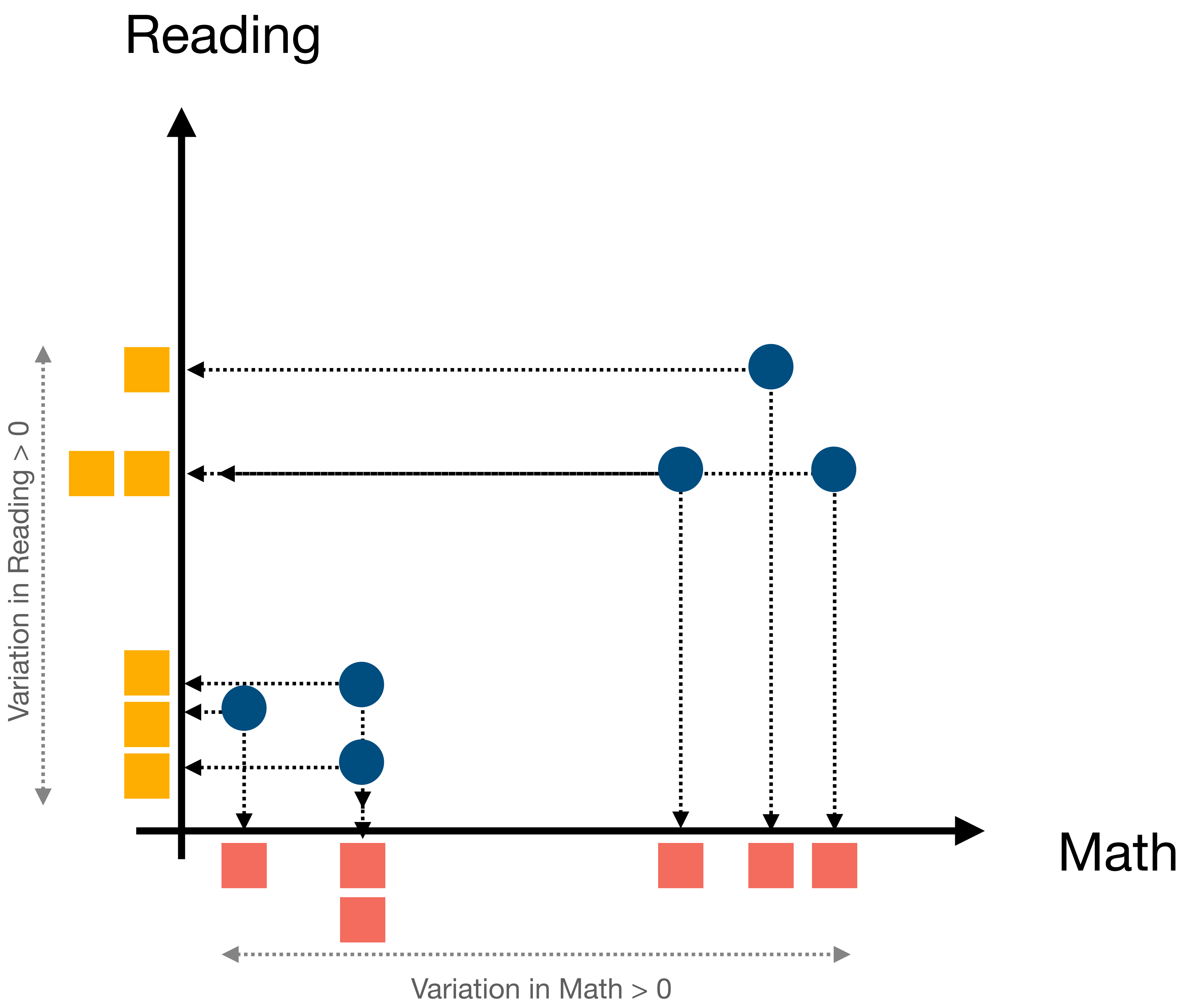
PCA - concept

Student	Reading	Math
บุญมี	8	99
บุญมา	5	43
บุญหนัก	1	16
บุญทับ	4	49
บุญถึก	7	83
บุญถึง	3	55

PCA - concept



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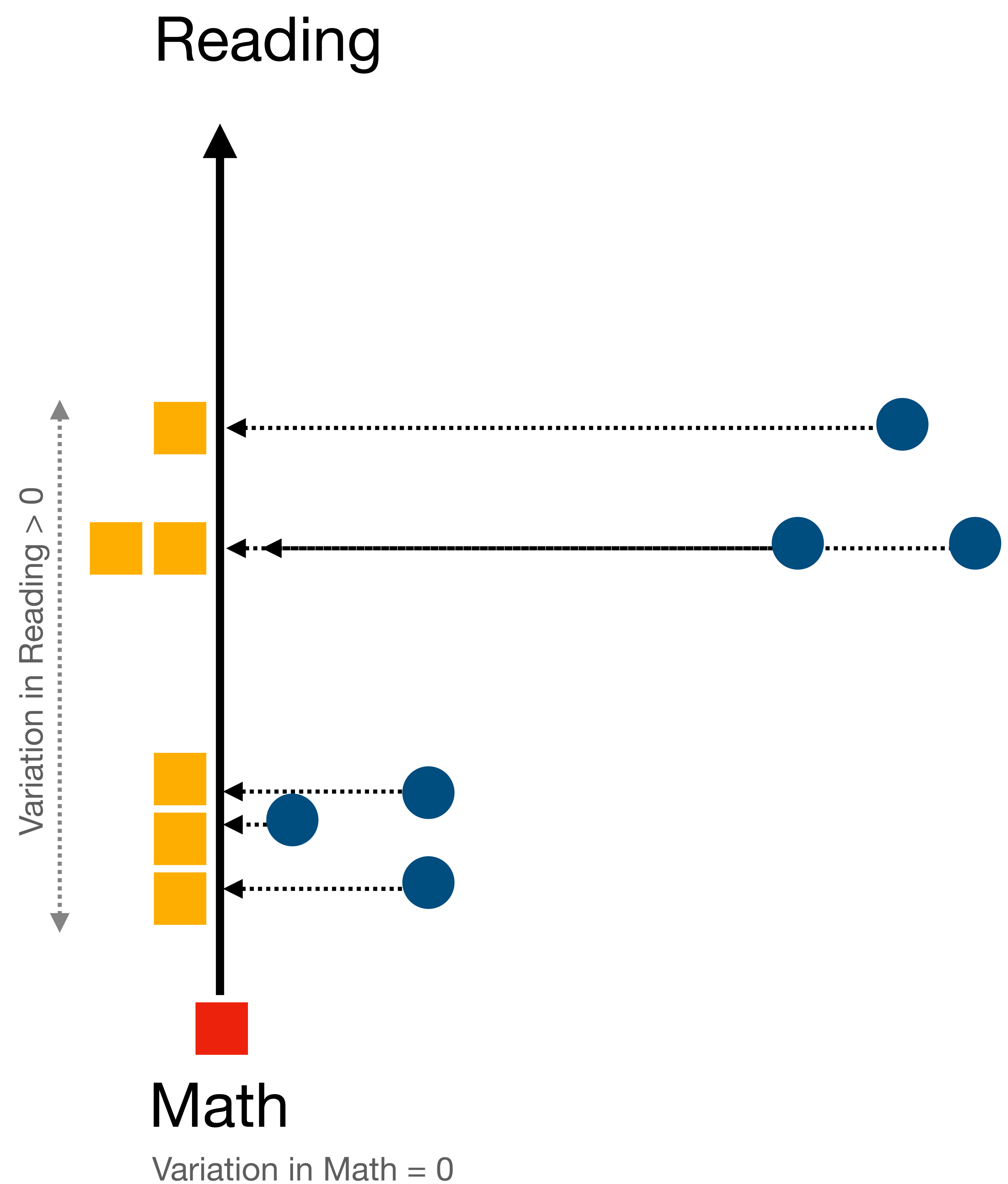

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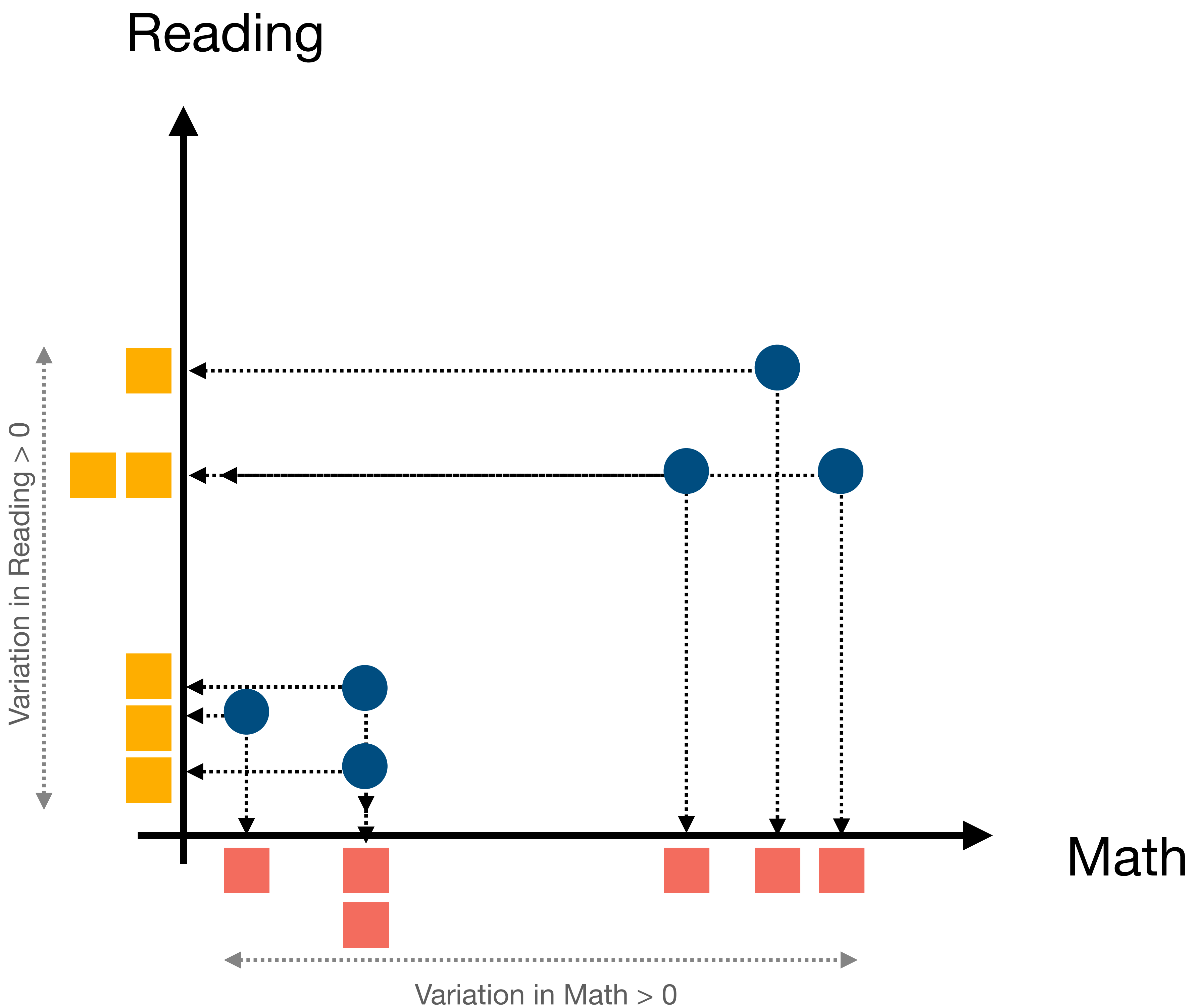

