusters or choose to join preexisting ones, leading to cluster growth, where sharing norms close to optimal allow clusters to grow at an advantage to same-sized clusters with less optimal norms.

well as what sort of processes can lead to sharing norm evolution at the group level. These approximations are then compared to simulated payoffs to show they are appropriate. Overall, we find that

1. Environmental stochasticity itself is not a sufficient condition for the emergence of sharing networks. Significant variability in expected returns is necessary for agents to perceive benefits from costly networked risk-pooling. Sharing collapses at both extremes of the risk spectrum: when environments are too risky or when they are too secure.
2. Optimal network size and maximum network size are affected by risk in distinct ways. If agents optimize number of sharing partners in a stable network, then the largest sharing clusters will emerge in riskier environments. If agents let their sharing networks grow indiscriminately, until the payoff to a cluster agent equals the payoff of a lone individual, then the largest sharing clusters will emerge in more secure environments, mediated by the stake size.
3. If agreement on a sharing norm is assumed to be a requirement for stable sharing cooperation, then stable reciprocal sharing clusters should minimize variance in network degree. In other words, inequality in number of sharing partners within a sharing network might decrease its stability, as optimal sharing norms for a particular agent depend on their position in the network hierarchy (e.g. how their network degree compares to

their personal network’s average degree). Individuals who are “lower” in the degree hierarchy also have lower associated maximum mean degrees, choosing to abandon networks

4. In the presence of heterogeneity in individual success rates and stakes, agents are likely to cluster with others in their same productive “stratum”. This is because there is an advantage to “sharing-up” with individuals of a higher stratum and a disadvantage to “sharing-down”, when reciprocal sharing is used for risk-pooling. Therefore, if individuals sort themselves into groups that cannot grow larger than the whole population, the resulting assortment will display homophily in individual productive capacities. This form of assortment, where agents pay close attention to the conditions of their partners, can also be a way to mitigate the success of defecting/parasitic strategies through credible threats of defection. We note that there are cases where sharing between strata might be stabilized, although we do not think these arrangements are likely to emerge in a purely risk-pooling context. In this way, reciprocal sharing can actually tap preexisting inequalities in a population of Sharers, and possibly feed back into these inequalities, exacerbating them by making the agents with better opportunities also the better insured ones.
5. In groups with low inequality in network degree, optimal sharing norms depend on the average size of the sharing cluster and its interaction with the size of the stake/resource package with respect to the baseline stake. At small network sizes, sharing norms can be significant but still far from full, as stake size increases. However, as networks grow larger/denser, the optimal sharing norm becomes more dependent on the size of stakes with respect to the baseline, converging to full sharing as the former increases.

We build the argument progressively, first reviewing relevant literature on risk, insurance and reciprocal sharing to motivate the model. After that, we explore the characteristics of sharing networks predicted by the model and discuss some of their potential real-life analogues. Finally, we summarize the conclusions and predictions from the model analysis, and we review their utility for thinking about how human groups form, fission, propagate and evolve functional strategic cultural norms.

2 Risk and reciprocity in theory and across societies

IGNORE, GOTTA EXPAND

Cooperation is aided by the presence of risk and uncertainty, and it often involves reciprocity relations (Offer, 1997; Andras, Lazarus, & Roberts, 2007; Buston & Balshine, 2007; Delton, Krasnow, Cosmides, & Tooby, 2011). Aversion to risk can be strategic when faced with stochastic environments (Price & Jones, 2020), and it can lead individuals to adopt strategies that minimize

variance at the expense of expected payoff, favoring the emergence of cooperative strategies. An extensive literature in economics studies the presence of risk-sharing through income sharing and/or consumption smoothing in low-income contexts lacking formal insurance markets (Alderman & Paxson, 1994; R. M. Townsend, 1994; Udry, 1994; Rosenzweig, 1988) as well as in the face of correlated regional risks (Blumenstock et al., 2011) and within family coalitions in urban contexts (Lee, Parish, & Willis, 1994).

As a complement, but also in contrast to this literature, a behavioral ecology perspective treats consumers as foragers interacting with an uncertain socio-ecological environment (Hantula, 2012). There are various treatments of sharing and risk that expose the risk-pooling benefits of reciprocal sharing in foraging contexts (Winterhalder, 1986, 1993; Smith & Boyd, 2019; Kaplan, Gurven, Hill, Hurtado, et al., 2005), as well as recent empirical analyses testing the implications of the resulting models (Ember, Skogsgard, Ringen, & Farrer, 2018). More broadly, the reduction in variance afforded by sharing can be crucial in scenarios that can be usefully described by models of multiplicative growth (Peters & Adamou, 2022).

In small-scale societies of hunter-gatherers, horticulturalists and pastoralists, risk-pooling through reciprocal sharing has been extensively documented by anthropologists and has shown to be mediated by environmental conditions, institutional structures (such as kinship networks and social norms about equality) and enduring relationships in the context of reciprocal altruism (Wiessner et al., 1982; Cashdan, 1985; R. B. Bird, Bird, Smith, & Kushnick, 2002; Nolin, 2010; Hames & McCabe, 2007; Koster, Leckie, Miller, & Hames, 2015; Gerkey, 2013; Ziker & Schnegg, 2005; Hames, 2017; Ready, 2018). This does not mean that all reciprocal sharing is done for risk-pooling reasons. In conditions where agents are not in need of the insurance provided by sharing strategies, reciprocal gift-giving can still be used as a way to maintain social relationships, mediate conflicts and/or acquire status and social capital (Offer, 1997; Platteau, 1997; Kent, 1993; Wiessner et al., 1982; Wu & Ma, 2017).

