Find $\int 2x + 2x^2 - \frac{1}{x}\,dx = (*)$

we know that:

$$\int f \pm g\,dx = \int f\,dx \pm \int g\,dx$$

Then,

$$(*) = \int 2x\,dx + \int 2x^2\,dx - \int \frac{1}{x}\,dx =$$

Now, $\int k f\,dx = k \cdot \int f\,dx$, k a constant

$$= 2\int x\,dx + 2\int x^2\,dx - \int \frac{1}{x}\,dx$$

$$\int f' \cdot f^n\,dx = \frac{f^{n+1}}{n+1} + C$$

$$= 2 \cdot \frac{x^2}{2} + 2 \cdot \frac{x^3}{3} - \ln|x| + C$$

$$\int \frac{f'}{f}\,dx = \ln|f| + C$$

$$= x^2 + \frac{2}{3}x^3 - \ln|x| + C$$

Find

$$\int \ln(\sqrt{x})\, dx =$$

$$= \int \underbrace{\ln(t)}_{g} \cdot \underbrace{2t}_{f}\, dt$$

$$\stackrel{\text{by parts}}{=} t^2 \cdot \ln(t) - \int t^2 \cdot \frac{1}{t}\, dt$$

$$= t^2 \cdot \ln(t) - \int t\, dt$$

$$= t^2 \cdot \ln(t) - \frac{t^2}{2} + C$$

$$= x \cdot \ln(\sqrt{x}) - \frac{x}{2} + C$$

substitution:

$$\boxed{x^{1/2} = t}$$

$$\Rightarrow \frac{1}{2} x^{-1/2}\, dx = dt$$

$$\Leftrightarrow \frac{1}{2x^{1/2}}\, dx = dt$$

$$\Leftrightarrow \boxed{dx = 2t\, dt}$$

- $f(t) = 2t \Rightarrow \int 2t\, dt = t^2 + C$
- $g(t) = \ln(t) \Rightarrow g'(t) = \frac{1}{t}$