EVALUATING THE *GENERAL CHORD TYPE* REPRESENTATION IN TONAL MUSIC AND ORGANISING GCT CHORD LABELS IN FUNCTIONAL CHORD CATEGORIES

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ABSTRACT

The General Chord Type (GCT) representation is appropriate for encoding tone simultaneities in any harmonic context (such as tonal, modal, jazz, octatonic, atonal). The GCT allows the re-arrangement of the notes of a harmonic sonority such that abstract idiom-specific types of chords may be derived. This encoding is inspired by the standard roman numeral chord type labelling and is, therefore, ideal for hierarchic harmonic systems such as the tonal system and its many variations; at the same time, it adjusts to any other harmonic system such as post-tonal, atonal music, or traditional polyphonic systems. In this paper the descriptive potential of the GCT is assessed in the tonal idiom by comparing GCT harmonic labels with human expert annotations (Kostka & Payne harmonic dataset). Additionally, novel methods for grouping and clustering chords, according to their GCT encoding and their functional role in chord sequences, are introduced. The results of both harmonic labelling and functional clustering indicate that the GCT representation constitutes a suitable scheme for representing effectively harmony in computational systems.

1. INTRODUCTION

Computational systems developed for harmonic analysis and/or harmonic generation (e.g. melodic harmonisation), rely on chord labelling schemes that are relevant and characteristic of particular idioms [7, 10, 20, 21, 26]. There exist various typologies for encoding note simultaneities that embody different levels of harmonic information/abstraction and cover different harmonic idioms. For instance, some commonly used chord notations in tonal music are the following: figured bass (pitch classes denoted above a bass note – no concept of 'chord'), popular music guitar style notation or jazz notation (absolute chord), roman numeral encoding (relative chord to a key) [18] - see, Harte's [12]

formal tonal chord symbol representation. For atonal and other non-tonal systems, pitch-class set theoretic encodings [8] may be employed. There exists no single chord encoding scheme that can be applied to all harmonic systems with sufficient expressiveness.

Preliminary studies on the General Chord Type (GCT) [3] representation (e.g. for probabilistic melodic harmonisation [15]) indicate that it can be used both as a means to represent accurately harmonic chords and to describe musically meaningful relations between different harmonic labels in diverse music idioms. The GCT provides accurate harmonic representation in a sense that it encompasses all the pitch-class-related information about chords. At the same time, for every pitch class simultaneity the GCT algorithm rearranges pitch classes so that it identifies a root pitch class and a chord base type and extension, leading to chord representations that convey musical meaning for diverse music idioms.

It is true that the main strength of the GCT representation is its application in non-tonal harmonic idioms; some such preliminary examples have been presented in [2, 3, 14]. This paper, however, focuses on the tonal idiom, as this provides a well-studied system with reliable ground truth data against which a chord labelling and grouping algorithm can be tested. If the GCT representation can cope with such a sophisticated hierarchical harmonic system as the tonal system, then it seems likely that it can deal with other non-tonal systems (even though other simpler representations may also be adequate). Applying and testing the GCT on other musics is part of ongoing research.

The paper at hand addresses two issues regarding the GCT representation. First, an evaluation of the GCT's ability to label chords is performed by comparing the chord roots and types it produces with human expert annotations (roman-numeral analysis) on the Kostka & Payne dataset. This analysis provides clear indications about the interpretational efficiency of the GCT (around 92% agreement with human annotations). Secondly, a grouping process is proposed, which allows the identification of the functional role of chord groups in GCT form. An initial grouping stage, solely based on the GCT expression of the chords, allowsn in a second stage, the identification of functional similarities according to first-order transitions of GCT chord groups. The results of this analysis on a set of Bach Chorales indicate that the functional role of GCT chord groups is determined in a reliable manner, agreeing with theoretic

functional characteristics of chords in this idiom.

2. THE GENERAL CHORD TYPE REPRESENTATION

Harmonic analysis is a rather complex musical task that involves not only finding roots and labelling chords within a key, but also segmentation (points of chord change), identification of non-chord notes, metric information and more generally musical context [27]. In this section, we focus on the core problem of labelling chords within a given pitch hierarchy (e.g. key). We assume, for simplicity, that a full harmonic reduction (main harmonic notes) is available as input to the model along with key/modulation annotations. It is suggested that the GCT representation scheme can be used in the future so as to facilitate the harmonic reduction per se of an unreduced musical surface (e.g. by identifying dissonant chord extensions in relation to a chord's consonant base).

