ThinkDSP

This notebook contains solutions to exercises in Chapter 9: Differentiation and Integration

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In [1]:
```
from __future__ import print_function, division

import thinkdsp
import thinkplot
import thinkstats2

import numpy as np
import pandas as pd
import scipy.signal

import warnings
warnings.filterwarnings('ignore')

PI2 = 2 * np.pi
GRAY = '0.7'

np.set_printoptions(precision=3, suppress=True)
%matplotlib inline
```

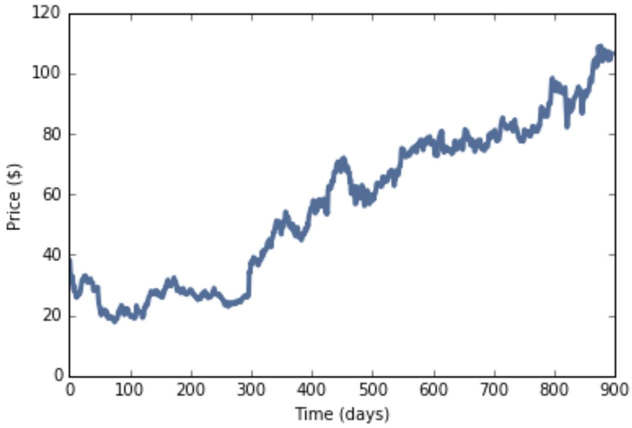
Exercise: In the section on cumulative sum, I mentioned that some of the examples don't work with non-periodic signals. Try replacing the sawtooth wave, which is periodic, with the Facebook data, which is not, and see what goes wrong.

Solution: I'll start by loading the Facebook data again.

In [2]:
```
names = ['date', 'open', 'high', 'low', 'close', 'volume']
df = pd.read_csv('fb.csv', header=0, names=names, parse_dates=[0])
ys = df.close.values[::-1]
```

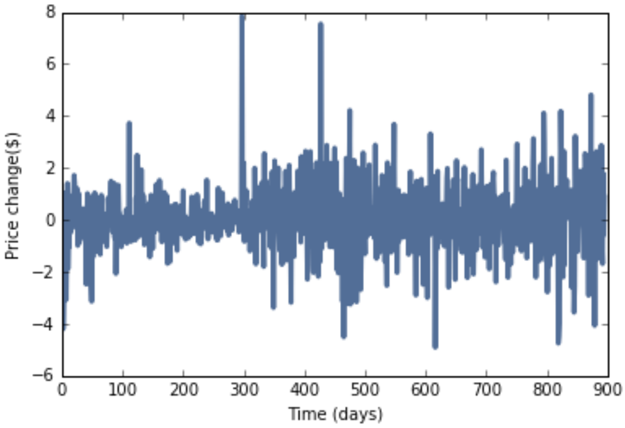
And making a Wave

In [3]:
```
close = thinkdsp.Wave(ys, framerate=1)
close.plot()
thinkplot.config(xlabel='Time (days)', ylabel='Price ($)')
```



I'll compute the daily changes using `Wave.diff` :

In [4]:
```
change = close.diff()
change.plot()
thinkplot.config(xlabel='Time (days)', ylabel='Price change($)')
```

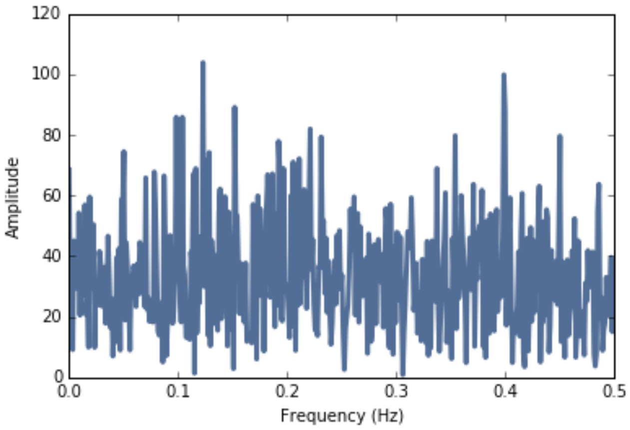


Now I'll run the `cumsum` example using `change` as the input wave:

