BML. 1 Intro

1.5 Recap

DataLab CSIC

Contents

Recap Bayesian theory for quick reference

Recap. Exchangeability

Of data, models and parameters…..

Observations from random phenomena: independent given a certain parameter (conditionally independent)→exchangeability

http://en.wikipedia.org/wiki/Exchangeable_random_variables

Finite set of rvs exchangeable: any two permutations have the same distribution

Infinite set of rvs exchangeable: any finite subset is exchangeable

De Finetti's theorem: set of rvs exchangeable iff ciid given a certain distribution

Recap: Classical vs Bayesian

Once model fixed, we want to learn about it (its parameters)

Classical	Bayesian
Parameters fixed	Parameters uncertain, prior
Given data, formulate likelihood	Given data, formulate likelihood
Maximize likelihood to find MLE (mimimum least squares, cross entropy,…)	Aggregate likelihood and prior to get posterior

Recap: ML inference

Likelihood

MLE

$$\ell(\theta|x) = \prod_{i=1}^{n} f(x_i|\theta)$$

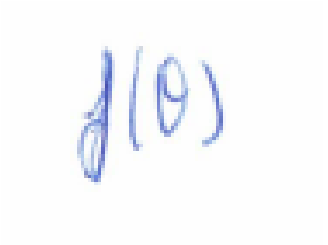
$$h(\theta) = \log\left(\ell(\theta|\underline{x})\right)$$

$$\max_{\theta} h(\theta) \longrightarrow \hat{\theta}$$

Recap: Bayesian inference

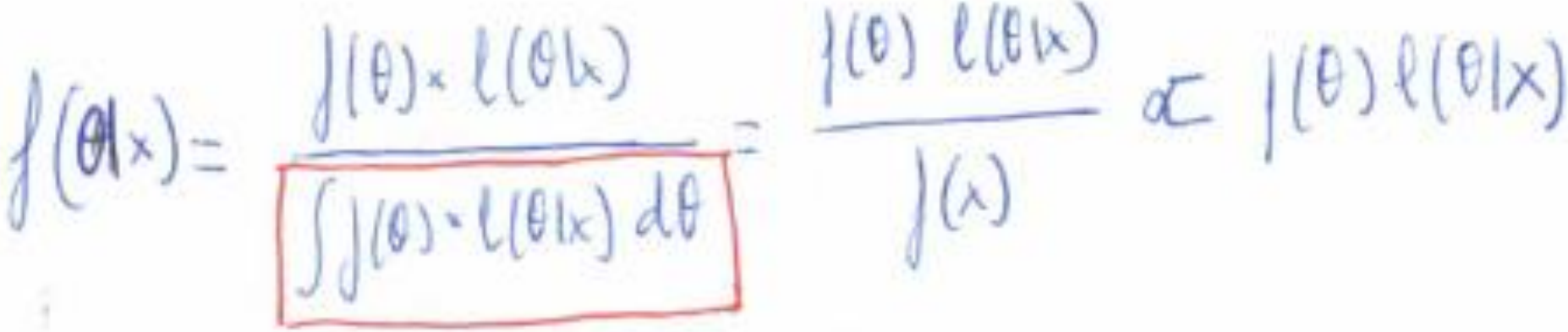
Prior

- Noninformative prior
- Conjugate prior
- Eliciting the prior



Recap: Bayesian inference

Posterior distribution. Bayes formula



Recap: Bayesian inference. Recall in parallel simulations for this

Point estimation

Posterior mean

$$E(\theta|x) = \int \theta \, f(\theta|x) \, d\theta$$

Posterior median

$$Pr(\theta \leq med|x) \geq \frac{1}{2} \qquad Pr(\theta \geq med|x) \geq \frac{1}{2}$$

