A model of reciprocal sharing clusters under risk and uncertainty

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Abstract

We model norms that prescribe behaviors related to the sharing of resources under environmental stochasticity and show that foragers with pessimistic risk attitudes maximize fitness by connecting with each other at a cost and by adopting reciprocal sharing norms that enable stability in time. The resulting risk-pooling networks will be at their most stable when foragers maintain a similar number of partners to the rest of their associates and when individuals preferentially assort with others in similar socioeconomic standing. The optimal network size and the limits to network growth are given by relationships between the costs of maintaining connections, the resource package size and the variability in resource production, while optimal sharing norms depend on benefit package size and sharing cluster density.

1 Introduction

Human societies run on social norms. Cooperation requires that individuals agree to "play by the rules", even when this means adopting (and enforcing) behaviors that are costly and that might trade off an individual's immediate returns in favor of maintaining cooperative relations in time. The maintenance of these sorts of cooperative schemes has inspired a field examining the conditions for the evolution of cooperation, and it is clear today that there are many pathways to cooperative assortment which are explored and exploited by human societies around the world.

When we ask ourselves how cooperative groups form and maintain themselves, we are engaging in a twofold problem. First, it is necessary to ask what sort of incentive structures lead people to join groups and adopt their norms over the other options provided by their environments and personal capacities. At the center of this concern is the question: why join a group if I could be on my own? Second, it is important to consider the psychology of decision-makers and how it interacts with the aforementioned incentive structures. Classical rational decision-making asks that agents choose strategies such that they maximize their expected utility. However, decisions do not happen in a vacuum, and decision-makers might factor their uncertainty about the world into their choices in order to deal with possibilities like unknown risks or incomplete knowledge about the consequences of choices. Given the possible edges of this problem, social and economic decision-making is usefully seen through the lenses of foraging and cultural evolutionary theory, where social learning and cultural transmission are modeled as evolutionary forces that can lead to different distributions of traits within and between populations of agents with incomplete information about their environments.

In this work, we focus on reciprocal sharing norms, in which agents (be they individuals or households) enter pairwise contracts with other agents that prescribe the sharing of benefits earned through each agent's individual exploitation of their environment. Sharing and reciprocity are ubiquitous in human societies, and often continuing relationships (like kinship ties and friendships) are maintained by stable benefit transfers like workload sharing, reciprocal gift-giving and resource lending (with or without interest), all practices which tend to develop in either explicit or implicit normative backgrounds which aid in setting the individual expectations that facilitate trust and coordination. In stochastic environments, reciprocal sharing enables tapping into other individuals' securities in ways that lead to variance-reduction in expected returns (risk pooling), while possibly requiring the payment of costs of interaction maintenance that cut down on the direct benefits earned by individual agents during each time period.

Previous work on the risk-mitigating benefits of reciprocal sharing has concentrated on showing how sharing behaviors can propagate and persist in different scenarios through reduction of payoff variance (McCloskey, 1991; Winterhalder, 1986; Alderman & Paxson, 1994; Blumenstock, Fafchamps, & Eagle, 2011). However, there is no work that examines the dynamics of the clusters of sharing cooperators that emerge from the adoption of sharing strategies and what these dynamics mean for the stability of sharing strategies in time. This is important, as reciprocity and resource-sharing has been extensively documented in societies around the world, and their effects often go beyond individual welfare and into the broader arena of group interactions. In this vein, we aim to explore the following questions:

1. What characteristics can we expect stable risk-sharing clusters to exhibit? What sort of network structures and assortment preferences are best supported by risk-pooling relationships?

2. In stable networks, what is the optimal number of sharing partners, and how large/dense can sharing networks grow before losing their risk-sharing benefits?

3. What are the optimal sharing norms to adopt at the group level? What sort of dynamic group processes can lead to the evolution of sharing norms?

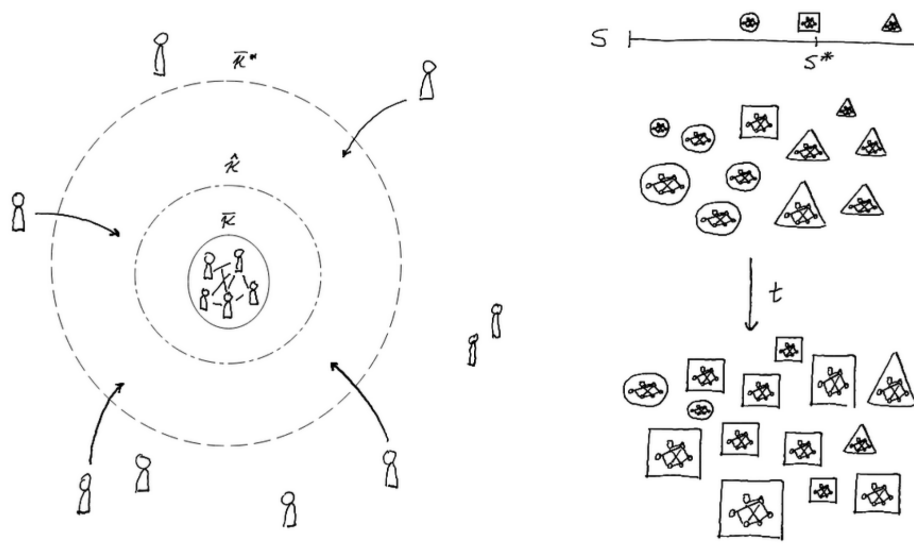

Figure 1: Schematic illustration of the sharing game and the evolution of sharing norms through the growth and propagation of sharing clusters. Agents form new clusters or choose to join preexisting ones, leading to cluster growth, where sharing norms close to optimal allow clusters to grow at an advantage to same-sized clusters with less optimal norms.

We attempt to answer these questions by building a model in which agents perform some productive activity with an uncertain benefit, consisting of either a baseline payoff or an surplus payoff with an associated success rate. Loners are agents that embark on this productive effort alone, while Sharers pay a per-connection cost in order to cluster in reciprocal sharing networks with each other. By using agreed-upon sharing norms that stipulate what proportion of the surplus benefit Sharers break up and distribute among their network peers (when they obtain it), clusters of Sharers can mitigate environmental stochasticity by (potentially) reducing the variance of their returns. Group pooling is a special case of this setup, where networks are cliques and individuals share out the whole surplus benefit when successful.

By examining when do Sharers achieve better results than Loners we can find approximate conditions for the existence of reciprocal sharing networks given fixed connection costs, surplus benefit package sizes and environmental variability in terms of cluster mean degree (which we also call cluster size, not to be confused with the number of cluster members, a measure we have no current use for except in the case of clique pooling, where it converges to our definition of cluster size). We can use these elements to construct a model of constraints on network emergence and growth. We can also find approximate conditions for the optimal network size and sharing norm, motivating a discussion on what sorts of dynamics can lead towards or away from optimality in network sizes, as well as what sort of processes can lead to sharing norm evolution at the group level. Overall, we find that

1. The most important determinant of the possibility, success and possible extent of reciprocal sharing networks, other than the surplus-benefit-to-connection-cost ratio, is environmental variability. When the desired surplus resources display significant variability, then sharing at a cost is a viable strategy. However, sharing collapses at both extremes of the risk spectrum: when environments are too risky or when they are too secure.