In [5]:
```
in_wave = change
```

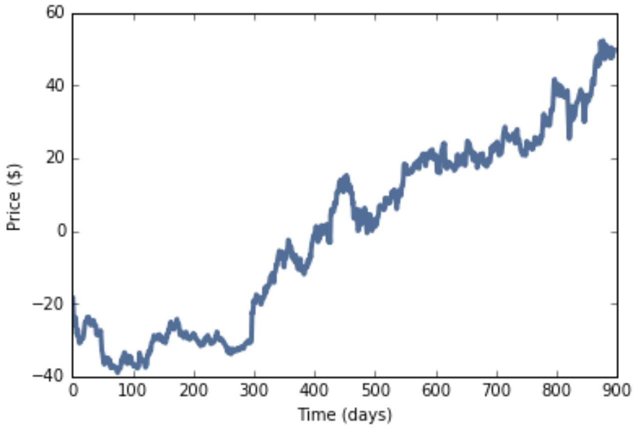
Here's the spectrum before the cumulative sum:

In [6]:
```
in_spectrum = in_wave.make_spectrum()
in_spectrum.plot()
thinkplot.config(xlabel='Frequency (Hz)',
                 ylabel='Amplitude')
```



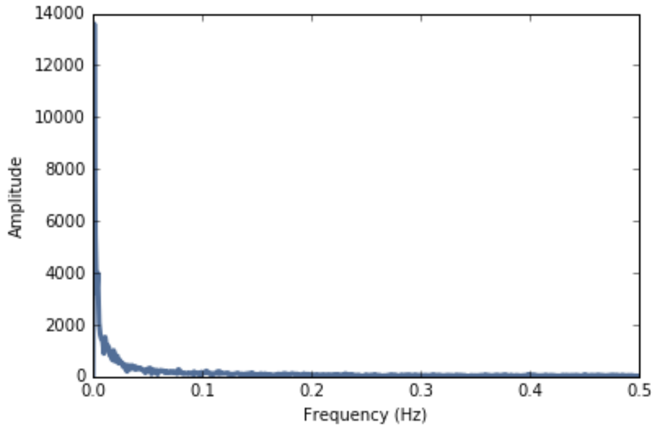
The output wave is the cumulative sum of the input

In [7]:
```
out_wave = in_wave.cumsum()
out_wave.unbias()
out_wave.plot()
thinkplot.config(xlabel='Time (days)', ylabel='Price ($)')
```



And here's its spectrum

In [8]:
```
out_spectrum = out_wave.make_spectrum()
out_spectrum.plot()
thinkplot.config(xlabel='Frequency (Hz)',
                 ylabel='Amplitude')
```



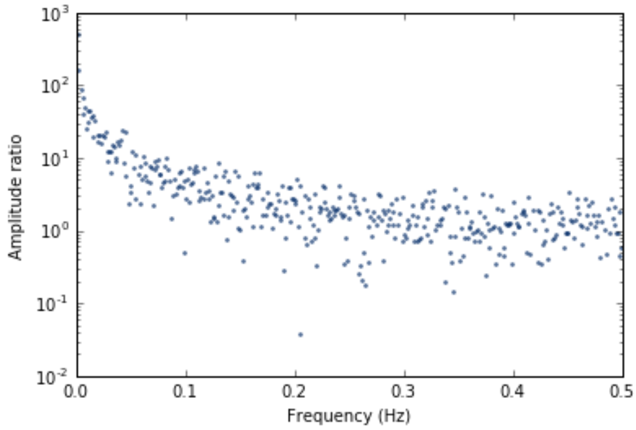
Now we compute the ratio of the output to the input:

In [9]:
```
sum(in_spectrum.amps < 10), len(in_spectrum)
```

Out[9]: (37, 448)

In between the harmonics, the input componenents are small, so I set those ratios to NaN.

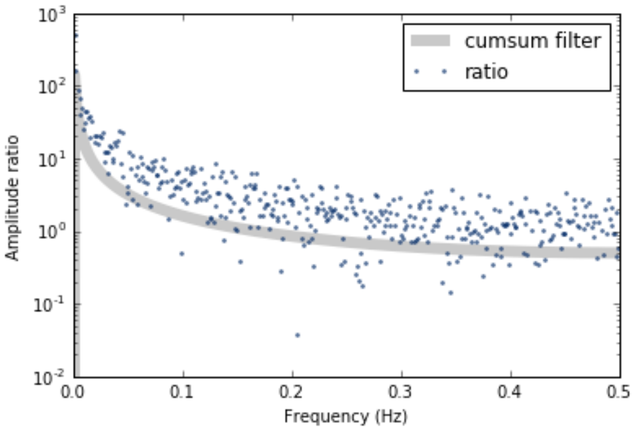
In [10]:
```
ratio_spectrum = out_spectrum.ratio(in_spectrum, thresh=10)
ratio_spectrum.hs[0] = 0
ratio_spectrum.plot(style='.', markersize=4)

thinkplot.config(xlabel='Frequency (Hz)',
                 ylabel='Amplitude ratio',
                 yscale='log')
```



Instead of falling in a neat line, the ratios are pretty noisy.

