

## ThinkDSP

This notebook contains solutions to exercises in Chapter 11: Modulation and sampling

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```
In [1]: from __future__ import print_function, division

import thinkdsp
import thinkplot

import numpy as np

import warnings
warnings.filterwarnings('ignore')

PI2 = 2 * np.pi

np.set_printoptions(precision=3, suppress=True)
%matplotlib inline
```

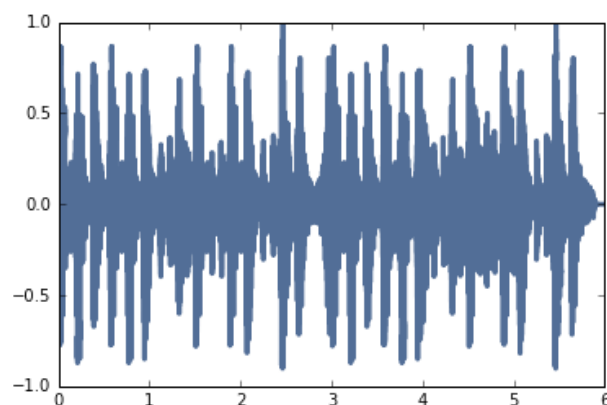
**Exercise:** As we have seen, if you sample a signal at too low a framerate, frequencies above the folding frequency get aliased. Once that happens, it is no longer possible to filter out these components, because they are indistinguishable from lower frequencies.

It is a good idea to filter out these frequencies *before* sampling; a low-pass filter used for this purpose is called an “anti-aliasing filter”.

Returning to the drum solo example, apply a low-pass filter before sampling, then apply the low-pass filter again to remove the spectral copies introduced by sampling. The result should be identical to the filtered signal.

*Solution:* I'll load the drum solo again.

```
In [2]: wave = thinkdsp.read_wave('263868__kevcio__amen-break-a-160-bpm.wav')
wave.normalize()
wave.plot()
```



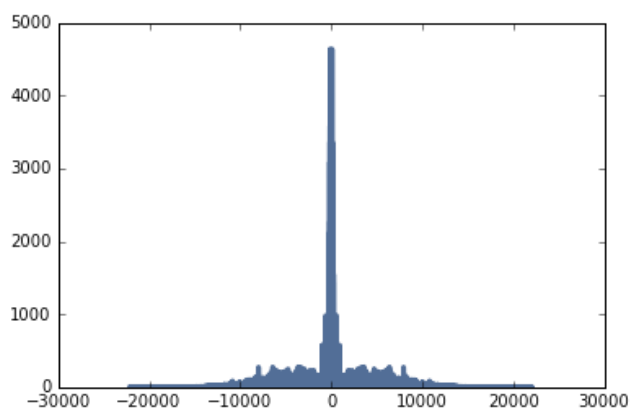
This signal is sampled at 44100 Hz. Here's what it sounds like.

```
In [3]: wave.make_audio()
```

Out[3]:  0:00:00 / 13:31:36

And here's the spectrum:

```
In [4]: spectrum = wave.make_spectrum(full=True)
spectrum.plot()
```

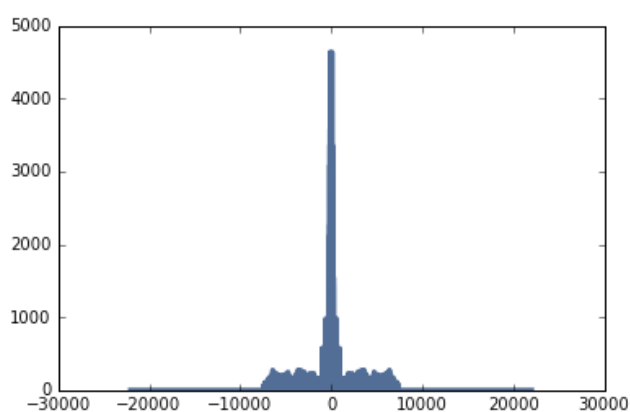


I'll reduce the sampling rate by a factor of 3 (but you can change this to try other values):

```
In [5]: factor = 3
framerate = wave.framerate / factor
cutoff = framerate / 2 - 1
```

Before sampling we apply an anti-aliasing filter to remove frequencies above the new folding frequency, which is  $\text{framerate}/2$  :

```
In [6]: spectrum.low_pass(cutoff)
spectrum.plot()
```



Here's what it sounds like after filtering (still pretty good).

