

MCS-242: APPLIED STATISTICAL METHODS
COMPARISON OF SIMPLE AND MULTIPLE LOGISTIC REGRESSION

	Simple Logistic Regression	Multiple Logistic Regression
Response Variable type	Binary Categorical Numerical	Binary Categorical Numerical
Explanatory Variable(s) type	Binary Categorical Numerical	Binary Categorical Numerical
Model Equation: Logit form	$\ln\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 x$	$\ln\left(\frac{\pi}{1-\pi}\right) = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k$
Model Equation: Probability form	$\pi = \frac{\exp(\beta_0 + \beta_1 x)}{1 + \exp(\beta_0 + \beta_1 x)}$	$\pi = \frac{\exp(\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k)}{1 + \exp(\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k)}$
Description of model terms	π = prob. of success β_0 = intercept of logit form β_1 = slope " "	β_i = slope of x_i
Model Assumptions	- linearity between logit (ln odds) + x - randomness of outcomes - independence of outcomes	- linearity between logit + x_i for each x_i } →
Graphs used to test model assumptions	empirical logit plot: ln odds vs. x	empirical logit plots for each x_i
Estimated model equation	$\hat{\text{logit}}(\pi) = \hat{\beta}_0 + \hat{\beta}_1 x$	$\hat{\text{logit}}(\pi) = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k$

	Simple Logistic Regression	Multiple Logistic Regression
Can you transform X?	<u>Yes</u> No	<u>Yes</u> No
Why would you do so?	If x shows a non-linear relationship with the empirical logits (that could be fixed thru transform).	
Interpretation of slope coefficient	The odds of success for $x+1$ are $\exp(\beta_1)$ the odds of success for x .	The odds of success for x_i+1 are $\exp(\beta_i)$ the odds of success for x_i , holding other vars. constant.
Wald test for individual coefficient (β_i):		
hypotheses	$H_0: \beta_i = 0$ $H_a: \beta_i \neq 0$	$H_0: \beta_i = 0$ $H_a: \beta_i \neq 0$
test statistic	$z = \hat{\beta}_i / SE_{\hat{\beta}_i} \sim N(0,1)$	$z = \hat{\beta}_i / SE_{\hat{\beta}_i} \sim N(0,1)$
CI for individual coefficient (β_i)	$\hat{\beta}_i \pm z^* SE_{\hat{\beta}_i}$	$\hat{\beta}_i \pm z^* SE_{\hat{\beta}_i}$
CI for odds ratio for 1-unit change in X	$\exp(\quad)$	$\exp(\quad)$
G-test (drop-in-deviance test, LRT) for utility of the model:		
hypotheses	H_0 : model is useless ($\beta_i=0$) H_a : model is useful ($\beta_i \neq 0$)	H_0 : model is useless (all $\beta_i=0$) H_a : model is useful (not all $\beta_i=0$)
test statistic (and how to find on R)	$G = \text{null deviance} - \text{residual deviance}$	same
distribution & degrees of freedom of test stat	$\sim \chi^2_1$	$\sim \chi^2_k$ ($k = \# \text{vars. in model}$)
Nested drop-in-deviance test (LRT):		
hypotheses	/	H_0 : reduced model (k_1 vars.) H_a : full model ($k_1 + k_2$ vars.)
test statistic (and how to find on R)		$G = \text{resid. deviance reduced} - \text{resid. deviance full}$
distribution & degrees of freedom of test stat		$\sim \chi^2_{k_2}$