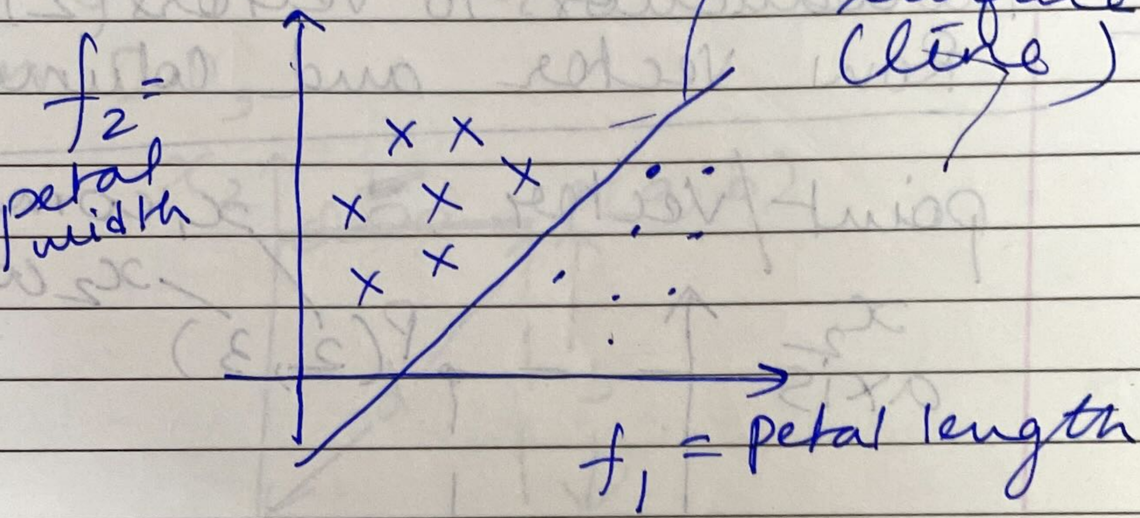
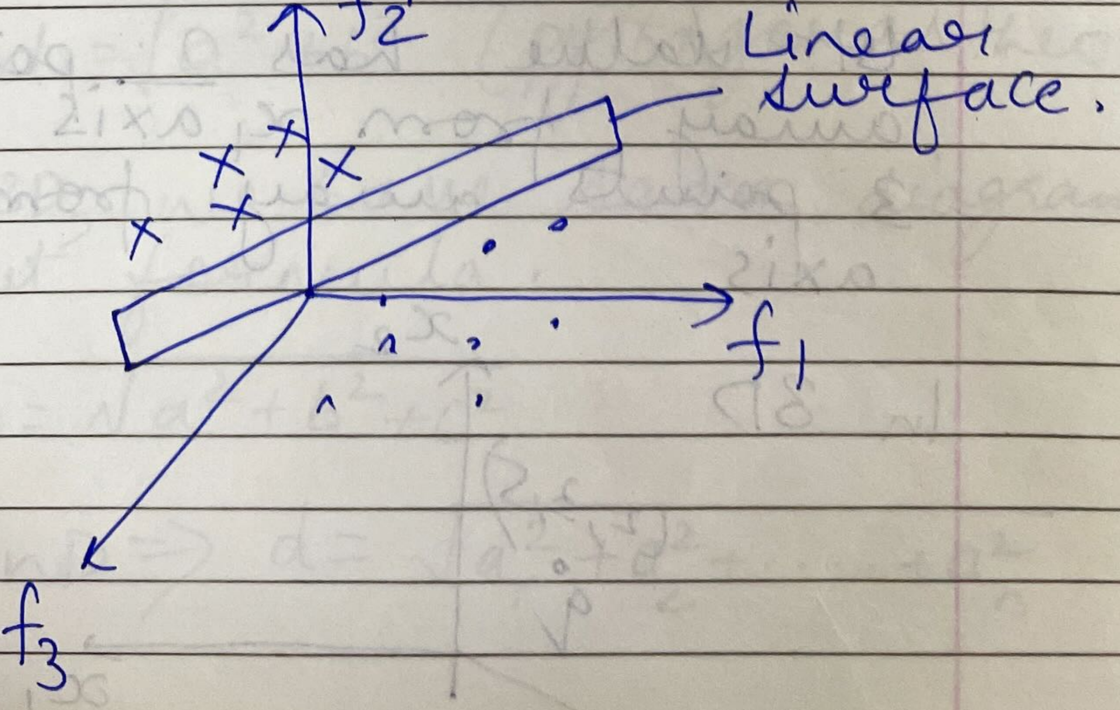
Linear Algebra

V1: Why learn it?