The General Chord Type (GCT) representation, allows the re-arrangement of the notes of a harmonic simultaneity such that a maximal consonant part determines the base of the chord, and the rest of the dissonant notes form the chord extension; the lowest note of the base is the root of the chord. The GCT representation has common characteristics with the stack-of-thirds and the virtual pitch root finding methods for tonal music, but has differences as well (see [3]). This encoding is inspired by the standard roman numeral chord type labelling, but is more general and flexible. A brief description of merely the GCT core algorithm is presented below (due to space limitations); a more extended discussion on the background concepts necessary for the GCT model as well as a more detailed description of the GCT representation are presented in [3].

2.1 Description of the GCT Algorithm

Given a classification of intervals into consonant/dissonant (binary values) and an appropriate scale background (i.e. scale with tonic), the GCT algorithm computes, for a given multi-tone simultaneity, the 'optimal' ordering of pitches such that a maximal subset of consonant intervals appears at the 'base' of the ordering (left-hand side) in the most compact form; the rest of the notes that create dissonant intervals to one or more notes of the chord 'base' form the chord 'extension'. Since a tonal centre (key) is given, the position within the given scale is automatically calculated.

Input to the algorithm is the following:

- Consonance vector: a Boolean 12-dimensional vector is employed indicating the consonance of pitch-class intervals (from 0 to 11). E.g., the vector $[1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0]$ means that the unison, minor and major third, perfect fourth and fifth, minor and major sixth intervals are consonant – dissonant intervals are the seconds, sevenths and the tritone; this specific vector is referred to in this text as the tonal consonance vector.
- Pitch Scale Hierarchy: is given in the form of scale tones and a tonic. E.g., a D major scale is given as:

Table 1. GCT chord labelling example

Input: Bb major scale: $[10, [0, 2, 4, 5, 7, 9, 11]]$
Input: Consonance vector: $[1, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 0]$
– input $[53, 63, 69, 72, 75]$
– input converted to pc-set: $[0, 3, 5, 9]$
– maximal consonant subset: $[0, 5, 9]$
– rewrite in narrowest range: $[5, 9, 0]$
– Dissonant tone 3 goes to the end: $[5, 9, 0, 3]$
– Lowest tone is root, i.e. 5 (note F)
– Chord with root 0: $[0, 4, 7, 10]$ (i.e., dominant seventh)
– Absolute chord: $[5, [0, 4, 7, 10]]$ (i.e., F7)
– Relative position: root is 7 semitones above the tonic Bb
– Chord in relative position: $[7, [0, 4, 7, 10]]$
– No other maximal subset exists.
Output: $[7, [0, 4, 7, 10]]$ (i.e. V7)

$2, [0, 2, 4, 5, 7, 9, 11]$, or an A minor pentatonic scale as: $9, [0, 3, 5, 7, 10]$

- Input chord: list of pitch classes (MIDI pitch numbers modulo 12).

Algorithm 1 GCT algorithm (core) – computational pseudocode

Require: (**i**) the pitch scale (tonality), (**ii**) a vector of the intervals considered consonant, (**iii**) the pitch class set (pc–set) of a note simultaneity

Ensure: The roots and types of the possible chords describing the simultaneity

1: find all maximal subsets of pairwise consonant tones
2: select maximal subsets of maximum length
3: **for** all selected maximal subsets **do**
4:      order the pitch classes of each maximal subset in the most compact form (chord 'base')
5:      add the remaining pitch classes (chord 'extensions') above the highest of the chosen maximal subset's (if necessary, add octave – pitches may exceed the octave range)
6:      the lowest tone of the chord is the 'root'
7:      transpose the tones of the chord so that the lowest becomes 0
8:      find position of the 'root' in regards to the given tonal centre (pitch scale)
9: **end for**

Since the aim of this algorithm is not to perform sophisticated harmonic analysis, but rather to find a practical and efficient encoding for tone simultaneities (to be used, for instance, in statistical learning and automatic harmonic generation in the context of the project COINVENT [25]), we decided to extend the algorithm so as to reach a single chord type for each simultaneity (no ambiguity) in every case . These additional steps are described in [3] and take into account overlapping of maximal subsets and avoidance of non-scale notes in the base of chord types.