2. Optimal network size and maximum network size are affected by risk in distinct ways. If agents optimize number of sharing partners in a stable network, then the largest sharing clusters will emerge in riskier environments. If agents let their sharing networks grow indiscriminately, until the payoff to a cluster agent equals the payoff of a lone individual, then the largest sharing clusters will emerge in more secure environments.

3. If agreement on a sharing norm is assumed to be a requirement for stable sharing cooperation, then stable reciprocal sharing clusters should minimize variance in network degree. In other words, inequality in number of sharing partners within a sharing network decreases its stability, as optimal sharing norms for a particular agent depend on their position in the network hierarchy (e.g. how their network degree compares to the network's average degree).

4. In the presence of heterogeneity in individual success rates and surplus benefits, agents are likely to cluster with others in their same productive "stratum". This is because there is an advantage to "sharing-up" with individuals of a higher stratum and a disadvantage to "sharing-down", when reciprocal sharing is used for risk-pooling. Therefore, if individuals sort themselves into groups that cannot grow larger than the whole population, the resulting assortment will display homophily in productive capacities. We note that there are cases where sharing between strata might be stabilized, although we do not think these arrangements are likely to emerge in a purely risk-pooling context. In this way reciprocal sharing can actually tap preexisting inequalities in a population of Sharers, and possibly feed back into this inequality, exacerbating it. This form of assortment, where agents pay close attention to the conditions of their partners, can also be a way to mitigate the success of defecting/parasitic strategies.

5. In groups with low inequality in network degree, optimal sharing norms depend on the average size of the sharing cluster and its interaction with the size of the surplus benefit package with respect to the baseline benefit. At small network sizes, sharing norms can be significant but still far from full, as surplus benefit package size increases. However, as networks grow larger on average, the optimal sharing norm becomes more fully dependent

on the size of the surplus benefit with respect to the baseline, converging to full sharing as the former increases.

We build the argument progressively, first reviewing relevant literature on risk, insurance and reciprocal sharing to motivate the model. After that, we explore the characteristics of sharing networks predicted by the model and discuss some of their potential real-life analogues. Finally, we summarize the conclusions and predictions from the model analysis, and we review their utility for thinking about how human groups form, fission and propagate and evolve functional strategic norms.

2 Risk and reciprocity in theory and across societies

Cooperation is aided by the presence of risk and uncertainty, and it often involves reciprocity relations (Offer, 1997; Andras, Lazarus, & Roberts, 2007; Buston & Balshine, 2007; Delton, Krasnow, Cosmides, & Tooby, 2011). Aversion to risk can be strategic when faced with stochastic environments (Price & Jones, 2020), and it can lead individuals to adopt strategies that minimize variance at the expense of expected payoff, favoring the emergence of cooperative strategies. An extensive literature in economics studies the presence of risk-sharing through income sharing and/or consumption smoothing in low-income contexts lacking formal insurance markets (Alderman & Paxson, 1994; Townsend, 1994; Udry, 1994; Rosenzweig, 1988) as well as in the face of correlated regional risks (Blumenstock et al., 2011) and within family coalitions in urban contexts (Lee, Parish, & Willis, 1994).

As a complement, but also in contrast to this literature, a behavioral ecology perspective treats consumers as foragers interacting with an uncertain socio-ecological environment (Hantula, 2012). There are various treatments of sharing and risk that expose the risk-pooling benefits of reciprocal sharing in foraging contexts (Winterhalder, 1986, 1993; Smith & Boyd, 2019; Kaplan, Gurven, Hill, Hurtado, et al., 2005), as well as recent empirical analyses testing the implications of the resulting models (Ember, Skoggard, Ringen, & Farrer, 2018). More broadly, the reduction in variance afforded by sharing can be crucial in scenarios that can be usefully described by models of multiplicative growth (Peters & Adamou, 2022).

In small-scale societies of hunter-gatherers, horticulturalists and pastoralists, risk-pooling through reciprocal sharing has been extensively documented by anthropologists and has shown to be mediated by environmental conditions, institutional structures (such as kinship networks and social norms about equality) and enduring relationships in the context of reciprocal altruism (Wiessner et al., 1982; Cashdan, 1985; R. B. Bird, Bird, Smith, & Kushnick, 2002; Nolin, 2010; Hames & McCabe, 2007; Koster, Leckie, Miller, & Hames, 2015; Gerkey, 2013; Ziker & Schnegg, 2005; Hames, 2017; Ready, 2018). This does not mean that all reciprocal sharing is done for risk-pooling reasons. In conditions where agents are not in need of the insurance provided by sharing strategies, reciprocal gift-giving can still be used as a way to maintain social relationships, mediate conflicts and/or acquire status and social capital (Offer, 1997; Platteau, 1997; Kent, 1993; Wiessner et al., 1982; Wu & Ma, 2017).

At the group level, reciprocal sharing can serve egalitarian ends (Woodburn, 1982; Dowling, 1968; Aspelin, 1979), but it can also impose pressures on the scale that a sharing group can achieve. Destabilization related to scale has been examined through theoretical models (Boyd & Richerson, 1988) and through ethnographic accounts of the relationship of sharing behavior with village scale and group fission (Ramos, 1995; Chagnon, 2012). Reciprocal sharing patterns can also track differences in social status and access to resources, such that agents in advantageous positions can use reciprocal sharing norms in their favour, in order to attain better positions on a social hierarchy (Sahlins, 1963; Betzig, 1988; R. L. B. Bird & Bird, 1997).

3 The model

3.1 The setup

Parameter List		
Parameter Name	Symbol	Parameter Space
Surplus Benefit	$\boldsymbol{B}$	$(1, \infty)$
Per-Connection Cost	$\boldsymbol{C}$	$(0, \infty)$
Security	$\boldsymbol{u}$	$(0, 1)$
Risk	$\boldsymbol{1-u}$	$(0, 1)$
Sharing Norm	$\boldsymbol{s}$	$(0, 1)$
Agent Degree	$\boldsymbol{k}$	$\mathbb{N}_{>0}^{\leq N-1}$
Network Expected Degree	$\boldsymbol{\bar{k}}$	$(0, N-1]$
Risk Preference	$\boldsymbol{\delta}$	$(0, \infty)$
Sharing Group Size	$\boldsymbol{n}$	$\mathbb{N}_{>0}$
Population Size	$\boldsymbol{N}$	$\mathbb{N}_{>0}$