In a 2-D dataset lets have graph



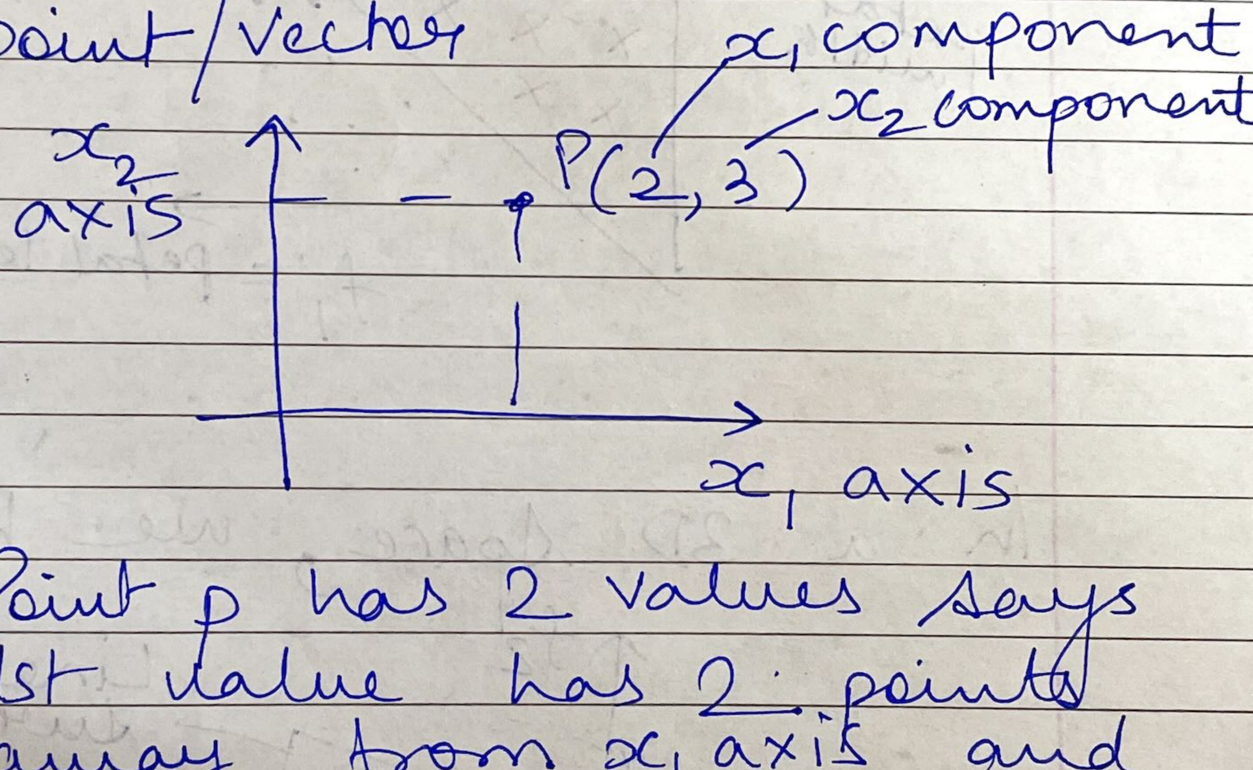
In a 3D Space, we have



How do we generalize linear surface to high dimensional space.

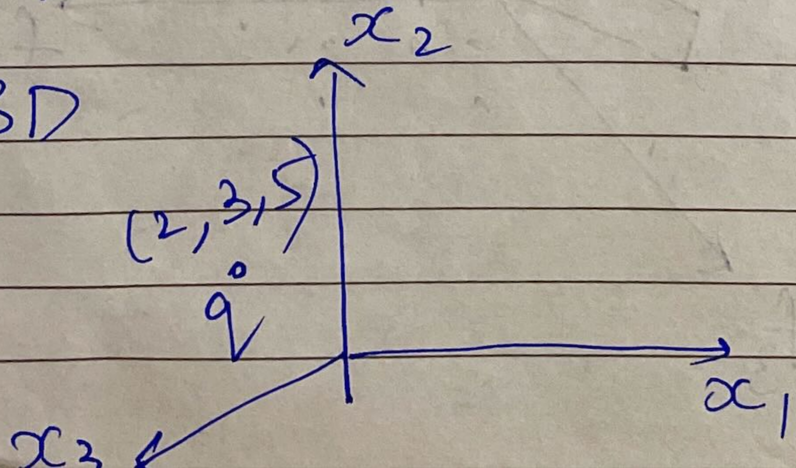
V2: Introduction to Vectors (2D, 3D, nD)
Row vector and column vector

point / vector



Point p has 2 values says 1st value has 2 points away from x_1 axis and 3 points away from x_2 axis

In 3D



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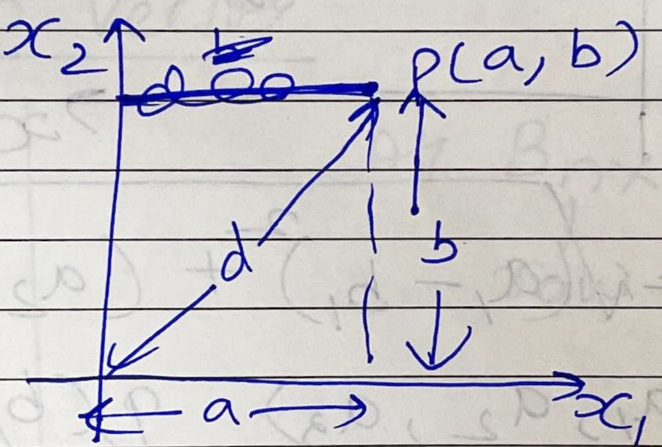
what about nD point?

Distance of a point from Origin?

In 2D,



d = distance from origin & p.

$d = \sqrt{a^2 + b^2}$ (pythagoreas theom.)

In 3D, we have same diagram but formula.

$d = \sqrt{a^2 + b^2 + c^2}$

In nD $\Rightarrow$ $d = \sqrt{a_1^2 + a_2^2 + \dots + a_n^2}$

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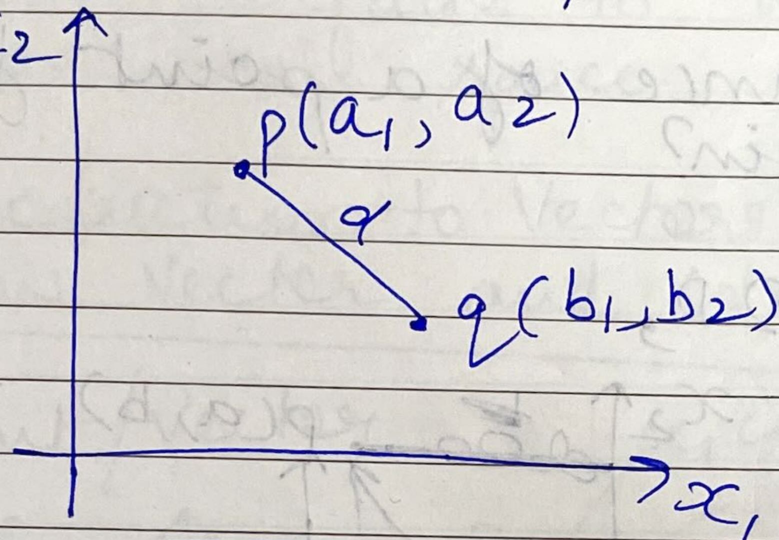
Distance b/w 2-points