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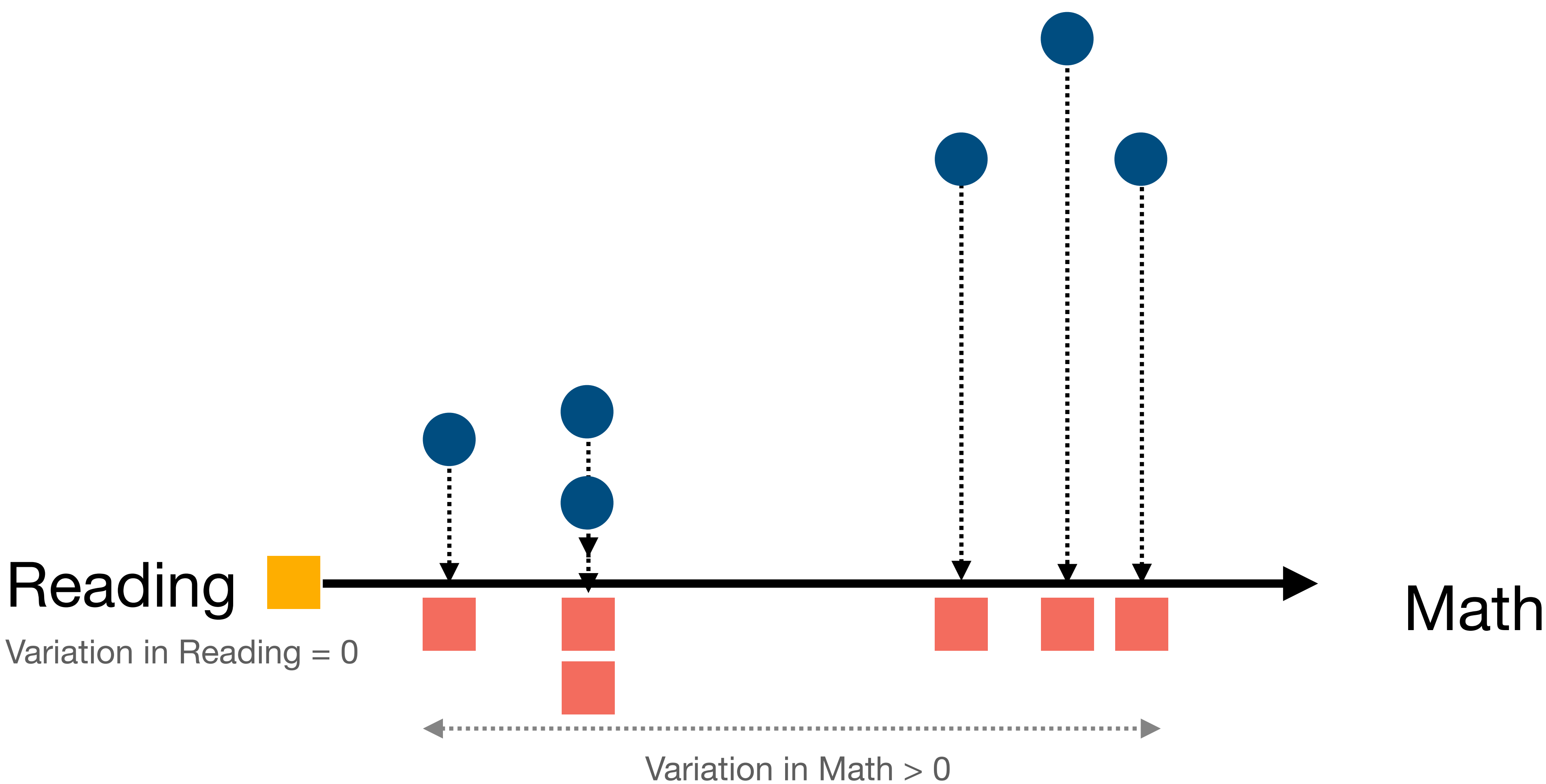
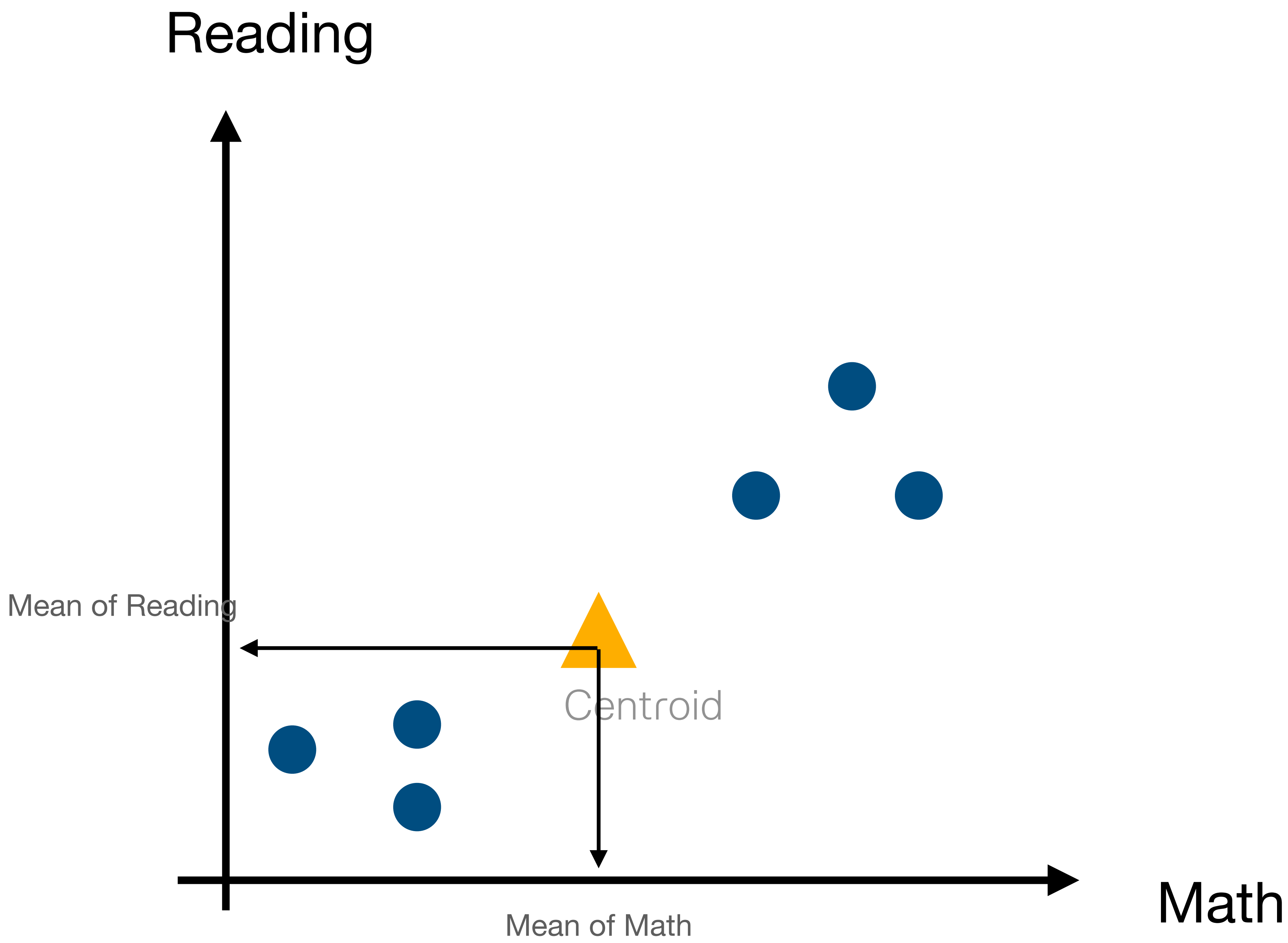


PCA - concept

Calculate centroids and standard deviations of the data

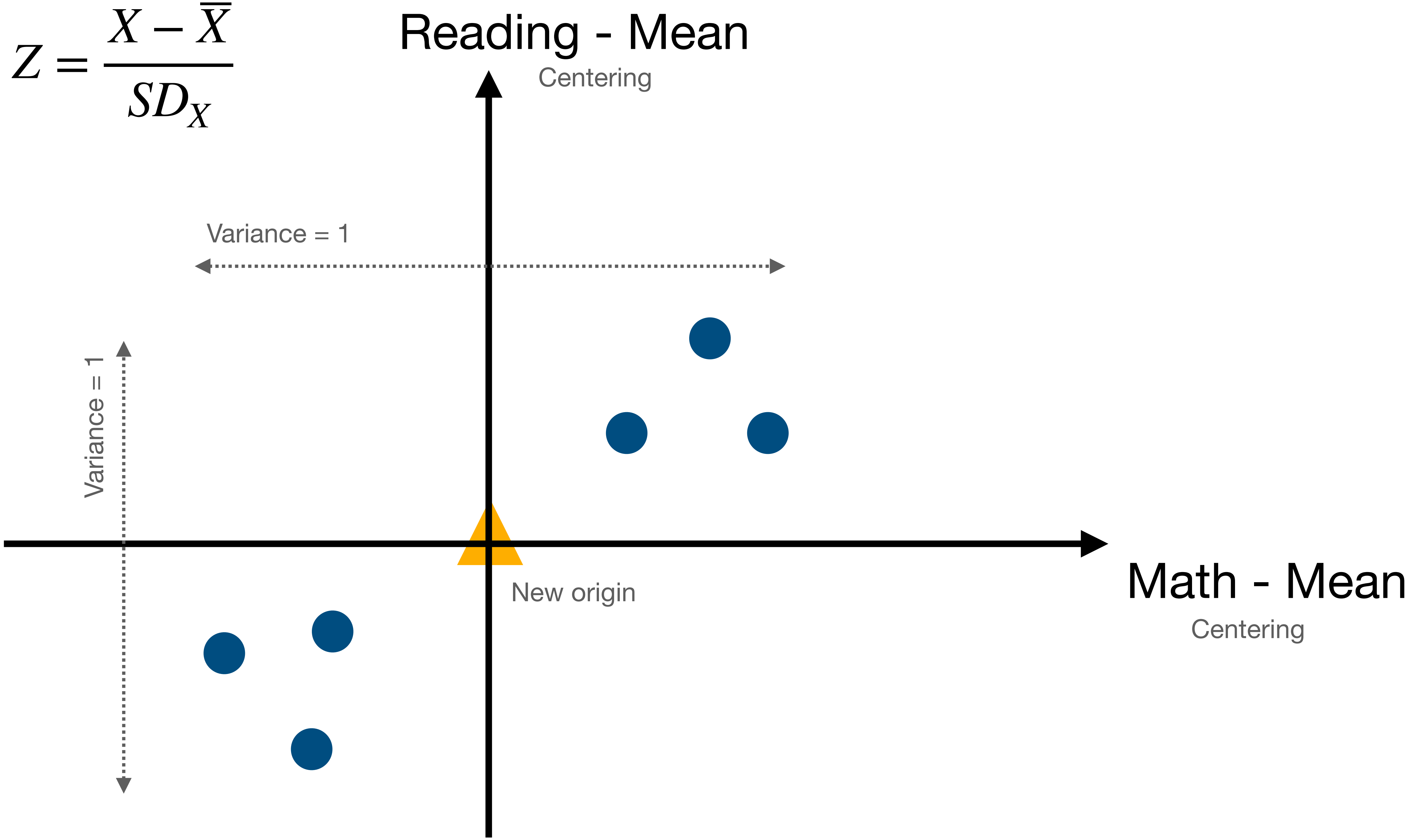


PCA - concept

We will see that standardization did not change the relationship between reading and math scores.