We begin with a population of N agents who perform some productive activity every time period, for a total of T time periods. For each time period, the agent can get a baseline payoff of v_0 with a probability $1-u$, which we denominate risk, or a surplus payoff $B > w_0$ with probability u, which we denominate security. For simplicity, we assume $v_0 = 1$. We also assume agents invest their payoffs multiplicatively into a currency reserve w which, for the sake of simplicity, maps directly onto their (cultural) fitness, with baseline fitness w_0. At the end of every productive period of T time periods, a number of agents will choose to imitate others in the population, with the probability of agent i for being chosen as a model given by $\phi_i = \frac{w_i}{\sum_j^N w_j}$. Assuming T is large enough, then the measure of replication success of agent i is their geometric mean fitness $w_i = w_0 e^{G_i}$, where $G_i = \mathbb{E}(\log V_i)$ is the expected growth rate of agent i and V_i is a random variable representing the outcome of the productive effort of agent i in a single time period. By assuming w_0 is constant across agents in the population, we can use G, the logarithm of the geometric mean fitness, as the relevant measure of success to optimize. Thus, an agent who works alone (Loner) and reaps their benefits will have an average growth rate

$$G(\text{Loner}) = u\log(B) + (1-u)\log(1) = u\log(B) \tag{1}$$

Assume now that $n \leq N$ agents come together into a networked cluster of mean degree $\bar{k}$, where a connection between a pair of agents has a maintenance cost C. The cluster establishes a sharing norm s, a proportion of their payoff that a successful agent (one who successfully obtained B in the time period in question) shares with their connections by breaking it up equally among them. Every cluster agent i thus receives $\frac{sB}{k_j}$ from every one of their successful connections j, regardless of their success or failure in the time period in question. Each agent then invests the total payoff from this process into their cultural fitness. We call this strategy Sharer. Sharers pay a cost to connect with each other, but they enjoy the benefits brought forth by variance reduction in a multiplicative dynamic. We write the average growth rate of a Sharer i as

$$G_i(\text{Sharer}) = u\log\{(1-s)B + S_i\} + (1-u)\log\{1 + S_i\} \tag{2}$$

with S_i the average net received shared benefits,

$$S_i = \frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{n} a_{ij}\epsilon_{j,t}\frac{sB}{k_j} - k_i C \tag{3}$$

where a_{ij} equals 1 if agents i and j share a connection and 0 otherwise, and $\epsilon_{j,t}$ equals 1 if agent j is successful in time period t and 0 whenever they aren't. In order to make this expression more tractable for our purposes, we assume that each k_j is a draw from a random variable with variance small enough so that each $k_j \approx \bar{k}$. This means we assume that the cluster members agent i is connected to don't (on average) differ in their degree between each other. We expand on and justify this assumption in one of the sections below. Mathematically, it allows us to treat the first expression in S_i as the expectation of a binomial distribution, over-weighted by a term accounting for potentially small network size:

$$
\begin{aligned}
S_i &= \left(\frac{1}{T}\sum_{t=1}^{T}\sum_{j=1}^{n} a_{ij}\epsilon_{j,t}\right)\frac{sB}{\bar{k}} - k_i C \\
&\approx \rho u k_i \frac{sB}{\bar{k}} - k_i C \\
&= \beta_i(\underbrace{\rho u s B}_{\text{average shared benefit}} - \underbrace{\bar{k}C}_{\text{average network cost}}) \\
&= \beta_i \tilde{S}
\end{aligned} \tag{4}
$$

where $\beta_i = \frac{k_i}{\bar{k}}$ is agent i's position with respect to the mean degree of the cluster, $\rho = 1-(1-u)^{\beta\bar{k}}$ is the probability that at least one cluster member is successful and $\tilde{S}$ is the (average) per-capita received shared benefit, a term that accounts for the higher risk involved in smaller sharing networks. This leads to the approximate average growth rate for a Sharer in position β:

$$
\tilde{G}(\text{Sharer}|\beta) = u\log\left\{(1-s)B + \beta\tilde{S}\right\} + (1-u)\log\left\{1+\beta\tilde{S}\right\} \tag{5}
$$

It is important to note that this expression, and the procedure we used to construct it, does not consider the full effects of volatility, which would require an approach that explicitly considers the variance and its effect on fitness. Other works on sharing that have used approaches where variance is explicitly considered have shown that the benefits from reduction of volatility increase as $\frac{1}{n}$ (McCloskey, 1991; Winterhalder, 1986; Peters & Adamou, 2022). Hence, there is a diminishing return in volatility reduction for every new cluster member that joins, which is here represented through the effect of ρ. This means that when costs are absent clusters can grow indefinitely and still enjoy an additional benefit, however small, for each new member. But in the presence of connection costs, the diminishing returns from variance reduction eventually become negligible in the face of increasing total costs. The net effect of this is that a cluster's *growth horizon* (the point where the cluster's average growth rate meets the average growth rate of a loner) will be determined, on the most part, by what happens to the average payoff as the network's average degree grows (with respect to the average payoff of Loners). By relying on the (ρ-weighted) average shared benefits received from connections, we disregard the full effects of fluctuations on these received shared benefits, leading to an overall overestimate of growth rates with respect to explicitly simulated payoffs.

Another point of importance is that we are considering here only the case of uncorrelated risk. In the present framework, a simple way of introducing correlation in success between sharers in a cluster would be to replace the probability of success u in (4) with $\upsilon = (1-R)u$ where $R \in [0,1]$ is the Pearson correlation of Sharers, restricted to null or positive correlations. We do not consider the case of anti-correlation here, as it is not a relevant problem for groups of

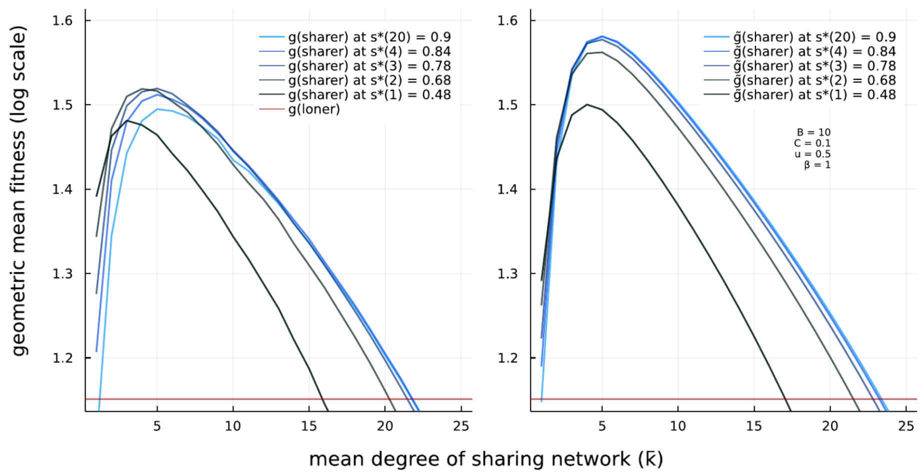

Figure 2: Simulated growth rate trajectories with increasing mean degree (left) versus trajectories from the approximate model (right). The approximation overestimates the growth rates as well as the optimal sharing norms, but remains an informative model.

3 or more individuals, which are the main object of our focus. However, Winterhalder previously examined the effects of anti-correlation of sharers using a Z-score model (Winterhalder, 1986).