At the group level, reciprocal sharing can serve egalitarian ends (Woodburn, 1982; Dowling, 1968; Aspelin, 1979), but it can also impose pressures on the scale that a sharing group can achieve. Destabilization related to scale has been examined through theoretical models (Boyd & Richerson, 1988) and through ethnographic accounts of the relationship of sharing behavior with village scale and group fission (Ramos, 1995; Chagnon, 2012). Reciprocal sharing patterns can also track differences in social status and access to resources, such that agents in advantageous positions can use reciprocal sharing norms in their favour, in order to attain better positions on a social hierarchy (Sahlins, 1963; Betzig, 1988; R. L. B. Bird & Bird, 1997).

3 The model

3.1 The setup

Parameter List		
Name	Symbol	Range
Baseline payoff	\mathbf{b}	$(1, \infty)$
Surplus payoff	\mathbf{B}	$(1, \infty)$
Stake size	$\mathbf{B} - \mathbf{b}$	$(0, \infty)$
Per-connection cost	\mathbf{C}	$(0, \infty)$
Rate of success	\mathbf{u}	$(0, 1)$
Risk	$\mathbf{1} - \mathbf{u}$	$(0, 1)$
Sharing norm	\mathbf{s}	$(0, 1)$
Agent Degree	\mathbf{k}_i	$\mathbb{N}_{\geq 0}^{\leq N-1}$
Agent's network mean degree	$\bar{\mathbf{k}}_i$	$(0, N - 1]$
Risk preference	δ	$(0, \infty)$
Population size	\mathbf{N}	$\mathbb{N}_{>0}$

We assume a population of N agents who perform some productive activity every season, for a total of T time periods. For each time period, the agent can get a baseline payoff of b with a probability $1 - u$, which we denominate risk, or a surplus payoff $B > b$ with probability u , which we denominate security. For simplicity, we assume $b = 1$.

We also assume agents invest their payoffs into their (cultural) fitness, with baseline fitness w_0 . At the end of every productive season of T time periods, a number of agents will choose to imitate the strategies of others in the population, with the probability of agent i for being chosen as a model given by $\phi_i = \frac{w_i}{\sum_j^N w_j}$. Assuming T is large enough, then the measure of replication success of agent i 's strategy is their geometric mean fitness $w_i = w_0 e^{G_i}$, where $G_i = \mathbb{E}(\log V_i)$ is the expected growth rate of agent i and V_i is a random variable representing the outcome of the productive effort of agent i in a single time period. By assuming w_0 is constant across agents in the population, we can use G_i , the logarithm of the geometric mean fitness, as the relevant measure of success of agent i 's

strategy. Thus, an agent who works alone (Loner) and reaps their benefits alone will have an average growth rate

$$G(\text{Loner}) = u \log(B) + (1 - u) \log(1) = u \log(B) \quad (1)$$

Assume now that $n \leq N$ agents come together into a networked cluster, where a connection between a pair of agents has a maintenance cost C . The cluster members agree on a sharing norm s , a proportion of their payoff that any successful agent (one who successfully obtained B in the time period in question) shares with their connections by breaking it up equally among them. Every cluster agent i thus receives $\frac{sB}{k_{i,j}}$ from every one of their successful connections j , regardless of their success or failure in the time period in question, where $k_{i,j}$ is the degree (number of connections) of agent i 's peer j . Each agent then invests the total payoff from this process into their cultural fitness. We call this strategy Sharer. Sharers pay a cost to connect with each other, but they enjoy the benefits brought forth by variance reduction in payoffs. We approximate the average growth rate of a Sharer i as

$$G_i(\text{Sharer}) = u \log \{(1 - s)B + S_i\} + (1 - u) \log \{1 + S_i\} \quad (2)$$

with S_i the (average) received shared benefits,

$$S_i = \mathbb{E} \left(\sum_{j \neq i}^n a_{ij} \epsilon_j \frac{sB}{k_j} \right) - k_i C \quad (3)$$

where a_{ij} equals 1 if agents i and j share a connection and 0 otherwise, and ϵ_j equals 1 if agent j is successful (and thus has something to share) and 0 whenever they aren't. In order to make this expression simpler, we assume that each $k_{i,j} \approx \bar{k}_i$. This means we assume that the degree of cluster members agent i is connected to can be meaningfully approximated by a *network mean degree* \bar{k}_i , which represents the mean degree of agent i 's ego network. This allows us to treat the first expression in S_i as the expectation of a binomial distribution. However, doing so implicitly assumes that k_i is large (agent i has many connections). To circumvent this, we over-weight this expectation by a term ρ , in order to account for effects like correlation in success rates among Sharers and higher probability of network failure at small network sizes:

$$\begin{aligned} S_i &\approx \mathbb{E} \left(\sum_{j \neq i}^n a_{ij} \epsilon_j \right) \frac{sB}{\bar{k}_i} - k_i C \\ &\approx \rho u k_i \frac{sB}{\bar{k}_i} - k_i C \end{aligned}$$

Finally, we define the *relative degree* of agent i as $\beta_i = \frac{k_i}{\bar{k}_i}$. This measures the degree of agent i with respect to the mean degree of their ego network, allowing us to examine the consequences of agents being above or below the average degree that their network peers display.