```
In [7]: filtered = spectrum.make_wave()  
filtered.make_audio()
```

Out[7]:  0:00:00 / 13:31:36

Here's the function that simulates the sampling process:

```
In [8]: def sample(wave, factor):  
        """Simulates sampling of a wave.  
  
        wave: Wave object  
        factor: ratio of the new framerate to the original  
        """  
        ys = np.zeros(len(wave))  
        ys[::factor] = wave.ys[::factor]  
        return thinkdsp.Wave(ys, framerate=wave.framerate)
```

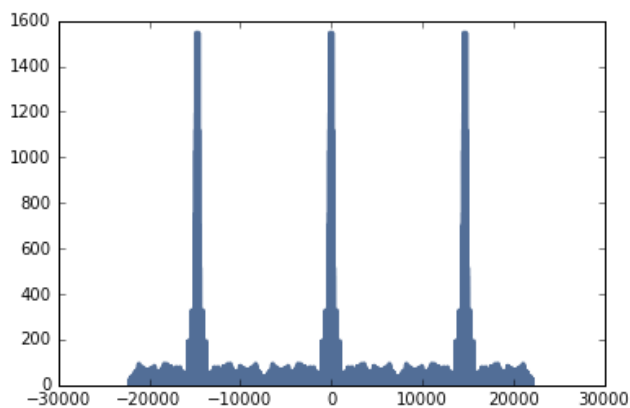
The result contains copies of the spectrum near 20 kHz; they are not very noticeable:

```
In [9]: sampled = sample(filtered, factor)  
sampled.make_audio()
```

Out[9]:  0:00:00 / 13:31:36

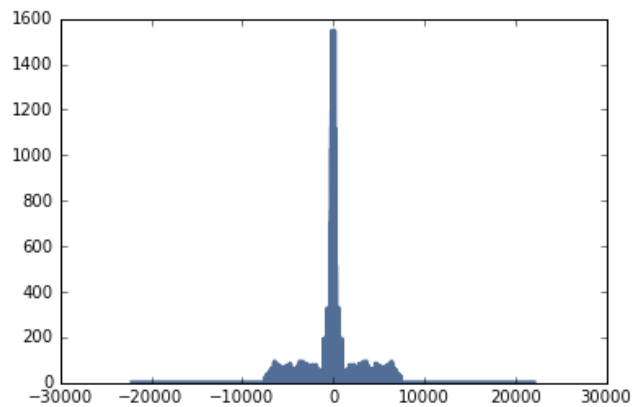
But they show up when we plot the spectrum:

```
In [10]: sampled_spectrum = sampled.make_spectrum(full=True)  
sampled_spectrum.plot()
```



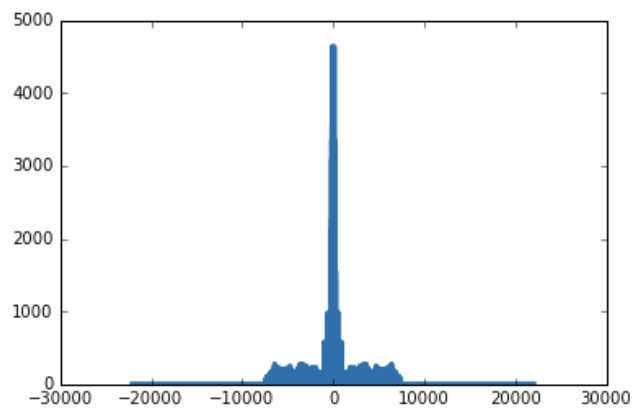
We can get rid of the spectral copies by applying the anti-aliasing filter again:

```
In [11]: sampled_spectrum.low_pass(cutoff)
sampled_spectrum.plot()
```



We just lost half the energy in the spectrum, but we can scale the result to get it back:

```
In [12]: sampled_spectrum.scale(factor)
spectrum.plot()
sampled_spectrum.plot()
```



Now the difference between the spectrum before and after sampling should be small.

```
In [13]: spectrum.max_diff(sampled_spectrum)
```

```
Out[13]: 9.0949470177292824e-12
```

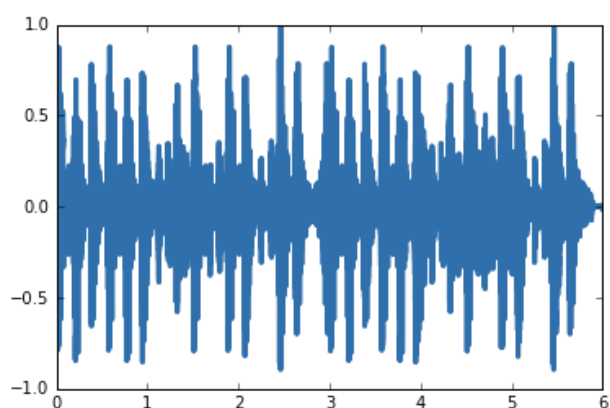
After filtering and scaling, we can convert back to a wave:

```
In [14]: interpolated = sampled_spectrum.make_wave()
interpolated.make_audio()
```

```
Out[14]:  0:00:00 / 13:31:36
```

And the difference between the interpolated wave and the filtered wave should be small.

```
In [15]: filtered.plot()  
interpolated.plot()
```



Multiplying by `impulses` makes 4 shifted copies of the original spectrum. One of them wraps around from the negative end of the spectrum to the positive, which is why there are 5 peaks in the spectrum of the sampled wave.

```
In [16]: filtered.max_diff(interpolated)
```

```
Out[16]: 3.663750800998542e-15
```

```
In [ ]:
```

```
In [ ]:
```