2D $\quad d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$

3D $\quad p(a_1, a_2, a_3) \quad q(b_1, b_2, b_3)$

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

nD $\quad p(a_1, a_2, \dots, a_n) \quad q(b_1, b_2, \dots b_n)$

$$\cancel{d = \sqrt{(a_i - b_i)^2}}$$

$$d_{pq} = \mathrm{sqrt}\left(\sum_{i=1}^{n} (a_i - b_i)^2\right)$$

Row vector

$$A = [a_1, a_2, a_3, \dots, a_n]_{1\times n}$$

1 rows, n columns

$A_{1\times n}$

Column vector

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{n\times 1}$$

or $B_{n\times 1}$ — n row, 1 column

Matrices are represented as $M_{m\times n}$

" are double array

or array of arrays.

V3 Dot product and Angle b/w 2 vectors

Addition of vectors

$a = [a_1, a_2 - - - . a_n]$
$b = [b_1, b_2 - - - - . b_n]$

$C = a + b = [a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n]$

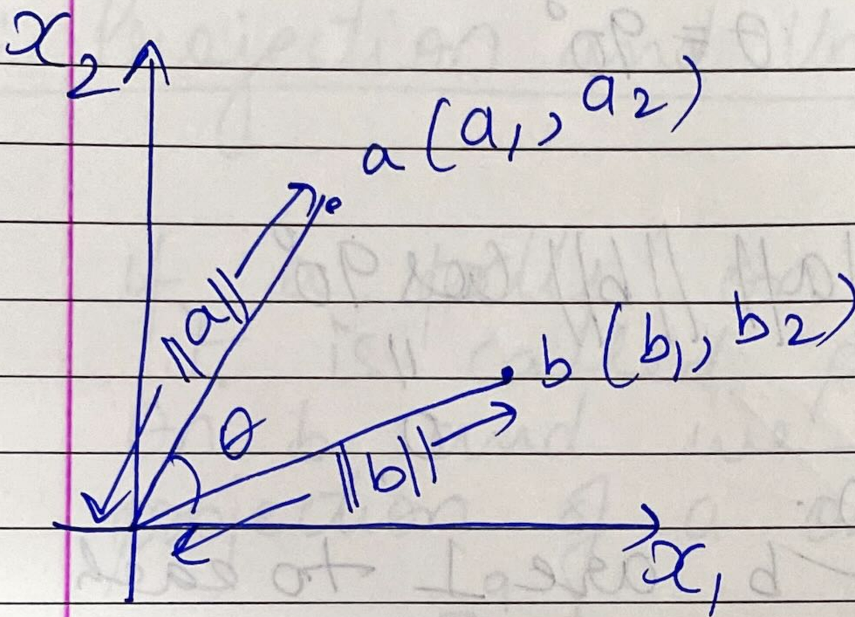
Multiplication - Dot product in ML

$a \cdot b = [a_1 b_1 + a_2 b_2 + \cdots a_n b_n]$

$$= \underset{1 \times n}{\begin{bmatrix} a_1, a_2, \cdots a_n \end{bmatrix}} \cdot \underset{n \times 1}{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}}$$

n should be same

If a_n it means it is a column vector
That is why we take transpose to do Dot product.



$a \cdot b = \|a\| \|b\| \cos\theta$

length of a = distance of a from origin.

length of b = distance of b from origin.

Or $a \cdot b = a_1 b_1 + a_2 b_2$

$= \|a\| \|b\| \cos\theta$

$$\theta = \cos^{-1}\left\{\frac{a_1 b_1 + a_2 b_2}{\|a\| \|b\|}\right\}$$

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what if $\theta = 90^\circ$

then

$a \cdot b = ||a|| \, ||b|| \underset{\substack{\parallel \\ 0}}{\cos 90^\circ}$

$a \cdot b = 0$.

So a & b are $\perp$ to each other.

In nD $\theta = \cos^{-1}\left(\dfrac{\sum_{i=1}^{n} a_i b_i}{||a|| \, ||b||}\right)$

& $a, b = \sum_{i=1}^{n} a_i b_i = 0$

if $a \cdot b = 0$, they are $\perp$

To find $a \cdot a = a_1 a_1 + a_2 a_2 + \dots + a_n a_n$

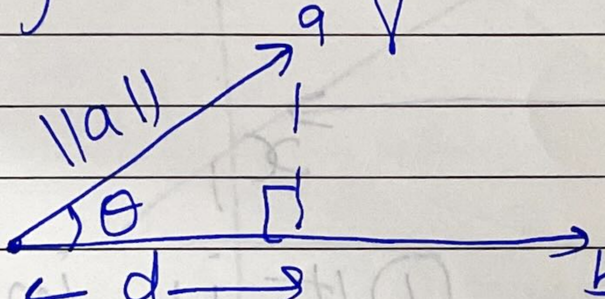
$= a_1^2 + a_2^2 + \dots + a_n^2$

$= ||a||^2$ which is square of distan

V4: Projection and Unit Vector

If a falls on b ⊥ly, it is called projecting a on b and we get projection of a on b as

$$d = ||a|| \cos\theta \quad —(1)$$



$$a \cdot b = \sum_{i=1}^{n} a_i b_i = ||a|| \, ||b|| \cos\theta$$

$$d = \frac{a \cdot b}{||b||} = \frac{||a|| \cancel{||b||} \cos\theta}{\cancel{||b||}}$$