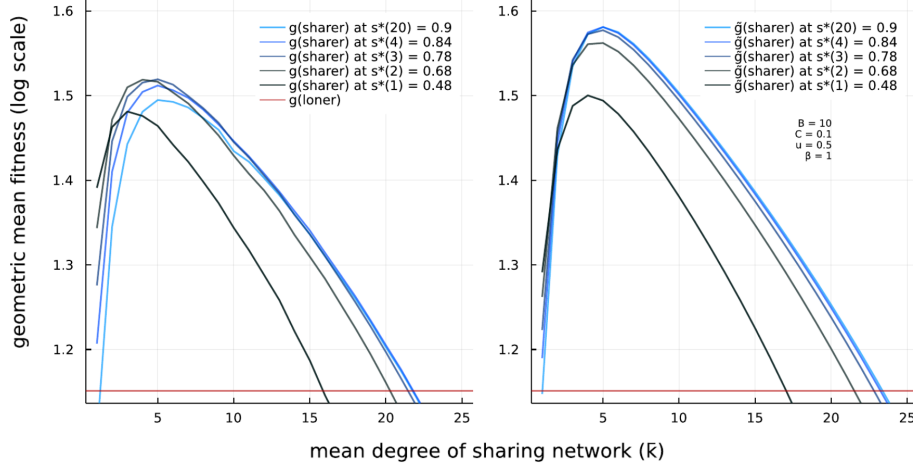


Figure 2: Simulated growth rate trajectories with increasing mean degree (left) versus trajectories from the approximate model (right). The approximation overestimates the growth rates as well as the optimal sharing norms, but remains an informative model.

$$S_i \approx \beta_i \left(\underbrace{\rho u s B}_{\text{average received shared benefit}} - \underbrace{\bar{k}_i C}_{\text{average network cost}} \right) \quad (4)$$

Work on resource pooling that uses where variance is explicitly considered have shown that the benefits from reduction of volatility increase as $\frac{1}{n}$, where n is the number of individuals in a resource-sharing pool (McCloskey, 1991; Winterhalder, 1986; Peters & Adamou, 2022). Hence, assuming cluster members strive to keep their relative degree (β) fixed, there is a diminishing return in volatility reduction for every new cluster member that joins, which is here represented through the effect of ρ . This means that when costs are absent clusters can grow indefinitely and still enjoy an additional benefit, however small, for each new member, as the addition of new cluster members means older cluster agents have to increase their number of connections in order to avoid “falling behind” their network’s average number of connections. But in the presence of connection costs, the diminishing returns from variance reduction eventually become negligible in the face of increasing total costs. The net effect of this is that a cluster’s *growth horizon* (the point where the cluster’s average growth rate meets the average growth rate of a Loner) will be determined by what happens to the average payoff as the network’s average degree grows. By relying on the (ρ -weighted) average shared benefits received from connections, we disregard the full effects of fluctuations on these received shared benefits, leading to a slight overestimate of growth rates with respect to explicitly simulated payoffs. This can be appreciated in Figure 2.

3.2 Model analysis

We can now ask questions from this model. Of particular interest is finding the aforementioned growth horizon, which is equivalent to asking when does the growth rate of the average Sharer’s payoff exceed that of the average Loner in terms of \bar{k} , given some values for B , C , u , β and s . We concentrate on the case of an agent who “keeps up” with the overall degree increases of their network partners. This means that if the mean degree \bar{k}_i of the focal agent’s network grows, the agent always maintains a degree $\beta_i \bar{k}_i$. This leaves us with the condition

$$G_i(\text{Sharer}) > G(\text{Loner}) \quad (5)$$

We can think of a growing cluster of Sharers as a trajectory of the average Sharer’s payoff as the mean degree of their ego network, \bar{k}_i , grows. Simulating this dynamic explicitly in a clique of Sharers and plotting growth rate as a function of mean degree shows this trajectory: there is an initial gain in fitness as the network recruits more members through its fitness advantage over lone production, followed by a peak (which we denominate \bar{k}_i^*) and then a continued decrease until the average Sharer’s fitness intersects the fitness of a Loner. This point of intersection, which we have also called the growth horizon and we identify with the symbol \bar{k}_i^* , is the point at which an agent will be receiving a payoff equal to a Loner, given that their ego network has exhibits a particular mean degree. If the cluster is particularly regular, so that most individuals are experiencing the same network conditions, then this is the value for mean degree for which a cluster would not be able to sustain any more recruitment, as an increase in the cluster’s average degree beyond this point would lead to a lower payoff than that of a Loner. By solving (6) for \bar{k}_i , we can find this “maximum density” of sharing networks. While, at the group level, this applies to any network with high degree regularity in its members, it is useful to think about it through the case of cliques, where $\bar{k} = n - 1$, and n is the size of the clique/cluster. In this case, \bar{k}^* represents the maximum size a sharing clique can attain before it becomes too “congested”: in approximate terms, this is when there are so many members that the average shared benefit (usB) is cancelled out by the total network cost ($\bar{k}C = (n - 1)C$).