Posterior mode. MAP

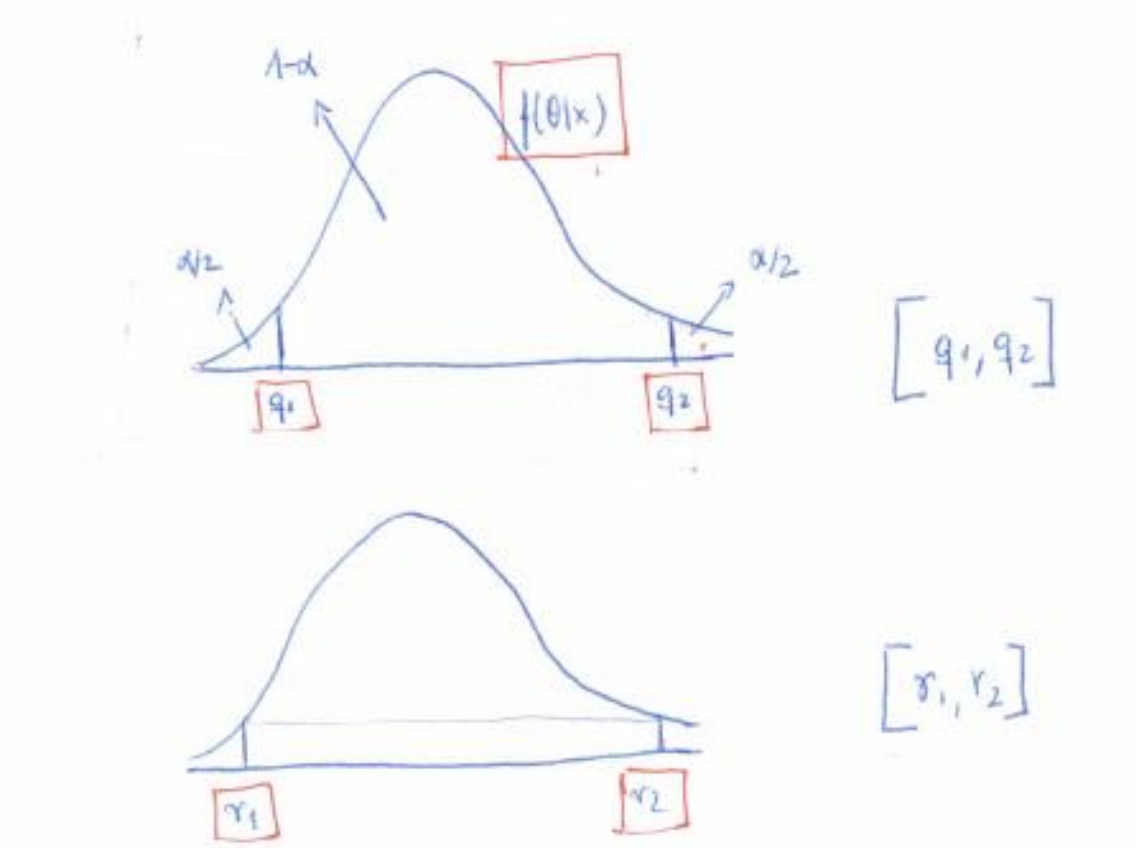
$$\begin{aligned} &\operatorname{argmax} f(\theta|x) \\ =\ &\operatorname{argmax} \ell(\theta|x) f(\theta) \\ =\ &\operatorname{argmax} h(\theta) + \log f(\theta) \end{aligned}$$