3.2 Model analysis

We can now ask questions from this model. Of particular interest is finding the aforementioned growth horizon, which is equivalent to asking when does the growth rate of the average Sharer's payoff exceeds that of the average Loner in terms of $\bar{k}$, given some values for B, C, u and s. Defining $G(\text{Sharer})$ as the average fitness of a Sharer with degree $k = \beta\bar{k}$, we have $G(\text{Sharer}) \approx \tilde{G}(\text{Sharer}|\beta) > G(\text{Loner})$. We concentrate on the case of an agent who "keeps up" with the overall degree increases of their network partners. This means that if the mean degree $\bar{k}$ of the cluster grows, the focal agent always maintains a degree $\beta\bar{k}$. This leaves us with the condition

$$\tilde{G}(\text{Sharer}|\beta) > G(\text{Loner}) \tag{6}$$

We can think of a growing cluster of Sharers as a trajectory of the average Sharer's payoff as the mean degree of the network, $\bar{k}$, grows. Simulating this dynamic explicitly in a clique of Sharers and plotting growth rate as a function of mean degree shows this trajectory: there is an initial gain in fitness as the network recruits more members through its fitness advantage over lone production, followed by a peak (which we denominate $\hat{k}$) and then a continued decrease until the average Sharer's fitness intersects the fitness of a Loner. This point of intersection, which we have also called the growth horizon and we identify with the symbol $\bar{k}^*$, is the point at which a cluster would not be able to sustain any more recruitment, as an increase in the network's average degree beyond this point would lead to a lower payoff than that of a Loner. By solving (6) for $\bar{k}$, we can find this "maximum density" of sharing clusters. While this applies to any network with high degree regularity in its members, it is useful to think about it through the case of cliques, where $\bar{k} = n - 1$. In this case, $\bar{k}^*$ represents the maximum size a sharing cluster can attain before it becomes too "congested": in approximate terms, this is when there are so many members that the average shared benefit (usB) is cancelled out by the average network cost ($\bar{k}C$).

3.2.1 A rank-dependent representation

We can gain further insight into $\bar{k}^*$ by translating it to the scenario of Rank-Dependent Expected Utility Theory (henceforth RDEUT), where an explicit expression for $\bar{k}^*$ can be obtained at the cost of introducing an additional parameter representing a decision-maker's risk preference. In a decision-making scenario where decision-makers can over- or under-weight the probabilities of ranked outcomes, pessimism (under-weighting of the probabilities of the outcomes ranked as better) can approximate the solution to a problem of probabilistic optimization under environmental uncertainty (Price & Jones, 2020). For a two outcome case, like the one we are exploring here, the rank-dependent utility for a strategy r is given by

$$V(r) = w(p)r_h + [1 - w(p)]r_l \tag{7}$$

where r_h and r_l are the high- and low-ranked outcomes, p is the probability of achieving the high-rank outcome and w is a probability weighting function. While there are several forms that w can take, we use a simple form $w(p) = p^\delta$, where δ is a parameter representing an agent's risk preference. When $\delta = 1$ the agent is risk-neutral, while for $\delta > 1$ they display increasing pessimism, and for $0 < \delta < 1$ they are optimistic, over-weighting the given probability of the higher outcome. Probability weighting can happen when a decision-maker's model of an outcome's distribution differs from the "objective" distribution of that outcome (Peters & Adamou, 2022). For example, an agent that assumes there is additional uncertainty about the possible benefits than what the current available information suggests can be represented with a decision-making model incorporating probability weighting. In the current framework, this just means that an agent that receives information about available benefits and their associated probabilities proceeds to rank the possible outcomes and then assume some outcomes are more or less likely than others based on their rank. Pessimism shifts probability mass towards the lower ranks, while optimism does the opposite. Risk neutrality performs an "unbiased" weighted average of the outcomes.

3.2.2 Growth horizon

The rank-dependent utility representation of both Loner and Sharer approximate payoffs becomes

$$V(\text{Loner}) = u^{\delta} B + (1 - u^{\delta}) \tag{8}$$

$$\tilde{V}(\text{Sharer}|\beta) = u^{\delta} \left\{ (1-s)B + \beta \tilde{S} \right\} + (1 - u^{\delta}) \left\{ 1 + \beta \tilde{S} \right\} \tag{9}$$

from which we can solve $\tilde{V}(\text{Sharer}|\beta) = V(\text{Loner})$ for the growth horizon, yielding

$$\bar{k}^{*} = \frac{\frac{u\left(\beta - u^{\delta-1}\right)sB}{C} - \frac{W(\Sigma)}{\log(1-u)}}{\beta} \tag{10}$$

with

$$\Sigma = \frac{u(1-u)^{\frac{u\left(\beta - u^{\delta-1}\right)sB}{C}} s\beta B \log(1-u)}{C} \tag{11}$$

and W the Lambert W function, also known as the product logarithm. Finally, whenever $\bar{k}^{*} < 0$ we set $\bar{k}^{*} = 0$. While at plain sight this expression might seem complicated, we note that the term involving the product logarithm serves as a delimiter of the range of parameter values for which sharing is possible. In other words, it draws the boundaries of the region of parameter space that allows for sharing to emerge. If we assume that we are within this region, then $W(\Sigma) \approx 0$ and

$$\bar{k}^{*} \approx \frac{u\left(\beta - u^{\delta-1}\right) sB}{\beta C} \tag{12}$$

which is a more approachable expression. It is worth examining this expression in the case of an agent who sits at the mean degree of the network, so that $\beta = 1$. Then

$$\left. \bar{k}^{*} \right|_{\beta=1} \approx \frac{u\left(1 - u^{\delta-1}\right) sB}{C} \tag{13}$$

which makes evident that the growth horizon, for an average agent in the network, depends on the surplus benefit to per-connection cost ratio, weighted by the sharing norm and the *probability-weighted variance* $\left(u\left(1 - u^{\delta-1}\right)\right)$. The probability-weighted variance demonstrates the effect of risk preferences on the average agent's perception of the growth horizon: in its present form it says that an agent will only consider sharing when they are pessimistic ($\delta > 1$) as expected by the link between RDEUT and probabilistic optimization under environmental uncertainty. Furthermore, as the agent grows in pessimism, the more this variance term resembles the raw success rate u, so that the peak of

$\bar{k}^*$ with respect to u shifts towards the values representing higher security. This can be interpreted as more pessimistic agents allowing themselves to be part of more crowded sharing clusters when the chances of obtaining benefits are favorable. However, it is also evident that sharing collapses at both ends of the risk spectrum, with the growth horizon tending to 0 as u approaches either 1 or 0. If we account for the effect of the $W(\Sigma)$ term, sharing might collapse at intermediate values of u, not just the endpoints. Finally, comparing this growth horizon to explicitly simulated ones, while also keeping the surplus benefit to per-connection cost ratio fixed, suggests that δ can be assumed to be a concave function of the magnitude of the surplus benefit B. In other words, as the size of the benefit increases, so does the pessimism of a fitness-maximizing agent, reflecting the higher stakes involved in a game where failing to obtain a sizeable benefit might set an agent back considerably with respect to those agents who did obtain the benefit (Fehr-Duda, Bruhin, Epper, & Schubert, 2010). We choose $\delta(B) = \alpha + \log(B)$, where α is a baseline risk preference value. This provides a good first approximation of $\bar{k}^*$ for the growth rate representation of the problem.