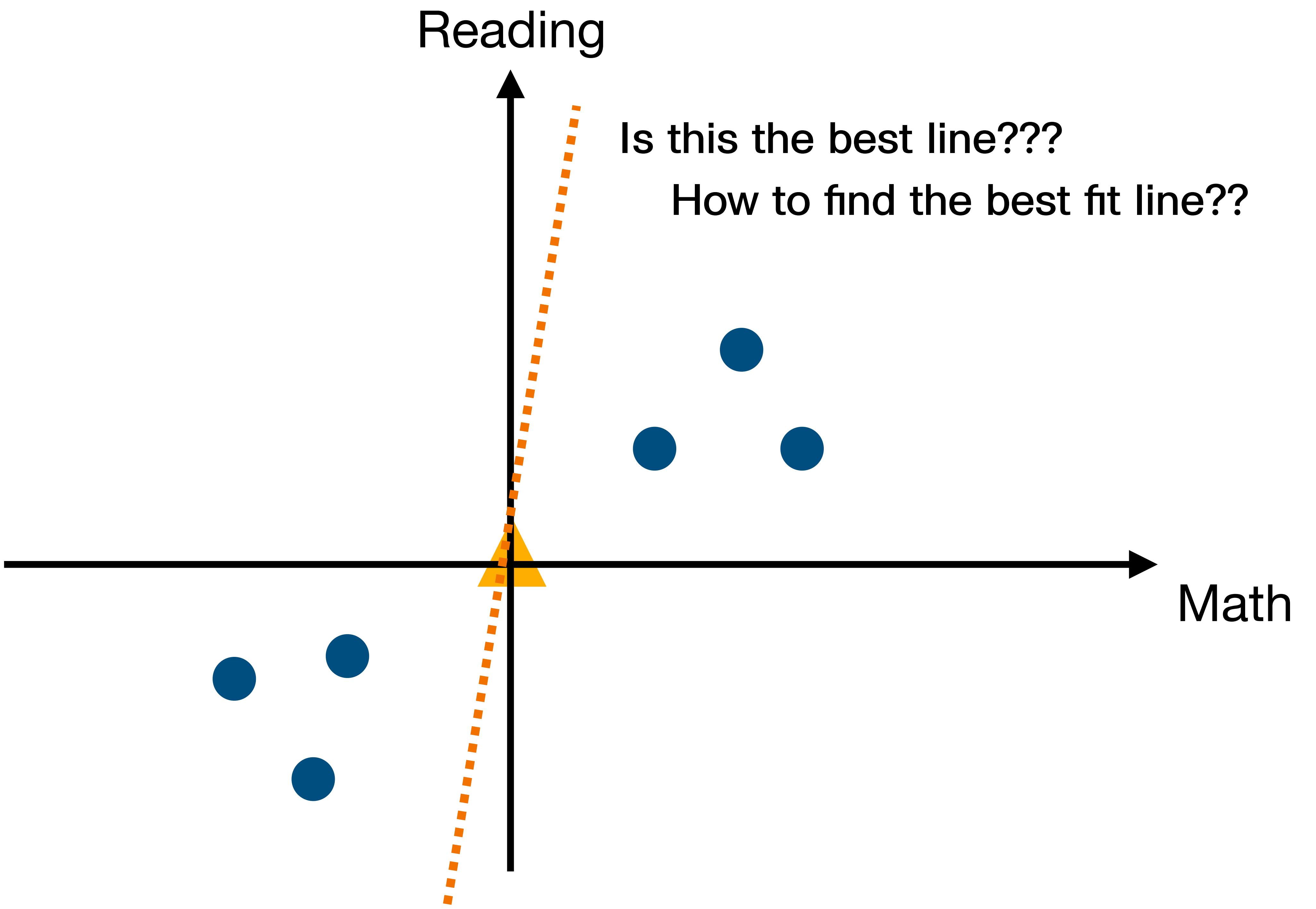
Standardized the variables

$$Z = \frac{X - \overline{X}}{SD_X}$$



PCA - concept

Fit a line to the data…



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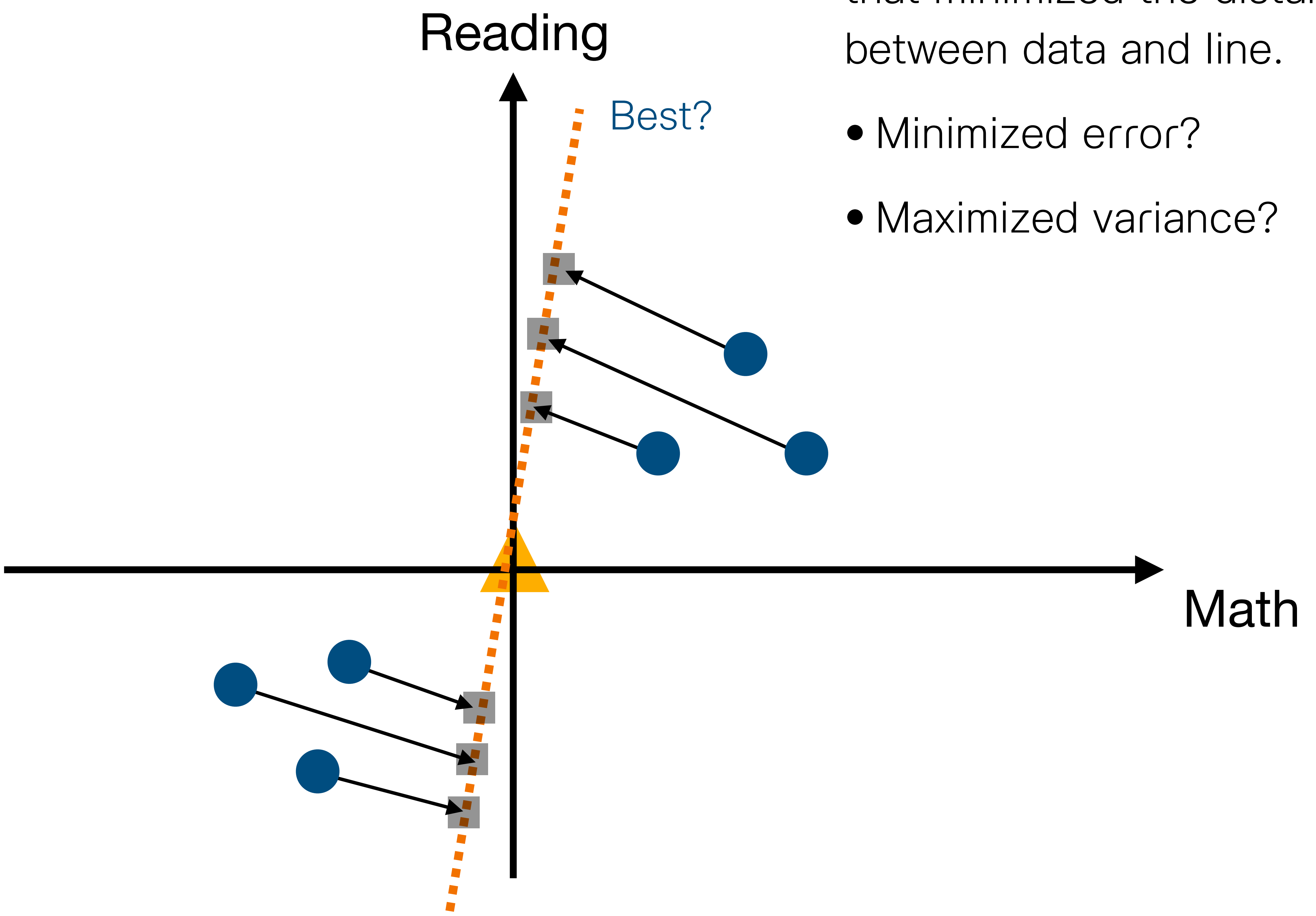

The best fit line is the line that minimized the distances between data and line.

- Minimized error?
- Maximized variance?

PCA - concept

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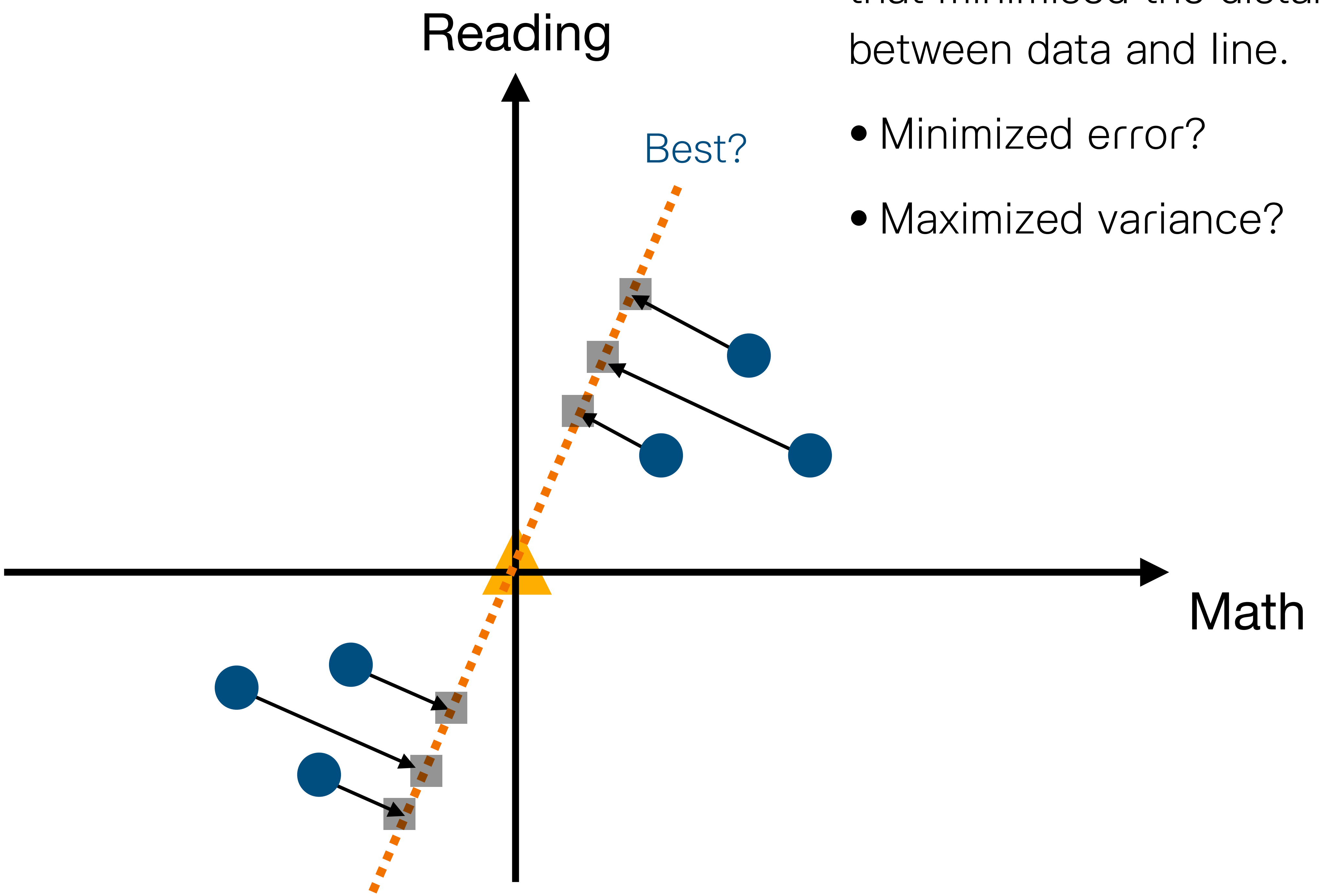

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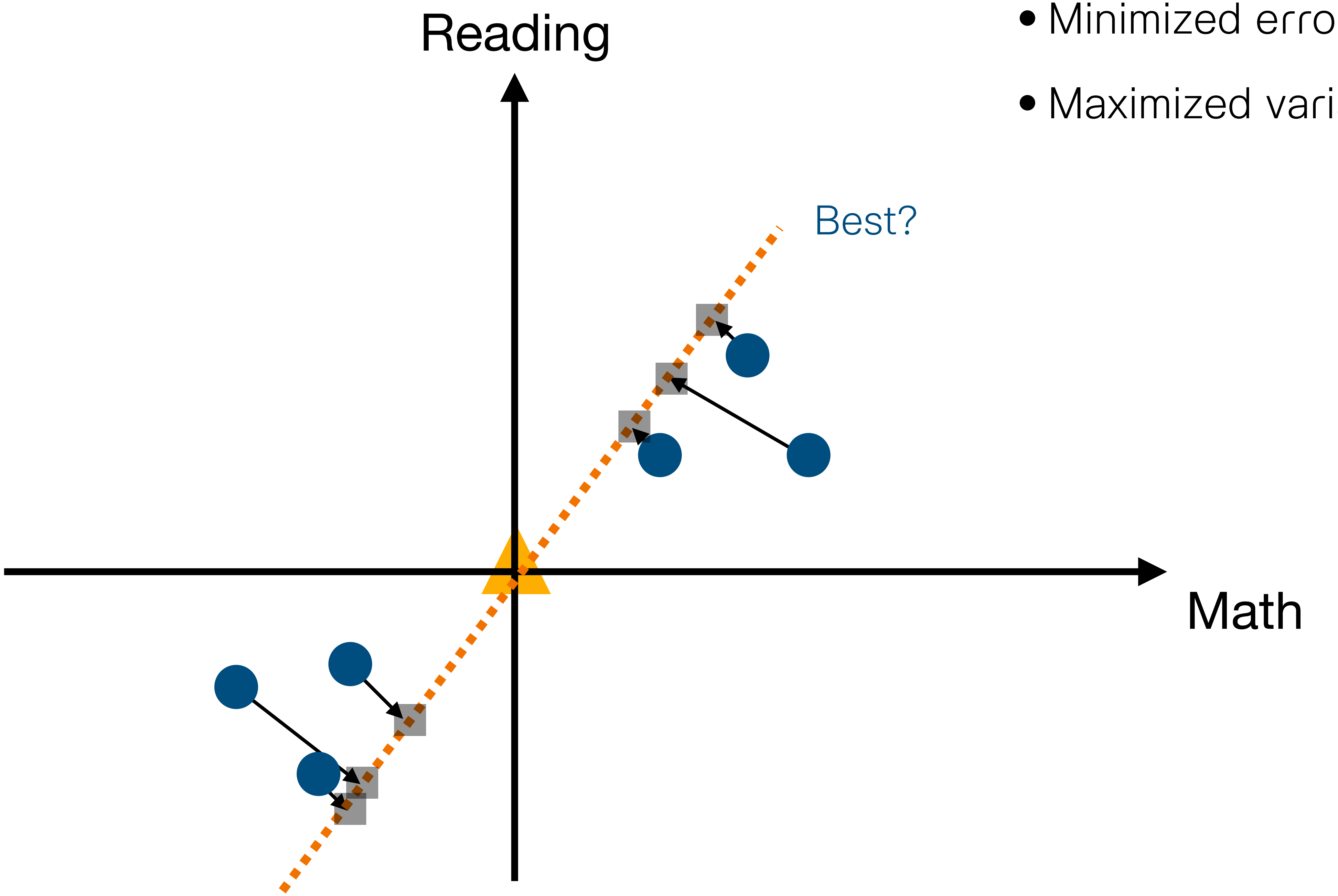