So projection of a on b is

$$\boxed{d = \frac{a \cdot b}{||b||}}$$

a from origin

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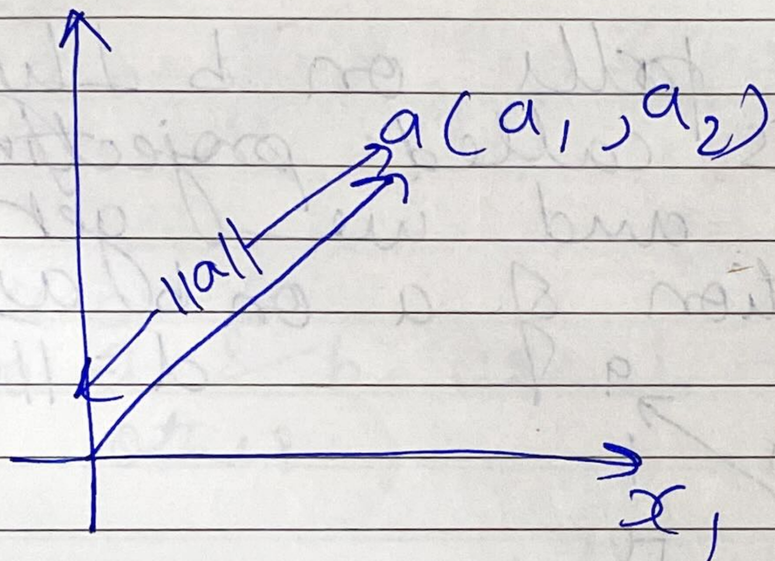
Unit Vector



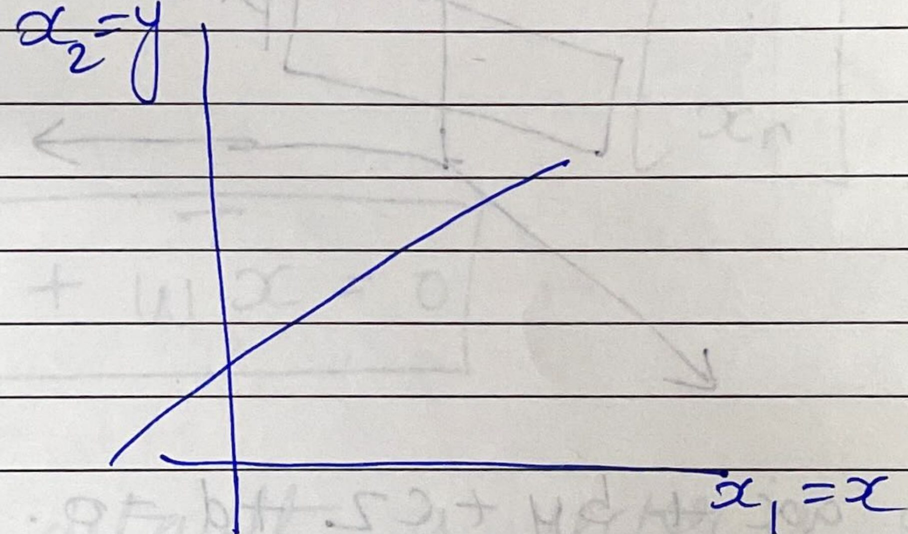
$$\hat{a} = \frac{a}{||a||}$$

Unit vector

① It is in same direction to the vector a.

② length of unit vector = 1.
So $||\hat{a}|| = 1$.

V5 Equation of a line (2-D), Plane (3D) and Hyperplane (nD), Plane passing through origin, Normal to a plane



In 2D $\quad y = mx + c$

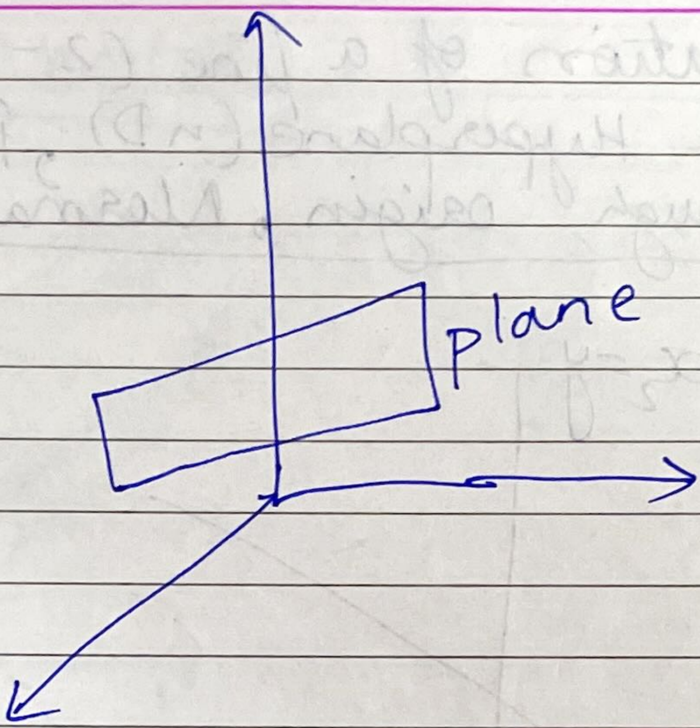
General form $\Rightarrow ax + by + c = 0$.

$$y = \underbrace{\boxed{-\frac{c}{b}}}_{c} - \underbrace{\boxed{\frac{a}{b}}}_{m} x$$

$$ax_1 + bx_2 + c = 0$$

$w_1 x_1 + w_2 x_2 + w_0 = 0$. Eq of line in 2D.