An example taken from Beethoven's *Andante Favori* (Figure 1) illustrates the application of the GCT algorithm for different consonance vectors. For the tonal vector, GCT encodes classical harmony in a straightforward manner.

All instances of the tonic chord are tagged as $[0, [0, 4, 7]]$; the dominant seventh (inverted or not) is $[7, [0, 4, 7, 10]]$; the third to last chord is a minor seventh on the second degree encoded as $[2, [0, 3, 7, 10]]$; the second and fourth chord is a Neapolitan sixth chord encoded as $[1, [0, 4, 7]]$ (which means major chord on lowered second degree) with a secondary dominant in between (the pedal G flat note in the third chord is not taken into account). This way we have an encoding that is analogous to the standard roman numeral encoding (Figure 1, 'tonal'). If the tonal context is changed to a chromatic scale context and all intervals are considered equally consonant, i.e. all entries in consonance vector are 1s, we get the second 'atonal' GCT analysis (Figure 1, 'atonal') which amounts to normal orders (not prime forms) in standard pc-set analysis. In pitch class set theory normal orders do not have roots – however, they have transposition values (T0-T11) in relation to a reference pc (normally pc 0); the normal orders with transposition values of pc-set theory are equivalent to the GCT for the atonal consonance vector. Obviously, for tonal music, this pc-set-like analysis is weak as it misses out or obscures important tonal hierarchical relationships; however, it can encode efficiently non-tonal musics. More examples from non-tonal music in $[2, 3, 14]$.

2.2 Qualitative evaluation of the GCT in tonal music

We tested the GCT algorithm on the Kostka-Payne dataset created by David Temperley. This dataset consists of the 46 excerpts that are longer than 8 measures from the workbook accompanying Kostka and Payne's theory textbook Tonal Harmony, 3rd edition (McGraw-Hill, 1995) [1] . Given the local tonality (key), the GCT algorithm was applied to all the Kostka-Payne excerpts. Then, the resulting GCTs were compared to the Kostka-Payne ground truth (i.e. the roman numeral analysis included in the Instructor's Manual, not taking into account chord inversions). From the 919 chords of the dataset, GCT successfully encodes 847 chords, and 72 chords are labelled differently. This means that the algorithm labels 92.16% of all chords correctly.

The identified mistakes can be categorised as follows:

a) Twenty three (23) mislabeled chords were diminished seventh chords $[0, 3, 6, 9]$. As explained earlier, these symmetric chords can have as their root any of the four constituent notes. In most cases these were vii^{o7} chords in various inversions, referring either to the main key or to other keys as applied chords, but in some cases they were embellishing (non functional) chords.

b) Twenty two (22) half-diminished chords $[0, 3, 6, 10]$ were labelled as minor chords with added sixth $[0, 3, 7, 9]$; e.g. $[B, D, F, A]$ was re-ordered as $[D, F, A, B]$. As a consequence, all ii$^{ø6/5}$ chords in minor keys were identified as iv^{add6} chords, and all vii^{ø7}-type chords in major keys were identified as ii^{add6} chords.

c) Seventeen (17) cases had a salient note missing (e.g. diminished chord without root, dominant seventh without third, half-diminished seventh without third, etc) and this

resulted in finding a wrong root; e.g. $[G\sharp, D, F]$, vii^{o7} in A minor without 3rd, was identified as $[D, F, A\flat]$, i.e. as iv$^{5\flat}$; $[B, F, A]$, vii^{ø7} in C major without 3rd, appears as $[F, A, B]$, i.e. IV$^{5\flat}$; $[C, E, B\flat, D\flat]$, V$^{7/9}$ in F minor, is identified as $[B\flat, D\flat, F\flat, C]$, i.e. iv$^{5\flat/9}$; $[E\flat, G, D\flat, C]$, i.e. V$^{7/13}$ in A$\flat$ major erroneously appeared as $[C, E\flat, G, D\flat]$, i.e. iii$^{9\flat}$, while $[C, E, B\flat, A\flat]$, i.e. V$^{7/13}$ in F minor appears almost correctly as $[C, E, G\sharp, B\flat]$, i.e. as V$^{5\sharp/7}$ (the difference is that in the first case the 13th interval was major).

d) Eight (8) chords were misspelled because they appeared over a pedal note (pedal notes were included in our GCT analysis, while they were omitted in Temperley's analysis); e.g. $[D, A, C\sharp, G]$, a V^7 over a tonic pedal in D major, appeared as $[A, D, G, C\sharp]$, i.e. as V$^{4/7/10}$, and $[D, C\sharp, G, B]$, a vii^{ø7} over a tonic pedal, is described as $[G, B, D, C\sharp]$, i.e. IV$^{11\sharp}$.

e) Two (2) sus4 chords $[0, 5, 7]$ were identified incorrectly as $[0, 5, 10]$; e.g. $[C, F, G]$, V^{sus4} in F major contains the dissonant interval $[F, G]$ and was erroneously re-ordered as $[G, C, F]$, i.e. as ii$^{4/7}$ (quartal chord).

On the other hand, the GCT algorithm correctly identified numerous functionally ambiguous chords, such as various cases of augmented 6th chords (mainly German types, but also Italian and French types) formed over a variety of scale degrees ($6\flat$, $2\flat$, 4, etc.). It also correctly identified most harmonic circles of fifths, applied dominants, neapolitan chords, chords produced by modal mixture and complex triadic chords (with more than four members).

Overall, in the context of tonal music and the for standard tonal consonance vector, the GCT algorithm produces quite satisfactory results. However, it makes primarily the following types of mistakes: firstly, it yields ambiguous results regarding the root of symmetric chords such as the full diminished seventh and augmented chords – to disambiguate the root for symmetrical chords (mainly for diminished seventh chords), harmonic context has to be taken into account (e.g. the root of the following chord); secondly, it assigns the wrong root to chords that have 'dissonant' intervals at their triadic base, such as diminished fifths in half-diminished chords or major second in sus4 chords; thirdly, tertian chords that have notes missing from their base (e.g. missing third in seventh chords) are misinterpreted as their upper denser part is taken as the chords base and the lower root as an extension; and, finally, pedal notes, when taken into account for the identification of the GCT type, produce complex and functionally incorrect results.

In order to correct such cases, a more sophisticated model for harmonic analysis is required, which extends the purely representational scope of the current proposal. Such a model would take into account voicing (e.g. the bass note), chord transition probabilities (functions), and, even, higher-level domain-specific harmonic knowledge (e.g. specific types of chords used in particular idioms).

The GCT algorithm captures the common-practice roman-numeral harmonic analysis encoding scheme (for the 'standard' consonance vector) reasonably well. Additionally, it

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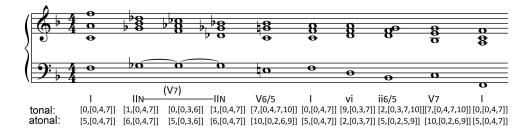|	I	IIN	(V7)	IIN	V6/5	I	vi	ii6/5	V7	I
tonal:	[0,[0,4,7]]	[1,[0,4,7]]	[0,[0,3,6]]	[1,[0,4,7]]	[7,[0,4,7,10]]	[0,[0,4,7]]	[9,[0,3,7]]	[2,[0,3,7,10]]	[7,[0,4,7,10]]	[0,[0,4,7]]
atonal:	[5,[0,4,7]]	[6,[0,4,7]]	[5,[0,3,6]]	[6,[0,4,7]]	[10,[0,2,6,9]]	[5,[0,4,7]]	[2,[0,3,7]]	[5,[0,2,5,9]]	[10,[0,2,6,9]]	[5,[0,4,7]]

Figure 1. Beethoven, *Andante Favori*, reduction of mm.189-198. Tonal and atonal GCT analysis (see text).

adapts to non-tonal systems, such as atonal, octatonic or traditional polyphonic music. The question is whether the GCT representation works well on such non-tonal systems. The GCT representation has been employed in the case of traditional polyphonic music from Epirus [14], whereby, song transcriptions were initially converted to the GCT encoding, followed by a learning HMM scheme. This scheme was then employed to learn chord transitions, which was finally used to create new harmonisations in the polyphonic style of Epirus. Ongoing research is currently studying the application of GCT on various harmonic idioms, from medieval music to 20th century music, and various pop and folk traditions.

3. GROUPING GCT CHORDS

Chord relationships, and more specifically chord similarity/distance in tonal and non-tonal music, have been studied by various music theorists/researchers; some notable examples are the work by Hindemith [13], the classification scheme by Harris [11], pitch-class set (pcset) theory [8, 9], neo-riemannian theory [4, 5], tonal pitch space theory [19] and the work by Quinn [22]. Empirical studies have attempted to evaluate aspects of such theories in an empirical manner - see, for instance, [1, 16, 17, 24]. Apart from sensory, cognitive and musicological factors that play a significant role in such studies (and also in the first chord grouping algorithm below), the work herein makes additional use of data-driven information derived from statistical harmonic analysis in order to tackle similarity of different chord groups based on their functionality (i.e. transitions between chords) cf. related work by Quinn and Mavromatis [23].

A large number of unique note simultaneities may appear in a certain musical style. These simultaneities, however, are organised into fewer more cognitively manageable chord families/categories. Things like octave equivalence, interval inversion equivalence, root, tonal centre and so on, enable a parsimonious 'packing' of the great variety of actual note simulaneities into a relatively small number of musically meaningful chord categories. This categorical organisation of chords is probably most apparent in the case of tonal music; for instance, 'major chord' applies to many vertical note configurations that may appear in different guises such as open/closed position, different registers and keys, with doubled or missing or, even, extra notes.

The GCT algorithm re-organises note simultaneities in

terms of 'root', 'base', 'extension' and relative root to local key, giving the same label to pitch collections that have identical structure in relation to a tonal centre. However, missing or extra notes are not taken into account, resulting in a larger number of chords than what is musically acceptable (at least for tonal music). For instance, the GCTs: $[7, [0, 4, 7]]$, $[7, [0, 4]]$, $[7, [0, 4, 10]]$, $[7, [0, 4, 7, 10]]$ are all independent chord labels whereas they could be grouped under one dominant chord label (these share the same relative root and are all subsets of the $[0, 4, 7, 10]$ chord type). Additionally, the GCTs: $[11, [0, 3, 6]]$ and $[11, [0, 3, 6, 9]]$ are diminished chords on the seventh scale degree; these cannot be grouped with the previous GCTs because of the different relative root and chord type, even though we know that they also belong to the dominant chord functional category.

In the next two subsections, firstly, a simple algorithm is presented that groups raw GCTs into GCT chord categories based on GCT properties, such as, relative root, type similarity and relationship to underlying scale/key; secondly, an algorithm is developed that further organises the above GCT categories into functional chord categories by examining the function of chords, i.e., chords that tend to be followed by the same chords (similar rows in a chord transition matrix) are considered to have the same function. These two algorithms tidy up the initial raw GCTs into meaningful chord categories, each represented by the most frequently occurring instance (exemplar) .

3.1 Grouping chords based on their GCT properties

Following the aforementioned example, the 'exemplar $[7, [0, 4, 7]]$ might be found in several 'reduced' (e.g. $[7, [0, 4]]$) or 'expanded' (e.g. $[7, [0, 4, 7, 11]]$) forms, that actually represent the same chord label. According to the GCT representation, further abstraction can be achieved through grouping GCT expressions of simultaneities that 'evidently' concern the same chord.

Grouping of GCTs has been studied under some basic assumptions about the chord characteristics that are reflected by the root scale degree, the base and the scale notes underlying a GCT expression. Specifically, GCT expressions are grouped into more general GCT categories that potentially contain several GCT members according to the criteria described below: two chords belong to the same group if

1. they have the same scale degree root,

2. their GCT bases are subset-related and
3. they both contain notes that either belong or not to the given scale context.

Regarding criterion 2, two bases B_1 and B_2 are considered subset-related if $B_1 \subseteq B_2$ or $B_2 \subseteq B_1$, e.g. $[0,4] \subseteq [0,4,7]$ while $[0,4] \not\subset [0,3,7]$. Criterion 3 is utilised to identify and group together chords that belong to secondary tonalities within the primary tonality of the piece. For instance, in a diatonic major context, while $c_1 = [0, [0,4,7]]$ and $c_2 = [0, [0,4,7,10]]$ fulfil criteria 1 and 2, according to criterion 3 they are not grouped together since c2 includes value 10, which is mapped to the non-diatonic 10 pitch class value. In a major context $[0,[0,4,7,10]]$ is secondary dominant to the IV (V/IV) and is differentiated from the I major chord.

Each GCT group includes the GCT types that satisfy the aforementioned three criteria. Furthermore, each group is represented by the 'exemplar' GCT type, which is the one that is more often met in the datasets under study. Some common chord groups in the major scale Bach Chorales are illustrated in Table 2. This table also includes the functional naming of each group in order to assist the comparison of the derived GCT types and the standard roman-numeral labelling. Testing this simple algorithm on sets of both major and minor Bach chorales gives a reasonable first classification of the 'raw' GCTs.

3.2 Functional similarity of chords

According to functional harmony each chord can be viewed not only in terms of its actual pitches, roots, chord type and so on, but also in terms of its 'dynamic' attributes according to its position in a chord sequence and to the chords that usually follow [18]. For instance, in the tonal idiom, dominant chords are 'expected' to resolve to a (relative) tonic chord. Therefore, different chords can be similar according to the purpose they serve in terms of their functionality within chord sequences.

In this Section, a first approach to derive the functionality of the GCT chord groups is addressed by observing their succeeding chords in chord sequences extracted from specific idioms. In order to capture the functional relations between GCT groups of specific music idioms, the first-order Markov transition table is considered for all the GCT chord sequences that pertain to a certain idiom. The proposed approach below, tackles chord similarity by employing the Euclidean distance metrics related to the probability distribution for each chord group to precede any other (i.e., euclidean distance between rows of the transition matrix).

Figure 2 illustrates a colour-based graphic interpretation of the transition matrix obtained from a collection of Bach Chorales in major mode (darker areas indicate higher probabilities); transitions between chords that pertain to the same GCT chord group are disregarded (this neutralises the diagonal). Furthermore, GCT chord groups that occurred 4 times or less in the entire dataset were discarded, since their functional role can hardly be determined by so few observations. The probability that a GCT chord group is

followed by another (a row of the transition matrix in Figure 2) is regarded as a vector that defines the position of this group into the 'space of transitions'. Thereby, functional relations between GCT groups according to their most common successors can be deduced by employing distance metrics between rows of the transition matrix.
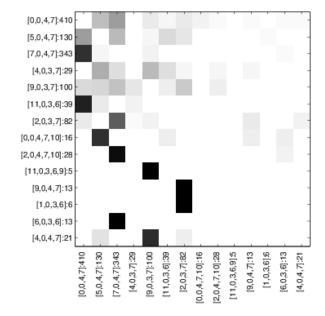

Figure 2. The first-order Markov transition matrix of GCT groups in the major Bach Chorales. The numbers after the colon indicate the number of times a representative of a GCT group was found in the data.

3.3 Functional similarity results

The Euclidean distance between transitions of GCT groups (rows in the transition matrix depicted in Figure 2) in a set of major Bach chorales has been utilised to produce the dendrogram of distances illustrated in Figure 3. For clarity of presentation, GCT groups with rare occurrences (less than 4) were not considered, although their placement in the grouping results was explainable. The six annotated clusters underpin interesting functional relations between the chords involved (the comments are presented in diminishing cluster coherence order):

Cluster 1 comprises the double dominant V/V $[2, [0,4,7, 10]]$ and its subset vii^o/V $[6, [0,3,6]]$. Both chords have identical harmonic function (pre-dominant) and they always lead to the dominant V chord as applied dominants.

Cluster 4 contains the dominant V $[7, [0,4,7]]$ and the leading-tone triad vii^o $[11, [0,3,6]]$, which is a subset of the dominant 7th chord. Both chords have strong dominant function.

Cluster 6 contains the applied dominant of the sub-mediant, i.e. V/vi, and the corresponding applied diminished 7th chord, i.e. vii^{o7}/vi. The GCT algorithm erroneously describes the second chord as its enharmonic equivalent $[11, [0,3,6,9]]$, i.e. as vii^{o7} $[B, D, F, A\flat]$, while it should be $[8, [0,3,6,9]]$, i.e. vii^{o7}/vi$[G\sharp, B, D, F]$. However, the strong clustering relation could help to disambiguate the

functional name	exemplar	Group members			
tonic	$[0, [0, 4, 7]]$	$[0, [0, 4, 7]]$	$[0, [0, 4]]$	$[0, [0, 4, 7], [11]]$	
dominant	$[7, [0, 4, 7]]$	$[7, [0, 4, 7]]$	$[7, [0, 4, 7], [10]]$	$[7, [0, 4], [10]]$	$[7, [0, 4]]$
subdominant	$[5, [0, 4, 7]]$	$[5, [0, 4, 7]]$	$[5, [0, 4]]$	$[5, [0, 4, 7], [11]]$	
V / IV	$[0, [0, 4, 7], [10]]$	$[0, [0, 4, 7], [10]]$	$[0, [0, 4], [10]]$		

Table 2. Four tonal chord groups and their exemplar GCTs. Notice how the group of $[0, [0, 4, 7]]$ has been separated from the group of $[0, [0, 4, 7], [10]]$, due to the non-diatonic pitch class 10 of the latter.

root of the diminished 7th chord; this is future work for improving the descriptiveness efficiency of the GCT representation.

Cluster 5 groups the applied dominant of the supertonic, i.e. V/ii$[9, [0, 4, 7]]$, and the corresponding applied diminished triad, i.e. vii^o/ii$[1, [0, 3, 6]]$. Clusters 1, 4, 5, 6 are of the same category, as they share the same dominant function.

Cluster 2 is different, as it groups three chords that have (or may have) tonic harmonic function, the tonic I $[0, [0, 4, 7]]$, the submediant vi$[9, [0, 3, 7]]$ and the mediant iii$[4, [0, 3, 7]]$. In functional harmony [6], these chords are labeled as T, Tp and Tg accordingly and the last two chords have a diatonic (with two common tones) third-relation with the first.

Cluster 3 is similar to cluster 2, as it groups two chords with diatonic third-relation, however in this case the chords share subdominant harmonic function: the subdominant IV $[5, [0, 4, 7]]$ and the supertonic ii$[2, [0, 3, 7]]$. In functional harmony, they are described as S and Sp accordingly.

Overall, the proposed data-driven functional approach to chord grouping seems to be quite reliable. Further testing is necessary on larger and more varied corpora.

4. CONCLUSIONS

The paper at hand examines two main topics: a) the ability of the GCT algorithm to analyse chord sequences (in comparison to roman numeral analysis) and b) the possibilty to organise the 'raw' GCT labels in higher-level chord families according to the internal GCT properties and to dynamic functional properties in terms of chord successions in harmonic corpora. The first study was based on comparing the annotations of chords produced by the GCT algorithm with the harmonic annotations of human experts (around 92% accuracy in the Kostka-Payne dataset). So, with its ability to identify roots and chord types, the GCT can be used as an interpretation/analytic tool allowing it to be classified as a hybrid between neutral representations (e.g. Forte pc-set theory analysis) and interpretative ones (e.g. roman numeral analysis). For the second study, information about transitions of the GCT chord groups were utilised to identify similarities between these groups according to their successors, thus, reflecting functional relations.

The results are promising, since they illustrate the ability of the GCT to accurately label chords, but also to reveal chord groups according to (higher) functional meaning in the tonal system. It is maintained that if the GCT representation can cope with such a sophisticated hierar-
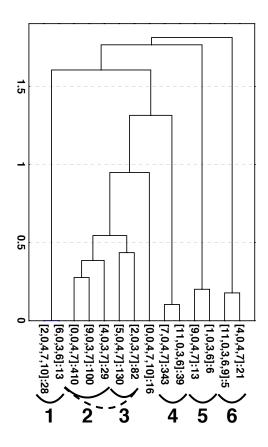

Figure 3. (a) The dendrogram derived from the Euclidean distances between rows of the transition matrix (Figure 2).

chic harmonic system as the tonal system, then it seems likely that it can deal with other non-tonal systems as well. Preliminary examples presented in [2, 3, 14] illustrate the potential of the GCT to represent non-tonal harmonic idioms; further reseach is under way to unveil the potential of the proposed representation in other musics.

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