3.2.3 Optimal network size

The rank-dependent representation of the model can also be used to approximate the optimal cluster size $\hat{k}$, by differentiating with respect to $\bar{k}$:

$$\frac{\partial \tilde{V}(\text{Sharer}|\beta)}{\partial \bar{k}} = 0 \iff \hat{k} = \frac{\log\left(-\frac{C}{us\beta B \log(1-u)}\right)}{\beta \log(1-u)} \tag{14}$$

The effect of risk on optimal network size is logarithmic. The *log-risk* appears in both the numerator and denominator, driving the overall shape of the relationship. As expected from the insight gained from approximating the growth horizon, optimal sharing breaks down at too high or too low values of risk, and the location of these boundaries depends on the cost-benefit ratio as well as the sharing norm and the position with respect to the mean degree. Also of note is that the optimal network size does not depend on the agent's risk preference.

The derived approximations for these two network size measures preserve the overall shape of the relationships produced by numerical simulations of the actual payoffs of the sharing game, while also providing insight in the "language" of an agent making decisions under uncertainty. It is worth noting that the skew of the relationship between risk and optimal network size is distinct from that of risk and growth horizon: while in the latter the peak of the curve skews towards higher security values, in the former the peak skews towards higher risk values. In other words, if sharing clusters tend to optimality in the average number of connections, then they will be larger wherever the risk is higher (but not high enough to make sharing collapse). However, if sharing clusters grow until their reach their limit, then the most sizeable clusters will be observed at the lower end of risk values, especially as the size of the surplus benefit grows larger (but, again, not low enough risk that sharing breaks down). A priori, there is no

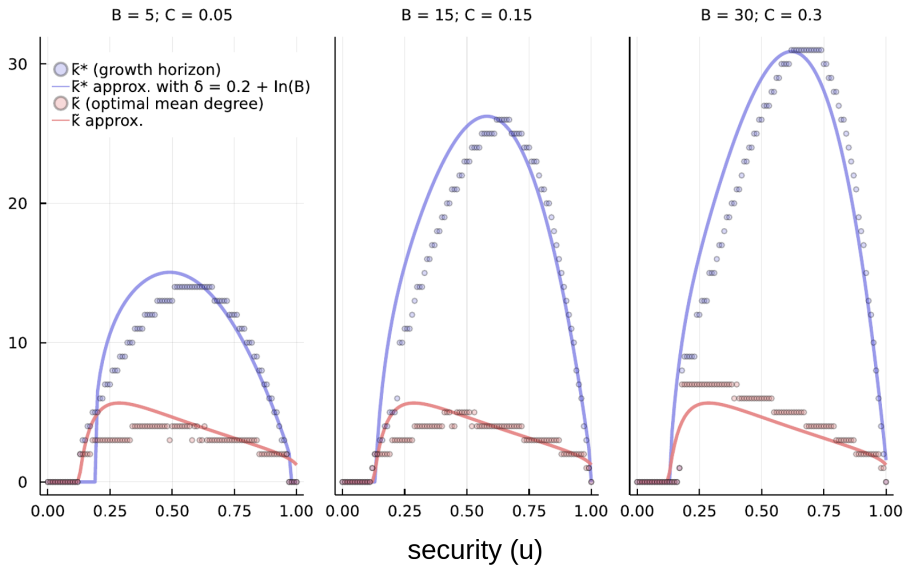

Figure 3: RDEUT approximations of $\hat{k}$ and $\bar{k}^*$ as functions of u (smooth) versus simulated values (dots) for increasing values of B and fixed surplus-to-connection-cost ratio. As B increases with respect to the baseline, so does the height and skew of the $\bar{k}^*$ curve, which in the RDEUT representation is due to surplus/stake dependence of the agent's risk attitudes. Also, with the increase of B the simulated $\hat{k}$ transitions from values below the RDEUT approximation to values above it. This is due to a valley in the payoff curve at network sizes that are not small enough to avoid high costs but not large enough to hedge for the risk of group failure. As B increases, the peak to the right of this valley overtakes the one to the left, leading to higher optimal network sizes and to the curve's final shape.

reason that sharing clusters will achieve optimality or hit their constraints. To give a strong argument for any of these it is necessary to model actual dynamics of sharing cluster growth. However, the distinct shapes of these relationships with respect to risk already provide a way to contrast two broad predictions, based on different hypotheses on the consequences of sharing cluster formation and growth under costly connections.

3.2.4 Optimal sharing norms

The approximate growth rate representation of the game can be successfully poked for an approximation of the optimal sharing norms:

$$\frac{\partial \tilde{G}(\text{Sharer}|\beta)}{\partial s} = 0 \iff s^* = \frac{\frac{(1-u)(B-\beta\bar{k}C)}{\beta u[(1-u)^{\beta k}-1]+1} + \frac{1-\beta\bar{k}C}{\beta[(1-u)^{\beta k}-1]}}{B} \tag{15}$$

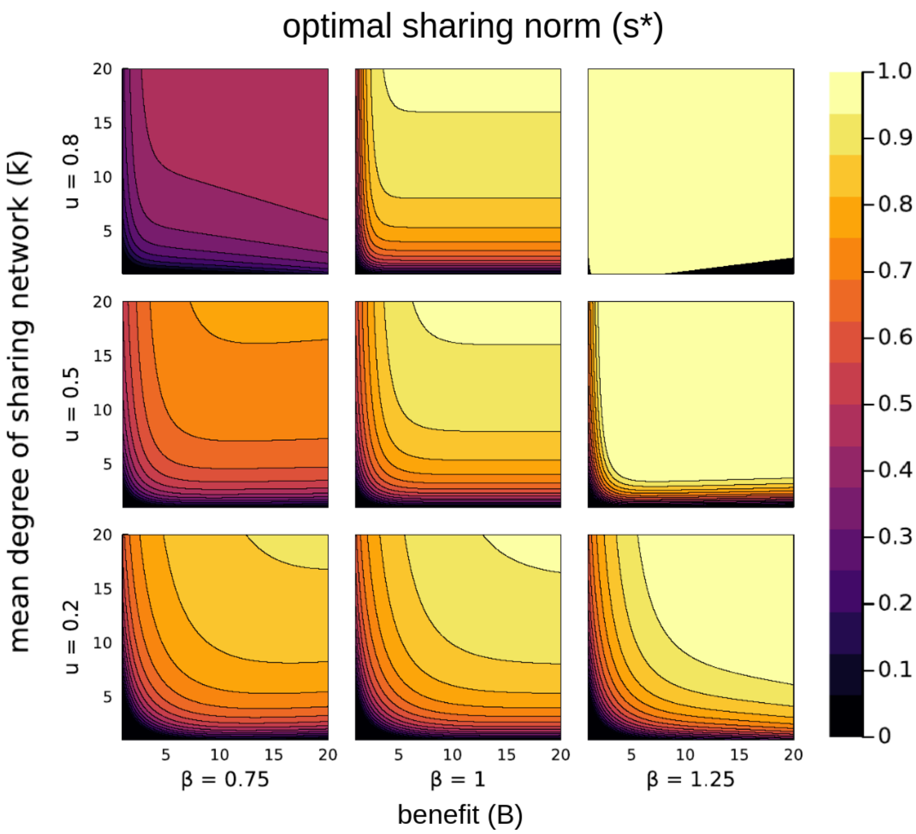

Figure 4: Optimal sharing norm as a function of $\bar{k}$ and B, for different values of u and β. Riskier environments allow for less divergence in optimal sharing norms across agents in a hierarchical cluster, where individuals differ in degree. More secure environments, on the other hand, exacerbate the difference in optimal sharing norms, and thus possible conflicts of interest between members of cluster with significant inequalities in sharing partners.