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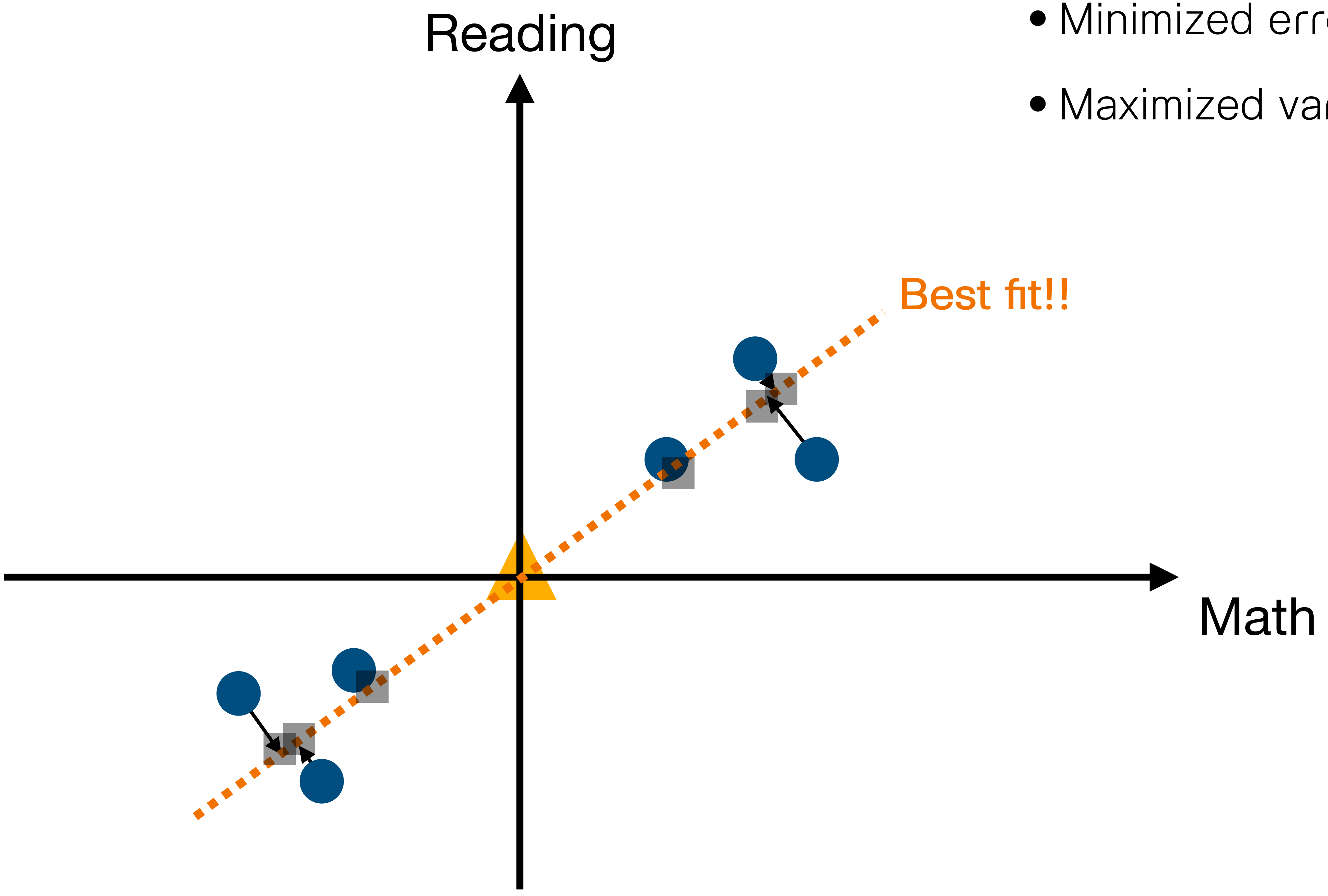

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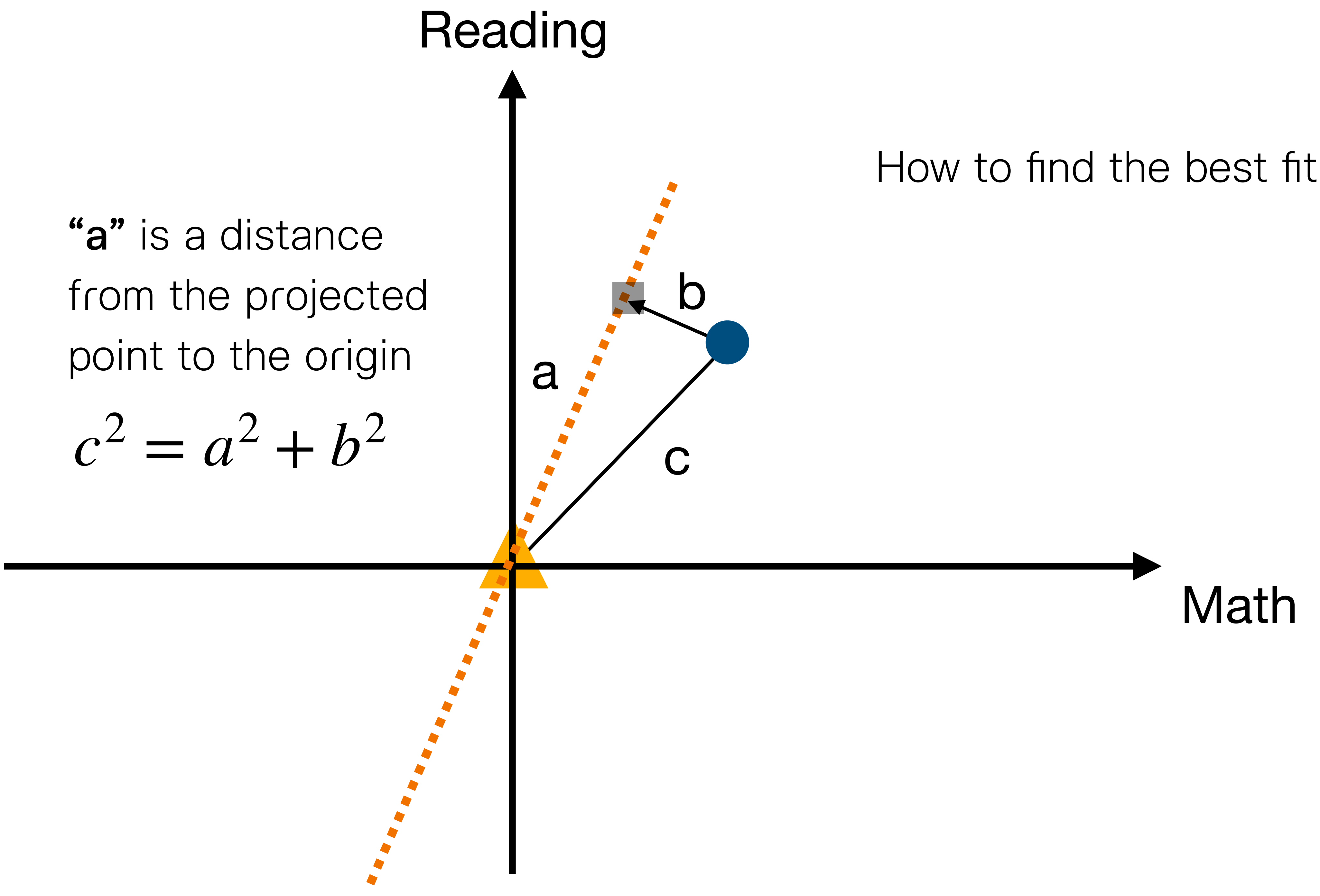
PCA - concept

Fit a line to the data…

How to find the best fit line??

“a” is a distance from the projected point to the origin

$$c^2 = a^2 + b^2$$



PCA - concept

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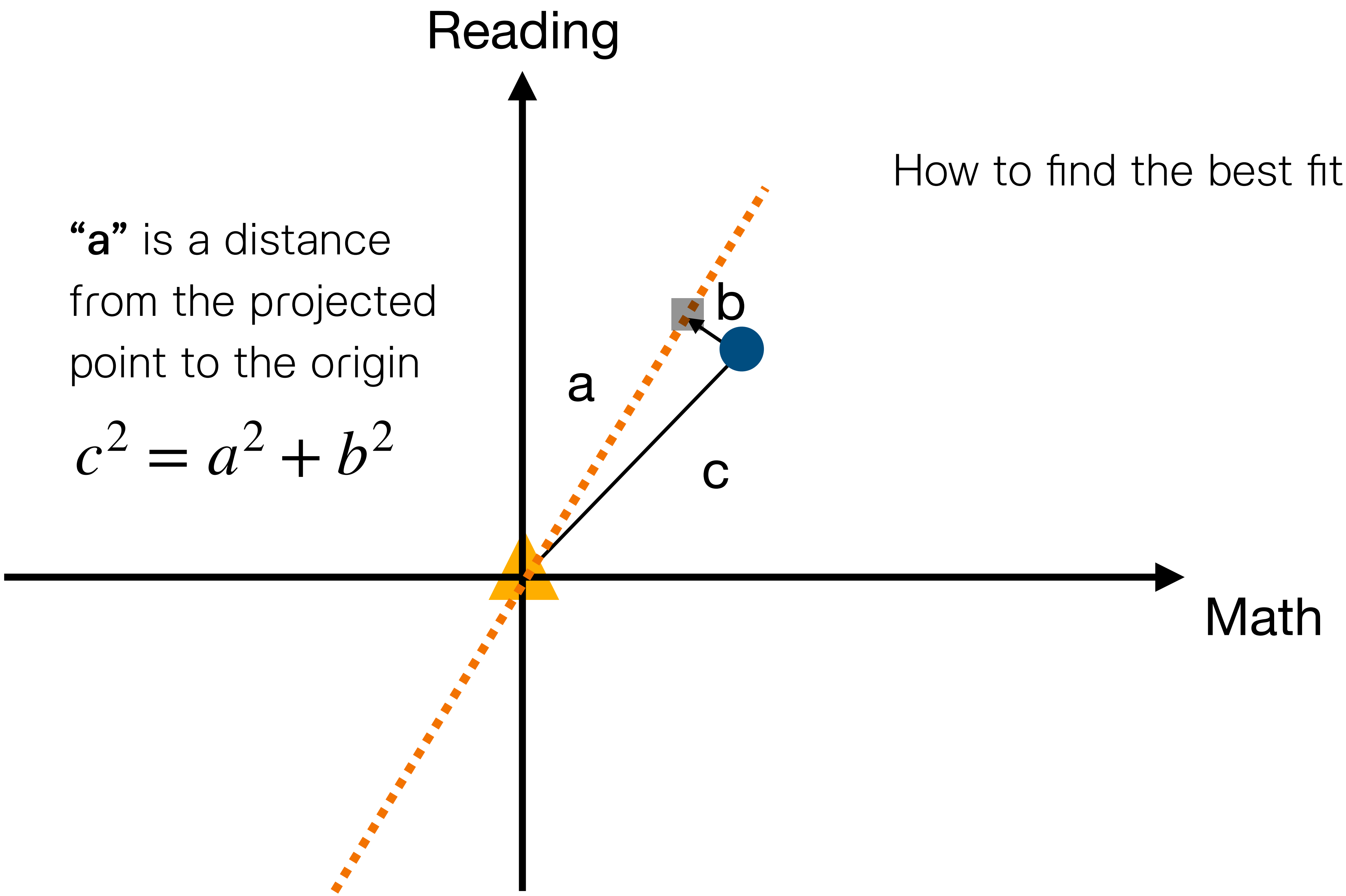