We can compare them with the cumsum filter:

In [11]:
```
diff_window = np.array([1.0, -1.0])
padded = thinkdsp.zero_pad(diff_window, len(in_wave))
diff_wave = thinkdsp.Wave(padded, framerate=in_wave.framerate)
diff_filter = diff_wave.make_spectrum()

cumsum_filter = diff_filter.copy()
cumsum_filter.hs = 1 / cumsum_filter.hs
cumsum_filter.hs[0] = 0
cumsum_filter.plot(label='cumsum filter', color=GRAY, linewidth=7)

ratio_spectrum.plot(label='ratio', style='.', markersize=4)
thinkplot.config(xlabel='Frequency (Hz)',
                 ylabel='Amplitude ratio',
                 yscale='log', legend=True)
```



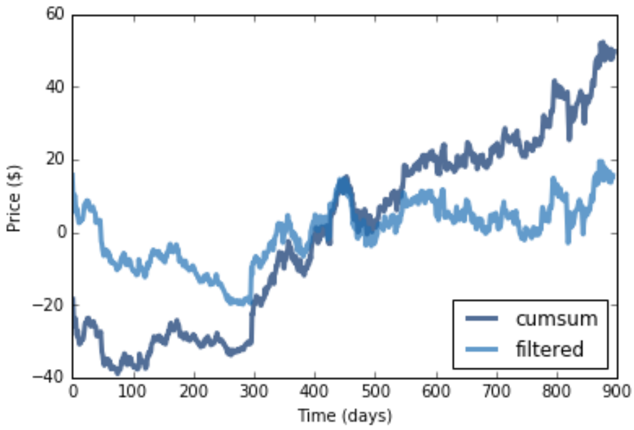
The ratios follow the general shape of the filter, but they are not in accord.

Now we can compute the output wave using the convolution theorem, and compare the results:

In [12]:
```
len(in_spectrum), len(cumsum_filter)
```

Out[12]: (448, 448)

In [13]:
```
thinkplot.preplot(2)

out_wave.plot(label='cumsum')

in_spectrum = in_wave.make_spectrum()
out_wave2 = (in_spectrum * cumsum_filter).make_wave()
out_wave2.plot(label='filtered')

thinkplot.config(legend=True, loc='lower right')
thinkplot.config(xlabel='Time (days)', ylabel='Price ($)')
```

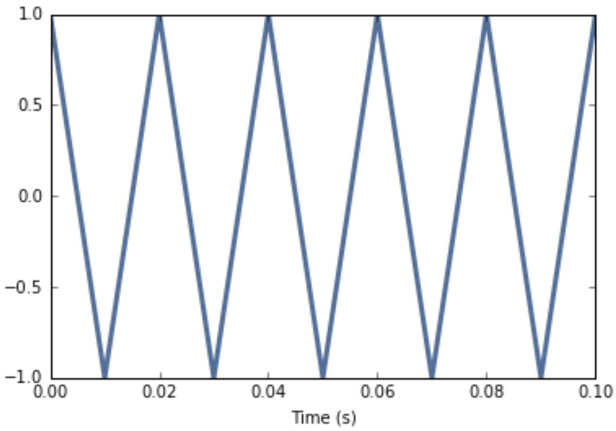


They are clearly different.

Exercise: The goal of this exercise is to explore the effect of `diff` and `differentiate` on a signal. Create a triangle wave and plot it. Apply the `diff` operator and plot the result. Compute the spectrum of the triangle wave, apply `differentiate`, and plot the result. Convert the spectrum back to a wave and plot it. Are there differences between the effect of `diff` and `differentiate` for this wave?

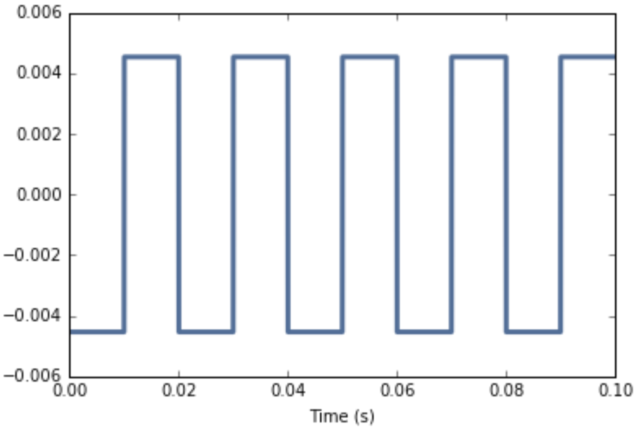
Solution: Here's the triangle wave.