In 3D



$$ax + by + cz + d = 0.$$

In 3D plane $\rightarrow$ $w_1 x_1 + w_2 x_2 + w_3 x_3 + w_0 = 0.$

In nD hyperplane eq. is :-

$\Rightarrow w_0 + w_1 x_1 + w_2 x_2 + \cdots + w_n x_n = 0.$

OR

$$\Rightarrow \boxed{w_0 + \sum_{i=1}^{n} w_i x_i = 0.}$$

OR
Vector notation

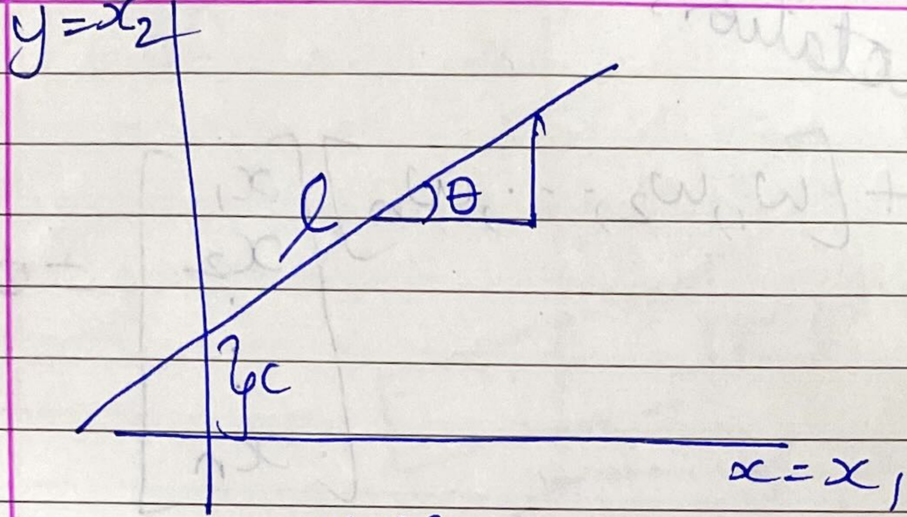
$$w_0 + [w_1, w_2, \cdots, w_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = 0.$$

$$\boxed{w_0 + w^T x = 0}$$

By default a vector is a column vector. So if I write a vector in ~~row~~ column form I have to multiply then I take transpose of it.

Planes are written as Π known as Pie.

Now $\boxed{w_0 + w^T_{n\times1} x_{n\times1} = 0.}$



$$y = \overset{\text{slope}}{m}x + \underset{\text{y intercept}}{c}$$

In 2D $\quad w_1 x_1 + w_2 x_2 + w_0 = 0$

$$\underset{y}{x_2} = \underset{c}{\boxed{\frac{-w_0}{w_2}}} - \underset{m}{\boxed{\frac{w_1}{w_2}}} \underset{x}{x_1}$$

$y \quad c + m x$

l is passing through $c = 0$.

$$c = -\frac{w_0}{w_2}$$

If $c = 0 \Rightarrow w_0 = 0$.

So line passing through origin is $w_1 x_1 + w_2 x_2 = 0$ (2D)

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$3D = w_1 x_1 + w_2 x_2 + w_3 x_3 = 0.$

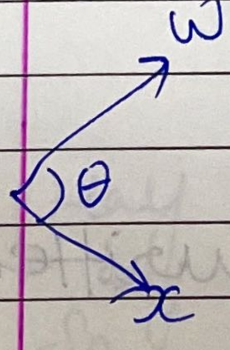
hyperplane $nD =$
$w_1 x_1 + w_2 x_2 + \dots + w_n x_n = 0$

$\boxed{w^T x = 0}$ — plane passing through origin.

We know

$\Pi_n : w^T x = 0.$

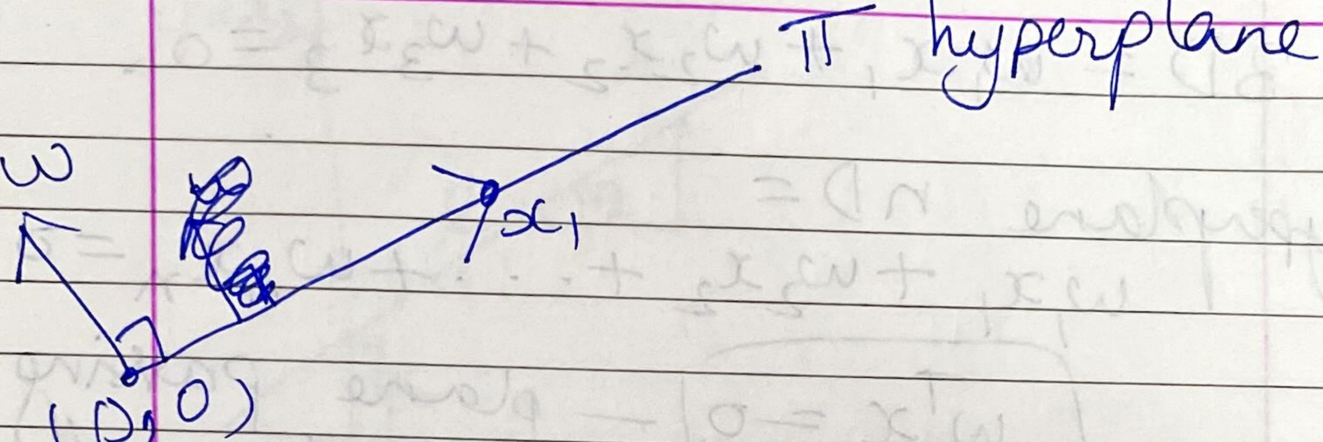
$$w = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$



$w \cdot x = w^T x = ||w|| \, ||x|| \cos\theta_{w,x}$

$= 0.$

$w \perp x \Rightarrow \theta_{w,x} = 90^\circ$



$w \cdot x_1 = 0.$ (since ⊥)