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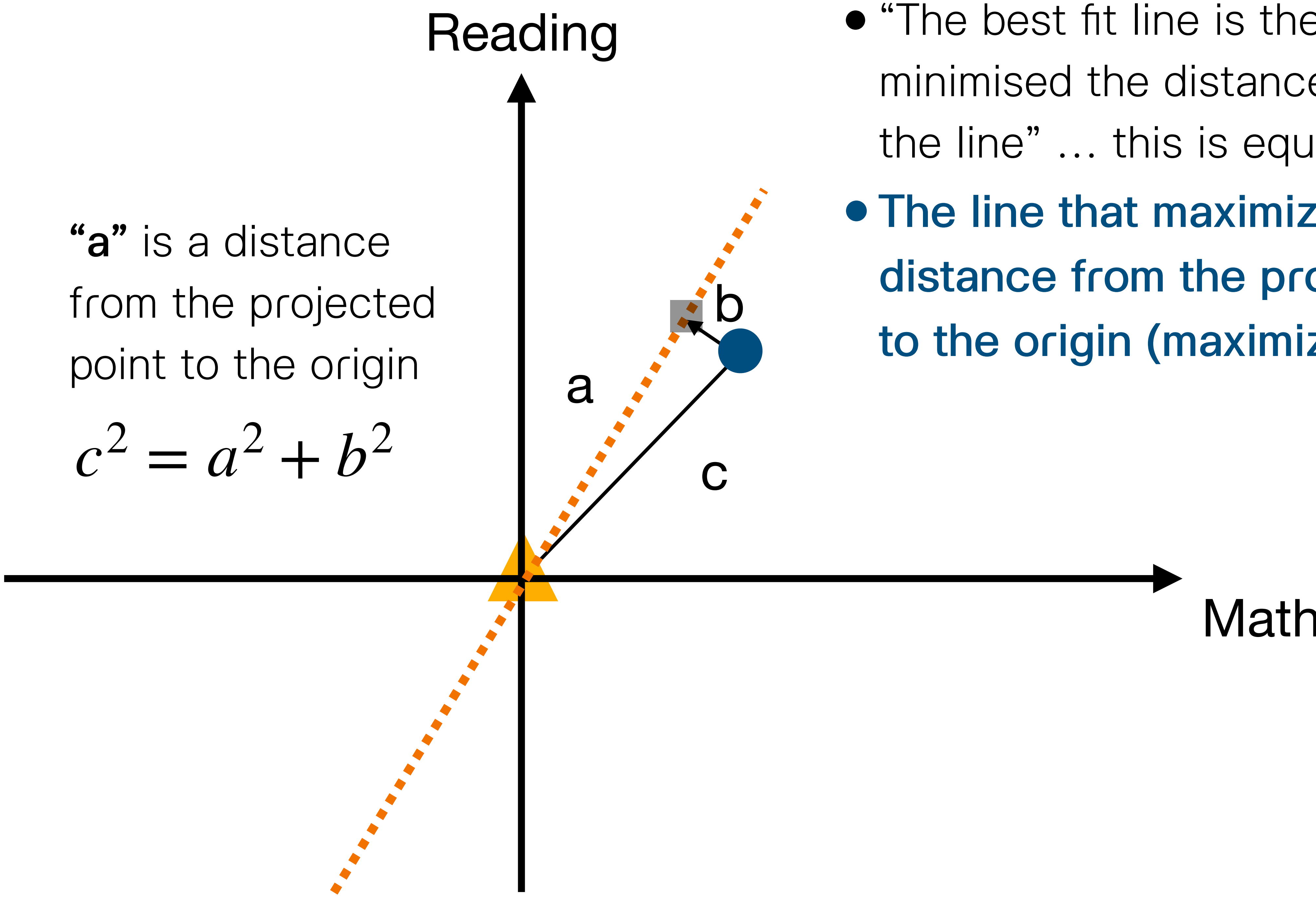


“a” is a distance from the projected point to the origin

$$c^2 = a^2 + b^2$$

How to find the best fit line??

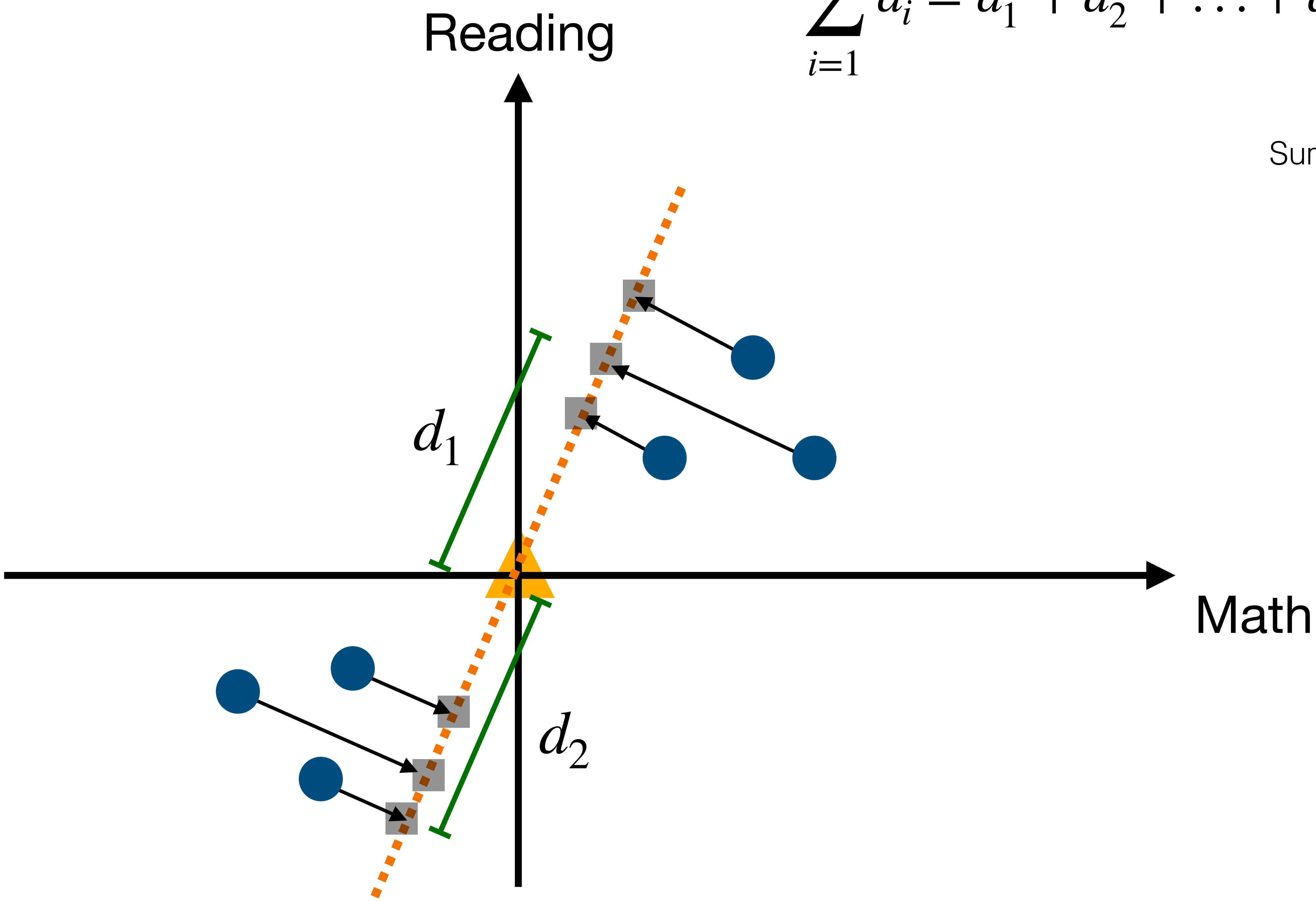
- “The best fit line is the line that minimised the distances from data to the line” … this is equivalent to….
- **The line that maximized the distance from the projected points to the origin (maximizes “a”)**

PCA - concept

Let d_i be a distance from the projected point i to the origin.

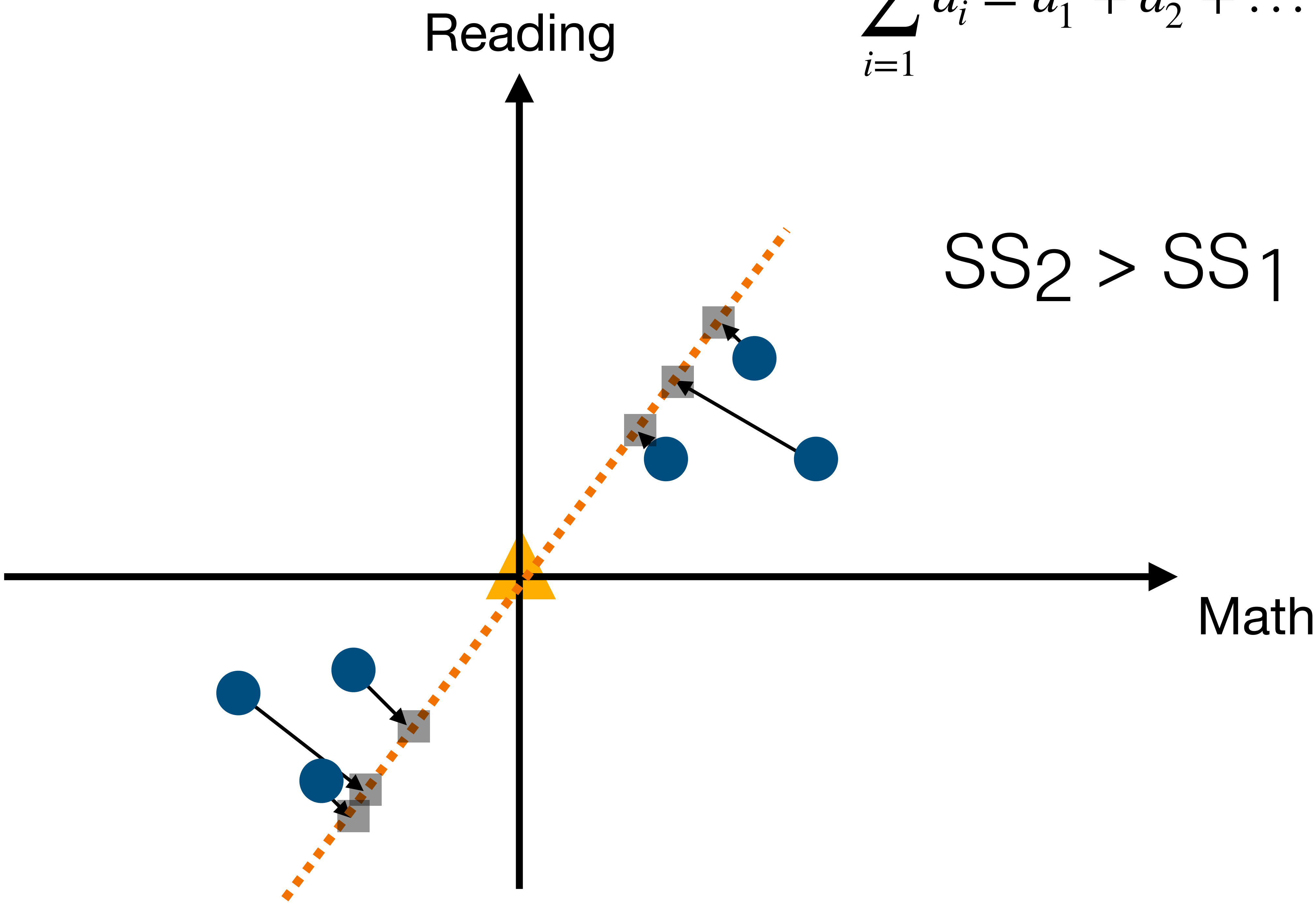
$$\sum_{i=1}^{6} d_i = d_1^2 + d_2^2 + \ldots + d_6^2 = \mathrm{SS}_1$$

Sum squares distance



PCA - concept

Fit a line to the data…



Let d_i be a distance from the projected point i to the origin.

$$\sum_{i=1}^{6} d_i = d_1^2 + d_2^2 + \ldots + d_6^2 = SS_2$$

$$SS_2 > SS_1$$

PCA - concept

Fit a line to the data...