In [14]:
```
in_wave = thinkdsp.TriangleSignal(freq=50).make_wave(duration=0.1, framerat
e=44100)
in_wave.plot()
thinkplot.config(xlabel='Time (s)')
```



The diff of a triangle wave is a square wave, which explains why the harmonics in a square wave drop off like $1/f$, compared to the triangle wave, which drops off like $1/f^2$.

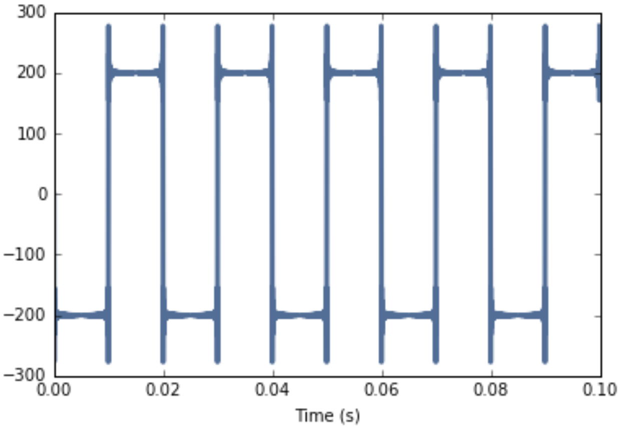
In [15]:
```
out_wave = in_wave.diff()
out_wave.plot()
thinkplot.config(xlabel='Time (s)')
```



When we take the spectral derivative, we get "ringing" around the discontinuities: https://en.wikipedia.org/wiki/Ringing_(signal (https://en.wikipedia.org/wiki/Ringing_(signal))

Mathematically speaking, the problem is that the derivative of the triangle wave is undefined at the points of the triangle.

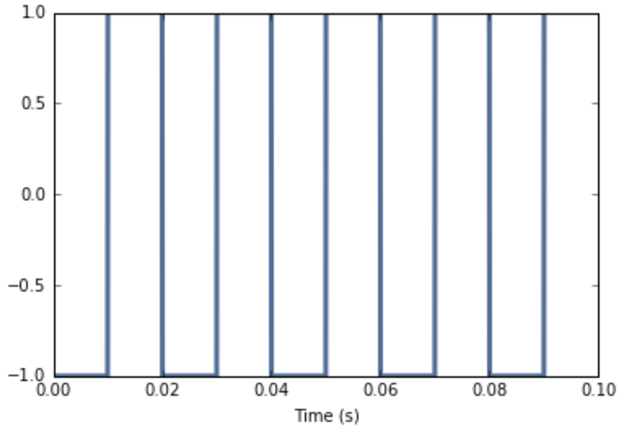
In [16]:
```
out_wave2 = in_wave.make_spectrum().differentiate().make_wave()
out_wave2.plot()
thinkplot.config(xlabel='Time (s)')
```



Exercise: The goal of this exercise is to explore the effect of `cumsum` and `integrate` on a signal. Create a square wave and plot it. Apply the `cumsum` operator and plot the result. Compute the spectrum of the square wave, apply `integrate`, and plot the result. Convert the spectrum back to a wave and plot it. Are there differences between the effect of `cumsum` and `integrate` for this wave?

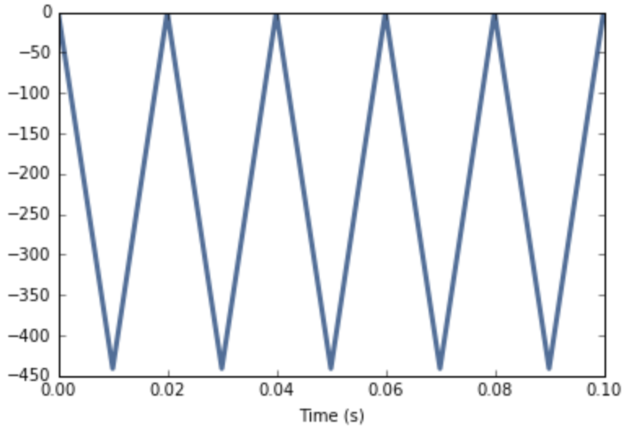
Solution: Here's the square wave.