This approximation, much like the one for the growth horizon that can be calculated using the approximate growth rate, systematically overestimates the optimal sharing norm in the explicitly simulated payoff, but is still an informative estimate of the way optimal sharing norms are affected by average network size, position with respect to mean degree, size of surplus benefit, etc. A useful insight from this expression comes from the difference in optimal sharing norms in the large versus small network cases. That is, the case where $\beta\bar{k}$ is large enough so that $(1-u)^{\beta\bar{k}} \approx 0$ and the one where it is not, respectively. In the small network case, the growth of the optimal sharing norm with the increase of the surplus benefit B is muted: when there is a non-negligible probability that an agent wins B and every one of their cluster partners fails in doing so, giving away a high proportion of the benefit is not as beneficial as staying with a significant portion of it. By staying with such a portion, the agent hedges against their own failure (by receiving a non-negligible quantity of resource from successful partners) and against their partners' failure (by keeping a significant portion when successful). As an agent's number of partners grows, such that the probability that they all fail becomes negligible, the effect of B on the optimal sharing norm becomes more pronounced:

$$\lim_{\rho \to 1} s^* = \frac{\beta\left\{(\beta-1)\bar{k}C - (1-u)B - u\right\} + 1}{\beta(\beta u - 1)B} \tag{16}$$

which is even easier to appreciate in the case where the agent sits at the mean degree. In this case, the effects of costs vanish and the relationship reduces to:

$$\left.\lim_{\rho \to 1} s^*\right|_{\beta=1} = \frac{B-1}{B} \tag{17}$$

which indicates that in large enough clusters where Sharers can expect their sharing partners to have the same amount of partners as them on average, the optimal sharing norm will grow asymptotically to 1 as the size of the surplus increases (with respect to the baseline).

3.3 Inequality, degree variance and assortment

In this section we argue that the case of an agent sitting at the average degree ($\beta = 1$), combined with the assumption of low variance in number of connections within a cluster, is not only convenient for model analysis but also of particular importance, as it minimizes conflicts of interest between agents within a cluster.

Individuals that accrue more connections with respect to the mean degree of the cluster will be at an overall advantage over most of the other cluster members. This is due to them getting more shared benefits out of the networks than what they are "sharing in" (represented here by $\beta > 1$), although they are also paying more costs. Individuals below the mean degree, on the other hand, will on average be getting less from their sharing partners than what they are sharing in ($\beta < 1$). This situation of inequality in social capital is plausible

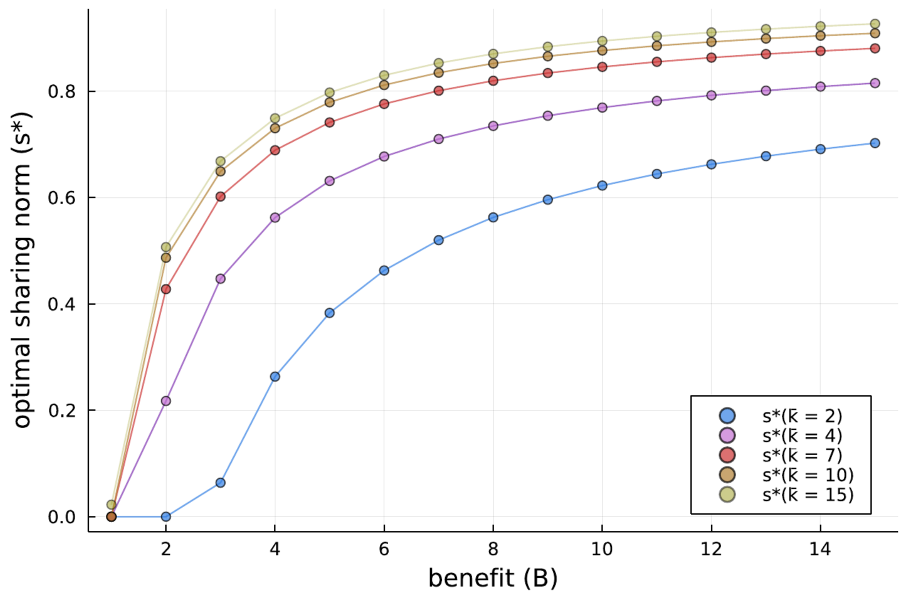

Figure 5: Increase of optimal sharing norms with increasing B at different average network size $\bar{k}$. Notice how network size mediates the asymptotic relationship between sharing norms and surplus. Plotted for $u = 0.5$ and $\beta = 1$.

in real life, as when some individuals hold special privileges that do not derive from the sharing game itself or, equivalently, when some individuals are in precarious conditions that lead them to consider accepting unequal treatment without pursuing a better deal. But in a playing field where individuals have the possibility to alter their number of connections (by, for example, joining a different cluster) then we can expect agents to seek out better opportunities for risk-pooling, leading to less tolerance for significant differences in number of connections between an agent and their partners. To add to this, the more the Sharers within a cluster differ in number of sharing partners, the more their optimal sharing norms differ from each other (this can be read from equation 16). If keeping a consensus on sharing norms is an important part of keeping a stable sharing cluster (recall that sharing norm magnitude is a factor in both optimal cluster size and growth horizon), then degree inequality is a direct destabilizing influence on sharing clusters, alongside any emerging payoff differences. Maximizing stability is not a trivial concern: the strategic advantage of reciprocal risk-pooling arrangements comes from their ability to smooth out resource variability, which can be seriously impaired by the potential social instability of possible conflicts with sharing partners.

Another concern comes from asking what should happen when agents vary in their productive conditions. Here it is useful to imagine a simple scenario: an individual X can initiate a reciprocal sharing partnership with three other individuals α, γ and ζ, such that $u_\alpha B_\alpha < u_\gamma B_\gamma = u_X B_X < u_\zeta B_\zeta$, where the subscript represents a particular agent's productive conditions, given by their surplus weighted by their success rate. It is clear that α is the least desirable partner for risk-pooling and *zeta* the most desirable one. X will then try to assort with ζ. However, if we assume that ζ is facing a similar scenario, where they can choose between three individuals from different strata, one of them being equal to ζ's and the other one of a higher stratum, then X becomes the least desirable choice. In other words, given that the population of possible sharing partners is large enough so that the number of individuals in any stratum is larger than the amount of sharing partners any particular individual can afford, then agents will be rejected when they try to "share up" with higher strata individuals, and they will be unwilling to "share down" with individuals of lower strata. Hence, assortment for risk pooling will happen preferentially with individuals in the same productive stratum.