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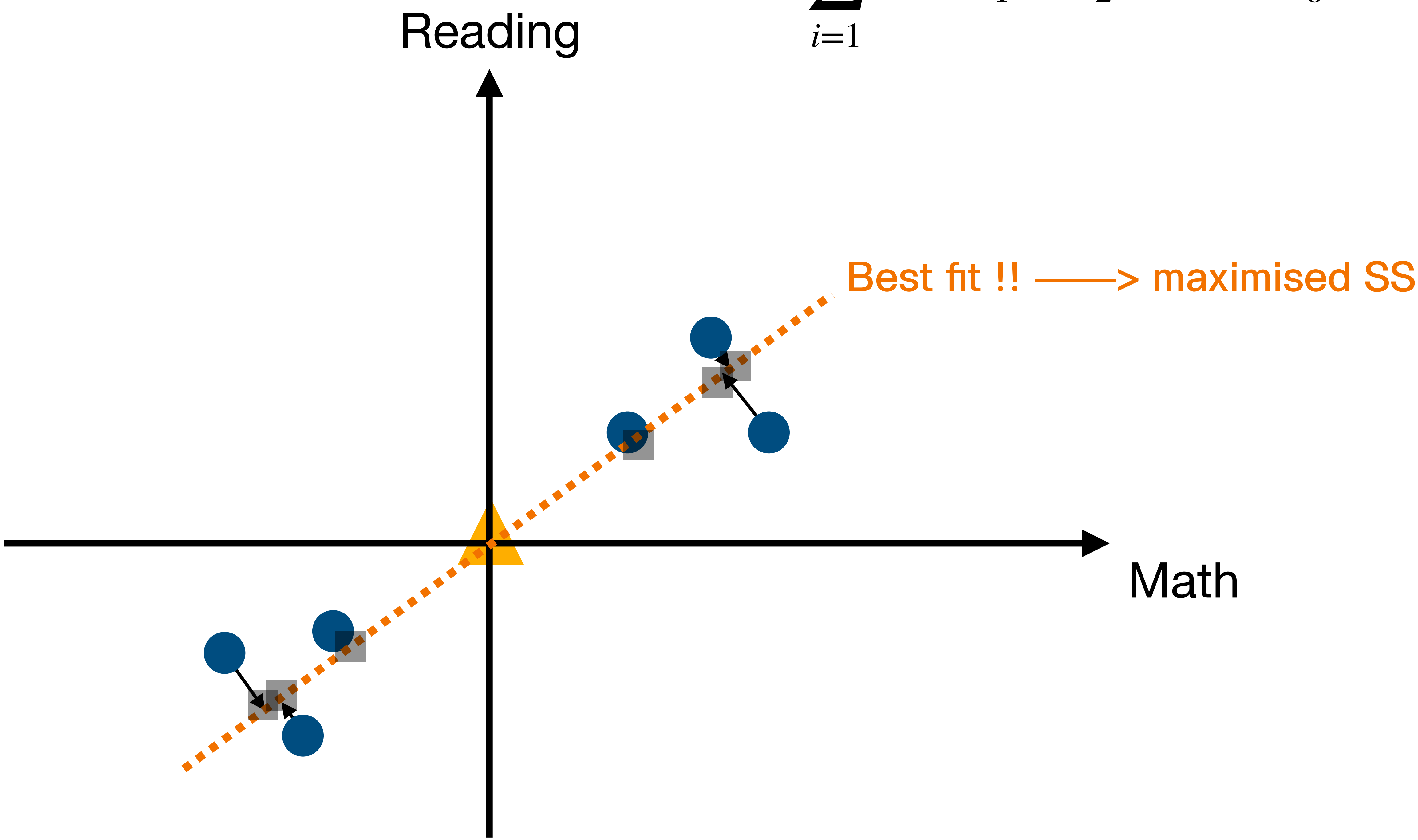

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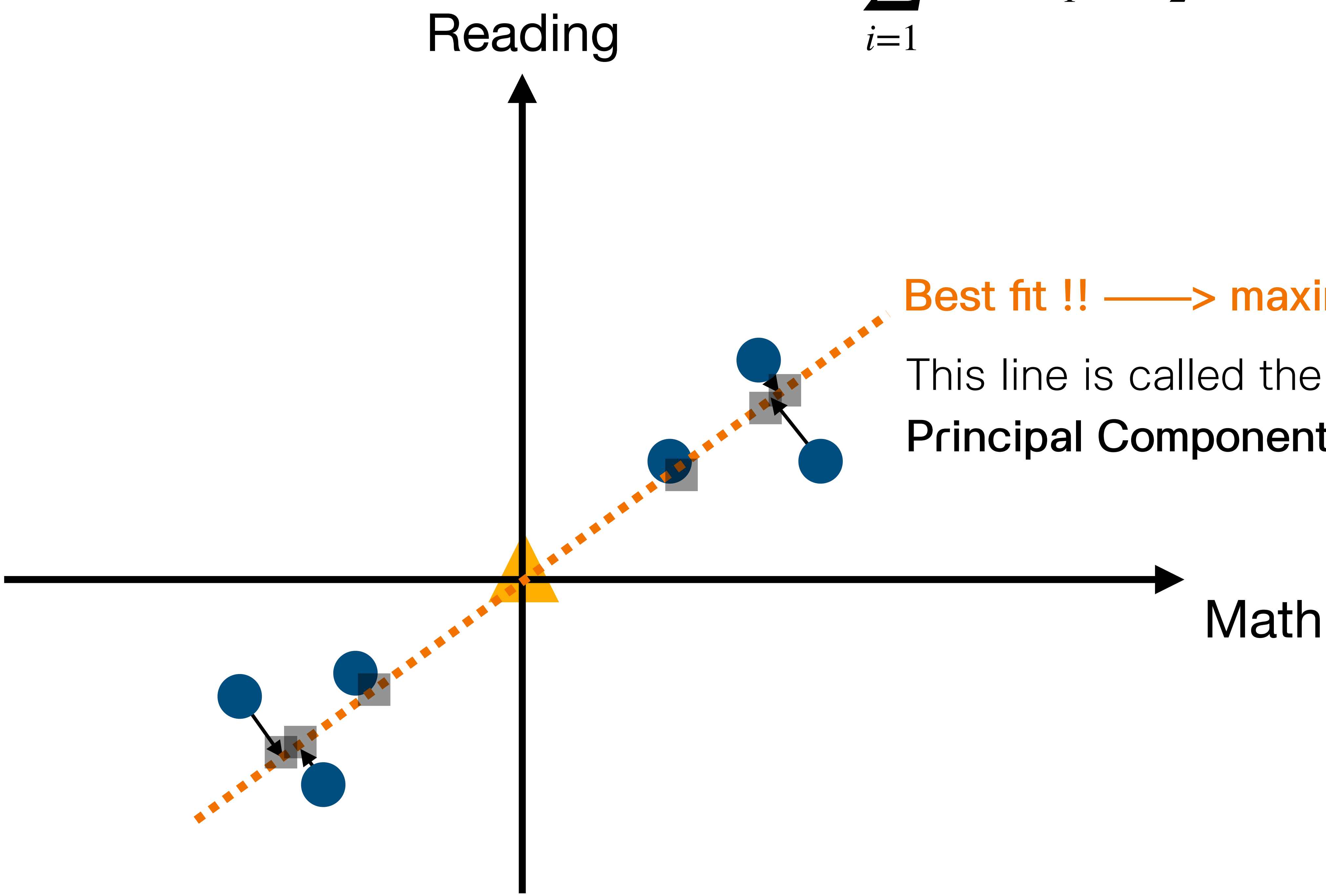
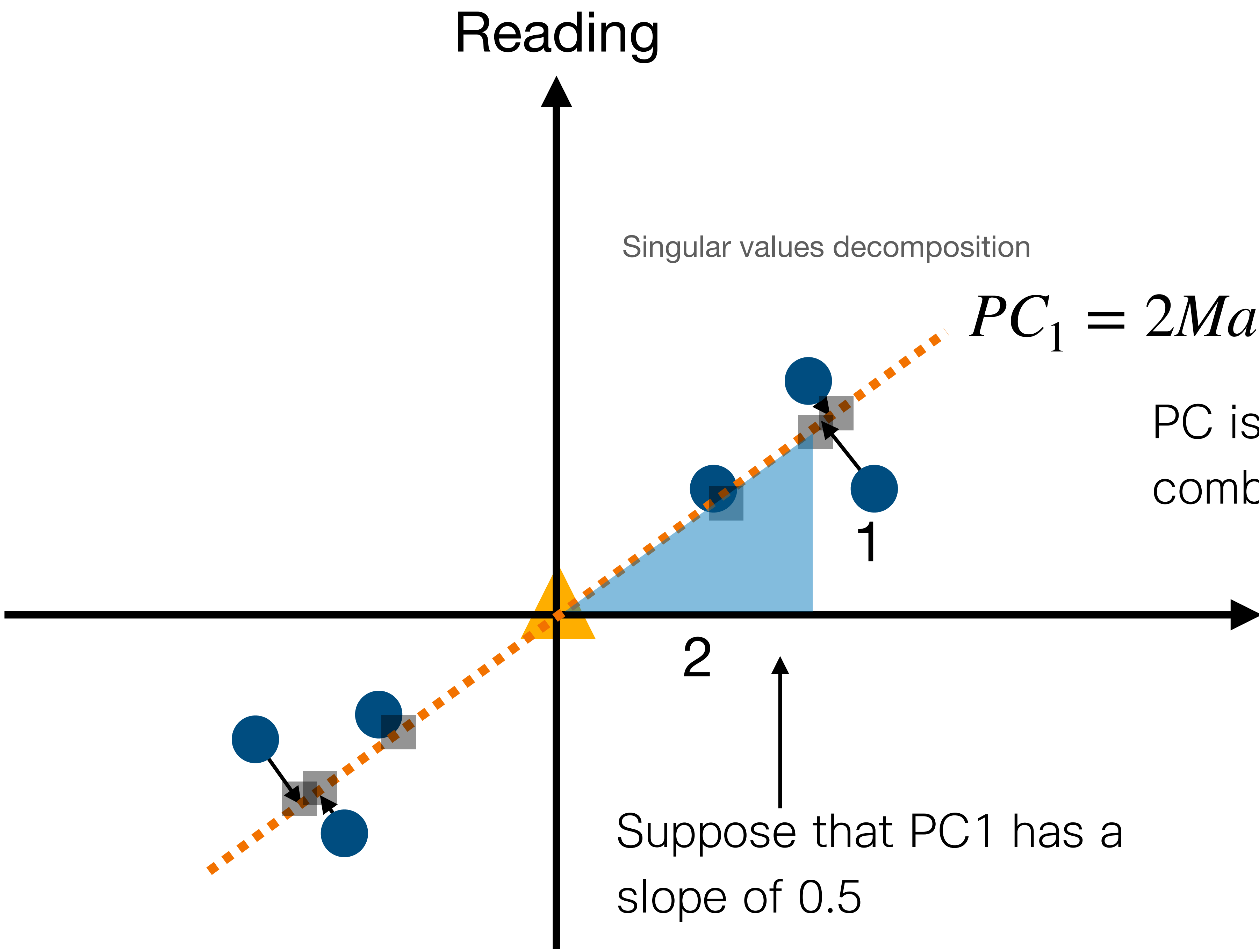


Best fit !! ——> maximised SS

This line is called the first **Principal Component (PC1).**

PCA - concept



$$PC_1 = 2Math + Reading$$

PC is a linear combination of variables.

Suppose that PC1 has a slope of 0.5

Figure not to scale.