In [17]:
```
in_wave = thinkdsp.SquareSignal(freq=50).make_wave(duration=0.1, framerate=4
4100)
in_wave.plot()
thinkplot.config(xlabel='Time (s)')
```



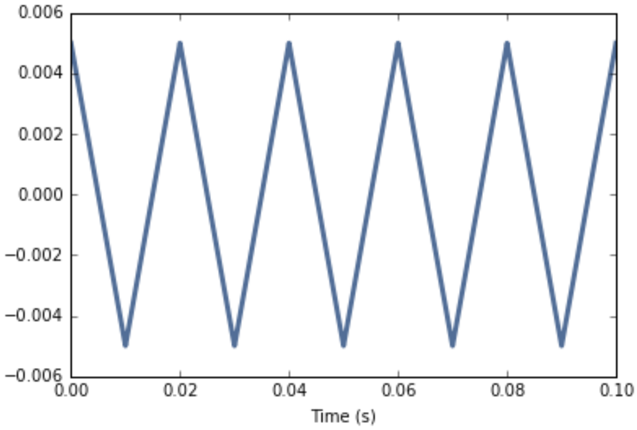
The cumulative sum of a square wave is a triangle wave. After the previous exercise, that should come as no surprise.

In [18]:
```
out_wave = in_wave.cumsum()
out_wave.plot()
thinkplot.config(xlabel='Time (s)')
```



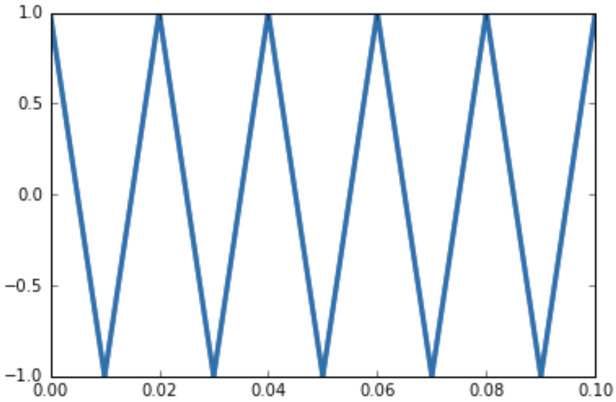
The spectral integral is also a triangle wave, although the amplitude is very different.

In [19]:
```
spectrum = in_wave.make_spectrum().integrate()
spectrum.hs[0] = 0
out_wave2 = spectrum.make_wave()
out_wave2.plot()
thinkplot.config(xlabel='Time (s)')
```



If we unbias and normalize the two waves, they are visually similar.

In [20]:
```
out_wave.unbias()
out_wave.normalize()
out_wave2.normalize()
out_wave.plot()
out_wave2.plot()
```



And they are numerically similar, but with only about 3 digits of precision.

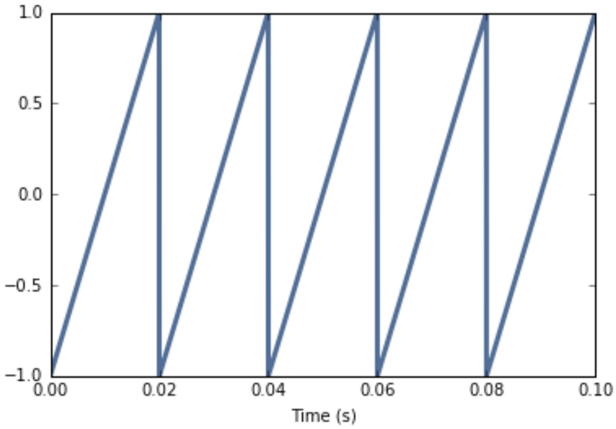
In [21]:
```
max(abs(out_wave.ys - out_wave2.ys))
```

Out[21]: 0.0045351473922902175

Exercise: The goal of this exercise is the explore the effect of integrating twice. Create a sawtooth wave, compute its spectrum, then apply `integrate` twice. Plot the resulting wave and its spectrum. What is the mathematical form of the wave? Why does it resemble a sinusoid?

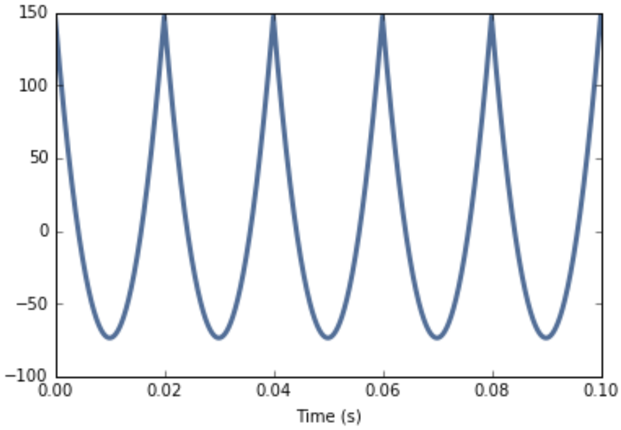
Here's the sawtooth.