3.2.1 A rank-dependent representation

We can gain further insight into \bar{k}_i^* by translating it to the scenario of Rank-Dependent Expected Utility Theory (henceforth RDEUT), where an explicit expression for it can be obtained at the cost of introducing an additional parameter representing a decision-maker’s risk preference. In a decision-making scenario where decision-makers can over- or under-weight the probabilities of ranked outcomes, pessimism (under-weighting of the probabilities of the outcomes ranked as better) can approximate the solution to an evolutionary dynamic under environmental stochasticity (Price & Jones, 2020).

We assume a linear utility function and a probability weighting function w in order to re-write our payoff expressions. While there are several forms that w can take, we use a simple form $w(p) = p^\delta$, called a power weighting function, where δ is a parameter representing an agent's risk preference. When $\delta = 1$ the agent is risk-neutral, while for $\delta > 1$ they display increasing pessimism, over-weighting the probability of the least desired outcomes, and for $0 < \delta < 1$ they are optimistic, over-weighting the probability of the more desired outcomes. Probability weighting can happen when a decision-maker's model of an outcome's distribution differs from the "objective" distribution of that outcome (Peters & Adamou, 2022). For example, a decision-making agent that assumes there is additional uncertainty about the possible benefits than what the current available information suggests can be represented with a decision-making model incorporating probability weighting. In the current framework, this just means that an agent that receives information about available benefits and their associated probabilities proceeds to rank the possible outcomes and then assume some outcomes are more or less likely than others based on their rank. Pessimism shifts probability mass towards the lower ranks, while optimism does the opposite. Risk neutrality performs an "unbiased" weighted average of the outcomes.

3.2.2 Deriving the growth horizon

The rank-dependent utility representation of both Loner and Sharer approximate payoffs becomes

$$V(\text{Loner}) = u^\delta B + (1 - u^\delta) \quad (6)$$

$$V_i(\text{Sharer}) = u^\delta \{(1 - s)B + \beta S_i\} + (1 - u^\delta) \{1 + \beta S_i\} \quad (7)$$

from which we can solve $V_i(\text{Sharer}) = V(\text{Loner})$ for the growth horizon, yielding

$$\bar{k}_i^* = \frac{\frac{u(\beta_i - u^{\delta-1})sB}{C} - \frac{W(\Sigma)}{\log(1-u)}}{\beta_i} \quad (8)$$

with

$$\Sigma = \frac{u(1-u) \frac{u(\beta_i - u^{\delta-1})sB}{C} s\beta_i B \log(1-u)}{C} \quad (9)$$

and W the Lambert W function, also known as the product logarithm. Finally, whenever $\bar{k}^* < 0$ we set $\bar{k}^* = 0$. While at plain sight this expression might seem complicated, we note that the term involving the product logarithm serves as a delimiter of the range of parameter values for which sharing is possible. In other words, whenever this function is not 0, it makes sharing collapse. Therefore, it draws the boundaries of the region of parameter space that allows for sharing to emerge, and it is negligible otherwise. If we assume that we are

within the region of parameter space that allows for sharing to happen, then $W(\Sigma) \approx 0$ and

$$\bar{k}_i^* \approx u (\beta_i - u^{\delta-1}) \frac{sB}{\beta C} \quad (10)$$

which is a more approachable expression. It is worth examining this expression in the case of an agent who sits at the mean degree of the network, so that $\beta = 1$. Then

$$\bar{k}_i^* \Big|_{\beta_i=1} \approx u (1 - u^{\delta-1}) \frac{sB}{C} \quad (11)$$

which makes evident that the growth horizon, for an average agent in the network, depends on the shared benefit to per-connection cost ratio, weighted by the *probability-weighted variance* ($u (1 - u^{\delta-1})$). The probability-weighted variance demonstrates the effect of risk preferences on the agent's perception of the growth horizon: in its present form it says that an agent will only consider sharing when they are pessimistic ($\delta > 1$) as expected by the link between RDEUT and probabilistic optimization under environmental uncertainty. Furthermore, as the agent grows in pessimism, the more this variance term resembles the raw success rate u , so that the peak of \bar{k}_i^* with respect to u shifts towards the values representing higher security. This can be interpreted as more pessimistic agents allowing themselves to be part of more crowded sharing clusters when the chances of obtaining benefits are favorable. However, it is also evident that sharing collapses at both ends of the risk spectrum, with the growth horizon tending to 0 as u approaches either 1 or 0. If we account for the effect of the $W(\Sigma)$ term, sharing might collapse at intermediate values of u , not just the endpoints. Finally, comparing this growth horizon to explicitly simulated ones, while also keeping the shared benefit to per-connection cost ratio fixed, suggests that δ can be assumed to be a concave function of stake size $B - 1$. In other words, as the difference between baseline and surplus payoffs increases, so does the pessimism of a fitness-maximizing agent, reflecting the higher stakes involved in a game where failing to obtain a sizeable benefit might set an agent back considerably with respect to those agents who did obtain the benefit (Fehr-Duda, Bruhin, Epper, & Schubert, 2010). We choose $\delta(B) = \alpha + (B - 1)^\gamma$, where α is a baseline risk preference value and γ is an elasticity. This provides a good first approximation of \bar{k}_i^* for the growth rate representation of the problem. Note: add the elasticity values.