PCA - concept

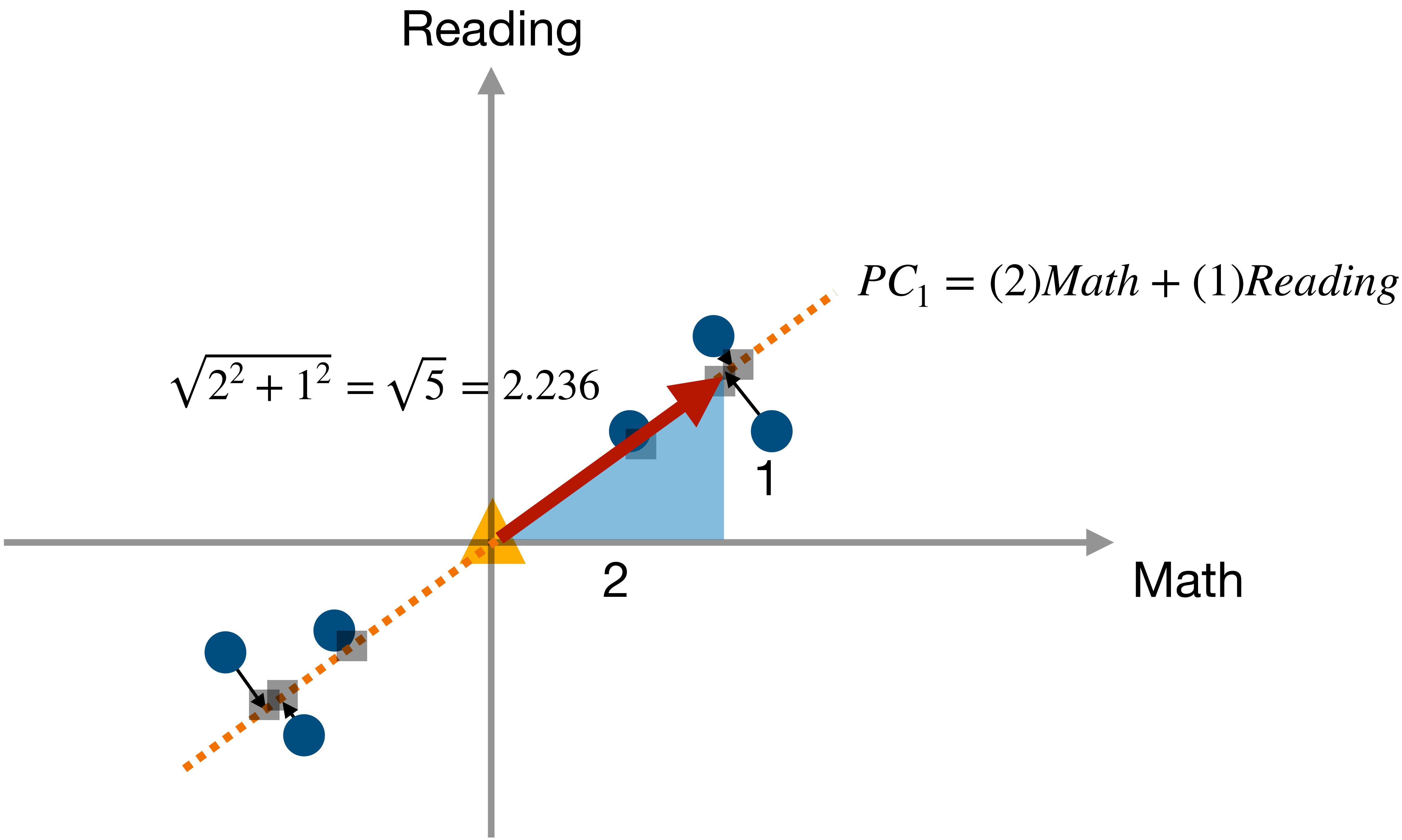
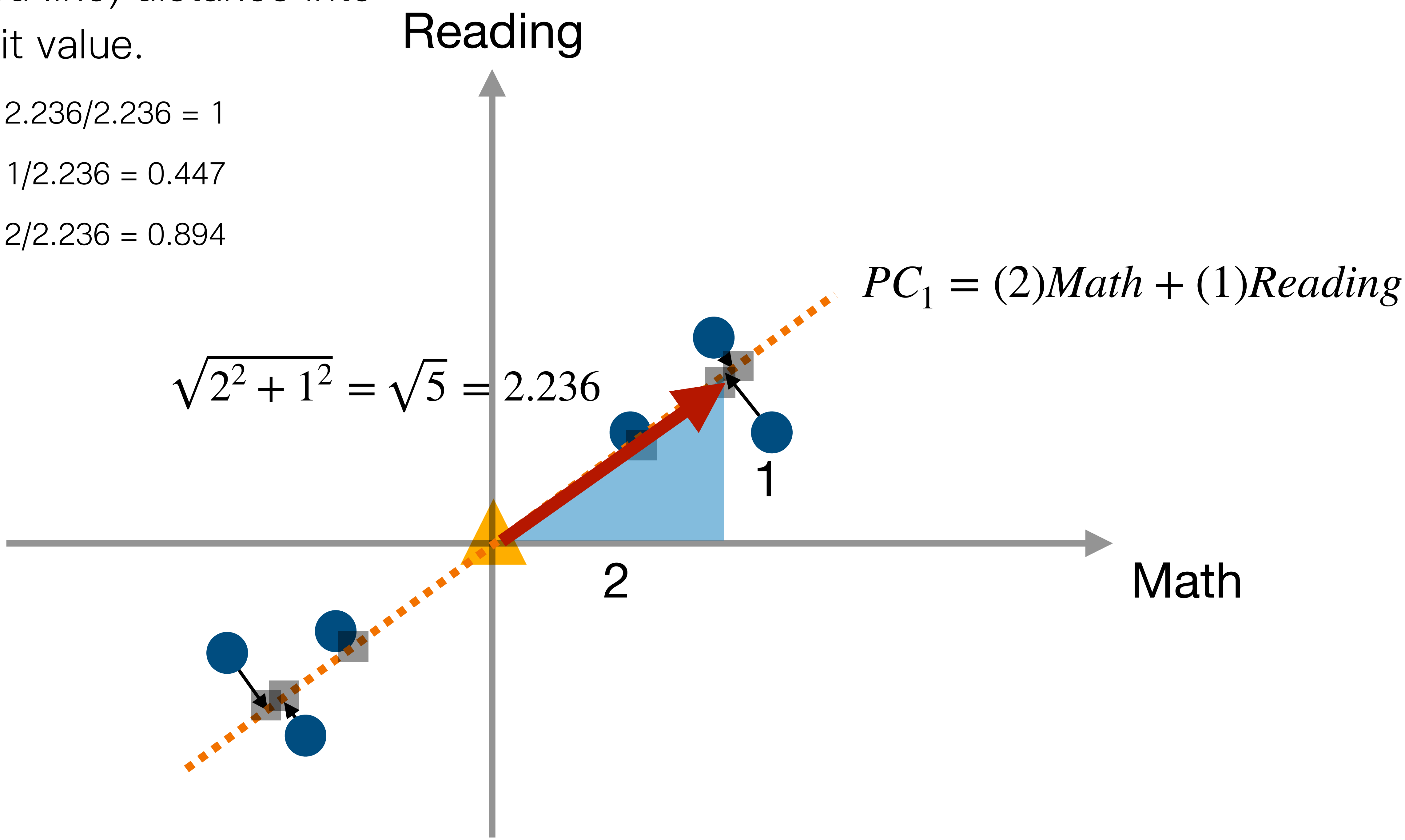


Figure not to scale.

PCA - concept

In PCA, we scaled the (red line) distance into unit value.

- 2.236/2.236 = 1
- 1/2.236 = 0.447
- 2/2.236 = 0.894



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PCA - concept

In PCA, we scaled the (red line) distance into unit value.

- 2.236/2.236 = 1
- 1/2.236 = 0.447
- 2/2.236 = 0.894

Eigenvector

(0.894,0.447) is called **"eigenvector" for PC1**

$$PC_1 = (0.894)Math + (0.447)Reading$$

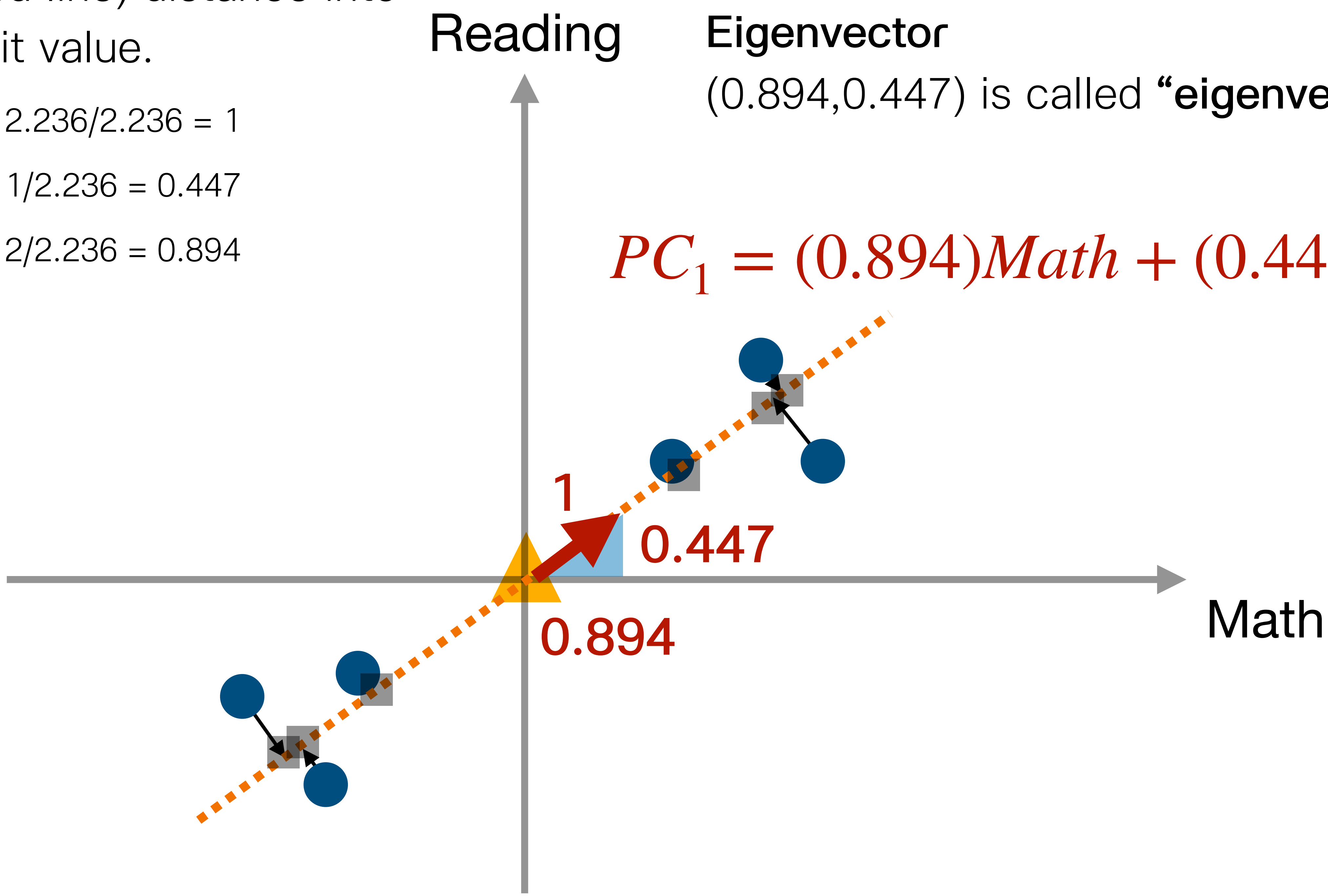


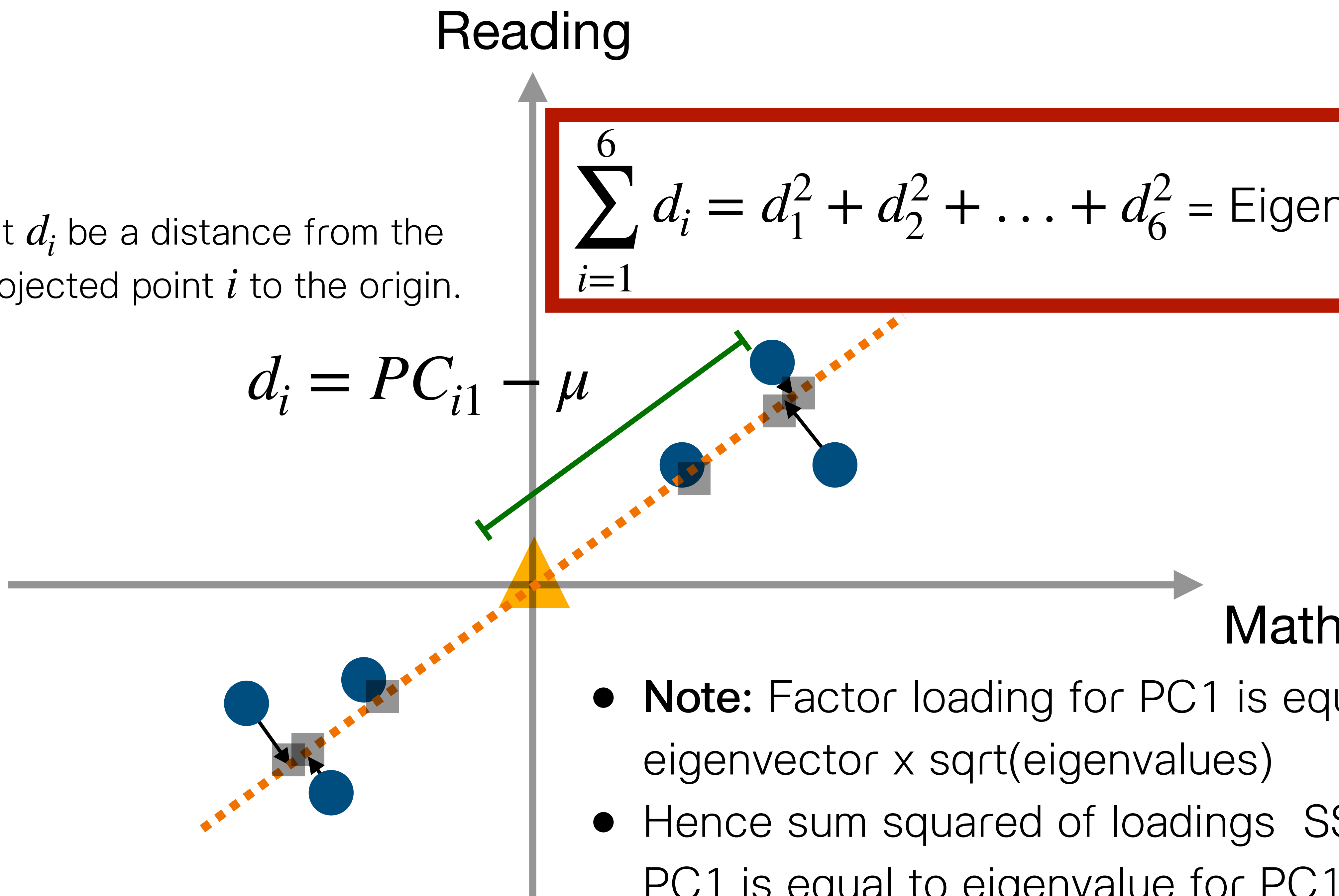
Figure not to scale.