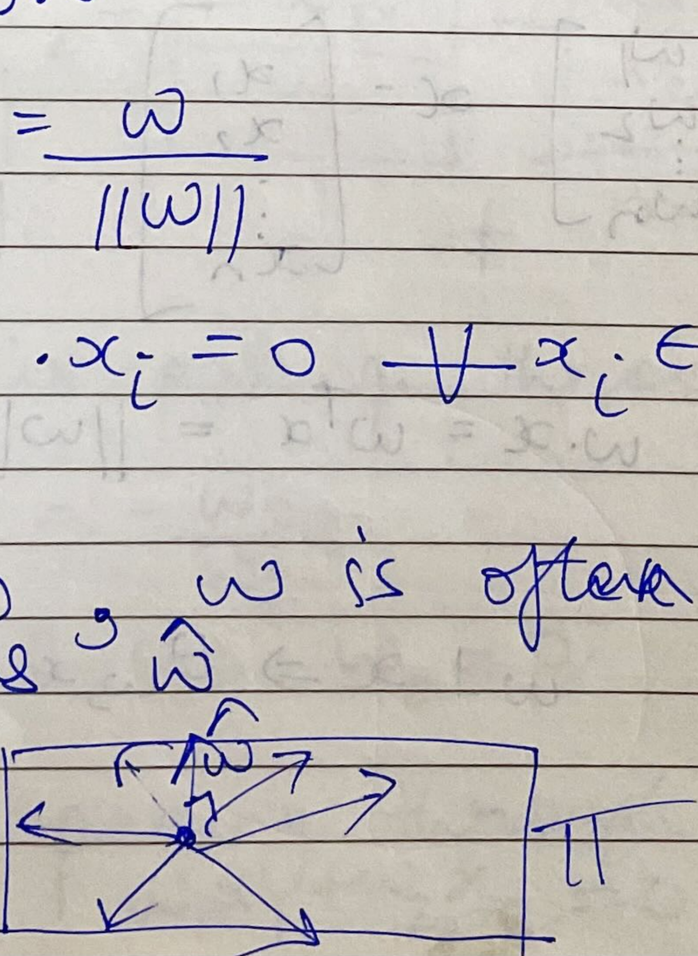
If $w \perp \pi$
then $w \cdot x_i = 0 \ \forall \ x_i \in \pi_{\text{plane}}$

we know.

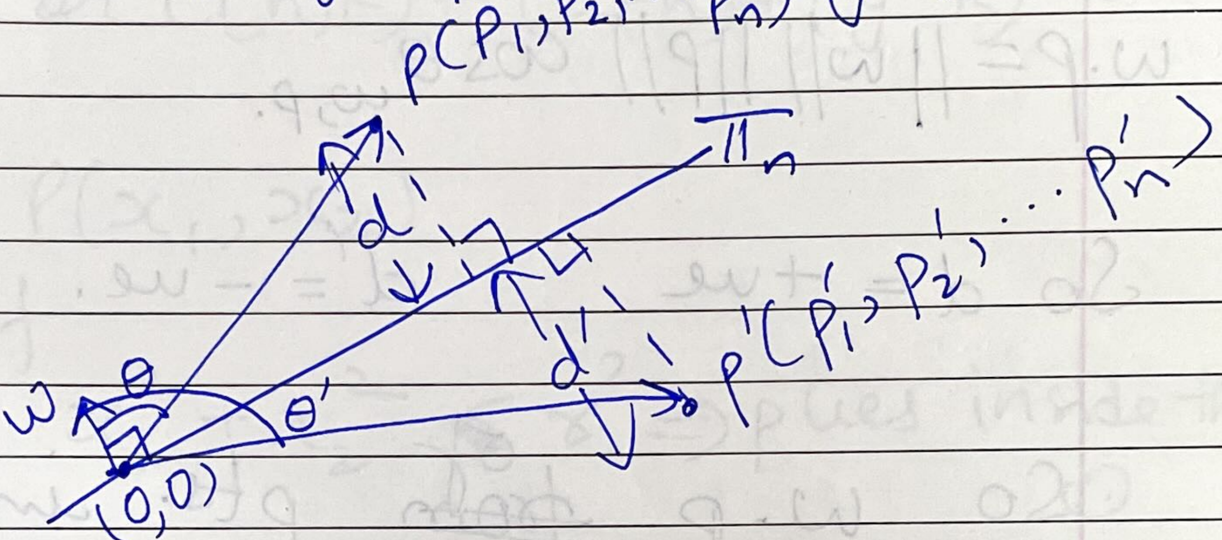
$$\hat{w} = \frac{w}{\|w\|}$$

$\hat{w} \cdot x_i = 0 \ \forall \ x_i \in \pi$

So, w is often written as, $\hat{w}$



V6: Distance of a point from a Plane/Hyperplane, Half Spaces



$w^T x = 0.$

Draw $\perp$ from pt. to plane to find distance.

So, $\boxed{d = \frac{w^T p}{\|w\|}}$

Half spaces – 2 regions separated by planes in nD.
So Hyperplane creates half spaces.