3.2.3 Optimal network size

The rank-dependent representation of the model can also be used to approximate the optimal network size \hat{k}_i , by differentiating with respect to \bar{k}_i :

$$\frac{\partial V_i(\text{Sharer})}{\partial \bar{k}_i} = 0 \iff \hat{k}_i = \frac{\log \left(-\frac{C}{us\beta_i B \log(1-u)} \right)}{\beta_i \log(1-u)} \quad (12)$$

In accordance with the insight gained from approximating the growth horizon, optimal sharing breaks down at too high or too low values of risk, and the location of these boundaries depends on the cost-benefit ratio as well as the sharing norm and the focal agent’s position with respect to the mean degree of their network. Also of note is that the optimal network size does not depend on the agent’s risk preference.

The derived approximations for these two network distinct density predictions preserve the overall shape of the relationships produced by numerical simulations of the actual payoffs of the sharing game, while also providing insight in the “language” of an agent making decisions under uncertainty. It is worth noting that the skew of the relationship between risk and optimal network size is distinct from that of risk and growth horizon: while in the latter the peak of the curve skews slightly towards higher security values as the stake size grows, in the former the peak skews towards higher risk values. In other words, if sharing clusters tend to optimality in the average number of connections, then they will be larger wherever the risk is higher (but not high enough to make sharing collapse). However, if sharing clusters grow until they reach their limit, then the most sizeable clusters will be observed at the lower end of risk values, especially as the size of the surplus payoff grows larger with respect to the baseline payoff (but, again, not low enough risk that sharing breaks down). A priori, there is no reason that sharing clusters will achieve optimality or grow until they hit their constraints. Sharing clusters are more complex structures than simple replicators, composed of individuals brokering sharing relationships, stabilizing norms, recruiting new Sharers and losing others to abandonment and fission as they face an uncertain environment. To give a strong argument for any distribution of sharing cluster characteristics, it is necessary to model actual dynamics of sharing cluster growth and development. However, the distinct shapes of the relationships explored here with respect to risk, payoffs and network structure already provide a way to contrast two broad simple predictions, based on different hypotheses on the consequences of sharing cluster formation and growth under costly connections. They also reveal important details about the “hard constraints” cluster development faces with respect to environmental risk. These findings are summarized in Figure 3.

3.2.4 Optimal sharing norms

Finally, by going back to the approximate growth rate representation of the game can be successfully poked for an approximation of the optimal sharing norms:

$$\frac{\partial G_i(\text{Sharer})}{\partial s} = 0 \iff s^* = \frac{\frac{(1-u)(B-\beta_i \bar{k}C)}{\beta_i u [(1-u)^{\beta_i \bar{k}} - 1] + 1} + \frac{1-\beta_i \bar{k}C}{\beta_i [(1-u)^{\beta_i \bar{k}} - 1]}}{B} \quad (13)$$

This approximation, much like the one for the growth horizon that can be calculated using the approximate growth rate, systematically overestimates the optimal sharing norm in the explicitly simulated payoff, but is still an informa-

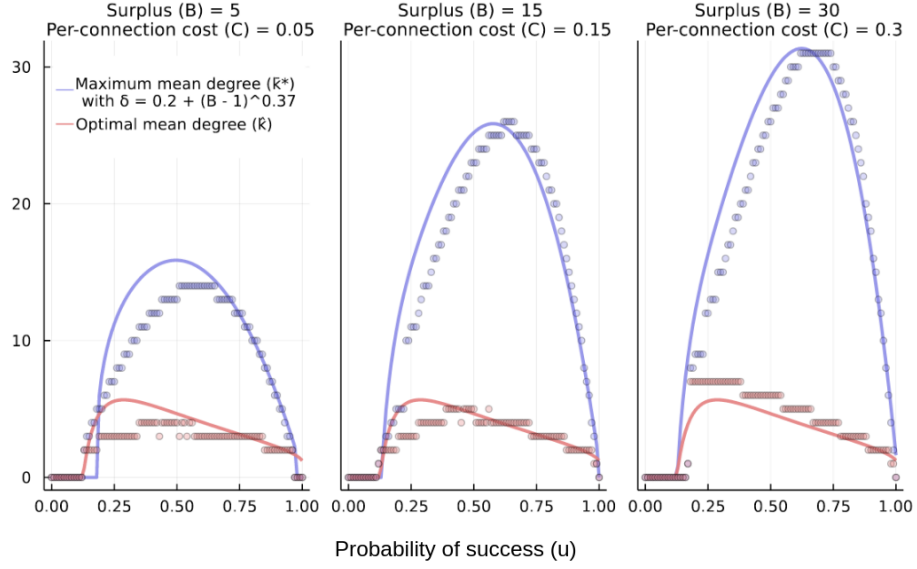


Figure 3: RDEUT approximations of \hat{k}_i and \bar{k}_i^* as functions of u (smooth) versus simulated values (dots) for increasing values of B and fixed surplus-to-connection-cost ratio. As B increases with respect to the baseline, so does the height and skew of the \bar{k}_i^* curve, which in the RDEUT representation is due to surplus/stake dependence of the agent's risk attitudes. Also, with the increase of B the simulated \hat{k}_i transitions from values below the RDEUT approximation to values above it. This is due to a valley in the payoff curve at network sizes that are not small enough to avoid high costs but not large enough to hedge for the risk of group failure. As B increases, the peak to the right of this valley overtakes the one to the left, leading to higher optimal network sizes and to the curve's final shape. Both shapes are well approximated by the optimal network density.

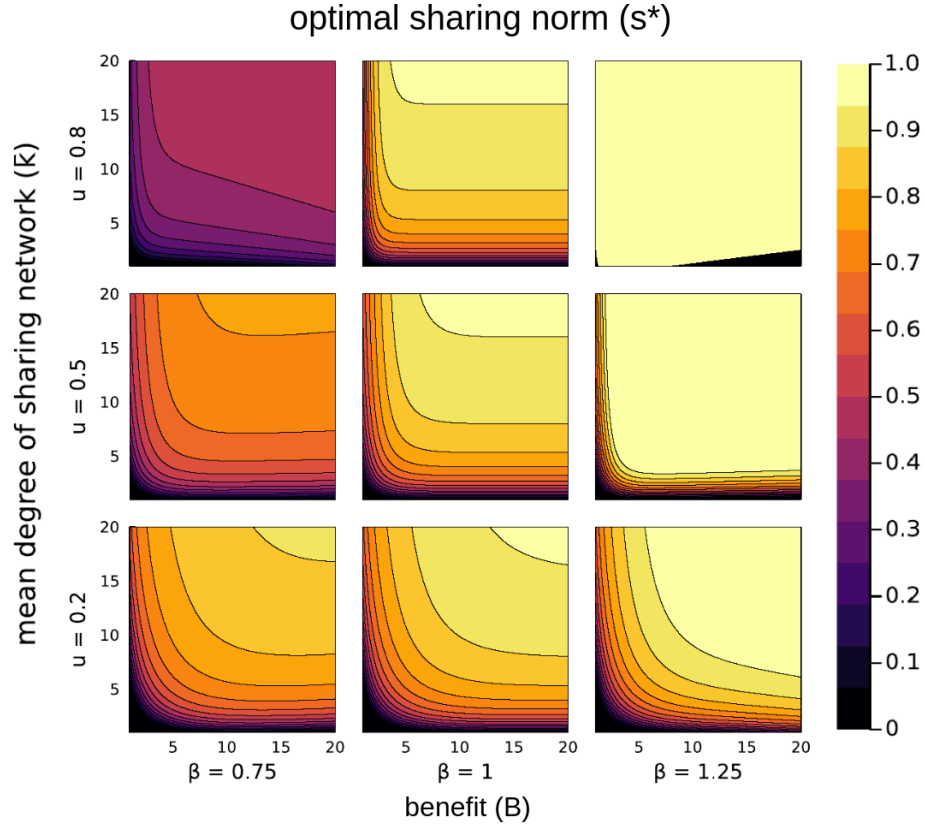


Figure 4: Optimal sharing norm as a function of \bar{k} and B , for different values of u and β . Riskier environments allow for less divergence in optimal sharing norms across agents in a hierarchical cluster, where individuals differ in degree. More secure environments, on the other hand, exacerbate the difference in optimal sharing norms, and thus possible conflicts of interest between members of cluster with significant inequalities in sharing partners.

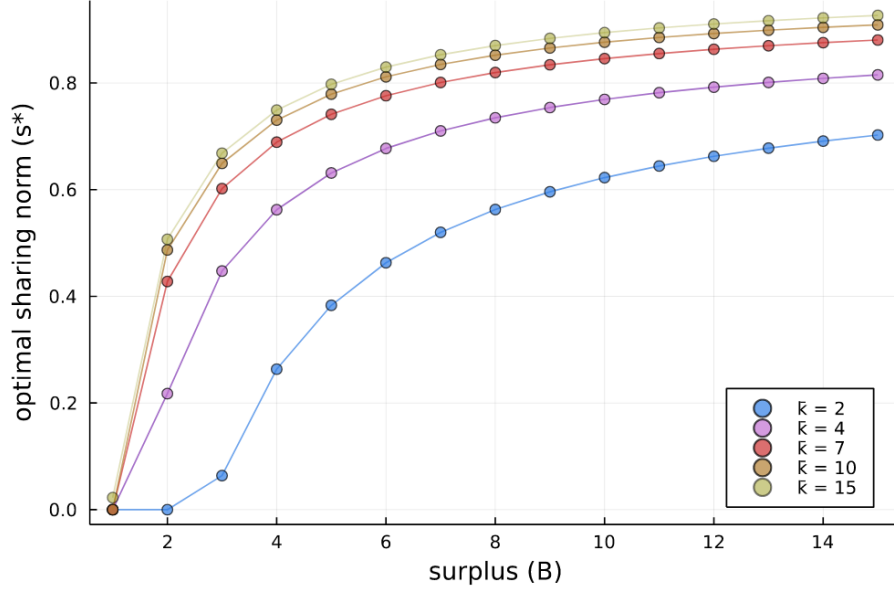


Figure 5: Increase of optimal sharing norms with increasing B at different average network size \bar{k} . Notice how network size mediates the asymptotic relationship between sharing norms and surplus. Plotted for $u = 0.5$ and $\beta = 1$.

tive estimate of the way optimal sharing norms are affected by average network size, relative degree, size of surplus benefit, etc. A useful insight from this expression comes from the difference in optimal sharing norms in the large versus small network cases. That is, the case where $\beta_i \bar{k}_i$ is large enough so that $(1 - u)^{\beta_i \bar{k}_i} \approx 0$ and the one where it is not, respectively. In the small network case, the growth of the optimal sharing norm with the increase of the surplus benefit B is muted: when there is a non-negligible probability that an agent wins B and every one of their cluster partners fails in doing so, giving away a high proportion of the benefit is not as beneficial as staying with a significant portion of it. By staying with such a portion, the agent hedges against their own failure (by receiving a non-negligible quantity of resource from successful partners) and against their partners' failure (by keeping a significant portion when successful). As an agent's number of partners grows, such that the probability that they all fail becomes negligible, the effect of B on the optimal sharing norm becomes more pronounced:

$$\lim_{\rho \rightarrow 1} s^* = \frac{\beta_i \{(\beta_i - 1)\bar{k}C - (1 - u)B - u\} + 1}{\beta_i(\beta_i u - 1)B} \quad (14)$$

which is even easier to appreciate in the case where the agent sits at the mean degree. In this case, the effects of costs vanish and the relationship reduces to:

$$\lim_{\rho \rightarrow 1} s^* \Big|_{\beta_i=1} = \frac{B-1}{B} \quad (15)$$

which indicates that in large enough clusters where Sharers can expect their sharing partners to have the same amount of partners as them on average, the optimal sharing norm will grow asymptotically to 1 as the size of the surplus increases (with respect to the baseline). The effects of mean degree and surplus payoff on optimal sharing norm can be seen in Figure 5 for fixed values of β_i and u , while Figure 4 shows a more complete picture of the relationships.

3.3 Inequality, degree variance and assortment

In this section we argue that the case of an agent sitting at the average degree ($\beta_i = 1$), combined with the assumption of low variance in number of connections within a cluster, is not only convenient for model analysis but also of particular importance, as it minimizes conflicts of interest between agents within a cluster, which might be important for continued cooperation.

Individuals that accrue more connections with respect to the mean degree of the cluster will be at an overall advantage over most of the other cluster members. This is due to them getting more shared benefits out of the networks than what they are “sharing in” (represented here by $\beta_i > 1$), although they are also paying more costs. Individuals below the mean degree, on the other hand, will on average be getting less from their sharing partners than what they are sharing in ($\beta < 1$). This situation of inequality in social capital is plausible in real life, as when some individuals hold special privileges that do not derive from the sharing game itself or, equivalently, when some individuals are in precarious conditions that lead them to consider accepting unequal treatment without pursuing a better deal. But in a playing field where individuals have the possibility to alter their number of connections (by, for example, joining a different cluster) then we can expect agents to seek out better opportunities for risk-pooling, leading to less tolerance for significant differences in number of connections between an agent and their partners. To add to this, the more the Sharers within a cluster differ in number of sharing partners, the more their optimal sharing norms differ from each other (this can be read from equation 16). If keeping a consensus on sharing norms is an important part of keeping a stable sharing cluster (recall that sharing norm magnitude is a factor in both optimal cluster size and growth horizon), then degree inequality is a direct destabilizing influence on sharing clusters, alongside any emerging payoff differences. Maximizing stability is not a trivial concern: the strategic advantage of reciprocal risk-pooling arrangements comes from their ability to smooth out resource variability, which can be seriously impaired by the potential social instability of possible conflicts with sharing partners.

Another concern comes from asking what should happen when agents vary in their productive conditions. Here it is useful to imagine a simple scenario: an individual X can initiate a reciprocal sharing partnership with three other

individuals 1, 2 and 3, such that $u_1 \log B_1 < u_2 \log B_2 = u_X \log B_X < u_3 \log B_3$, where the subscript represents a particular agent’s productive conditions, given by their (loner) geometric growth rate. It is clear that 1 is the least desirable partner for risk-pooling and 3 the most desirable one. X will then try to assort with 3. However, if we assume that 3 is facing a similar scenario, where they can choose between three individuals from different strata, one of them being equal to 3’s and the other one of a higher stratum, then X becomes the least desirable choice. In other words, given that the population of possible sharing partners is large enough so that the number of individuals in any stratum is larger than the amount of sharing partners any particular individual can afford, then agents will be rejected when they try to “share up” with higher strata individuals, and they will be unwilling to “share down” with individuals of lower strata. Hence, assortment for risk pooling will happen preferentially with individuals in the same productive stratum. Although we do not pursue the task in the current work, this dynamic can be interpreted as a “stable roommates problem” for settings in which groups can be larger than dyads (Irving, 1985; Huang, 2007).