PCA - concept

Let d_i be a distance from the projected point i to the origin.

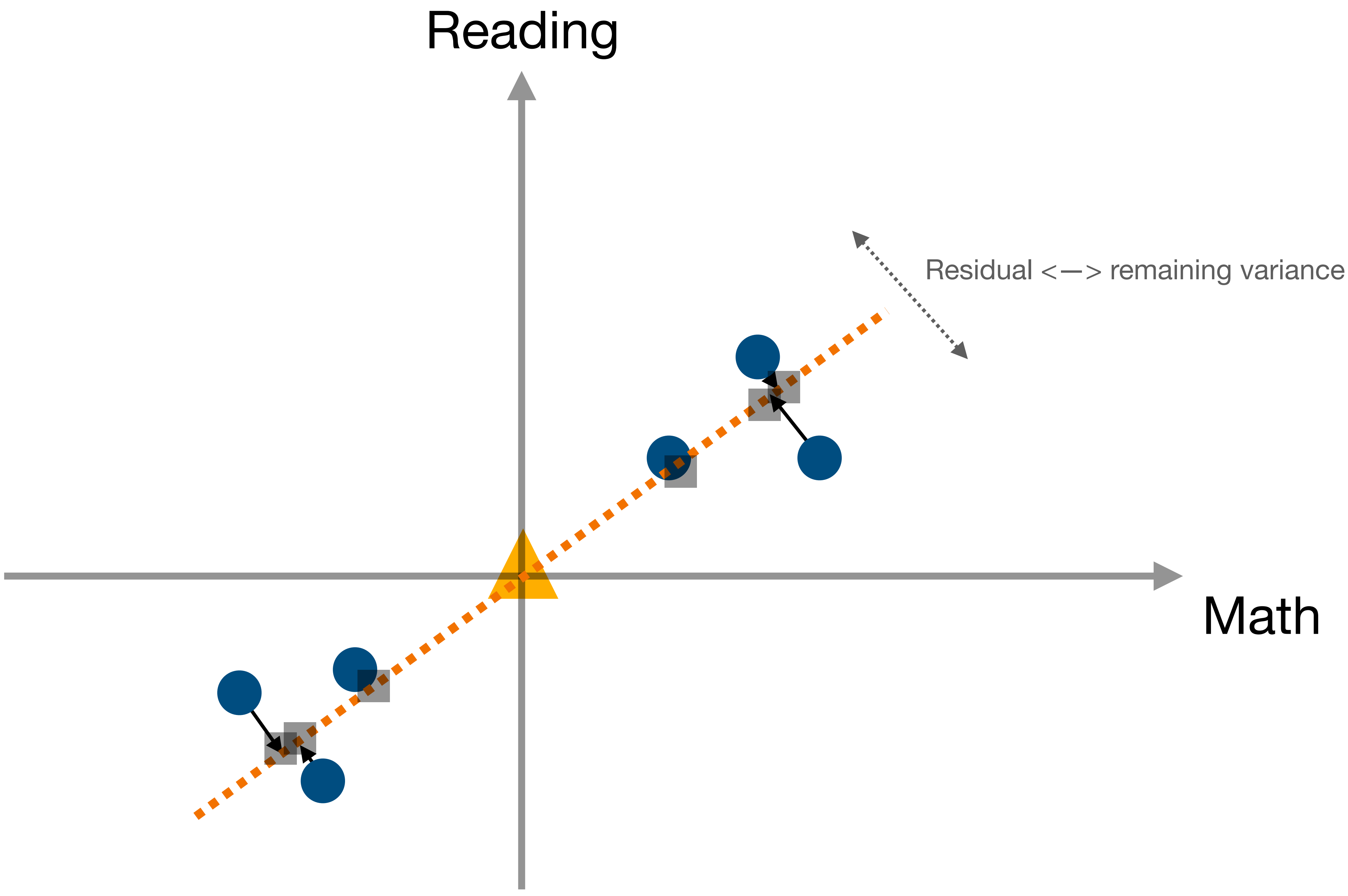
$$d_i = PC_{i1} - \mu$$

$$\boxed{\sum_{i=1}^{6} d_i = d_1^2 + d_2^2 + \ldots + d_6^2 = \text{Eigenvalue for PC1}}$$



- **Note:** Factor loading for PC1 is equal to eigenvector x sqrt(eigenvalues)
- Hence sum squared of loadings SS-loading for PC1 is equal to eigenvalue for PC1

PCA - concept



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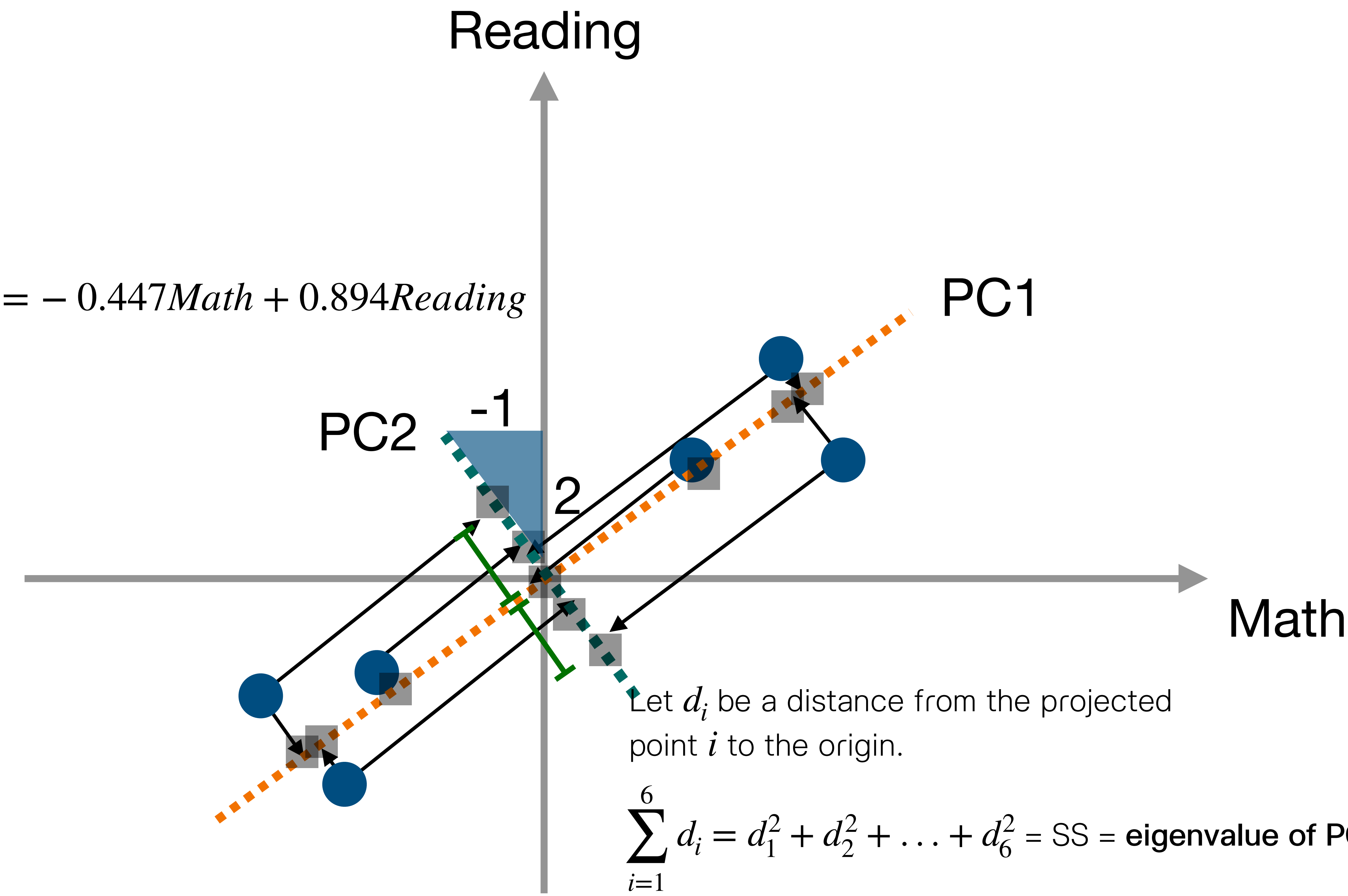
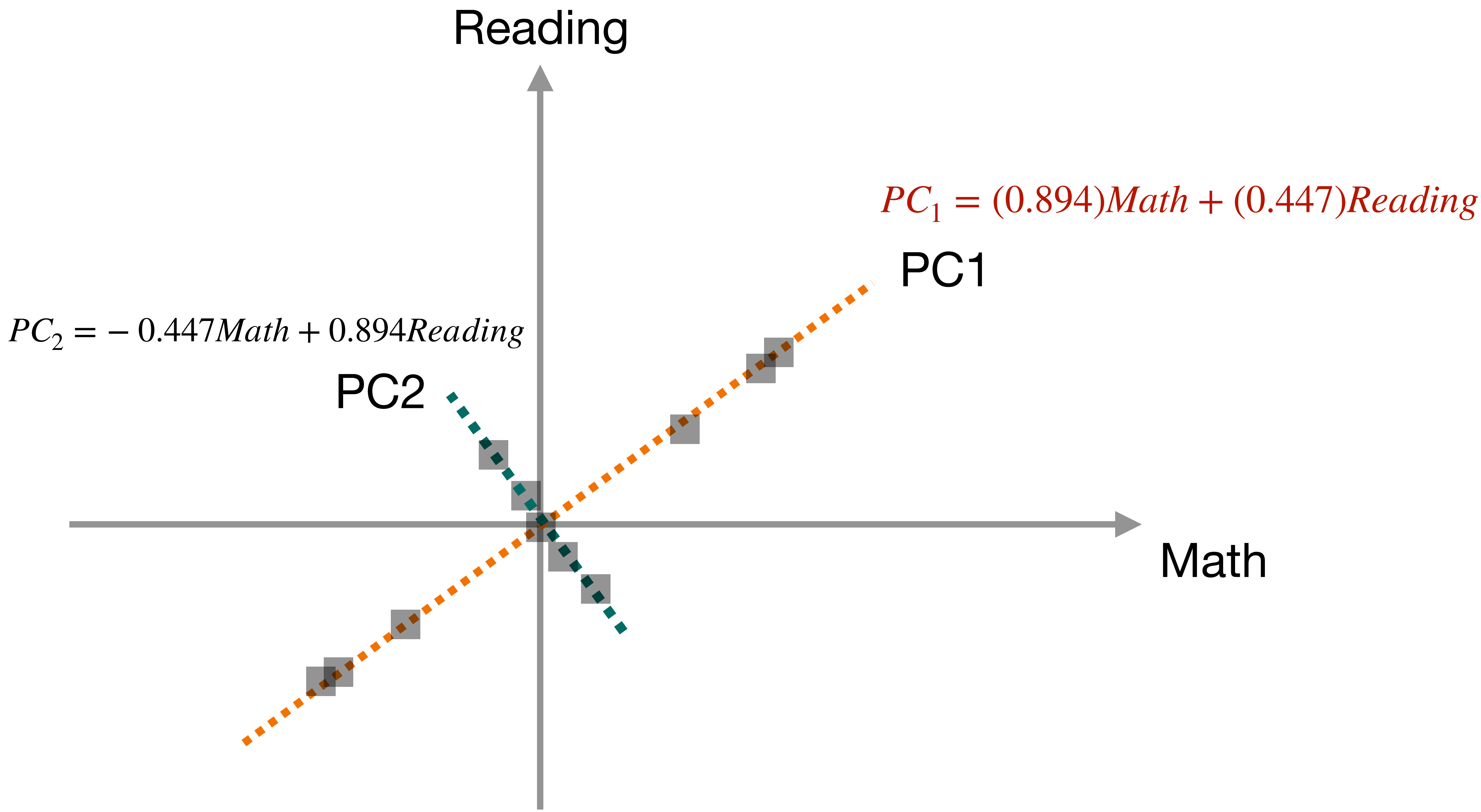


Let d_i be a distance from the projected point i to the origin.

$\sum_{i=1}^{6} d_i = d_1^2 + d_2^2 + \ldots + d_6^2$ = SS = **eigenvalue of PC2**

Figure not to scale.

PCA - concept



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