$d = \frac{w \cdot p}{\|w\|}$ $\qquad$ $d' = \frac{w \cdot p'}{\|w\|}$

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$\theta < 90°$ $\quad\quad$ $\theta' > 90°$

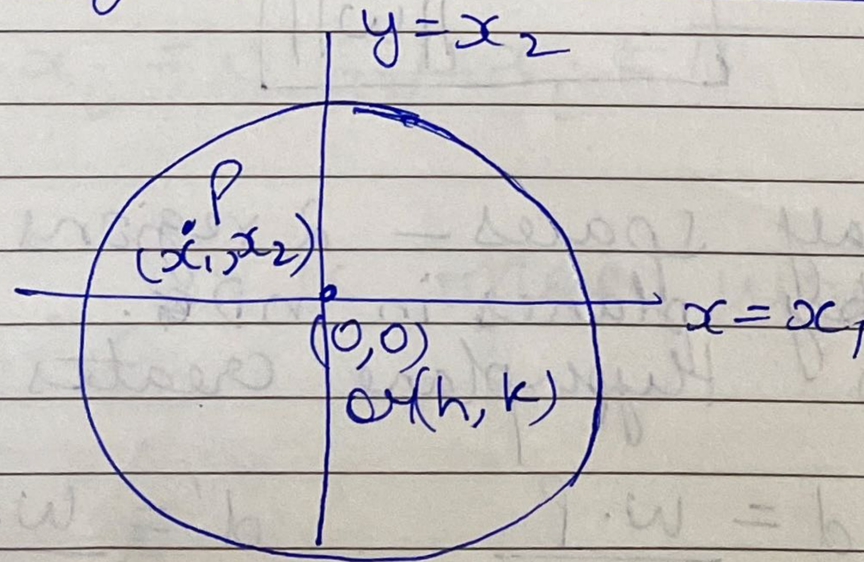
$w.p = \|w\| \, \|p\| \cos\theta_{w,p}$

So $d = +ve$ $\quad\quad$ $d' = -ve$.

So $w.p$ ~~from~~ of pts. in same direction as $\perp$ to plane are $+ve$.
and $w.p'$ of pts in opp direction as $\perp$ to plane are $-ve$.

V7: Equation of Circle (2D), Sphere (3D), and Hypersphere (nD)

In 2D



Then eq. of ⊙ $\Rightarrow x^2+y^2=r^2$

or $C(h,k) \div (x-h)^2+(y-k)^2=r^2$

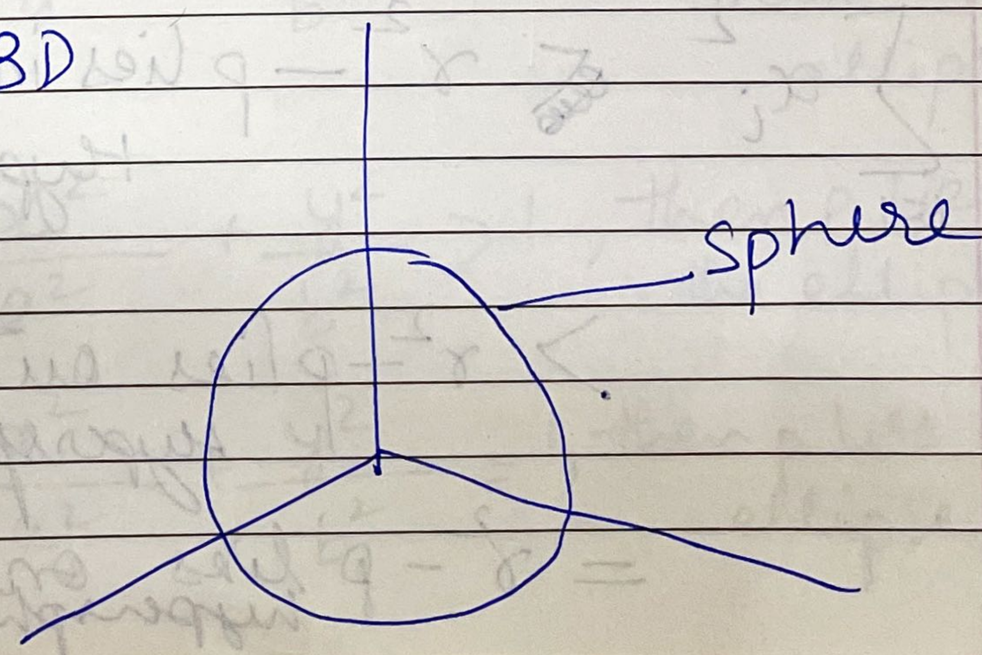
$P(x_1, x_2)$

If
$x_1^2+x_2^2 < r^2 \Rightarrow$ P lies inside the ⊙

If $x_1^2+x_2^2 > r^2 \Rightarrow$ P lies outside the ⊙

If $x_1^2+x_2^2 = r^2 \Rightarrow$ P lies on the ⊙

In 3D



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Eq. of sphere,

$$x_1^2 + x_2^2 + x_3^2 = r^2$$

nD $x_1, x_2, \cdots x_n$.

Hypersphere.

$$x_1^2 + x_2^2 + \cdots + x_n^2 = r^2$$

Or $$\boxed{\sum_{i=1}^{n} x_i^2 = r^2}$$

4 point $p = (x_1, x_2, \cdots x_n)$

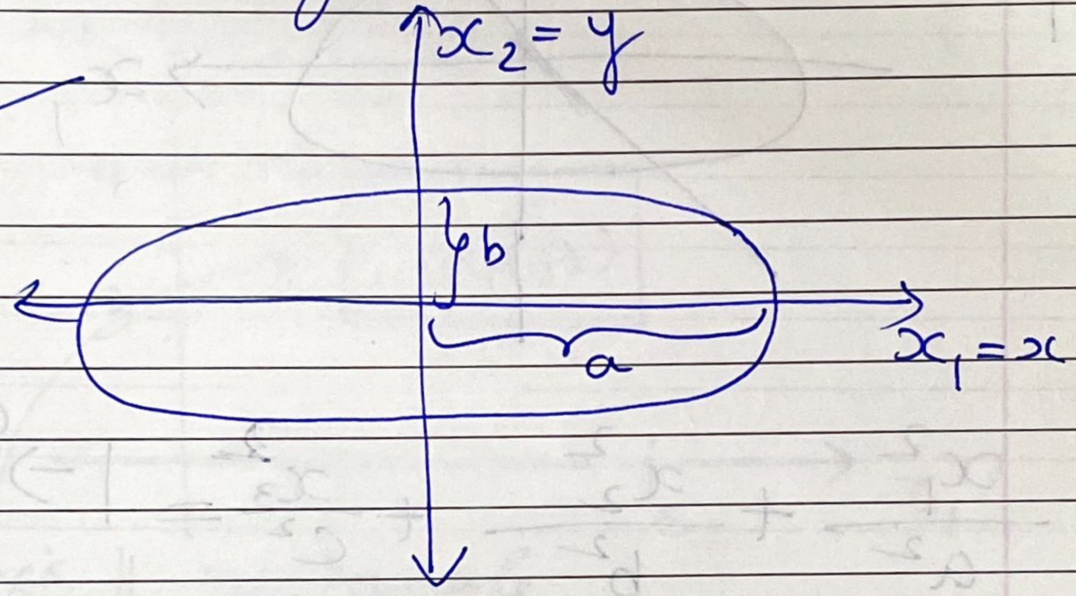
$\sum_{i=1}^{n} x_i^2 < r^2$ – p lies inside Hypersphere

