Integration by Trigonometric Substitution

Example

$$\int \sqrt{1-x^2} \, dx = ... = \int \sqrt{1-\sin^2(t)} \cos(t) \, dt = ...$$

Do the following substitution:

$$\sin(t) = x$$

$$\Rightarrow \cos(t) dt = dx$$

- This substitution leads to a simplification in the radicand.
- At the end it is necessary to apply an inverse substitution (to obtain the first variable).

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Table of Trigonometric Substitution

Integrand	Substitution	Trigonometric Identity
$\sqrt{1-f^2}$	$f = \sin(t) \\ -\frac{\pi}{2} \le t \le \frac{\pi}{2}$	$1-\sin^2(t)=\cos^2(t)$
$\sqrt{1+f^2}$	$f= ext{tg}(t) \ -rac{\pi}{2} < t < rac{\pi}{2}$	$1+tg^2(t)=sec^2(t)$
$\sqrt{f^2-1}$	$f=\sec(t) \ 0 \leq t < rac{\pi}{2} ext{ ou } \pi \leq t < rac{3\pi}{2}$	$\sec^2(t) - 1 = \operatorname{tg}^2(t)$