In [22]:
```
in_wave = thinkdsp.SawtoothSignal(freq=50).make_wave(duration=0.1, framerat
e=44100)
in_wave.plot()
thinkplot.config(xlabel='Time (s)')
```



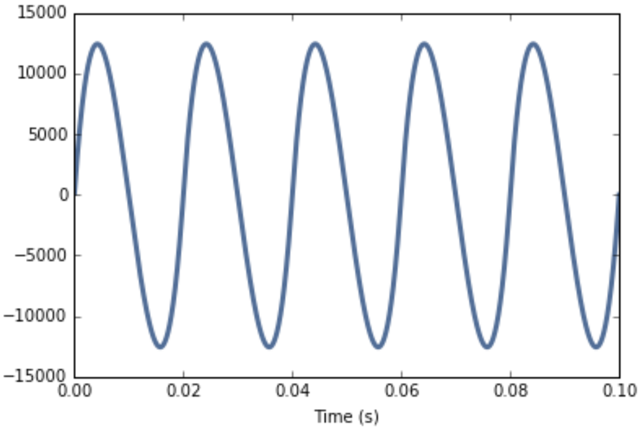
The first cumulative sum of a sawtooth is a parabola:

In [23]:
```
out_wave = in_wave.cumsum()
out_wave.unbias()
out_wave.plot()
thinkplot.config(xlabel='Time (s)')
```



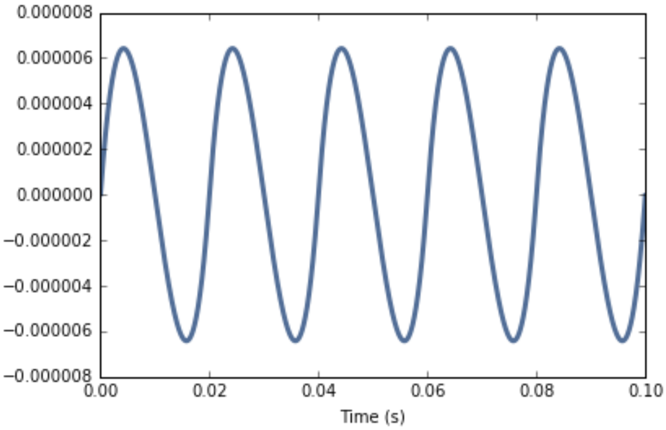
The second cumulative sum is a cubic curve:

In [24]:
```
out_wave = out_wave.cumsum()
out_wave.plot()
thinkplot.config(xlabel='Time (s)')
```



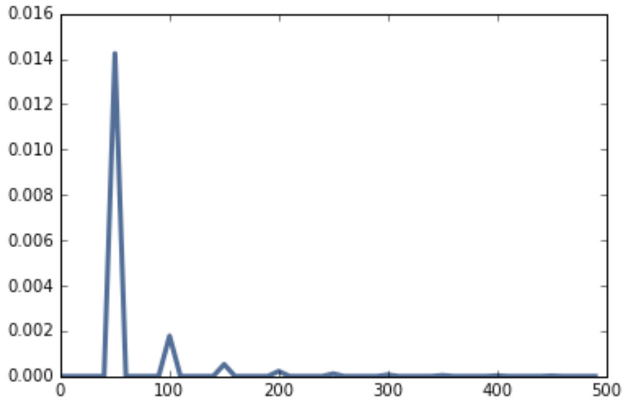
Integrating twice also yields a cubic curve.

In [25]:
```
spectrum = in_wave.make_spectrum().integrate().integrate()
spectrum.hs[0] = 0
out_wave2 = spectrum.make_wave()
out_wave2.plot()
thinkplot.config(xlabel='Time (s)')
```



At this point, the result looks more and more like a sinusoid. The reason is that integration acts like a low pass filter. At this point we have filtered out almost everything except the fundamental, as shown in the spectrum below:

In [26]:
```
out_wave2.make_spectrum().plot(high=500)
```

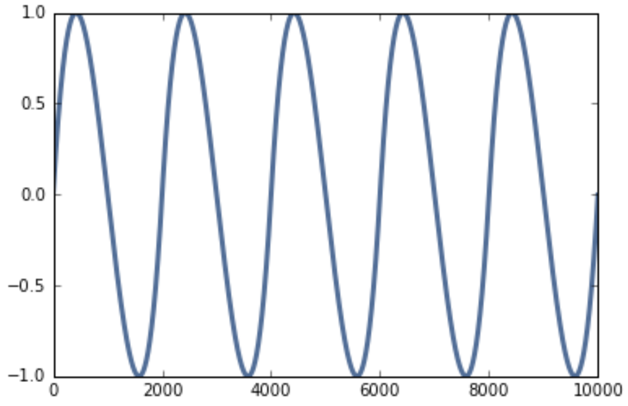


Exercise: The goal of this exercise is to explore the effect of the 2nd difference and 2nd derivative. Create a `CubicSignal`, which is defined in `thinkdsp`. Compute the second difference by applying `diff` twice. What does the result look like. Compute the second derivative by applying `differentiate` twice. Does the result look the same?

Plot the filters that corresponds to the 2nd difference and the 2nd derivative and compare them. Hint: In order to get the filters on the same scale, use a wave with framerate 1.

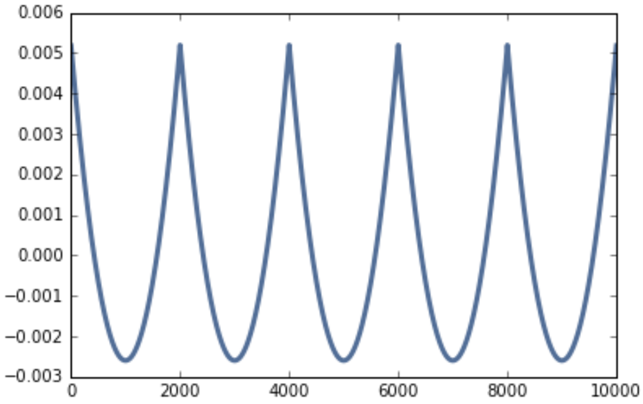
Solution: Here's the cubic signal

In [27]:
```
in_wave = thinkdsp.CubicSignal(freq=0.0005).make_wave(duration=10000, framer
ate=1)
in_wave.plot()
```

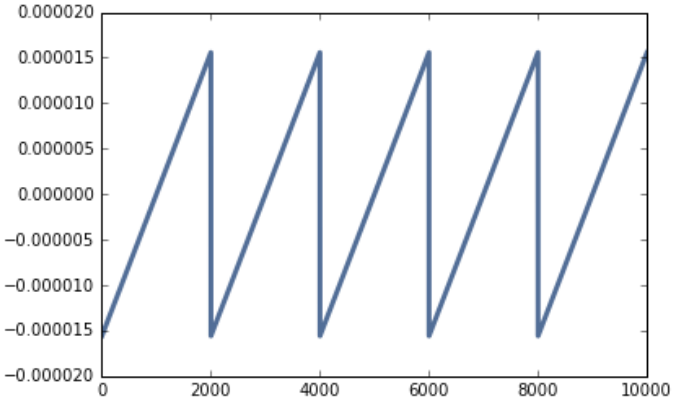


The first difference is a parabola and the second difference is a sawtooth wave (no surprises so far):