There are imaginable situations in which it might not be the case that individuals assort themselves in a such a homophilic, pseudo-egalitarian manner when their objective is to risk-pool, as when the number of possible partners is low with respect to the maximum/desired number of sharing partners but at the same time productive heterogeneity is maintained, or when a high stratum individual might trade off having fewer connections with individuals in their same stratum in order to form more connections with individuals in lower strata, provided these lower strata individuals also hold a lower number of sharing partners on average than the higher stratum individual. These hierarchical situations are likely to require special conditions or additional power dynamics to stabilize in the face of more "egalitarian" alternatives, and a closer study of them is merited. However, we believe that the stability of the pseudo-egalitarian clusters described here (of which resource pooling is a special case, that of clusters structured as cliques, where everyone shares with everyone) makes it a likely form of risk-sharing clustering to be employed by individuals, households and communities, enough to be considered an initial assumption. Furthermore, its long-term stability and success over lone production could perhaps drive other aspects of social structure, like household size. This last consideration, however, is at this point merely speculative.

4 Discussion

The model here exposed and analysed deviates from common ways of modeling risk-sharing by taking an (admittedly minimal) network perspective and incorporating per-connection costs of relationship maintenance. This imposes constraints on cooperation that go on to have important effects on both the optimal and the maximum possible sizes (average number of connections) a sharing cluster can achieve. It also allows for an initial exploration of the effects of inequality within clusters and the conflicts of interest that may arise, allowing us to pursue the argument that reciprocal sharing clusters can be expected, at least as an initial assumption, to involve individuals in similar productive conditions who do not greatly differ between each other in number of sharing partnerships.

Extending this work will have to involve incorporating other aspects of network structure, such as clustering, especially when it comes to maintaining stability of the cooperation within networks. Certain network structures can favor the monitoring and enforcement of norms necessary to keep a working cluster of cooperators, as often the same ties that support resource flows will also give way to the information flows that form the basis of reputations, monitoring and punishment, and the characteristic costs of these actions can affect what kind of structure supports cooperation. While a model extension in this direction is warranted, it can be noted that Zefferman has explored the network structures that support cooperation promoted by monitoring and punishment, showing that when punishment costs are high and monitoring costs low, distributed organizations, characterized by higher density networks with low variance in degree, support effective cooperation better than more hierarchical structures (Zefferman, 2022). This opens the possibility that cooperation maintenance itself might be a driver of low degree variance, distributed sharing clusters, in addition to the conflicts of interests that hierarchy can spark.

The current model produces expressions that can be used as distinct hypotheses of how sharing clusters change with socioecological parameters like resource variability. This permits us to make gross predictions of what we should observe if clusters are forming in one way or another. What happens if clusters grow until they reach their limits? What happens if they do not grow beyond their optimal size? What happens if they reach their optimal sharing norms? The derived approximations can be used to address these questions empirically. However, the group-level cultural evolutionary processes that drive the formation of sharing clusters might not lead to population convergence on these points of interest, even if they are important for the processes involved. In order to address this, dynamics must be introduced that specify the way clusters grow and compete in an ecology of clusters. What would happen if individuals compete for connections within clusters, or if clusters can fission into smaller "offspring" clusters at particular stages in their growth trajectory, or if newly-formed clusters imitate the sharing norms of currently-successful clusters? A future extension of this work will address these questions using an Agent-Based framework, in order to simulate the evolution of distributions of sharing cluster sizes and norms, and using these estimates as additional models to be explored with existing data on sharing networks.

At a glance, the model seems to be consistent with past and present studies of sharing in human communities. When it comes to food sharing, an observed relationship of increase in the proportion of resource shared with increased package size (Kaplan et al., 2005; Gurven, Allen-Arave, Hill, & Hurtado, 2001; Gurven, Hill, & Kaplan, 2002) is a special case of our estimate of optimal sharing norm. In its more general form, this increase is mediated by number of sharing partners, which can motivate further empirical inquiry on this subject. The breakdown of sharing under conditions of high risk or high security also has some empirical precedent (Wiessner et al., 1982), as does an increased number of sharing partnerships with increases in productivity and variability (Gurven, Jaeggi, Von Rueden, Hooper, & Kaplan, 2015; Howe, Murphy, Gerkey, & West, 2016; Scaggs, Gerkey, & McLaughlin, 2021; Ready, 2018; Ready & Power, 2018) and a tendency for assortment by productive conditions. Size of sharing clusters has also been observed to be independent of population size, with a study observing more, but not larger, sharing clusters with larger population size in Agta and BaYaka communities (Dyble et al., 2016), which is consistent with the model's prediction that neither optimal nor maximum cluster size will depend on population size (unless population size is smaller than these quantities), but the maximum number of sharing clusters in a given population will.

While these observed parallels between model and empirical research are compelling, they must be examined more closely and with more explicit connections to both model and data structure before the links between them can be given proper consideration. We think this is enough to motivate further exploration of these relationships in light of the model, and that the observed preliminary links between model and empirical research make it a promising enterprise.

References

Alderman, H., & Paxson, C. H. (1994). Do the poor insure? a synthesis of the literature on risk and consumption in developing countries. *Economics in a changing world*, 48–78.

Andras, P., Lazarus, J., & Roberts, G. (2007). Environmental adversity and uncertainty favour cooperation. *BMC Evolutionary Biology*, *7*(1), 1–8.

Aspelin, P. L. (1979). Food distribution and social bonding among the mamainde of mato grosso, brazil. *Journal of Anthropological Research*, *35*(3), 309–327.

Betzig, L. (1988). Redistribution: Equity or exploitation. *Human reproductive behavior: A Darwinian perspective*, 49–63.

Bird, R. B., Bird, D. W., Smith, E. A., & Kushnick, G. C. (2002). Risk and reciprocity in meriam food sharing. *Evolution and Human Behavior*, *23*(4), 297–321.

Bird, R. L. B., & Bird, D. W. (1997). Delayed reciprocity and tolerated theft: The behavioral ecology of food-sharing strategies. *Current anthropology*, *38*(1), 49–78.

Blumenstock, J. E., Fafchamps, M., & Eagle, N. (2011). Risk and reciprocity over the mobile phone network: evidence from rwanda. *Available at SSRN 1958042*.

Boyd, R., & Richerson, P. J. (1988). The evolution of reciprocity in sizable groups. *Journal of theoretical Biology*, *132*(3), 337–356.

Buston, P. M., & Balshine, S. (2007). Cooperating in the face of uncertainty: a consistent framework for understanding the evolution of cooperation. *Behavioural Processes*, *76*(2), 152–159.

Cashdan, E. A. (1985). Coping with risk: Reciprocity among the basarwa of northern botswana. *Man*, 454–474.

Chagnon, N. A. (2012). *The yanomamo*. Cengage Learning.

Delton, A. W., Krasnow, M. M., Cosmides, L., & Tooby, J. (2011). Evolution of direct reciprocity under uncertainty can explain human generosity in one-shot encounters. *Proceedings of the National Academy of Sciences*, *108*(32), 13335–13340.

Dowling, J. H. (1968). Individual ownership and the sharing of game in hunting societies 1. *American Anthropologist*, *70*(3), 502–507.

Dyble, M., Thompson, J., Smith, D., Salali, G. D., Chaudhary, N., Page, A. E., ... Migliano, A. B. (2016). Networks of food sharing reveal the functional significance of multilevel sociality in two hunter-gatherer groups. *Current Biology*, *26*(15), 2017–2021.

Ember, C. R., Skoggard, I., Ringen, E. J., & Farrer, M. (2018). Our better nature: Does resource stress predict beyond-household sharing? *Evolution and Human Behavior*, *39*(4), 380–391.

Fehr-Duda, H., Bruhin, A., Epper, T., & Schubert, R. (2010). Rationality on the rise: Why relative risk aversion increases with stake size. *Journal of Risk and Uncertainty*, *40*(2), 147–180.

Gerkey, D. (2013). Cooperation in context: Public goods games and post-soviet collectives in kamchatka, russia. *Current Anthropology*, *54*(2), 144–176.

Gurven, M., Allen-Arave, W., Hill, K., & Hurtado, A. M. (2001). Reservation food sharing among the ache of paraguay. *Human Nature*, *12*(4), 273–297.

Gurven, M., Hill, K., & Kaplan, H. (2002). From forest to reservation: Transitions in food-sharing behavior among the ache of paraguay. *Journal of anthropological research*, *58*(1), 93–120.

Gurven, M., Jaeggi, A. V., Von Rueden, C., Hooper, P. L., & Kaplan, H. (2015). Does market integration buffer risk, erode traditional sharing practices and increase inequality? a test among bolivian forager-farmers. *Human ecology*, *43*(4), 515–530.

Hames, R. (2017). Reciprocal altruism in yanomamö food exchange. In *Adaptation and human behavior* (pp. 397–416). Routledge.

Hames, R., & McCabe, C. (2007). Meal sharing among the ye'kwana. *Human Nature*, *18*(1), 1–21.

Hantula, D. A. (2012). Consumers are foragers, not rational actors: Towards a behavioral ecology of consumer choice. In *Handbook of developments in consumer behaviour* (pp. 549–577). Edward Elgar Publishing.

Howe, E. L., Murphy, J. J., Gerkey, D., & West, C. T. (2016). Indirect reciprocity, resource sharing, and environmental risk: Evidence from field experiments in siberia. *PloS one*, *11*(7), e0158940.

Kaplan, H., Gurven, M., Hill, K., Hurtado, A. M., et al. (2005). The natural history of human food sharing and cooperation: a review and a new multi-individual approach to the negotiation of norms. *Moral sentiments and material interests: The foundations of cooperation in economic life*, *6*, 75–113.

Kent, S. (1993). Sharing in an egalitarian kalahari community. *Man*, 479–514.

Koster, J., Leckie, G., Miller, A., & Hames, R. (2015). Multilevel modeling analysis of dyadic network data with an application to ye'kwana food sharing. *American journal of physical anthropology*, *157*(3), 507–512.

Lee, Y.-J., Parish, W. L., & Willis, R. J. (1994). Sons, daughters, and intergenerational support in taiwan. *American journal of sociology*, *99*(4), 1010–1041.

McCloskey, D. (1991). The prudent peasant: new findings on open fields. *The Journal of Economic History*, *51*(2), 343–355.

Nolin, D. A. (2010). Food-sharing networks in lamalera, indonesia. *Human Nature*, *21*(3), 243–268.

Offer, A. (1997). Between the gift and the market: the economy of regard. *The Economic history review*, *50*(3), 450–476.

Peters, O., & Adamou, A. (2022). The ergodicity solution of the cooperation puzzle. *Philosophical Transactions of the Royal Society A*, *380*(2227), 20200425.

Platteau, J.-P. (1997). Mutual insurance as an elusive concept in traditional rural communities. *The Journal of Development Studies*, *33*(6), 764–796.

Price, M. H., & Jones, J. H. (2020). Fitness-maximizers employ pessimistic probability weighting for decisions under risk. *Evolutionary Human Sciences*, *2*.

Ramos, A. R. (1995). *Sanumá memories: Yanomami ethnography in times of crisis*. University of Wisconsin Press.

Ready, E. (2018). Sharing-based social capital associated with harvest production and wealth in the canadian arctic. *PloS one*, *13*(3), e0193759.

Ready, E., & Power, E. A. (2018). Why wage earners hunt: food sharing, social structure, and influence in an arctic mixed economy. *Current Anthropology*, *59*(1), 74–97.

Rosenzweig, M. R. (1988). Risk, implicit contracts and the family in rural areas of low-income countries. *The Economic Journal*, *98*(393), 1148–1170.

Sahlins, M. D. (1963). Poor man, rich man, big-man, chief: political types in melanesia and polynesia. *Comparative studies in society and history*, *5*(3), 285–303.

Scaggs, S. A., Gerkey, D., & McLaughlin, K. R. (2021). Linking subsistence harvest diversity and productivity to adaptive capacity in an alaskan food sharing network. *American Journal of Human Biology*, *33*(4), e23573.

Smith, E. A., & Boyd, R. (2019). Risk and reciprocity: Hunter-gatherer socioecology and the problem of collective action. In *Risk and uncertainty in tribal and peasant economies* (pp. 167–191). Routledge.

Townsend, R. M. (1994). Risk and insurance in village india. *Econometrica: journal of the Econometric Society*, 539–591.

Udry, C. (1994). Risk and insurance in a rural credit market: An empirical investigation in northern nigeria. *The Review of Economic Studies*, *61*(3), 495–526.

Wiessner, P., et al. (1982). Risk, reciprocity and social influences on !kung san economics. In *Politics and history in band societies* (pp. 61–84). Cambridge University Press.

Winterhalder, B. (1986). Diet choice, risk, and food sharing in a stochastic environment. *Journal of anthropological archaeology*, *5*(4), 369–392.
Winterhalder, B. (1993). Work, resources and population in foraging societies. *Man*, 321–340.
Woodburn, J. (1982). Egalitarian societies. *Man*, 431–451.
Wu, Z., & Ma, X. (2017). Money as a social currency to manage group dynamics: Red packet gifting in chinese online communities. In *Proceedings of the 2017 chi conference extended abstracts on human factors in computing systems* (pp. 2240–2247).
Zefferman, M. (2022). How cooperation constructs hierarchical and distributed organizations: a network approach.
Ziker, J., & Schnegg, M. (2005). Food sharing at meals. *Human nature*, *16*(2), 178–210.