$> r^2$ – p lies outside hypersphere.

$= r^2$ – p lies on the hypersphere.

V8: Equation of an Ellipse (2D), Ellipsoid (3D) and Hyperellipsoid (n-D)

~~2D~~



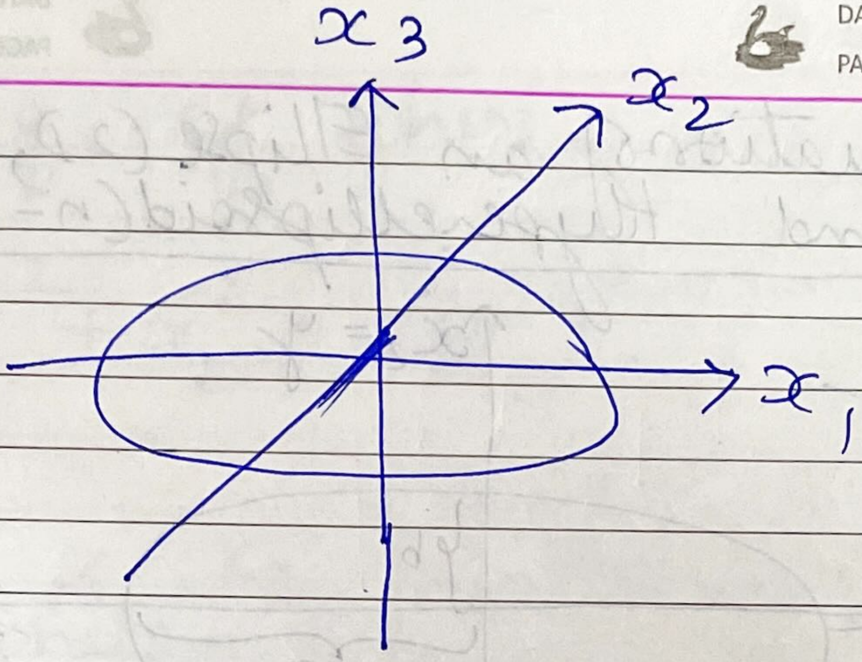
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \rightarrow \text{Eq. of ellipse.}$$

If $\frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$, then p lies inside the ellipse.

If $\frac{x^2}{a^2} + \frac{y^2}{b^2} > 1$, then p lies outside the ellipse

If $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then p lies on the ellipse.

In 3D
ellipsoid



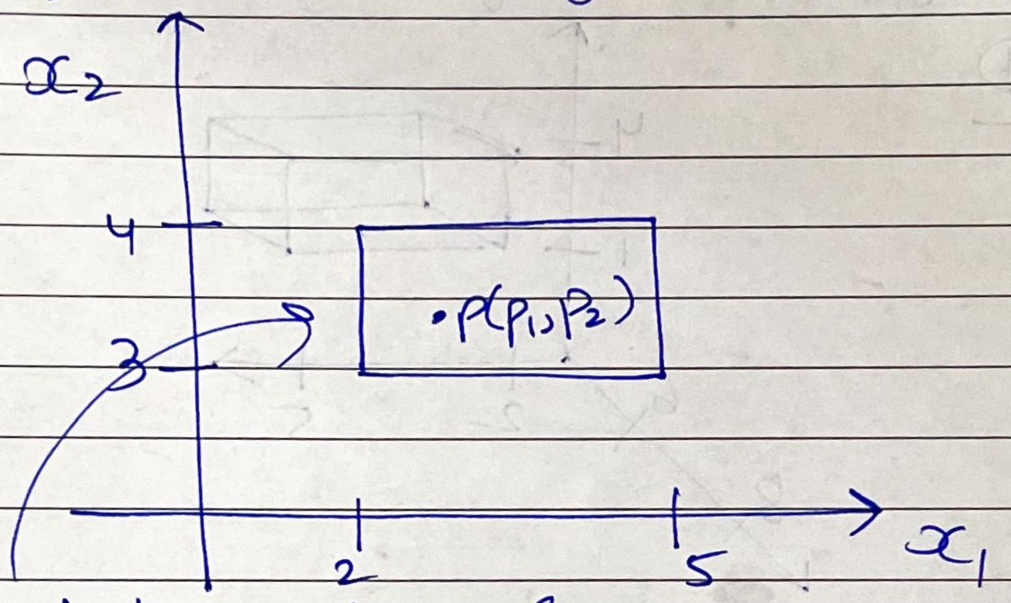
$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1 \rightarrow \text{Eq. of ellipsoid}$$

nD $$\frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2} + \cdots + \frac{x_n^2}{a_n^2} = 1$$

hyper ellipsoid.

V9: Square, Rectangle

In 2D



axis || rectangle

if $p_1 \leq 5$ && $p_1 > 2$

if $p_2 > 3$ && $p_2 < 4$

then p lies inside the rectangle.

V10: Hypercube, Hypercuboid

In 3D