Recap: Bayesian inference

Credible interval

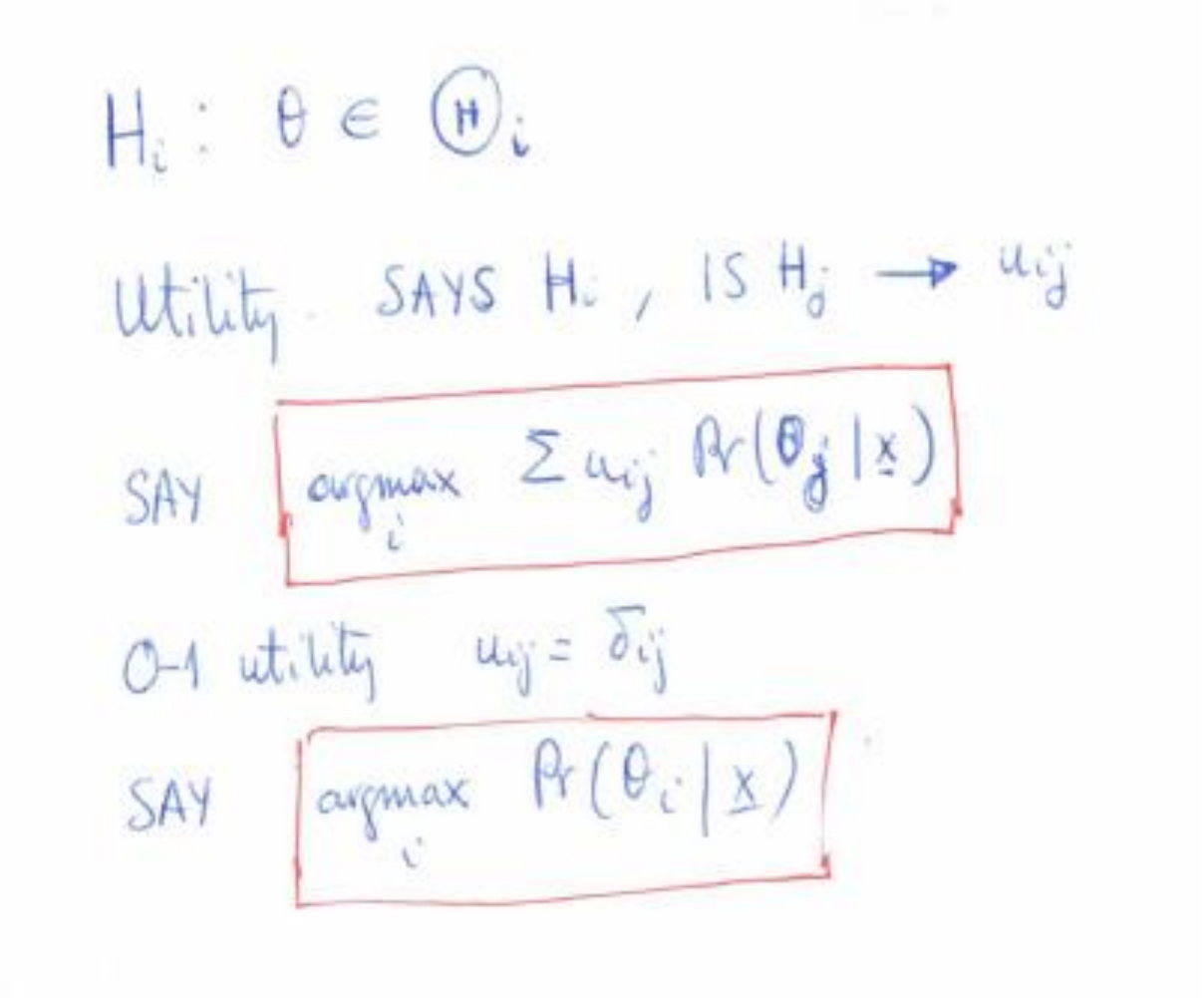
Symmetric interval

HPD



Recap: Bayesian inference

Hypothesis testing



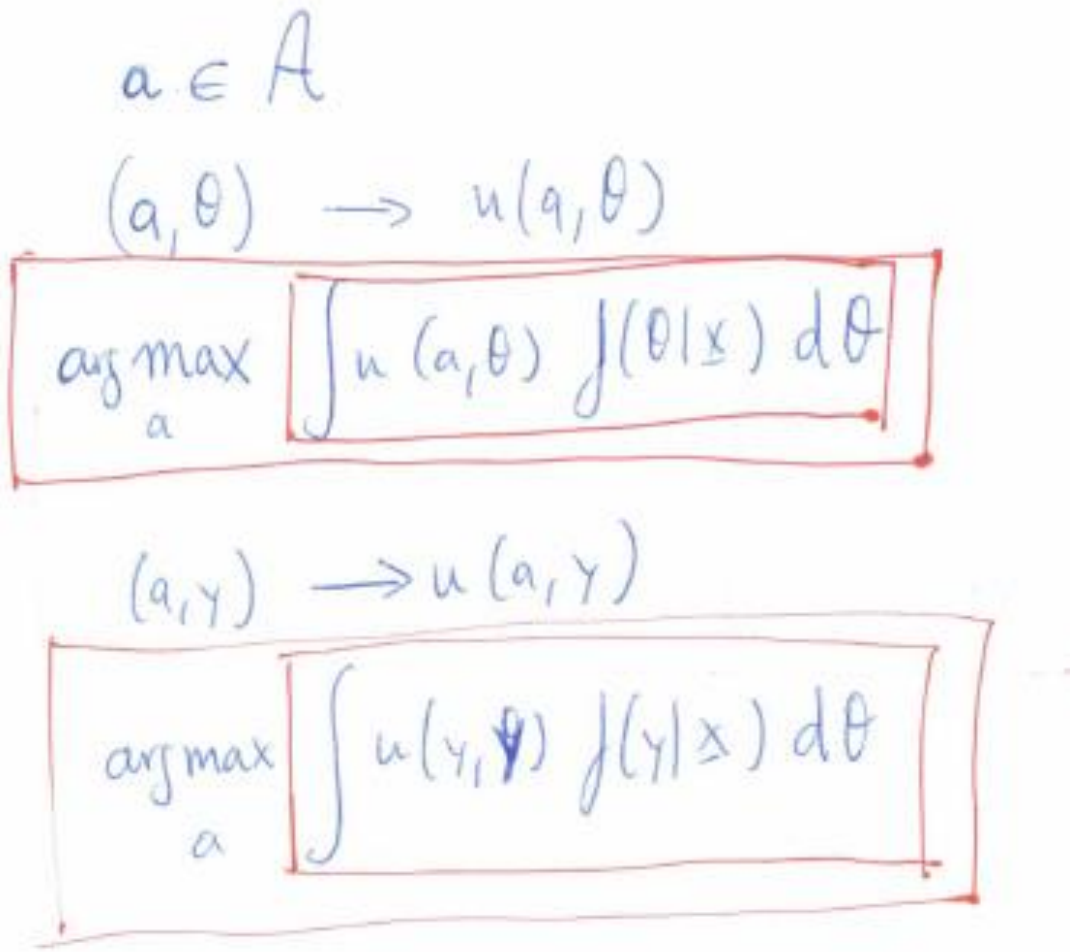
Recap: Bayesian inference

Forecasting

$$f(y|x) = \int f(y|\theta) f(\theta|x) \, d\theta$$

Recap: Bayesian decision analysis

Decision analysis



Recap: Inference as decision analysis. Example

$A = H$ $\quad \ell(a,\theta) = (\theta - a)^2$

$$\underset{a}{\operatorname{argmin}} \int (\theta - a)^2 f(\theta|x)\, d\theta = \underset{a}{\operatorname{argmin}} \int \theta^2 f(\theta|x) d\theta - 2a \int \theta f(\theta|x) d\theta + a^2 \int f(\theta|x) d\theta$$

$$= \underset{a}{\operatorname{argmin}} \; a^2 - 2a\, E(\theta|x) \Longrightarrow \hat{a} = E(\theta|x)$$

Recap. Computational problems in BML

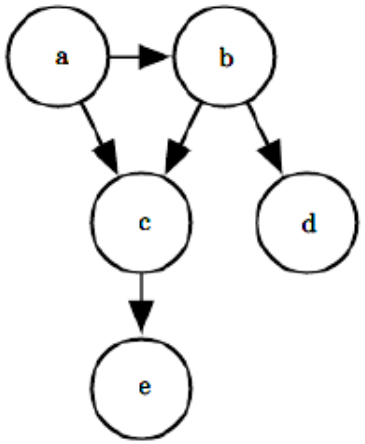
Plagued by complex integrals with complex integrands

+ optimisations

Easy conceptually… tough computationally

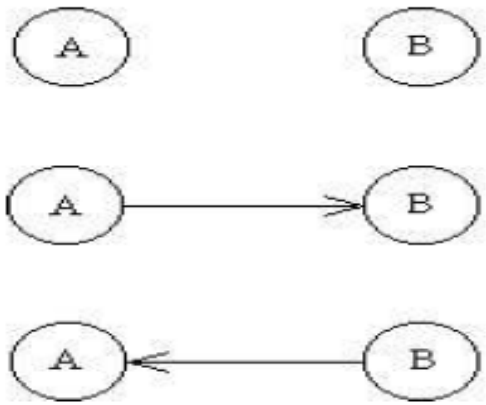
Appendix: Probabilistic graphical models

$$p(\mathbf{x}) = \prod_i p\left(\mathrm{x}_i \mid Pa_{\mathcal{G}}(\mathrm{x}_i)\right)$$



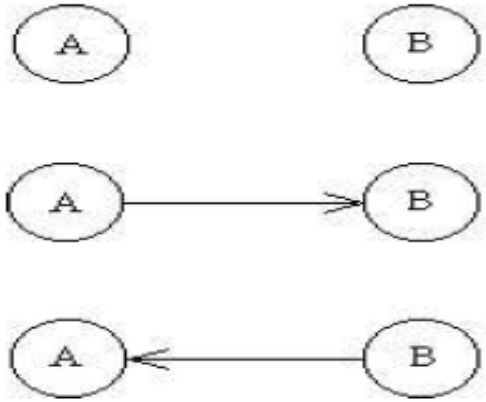
$$p(\mathrm{a},\mathrm{b},\mathrm{c},\mathrm{d},\mathrm{e}) = p(\mathrm{a})p(\mathrm{b} \mid \mathrm{a})p(\mathrm{c} \mid \mathrm{a},\mathrm{b})p(\mathrm{d} \mid \mathrm{b})p(\mathrm{e} \mid \mathrm{c})$$

PGMs with two nodes



PGMs with two nodes

Model for P(A,B)

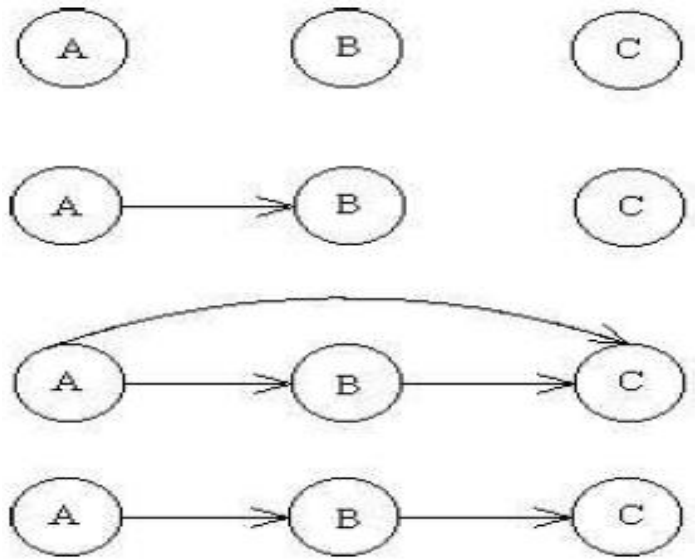


P(A)P(B)

P(A) P(B|A)

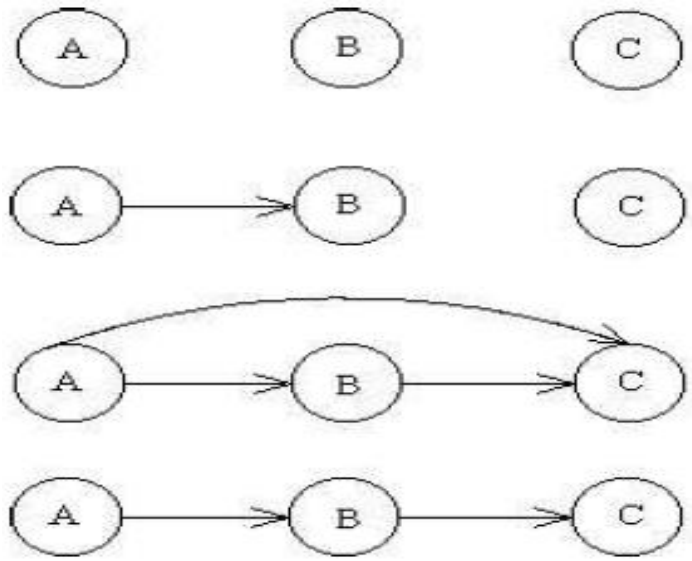
P(B) P(A|B)

Some MGPs with three nodes



PGMs with three nodes

Model P(A ,B ,C)



P(A)P(B)P(C)

P(A) P(B|A) P(C)

P(A)P(B|A)P(C|A,B)

P(A)P(B|A)P(C|B)

http://en.wikipedia.org/wiki/Conditional_independence

PGMs. Asia

A dispnea may due to tuberculosis, lung cancer or bronchitis, none of them or several of them. A recent visit to Asia increases the chances of tuberculosis, whereas smoking is a risk factor for lung cancer and bronchitis. An x-ray image cannot discern between cancer and tuberculosis, as neither the presence or absence of dispnea

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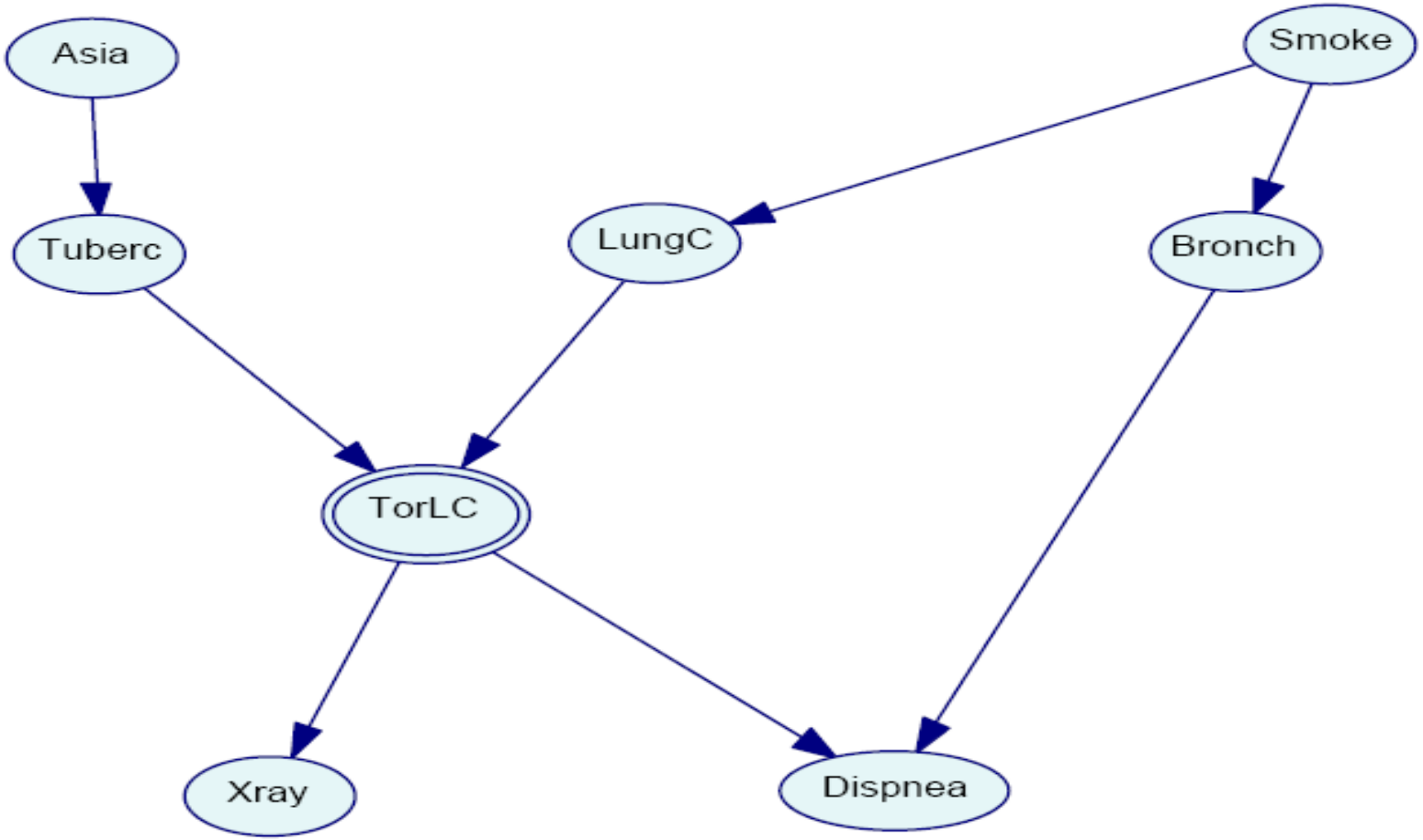
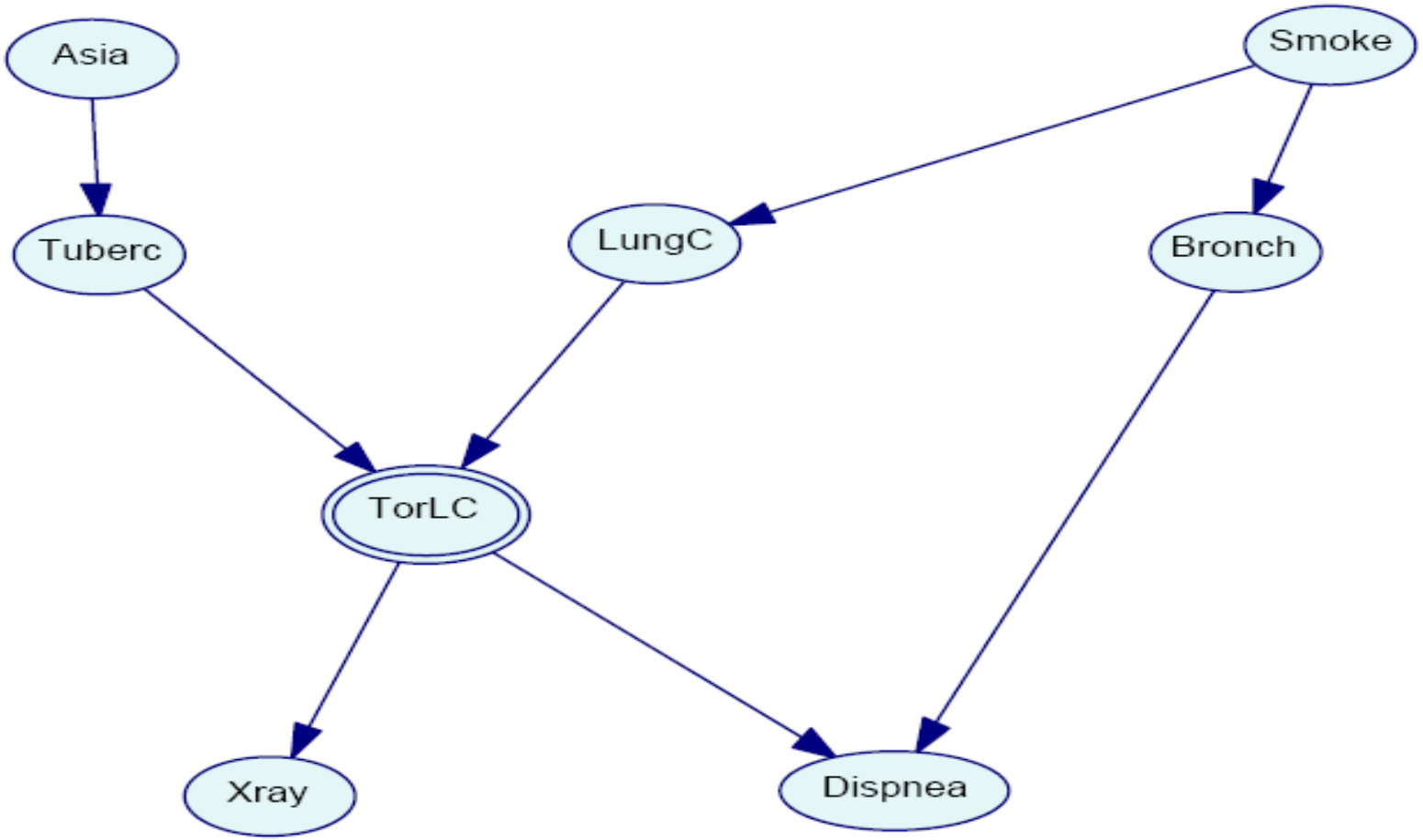
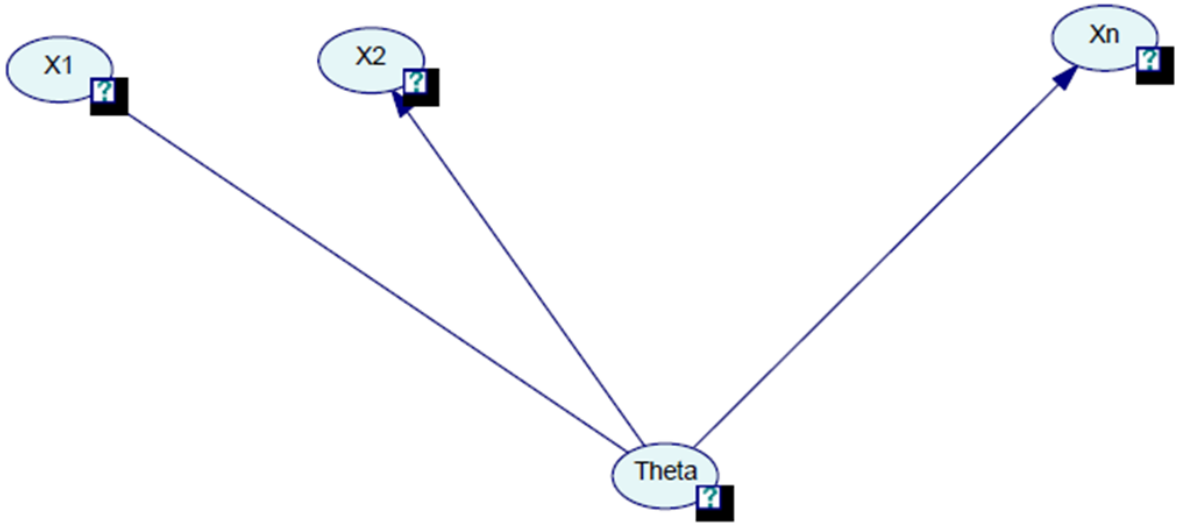


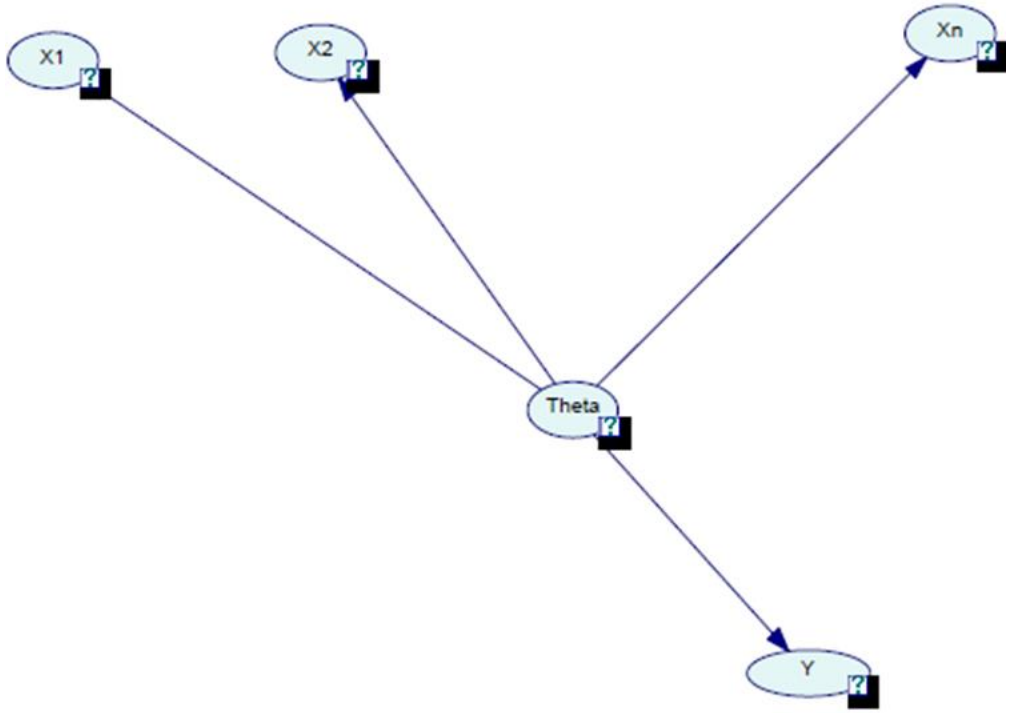
PGM. Asia



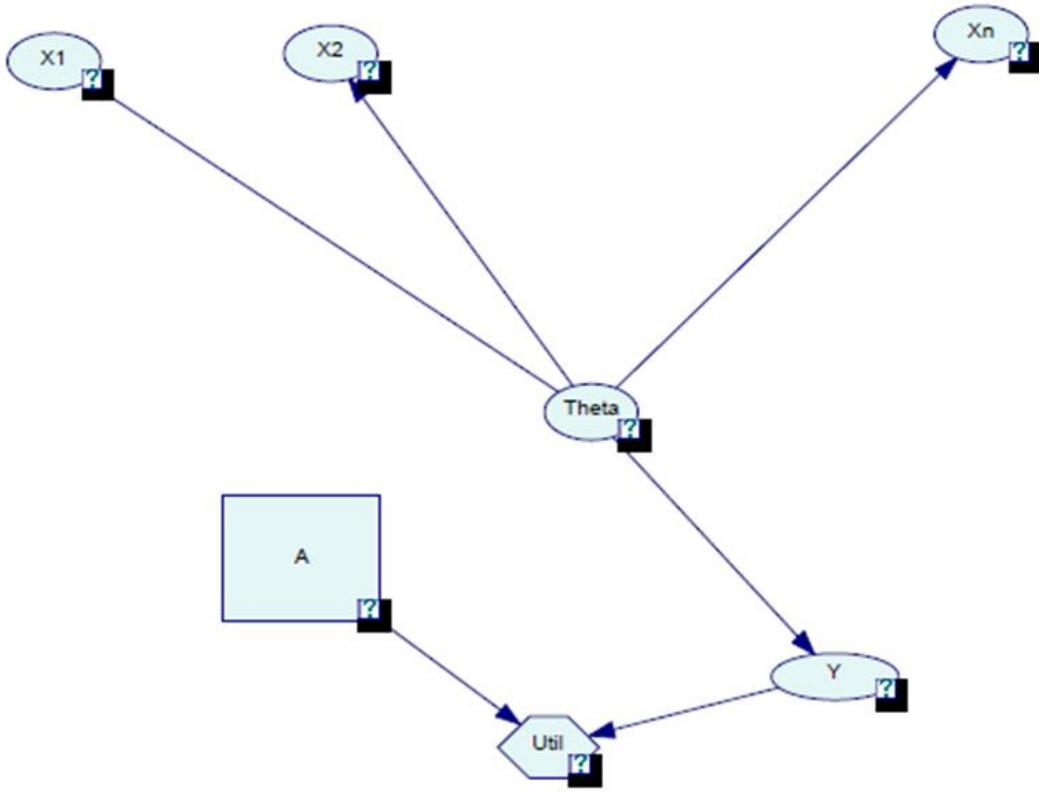
$P(A,T,S,L,B,O,X,D) = P(A)P(T|A)P(S)P(L|S)P(B|S)P(0|T,L)P(X|O)P(D|O,B)$ $P(t|no\ a, d)$

Appendix: Statistical models as PGMs. Inference and Prediction





Appendix: Statistical models as PGMs. Decision Analysis



Appendix: Statistical models as PGMs. Hierarchical models