There are imaginable situations in which it might not be the case that individuals assort themselves in a wealth homophilic, network pseudo-egalitarian manner when their objective is to risk-pool, as when the number of possible partners is low with respect to the maximum/desired number of sharing partners, or when a high stratum individual might trade off having fewer connections with individuals in their same stratum in order to form more connections with individuals in lower strata, provided these lower strata individuals also hold a lower number of sharing partners on average than the higher stratum individual. These hierarchical situations are likely to require special conditions or additional power dynamics to stabilize in the face of more “egalitarian” alternatives, and a closer study of them is merited. However, we believe that the stability of the pseudo-egalitarian clusters described here (of which resource pooling is a special case, that of clusters structured as cliques, where everyone shares with everyone) makes it a likely form of risk-sharing assortment to be employed by individuals, households and communities, enough to be considered an initial assumption. Furthermore, its long-term stability and success over lone production could perhaps drive other aspects of social structure, like household size. This last consideration, however, is at this point merely speculative.

NOTE: I AM REWRITING THE MATH IN THIS SECTION.

4 Discussion

The model here exposed and analysed deviates from common ways of modeling risk-sharing by taking a network perspective and incorporating per-connection costs of relationship maintenance. This imposes constraints on cooperation that go on to have important effects on both the optimal and the maximum possible sizes (average number of connections) a sharing cluster can achieve. It also allows for an initial exploration of the effects of inequality within clusters and the conflicts of interest that may arise, allowing us to pursue the argument that

reciprocal sharing clusters can be expected, at least as an initial assumption, to involve individuals in similar productive conditions who do not greatly differ between each other in number of sharing partnerships.

Extending this work will have to involve incorporating other aspects of network structure, such as clustering, especially when it comes to maintaining stability of the cooperation within networks. Certain network structures can favor the monitoring and enforcement of norms necessary to keep a working cluster of cooperators, as often the same ties that support resource flows will also give way to the information flows that form the basis of reputations, monitoring and punishment, and the characteristic costs of these actions can affect what kind of structure supports cooperation. While a model extension in this direction is warranted, it can be noted that Zefferman has explored the network structures that support cooperation promoted by monitoring and punishment, showing that when punishment costs are high and monitoring costs low, distributed organizations, characterized by higher density networks with low variance in degree, support effective cooperation better than more hierarchical structures (Zefferman, 2022). This opens the possibility that cooperation enforcement itself might be a driver of low degree variance, distributed sharing clusters, in addition to the conflicts of interests that hierarchy can spark.

The current model produces expressions that can be used as distinct hypotheses of how sharing clusters change with socio-ecological parameters like resource variability. This permits us to make predictions of what we should observe if clusters are forming in one way or another. What happens if clusters grow until they reach their limits? What happens if they do not grow beyond their optimal size? What happens if they reach their optimal sharing norms? The derived approximations can be used to address these questions empirically. However, the group-level cultural evolutionary processes that drive the formation of sharing clusters might not lead to population convergence on these points of interest, even if they are important for the processes involved. Instead, these relationships might be of particular use as a theory of the limits of risk-pooling through reciprocity in the face of costly connections.

To extend the current model, dynamics can be introduced that specify the way clusters grow and compete in an ecology of clusters. What would happen if individuals compete for connections within clusters, or if clusters can fission into smaller “offspring” clusters at particular stages in their growth trajectory, or if newly-formed clusters imitate the sharing norms of currently-successful clusters? A future extension of this work will address these questions in order to simulate the evolution of distributions of sharing cluster sizes and norms, and using these estimates as additional models to be explored with existing data on sharing networks.

At a glance, the model seems to be consistent with past and present studies of sharing in human communities. When it comes to food sharing, an observed relationship of increase in the proportion of resource shared with increased package size (Kaplan et al., 2005; Gurven, Allen-Arave, Hill, & Hurtado, 2001; Gurven, Hill, & Kaplan, 2002) is a special case of our estimate of optimal sharing norm. In its more general form, this increase is mediated by the mean degree

of sharing partners, which can motivate further empirical inquiry on this subject. The breakdown of sharing under conditions of high risk or high security also has empirical precedent (Wiessner et al., 1982; Wiessner & Huang, 2022; C. Townsend, Aktipis, Balliet, & Cronk, 2020), as does an increased number of sharing partnerships with increases in productivity and variability (Gurven, Jaeggi, Von Rueden, Hooper, & Kaplan, 2015; Howe, Murphy, Gerkey, & West, 2016; Scaggs, Gerkey, & McLaughlin, 2021; Ready, 2018; Ready & Power, 2018) and a tendency for cooperative assortment by material conditions or productive capacities (Redhead, Dalla Ragione, & Ross, 2022). Size of sharing clusters has also been observed to be independent of population size, with a study observing more, but not larger, sharing clusters with larger population size in Agta and BaYaka communities (Dyble et al., 2016), which is consistent with the model’s prediction that neither optimal nor maximum cluster size will depend on population size (unless population size is smaller than these quantities), but the maximum number of sharing clusters in a given population will.

MARKET INTEGRATION (GURVEN AND KRAMER PAPERS). (Gurven et al., 2015; Hackman & Kramer, 2021).

While these observed parallels between model and empirical research are compelling, they must be examined more closely and with more explicit connections to both model and data structure before the links between them can be given proper consideration. We think this is enough to motivate further exploration of these relationships in light of the model, and that the observed preliminary links between model and empirical research make it a promising enterprise.

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