In [28]:
```
out_wave = in_wave.diff()
out_wave.plot()
```

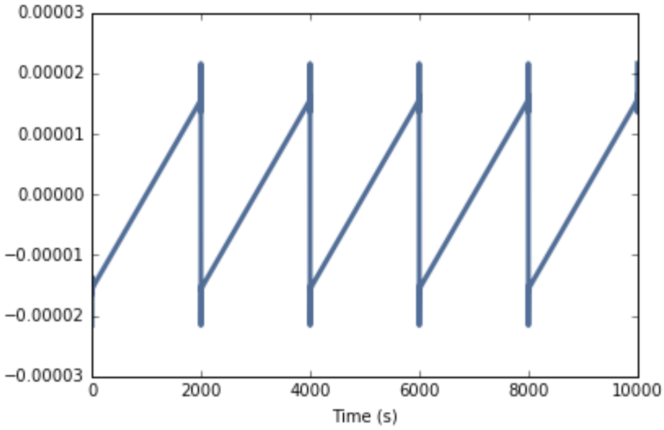


In [29]:
```
out_wave = out_wave.diff()
out_wave.plot()
```



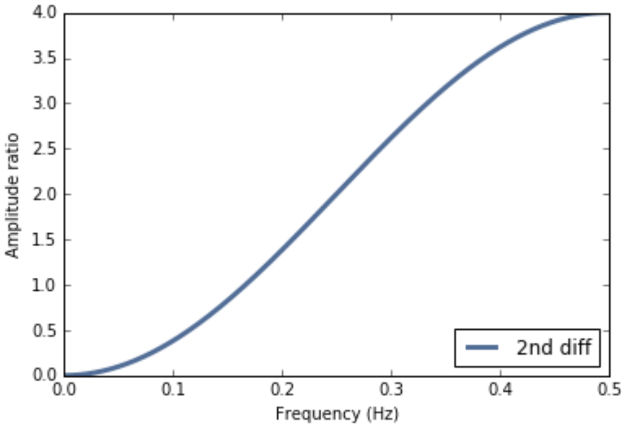
When we differentiate twice, we get a sawtooth with some ringing. Again, the problem is that the deriviative of the parabolic signal is undefined at the points.

In [30]:
```
spectrum = in_wave.make_spectrum().differentiate().differentiate()
out_wave2 = spectrum.make_wave()
out_wave2.plot()
thinkplot.config(xlabel='Time (s)')
```



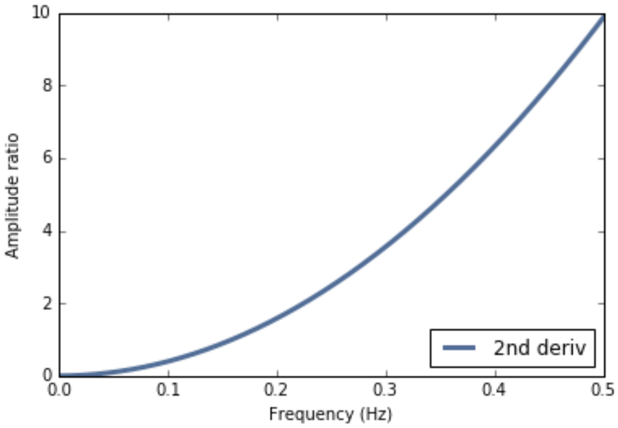
The window of the second difference is -1, 2, -1. By computing the DFT of the window, we can find the corresponding filter.

In [31]:
```
diff_window = np.array([-1.0, 2.0, -1.0])
padded = thinkdsp.zero_pad(diff_window, len(in_wave))
diff_wave = thinkdsp.Wave(padded, framerate=in_wave.framerate)
diff_filter = diff_wave.make_spectrum()
diff_filter.plot(label='2nd diff')

thinkplot.config(xlabel='Frequency (Hz)',
                 ylabel='Amplitude ratio',
                 legend=True, loc='lower right')
```



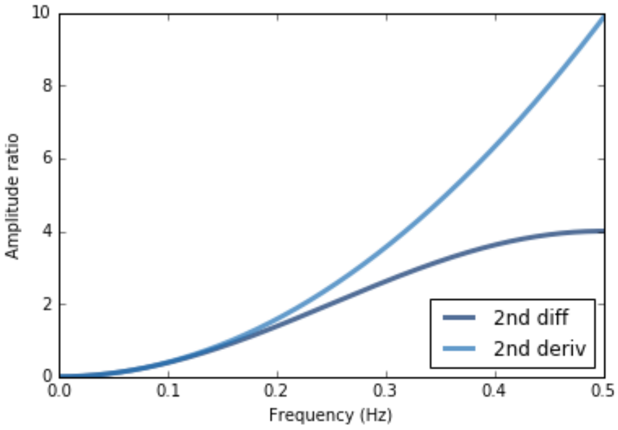
And for the second derivative, we can find the corresponding filter by computing the filter of the first derivative and squaring it.

In [32]:
```
deriv_filter = in_wave.make_spectrum()
deriv_filter.hs = (PI2 * 1j * deriv_filter.fs)**2
deriv_filter.plot(label='2nd deriv')

thinkplot.config(xlabel='Frequency (Hz)',
                 ylabel='Amplitude ratio',
                 legend=True, loc='lower right')
```



Here's what the two filters look like on the same scale:

In [33]:
```
diff_filter.plot(label='2nd diff')
deriv_filter.plot(label='2nd deriv')

thinkplot.config(xlabel='Frequency (Hz)',
                 ylabel='Amplitude ratio',
                 legend=True, loc='lower right')
```



Both are high pass filters that amplify the highest frequency components. The 2nd derivative is parabolic, so it amplifies the highest frequencies the most. The 2nd difference is a good approximation of the 2nd derivative only at the lowest frequencies, then the deviates substantially.

In [ ]: